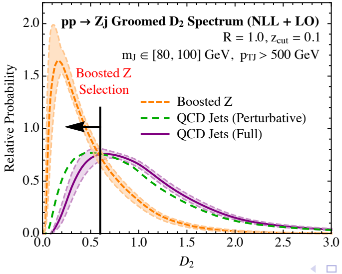
Selections from Jet Substructure Theory

Ian Moult

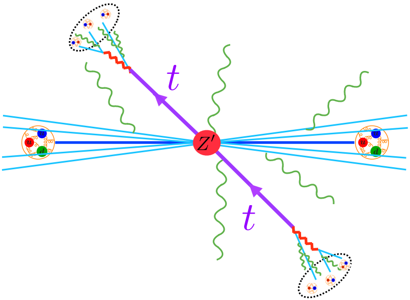
Berkeley



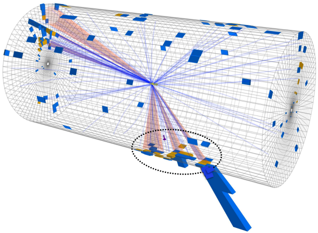
Jets at the LHC: Internal Structure

- Internal structure of jets resolved due to excellent detector resolution.
- Electroweak scale objects, $W/Z/H$ or t can have sufficiently high p_T to appear inside a jet.

Boosted Tops



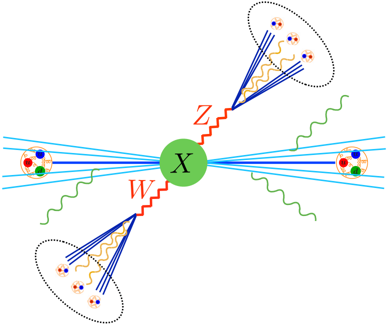
Event Display



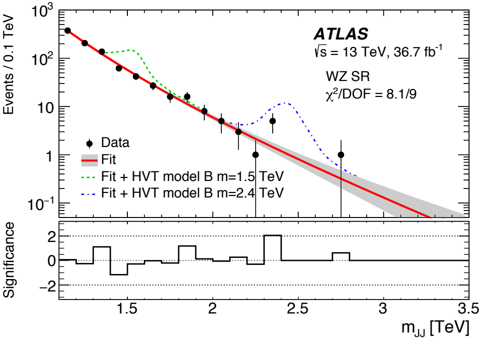
- Revolutionizes the types of questions we can/must ask about jets:
  $\Longrightarrow$ jets have substructure!

Jet Substructure

- Substructure techniques were primarily developed to search for new physics.
- Extremely succesful!



WZ Diboson Invariant Mass

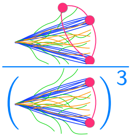


Jet Substructure

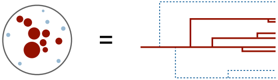
- What has come out of this?
  - New and stronger bounds on BSM physics.
  - A wealth of new and sophisticated techniques for studying and calculating properties of jets!

Substructure Observables

$$D_2^{(\beta)} = \frac{e_3^{(\beta)}}{\left(e_2^{(\beta)}\right)^3} =$$



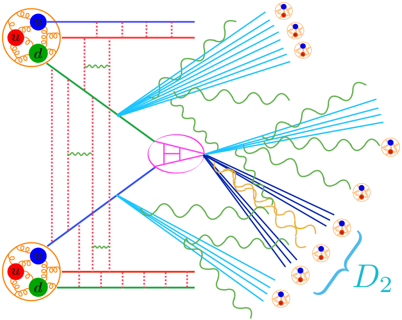
Grooming Strategies



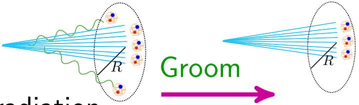
- Can be applied more generally to the study of hadronic final states.

Analytic Calculations with Grooming

- Difficulties in QCD calculations for *pp*:



- Global color correlations
- Hadronization corrections
- Pile-Up
- Underlying event



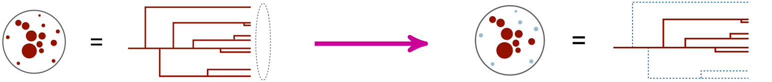
- All complications associated with soft radiation.
- Groomers remove soft radiation

$\Longrightarrow$ Makes calculations simpler and more universal.

