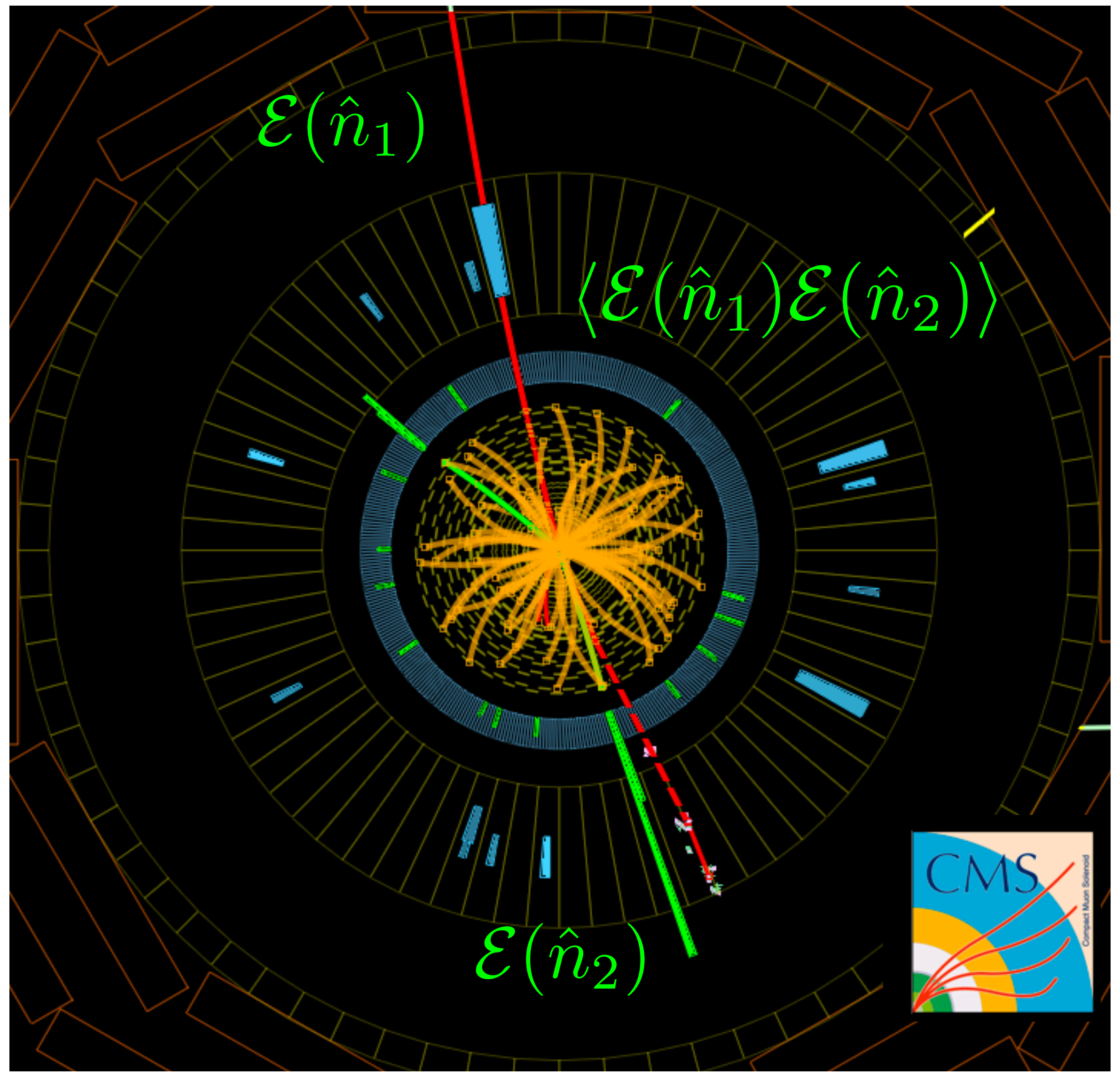
Energy Correlators taking Charge!

In Collaboration with Ian Moult

Kyle Lee
CTP, MIT

BOOST 2023

ENERGY FLOW OPERATOR

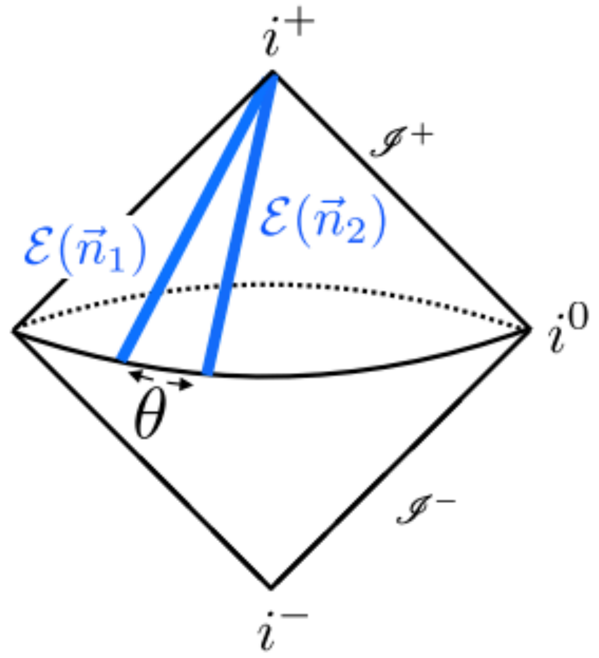


Energy Flow Operators **(Light Ray Operators)**

Basham, Brown, Ellis, Love, `78-79
Sveshnikov, Tkachov, `95
Korchemsky, Sterman, `01

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n})$$

$$\mathcal{E}(\hat{n})|X\rangle = \sum_a E_a \delta^{(2)}\left(\Omega_{\vec{p}_a} - \Omega_{\hat{n}}\right)|X\rangle$$

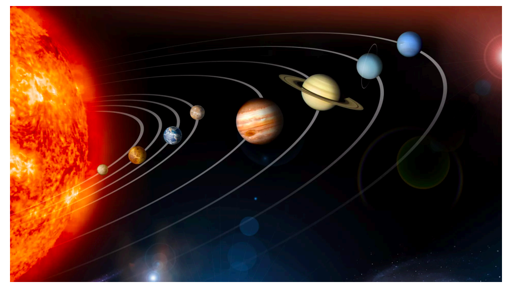


- **Much like cosmology, we observe asymptotic energy flux at the detectors that are placed at cosmic scale away from where the events originated.**

(Collision events happen at $\delta x \sim \frac{\hbar}{10\text{TeV}} \sim 2\times 10^{-20}$ **meters, and observed at** ~ 10 **meters away)**

