Understanding Jet Charge

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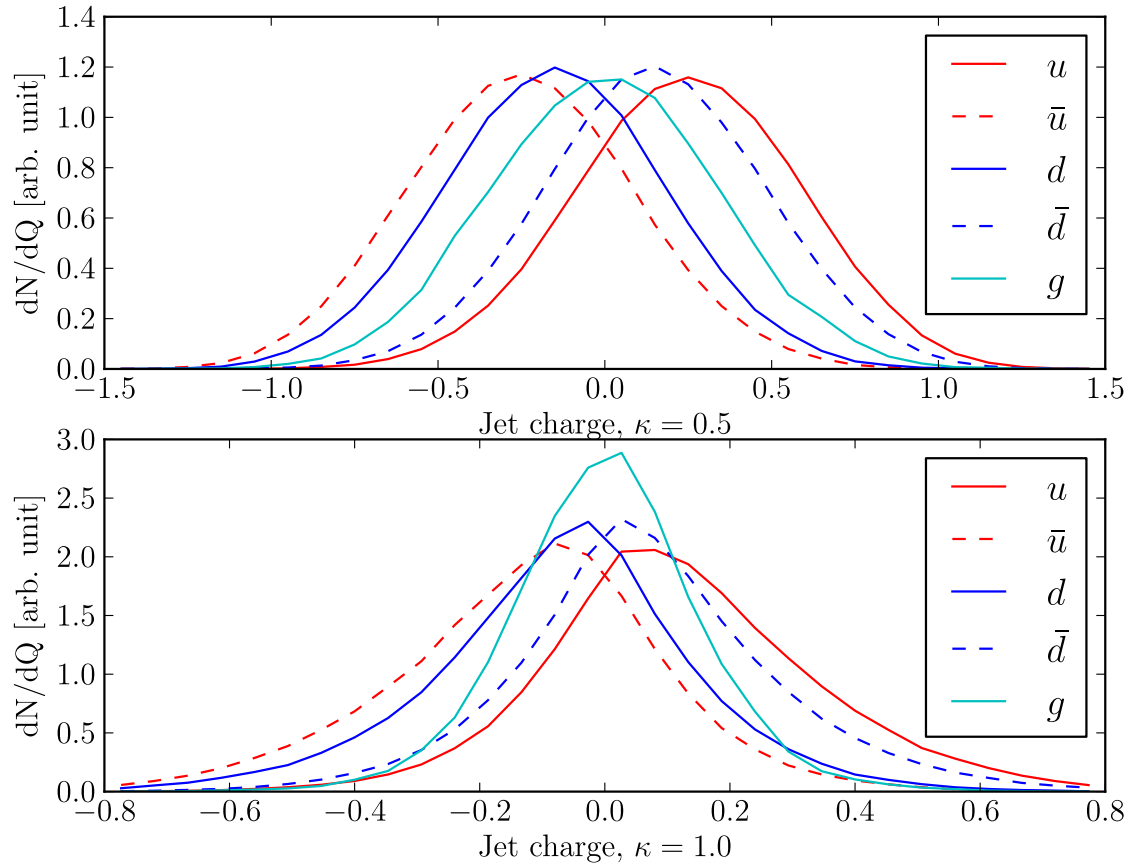
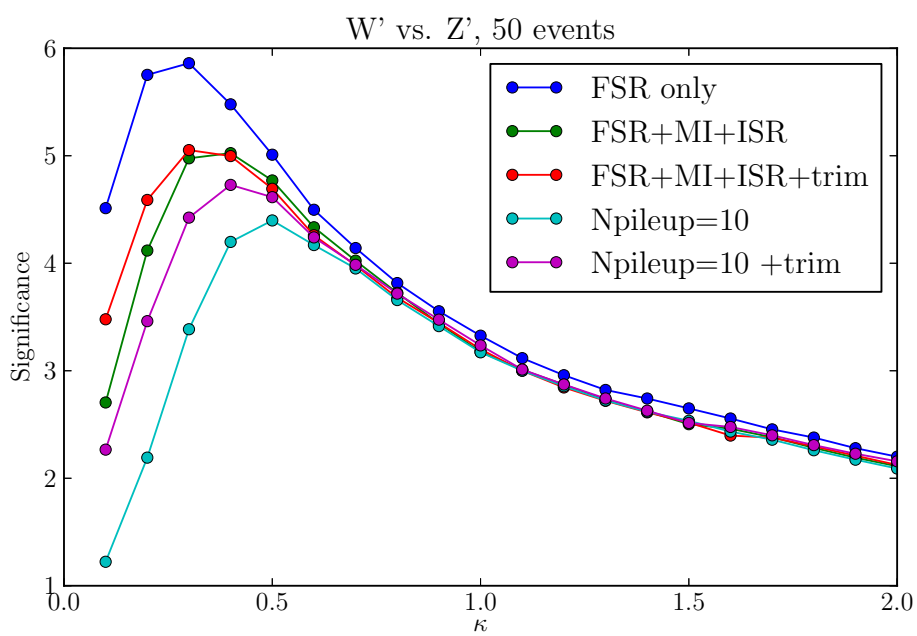
$$Q_\kappa \equiv \sum_{i \in J} z_i^\kappa \, Q_i$$