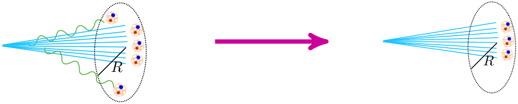
Soft Drop Grooming

[Larkoski, Marzani, Soyez, Thaler]
[Dasgupta, Fregoso, Marzani, Salam]

- Groomer are used to remove soft contamination.
- Soft Drop/ mMDT: Recurse through a Cambridge-Aachen clustering tree and remove particles that fail the condition: $\frac{\min[p_{Ti}, p_{Tj}]}{p_{Ti} + p_{Tj}} > z_{\text{cut}}$



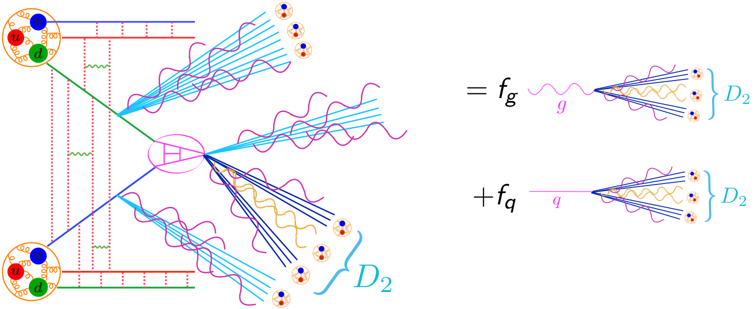
- Loosely speaking, reduces a jet to its collinear core.



- Any IRC safe observable measured on a groomed jet is IRC safe.

Analytic Calculations with Grooming

- Grooming removes all color correlations.

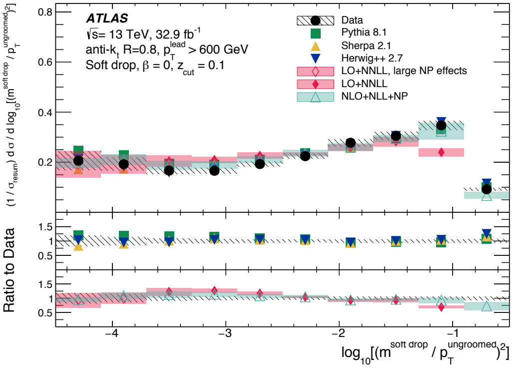


- Jet can be considered in isolation!
- Enables calculations in complicated LHC environment.

Soft Drop Jet Mass

- Measurement of the groomed jet mass in ATLAS!

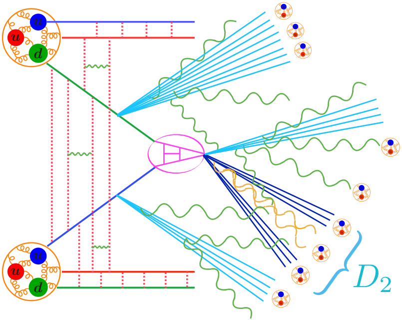
Soft Drop Mass in ATLAS



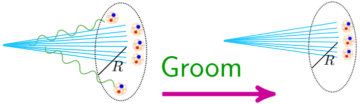
- Comparison of theory and data for a substructure observable!

Analytic Calculations with Grooming

- This has both pros and cons.



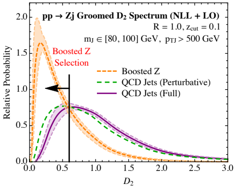
- Global color correlations
- Hadronization corrections
- Pile-Up
- Underlying event



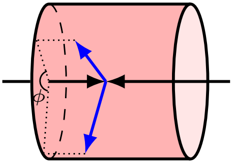
- If you remove soft radiation, you cannot learn about it.
- Effects associated with soft radiation are least well understood theoretically.

