Replica Wormholes and the Black Hole Interior

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Based on:

Entanglement Wedge Reconstruction and the Information Paradox. GP. *arXiv:1905.08255*.

Replica Wormholes and the Black Hole Interior. GP, S. Shenker, D. Stanford, Z. Yang. *arXiv:1911.11977*.

See also:

The Entropy of Bulk Quantum Fields and the Entanglement Wedge of an Evaporating Black Hole. A. Almheiri, N. Engelhardt, D. Marolf, H. Maxfield. *arXiv:1905.08762*.

The Page Curve of Hawking Radiation From Semiclassical Geometry. A. Almheiri, R. Mahajan, J. Maldacena, Y. Zhao. *arXiv:1908.10996.*

Replica wormholes and the entropy of hawking radiation. A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, A. Tajdini. *arXiv:1911.12333*.

Other important work by: Akers, Harlow, Bousso, Tomasevic, Chen, Fisher, Hernandez, Myers, Ruan, Rozali, Van Raamsdonk, Sully, Waddell, Wakeham

The claim:

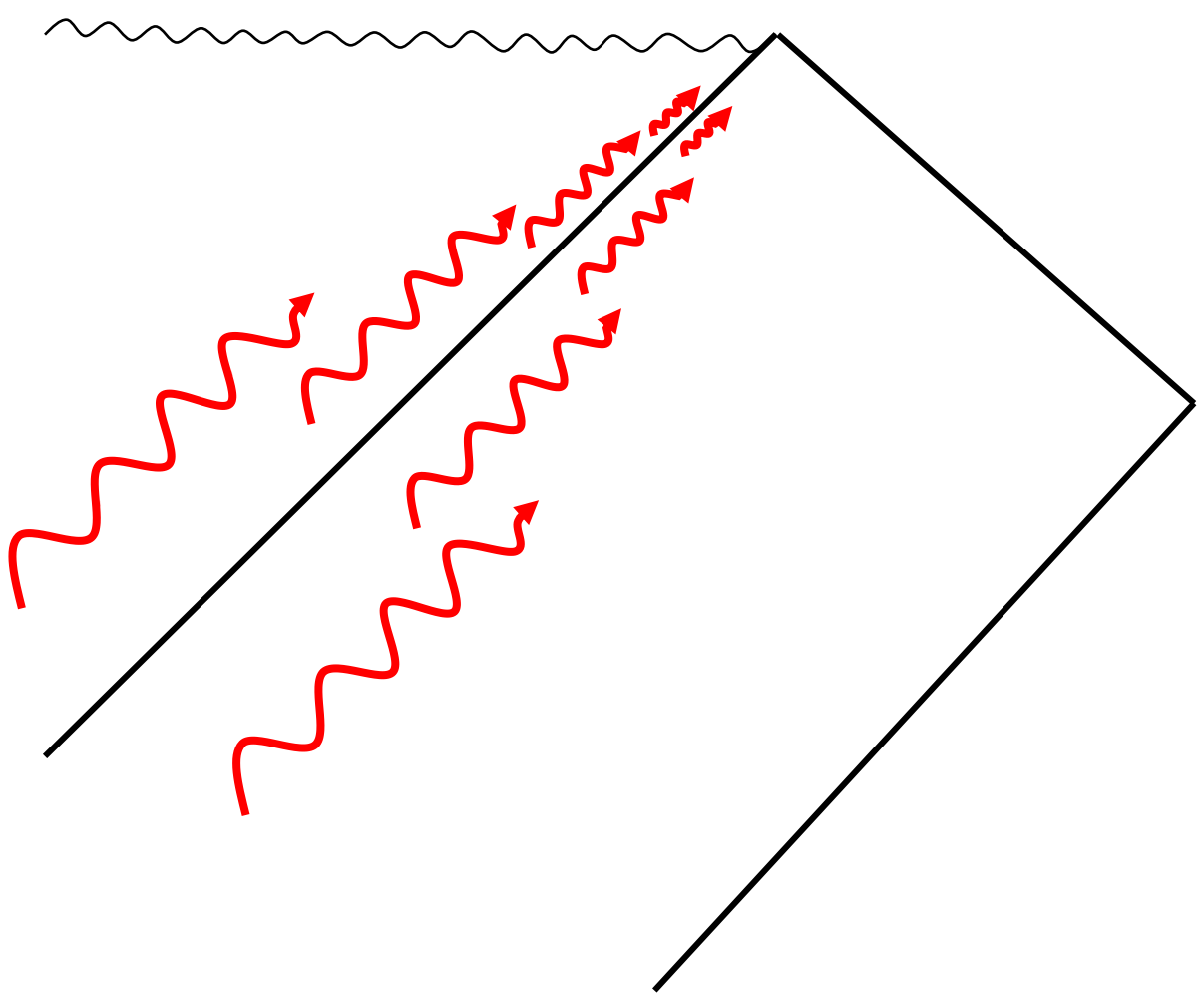
- We can derive a **unitary Page curve**, and the information escaping the black hole (specifically the **Hayden-Preskill decoding criterion**) from gravity.
- Original context: AdS/CFT. **However**, don't need a CFT, string theory, anti-de Sitter space, etc…
- All we need is a **gravitational path integral** (as the low energy **effective field theory**).
- **But,** when we do this path integral, we need to include **arbitrary topologies**, including **spacetime wormholes**.
- This is the "right" thing to do, but is known to cause problems of its own, particularly in AdS/CFT.
- Still plenty of mysteries that remain: **microscopic mechanism**, single unitary theory vs **ensemble** of unitary theories etc.