Feynman, Field 1977; 1209.2421

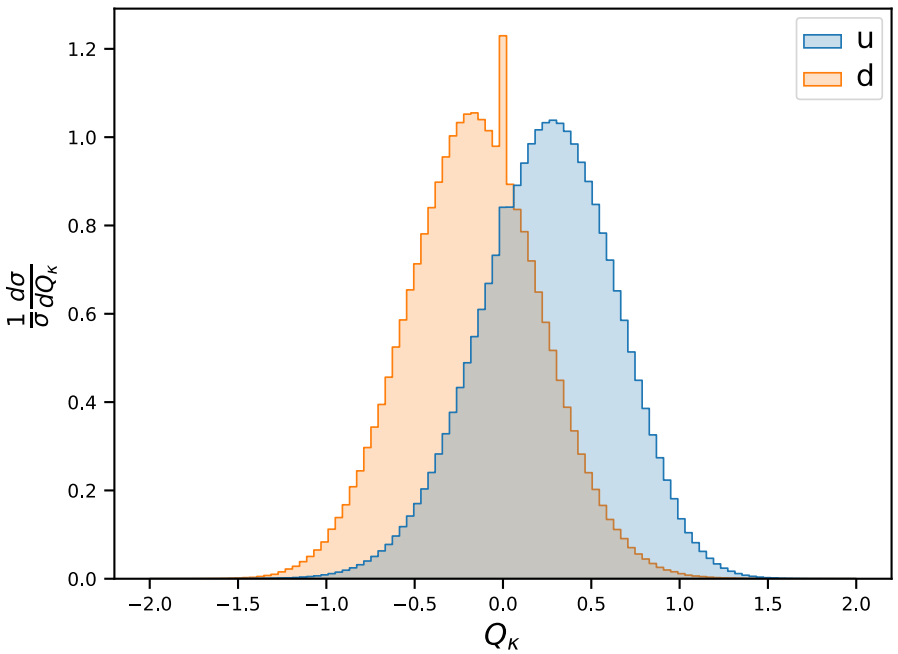




Powerful discriminant between parton flavors

Improved discrimination power as $\kappa \to 0$

Distribution narrows as κ increases



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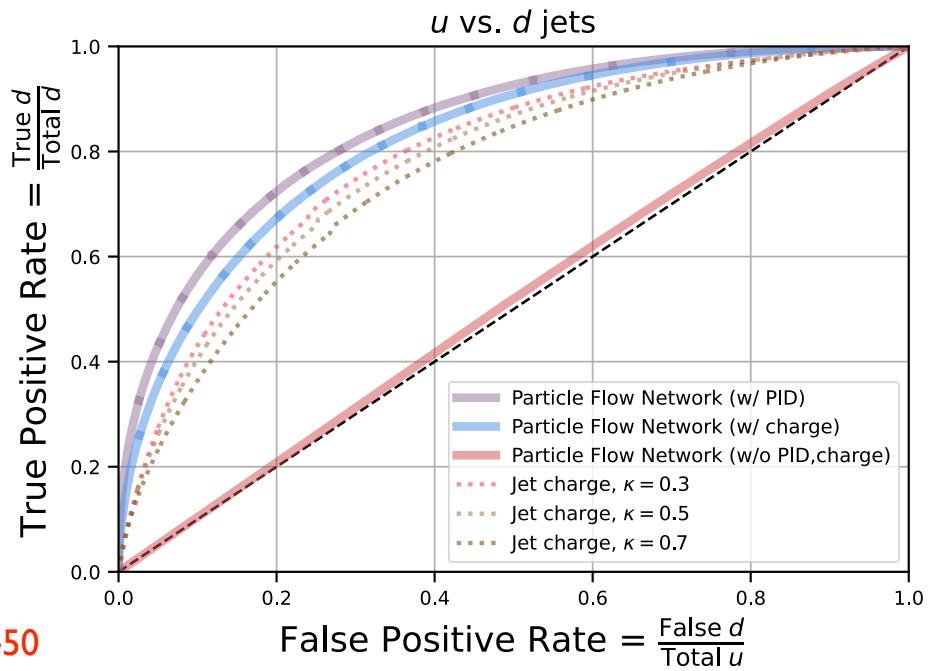


