Perturbatively Regularized Neural Networks

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Ian Moult
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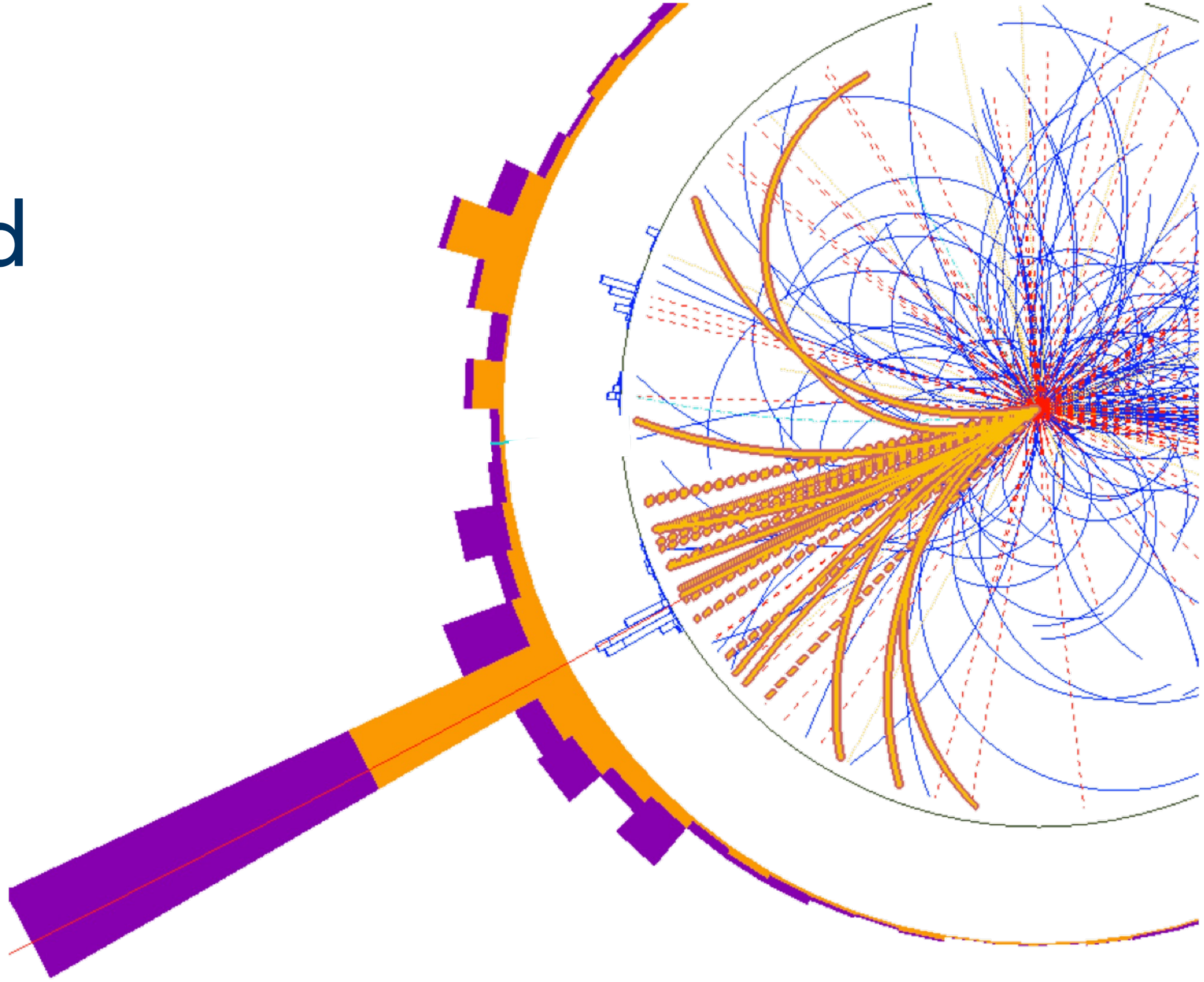
Yale University



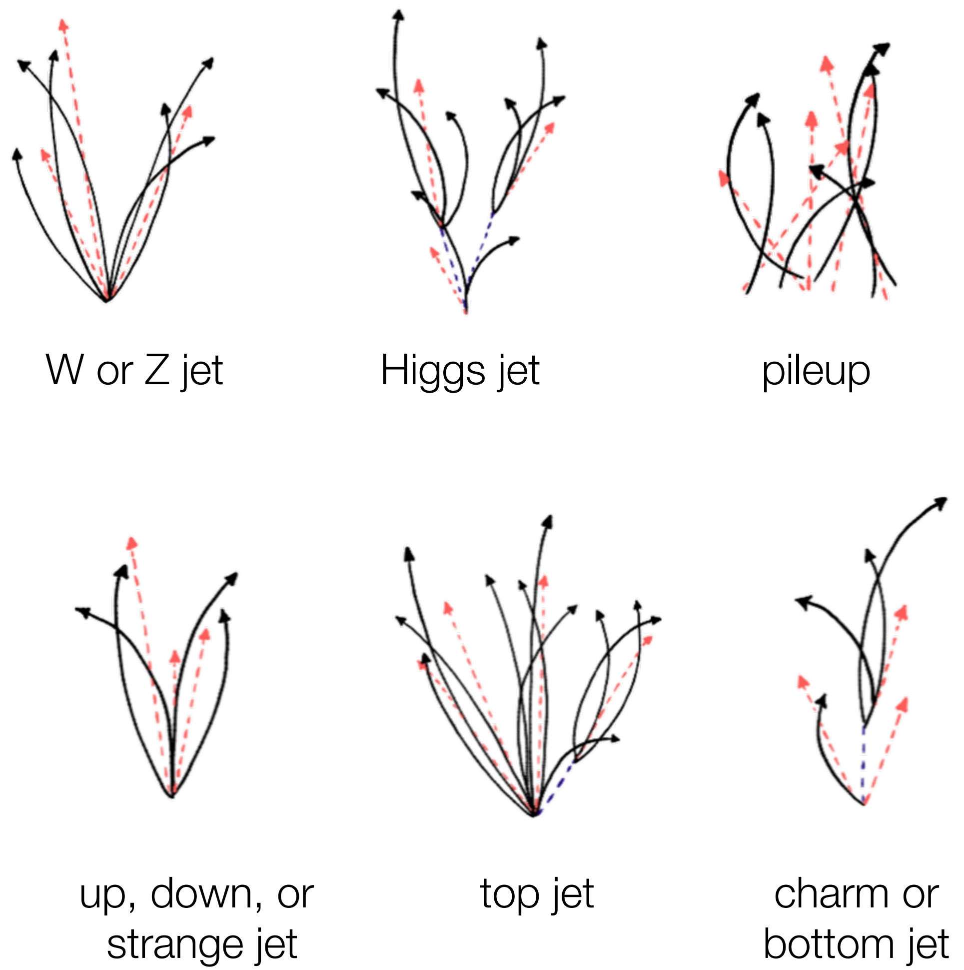
Jets Substructure

- Improve tagging for boosted objects
- Many successful observable measurements
- Gain insight into QCD

⇢ Done through NN jet taggers

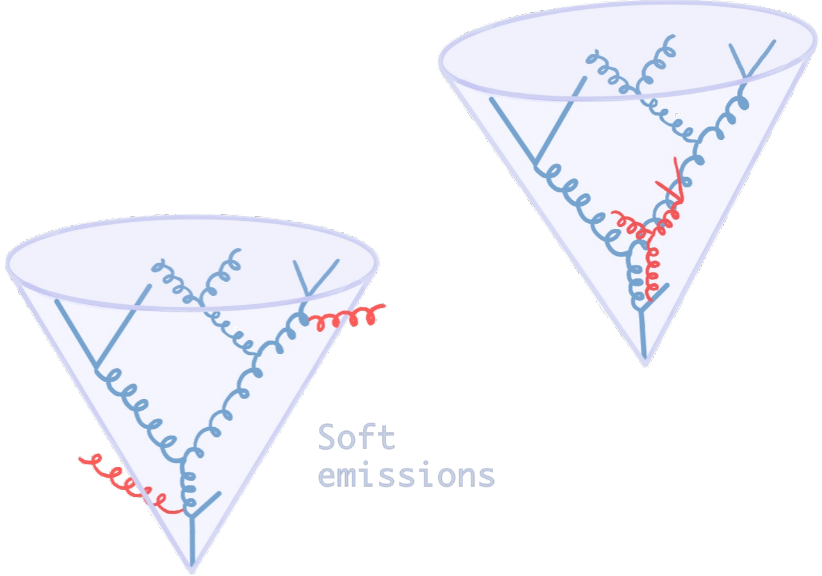
Higher jet detail ⟶ Higher sensitivity to theoretical uncertainties *

✻ Rooted in modeling of non-perturbative processes: hadronization



To take full advantage of jet substructure measurements we must have a good handle on the uncertainties

Energy Flow Networks (EFNs)



- Introduced by Komiske, Metodiev, Thaler
  - ⇢ To control non-perturbative corrections
- IRC safe
  - ⇢ Observable is unchanged under soft emissions and collinear splittings

$$EFN = F\left(\sum_{i}^{N} e_i \Phi\left(\hat{n}_i\right)\right)$$

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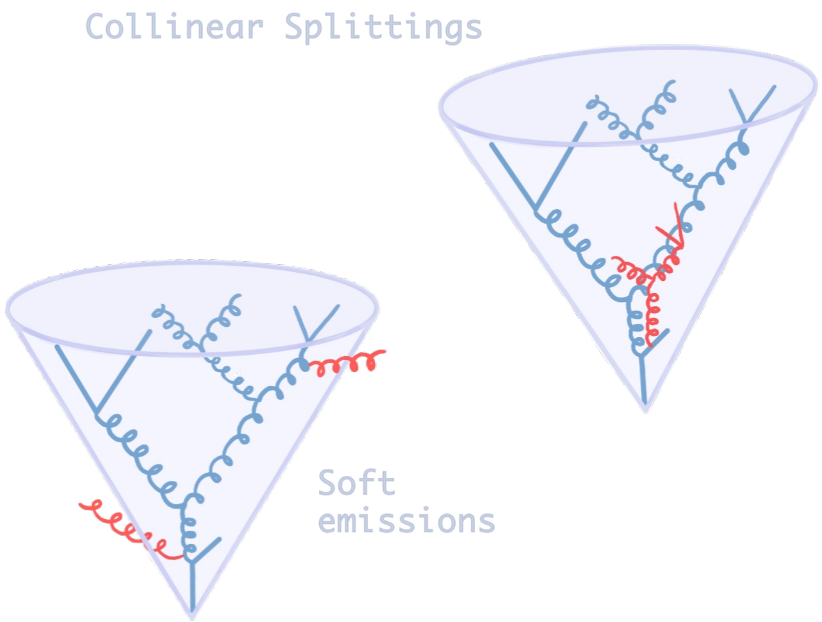

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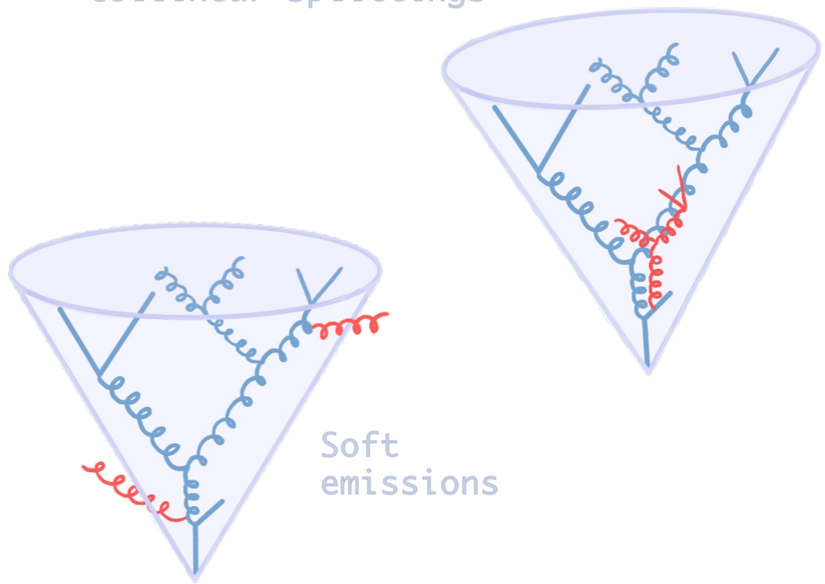

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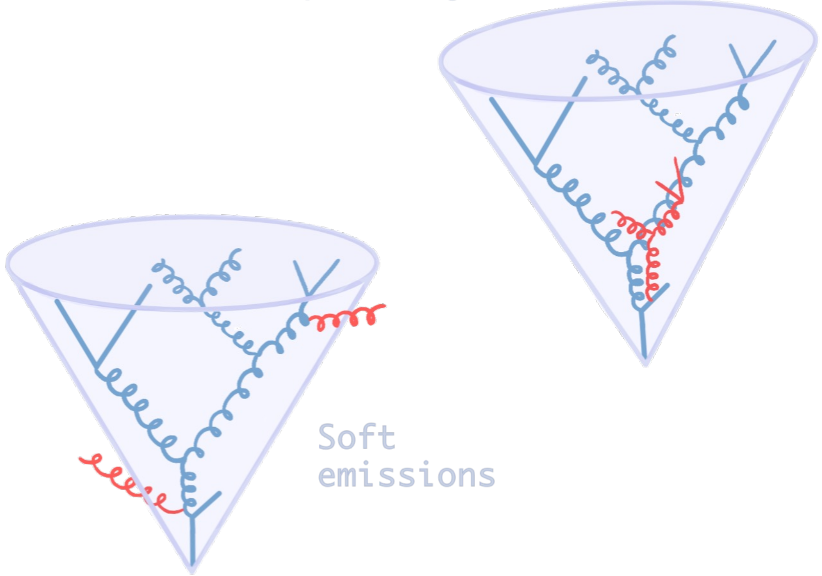

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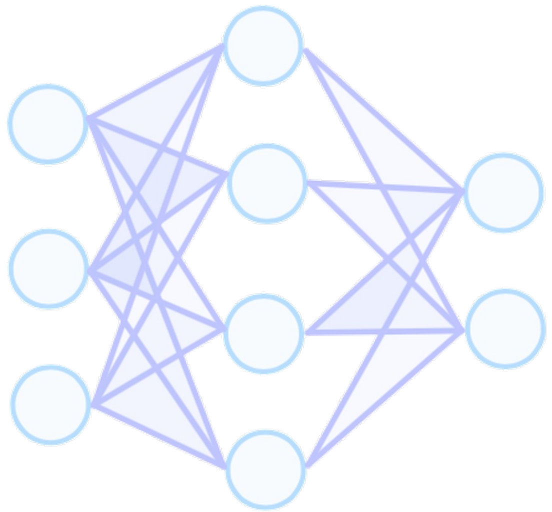
- Comparatively poor performance
- IRC benefits?

