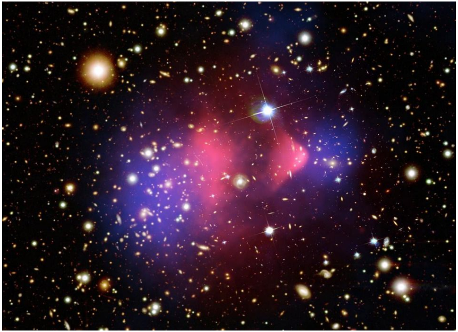
Axion Dark Matter

Phys 290e
Jeff Dror



Outline

- Strong CP problem
  - Qualitative picture
  - Slightly formal treatment
- Axion solution
  - Minimal requirements
  - Building an axion model
  - Axion electrodynamics
- Axion cosmology
  - Inflation/reheating/DM
  - Cosmology of scalar fields
  - Misalignment ($f_a \lessgtr T_{RH}$)
- Direct searches
  - Summary of constraints
- Dark matter searches
  - Summary of experiments

C? P? T?

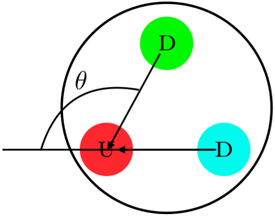
history lesson

cooling/LSW/
CAST/...
ADMX/CASPEr/
ABRACABRA/...

Strong CP

Neutron dipole moment

- **Puzzle:** electric dipole moment (EDM) of a neutron
- neutron = 1 up quark, 2 down quarks
- Recall: EDM is $\mathbf{d}_n = \sum q_i \mathbf{r}_i$



[Hook - 2018]

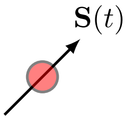
- Taking $r_n \sim \mathrm{fm}$...

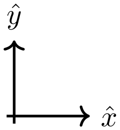
$$|\mathbf{d}_n| \simeq 10^{-13}\sqrt{1-\cos\theta}\, e\,\mathrm{cm}$$

- $\mathbf{d}_n$ is a vector: aligned with neutron spin

Neutron dipole moment

- To measure EDM
- Neutron with electric and magnetic fields:





Neutron dipole moment

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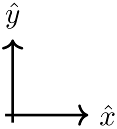
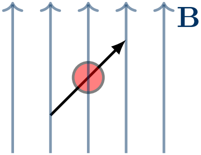
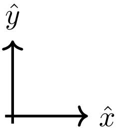


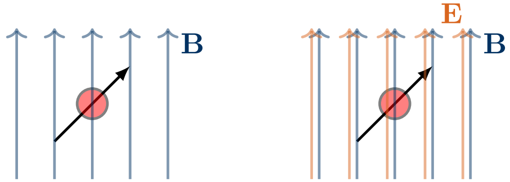


$$S_0\Big[\cos(2\pi\mu Bt)\hat{x} + \sin(2\pi\mu Bt)\hat{y}\Big]$$

Neutron dipole moment

- To measure EDM
- Neutron with electric and magnetic fields:



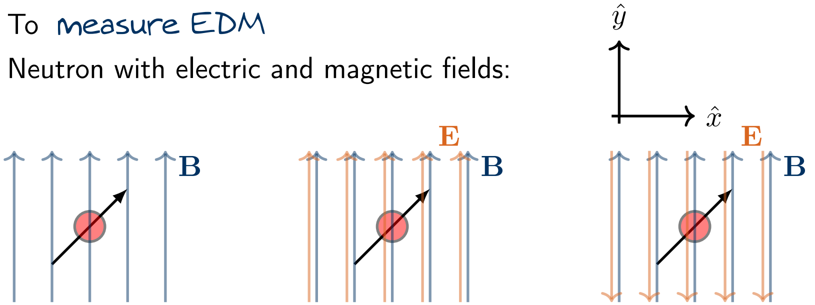


$$S_0\Big[\cos(2\pi\mu Bt)\hat{x} + \sin(2\pi\mu Bt)\hat{y}\Big]$$

$$S_0\Big[\cos(2\pi(\mu B + d_n E)t)\hat{x} + \sin(2\pi(\mu B + d_n E)t)\hat{y}\Big]$$

Neutron dipole moment

- To measure EDM
- Neutron with electric and magnetic fields:



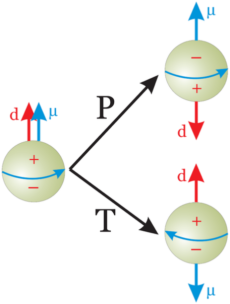
$$S_0\Big[\cos(2\pi\mu B t)\hat{x} + \sin(2\pi\mu B t)\hat{y}\Big]$$

$$S_0\Big[\cos(2\pi(\mu B + d_n E)t)\hat{x} + \sin(2\pi(\mu B + d_n E)t)\hat{y}\Big]$$

$$S_0\Big[\cos(2\pi(\mu B - d_n E)t)\hat{x} + \sin(2\pi(\mu B - d_n E)t)\hat{y}\Big]$$

- Take difference of oscillation frequencies
- Measured value: $|\mathbf{d}_n| \lesssim 6 \times 10^{-26} e \cdot \text{cm}$

EDM violates CP!



[- Wiki]

- Parity (P): flips space

$$\mathbf{x} \rightarrow -\mathbf{x}$$

- Time-reversal (T): flips time

$$t \rightarrow -t$$

- Charge conjugation (C): flips particle

particle → anti-particle

- Both $\mathbf{d}$ and $\boldsymbol{\mu}$ ⇒

$\cancel{P}$ and $\cancel{T}$

- CPT:good symmetry of Nature
  - QFT aficionado: “Due to spin-statistics theorem”
- neutron EDM ⇒ CP-violation

Strong CP

- If neutron had electric dipole moment it violates CP
- CP violation comes from imaginary components in Lagrangian
- Where can we find CP-violation at microscopic level?
- QCD with massless quarks:

$$\mathcal{L} \supset \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}(i\partial_\mu + eQ_q A_\mu + g_s G^A_\mu T^A)\gamma^\mu q$$

- $i\partial_\mu \rightarrow k$ (real)
- No CP violation so far
- Two sources of CP violation in QCD

"Theta term"

Mass terms

CP violation in gauge fields

- Why not add a term:

$$S \supset \int d^4x \frac{\theta}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} G_{\mu\nu} G_{\alpha\beta}$$

- Naive answer: its a total derivative
- Integrating by parts

$$S \supset \int d^4x \frac{\theta}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} \partial_\alpha \left(G^a_\beta \partial_\mu G^a_\nu + \frac{1}{3} f_{abc} G^a_\beta G^b_\mu G^c_\nu \right)$$

- Does not effect equations of motion
- QCD is weird: total derivatives can still contribute
- QFT aficionados: "instantons give non-trivial vacuum structure"

CP violation in quark masses

- If we add particle mass terms:

$$\mathcal{L} \supset m_{ij}\bar{q}_i q_j + \text{h.c.}$$

- Hermitian Lagrangian ⇒ $m_{ij} = m_{ji}^*$
- Imaginary component for m_{ij} ⇒ CP violation
- Subtlety: masses are real quantities
- More intuitive to rotate quarks to eliminate phase:

$$q \to e^{i\gamma_5 \alpha} q \qquad \text{(field redefinition)}$$

- Rotation has unintended consequences: results in additional term
- QFT aficionados: "Measure is not invariant under chiral rotation"

CP violation in both

- So what are those unintended consequences?
- Under a chiral field redefinition:

$$S \to S + \int d^4x \alpha \frac{g_s^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} G_{\mu\nu} G_{\alpha\beta}$$

- QFT aficionado: “chiral symmetry is anomalous”
- Total CP violation in Lagrangian can summarized with:

$$S \supset \int d^4x \frac{\bar{\theta}}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} G_{\mu\nu} G_{\alpha\beta}$$

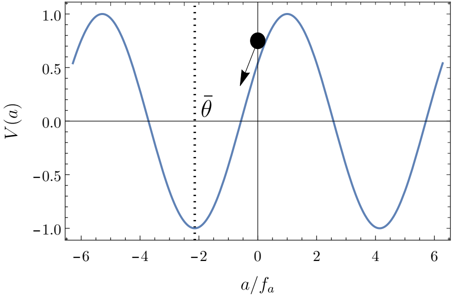
- $\bar{\theta}$ is sum of theta term and the mass term

Solution to strong CP (1/2)

- If new field, a, with an interaction to the offending operator:

$$\mathcal{L} \supset \frac{1}{16\pi^2}\left(\bar{\theta} + \frac{a}{f_a}\right)\epsilon^{\alpha\beta\mu\nu}G_{\mu\nu}G_{\alpha\beta}$$