The plan:

- **Part 0:** review the relevant features of the information paradox
- **Part I**: abstract everything away into a very simple but **unrealistic** toy model, where we can calculate everything completely **explicitly**.
- **Part II**: move to more realistic models, including **evaporating four-dimensional black holes**, at the cost of being somewhat less explicit.

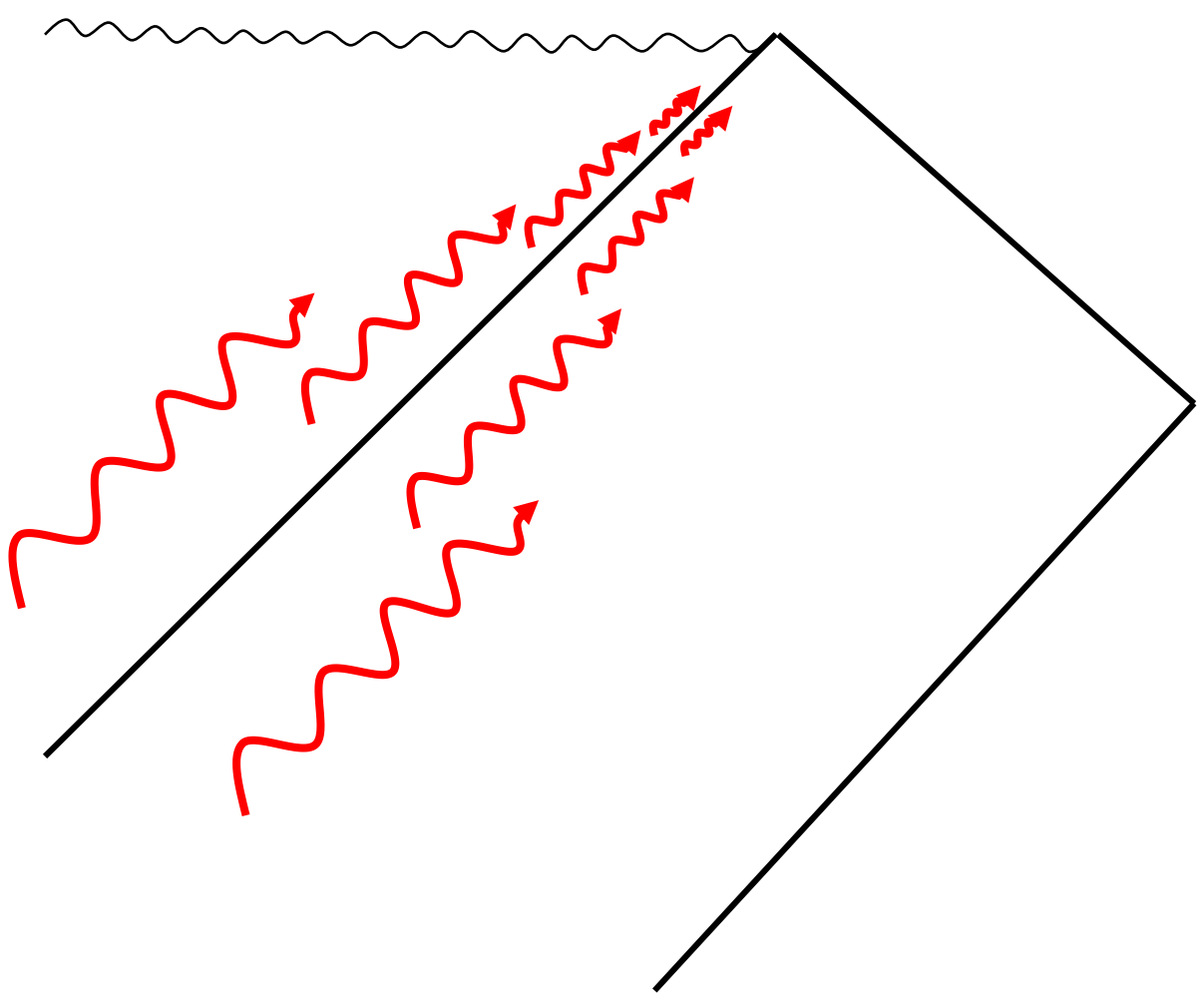
Part 0: The Information Paradox

Evaporating black holes



- **Quantum field theory:** short-range entanglement between outgoing modes slightly inside/outside the BH horizon
- BH dynamics **redshifts** these modes, creating **Hawking radiation** that is entangled with interior modes
- As the black holes evaporates, **more and more modes** escape. The entanglement (apparently) increases indefinitely