⁕ Introducing better performing IRC safe NN

⁕ Quantifying benefits IRC safety

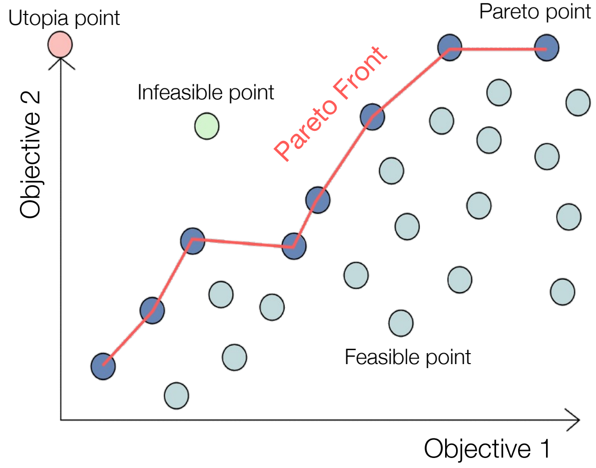
Outline

Beyond the EFN



- EFN and IRC safety
- E2FN architecture and features
- Performance Studies

Quantifying perturbative regularization



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- Overview of study
- Pareto Front

A Toy Study

- NNs significance comparison on toy boosted Z boson search
- Effects of theoretical uncertainties on significance

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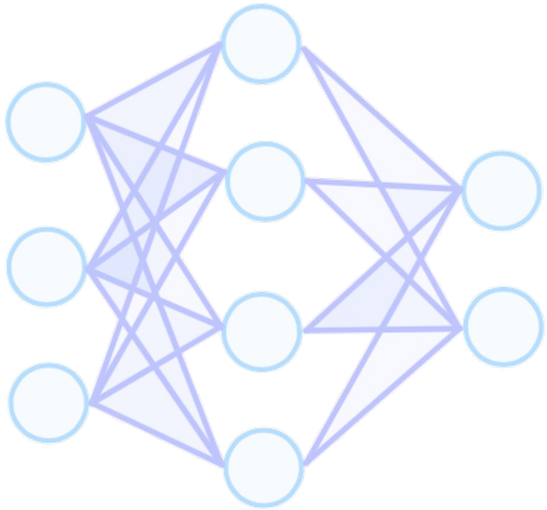

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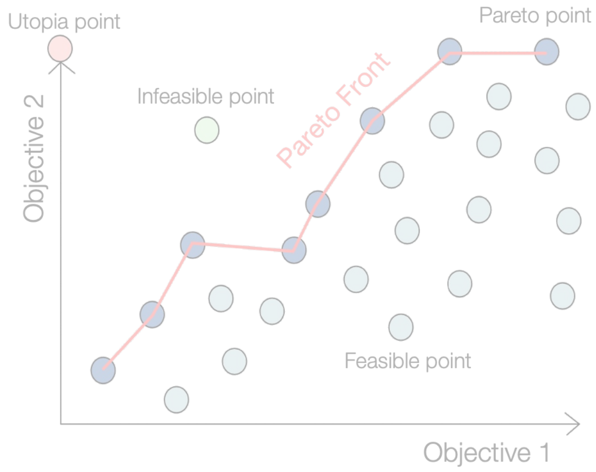

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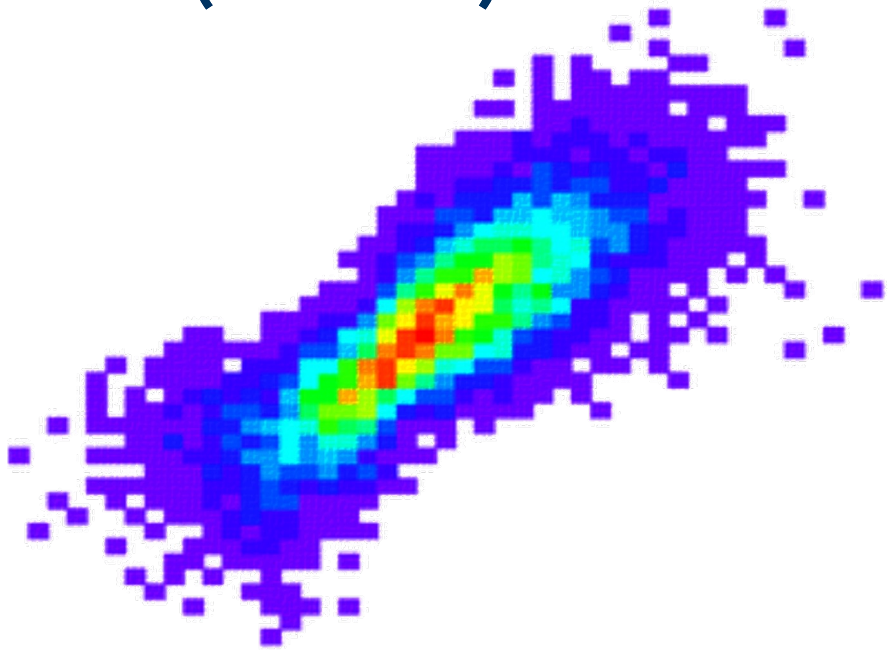

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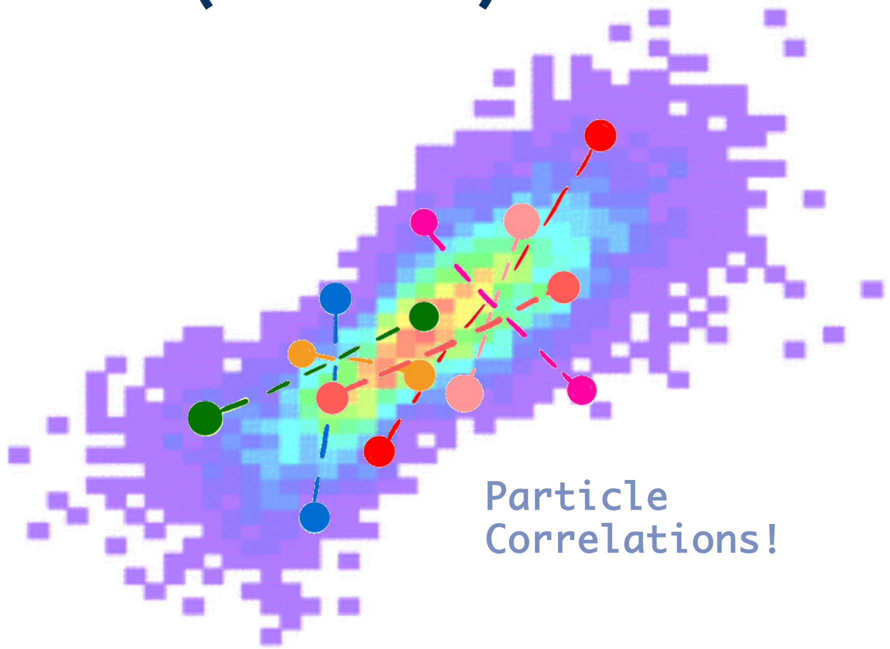

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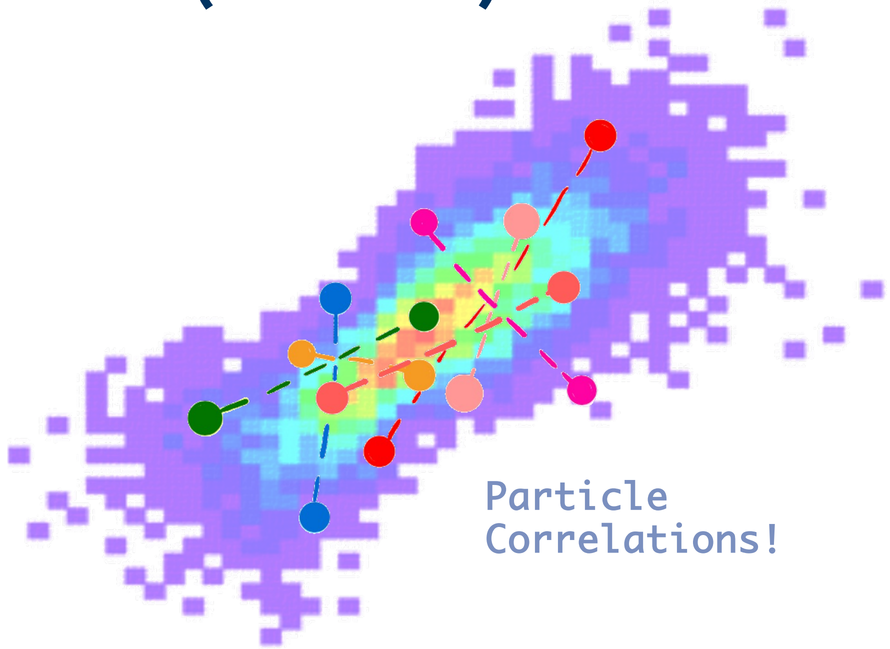

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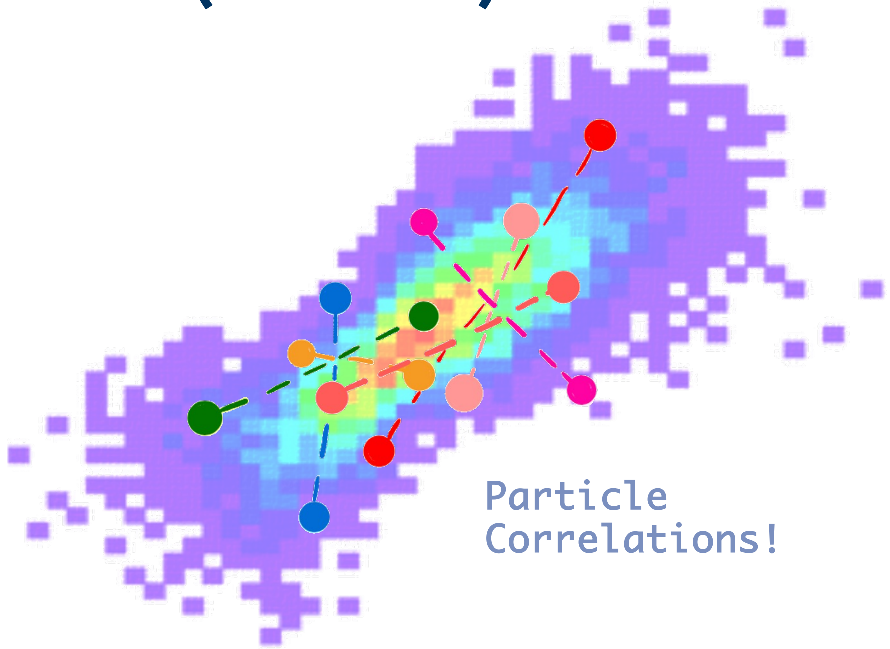

Inspiration:

- Tkachov's work on C-continuous observables
  - ⇢ Energy Correlator (n=1) is analogous to an EFN
  - ⇢ Set n=2 (EEC) and make F a NN ⟶ E2FN

Energy correlators 3.39

These have the form

$$f(\mathbf{P}) = \sum_{a_1 \ldots a_n} E_{a_1} \ldots E_{a_n} f_n(\hat{\mathbf{p}}_{a_1}, \ldots, \hat{\mathbf{p}}_{a_n}), \quad 3.40$$

where f_n is a symmetric continuous function of n arguments. Basic shape observables are special cases corresponding to $n = 1$.