- Think of a as a classical field with potential, $V(a)$
- If $V(a)$ is such that $a \to -\theta f_a$ then this term vanishes today



Solution to strong CP (2/2)

- Weird part: why potential depend on $\bar{\theta}$?
- Answer: QCD can create a $\bar{\theta}$-dependent potential
- Requires a to be Goldstone boson of anomalous symmetry "PQ"
- Lots of hard work...

$$V(a) \simeq m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{f_a} + \bar{\theta}\right)}$$

- Notable features
  - Minimizing potential ⇒ $\bar{\theta}$ angle vanishes today (and $|\mathbf{d}_n| \simeq 0$)
  - Expanding a (after shifting to true vacuum):

$$V(a) \simeq \frac{1}{2} \frac{m_\pi^2 f_\pi^2}{f_a^2} a^2 + \ldots \quad \Rightarrow \quad m_a^2 f_a^2 \simeq m_\pi^2 f_\pi^2$$

  - Higher order terms are small

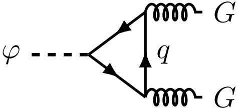
Axions

Axions - full theory (1/3)

- What is this field a?
- Goldstone boson of "PQ symmetry": anomalous under QCD
- Introduce complex field $\varphi = \phi + ia$

$$V(\varphi) = \frac{\lambda}{4}\left(|\varphi|^2 - f_a^2\right)^2$$

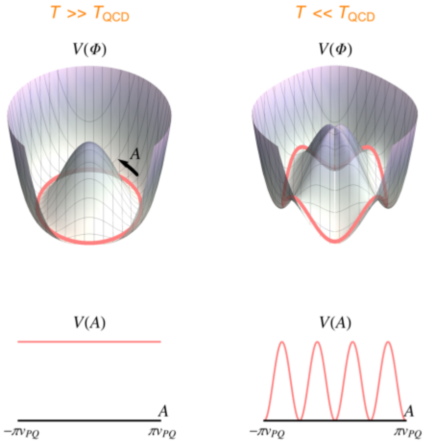
- Spontaneously breaks: $U(1)_{\rm PQ} \to \varnothing$
- φ coupling to quarks leads to interactions with gluons



- After PQ breaking axion (a) gets a coupling to $G\tilde{G}$
- The axion is a Goldstone boson ⇒ no potential

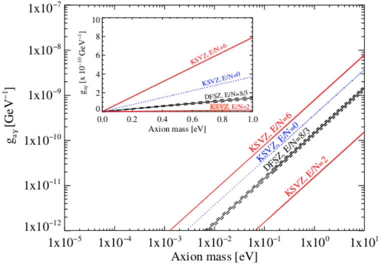
Axions - full theory (2/3)

- Visually:



Axions - full theory (3/3)

- Complete axion models?
- Classic: "electroweak axion"
  - Connects axion to electroweak scale
  - Just need to an electroweak doublet
  - Ruled out long ago
- Two classes of "invisible axion" models:
  - KSVZ: New scalar doublet + heavy quarks
  - DFSZ: New two scalar doublet model
- Useful benchmarks:



Photon coupling (1/2)

- Solving strong CP determines
    - Gluon coupling
    - Mass:

$$m_a^2 f_a^2 \sim m_\pi^2 f_\pi^2$$

- Additional photon interaction:

$$\mathcal{L} \supset \frac{C_{a\gamma\gamma}}{4} \frac{a}{f_a} \epsilon^{\alpha\beta\mu\nu} F_{\mu\nu} F_{\alpha\beta}$$

- Typically $C_{a\gamma\gamma} \sim \alpha/2\pi$ (but not fully determined)

$$C_{a\gamma\gamma} = \frac{\alpha}{2\pi}\left(\frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z}\right) \qquad z = m_u/m_d$$

- QFT aficionado: "E and N are anomaly coefficients"

Photon coupling (2/2)

- What does $C_{a\gamma\gamma}$ do?
- Can rewrite this term ($E_i = F_{0i}$, $B_i = -\frac{1}{2}\epsilon_{ijk}F_{jk}$)

$$\mathcal{L} \supset \frac{C_{a\gamma\gamma}}{4}\frac{a}{f_a}\left(\epsilon^{0ijk}F_{0i}F_{jk} + \epsilon^{ijk\ell}F_{ij}F_{k\ell}\right)$$

Photon coupling (2/2)

