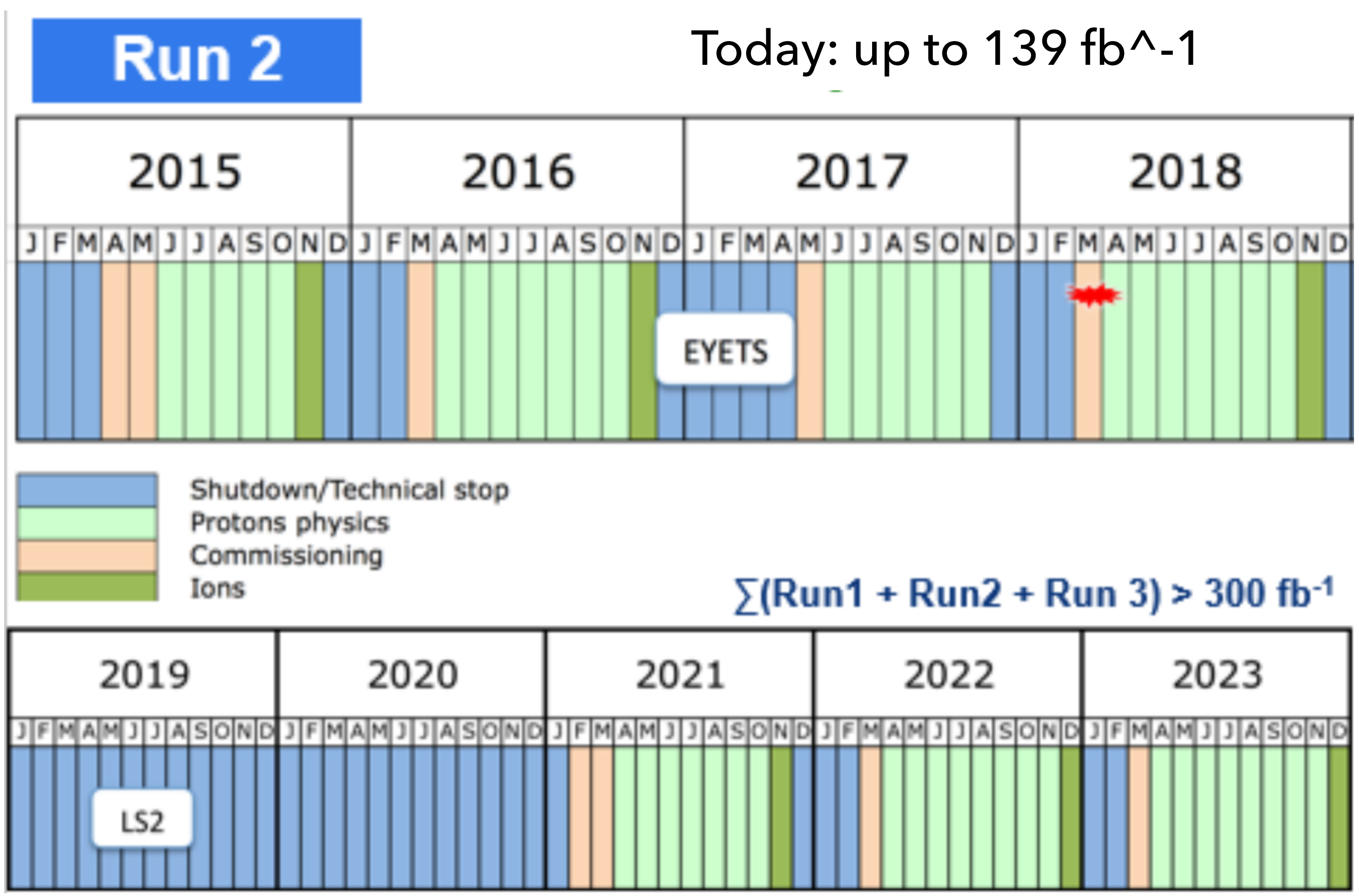
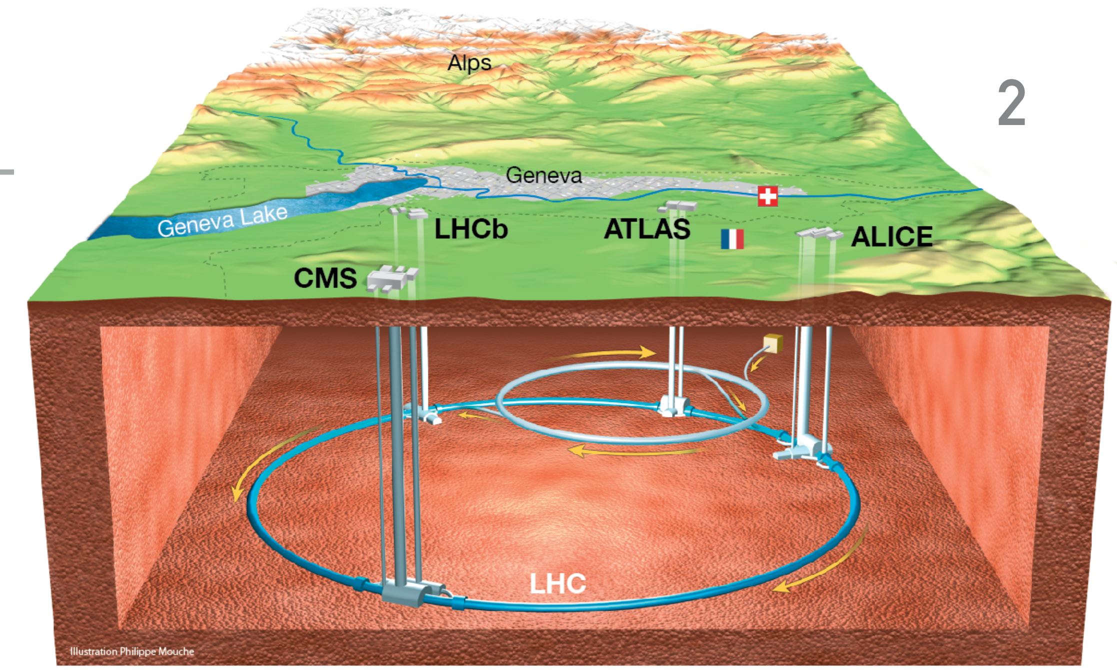
BERNHARD MISTLBERGER