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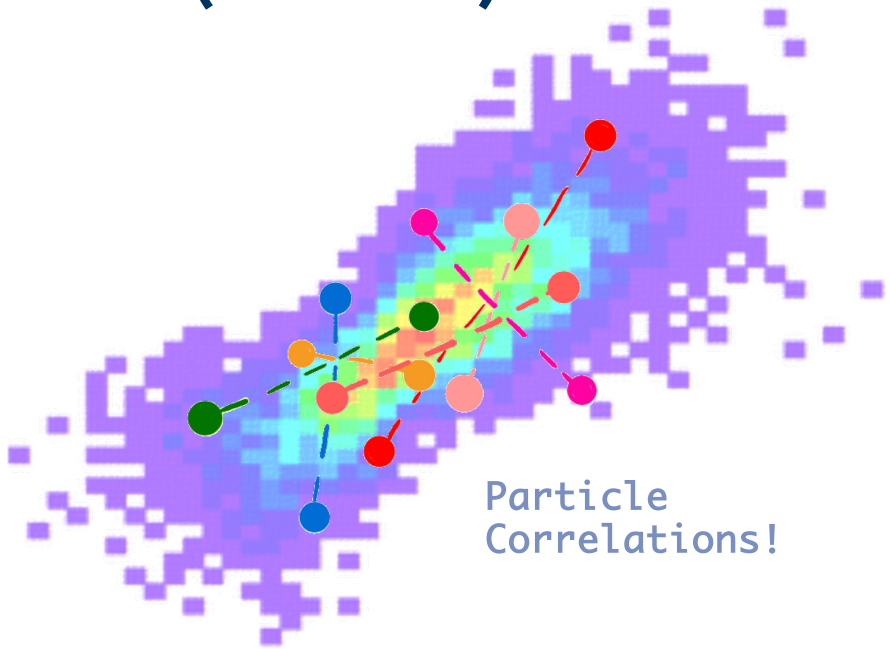

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$$P2FN = F\left(\sum_{ij}^{N} \Phi\left(p_i, p_j\right)\right)$$

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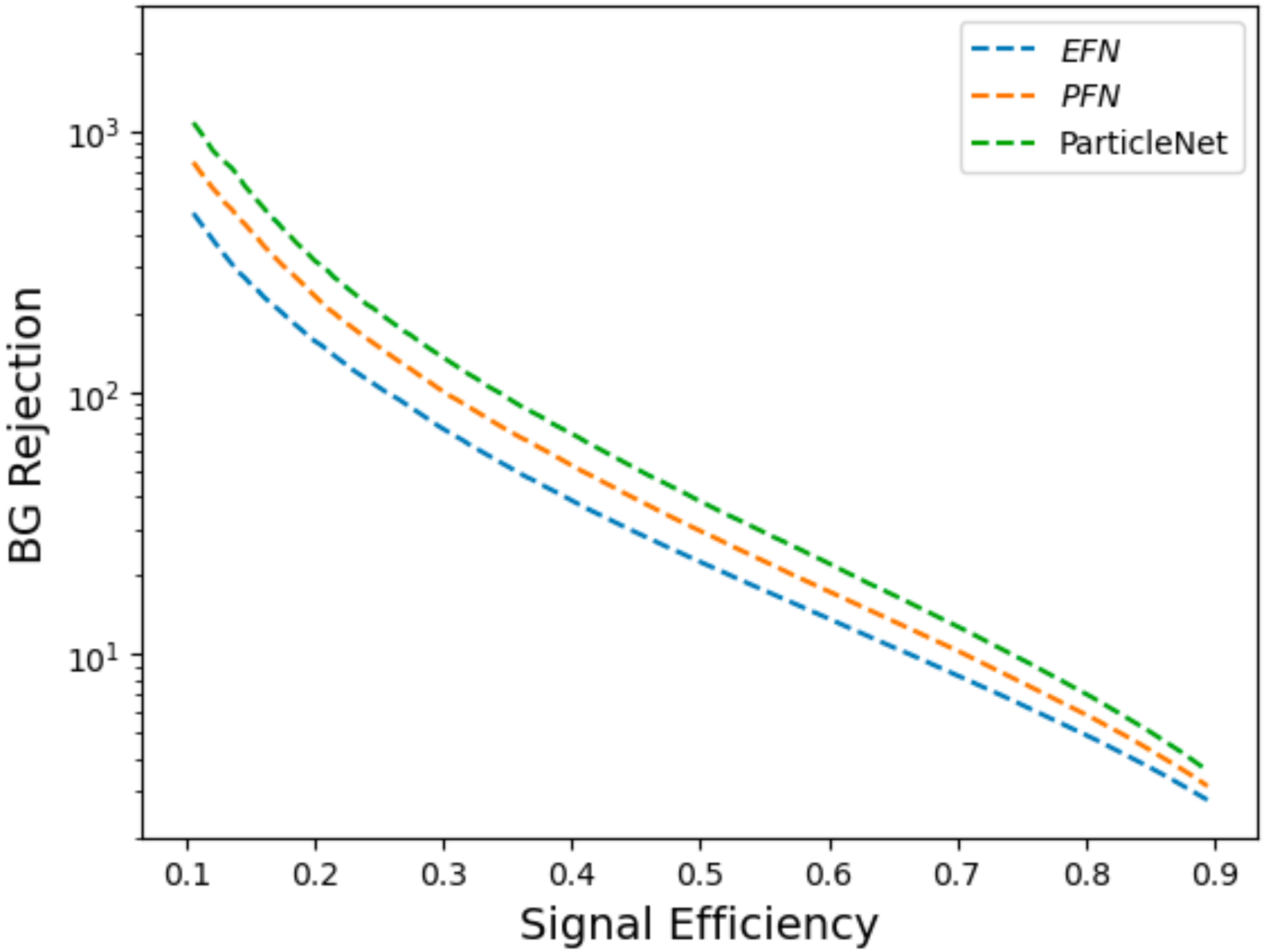

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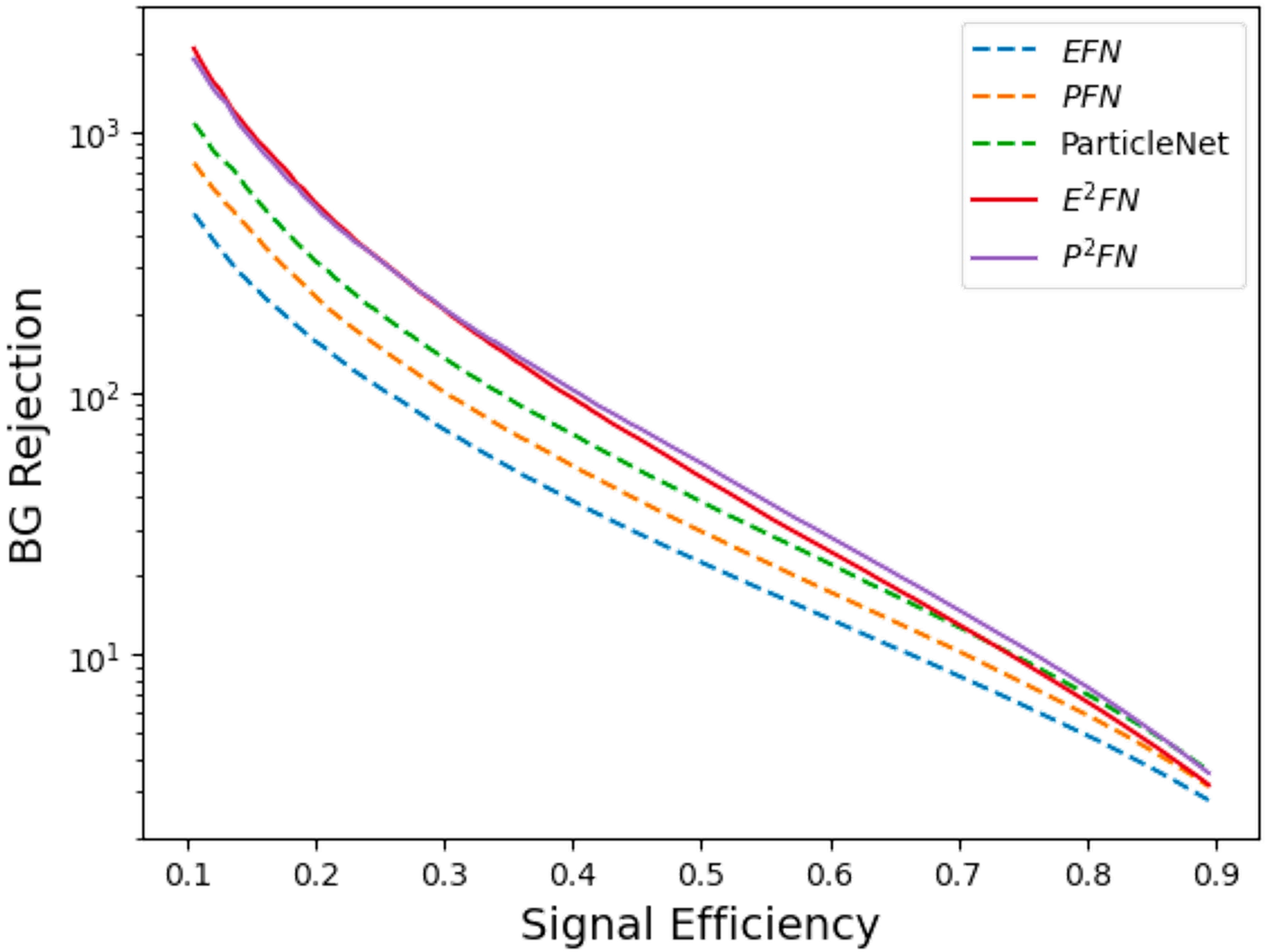
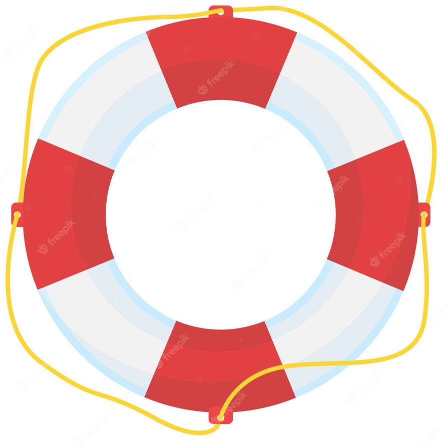


IRC Safety in E2FNs



- IRC safety conditions:
  1. Φ network is continuous
  2. $\Phi \to 0$ as the two studied particles become collinear

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Non-perturbative corrections of an IRC safe object can be arbitrarily large!

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→ Quantitative measurement of robustness of NN to hadronization effects

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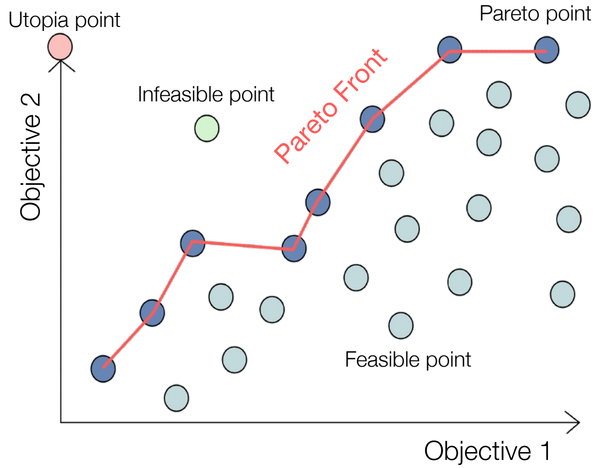
Quantifying perturbative regularization



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Perturbatively Regularized NNs

- Quantify the robustness of a NN to non-perturbative information:
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arXiv:1803.07977v1 [hep-ph] 21 Mar 2018

Les Houches 2017: Physics at TeV Colliders Standard Model Working Group Report

2 Performance versus robustness: Two-prong substructure taggers for the LHC [14]

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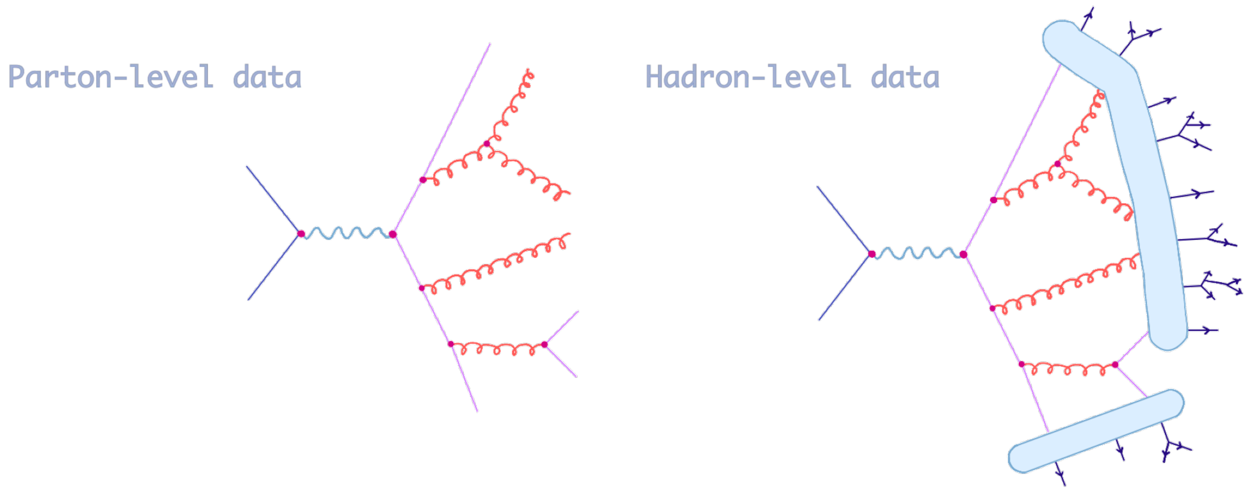
2 Performance versus robustness: Two-prong substructure taggers for the LHC 14