- What does $C_{a\gamma\gamma}$ do?
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$$\begin{aligned}\mathcal{L} &\supset \frac{C_{a\gamma\gamma}}{4}\frac{a}{f_a}\left(\epsilon^{0ijk}F_{0i}F_{jk} + \epsilon^{ijk\ell}F_{ij}F_{k\ell}\right) \\ &\supset \frac{C_{a\gamma\gamma}}{4}\frac{a}{f_a}\left(2E_iB_i + \cancel{\epsilon^{ijk\ell}F_{ij}F_{k\ell}}\right)\end{aligned}$$

Photon coupling (2/2)

- What does $C_{a\gamma\gamma}$ do?
- Can rewrite this term ($E_i = F_{0i},\ B_i = -\frac{1}{2}\epsilon_{ijk}F_{jk}$)

$$\begin{aligned}
\mathcal{L} &\supset \frac{C_{a\gamma\gamma}}{4}\frac{a}{f_a}\left(\epsilon^{0ijk}F_{0i}F_{jk} + \epsilon^{ijk\ell}F_{ij}F_{k\ell}\right) \\
&\supset \frac{C_{a\gamma\gamma}}{4}\frac{a}{f_a}\left(2E_iB_i + \cancel{\epsilon^{ijk\ell}F_{ij}F_{k\ell}}\right) \\
&\Rightarrow \frac{C_{a\gamma\gamma}}{f_a}a\mathbf{E}\cdot\mathbf{B}
\end{aligned}$$

Ordinary electrodynamics

- Maxwell's equations: $\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J^{\mu}A_{\mu}$
- Find Euler-Lagrange equations of motion:

$$\frac{\partial \mathcal{L}}{\partial A_{\nu}} = \partial_{\mu}\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}A_{\nu})}$$

- Using $E_i = F_{0i}$, $B_i = -\frac{1}{2}\epsilon_{ijk}F_{jk}$ (and $\epsilon^{\alpha\beta\mu\nu}\partial_{\mu}F_{\alpha\beta} = 0$)

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho \\ \nabla \times \mathbf{E} &= -\dot{\mathbf{B}} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{B} &= \dot{\mathbf{E}} + \mathbf{j} \end{aligned}$$

Axion electrodynamics

- Lagrangian w/ axion: $\mathcal{L} \supset -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu + \kappa a \mathbf{E} \cdot \mathbf{B}$
- Equations of motion:

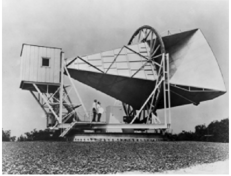
$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho - \kappa (\nabla a) \cdot \mathbf{B} \\ \nabla \times \mathbf{E} &= -\dot{\mathbf{B}} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{B} &= \dot{\mathbf{E}} + \mathbf{j} + \underbrace{\kappa (\dot{a} \mathbf{B} + \nabla a \times \mathbf{E})}_{\text{effective current}} \end{aligned}$$

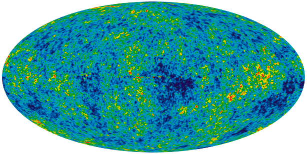
- New $\dot{a}$ and ∇a terms
- Axion dark matter: can look for these!

Axion cosmology

Horizon problem

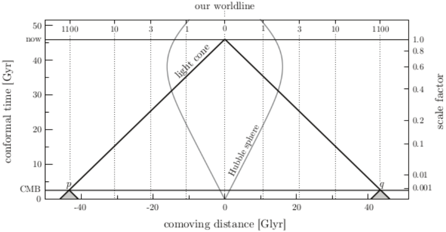
- Experimental fact: parts of universe that were never in causal contact have similar temperature





[WMAP - 2010]

- Space-time diagram of universe:

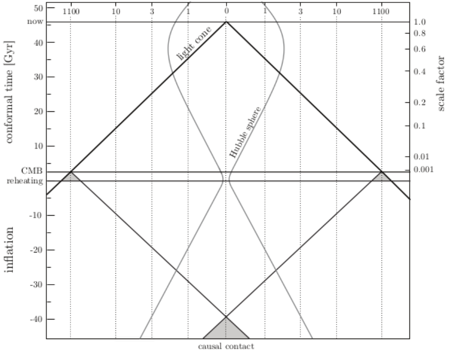


Simplest solution: inflation

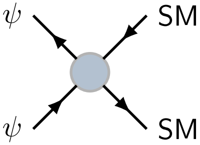
- Inflation: rapid expansion

$$R \propto e^{H_I t}$$

- R - size of universe
- H_I - "Hubble during inflation"
- Wipes out inhomogeneities
- inflation end: empty universe
- "Reheating": create SM
- Temperature: T_{RH}