Application of jet charge to EIC physics: *u* versus *d* jet identification

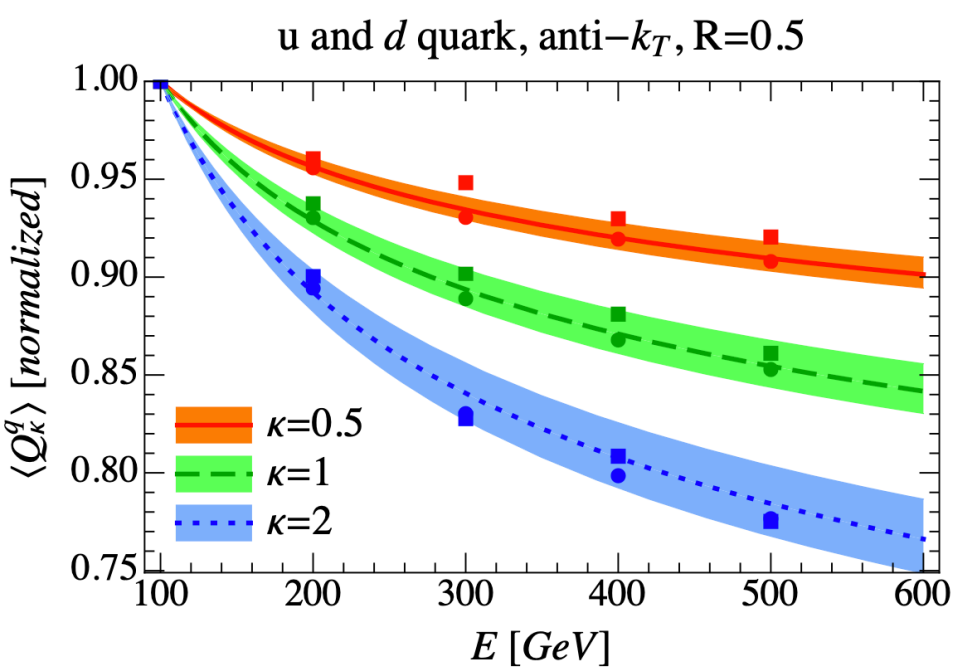
Essentially no useful discrimination information exclusively in distribution of particle momenta

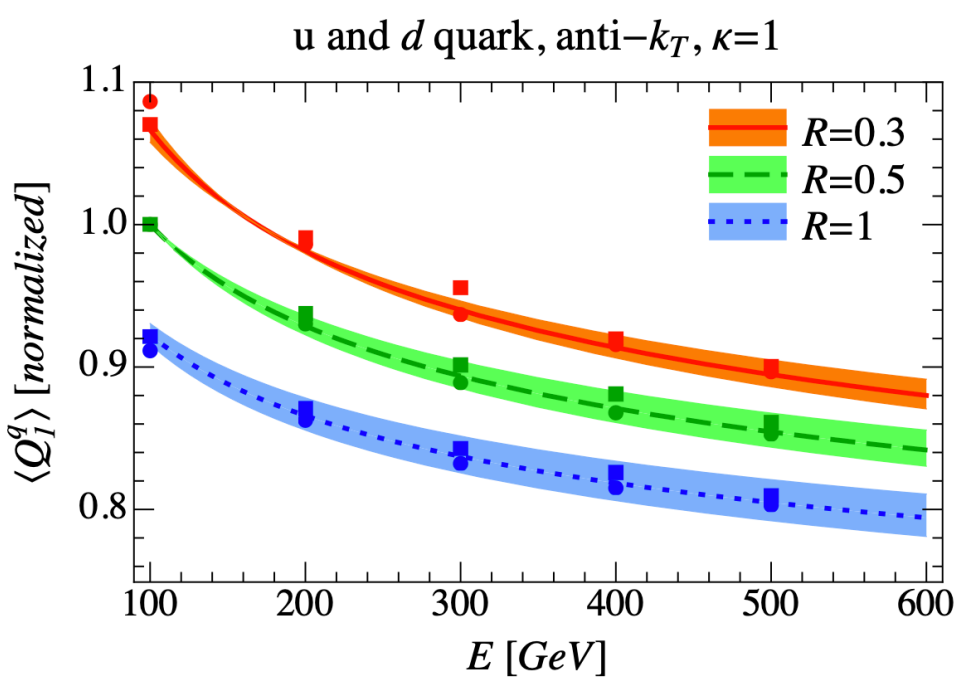
Jet charge is a (the only?) useful discriminant between these jets

Challenge: Jet Charge is not IRC Safe

Cannot calculate distributions from first principles in perturbation theory

Scale evolution of moments are well-understood, however





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Our Approach: Make Assumptions as Simple as Possible but No Simpler

1. Particles (hadrons) in the jet are produced though identical, independent processes.
2. The multiplicity of particles in the jet N is large.
3. The only particles are the pions: π^+, π^-, and π^0.
4. SU(2) isospin of the pions is an exact symmetry.

Non-perturbative assumptions that can be improved (i.e., SU(3) flavor)

Minimal assumptions regarding a short-distance description, like quark electric charge

Due to time, apply to measuring Jet Charge for u vs. d jet discrimination

(see backup for measuring Jet Charge on Inclusive Jets)

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Mean on u or d jets is set by fractional moment

$$\langle Q_\kappa \rangle = \left\langle \sum_{i \in J} z_i^\kappa Q_i \right\rangle = N \langle z^\kappa \rangle \langle Q \rangle = \langle z^\kappa \rangle (Q_u, Q_d)$$

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$$p_u(Q_\kappa|N) = \frac{1}{\sqrt{2\pi\frac{2}{3}N\langle z^{2\kappa}\rangle}}\, e^{-\frac{\left(Q_\kappa - \frac{2}{3}\langle z^\kappa\rangle\right)^2}{\frac{4}{3}N\langle z^{2\kappa}\rangle}}$$

$$p_d(Q_\kappa|N) = \frac{1}{\sqrt{2\pi\frac{2}{3}N\langle z^{2\kappa}\rangle}}\, e^{-\frac{\left(Q_\kappa + \frac{1}{3}\langle z^\kappa\rangle\right)^2}{\frac{4}{3}N\langle z^{2\kappa}\rangle}}$$