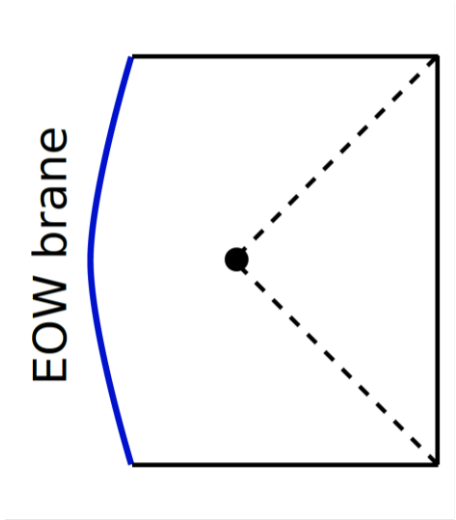
The Information Paradox



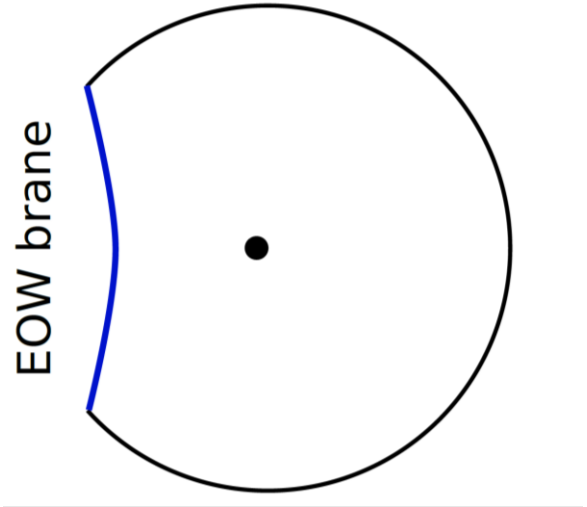
- Eventually the entanglement entropy becomes larger than the **Bekenstein-Hawking** entropy of the black hole (the **Page time**)
- If the BH entropy is truly the **statistical entropy** of black hole microstates (true in string theory, AdS/CFT), this is a **paradox**: not enough BH states to be able to purify the Hawking radiation
- Possible resolutions: a) **information loss** or b) entanglement entropy starts **decreasing** at/before Page time (**Page curve**)

Part 1: A Very Simple Model

A Very Simple Model: Pure JT gravity plus EOW Branes



Lorentzian



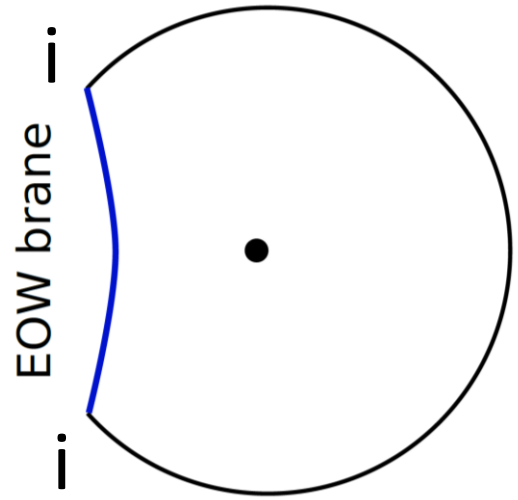
Euclidean

1+1-dimensional, one-sided eternal black hole, where the spacetime ends on an **'end-of-the-world' brane** in the BH interior

Analogue for Hawking radiation: add internal degrees of freedom to the EOW brane **(interior modes)** that are maximally entangled with a reference system

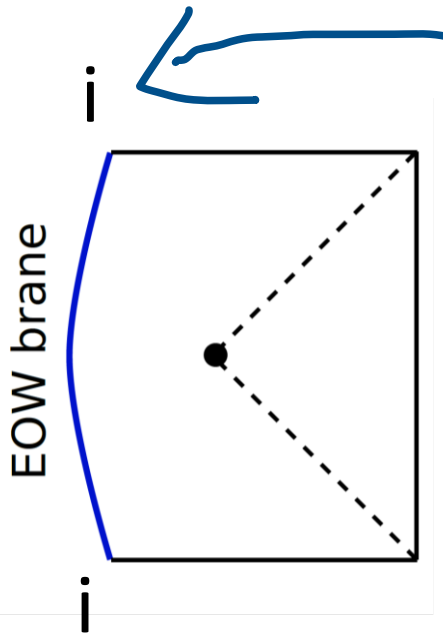
A Very Simple Information Paradox

$|\psi_i\rangle_{\mathrm{B}}$ =



Euclidean

=



Lorentzian

Orthogonal basis for **internal state** of EOW brane

$$|\Psi\rangle = \frac{1}{\sqrt{k}} \sum_{i=1}^{k} |\psi_i\rangle_{\mathrm{B}} |i\rangle_{\mathrm{R}}.$$

(Simple) **information paradox** when $k \gg e^{S_{BH}}$. Entanglement entropy seemingly becomes **larger** than the **Bekenstein-Hawking** entropy

The Replica Trick

- How do you calculate **von Neumann entropies** using a path integral?
- **Answer**: the **integer n Renyi entropies**

$$\frac{1}{1-n}\log \mathrm{Tr}\rho_R^n$$

‘Replica trick’

are proportional to the logarithm of an observable on **n copies of the system**.

- We can calculate the von Neumann entropy by **analytically continuing the Renyi entropies** to **n=1**.
- **The key idea**: the gravitational path integral includes topologies that connect the different replicas via **spacetime wormholes**.