[Baumann - 2013]

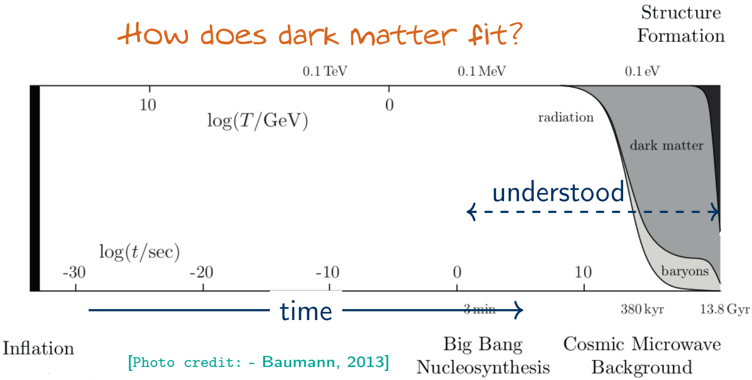


$$n \langle \sigma v \rangle \gtrsim H \equiv \dot{R}/R?$$

A lesson in history

- Post inflation all particles are generated ($n\langle\sigma v\rangle \gtrsim H \equiv \dot{R}/R$)
- Universe is thermal bath of particles
- As T drops below $m \to$ non-relavistic
- $n \propto e^{-m/T}$ until freeze-out

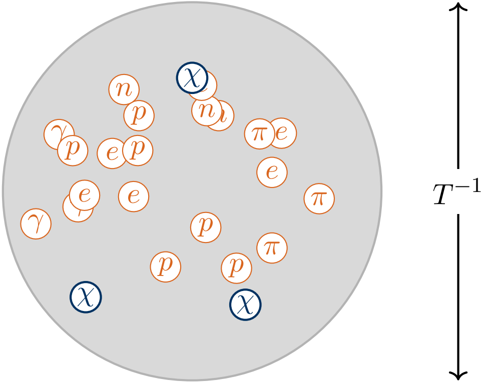
How does dark matter fit?



[Photo credit: - Baumann, 2013]

Weakly Interacting Massive Particles

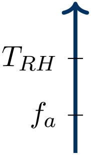
- WIMP dark matter: 1 new type of particle in the thermal bath

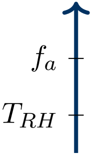


- Requires fast scattering between χ and the SM
- Axion: couplings suppressed so never thermalized

Axion cosmology

- Reheating temperature of universe?
- All we know is $T_{RH} > \text{ MeV}$
- Axion physics depends on if $T_{RH} > f_a$ or $T_{RH} < f_a$



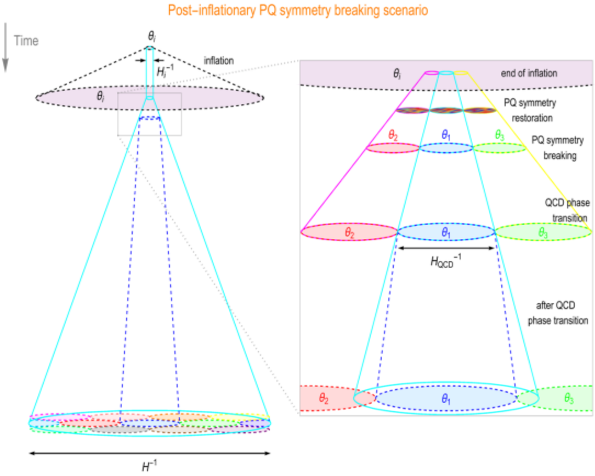


-PQ restored at high T

-PQ never restored

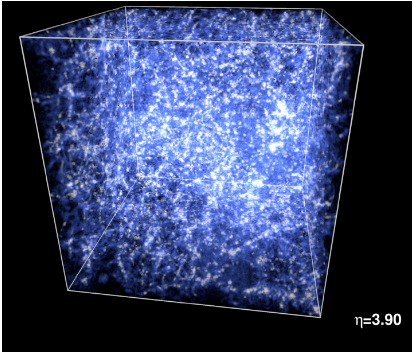
$f_a < T_{RH}$

- If PQ restored after inflation:



$f_a < T_{RH}$

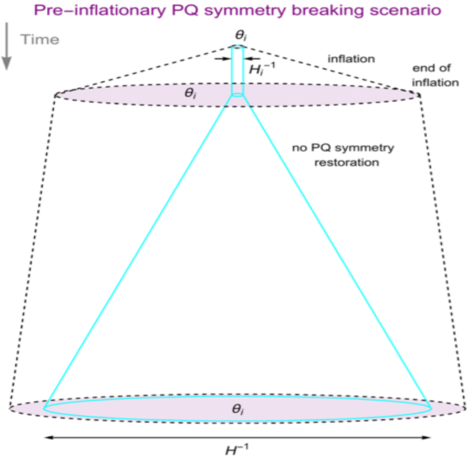
- Breaking of the PQ symmetry produces wealth of substructure
- “Topological defects” and “solitons” from phase transition
  - Domain walls
  - Strings
  - Oscillons
- Decay to axions



[Buschmann, Foster, Safdi - 2019]

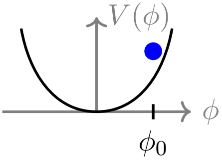
$f_a > T_{RH}$

- ○ PQ not restored after inflation:



Scalar evolution (1/2)

- Consider scalar with mass, m_ϕ



- Scalar action,

$$S \supset \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m_\phi^2\phi^2\right]$$

$$g^{\mu\nu} = \mathrm{diag}(1, -R, -R, -R)$$

- Euler-Lagrange equation of motion ($H = \dot{R}/R \sim T^2/M_{\rm pl}$):

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi = 0$$

friction

Scalar evolution (2/2)

- For $H \gg m_\phi$ $\Rightarrow$ field is stuck:

$$\ddot{\phi} + 3H\dot{\phi} \simeq 0 \quad \Rightarrow \quad \phi = \phi_0$$

- For $H = 0$: $\Rightarrow$ harmonic oscillator

$$\phi(t) = \phi_0 \cos(m_\phi t)$$

- For $H \propto t^{-1}$

$$\phi(t) = \phi_0 (R/R_0)^{3/2} \cos(m_\phi t)$$

- The energy density at time of transition is:

$$\rho_0 \sim V(\phi_0) \sim m_\phi^2 \phi_0^2$$

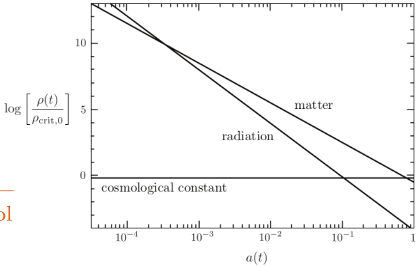
- After field gets unstuck it undergoes (damped) oscillations
- $\rho \propto R^{-3}$

Misalignment mechanism - axion like

- Relic density?
- DM: $\rho_{\rm DM}(T_{\rm eq}) \sim T_{\rm eq}^4 \sim {\rm eV}^4$:

$$\rho_{\rm DM}(T_{\rm eq}) \sim m_\phi^2 \phi_0^2 \left(\frac{T_{\rm eq}}{T_m}\right)^3$$

$$H(T_m) \sim m \Rightarrow T_m \sim \sqrt{m M_{\rm pl}}$$



- DM condition:

$$m_\phi^2 \phi_0^2 \left(\frac{\rm eV}{\sqrt{m_\phi M_{\rm pl}}}\right)^3 \sim {\rm eV}^4$$

- Solving gives,

$$\phi_0 \sim 10^{11}\ {\rm GeV} \times \left(\frac{\rm eV}{m_\phi}\right)^{1/4}$$

Misalignment mechanism - axion

- For axion: $\phi \to a$: $a_0 \sim \theta f_a$
- Turns out this is close but not sufficient estimate
- Subtlety: $m_a \to m_a(T)$ (not precise until QCD phase transition)
- Gives a revised estimate
- The energy density stored in axions:

$$\rho \sim m_a^2 f_a^2 \frac{m_a(T_c)}{m_a}$$

- T_c: temperature at which mass "turns on": $H(T_c) \sim m_a(T_c)$
- Somewhat lighter axions

m_a and f_a for axion dark matter?