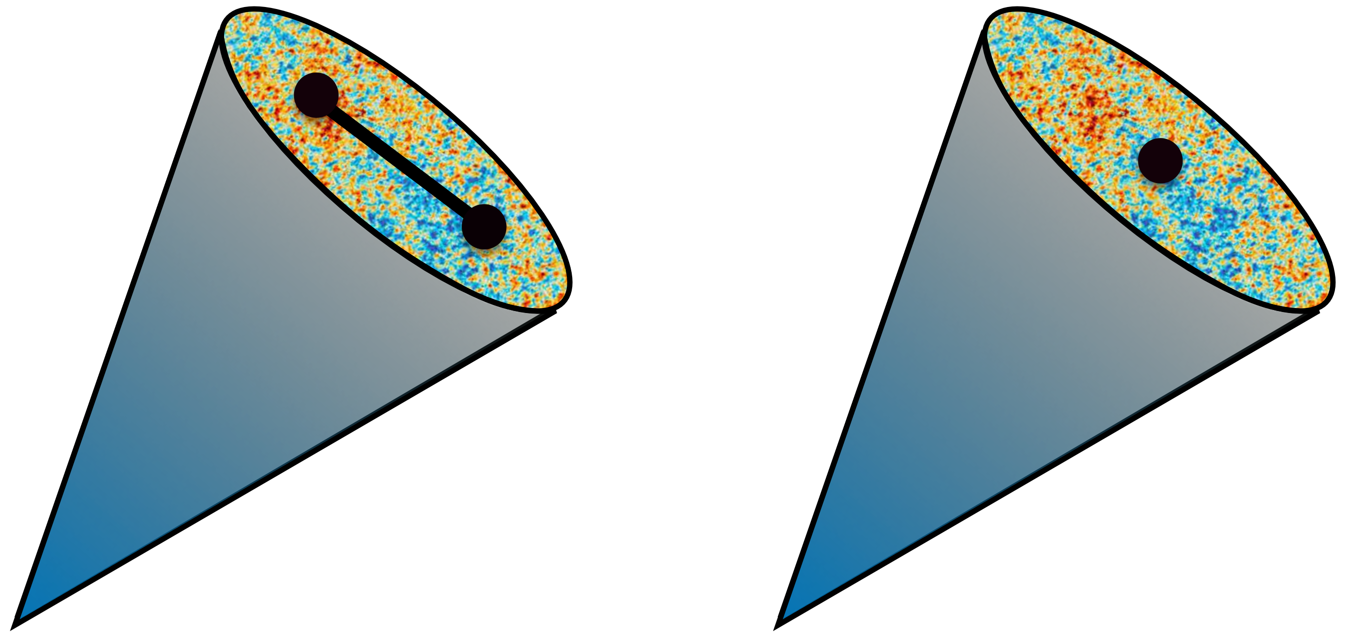
10^{21} **orders in distance!**

 $\mathcal{O}(1)$ **meters**

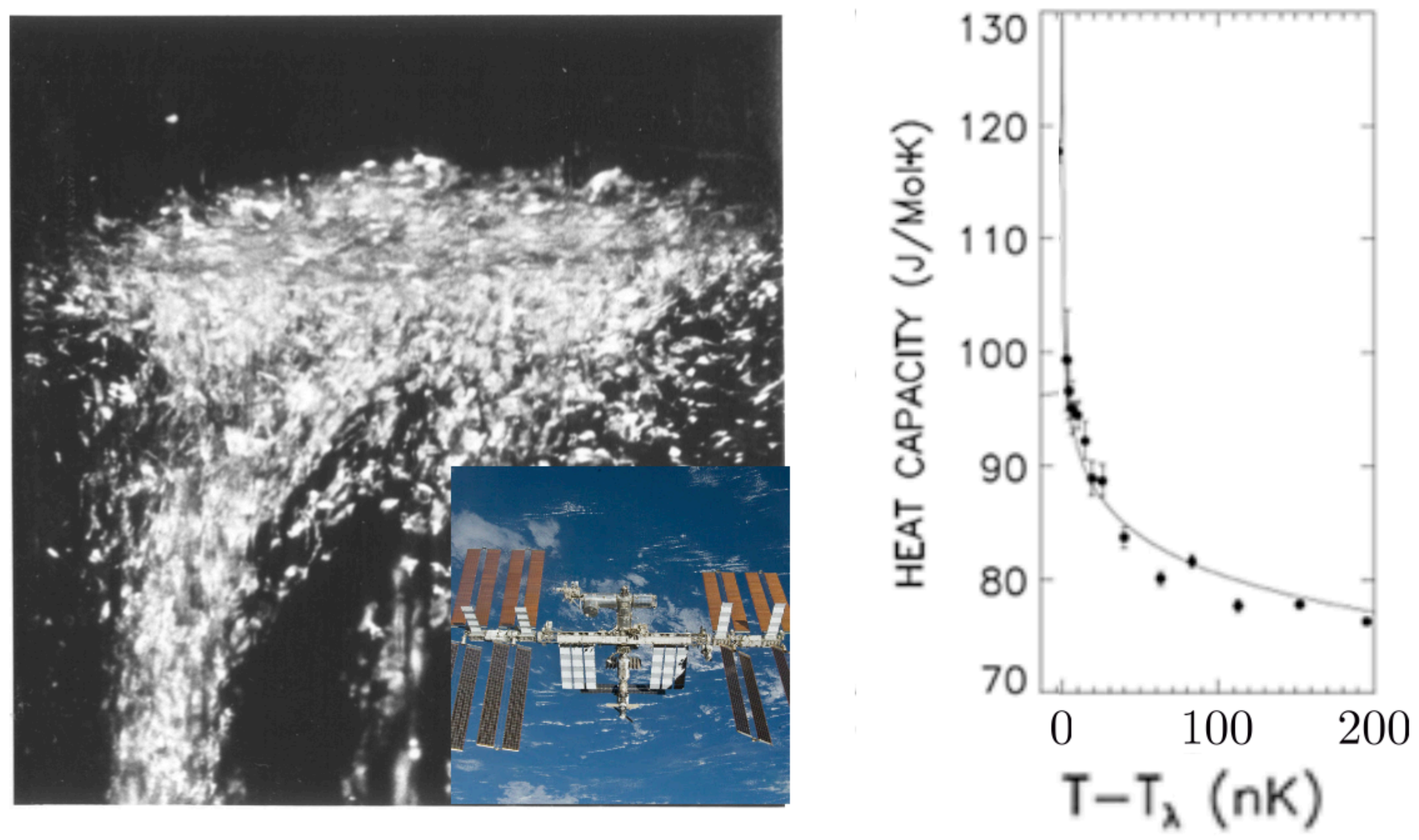
 $\mathcal{O}(10^{15})$ **meters**

 $\mathcal{O}(10^{21})$ **meters**

JET SUBSTRUCTURE AS CORRELATION FUNCTIONS

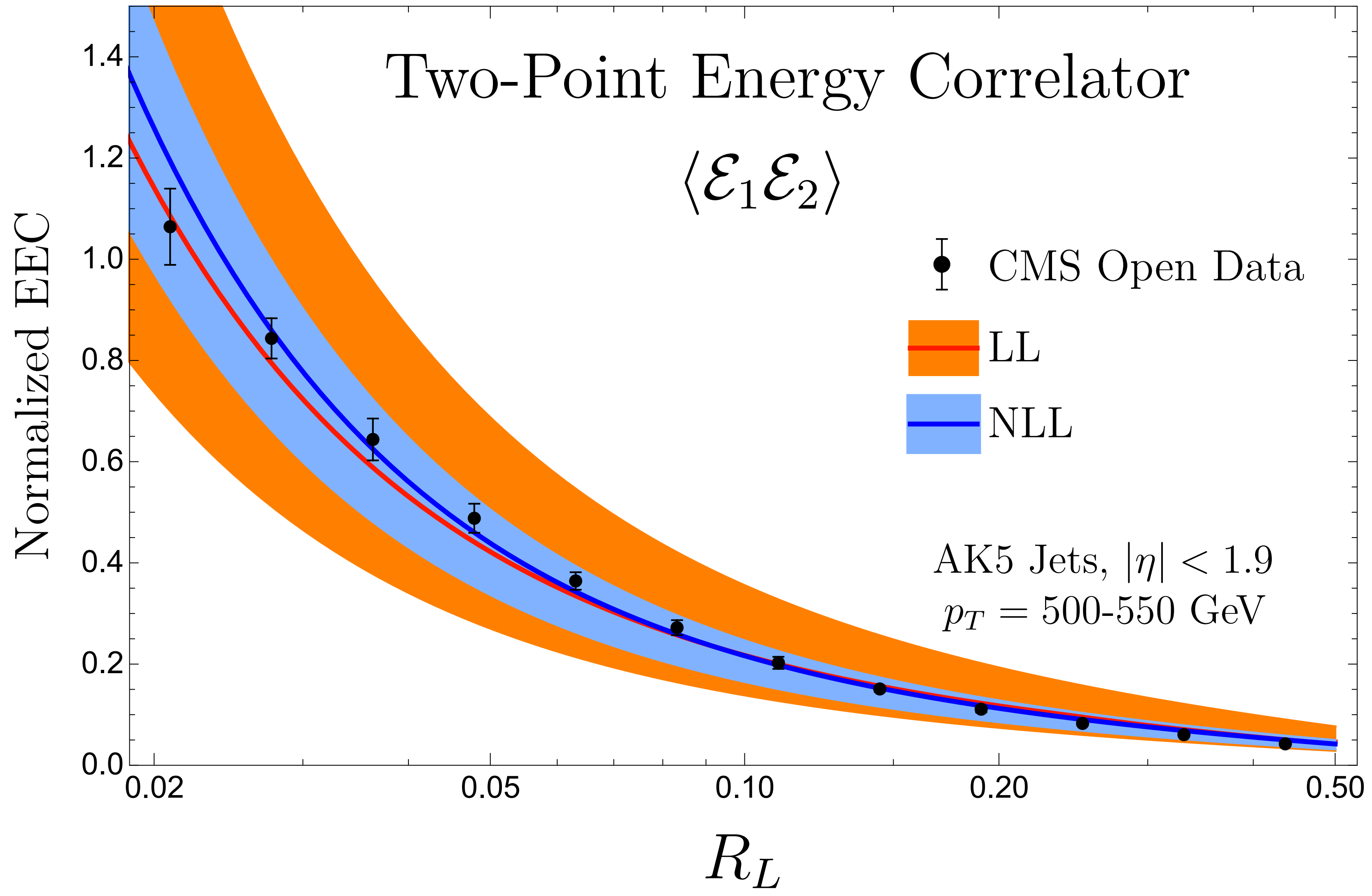


- **Field theory often predicts universal scaling as operators approach each other**
- **Jet limit corresponds to the collinear limit (OPE limit) of the correlation functions of the Energy Flow Operators**



$$\mathcal{O}(x)\mathcal{O}(0) = \sum x^{\gamma_i} c_i \mathcal{O}_i$$

$\Longleftrightarrow$



KL, Meçaj, Moult `22

$$\mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\, \mathbb{O}_i(\hat{n}_1)$$

Hofman, Maldacena, `08

- **Observation of the universal of QCD predicted at the operator levels from the light-ray operator product expansion!**

QUARK GLUON SCALING AND HADRONIZATION

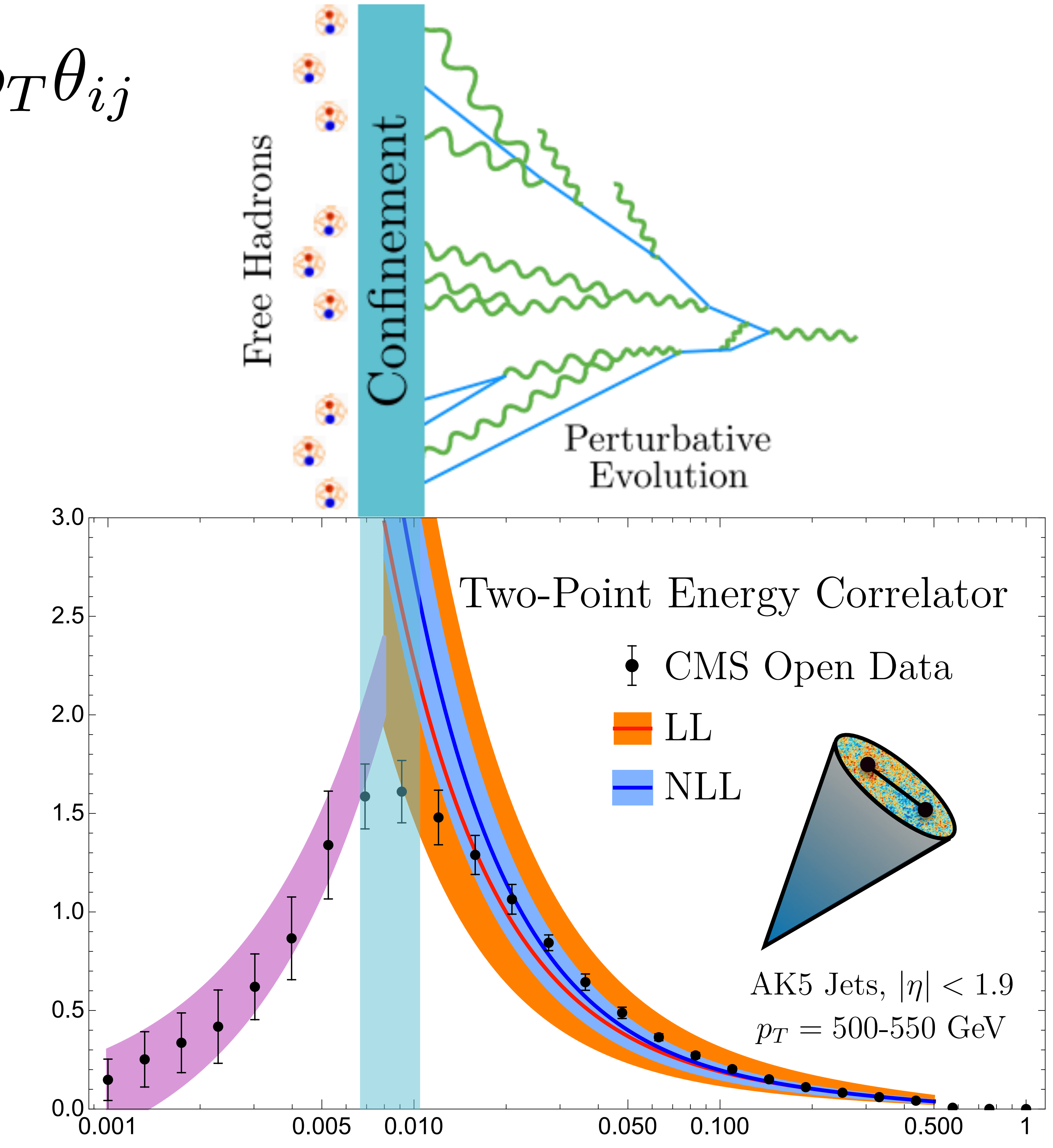
- **Energy correlators allow the hadronization process to be directly imaged inside high energy jets: transition from interacting quarks and gluons and free hadrons is clearly visible!**

EEC gives angular scale $\mu \sim p_T \theta_{ij}$

Free hadrons

$$\frac{d\sigma}{d\theta^2} = \text{const}$$

$$\frac{d\sigma}{d\theta} = \text{const} \times 2\theta$$



KL, Meçaj, Moult `22

Interacting quarks and gluons

$$\mathcal{E}(\hat{n}_1)\,\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$

Hofman, Maldacena, `08

WHAT IS A DETECTOR?

