Perturbatively Regularized Neural Networks

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Jets Substructure

- Improve tagging for boosted objects
- Many successful observable measurements
- Gain insight into QCD

Higher jet detail

*

----> Done through NN jet taggers



To take full advantage of jet substructure measurements we must have a good handle on the uncertainties

processes: hadronization

• Introduced by Komiske, Metodiev, Thaler

----> To control non-perturbative corrections

- IRC safe
 - ---> Observable is unchanged under soft emissions and collinear splittings

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- Introducing better performing IRC safe NN
- * Quantifying benefits IRC safety

Outline



- EFN and IRC safety
- E2FN architecture and features
- Performance Studies

Quantifying perturbative regularization



- Introduction to perturbative regularization
- Overview of study
- Pareto Front



- NNs significance comparison on toy boosted Z boson search
- Effects of theoretical uncertainties on significance

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Beyond the EFN

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Introducing Energy-Energy Flow Networks (E2FNs)

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Inspiration:

- Tkachov's work on C-continuous observables
 - ---> Energy Correlator (n=1) is analogous to an EFN

 \rightarrow Set n=2 (EEC) and make F a NN \rightarrow E2FN

orks (E2FNs	5)
	Particle Correlations!

Energy	corre	lator	S	3.39
	•••••			
		-		

These have the form

$$f(\mathbf{P}) = \sum_{a_1...a_n} E_{a_1}...E_{a_n} f_n(\hat{p}_{a_1},...,\hat{p}_{a_n}), \qquad 3.40$$

where f_n is a symmetric continuous function of *n* arguments. Basic shape observables are special cases corresponding to n = 1.

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. . .

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Raw Performance



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IRC Safety in E2FNs

- IRC safety conditions:
 - 1. Φ network is continuous
 - 2. $\Phi \rightarrow 0$ as the two studied particles become collinear

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Perturbatively Regularized NNs

- Quantify the robustness of a NN to nonperturbative information:
 - ---> How much NN's output changes when non-perturbative info is present in the input



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Les Houches 2017: Physics at TeV Colliders Standard Model Working Group Report

Performance versus robustness: Two-prong substructure taggers for the LHC ¹⁴



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 $\mathbf{2}$

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Control the type of information the NNs are sensitive to & restrict their access to non-perturbative information

Test Data

- PYTHIA data
 - ---> Each event generated once
 - ---> Data collected twice per event:
 - ---> Parton-level
 - ---> Hadron-level



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- PYTHIA data
 - ---> Each event generated once
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 - ---> Hadron-level
- Data pre-processing:
 - ---> Eta, phi coordinates centered to jet's eta, phi
 - \dashrightarrow Constituent's p_T normalized to jet p_T
 - ---> Exclusive kt reclustering by momentum splitting

$$k_T = p_{Tjet} \times \Delta \Theta$$





Regularization Metric & Pareto Plots

- Train on hadron-level data, test on parton-level data
 - RMS as regularization metric: $RMS = \sqrt{\frac{\sum_{i=1}^{N} \left(P_i(x \mid \text{parton-data}) - P_i(x \mid \text{hadron-data})\right)^2}{N}}$

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Regularization vs. Performance

Rejection at 50% eff vs RMS kT 0.6 GeV reclustering



- Pareto Front is dominated by EFN, E2FN at low RMS and high rejection
- P2FN contribution at higher RMS and rejection

To understand the effect of non-perturbative uncertainties on searches we want to relate RMS metric back to a physical quantity



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Objective 1
 Introduction to perturbative regularization

Feasible point

- Overview of study
- Pareto Plots



- NNs significance comparison on toy boosted Z boson search
- Effects of theoretical uncertainties on significance

Raw Discovery Significance in Toy Search

- Tagging boosted Z Bosons in QCD background
- NN represents the entire analysis work flow
 - Cut on NN signal score
 - Single bin counting experiment

Optimize analysis for best discovery significance



Significance with Theoretical Uncertainties

• Background Uncertainties:

 $\frac{1}{10} * [B(hadron) - B(parton)]$



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* IRC safety can help protect our measurements from theoretical systematics

* There might be other non-IRC safe NNs that might be well-behaved and outperform IRC safe nets!



Summary

- Extended the notion of EFNs to the more generalized E2FNs which allow for particle correlation and improve the NN performance while ensuring IRC safety
- Understanding the regularization of a NN plays an important role in jet substructure measurements. Especially as we move towards measuring more complicated and high-detail jet observables
- Theoretical uncertainties from non-perturbative processes can have a measurable impact on actual searches and should be further studied

Run: 282712 Event: 474587238 2015-10-21 06:26:57

Thank you!

Raw Discovery Significance in Toy Search





 P^2FN





Raw Discovery Significance in Toy Search



Regularization vs. Performance

Rejection at 50% eff vs RMS







Damping



kT 0.3 reclustering

Undamped E2FN RMS vs Performance

e2fn vR_pt (undamped)



e2fn R_pt



e2fn kt_pt



Damped E2FN RMS vs Performance

e2fn vR_pt (damped)



P2FN RMS vrs Performance



p2fn vR_2pt

Energy-Energy Flow Networks (E2FNs)

- Energy-Energy Correlators (EEC)
 - ---> Transition between perturbative and non-perturbative

physics at a visible angular scale between particles



Input Values	Definition
$\hat{n}_i,~\hat{n}_j$	$(\eta_i,\ \eta_j,\ \phi_i,\ \phi_j)$
R	$\left(\sqrt{\left(\eta_i - \eta_j ight)^2 + \left(\phi_i - \phi_j ight)^2} ight)$
k_T	$\left(\sqrt{\left(\eta_i-\eta_j ight)^2+\left(\phi_i-\phi_j ight)^2}*p_{Tjet} ight)$
Vector R	$\left(\ \left(\eta_i - \eta_j ight), \ \left(\phi_i - \phi_j ight) ight)$
R, p_{Tjet}	$\left(\sqrt{\left(\eta_i-\eta_j ight)^2+\left(\phi_i-\phi_j ight)^2},\ p_{Tjet} ight)$
$k_T, \ p_{Tjet}$	$\left(\sqrt{\left(\eta_i - \eta_j\right)^2 + \left(\phi_i - \phi_j\right)^2} * p_{Tjet}, p_{Tjet} ight)$
Vector R , $p_{T,i}$	$\left(\left(\eta_{i}-\eta_{j} ight),\left(\phi_{i}-\phi_{j} ight),p_{T_{jet}} ight)$

i,j

Hyperparameters & Trainable Parameters

- 2 layers in phi
- 3 layers in F
- Relu activation
- Latent Dim: 32
- Learning rate: 1e-4
- Damping:

$$damp = \frac{\left(\frac{k_T}{\tau}\right)^{\frac{1}{w}}}{\left(\frac{k_T}{\tau}\right)^{\frac{1}{w}} + 1}$$

Table 6: Number of Trainable Parameters

Architecture	Phi	F	Total
PFN	21.2 K	37.5 K	58.7 K
EFN	21.0 K	37.5 K	58.5 K
E2FN	21.2 K	37.5 K	58.7 K
P2FN	21.3 K	37.5 K	58.8 K
ParticleNet	-	-	366 K