- 3 different answers
  ① The true believer: "axions are misalignment axions"
    - Mass determined above with $\theta_0 \sim 1$
    - $m_a \sim 10^{-5}\mathrm{eV}$
    - $f_a \sim 10^{13}$ GeV

  ② The open minded axion theorist: "axions can be produced other ways"
    - Mass can be anywhere

$$10^{-21}\mathrm{eV} \lesssim m_a \lesssim \mathrm{eV}$$

    - Coupling determined by QCD line: $f_a \sim m_\pi^2 f_\pi^2 / m_a^2$

  ③ The anarchist: "axion-like particles are very reasonable"
    - Mass can be anywhere

$$10^{-21}\mathrm{eV} \lesssim m_a \lesssim \mathrm{eV}$$

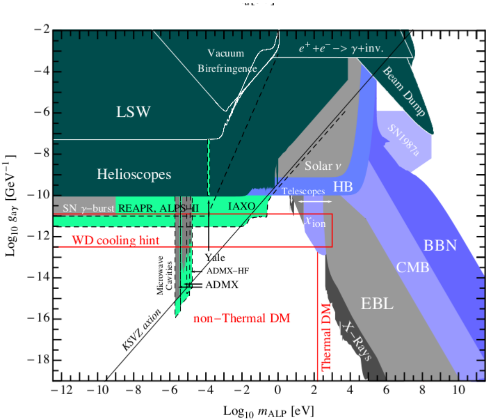
    - Coupling can be anywhere

$$f_a \gtrsim 10^{10} \text{ GeV}$$

Axion - direct searches

Axion-like particle

- ○ ALP-parameter space:

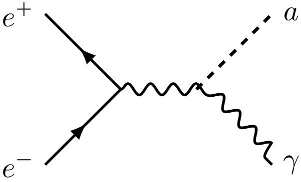


- ○ Lets go over every line in this plot...

[- 1205.2671]

Colliders

- ○ Make axions in collider experiments:



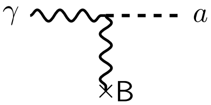
- ○ Can look for $e^+e^- \to \gamma + \text{inv}$
- ○ Requires $f_a \gtrsim \text{ TeV}$
- ○ $pp \to \gamma a$ vs $e^+e^- \to \gamma a$
- ○ General rule of thumb:

Higher dimensional operators: colliders probe around weak scale

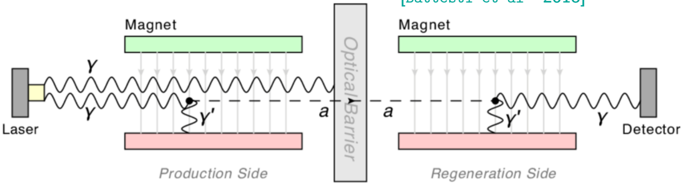
Renormalizable interactions: colliders probe around $\sim 10^{-3}$

Light shining through walls

- Oscillations of photons into axions and back
- Use $a\mathbf{E}\cdot\mathbf{B}$ interaction



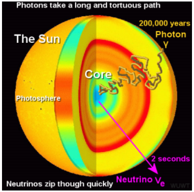
- Experimental setup:



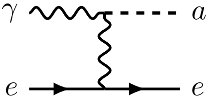
[Battesti et al - 2018]

Star cooling

- Stars have core temperatures $\sim 100\mathrm{keV}$
- Cooled by neutrino emission



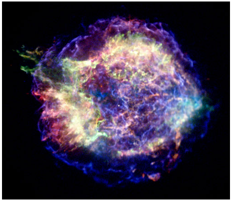
- Axions can cool in stars ("Primakoff process")



- Applicable for $m_a \lesssim 100\ \mathrm{keV}$
- Ruled out if cools more than neutrino emission

Supernova cooling

○ Supernovas reach much higher temperatures



[- https://www.space.com/6638-supernova.html]

○ $T \sim 30\ \mathrm{MeV}$

○ Densities in the core are near-nuclear

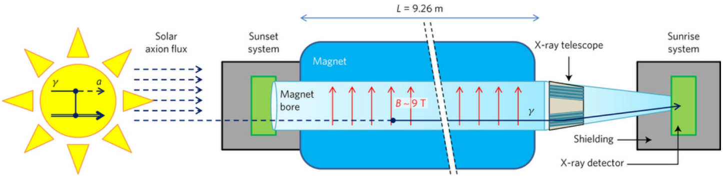
○ Dominant production through $n + n \rightarrow n + n + a$