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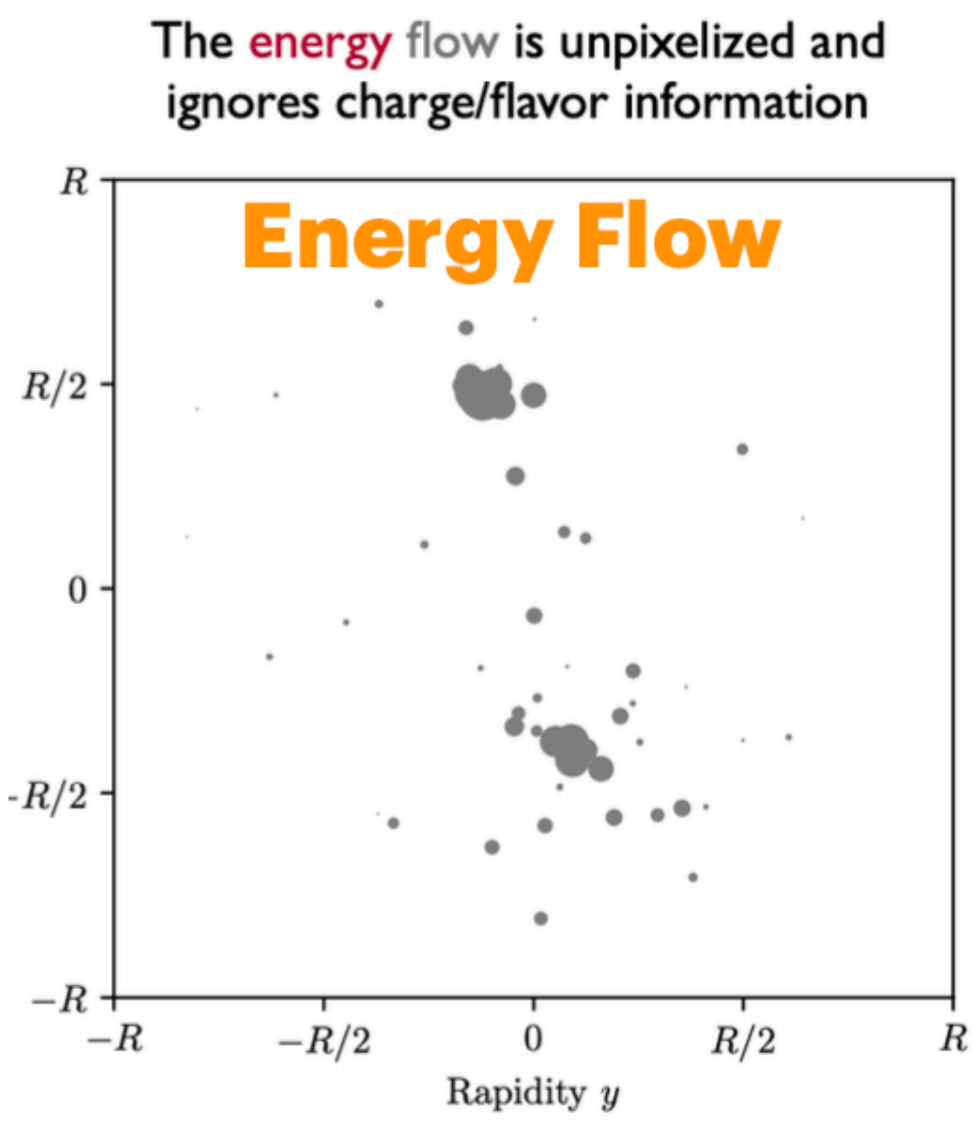
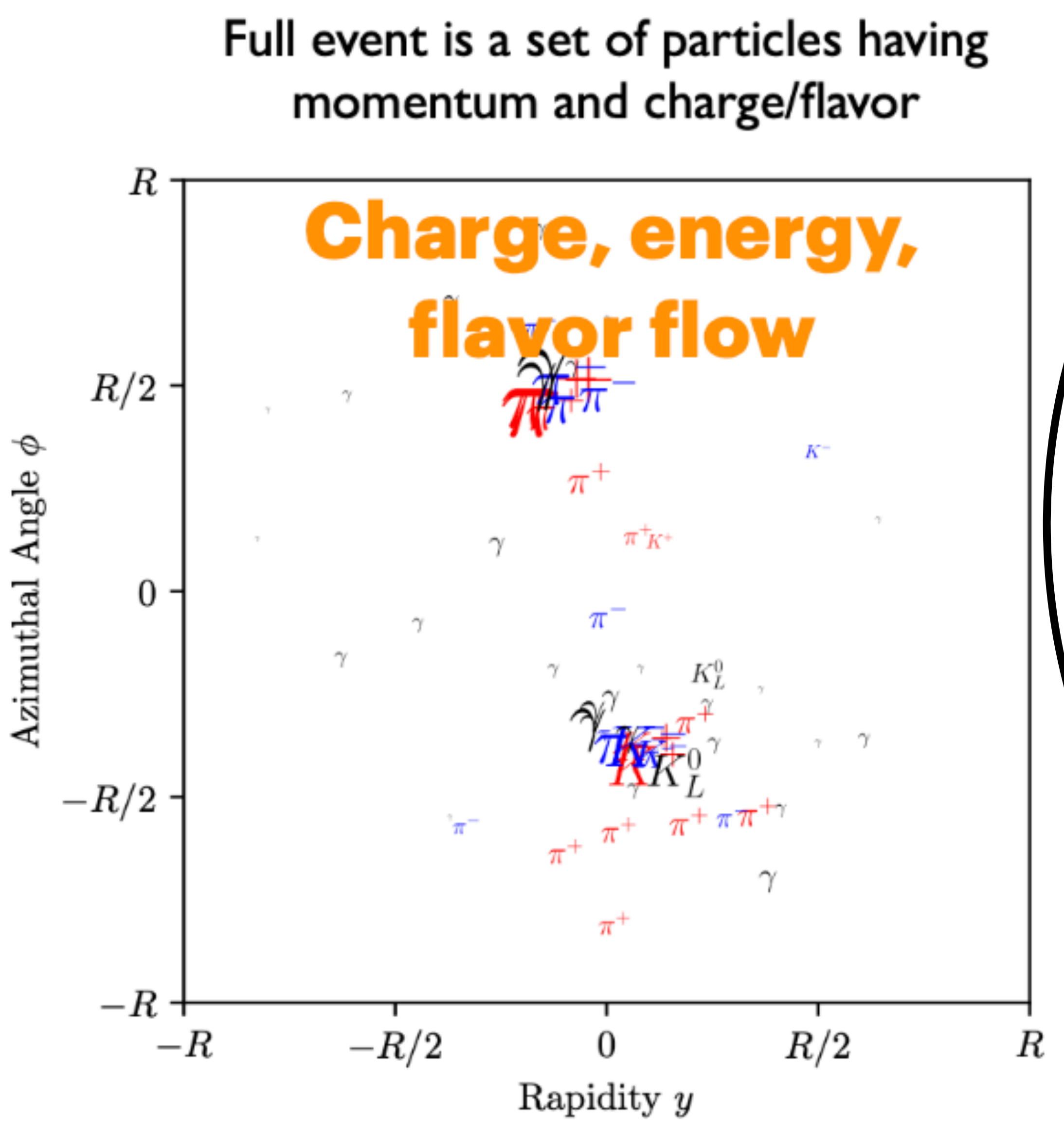
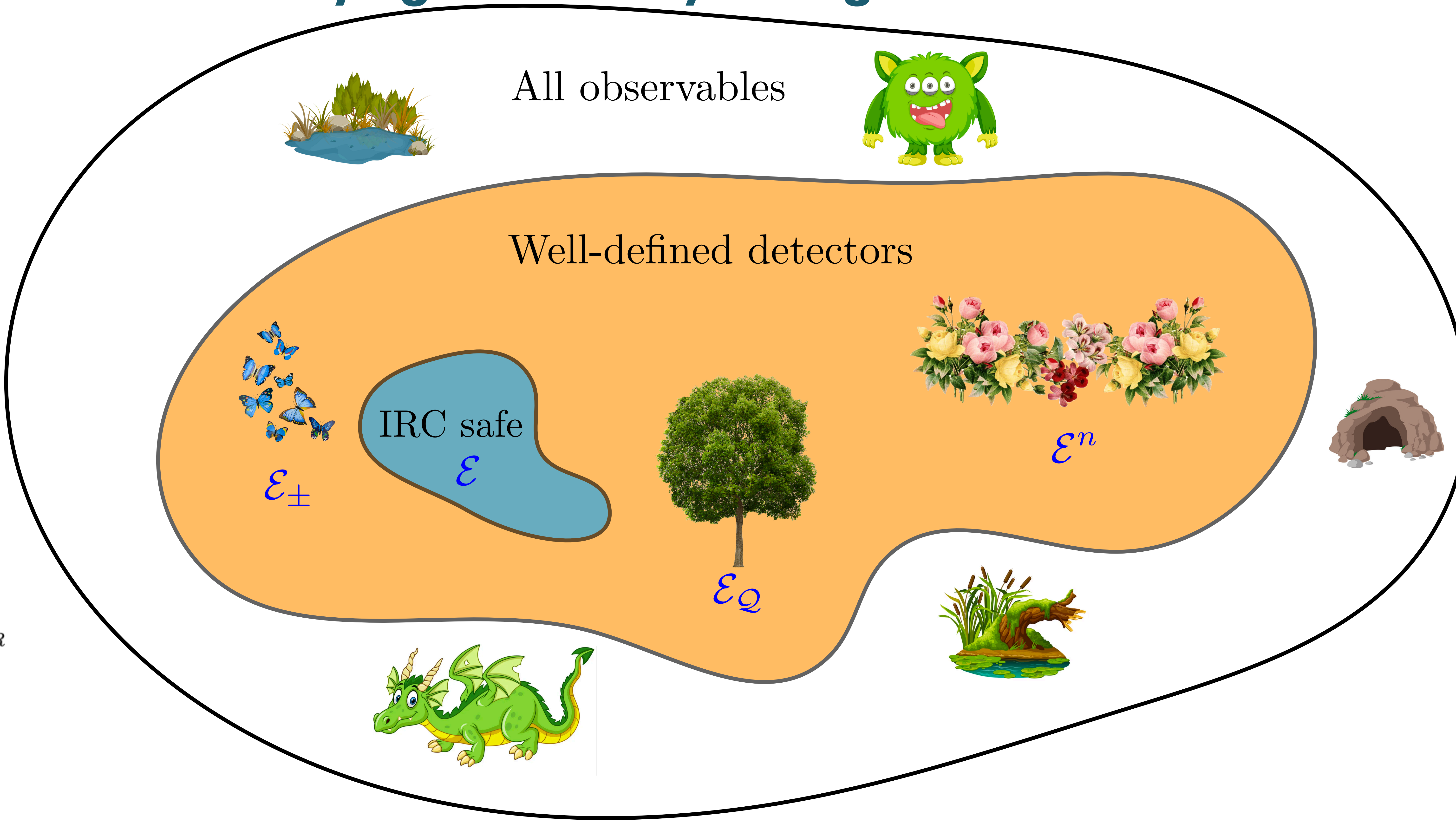
- Collider detector can give us **more than** just an energy flow. What constitutes a **field-theoretically well-defined detector?**

Caron-Huot, Kologlu, Kravchuk, Meltzer, Simmons-Duffin`22

- Well-defined detectors provide sharp link with underlying field theory through observables!







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- **Collider detector can give us more than just an energy flow. What constitutes a field-theoretically well-defined detector?**

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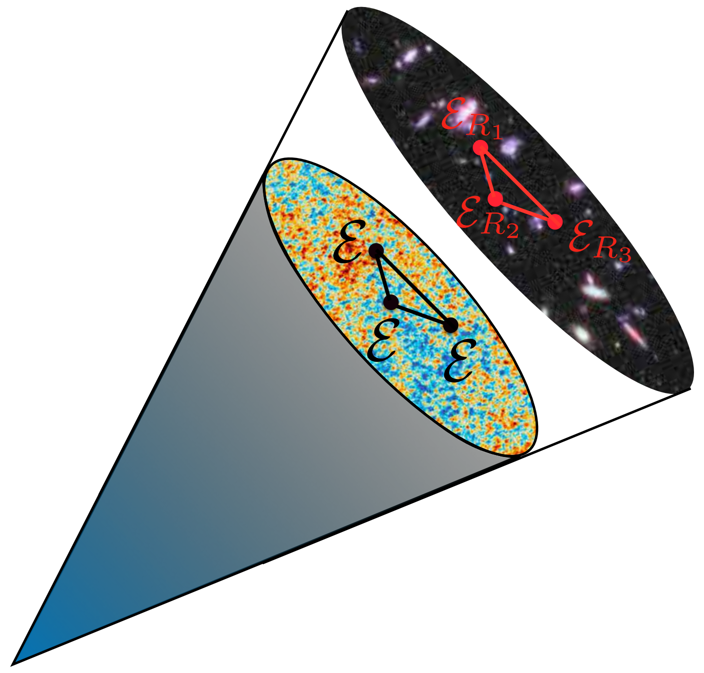
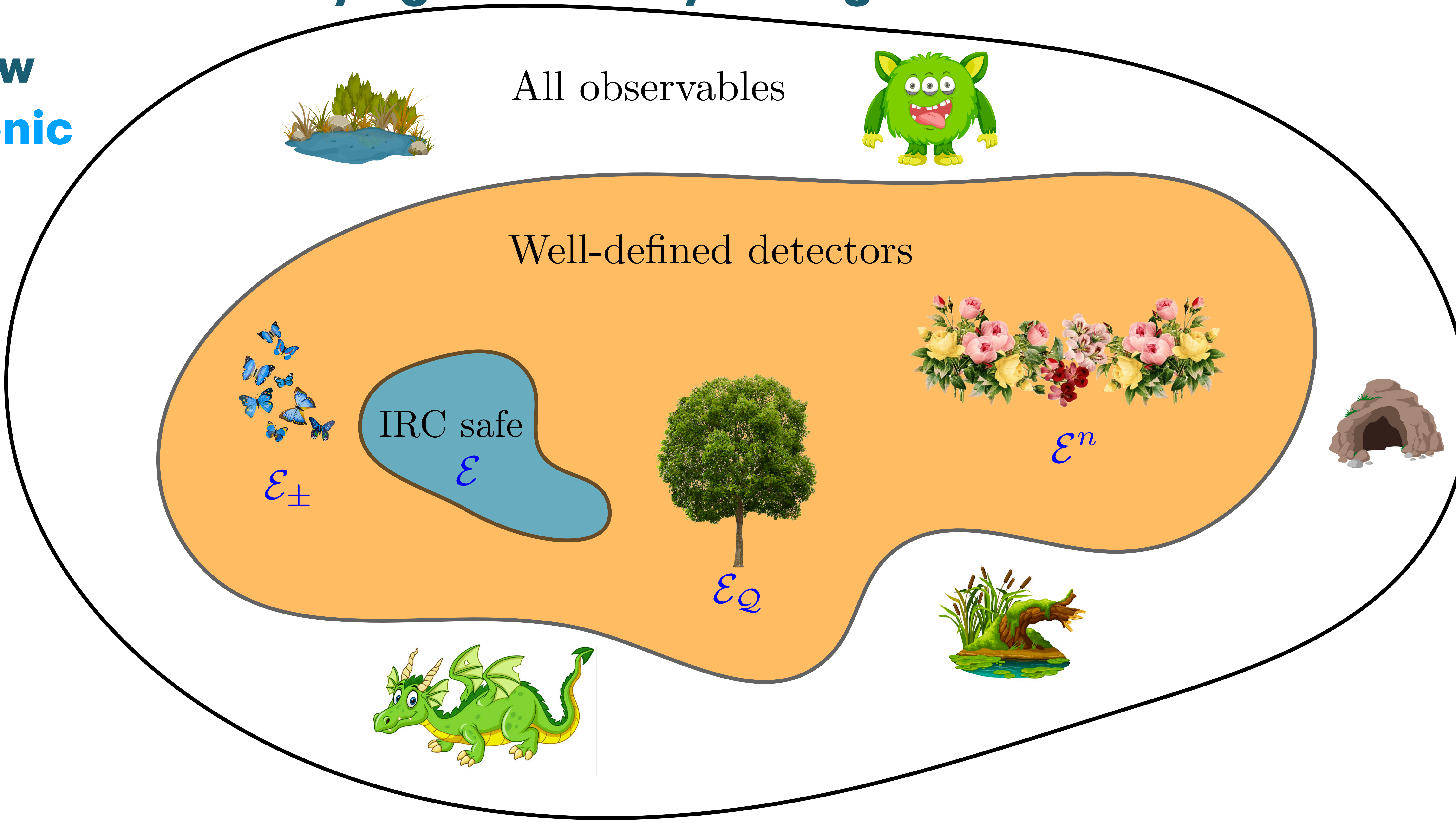
- **Well-defined detectors provide sharp link with underlying field theory through observables!**
- **Interesting measurements of energy flow can be made on a restricted set of hadronic states, R, for example, charged hadrons (tracks).**