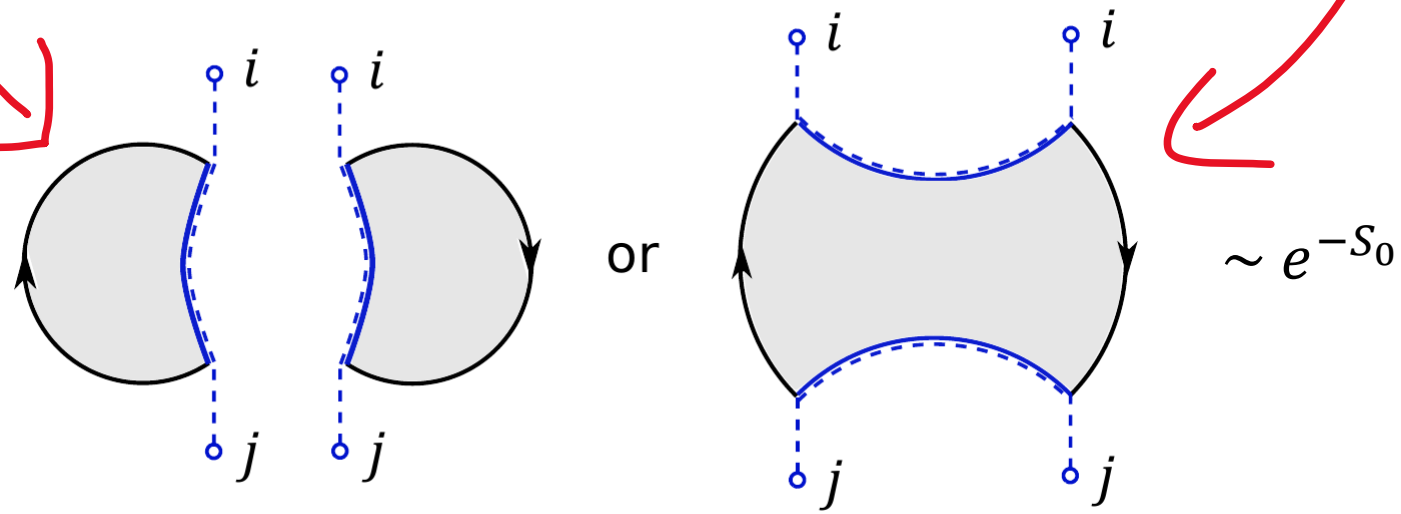
Calculating the Purity

Einstein-Hilbert term
~ Euler character

Calculate the purity $Tr(\rho_R^2)$ using a **Euclidean path integral**, where we sum over **all topologies** with the correct boundary conditions:

$$\mathrm{Tr}(\rho_{\mathrm{R}}^2) = \frac{1}{k^2}\sum_{i,j=1}^{k} |\langle\psi_i|\psi_j\rangle|^2.$$

$$= \frac{1}{\mathrm{k}} \sum_{i,j}$$



Very small but doesn't decay at large k

$\sim e^{-S_0}$

Two copies of the original black hole

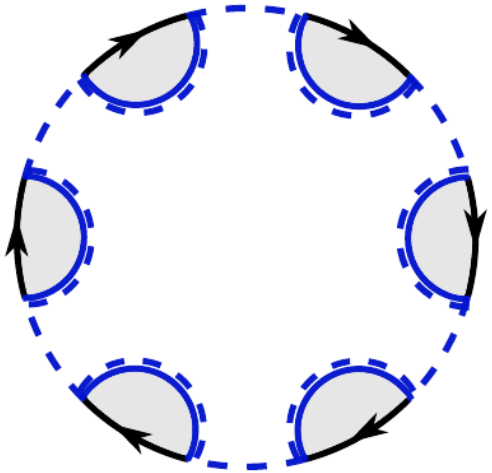
Two black holes connected by a wormhole

Calculating the von Neumann Entropy

In general, there are a lot of topologies that can contribute to $\mathrm{Tr}(\rho_R^n)$. However, in the limit where k is **very large/small** one of **two families of topologies** dominates