Outline

- Groomed Observables
  - Top Quark Mass
  - Groomed Two Prong Observables



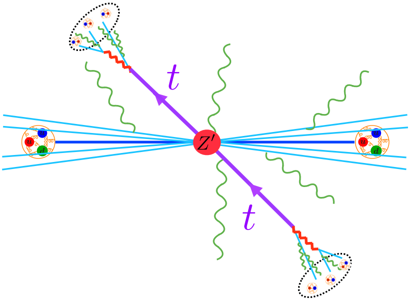
- Hadronic Event Shapes
  - Energy-Energy-Correlator



Substructure for the Standard Model

- Boosted electroweak scale objects, $W/Z/H$ and t are all interesting SM particles!

Boosted Tops



- Techniques that have been developed for use in jet substructure can provide innovative ways to study the SM.
- Here grooming can be highly beneficial.

Top Quark Mass

Top Quark Mass

- Top mass is an important parameter of the Standard Model.

Kinematic Top Mass Extractions:

Tevatron (2014): m_t = 174.34 ± 0.64 GeV

CMS Run 1 (2015): m_t = 172.44 ± 0.49 GeV

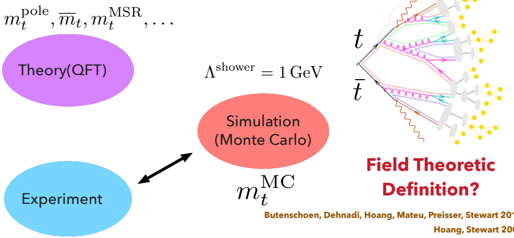
ATLAS Run 1 (2016): m_t = 172.84 ± 0.70 GeV

- Due to its strongly interacting nature, top mass is a renormalized parameter of the SM, needs to be defined in a given scheme.

What scheme is the top mass measured in?

See Also Talk by Czakon

CMS Run 1 (2015): m_t = 172.44 ± 0.49 GeV



[Pathak, Boost 2017]

Top Quark Mass

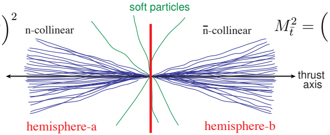
- For sufficiently inclusive event (jet) shape observables, top mass can be given a precise meaning through factorization formulas:

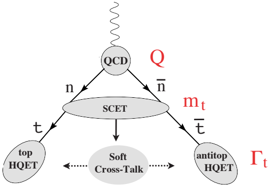
$$e^+e^- \to t\bar{t} \qquad \frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}$$

Peak Region:

$$M_{t,\bar{t}}^2 - m^2 \sim m\Gamma \ll m^2$$

$$M_t^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \qquad M_{\bar{t}}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$





Factorization Theorem:

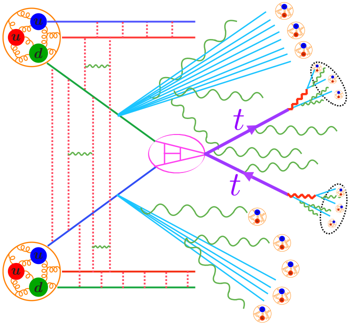
$$\left(\frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m)\, H_m\Big(m_J, \frac{Q}{m_J}, \mu_m, \mu\Big)$$
$$\times \int dl^+ dl^-\, J_B\Big(\hat{s}_t - \frac{Ql^+}{m_J}, \Gamma_t, \delta m, \mu\Big)\, J_B\Big(\hat{s}_{\bar{t}} - \frac{Ql^-}{m_J}, \Gamma_t, \delta m, \mu\Big)$$
$$\times S_{\text{hemi}}(l^+ - k, l^- - k', \mu)\; F(k, k')$$

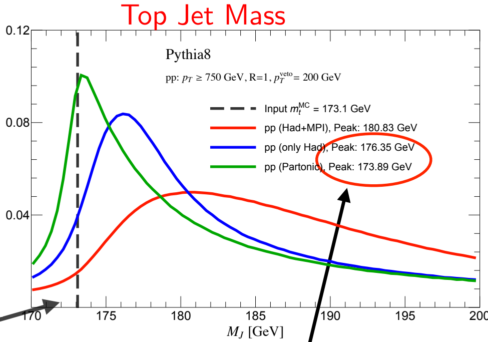
(boosted HQET) Jet Functions — Control Over Mass Scheme — Soft Function — Hadronization

Fleming, Hoang, Mantry, Stewart 2007

Top Jets in *pp*

- Can we directly measure top jet mass at *pp*?
- Naively seems impossible due to the very large contamination.