$$\mathcal{E}_R = \sum_{i \in R} \mathcal{E}_i$$





GENERALIZING ENERGY FLOW CORRELATIONS

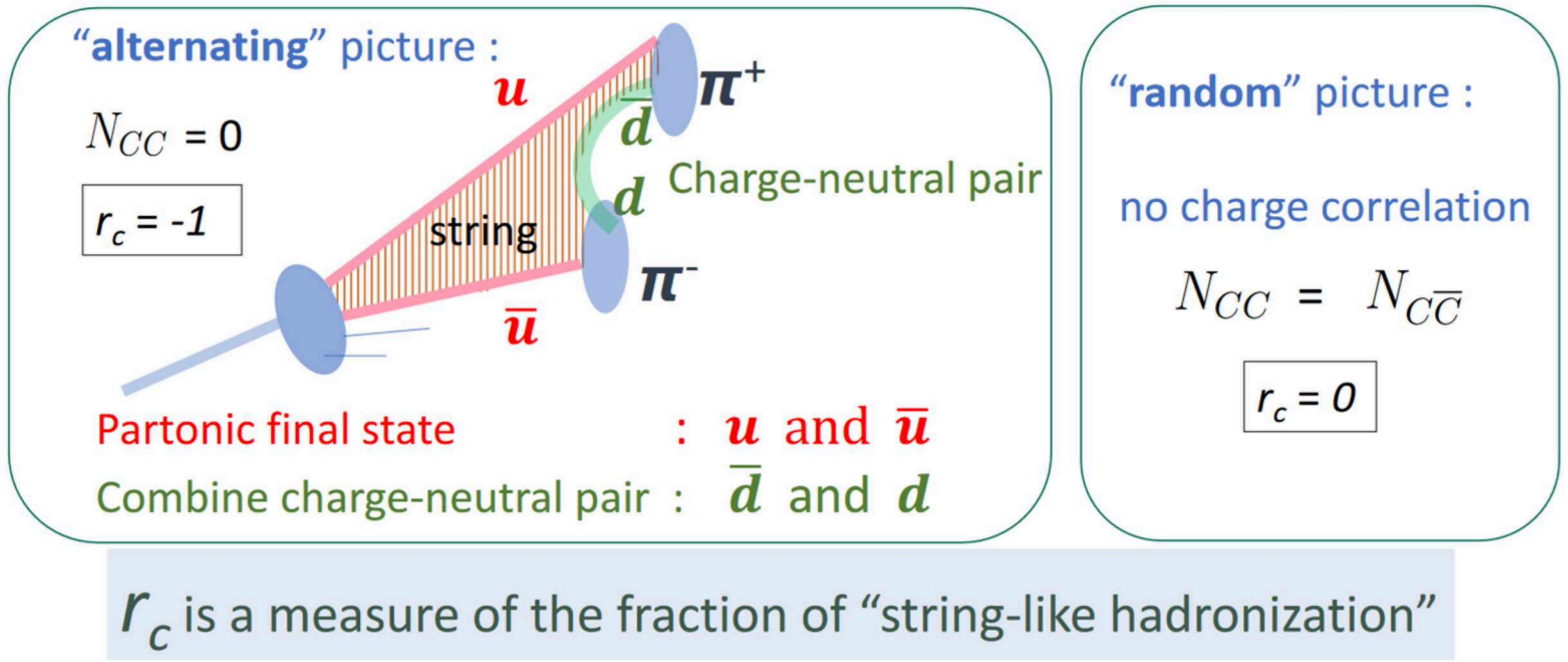
- **Writing down more general detectors allow us to consider more general correlation functions from**

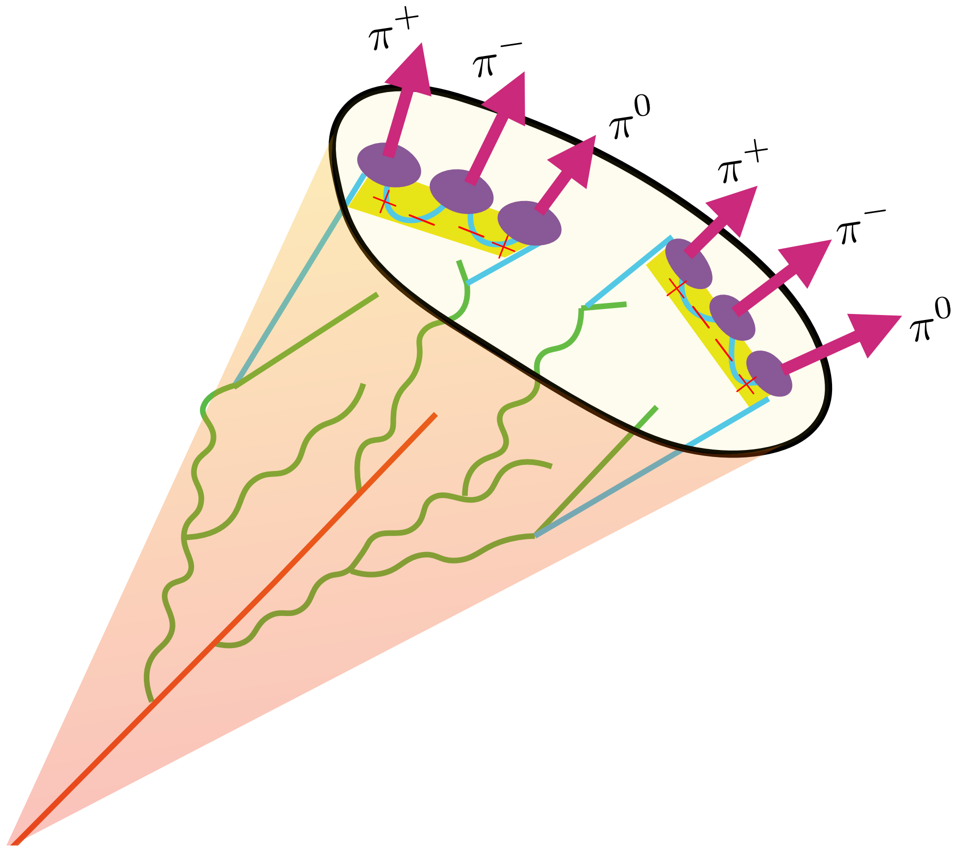
$$\langle \mathcal{E}(n_1)\,\mathcal{E}(n_2)\cdots\mathcal{E}(n_k)\rangle \rightarrow \langle \mathcal{E}_{R_1}(n_1)\,\mathcal{E}_{R_2}(n_2)\cdots\mathcal{E}_{R_k}(n_k)\rangle$$

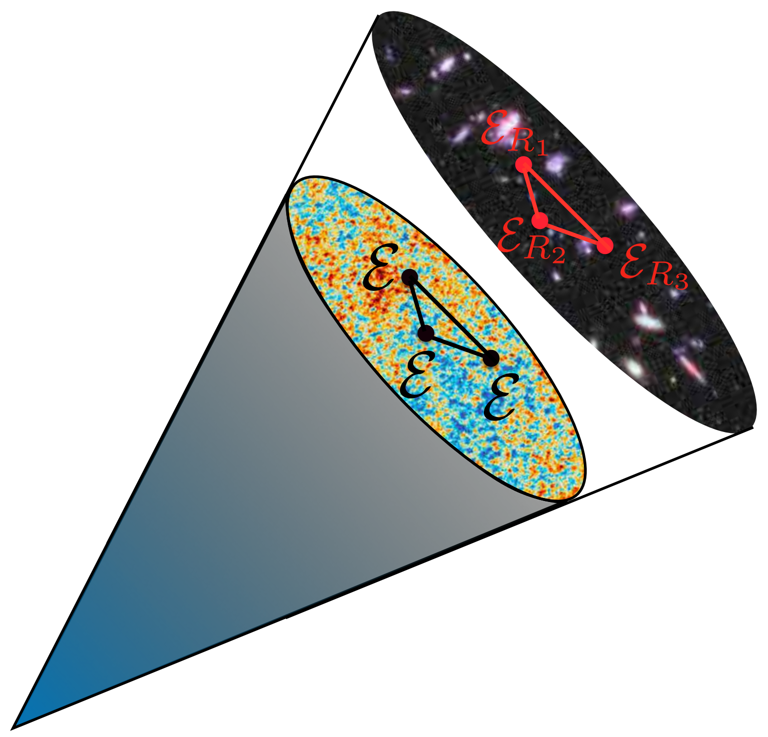
- **In general, restricting to a set of hadronic states to some particular quantum number R introduces sensitivity to the IR scale. As a concrete example, one can ask "is there more unlike-signed correlations compared with the like-signed correlations?"**

i.e. $\langle \mathcal{E}_+\mathcal{E}_-\rangle$ or $\langle \mathcal{E}_+\mathcal{E}_+\rangle + \langle \mathcal{E}_-\mathcal{E}_-\rangle$

Slide from Youqi Song







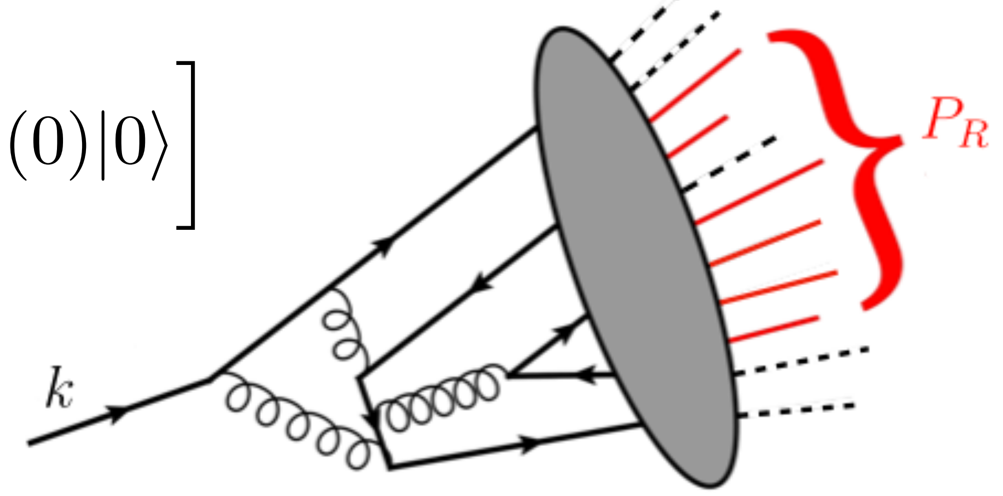
TRACK FORMALISM

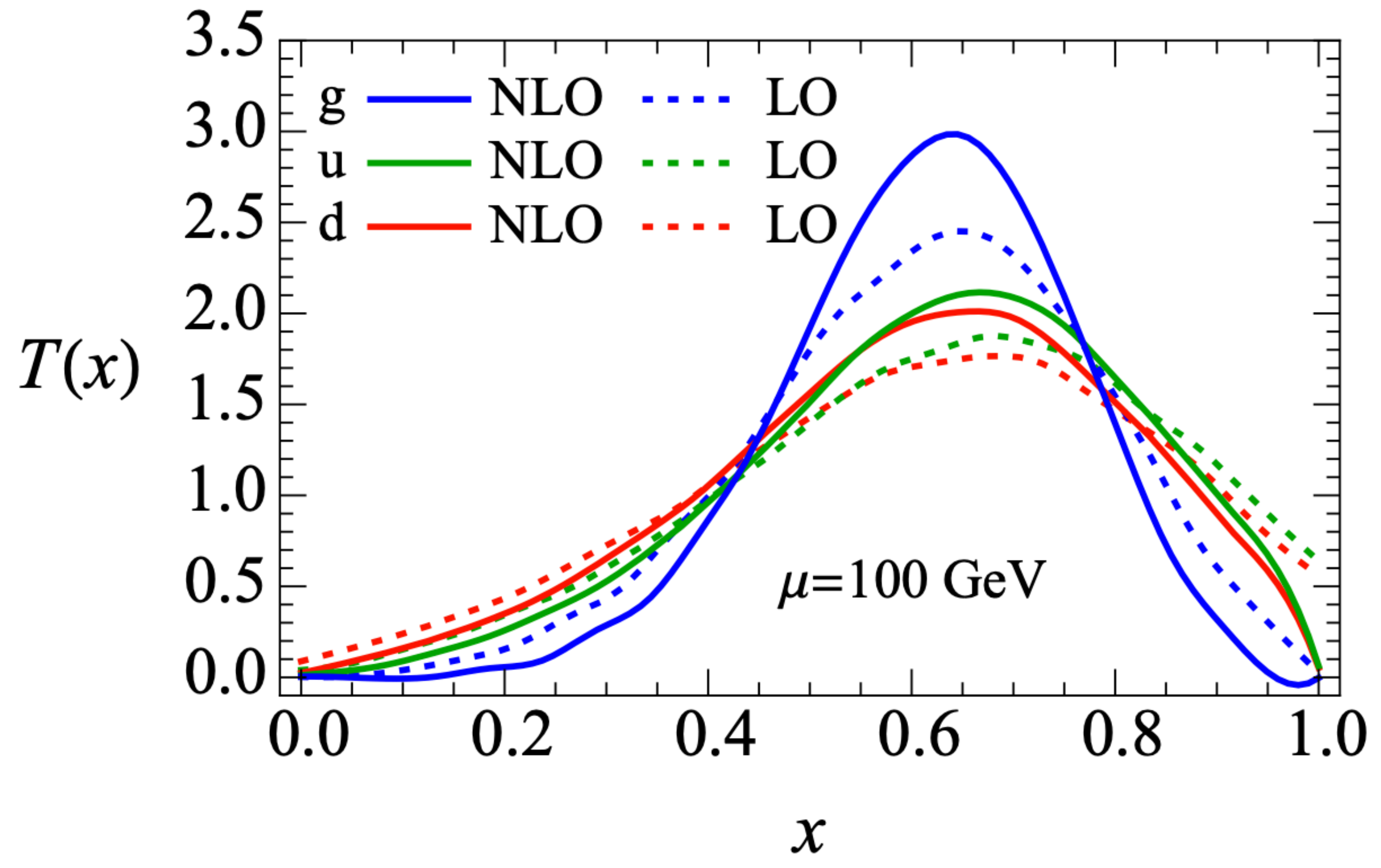
TRACK FUNCTIONS

Chang, Procura, Thaler, Waalewijn `13
Li, Moult, Waalewijn, Zhu et al `21, 22

- **Track functions are non-perturbative functions describing the momentum fraction of initial parton converted to hadrons with a particular property R.**

$$T_q(x) = \int dy^+ d^2 y_\perp e^{ik^- y^+/2} \frac{1}{2N_c} \sum_X \delta\left(x - \frac{P_R}{k_-}\right) \mathrm{tr}\left[\frac{\gamma^-}{2} \langle 0|\psi(y^+, 0, y_\perp)|X\rangle \langle X|\bar{\psi}(0)|0\rangle\right]$$