$$\frac{1}{k^{n-1}} =$$

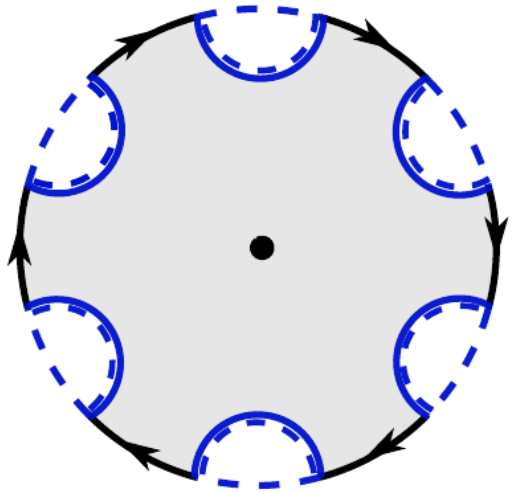
Small k



Disconnected topology
Gives $S = \log k$

or

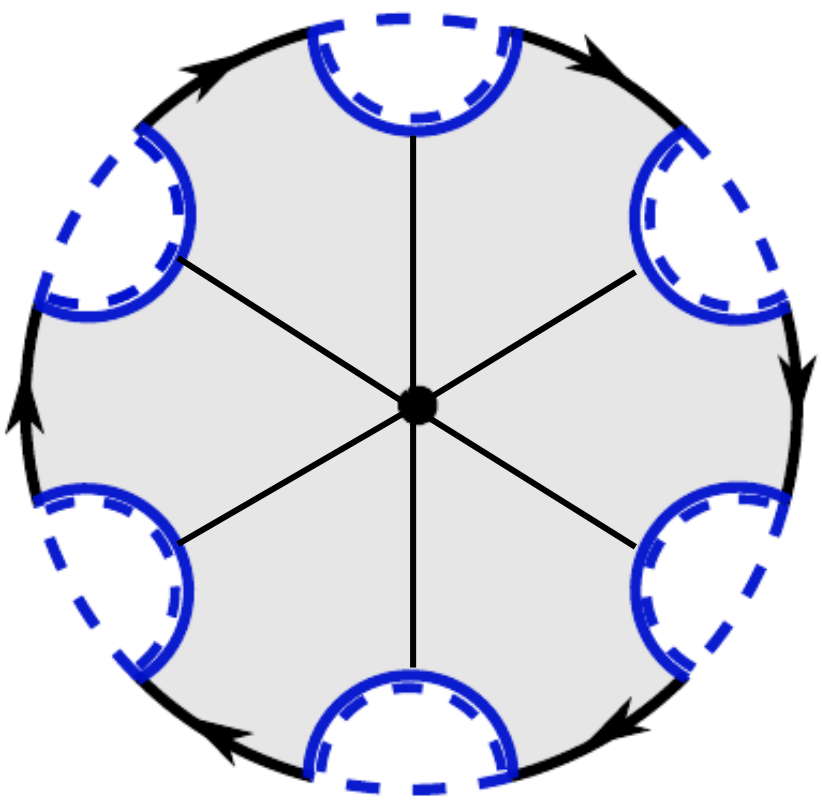
Large k



Connected topology
Gives $S = S_{BH}$

$$\sim e^{-(n-1)S_0}$$

Calculating the von Neumann Entropy



- Connected topology has a Z_n replica symmetry.
- After **quotienting** by this symmetry, we get *roughly* the original black hole geometry, except that there is a **conical singularity** at the fixed point of the replica symmetry
- In the limit $n \to 1$, the singularity **vanishes** (get original unbackreacted geometry)
- Von Neumann entropy given by the "area" of replica fixed point (in this case the **bifurcation surface**)

Much more to say in this model!

- Simple enough that we can do the **full path integral**, rather than just looking at classical saddle points
- Can use tools from **free probability theory** to find the corrections to the von Neumann entropy, and even the **full entanglement spectrum,** near the Page transition, when the Renyi entropies are not dominated by a single topology.
- Transition is **complicated**, with **seven distinct phases**. However the main qualitative features agree with previous expectations.
- (In particular, there are $O(1/\sqrt{G_N})$ corrections near the transition from **energy fluctuations**.)

Part 2: Actual Evaporating Black Holes

What about actual evaporating black holes?

- **Bad news:** no one has found analytic solution for the **replica wormhole** geometry at $\text{integer } n \geq 2$ in more realistic models.
- **Numerical results** for the **SYK model** suggest that the physics is inherently **messy**, with complicated backreaction related to the **fast scrambling** behaviour.
- **Good news**: we saw in the simple model that the **von Neumann entropy** was controlled by the area of the **replica fixed point** in the **unbackreacted** geometry.

Generally true

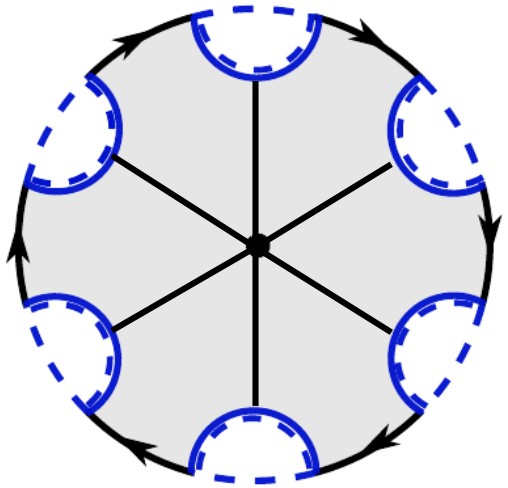
The Quantum Extremal Surface Prescription

1. In the limit $n \to 1$, the equations of motion imply that the replica fixed point needs to be a **quantum extremal surface**

$$\text{ext} \left[\frac{A(\chi)}{4G_N} + S_{\text{bulk}}(\chi) \right]$$

Generalised entropy

Entanglement between bulk fields on opposite sides of χ



2. **Von Neumann entropy** (from given family of saddles) is the generalised entropy of the corresponding QES.

3. The **dominant family of saddles** comes from the replica fixed point with **smallest generalised entropy** (the **minimal QES**)

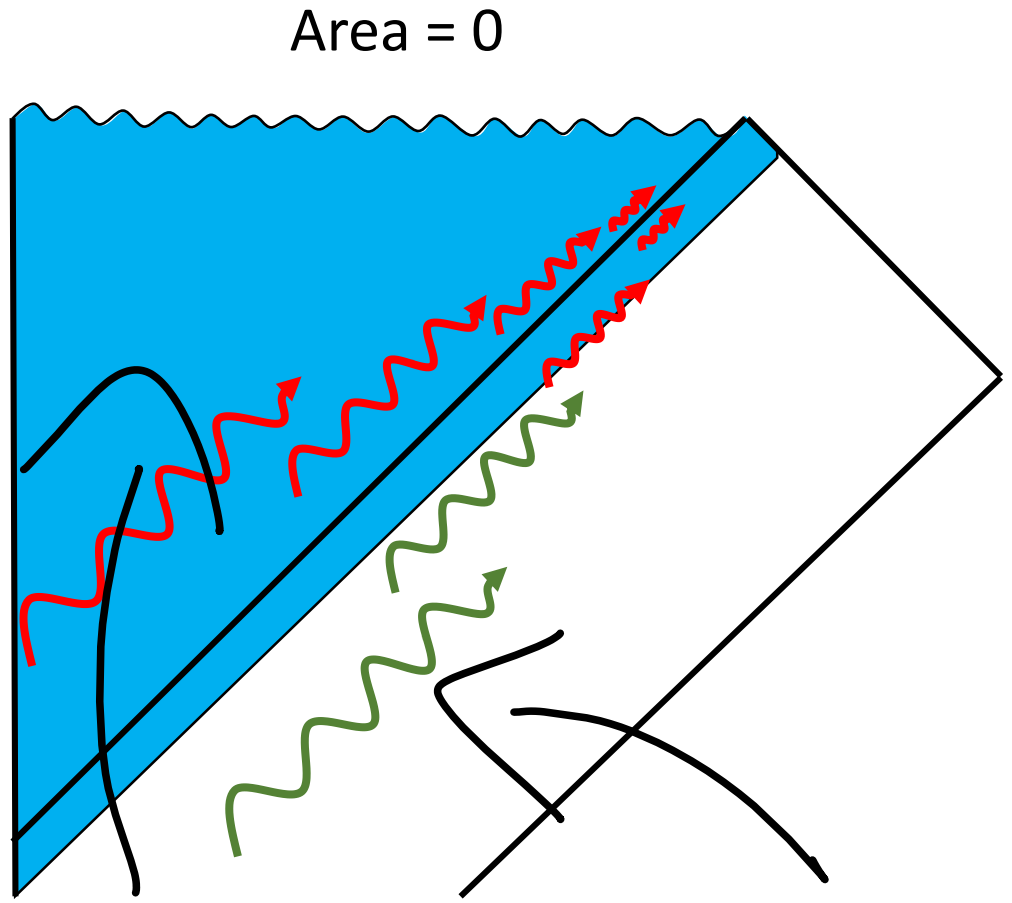
(Quantum-corrected) **Ryu-Takayanagi** formula