Let's calculate the mean/variance moments:

$$\langle Q_\kappa \rangle = Q_q \langle z^\kappa \rangle$$
$$\sigma_\kappa^2 = \frac{2}{3} N \langle z^{2\kappa} \rangle$$

$$\langle z \rangle = \int_0^1 dz\, z\, p(z|N) = \frac{1}{N}$$

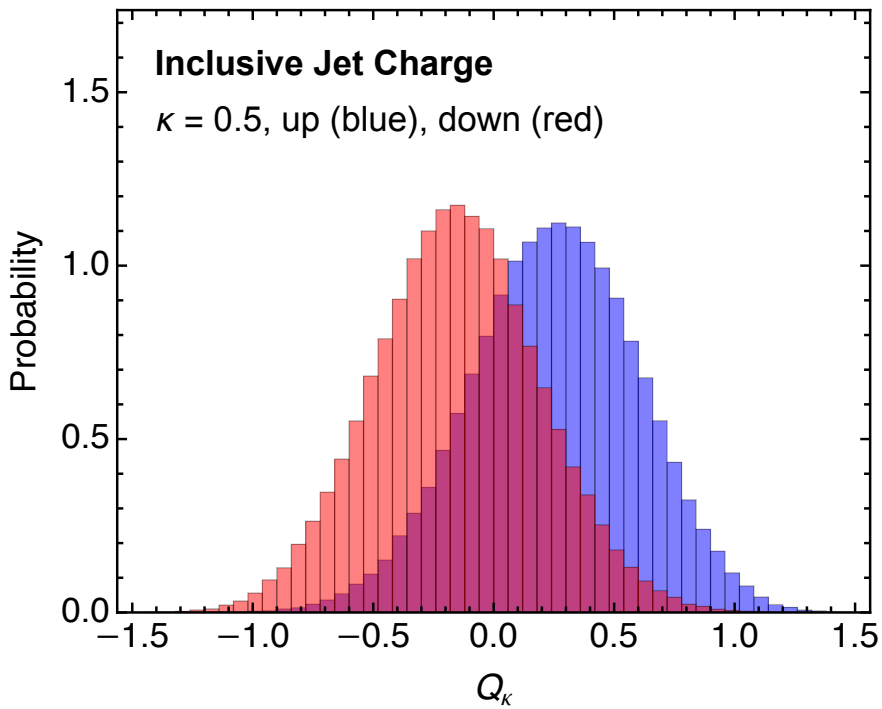
Make a central moment expansion:

$$p(z|N) = \delta\left(z - \frac{1}{N}\right) + \frac{\sigma_z^2}{2}\delta''\left(z - \frac{1}{N}\right) + \cdots$$

Fractional moments can be expressed as:

$$\langle z^\kappa \rangle = \int_0^1 dz\, z^\kappa\, p(z|N) = N^{-\kappa}\left(1 + \frac{\kappa}{2}(\kappa - 1)\sigma_z^2 N^2 + \cdots\right)$$