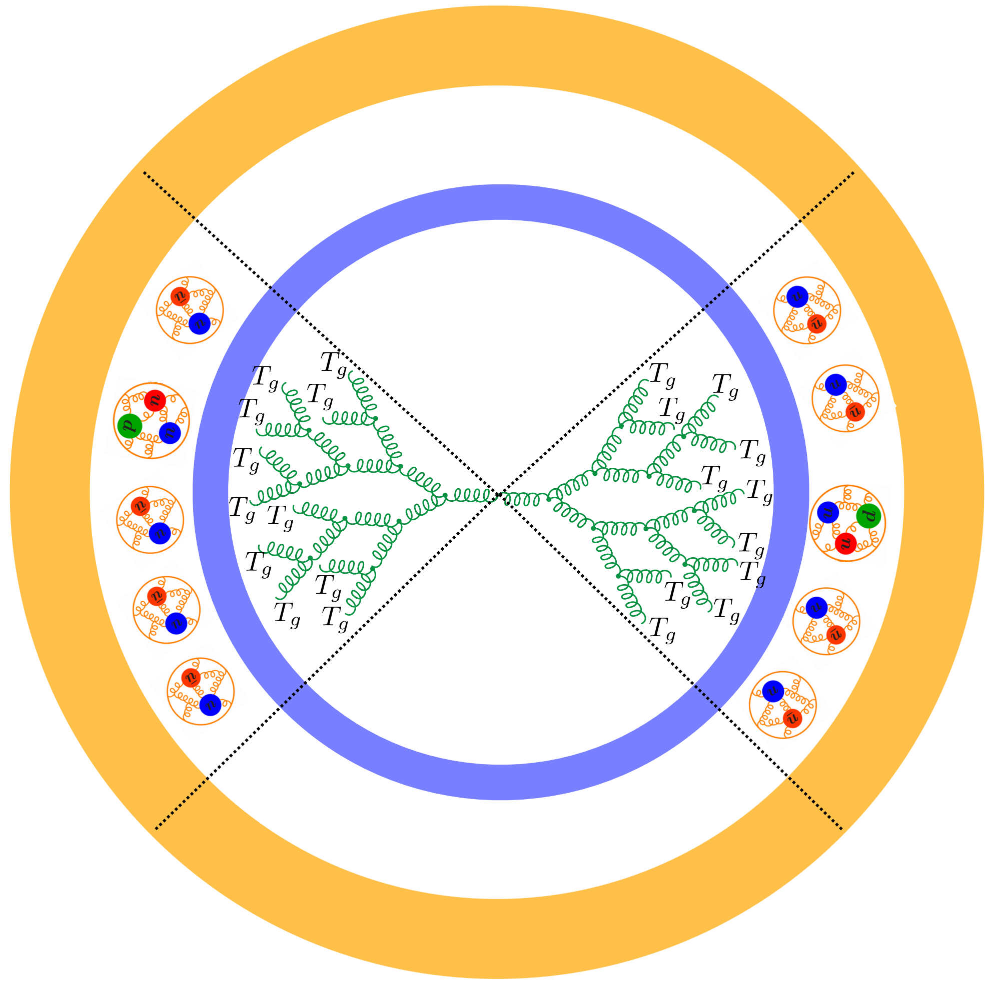




- **Track functions are technically what is needed for calculating the usual jet shape observables on track (jet mass, jet angularities, jet charge, etc...), but is technically challenging**

$$\delta(e - \hat{e}(\{p_i^\mu \in X_J\})) \rightarrow \delta(e - \hat{e}(\{x_i p_i^\mu \in X_J\}))$$

requires simultaneous knowledge of all the tracks in jet.



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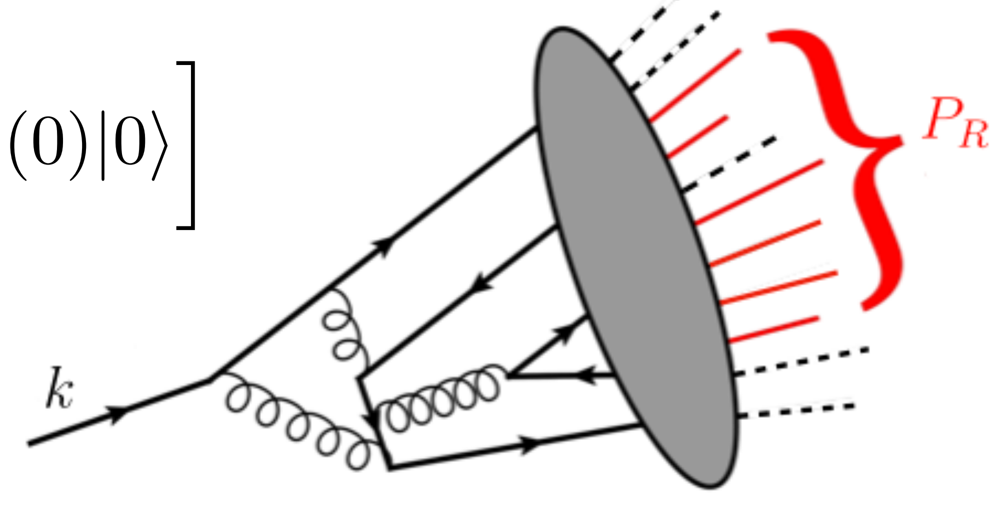
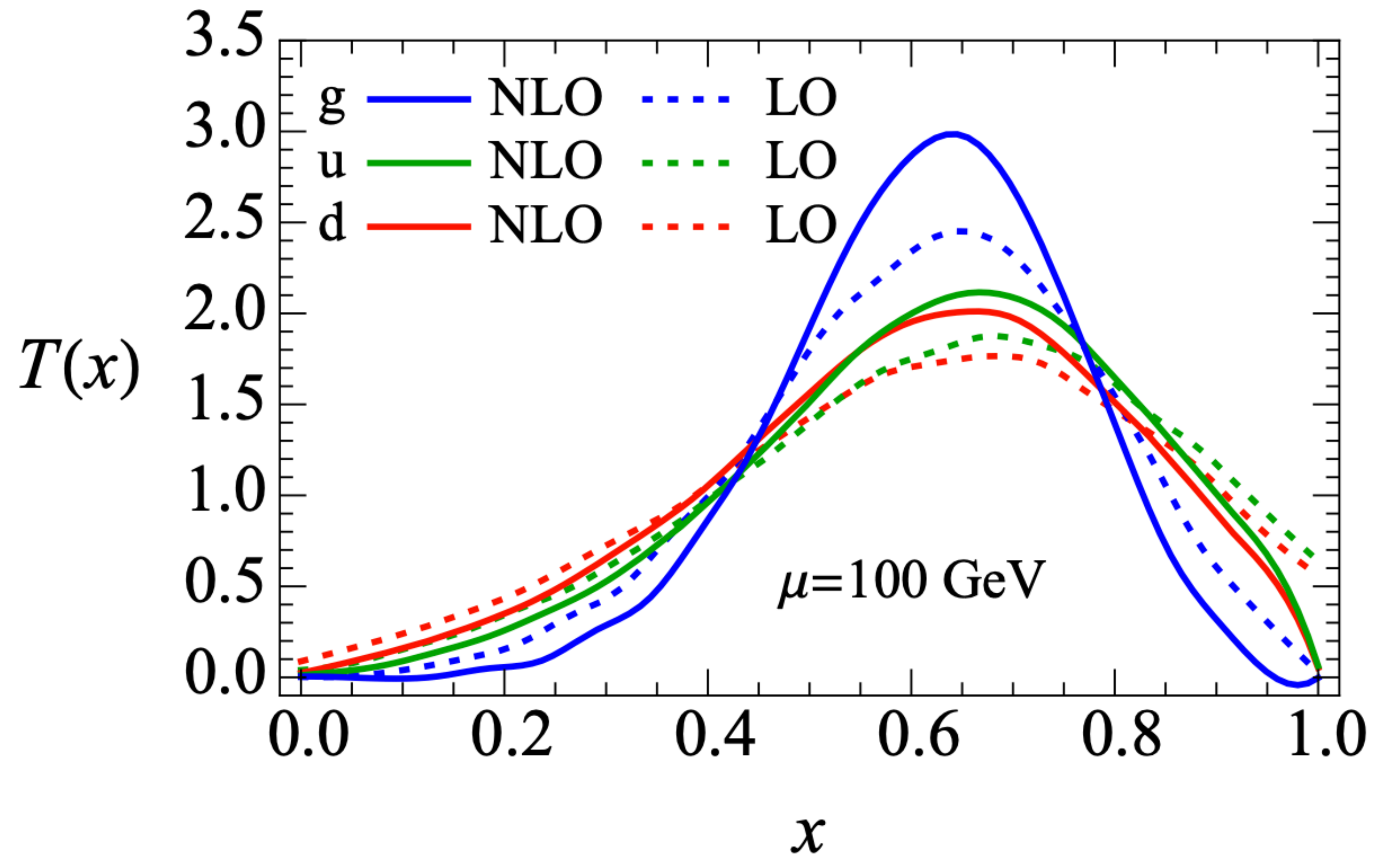




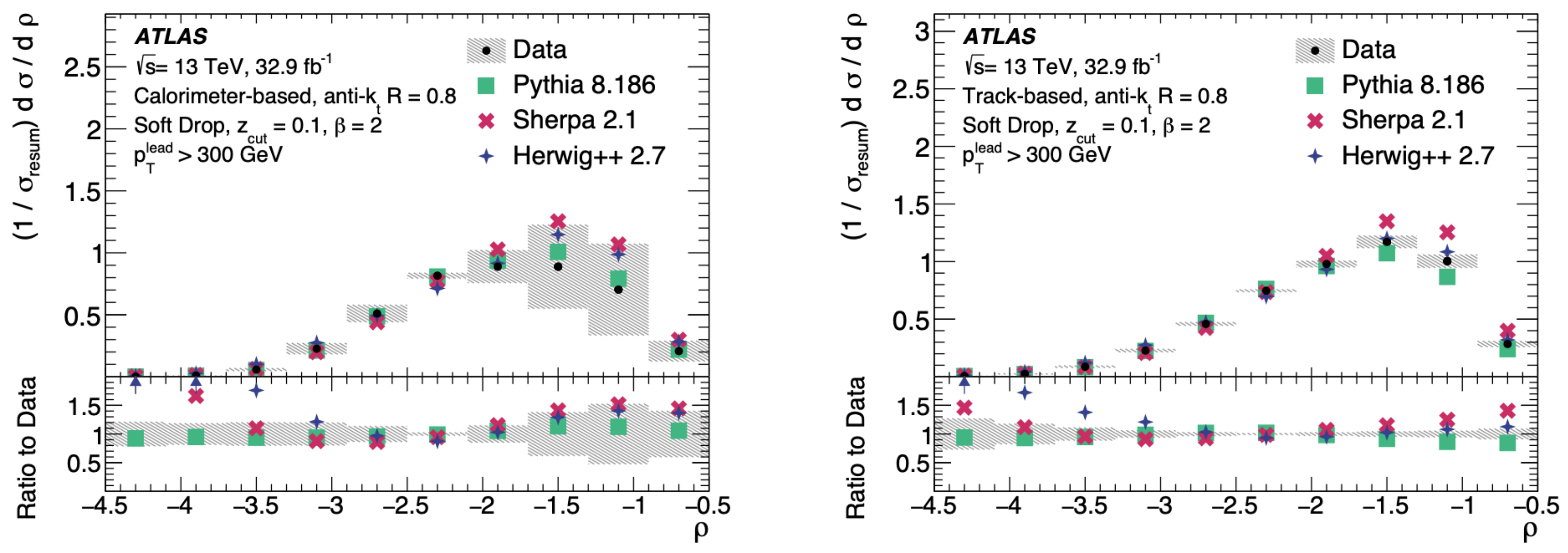
- **Ten years after track function formalism is developed, the computation still remains elusive.**

observables. For all of these observables, the uncertainties for the track-based observables are significantly smaller than those for the calorimeter-based observables, particularly for higher values of β, where more soft radiation is included within the jet. However, since no track-based calculations exist at the present time, calorimeter-based measurements are still useful for precision QCD studies.