The Page Curve in Evaporating Black Holes

At **early times**, disconnected replica topologies dominate

No replica fixed point = **empty QES**

Generalised entropy = A/4G_N= $A/4G_N + S_{bulk}$ = **semiclassical entropy** of the Hawking radiation



Entanglement between **blue region** and escaped **Hawking radiation** grows linearly with time

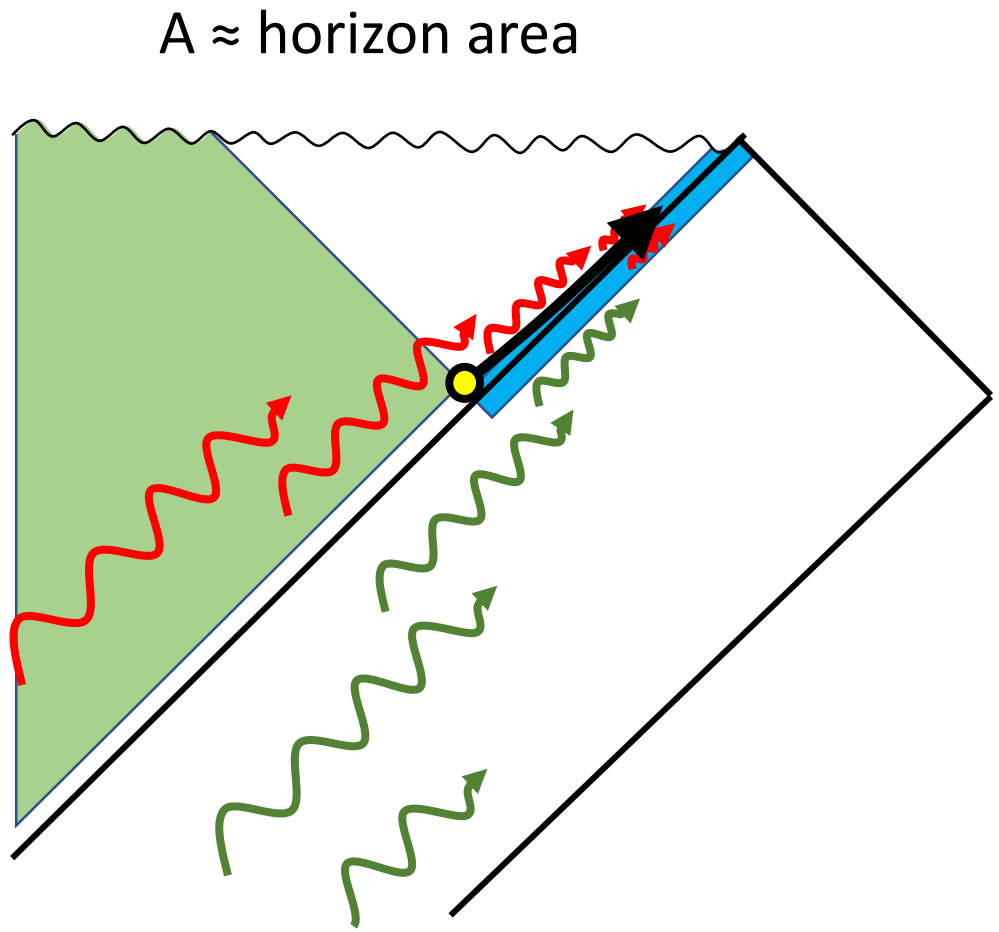
The Page Curve in Evaporating Black Holes

However, there also exists a **non-empty quantum extremal surface** that lies just inside the event horizon

Generalised entropy ≈ BH entropy

After the Page time, this becomes the **minimal QES** (this corresponds to the transition to a **fully connected replica wormhole topology**)

As the black hole continues to evaporate, the RT surface **tracks along the horizon**, travelling on a spacelike trajectory (generalised entropy **decreases** with time)