FIXED ORDER PRECISION FOR LHC PHENOMENOLOGY

THE LHC - AN INCREDIBLE SUCCESS

- Running since 2009
- Experimental performance excellent and exceeding expectations!





Period	Integrated Luminosity [fb^{-1}]
Run 1	29.2
Run 2: 2015	4.2
Run 2: 2016	39.7
Run 2: 2017	50.2
Run 2: 2018	66.0
Total Run1 + Run 2	189.3

- **We are still at the beginning of LHC physics!**
- **300 fb^-1 until end of 2023**
- **3000 fb^-1 in HL - LHC**

THE LHC – AN INCREDIBLE SUCCESS

- 4th of July 2012: **The Higgs Age begins!**

The quest p.H.

✳ **Explore a never before observed interaction: Yukawa!**

✳ **Gain insight in the mechanism of electro-weak symmetry breaking**

✳ **Investigate the generation of fundamental masses**

✳ **Determine couplings / interactions with established matter**

H ♡ μ ?

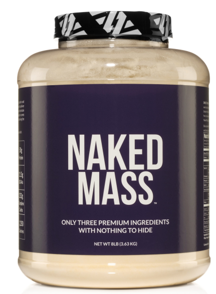
W ♡ W ♡ W ♡ W ?

✳ **Explore the limitations of the Standard Model of particle physics.**

hic svnt dracones

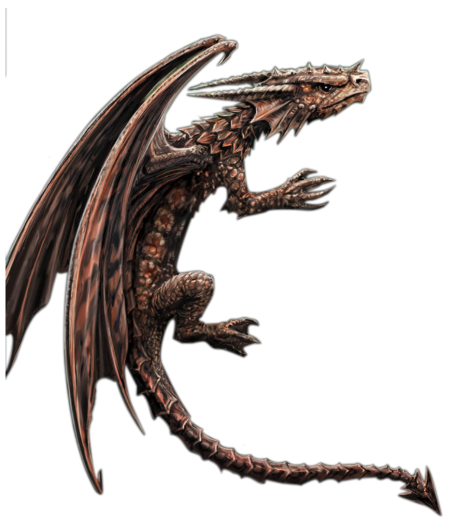
barrysworld.biz

THE LHC - AN INCREDIBLE SUCCESS



H ♡ μ ?