Control the type of information the NNs are sensitive to &
restrict their access to non-perturbative information

Test Data

- PYTHIA data
  - ⇢ Each event generated once
  - ⇢ Data collected twice per event:
    - ⇢ Parton-level
    - ⇢ Hadron-level



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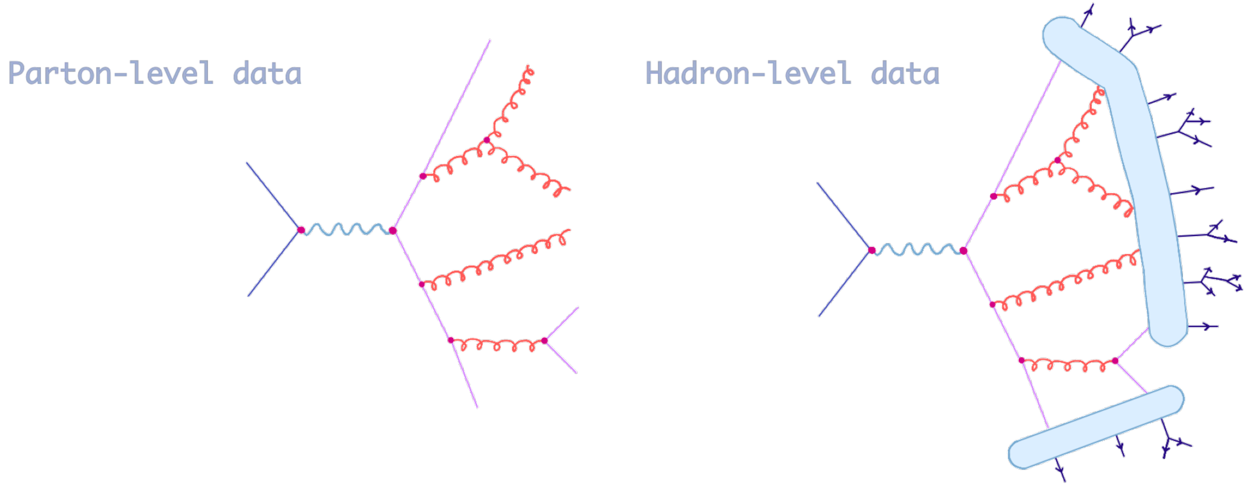
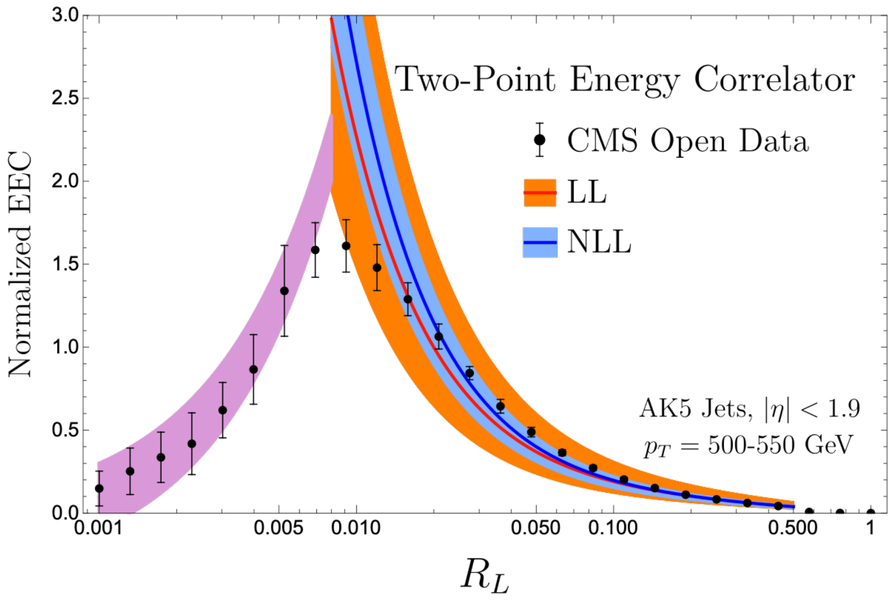


- Data pre-processing:
  - ⇢ Eta, phi coordinates centered to jet's eta, phi
  - ⇢ Constituent's p_T normalized to jet p_T
  - ⇢ Exclusive kt reclustering by momentum splitting

$$k_T = p_{T\,jet} \times \Delta\Theta$$

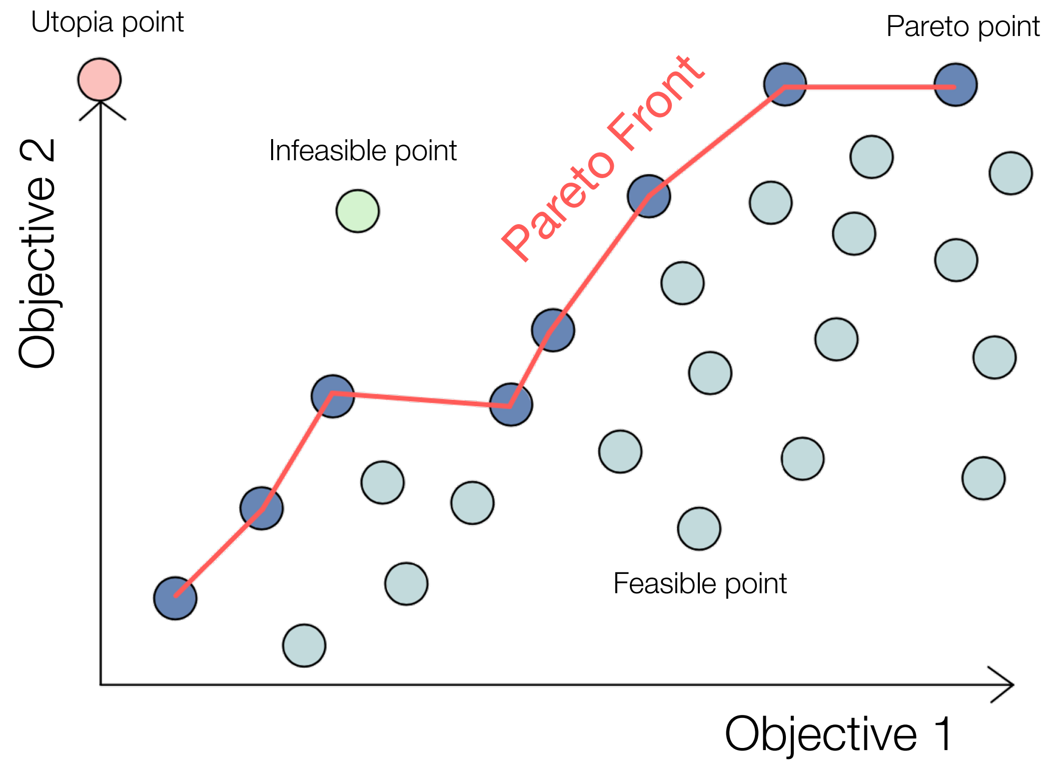


Regularization Metric & Pareto Plots

- Train on hadron-level data, test on parton-level data

- RMS as regularization metric:

$$RMS = \sqrt{\frac{\sum_{i}^{N}\left(P_i(x \mid \textbf{parton-data}) - P_i(x \mid \textbf{hadron-data}) \right)^2}{N}}$$



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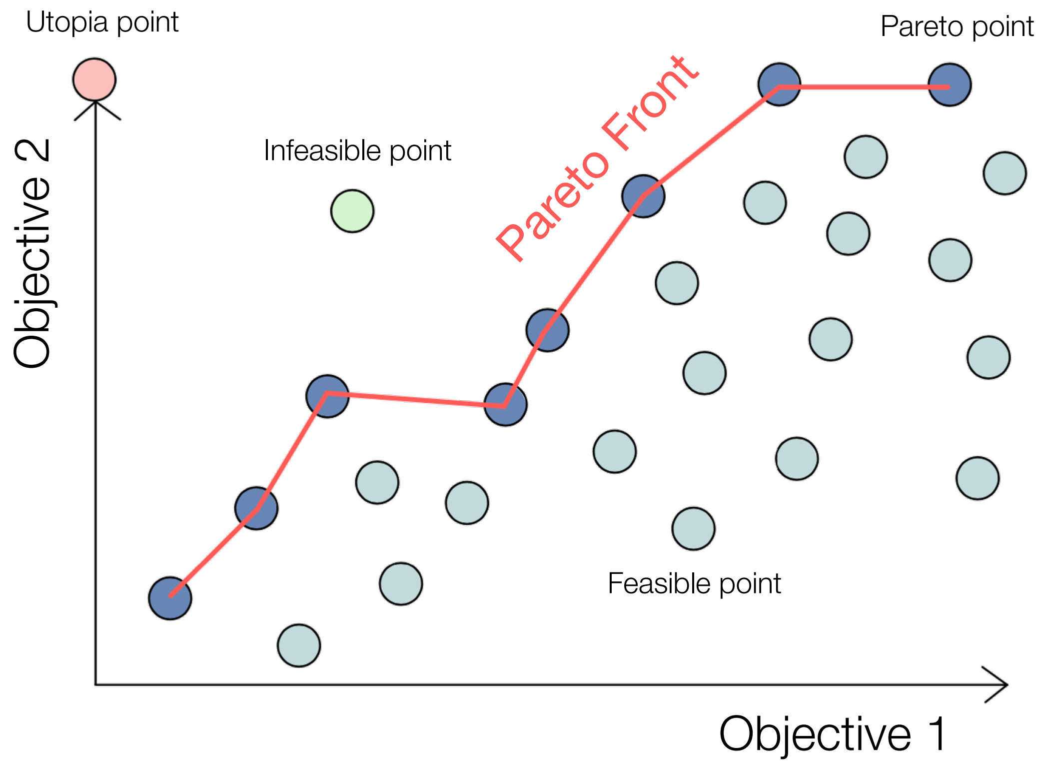
- Objectives:
  - ⇢ Better performance: higher rejection
  - ⇢ Better regularization: lower RMS