[ATLAS Collaboration, 1912.09837]

selection of charged particle jets. Note that track-based observables are IRC-unsafe. In general, nperturbative track functions can be used to directly compare track-based measurements to analytical culations [67–69]; however, such an approach has not yet been developed for jet angularities. Two

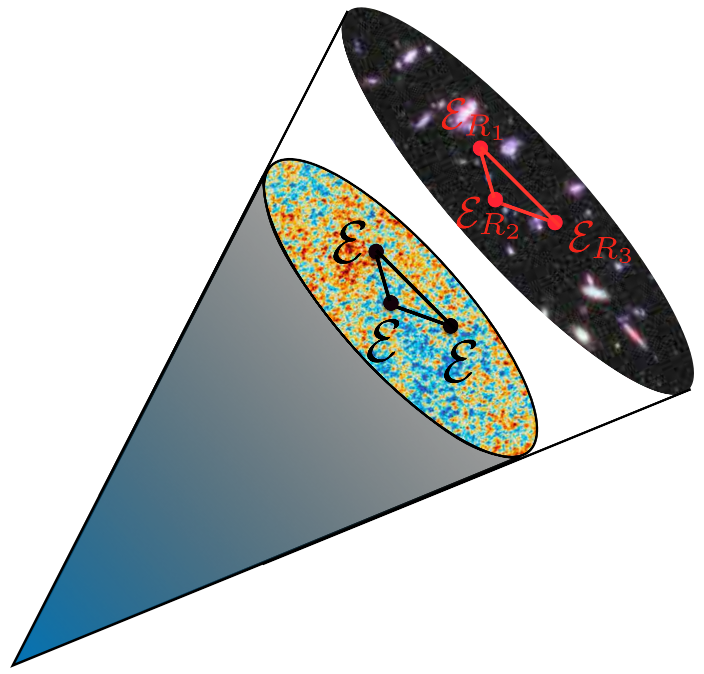
[ALICE Collaboration, 2107.11303]

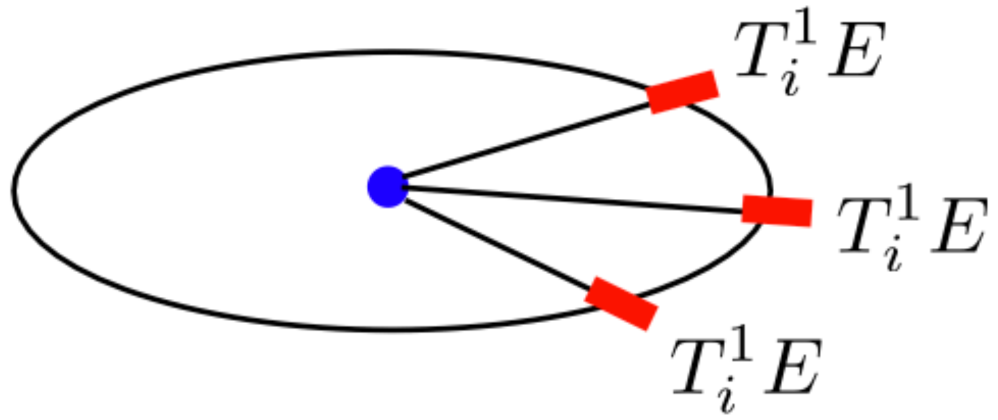


[ATLAS (2019)]

ENERGY CORRELATORS ON TRACK

- **From this "detector" as a fundamental operator perspective, track function formalism provides the essential matching between partonic and hadronic detectors.**

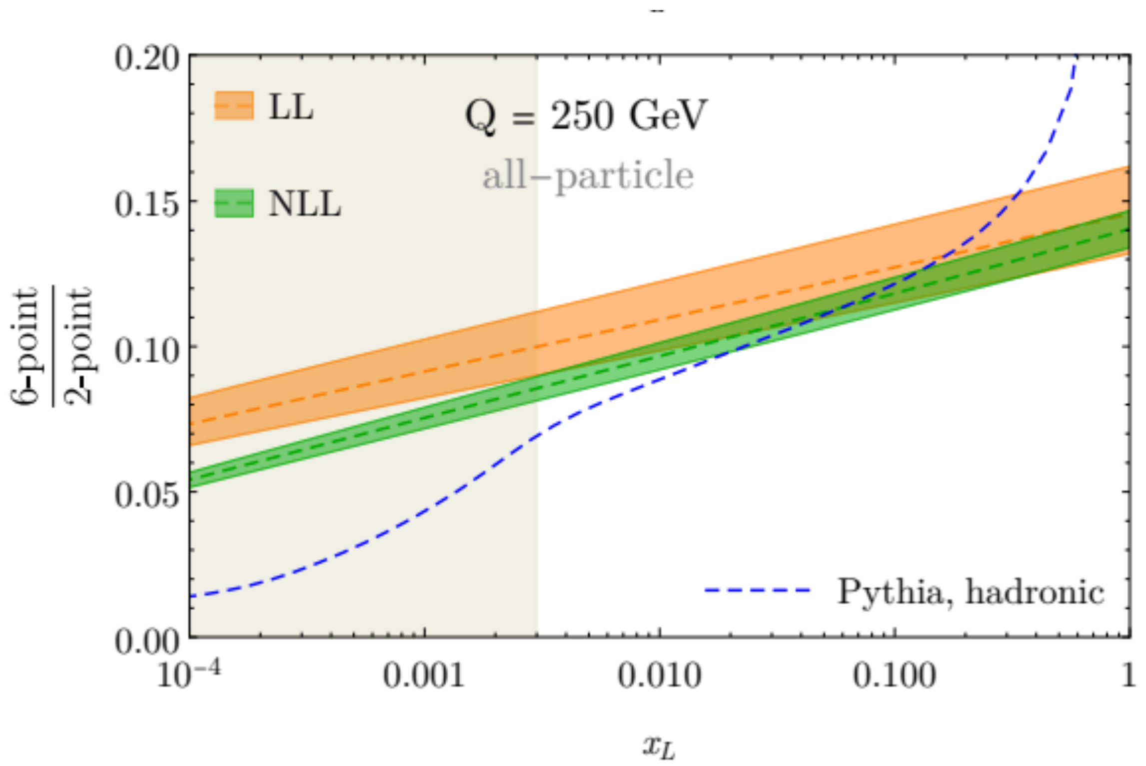


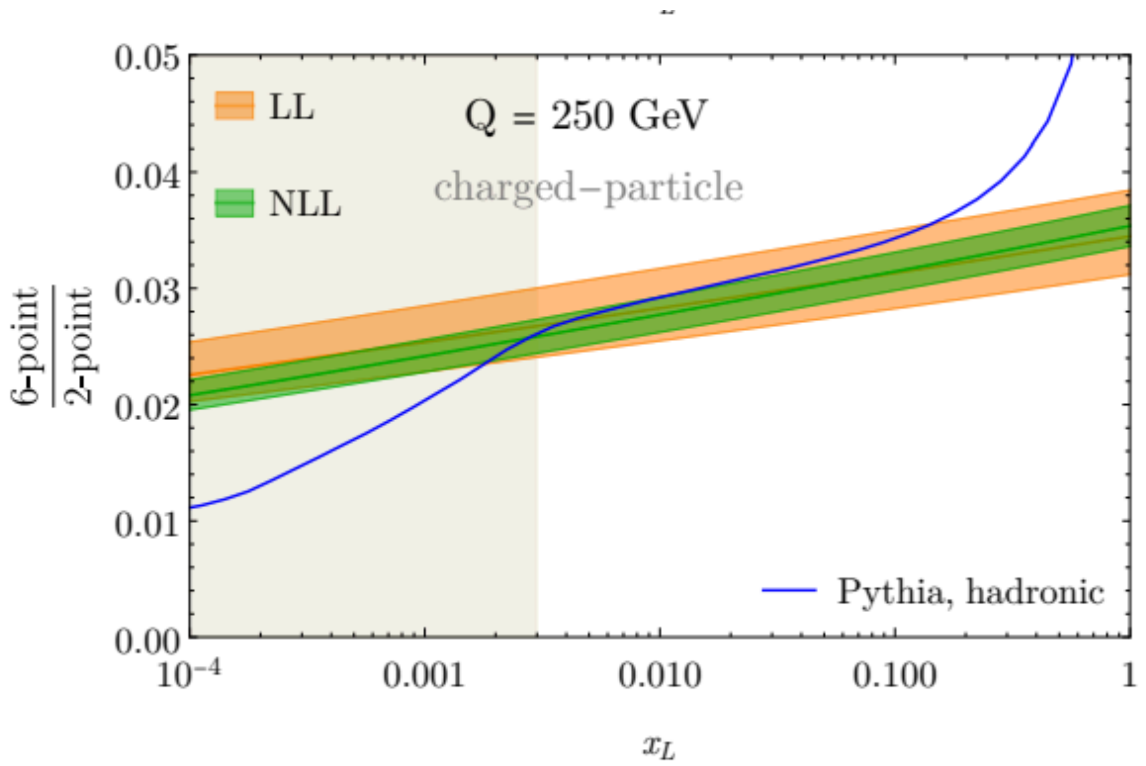


$$\langle \mathcal{E}_R(\vec{n}_1)\,\mathcal{E}_R(\vec{n}_2)\cdots\mathcal{E}_R(\vec{n}_k)\rangle = \sum_{i_1,i_2,\cdots i_k} T_{i_1}(1)\cdots T_{i_k}(1)\,\langle \mathcal{E}_{i_1}(\vec{n}_1)\,\mathcal{E}_{i_2}(\vec{n}_2)\cdots\mathcal{E}_{i_k}(\vec{n}_k)\rangle$$

+ contact terms. Requires up to k-th moment

- **Aside from the fact that this is technically much simpler, it only involves NP *numbers*, not functions.**





Up to 6 point EEC computed on tracks at NLL!

Jaarsma, Li, Moult, Waalewijn, Zhu `23

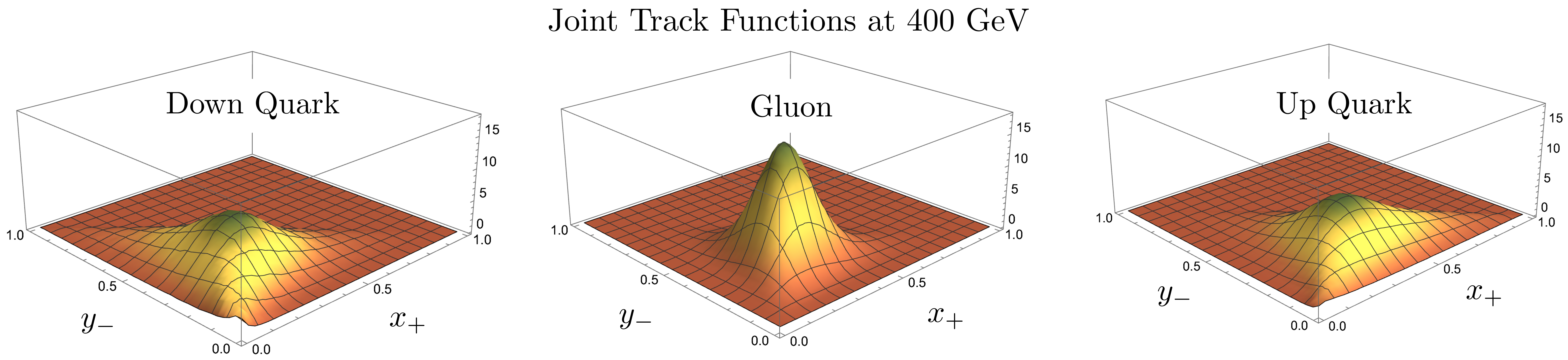
$$\langle \mathcal{E}(n_1)\,\mathcal{E}(n_2)\cdots\mathcal{E}(n_k)\rangle \to \langle \mathcal{E}_R(n_1)\,\mathcal{E}_R(n_2)\cdots\mathcal{E}_R(n_k)\rangle \to \langle \mathcal{E}_{R_1}(n_1)\,\mathcal{E}_{R_2}(n_2)\cdots\mathcal{E}_{R_k}(n_k)\rangle$$

JOINT TRACK FUNCTIONS

JOINT TRACK FUNCTION FORMALISM

- **Joint track functions are non-perturbative functions generalized to incorporate multiple quantum numbers. As a concrete example, one may consider + and - electromagnetic charges.**