Input mass in Pythia
m_t = 173.1 GeV

Significant contamination

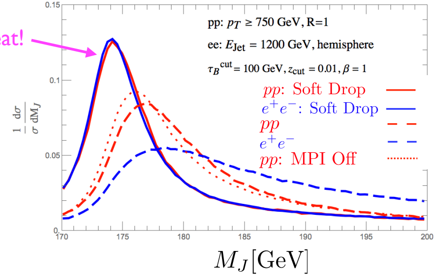
[Hoang, Mantry, Pathak, Stewart]

Soft Drop Top Mass

- Can we apply grooming to achieve a precision top mass measurement at the LHC?
  - Hadronization uncertainty reduced ✓
  - Poorly understood contribution from MPI minimal ✓

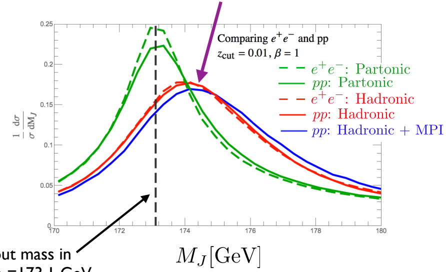
Groomed vs. Ungroomed

Great!



Soft Drop Top Mass

Only 0.19 GeV shift from MPI



input mass in Pythia =173.1 GeV

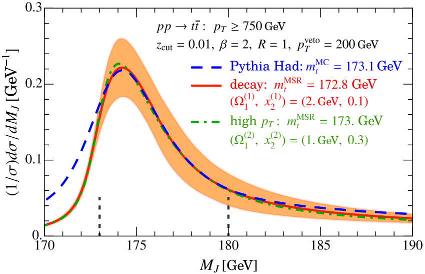
[Hoang, Mantry, Pathak, Stewart]

Soft Drop Top Mass

- All orders factorization theorem for Soft Drop Top Mass in *pp*:

$$\frac{d^2\sigma}{dM_J^2 d\mathcal{T}^{\rm cut}} = {\rm tr}\big[\hat{H}_{Qm}\hat{S}(\mathcal{T}^{\rm cut}, Qz_{\rm cut}, \beta, \ldots)\otimes F\big]\otimes J_B \otimes \mathcal{I}\mathcal{I} \otimes ff$$
$$\times \left\{ \int d\ell dk \, J_B\Big(\hat{s}_t - \frac{Q\ell}{m}, \Gamma_t, \delta m\Big) S_C\left[\ell - \Big(\frac{k^{2+\beta}}{2^\beta Q z_{\rm cut}}\Big)^{\frac{1}{1+\beta}}, Qz_{\rm cut}, \beta\right] F_C(k)\right\}$$

Soft Drop Top Mass NLL



- Calculation can be extended to NNLL for a precision top mass extraction $\Longrightarrow$ would be a massive success for jet substructure.

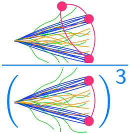
[Hoang, Mantry, Pathak, Stewart]

Groomed Two-Prong Observables

Analytic Boosted Boson Discrimination at the LHC

- Groomed mass measured and theoretically well understood.
- Groomed two-prong (probe three emissions) observables are next step:
    - Grooming and perturbative mass cut make them behave like a protected event shape inside the jet.
    - Clean even in LHC environment.
    - Interesting for QCD studies e.g. α_s, parton shower studies, etc
    - Understanding can be used as input into searches.
- Start with D_2:

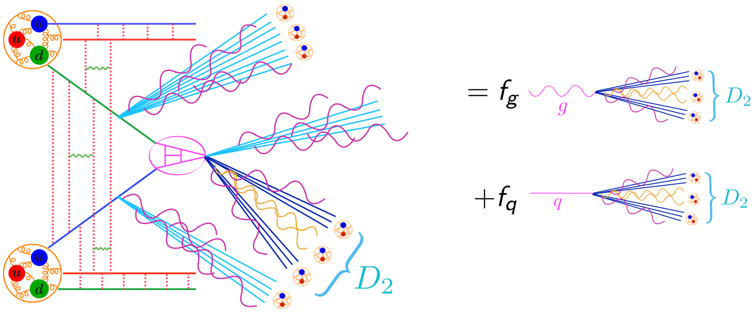
$$D_2^{(\beta)} = \frac{e_3^{(\beta)}}{\left(e_2^{(\beta)}\right)^3} =$$



[Larkoski, Moult, Neill]

Analytic Calculations with Grooming

- Grooming removes all color correlations.