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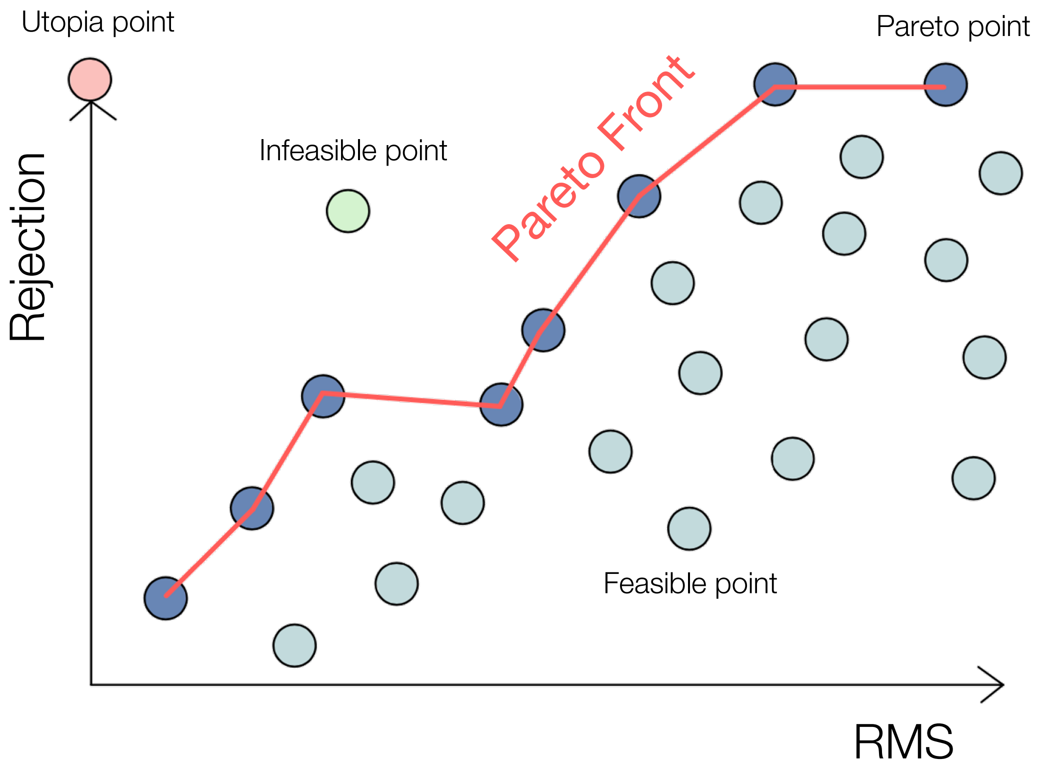
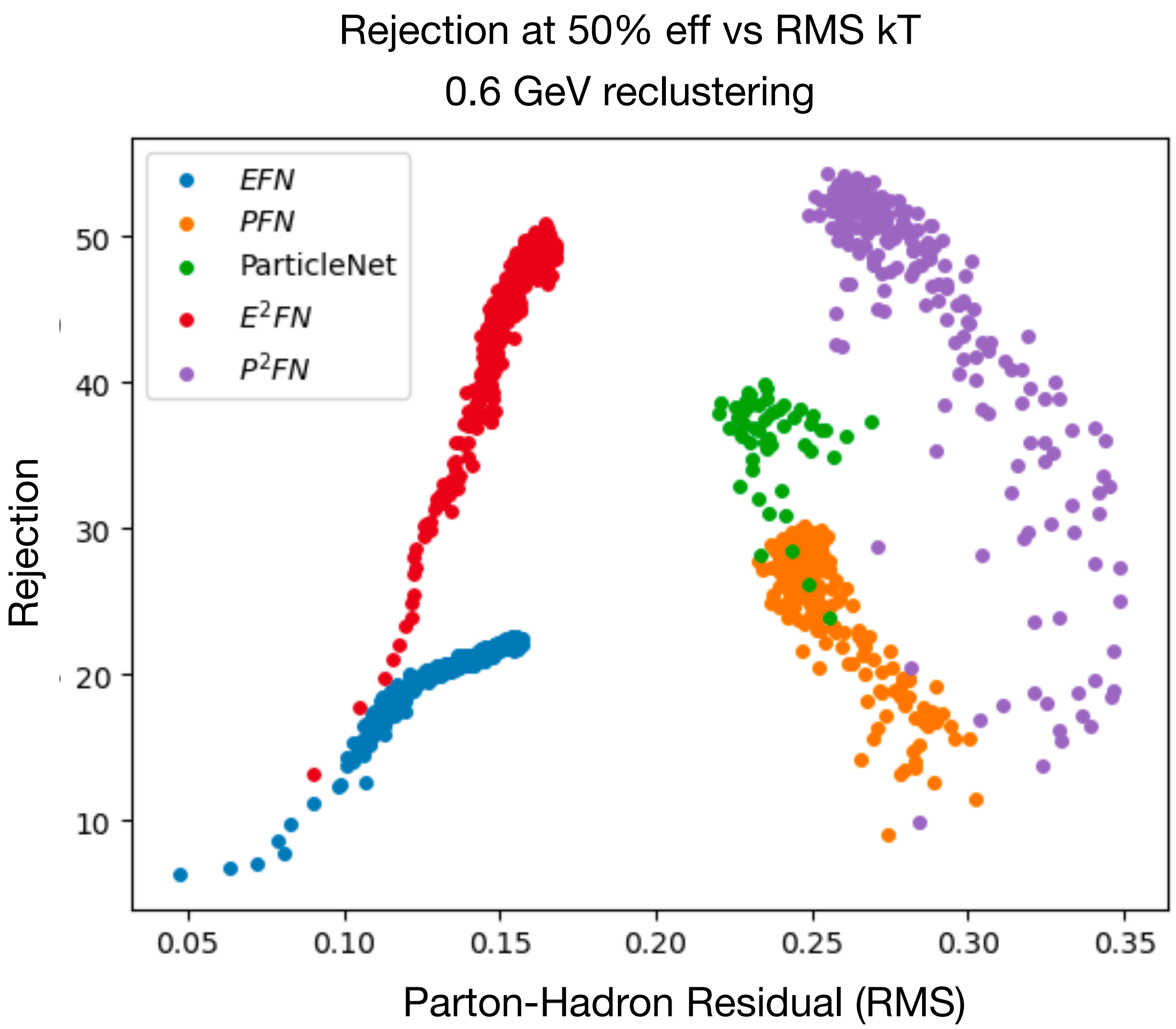
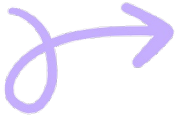


Regularization vs. Performance



- Pareto Front is dominated by EFN, E2FN at low RMS and high rejection
- P2FN contribution at higher RMS and rejection



To understand the effect of non-perturbative uncertainties on searches we want to **relate RMS metric** back **to a physical quantity**

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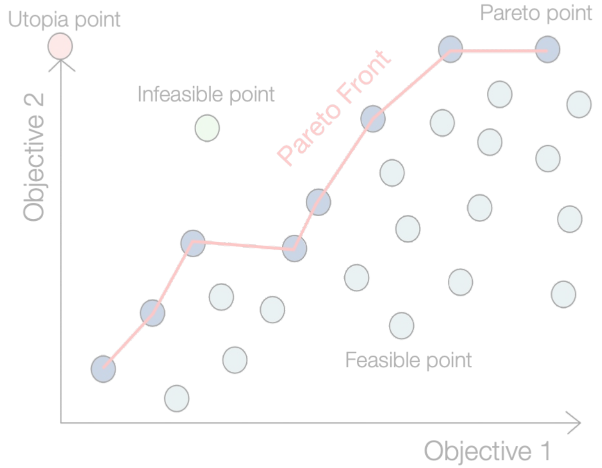

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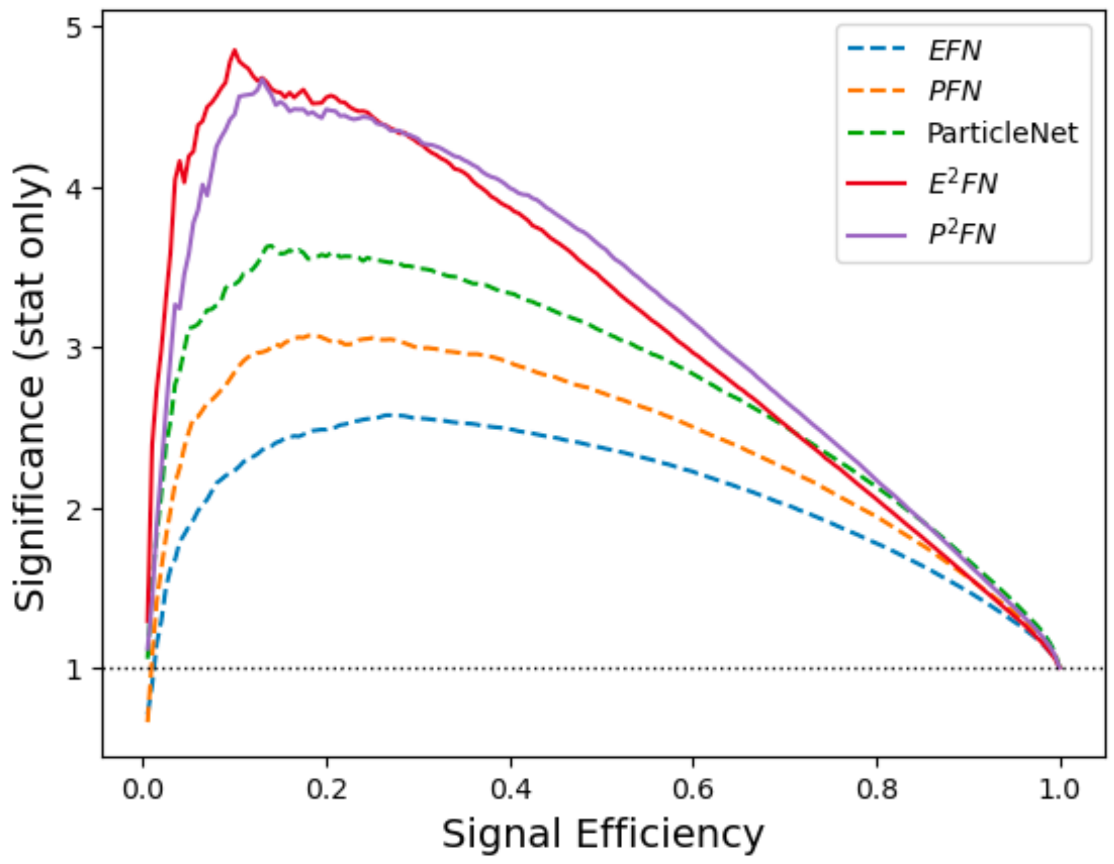
Raw Discovery Significance in Toy Search

- Tagging boosted Z Bosons in QCD background
- NN represents the entire analysis work flow
  - Cut on NN signal score
  - Single bin counting experiment

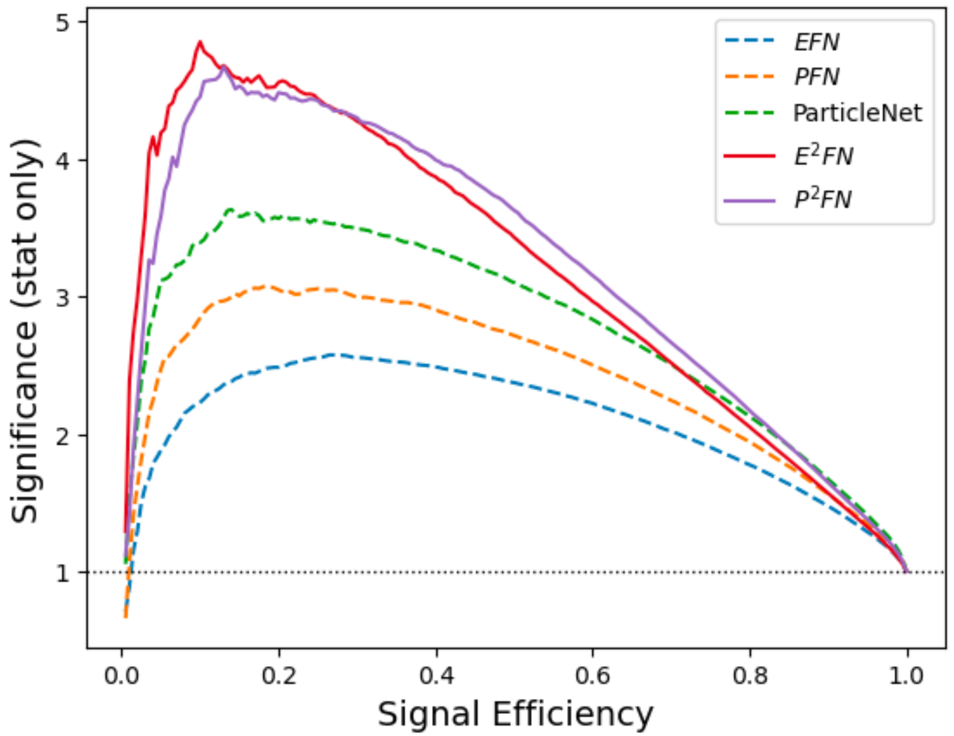
Optimize analysis for best discovery significance



Significance with Theoretical Uncertainties

- Background Uncertainties:

$$\frac{1}{10} * [B(\text{hadron}) - B(\text{parton})]$$



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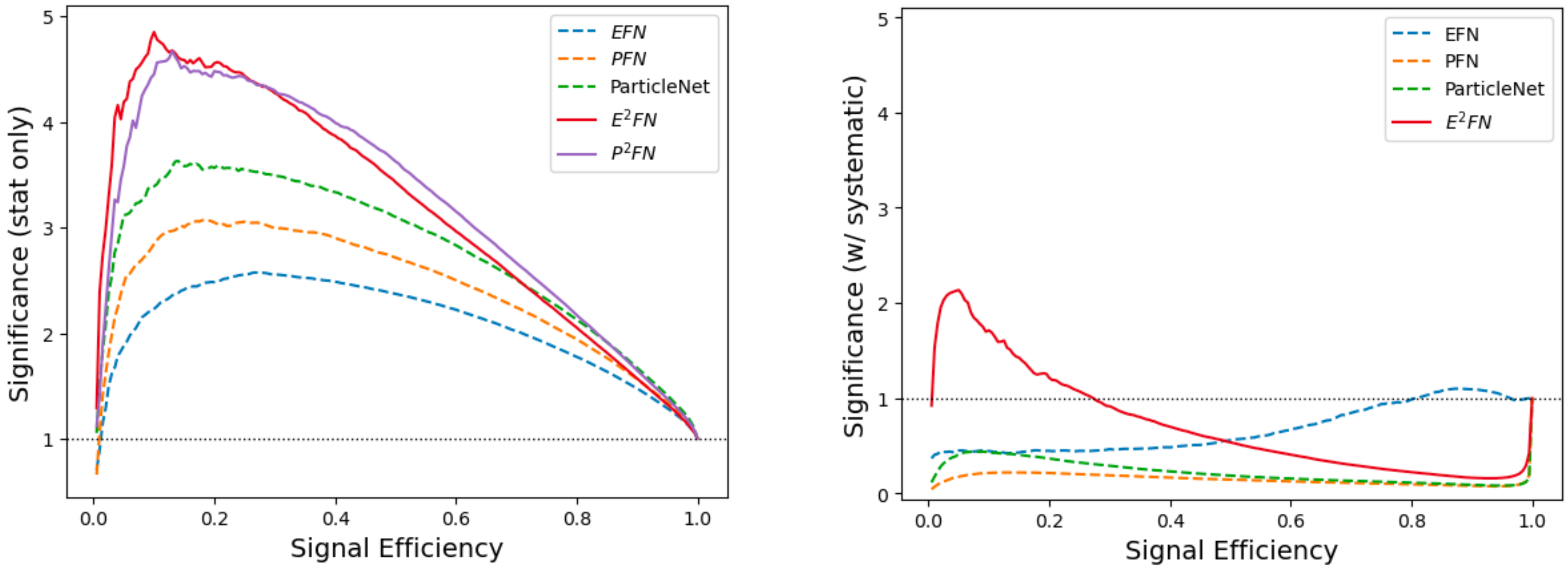
⁂ IRC safety can help protect our measurements from theoretical systematics