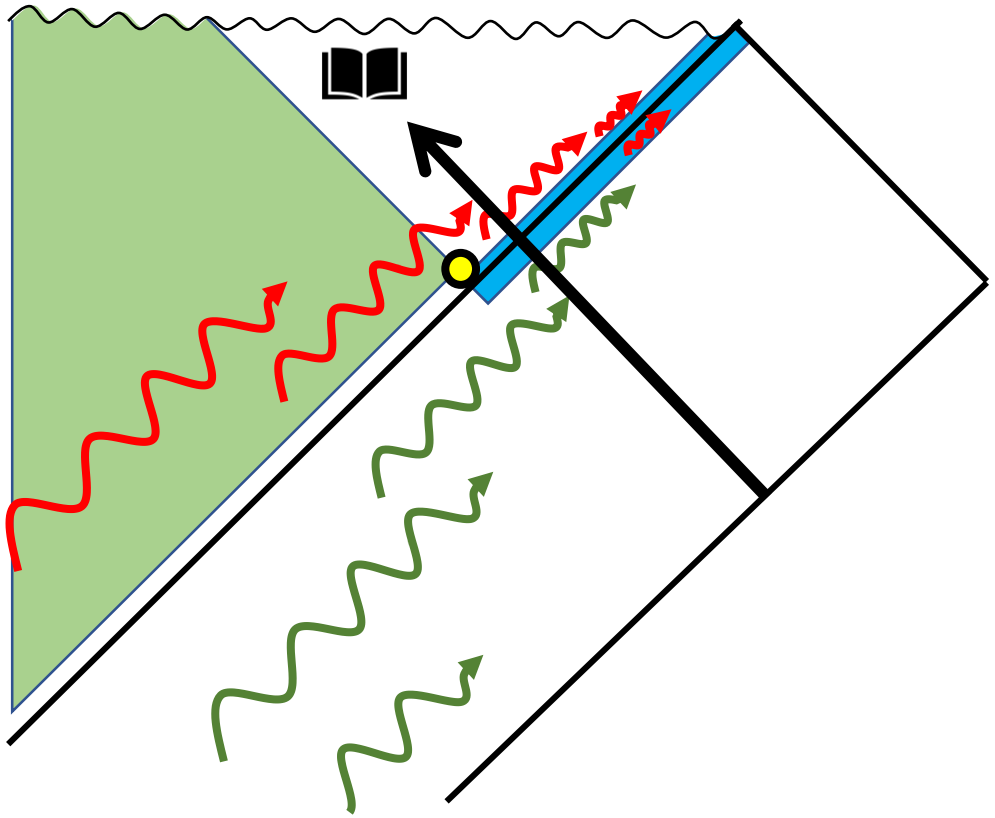


Entanglement between **blue region** and **green region + escaped Hawking radiation** is small (O(1))

Hayden-Preskill Decoding Criterion

Suppose we throw a **diary** into the black hole (after the Page time)

Initially, the worldline of the diary is in the **blue region**: this means that no information about the diary has escaped in the Hawking radiation



Can be shown by evaluating the action of the **Petz map**

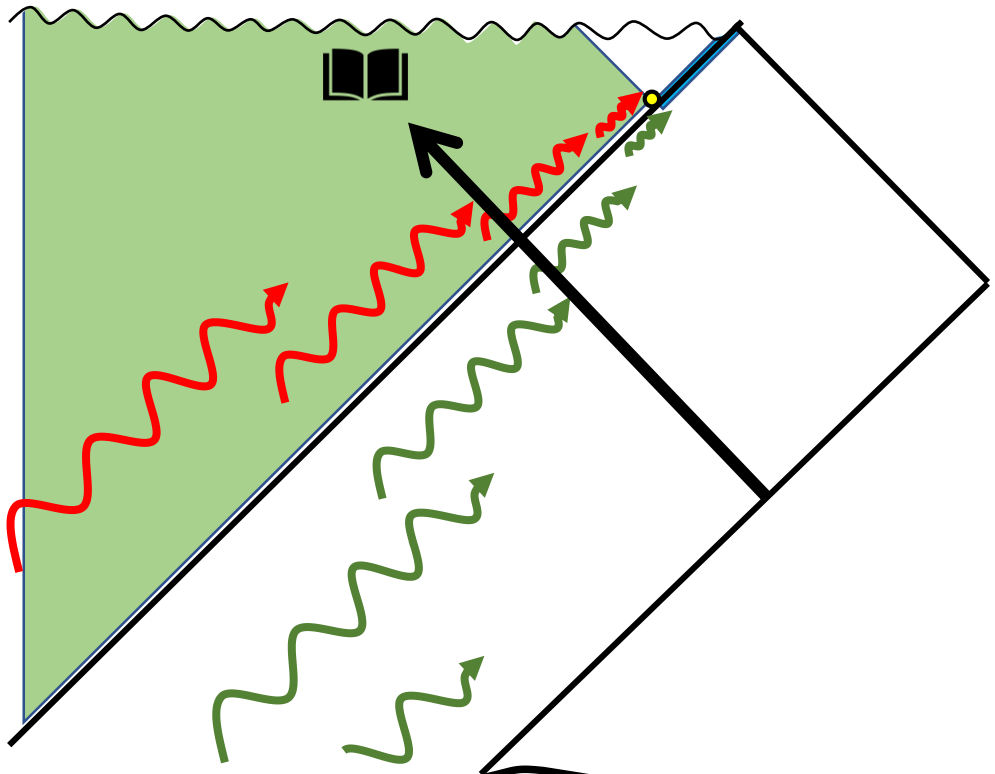
Hayden-Preskill Decoding Criterion

However as the black hole continues to evaporate, the RT surface continues to track along the horizon

After waiting for **more than the scrambling time**, the worldline of the diary will be in the **green region**

(Entanglement wedge of the Hawking radiation)

The information in the diary has escaped in the Hawking radiation (again shown via **Petz map**)