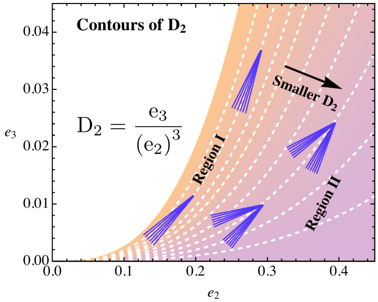


- Focus on two-prong substructure gives you "boosted e^+e^- event shapes"
- Radiation inside dipole is a remarkably clean, isolated system.

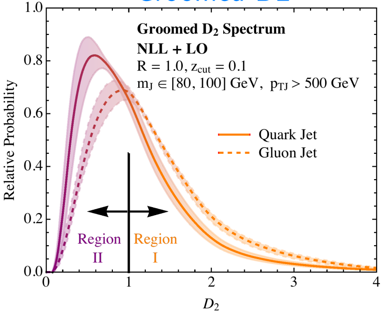
Perturbative Structure

- Calculation performed by piecing together different EFT descriptions.

Groomed D_2 Phase Space



Groomed D2



- Completely cover all asymptotic regions of phase space.

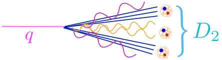
Non-Perturbative Behavior

- Non-perturbative effects are non-negligible, but under control.
- Provide interesting probe of color flow of non-perturbative radiation emitted from a dipole, and effects of grooming on this radiation.
- Would be very interesting to measure.

- Non-perturbative effects exhibit a number of interesting features
  - Non-perturbative power corrections are suppressed by the jet mass
  - Negligible contribution from MPI/Underlying Event
  - Independent of rest of event: Depends only on quark or gluon fraction, jet mass.
  - Independent of quark or gluon.

Non-Perturbative Behavior

- Non-Perturbative correction controlled by perturbative jet mass
  $\implies$ behaves like a (boosted) event shape with $Q = m_J$!

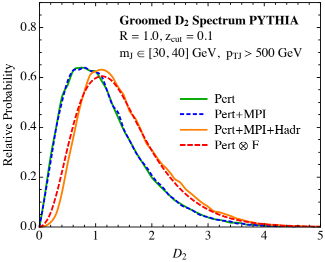


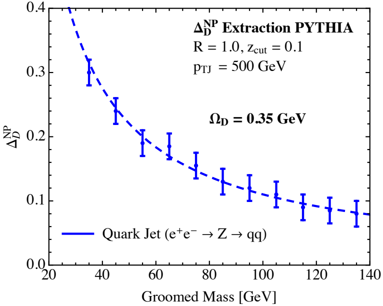
$$\frac{d\sigma_{\text{NP}}}{dD_2} = \int_0^\infty d\epsilon\, F(\epsilon) \frac{d\sigma}{dD_2}\left(D_2 - \frac{\epsilon}{m_J z_{\text{cut}}^{3/2}}\right)$$