Optimal Parameter Predictions



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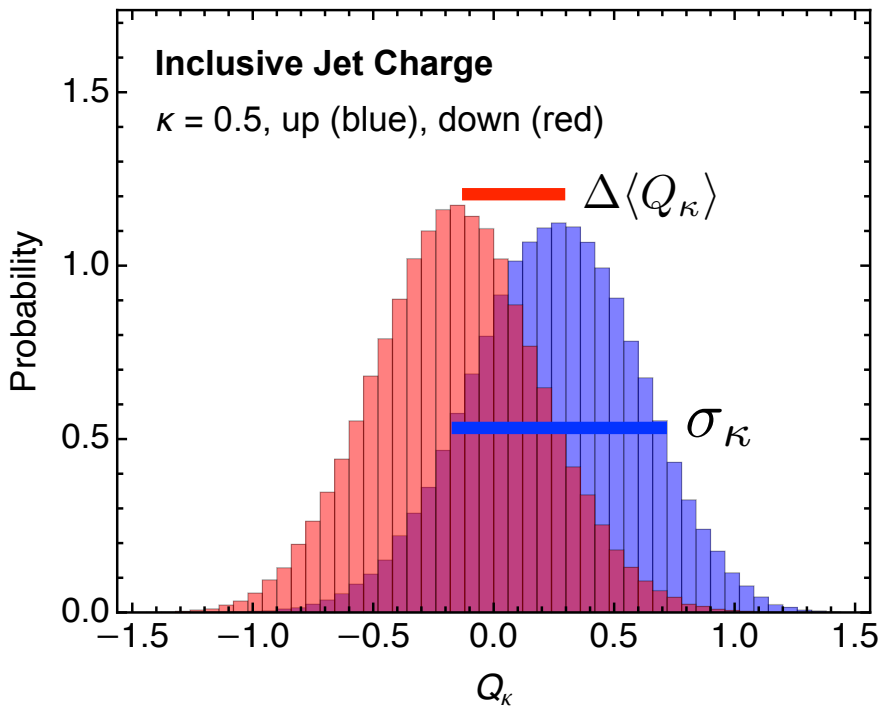
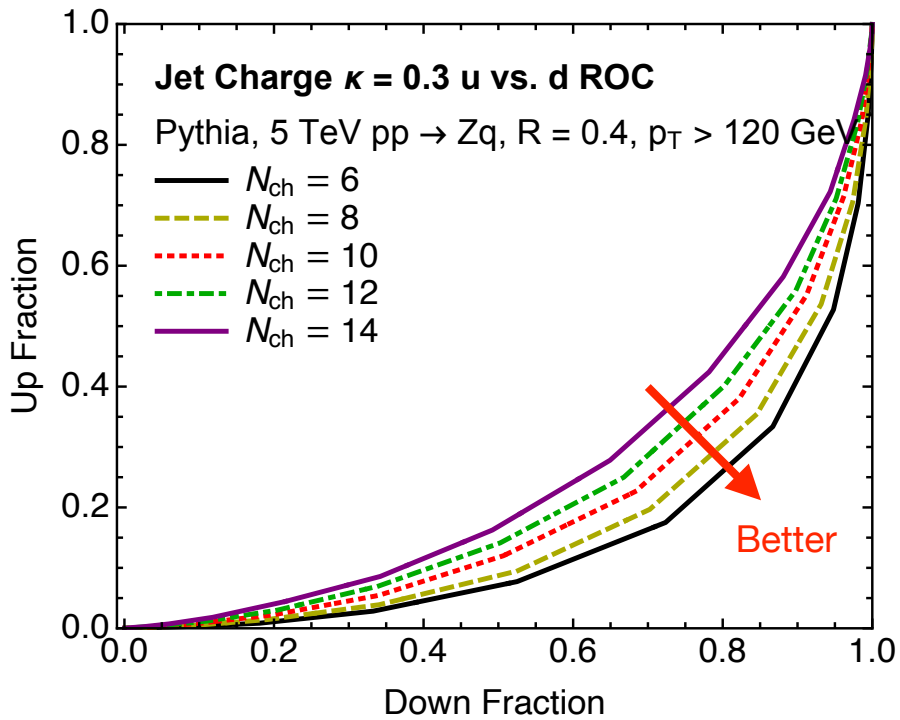


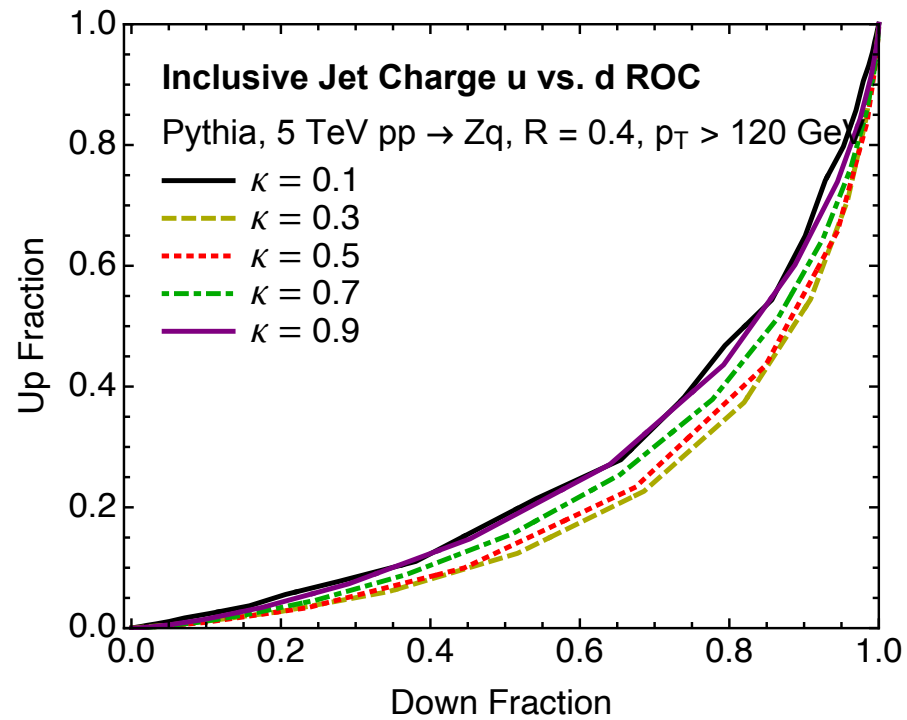
Discrimination Power is Maximized When

$$\arg\max_{\kappa,N} \frac{(\Delta\langle Q_\kappa\rangle)^2}{\sigma_\kappa^2} = \arg\max_{\kappa,N} \frac{\langle z^\kappa\rangle^2}{\frac{2}{3}N\langle z^{2\kappa}\rangle} = \arg\max_{\kappa,N} \frac{1}{\frac{2}{3}N}(1-\kappa^2\sigma_z^2N^2+\cdots)$$

Optimal discrimination when κ and N are small

Optimal Parameter Predictions





If κ is too small, then IR contamination overwhelms jet charge

One More Thing…

Optimal Discrimination Observable by Neyman-Pearson is Log-Likelihood:

$$\mathcal{O} = \log \frac{p_u(Q_\kappa, N)}{p_d(Q_\kappa, N)} = \log \frac{p_u(Q_\kappa|N)p(N)}{p_d(Q_\kappa|N)p(N)}$$