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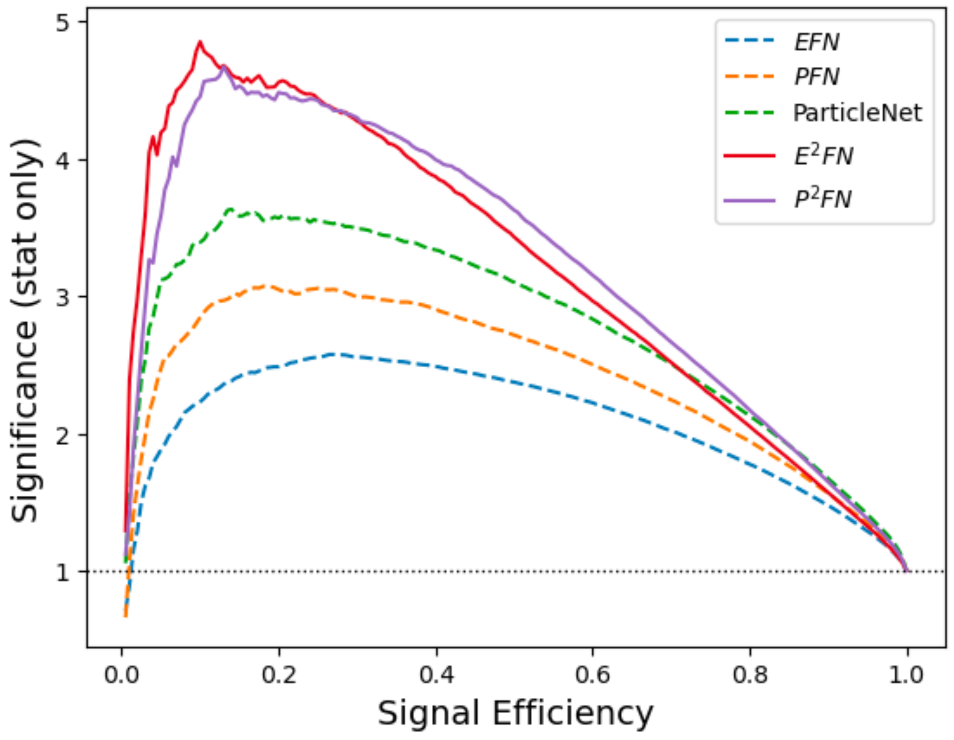
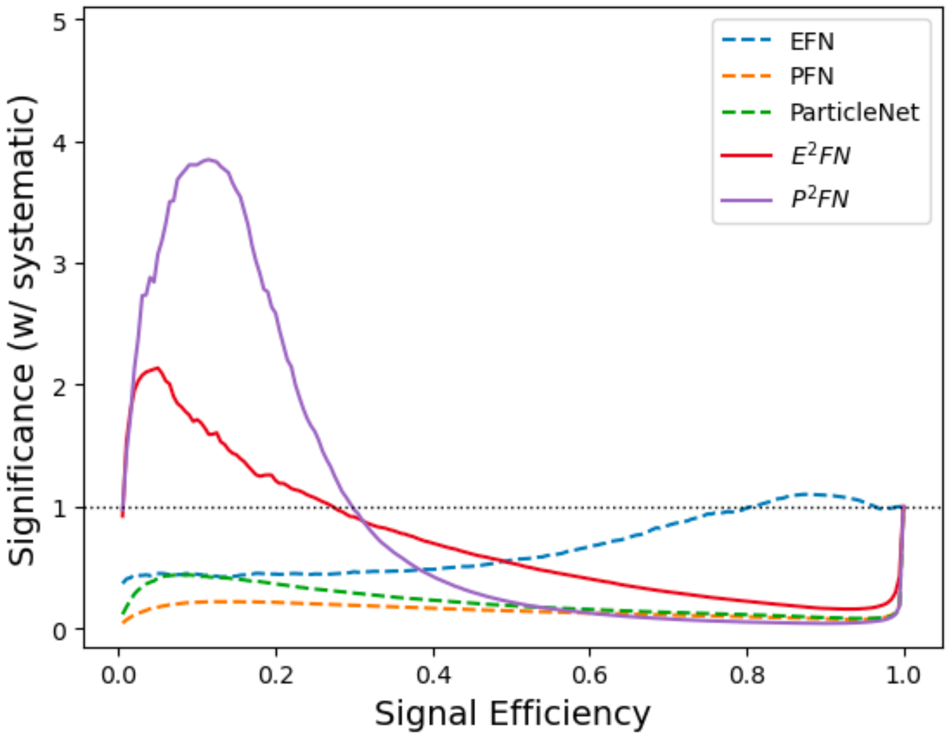
⁕ IRC safety can help protect our measurements from theoretical systematics

⁕ There might be other non-IRC safe NNs that might be well-behaved and outperform IRC safe nets!





Summary

- Extended the notion of EFNs to the more generalized E2FNs which allow for particle correlation and improve the NN performance while ensuring IRC safety
- Understanding the regularization of a NN plays an important role in jet substructure measurements. Especially as we move towards measuring more complicated and high-detail jet observables
- Theoretical uncertainties from non-perturbative processes can have a measurable impact on actual searches and should be further studied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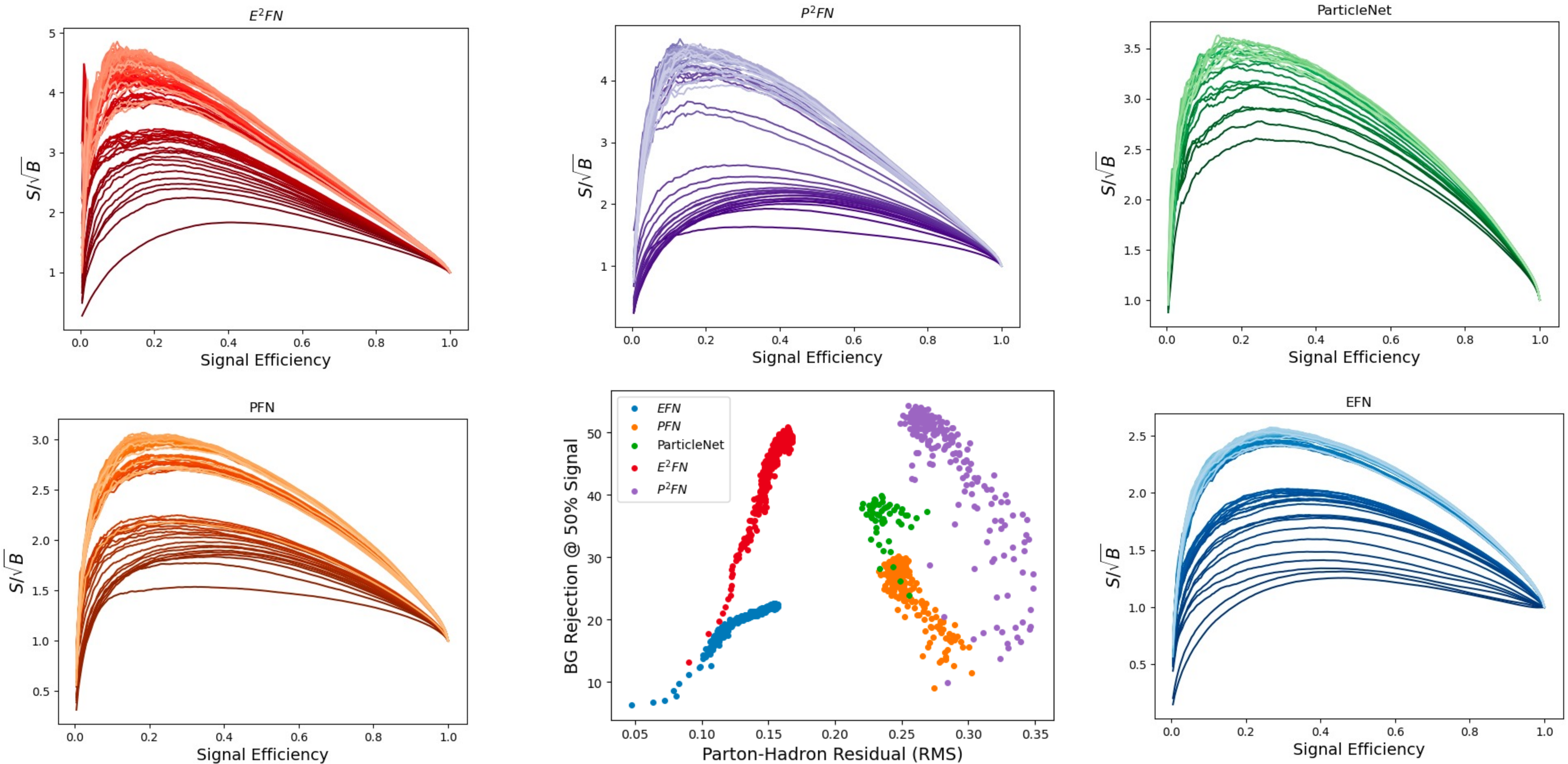
Raw Discovery Significance in Toy Search



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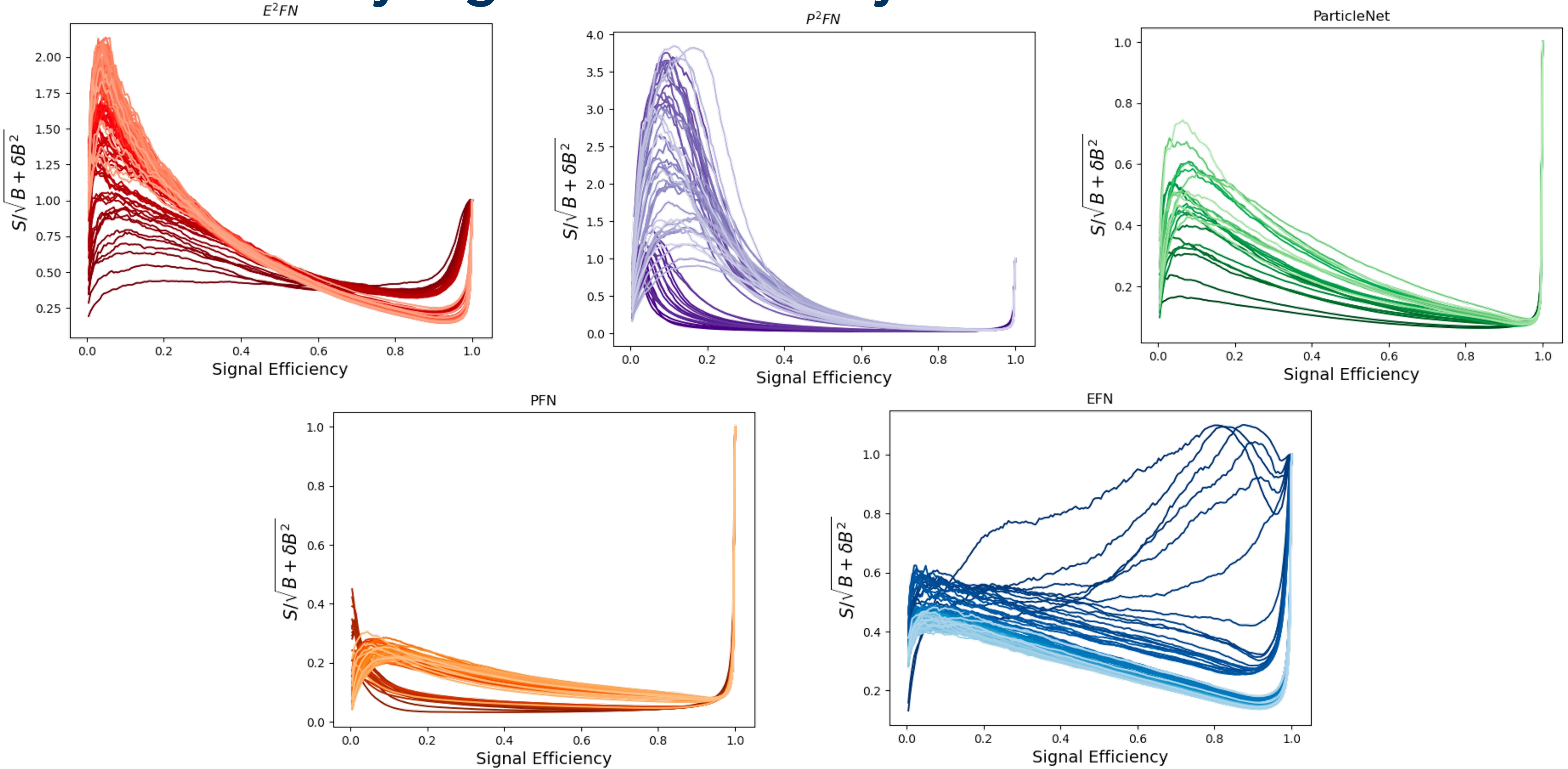