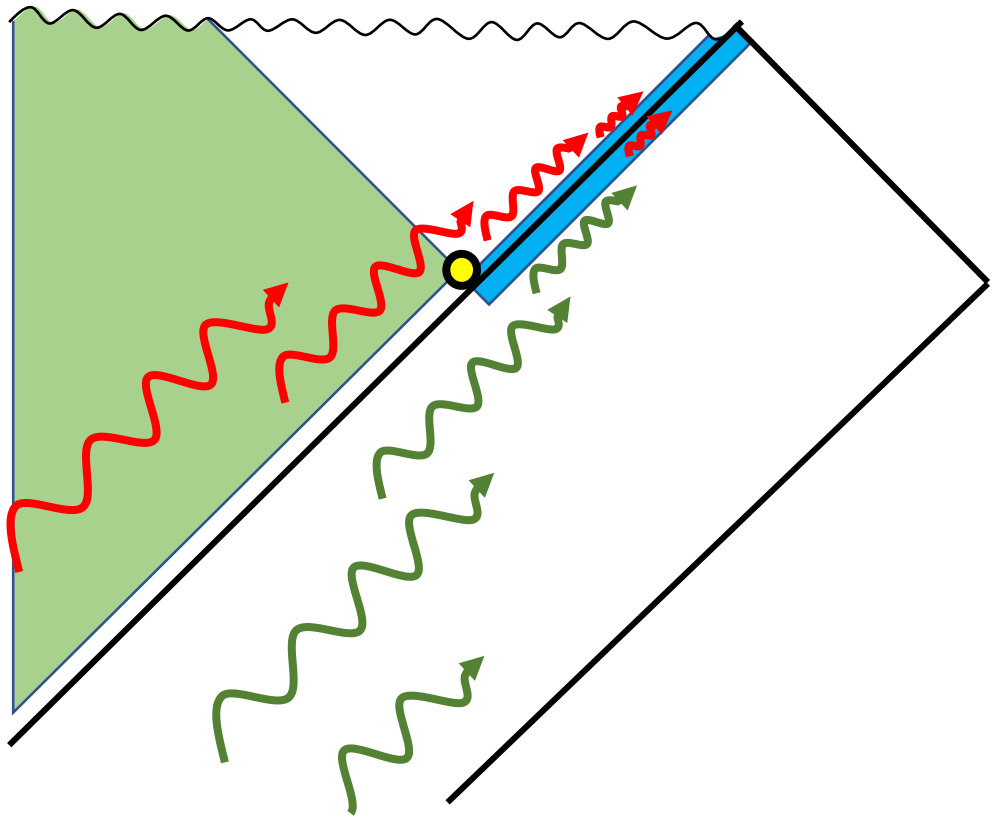
Famously predicted by **Hayden and Preskill**, based on simple toy models

The firewall paradox

AMPS Paradox: how can late time Hawking radiation be entangled with both **interior partner** and **early radiation** (given monogamy of entanglement)?

Answer: worldline of **interior partner** goes through **green region** so it is **encoded in** the early radiation (ER=EPR)

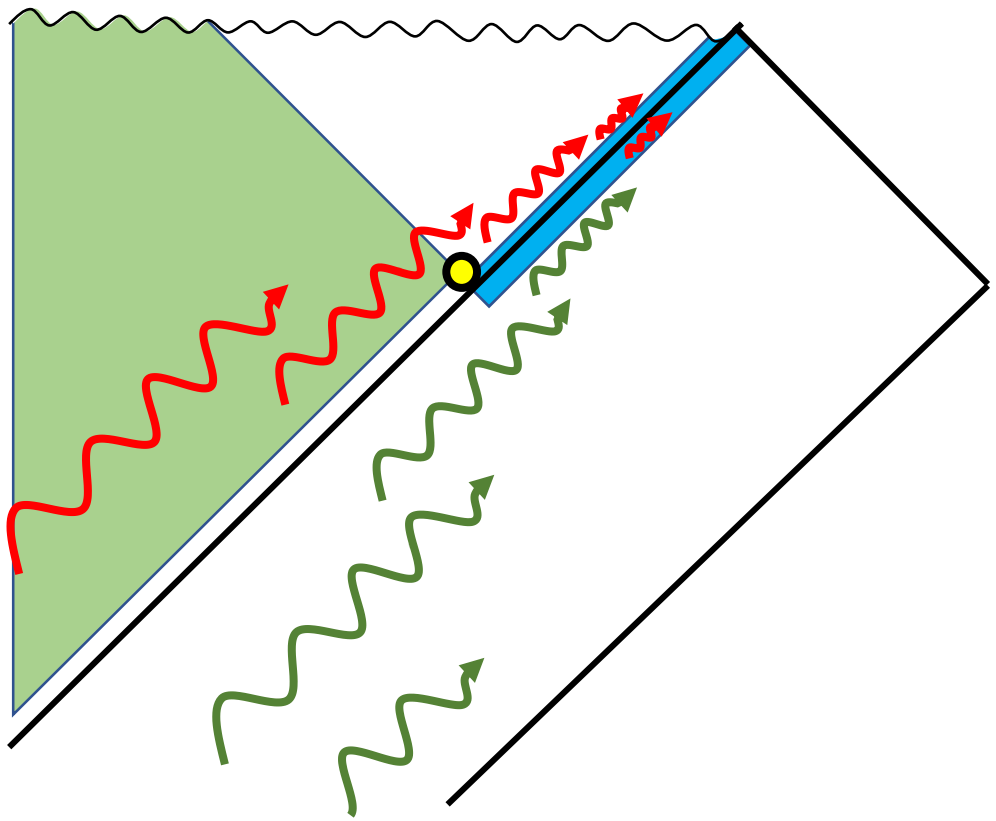
(Full story is more complicated with several important subtleties, but **quantum extremality** magically ensures that everything works out and you exactly avoid any firewall paradox.)



How does the information get out?

Still need to answer how the Hawking radiation can provide information about matter that fell into the black hole, when the state of the Hawking mode + interior partner is **fixed**

Answer: the **encoding** of the interior partner in the early radiation **depends** on the state of the matter that fell into the black hole



Thank you!