$$F(\epsilon) = \frac{4\epsilon}{\Omega_D^2} e^{-2\epsilon/\Omega_D}$$

$$\Delta_D^{\text{NP}} = \frac{\Omega_D}{m_J z_{\text{cut}}^{3/2}}$$

First Moment Shift

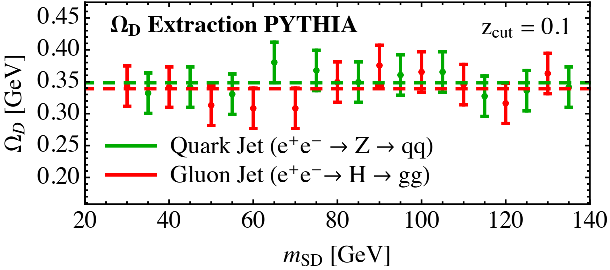


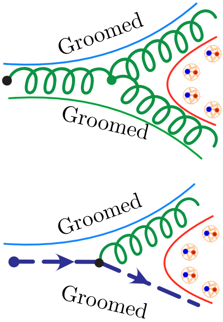


Non-Perturbative Behavior

- Collinear-soft radiation emitted from dipole:
  $\Longrightarrow$ In large N_c limit, independent of quark or gluon.
- Well born out in parton shower Monte Carlo.

NP Parameter Extraction

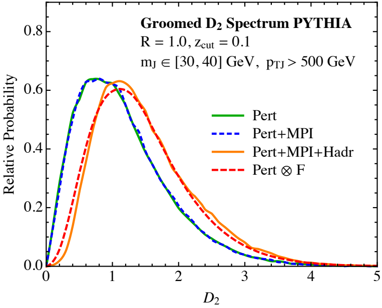




Non-Perturbative Behavior

- Contribution from MPI/Underlying Event completely negligible.
- Non-perturbative corrections are from hadronization within the jet.

Groomed D2

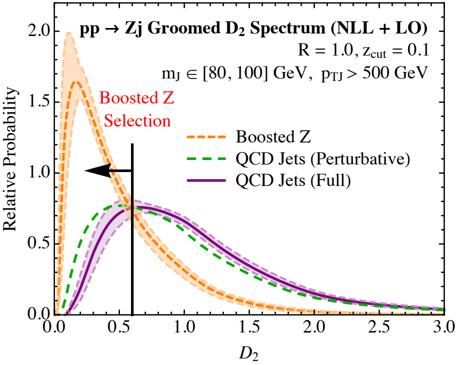


- Good control of non-perturbative contributions.

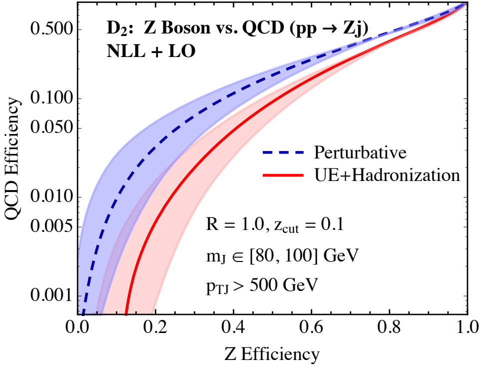
Analytic Boosted Boson Discrimination at the LHC

- Calculation of groomed D_2 at the LHC.

Groomed D_2 Distribution



Discrimination Power

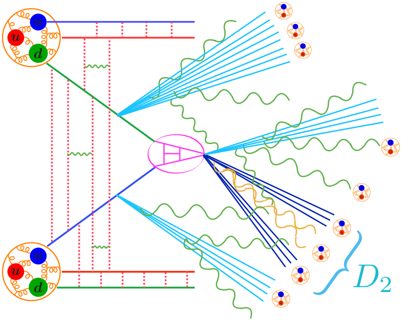


- Analytic understanding of modern jet substructure tools at LHC!
- Precision measurements useful!

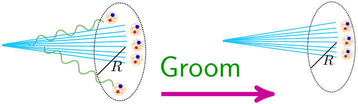
Hadronic Event Shapes

Analytic Calculations with Grooming

- Grooming has both pros and cons.



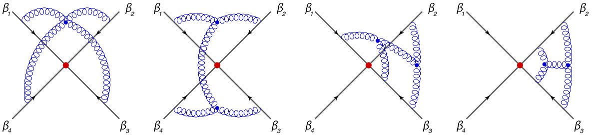
- Global color correlations
- Hadronization corrections
- Pile-Up
- Underlying event



- If you remove soft radiation, you cannot learn about it.
- Effects associated with soft radiation are least understood theoretically.

Color Evolution

- With four directions, one has "quantum color evolution".