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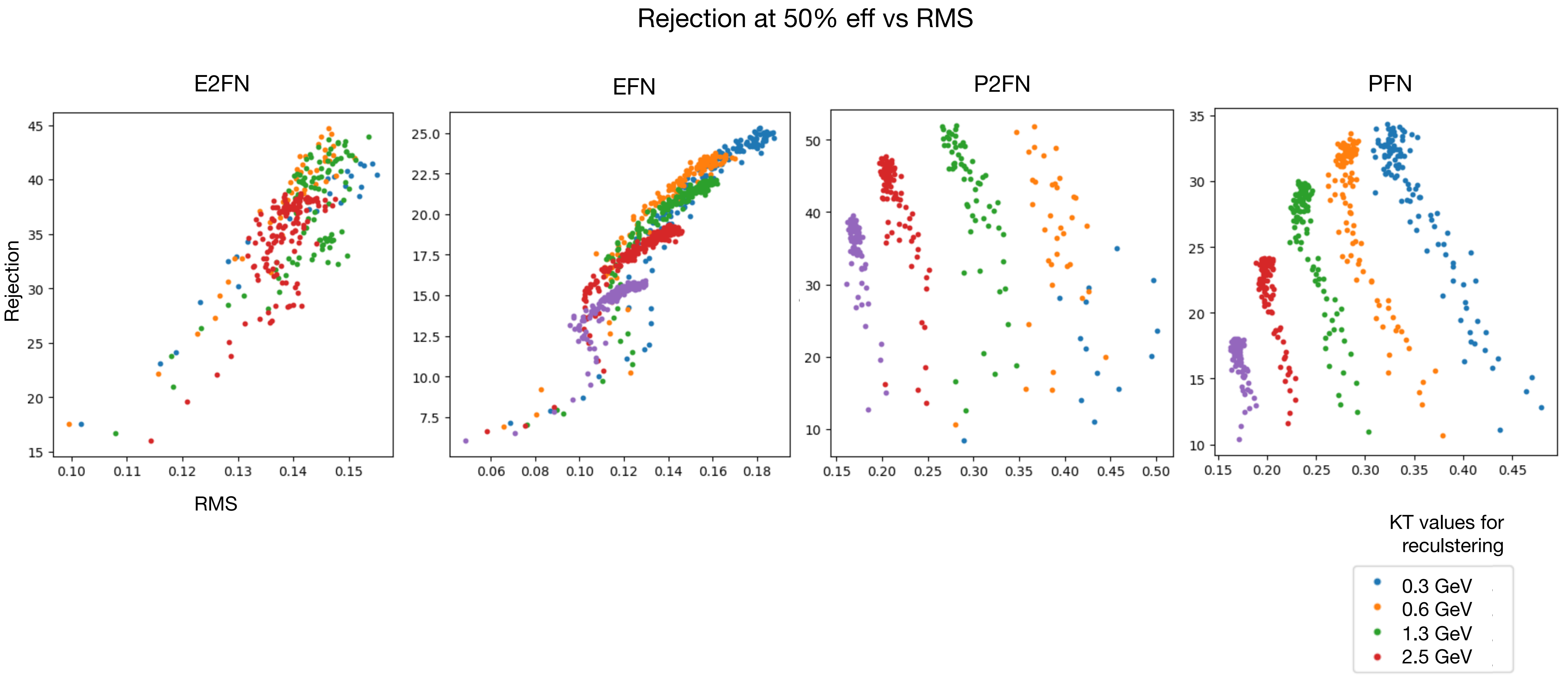
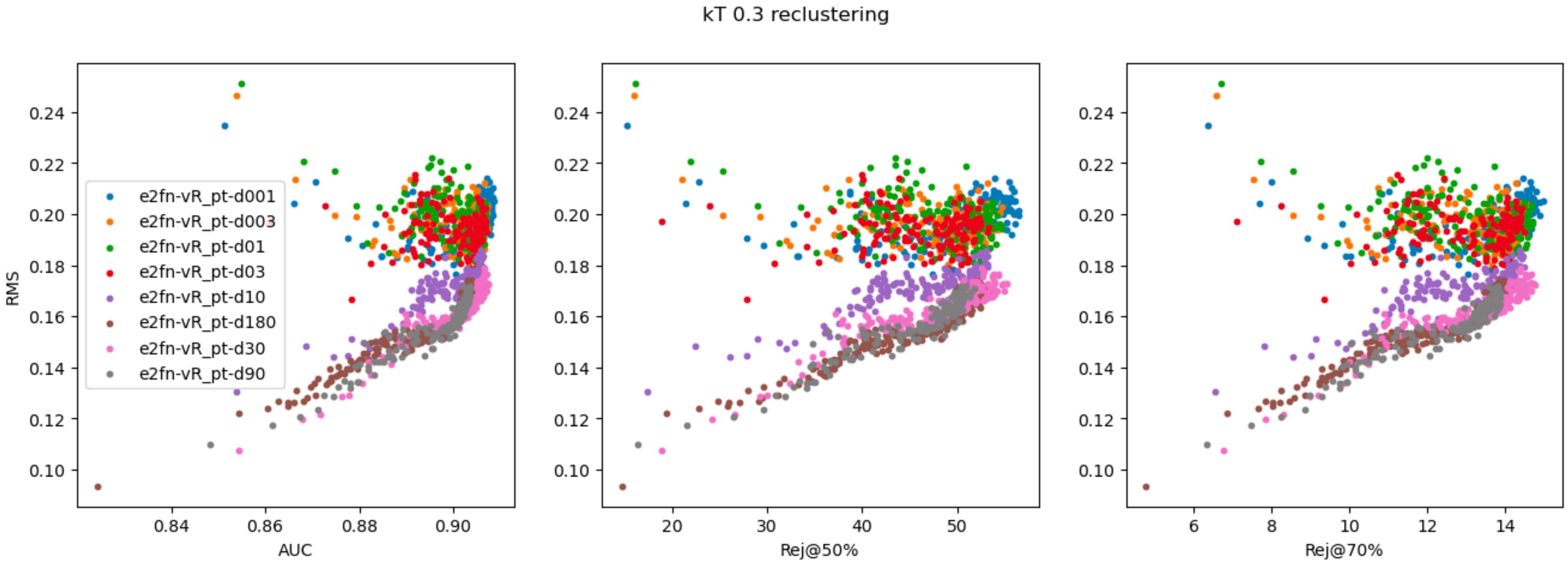


Damping


kT 0.3 reclustering
RMS
AUC
Rej@50%
Rej@70%
e2fn-vR_pt-d001
e2fn-vR_pt-d003
e2fn-vR_pt-d01
e2fn-vR_pt-d03
e2fn-vR_pt-d10
e2fn-vR_pt-d180
e2fn-vR_pt-d30
e2fn-vR_pt-d90
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
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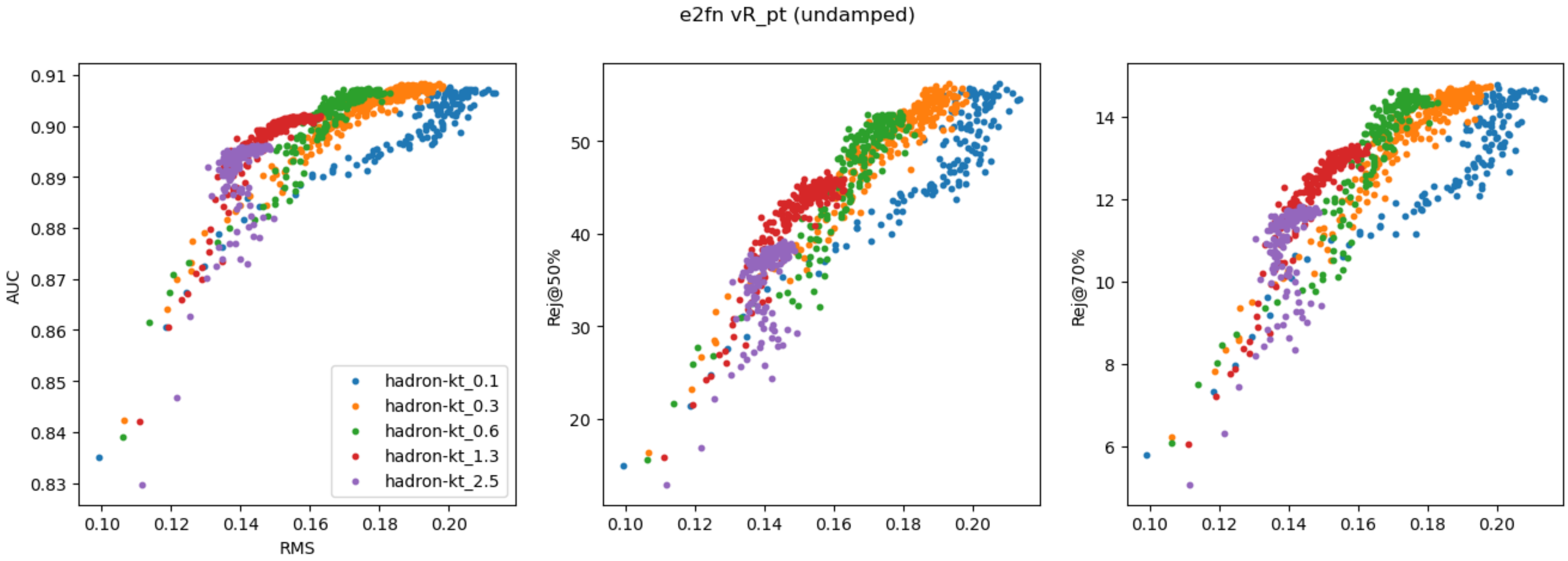
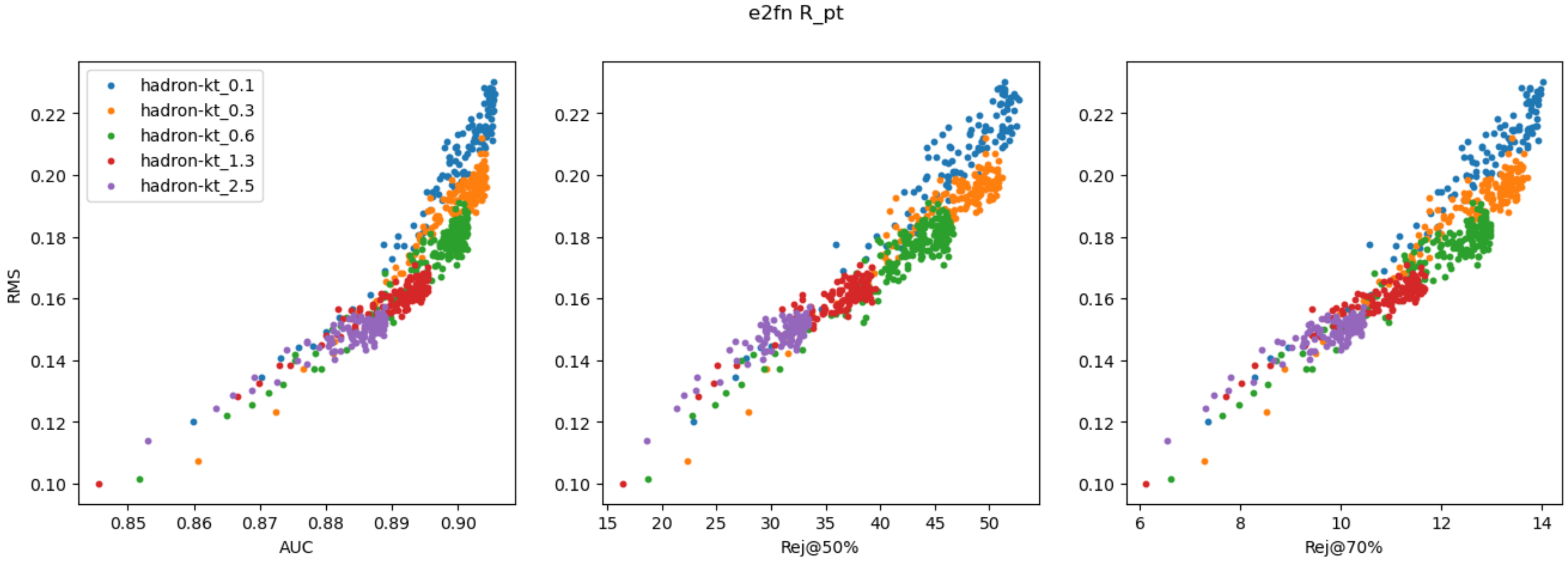
14