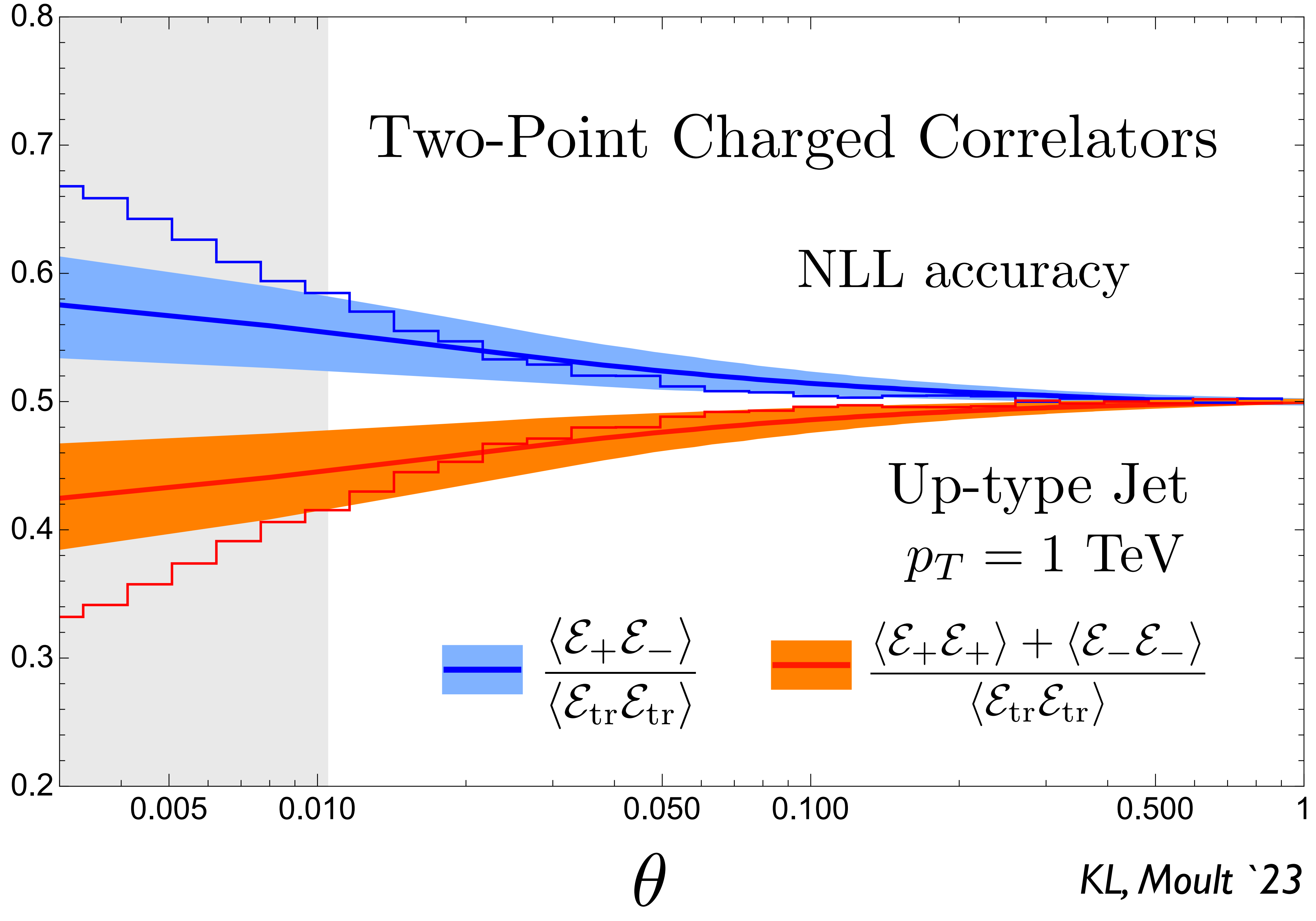
KL, Moult `23



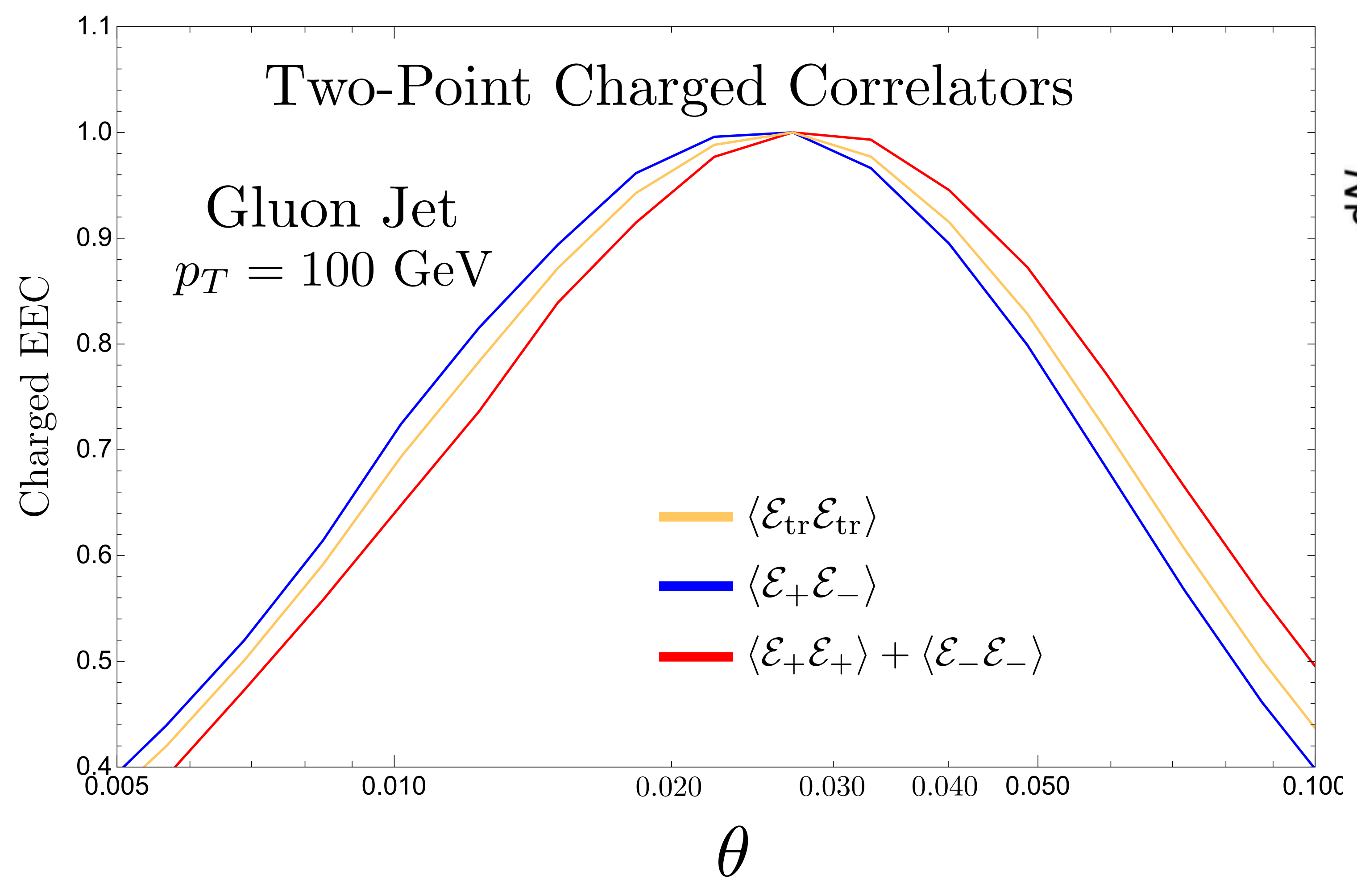
- **We see that their shapes obey charge conjugation invariance and up/down-type have more plus/minus charged hadrons as expected.**
- **These joint track functions then provide the means to study the energy correlators involving mixed quantum numbers!**

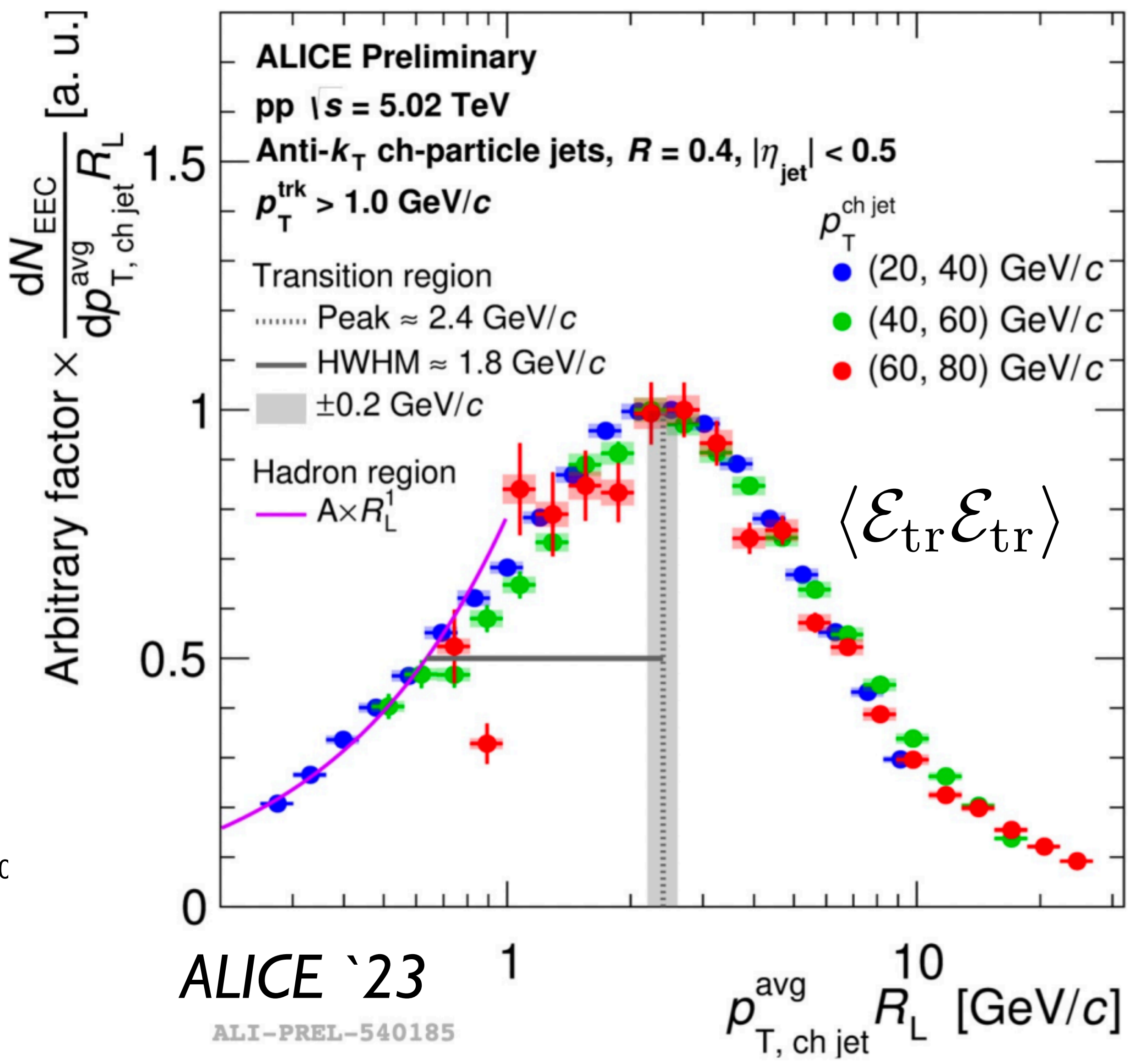
$\langle \mathcal{E}_{R_1}(n_1)\,\mathcal{E}_{R_2}(n_2)\cdots\mathcal{E}_{R_k}(n_k)\rangle$ **can be expressed in terms of moments of k-dimensional joint track functions!**

TWO-POINT CHARGED CORRELATORS



- Charged correlators have **modified scaling** in the perturbative region and unlike signed correlators are **correlated more** as the angle becomes smaller!



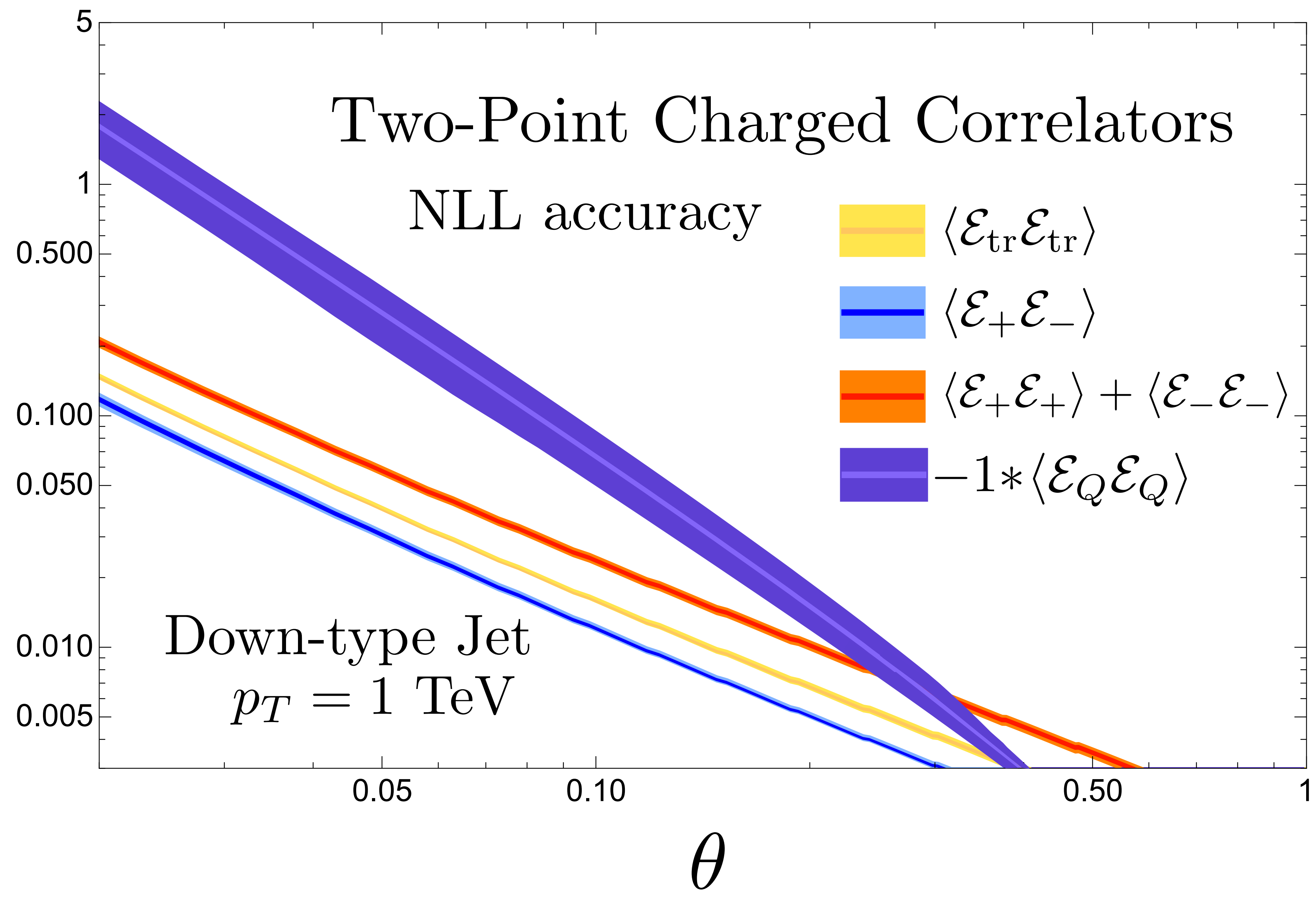


- We also observe that unlike signed correlators have **narrower transition** and the peak is also **shifted**!