○ Cooled by neutrino emission

○ Ruled out if cools more than neutrino emission

Solar axions

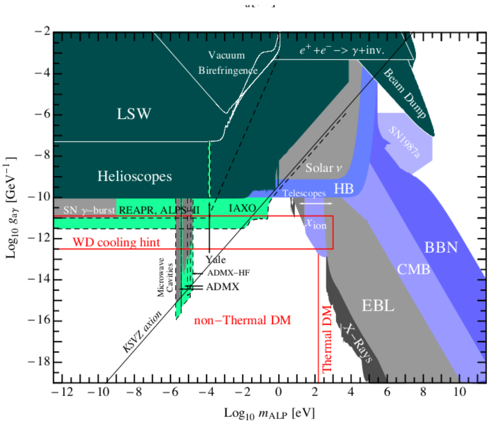
- The Sun is close by
- Can do better than just cooling bounds
- Look for axions produced in Sun
- CAST: Uses a LHC magnet pointed at the Sun



[CAST - 1705.02290]

Recap

- ○ ALP-parameter space:



[- 1205.2671]

Axion DM searches

Axion DM

- If axion makes up dark matter then:

$$a(t) = a_0 \cos(m_a t)$$

- What is the amplitude today? → fixed by known density!

$$\rho = \frac{1}{2} m_a^2 a_0^2 \Rightarrow \Rightarrow\Rightarrow a_0 = \sqrt{\frac{2\rho_{DM}}{m_a^2}}$$

- Coherence time for field?

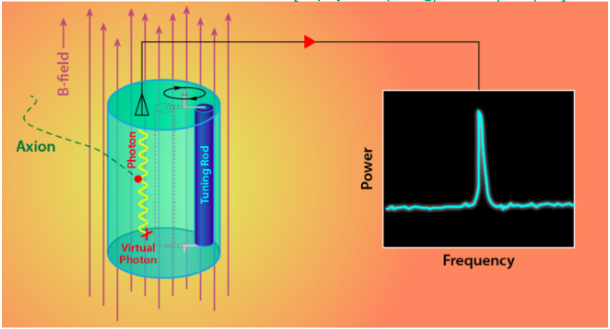
$$\tau \sim \frac{4\pi}{mv^2}$$

- After this no longer act as a simple trig function

ADMX

- "Haloscope" experiments (e.g., ADMX)
- Axion DM enters detector
- Converts in large B field

[- physics.aps.org/articles/v11/34]



- Conversion maximized when $L \sim m_a^{-1}$
- Specifically designed for $m_a \sim 10^{-5}\mathrm{eV}$

ABRACADABRA/DM-radio

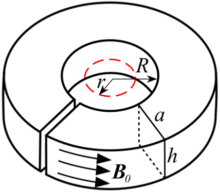
- Ampere's circuit law:

$$\nabla \times \mathbf{B} = \cancel{\dot{\mathbf{E}}} - C_{a\gamma\gamma}\left(\cancel{\mathbf{E} \times \nabla a} - \mathbf{B}\dot{a}\right)$$

- Static background B field ($\mathbf{B}_0$)
- Axion sources effective current

$$\mathbf{J}_{\mathrm{eff}} = C_{a\gamma\gamma}\sqrt{2\rho_{\mathrm{DM}}}\mathbf{B}_0 \cos(m_a t)$$

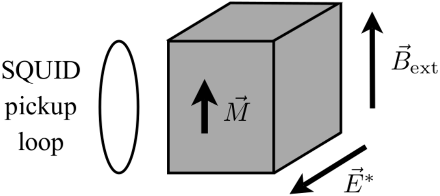
- Current sources a real oscillatory B field ($\perp$ to $\mathbf{B}_0$)



[- 1310.8545]
[- 1602.01086]
[- 1610.09344]

CASPEr

- DM-induced spin precision
- CASPEr experiment
- Test gluon coupling (though coupling to nucleons)



- ALPs in heavy ion collisions (1709.07110)
- ALPs in meson decays (1611.09355)
- CASPEr (1306.6089)
- DM-radio (1610.09344)
- Using RF cavities (1904.07245)
- New production mechanisms (1711.10486,1910.04163)
- Unified axion models (1702.00401)
- ...

Conclusions

- ○ Strong CP problem (one of the biggest mysteries in particle physics)
- ○ Axion provides a solution + facinating cosmology
- ○ Direct searches (many ideas in light dark matter needed)
- ○ Dark matter searchs ongoing
- ○ The search into the dark sector continues...