Undamped E2FN RMS vs Performance




e2fn R_pt
hadron-kt_0.1
hadron-kt_0.3
hadron-kt_0.6
hadron-kt_1.3
hadron-kt_2.5
RMS
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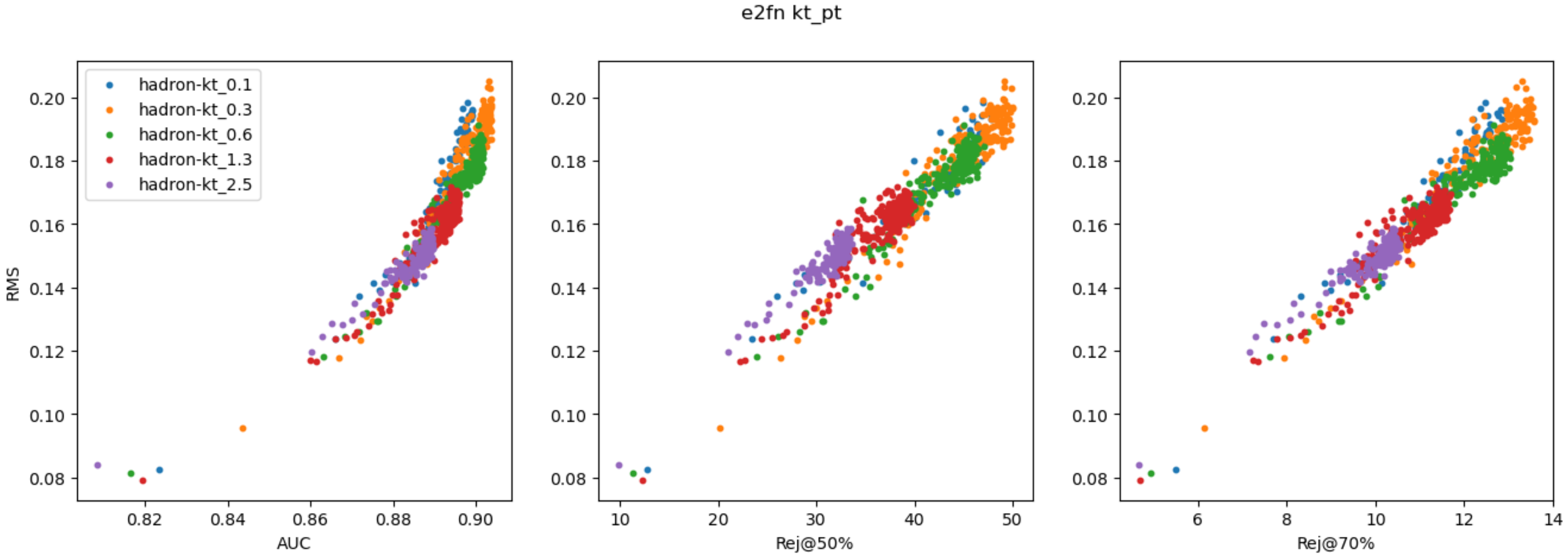
e2fn kt_pt
hadron-kt_0.1
hadron-kt_0.3
hadron-kt_0.6
hadron-kt_1.3
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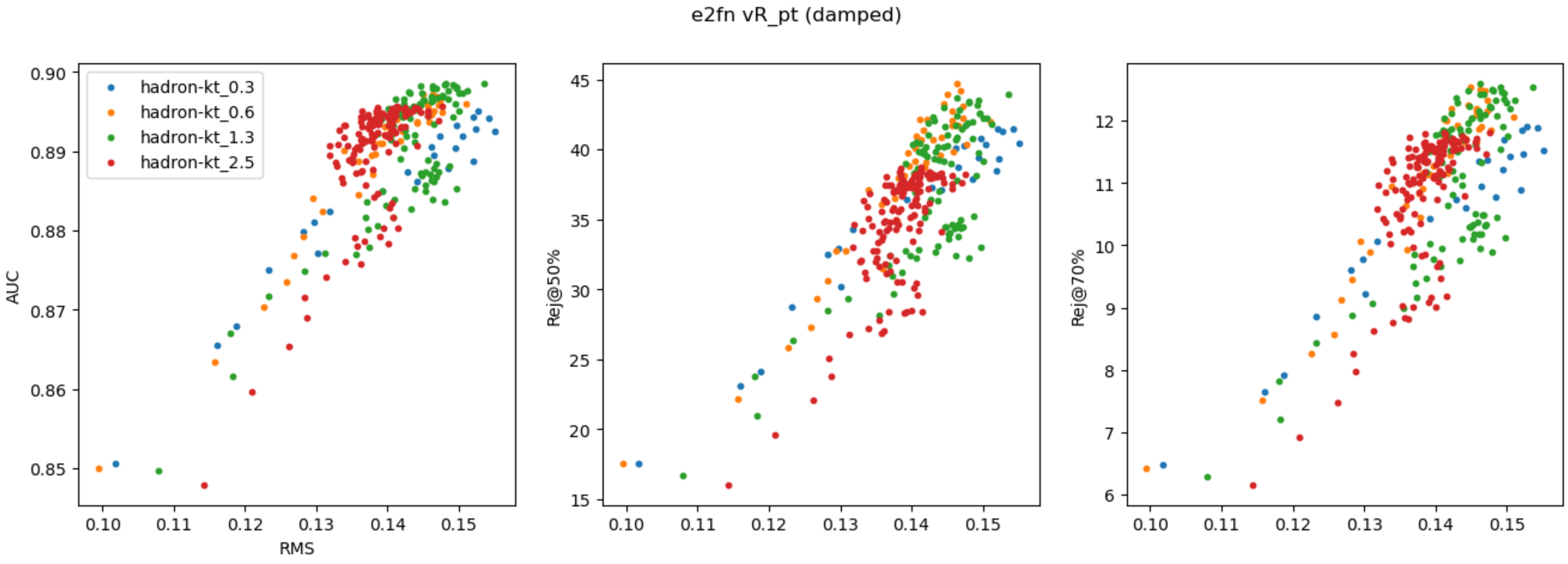
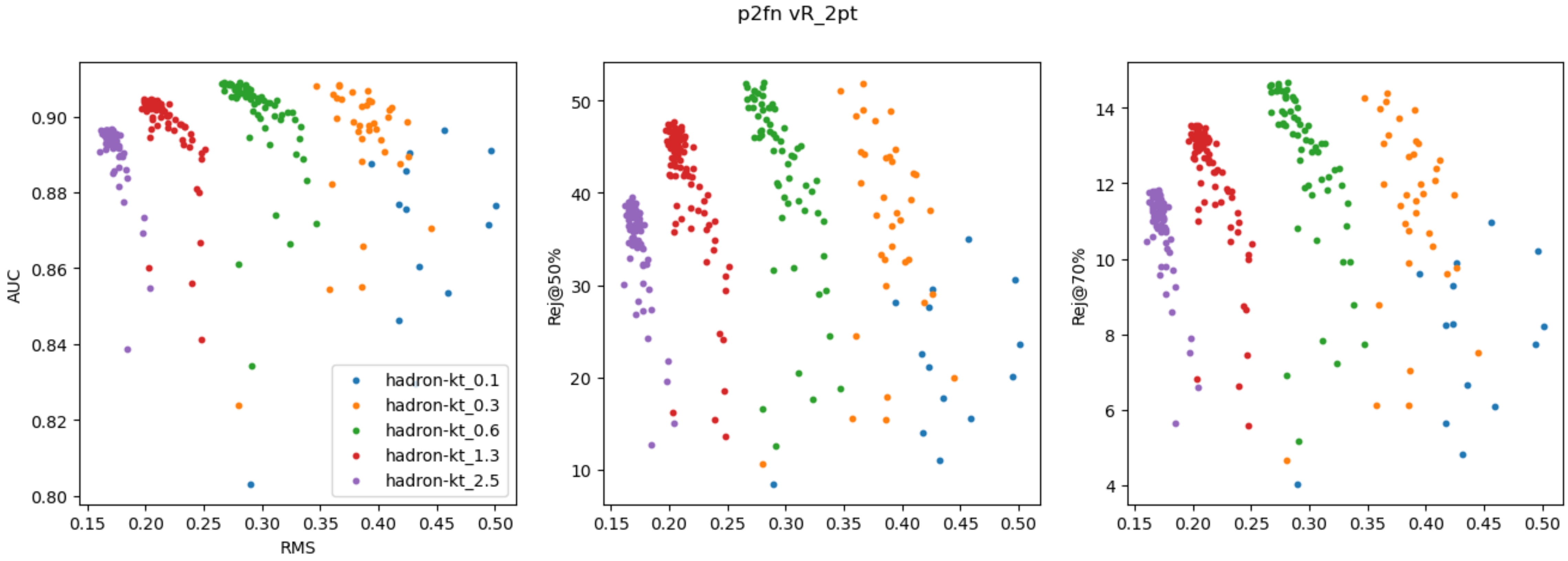
Damped E2FN RMS vs Performance



P2FN RMS vrs Performance



Energy-Energy Flow Networks (E2FNs)

- Energy-Energy Correlators (EEC)
  - ⇢ Transition between perturbative and non-perturbative physics at a visible angular scale between particles

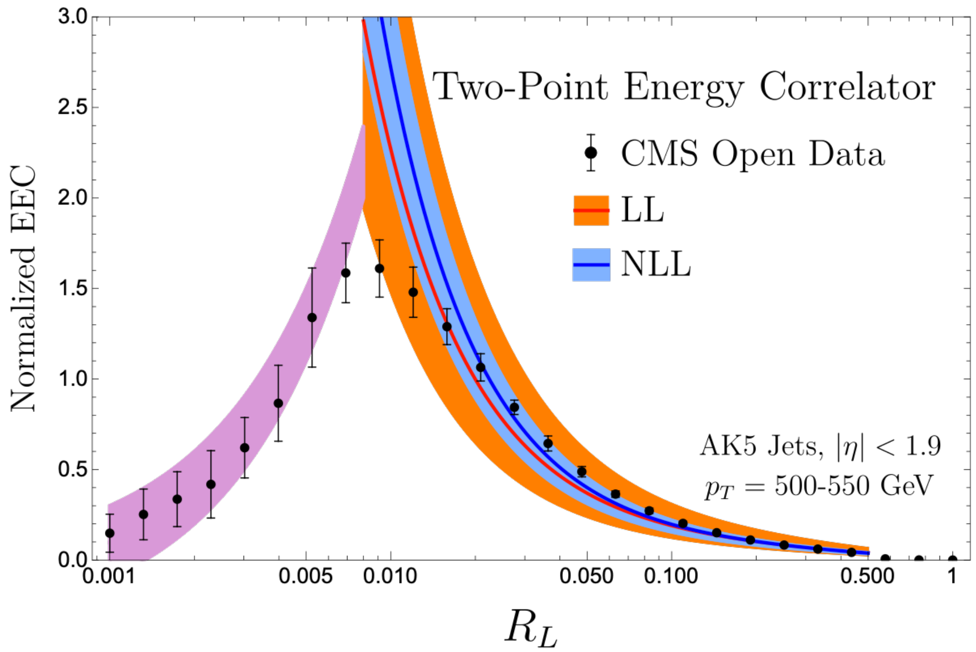


Table 3: Φ inputs for studied E2FN architectures

Input Values	Definition
$\hat{n}_i,\ \hat{n}_j$	$(\eta_i,\ \eta_j,\ \phi_i,\ \phi_j)$
R	$\left(\sqrt{(\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2}\right)$
k_T	$\left(\sqrt{(\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2} * p_{T\,jet}\right)$
Vector R	$(\ (\eta_i-\eta_j),\ (\phi_i-\phi_j)\ )$
$R,\ p_{T\,jet}$	$\left(\sqrt{(\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2},\ p_{T\,jet}\right)$
$k_T,\ p_{T\,jet}$	$\left(\sqrt{(\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2} * p_{T\,jet},\ p_{T\,jet}\right)$
Vector $R,\ p_{T\,jet}$	$(\ (\eta_i-\eta_j),\ (\phi_i-\phi_j),\ p_{T\,jet}\ )$

$$E2FN = F\left(\sum_{i,j}^{N} e_i e_j \Phi\left(\hat{n}_i, \hat{n}_j\right)\right)$$

Hyperparameters & Trainable Parameters

- 2 layers in phi
- 3 layers in F
- Relu activation
- Latent Dim: 32
- Learning rate: 1e-4
- Damping:

$$damp = \frac{(k_T/\tau)^{\frac{1}{w}}}{(k_T/\tau)^{\frac{1}{w}} + 1}$$

Table 6: Number of Trainable Parameters

Architecture	Phi	F	Total
PFN	21.2 K	37.5 K	58.7 K
EFN	21.0 K	37.5 K	58.5 K
E2FN	21.2 K	37.5 K	58.7 K
P2FN	21.3 K	37.5 K	58.8 K
ParticleNet	–	–	366 K