C-ODD DETECTOR

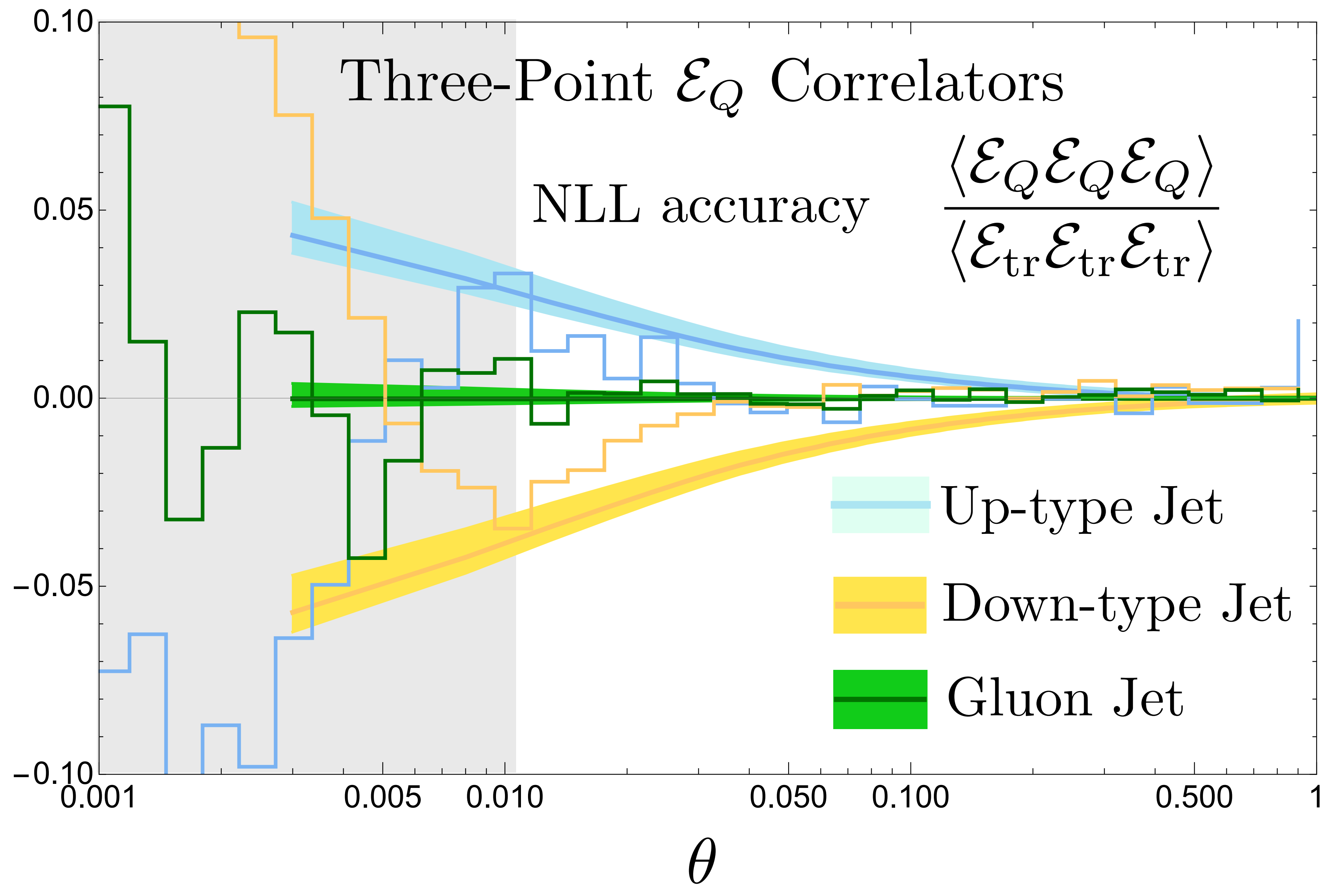
- **We can define also operator odd under charge conjugation: This operator is expected to be well-defined in a CFT as well**

$$\mathcal{E}_{\mathcal{Q}}\left(\vec{n}_1\right)|k\rangle = E_k Q_k \delta\left(\hat{n}_1 - \hat{k}\right)|k\rangle$$



KL, Moult `23

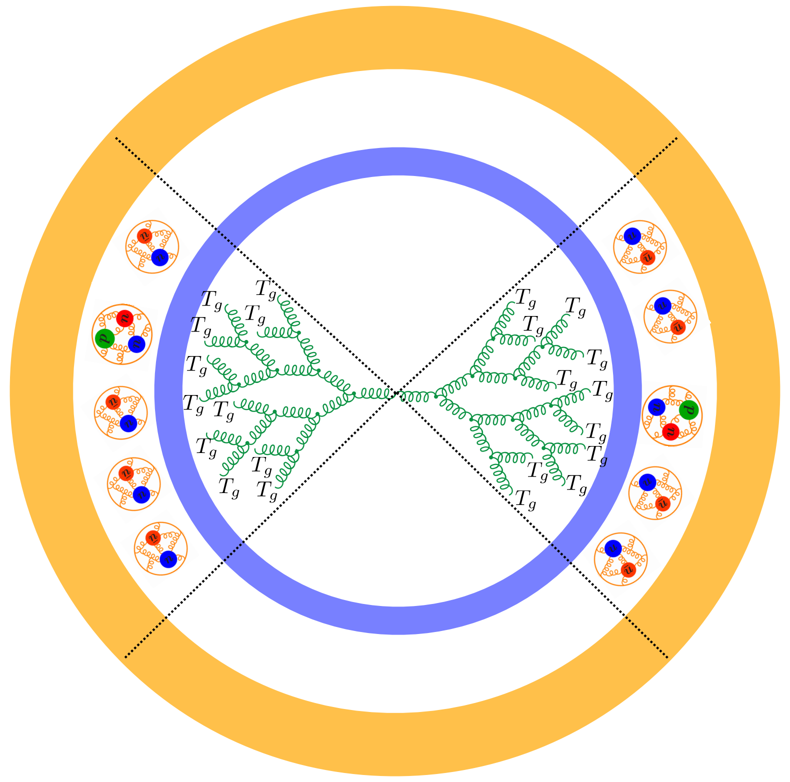
- **There is a large cancellation between like-sign and unlike-sign correlations, modifying the scaling by an integer amount.**



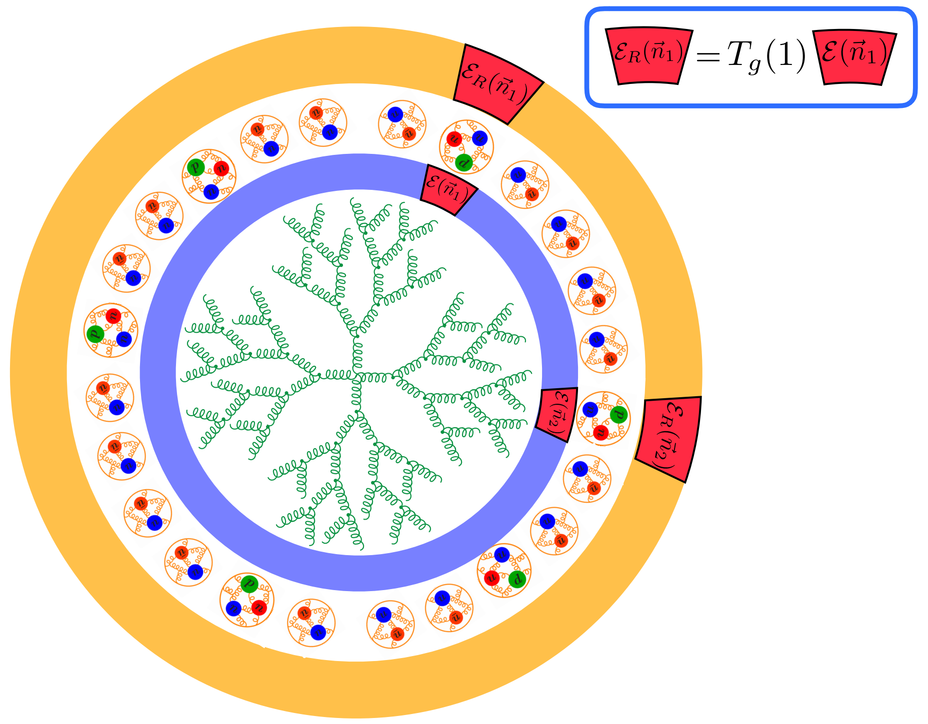
- **Due to C-odd nature, we observe that 3-pt correlators of EQ gives zero for gluon, while approximately opposite results between up and down-type due to approximate isospin symmetry. Furthermore, we observe nontrivial abrupt changes in structure in the transition region**

PHILOSOPHICAL DIFFERENCE TO JET SHAPE

Jet shape $\Leftrightarrow$ **Energy correlators**



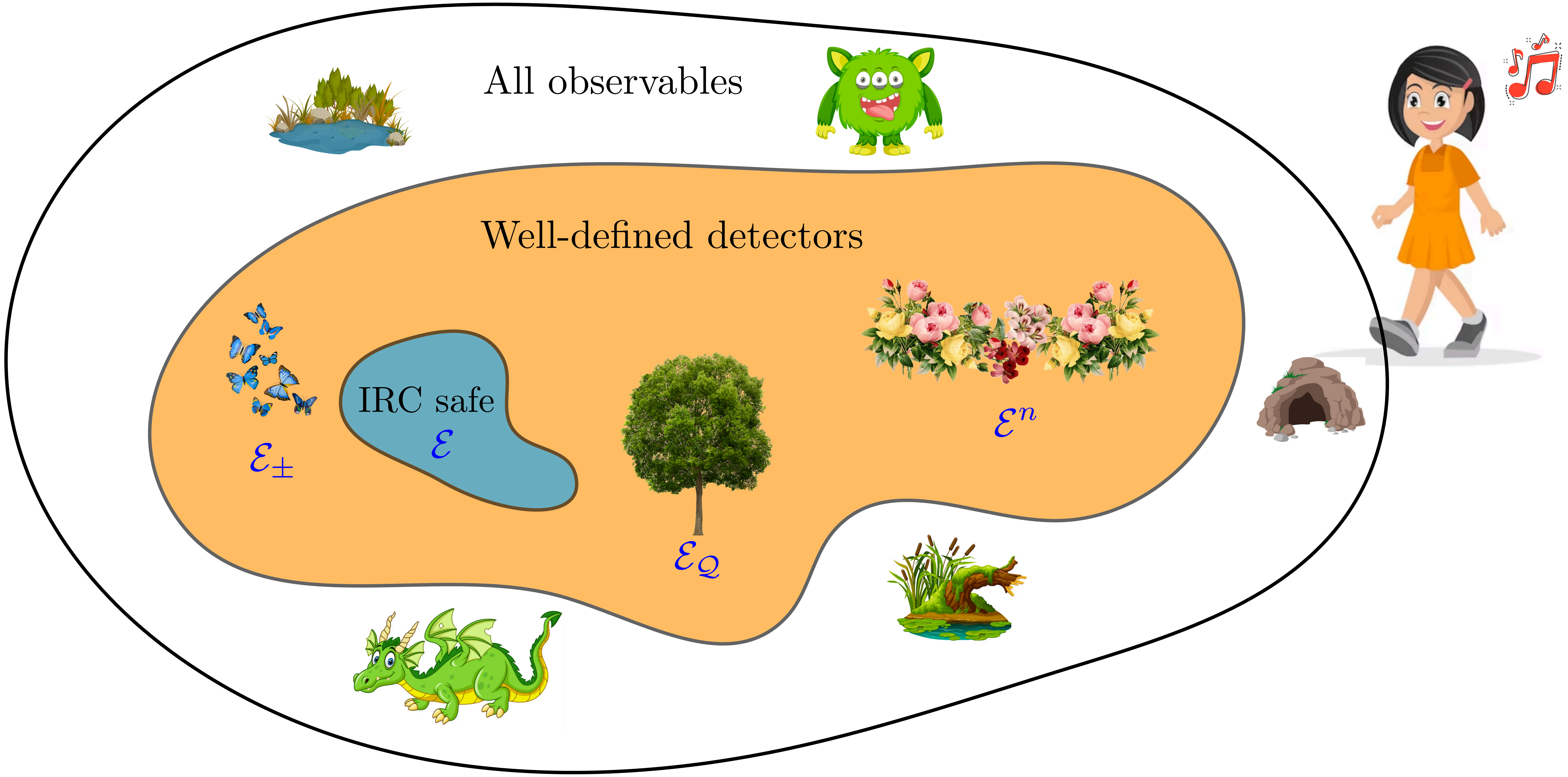
$$\delta(e-\hat{e}) = \delta(e) + \hat{e}\delta^{(1)}(e) + \cdots + \frac{\hat{e}^n}{n!}\delta^{(n)}(e) + \cdots ,$$



Jet charge $\Leftrightarrow$ **Charged energy correlators**

space of the states **VS** space of detectors

- **Jet shape / jet charge often useful for tagging purpose as you are characterizing the state of jets.**
- **For understanding the underlying charge dynamics, charged energy correlators are more useful.**
  Sensitive probe of hadronization and goes beyond the simple energy flow operator!
  Simple moment or number dependence allows enhanced sensitivity to the correlation of interest!
  Usual jet shape observable with convolution over full NP functions will wash out these information.



Let us explore the landscape of well-defined detectors and study its correlations!