- Grooming destroys this, leading to classical color evolution.
- At 3 loops, one encounters for the first time a "color quadrupole":

$$\Delta_n^{(3)}\left(\{\rho_{ijkl}\}\right) = 16\, f_{abe} f_{cde} \Big\{ \sum_{1\le i<j<k<l\le n} \Big[\mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_k^c \mathbf{T}_l^d \,\mathcal{F}(\rho_{ikjl}, \rho_{iljk}) + \mathbf{T}_i^a \mathbf{T}_k^b \mathbf{T}_j^c \mathbf{T}_l^d \,\mathcal{F}(\rho_{ijkl}, \rho_{ilkj}) + \mathbf{T}_i^a \mathbf{T}_l^b \mathbf{T}_j^c \mathbf{T}_k^d \,\mathcal{F}(\rho_{ijlk}, \rho_{iklj}) \Big] - C \sum_{i=1}^{n} \sum_{\substack{1\le j<k\le n \\ j,k\ne i}} \left\{ \mathbf{T}_i^a, \mathbf{T}_i^d \right\} \mathbf{T}_j^b \mathbf{T}_k^c \Big\},$$

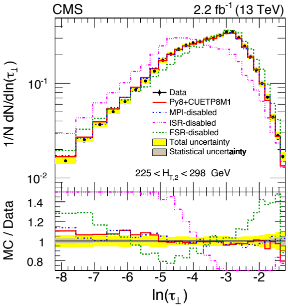
[Almelid, Duhr, Gardi]

- Can we get to this accuracy/ see this in a physical observable?

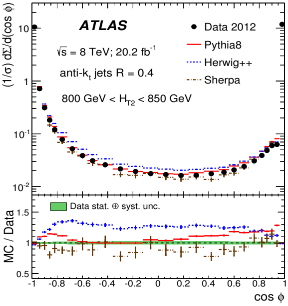
Choice of Observable

- Event Shapes are not all created equal.
- From increased study of observables, now much better understood which have properties enabling high order calculations.

Transverse Thrust in CMS

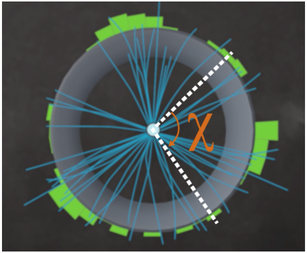


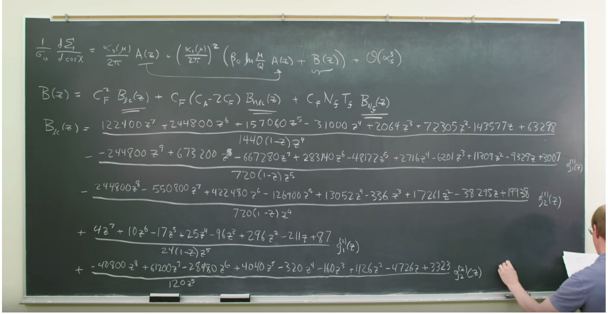
TEEC in ATLAS



Transverse Energy Energy Correlator

- Extensive ongoing theoretical work on understanding TEEC.
- See Lance Dixon at https://www.youtube.com/watch?v=WVC1ygsjZNc



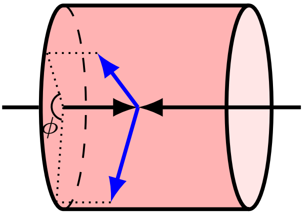


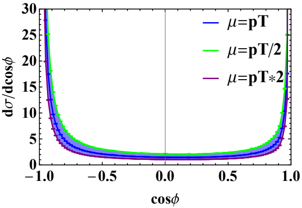
- Is now most accurately known dijet event shape NLL → N^{3}LL.

[Dixon, Gao, Li, Moult, Zhu]

Transverse Energy Energy Correlator

- TEEC interesting in both collinear and back-to-back region.
  - Back-to-back: color quadrupoles (Sudakov logarithms)
  - Collinear: Non-trivial probe of splitting (single logarithms).