Assuming that multiplicity distribution is identical for up and down jets (see bonus)

Just Take Ratio of Gaussian Distributions:

$$\mathcal{O} = \frac{3}{2} N^{-1+\kappa} Q_\kappa - \frac{N^{-1}}{4}$$

Not Monotonically Related to Jet Charge Q_κ

Necessarily Improve Discrimination Power by Measuring Jet Charge Differential in Multiplicity

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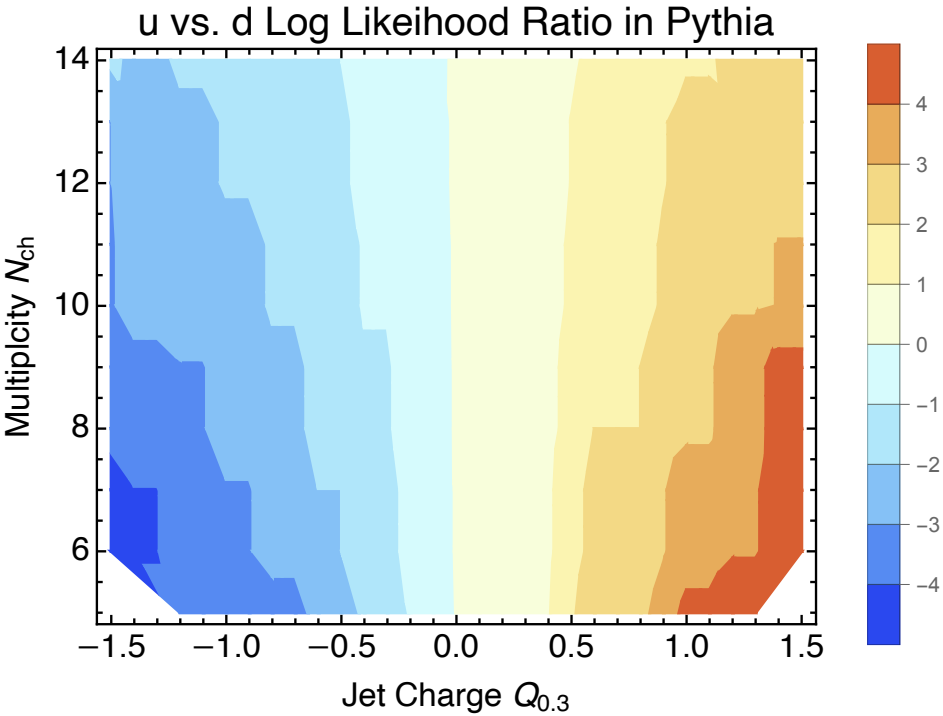
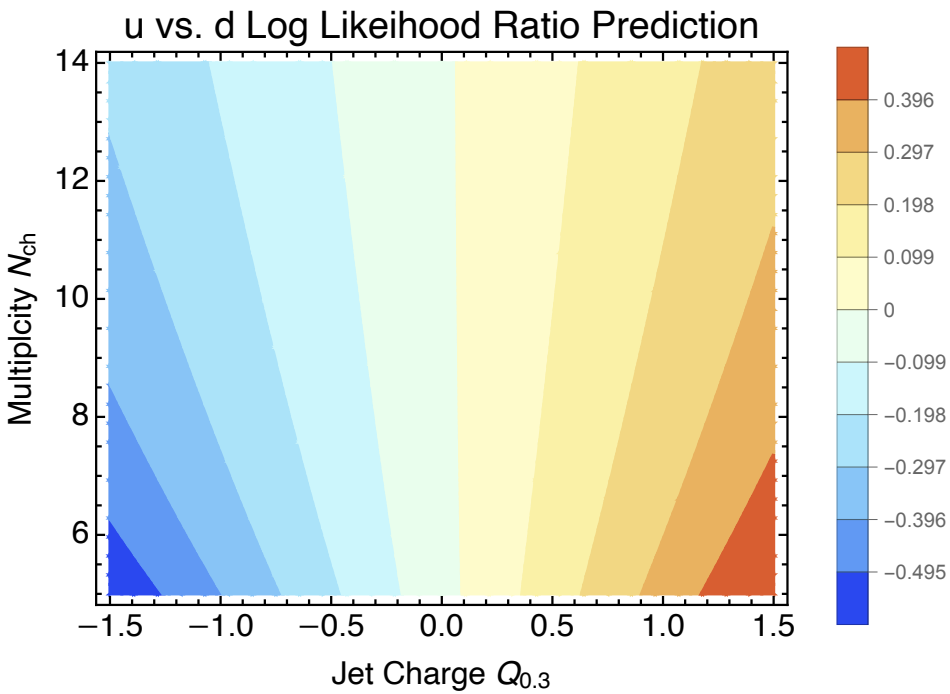


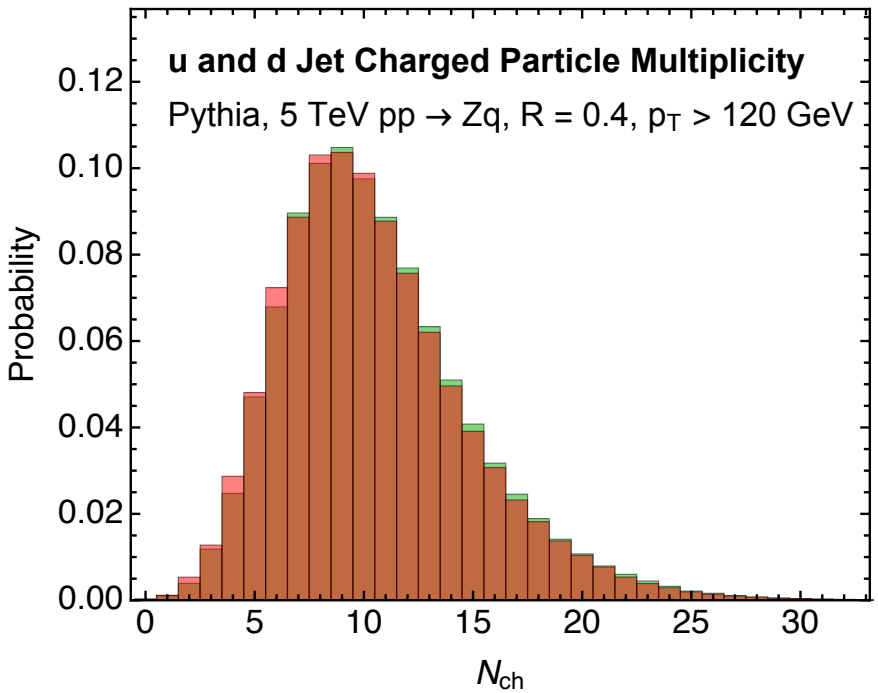


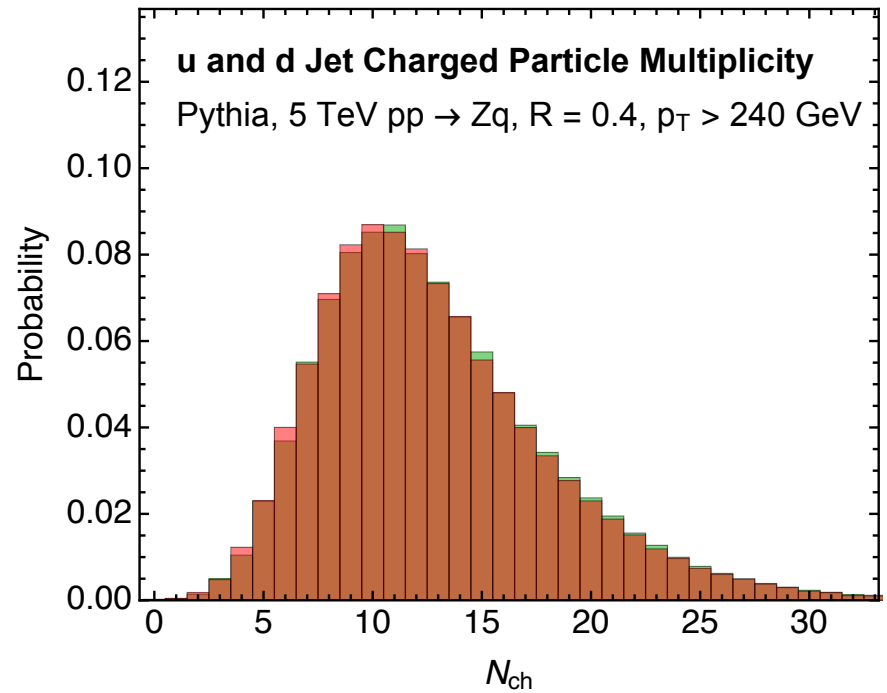
Takeaway:

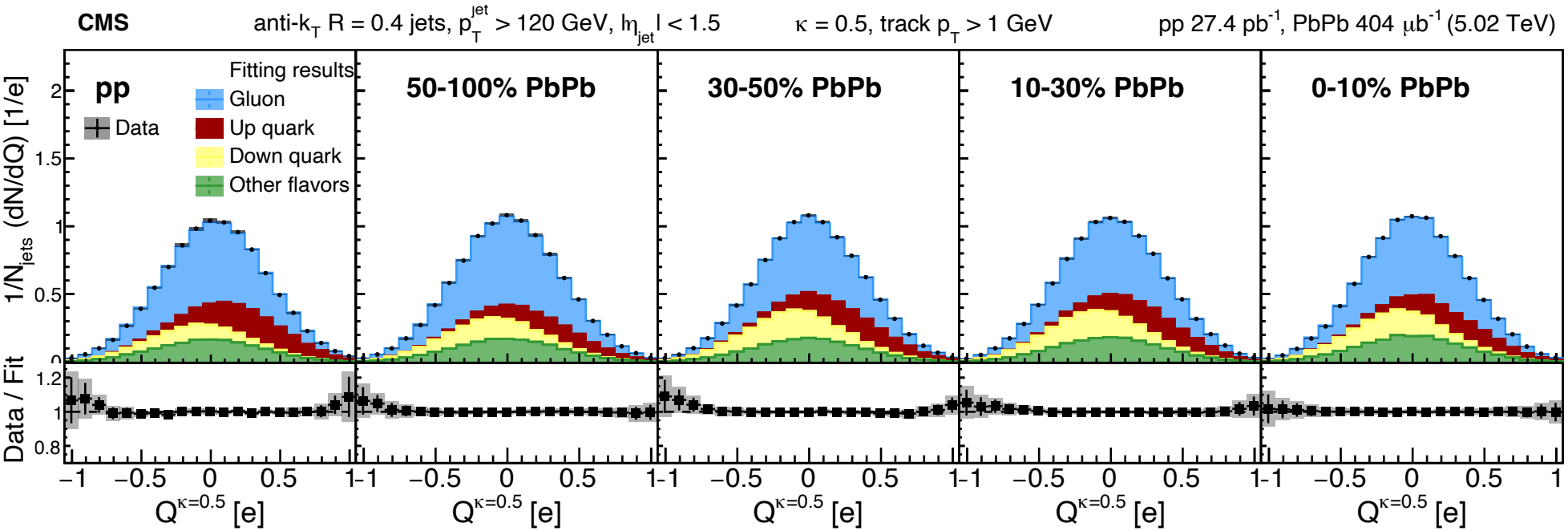
Measure Jet Charge Differential in Multiplicity!