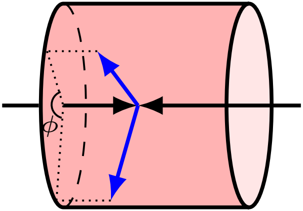


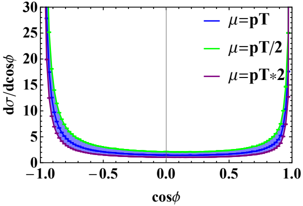


- ATLAS measurement currently uses NLO fixed order.
- Can be improved significantly (numerics not finished today:( ).
- Measurement with very fine bins would be nice!

TEEC in Collinear Limit

- TEEC in collinear ($\phi \to 0$) limit exhibits highly interesting behavior.
  - Is a self grooming, single logarithmic observable.
  - Does not exhibit Casimir scaling.
  - Can be resummed to high perturbative orders. [Dixon, Moult, Zhu]





- Has received almost zero experimental or theory interest.
- Would be very interesting to measure as a jet substructure observable.

TEEC in Collinear Limit

- Splitting function can be written as a classical piece + quantum piece:

$$= 4C_i \frac{\alpha}{4\pi}\left(\frac{x}{1-x} + (1-x)g_i(x)\right), \qquad \left\{ \begin{array}{l} g_\phi(x) = 0 \\ g_\lambda(x) = \frac{1}{2} \\ g_V(x) = x + \frac{1}{x} \end{array} \right. . \qquad (1)$$

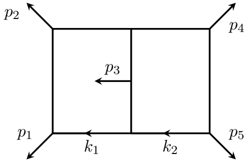
- All perturbatively calculable jet substructure observables that I am aware of (both groomed and ungroomed) can be computed at LL from the classical piece $\implies$ Casimir Scaling $\implies$
  Theorem: For an observable whose LL result can be computed using the eikonal splitting functions of (1) in the independent emission approximation (with or without running coupling), this LL result can not achieve better quark/gluon discrimination than multiplicity.
  [Metodiev, Moult, Nachman, Prestel]

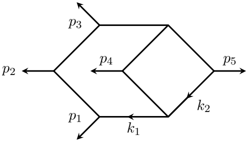
- Does not hold for TEEC. Is sensitive to "quantum" structure of splitting. Promising as q vs. g discriminant in simulation.
  [Dixon, Moult, Zhu]

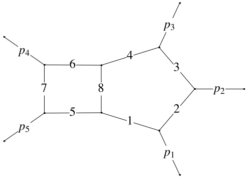
(Near?) Future Prospects

NNLO Jet Substructure/ Hadronic Event Shapes

- Next jump in precision will require $2 \to 3$ amplitudes at NNLO
  $\Longrightarrow$ substructure (mass) of jet described at NNLO!
  $\Longrightarrow$ hadronic event shape described at NNLO!
- Significant recent progress and numerical results
  [Abreu, Febres Cordero, Ita, Page, Sotnikov], [Gehrmann, Henn, Lo presti]





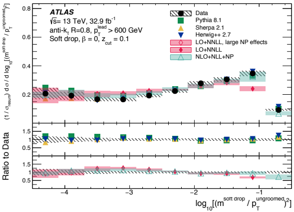


- Will be a game changer enabling precision substructure at the LHC!

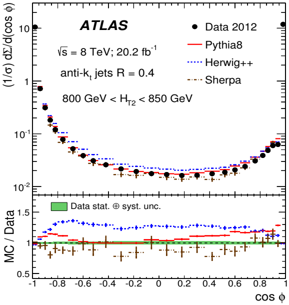
NNLO

- In the short term, only a few observables at hadronic colliders will get to NNLO+N^{3}LL.
- Two prime candidates are
  - Groomed Jet Mass
  - Transverse Energy-Energy-Correlator

Soft Drop Mass in ATLAS

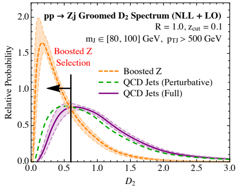


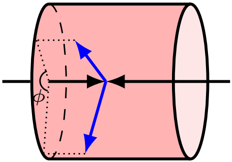
TEEC in ATLAS



Conclusions

- Jet substructure provides novel ways for studying hadronic final states.
- Grooming is useful, but don't groom everything!
- Hope for precision SM physics from substructure in the future.





Thanks!