Bonus

Up and Down Quark Jet Multiplicity Distributions







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Surprisingly little medium modification to jet charge from pp to PbPb

Aren't gluon jets quenched more than quark jets? How large is UE effect?

Can we understand this?

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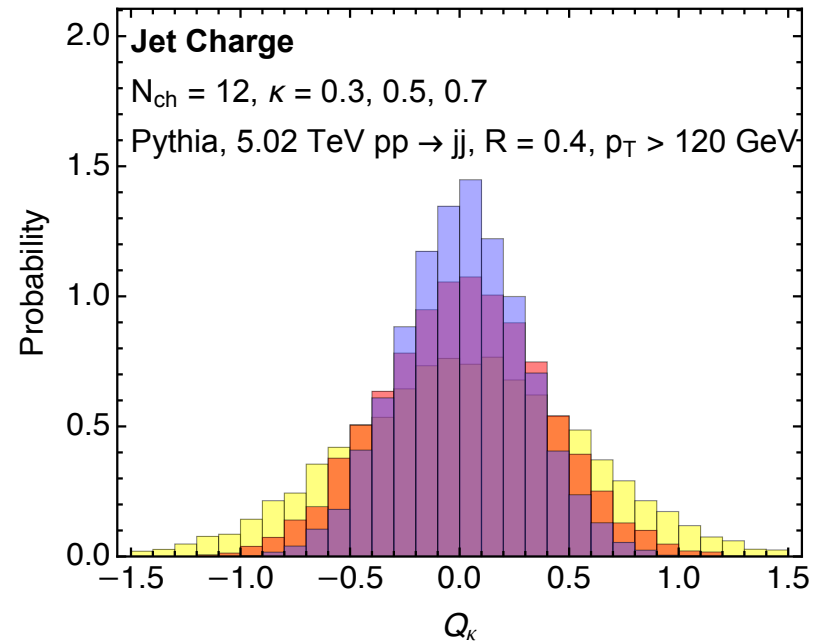
Predictions

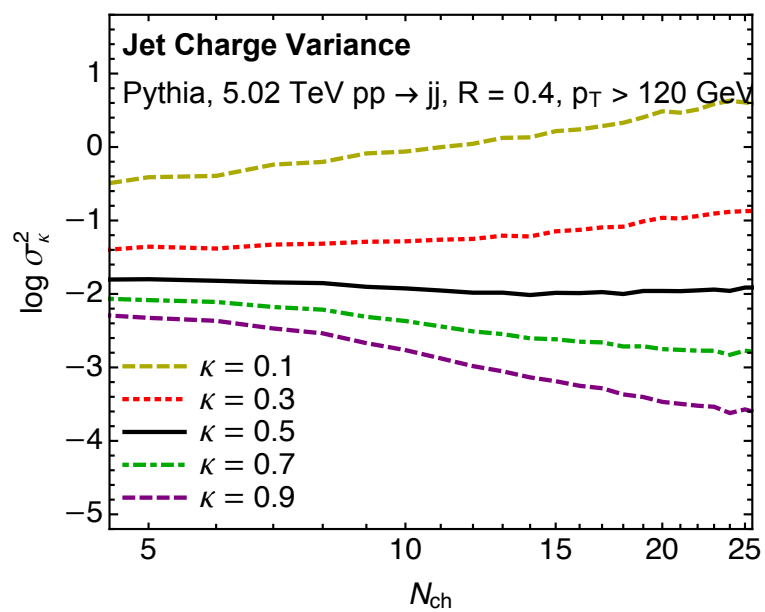
$$\sigma_\kappa^2 = \frac{2}{3} N^{1-2\kappa} \left(1 + \kappa(2\kappa - 1)\sigma_z^2 N^2 + \cdots\right)$$

Jet charge distribution narrows as κ increases



As multiplicity N increases (jet pT increases), distribution widens if $\kappa < 0.5$

As multiplicity N increases (jet pT increases), distribution narrows if $\kappa > 0.5$



Width is independent of N if $\kappa = 0.5$