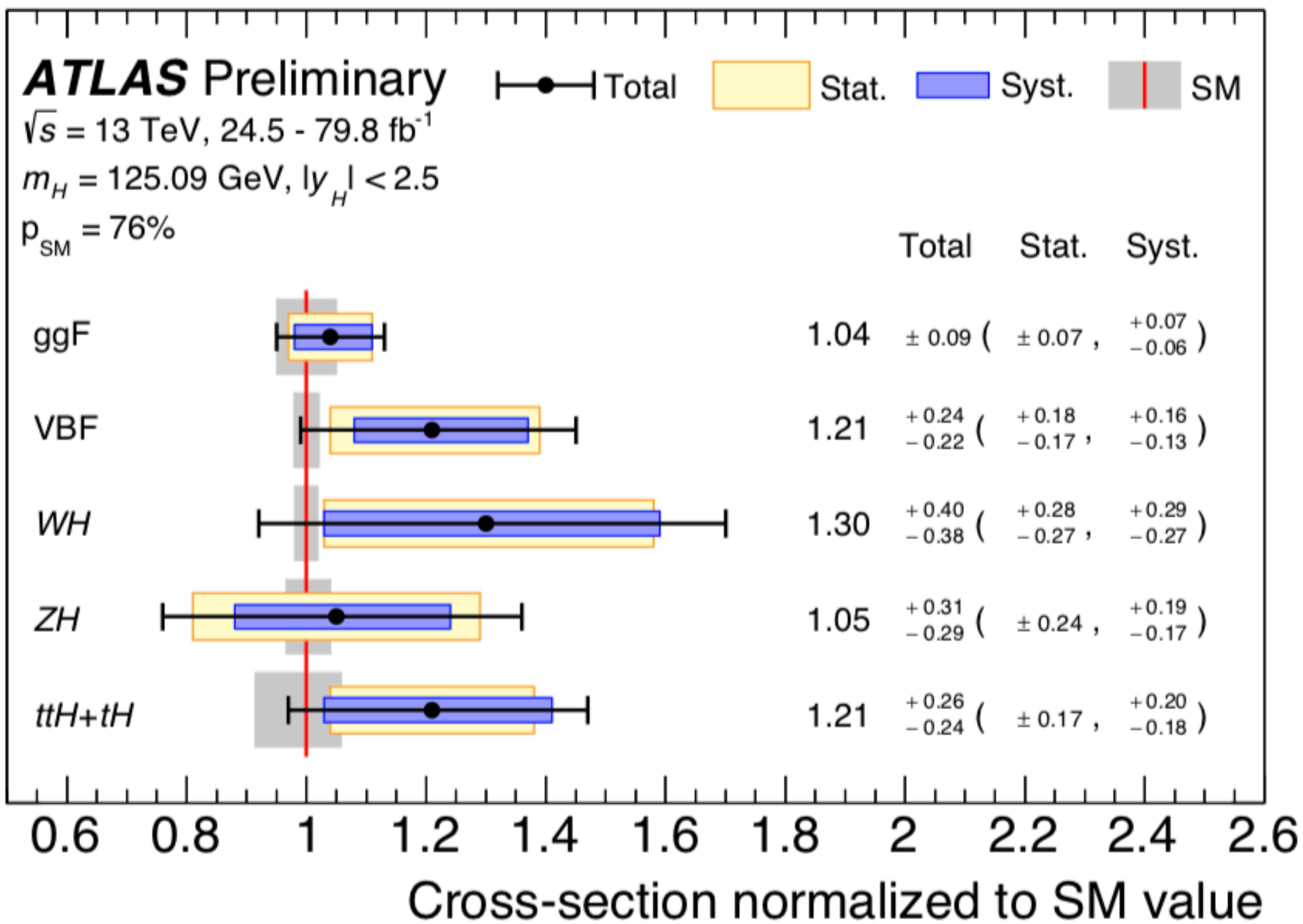
W ♡ W ♡ W ♡ W ?

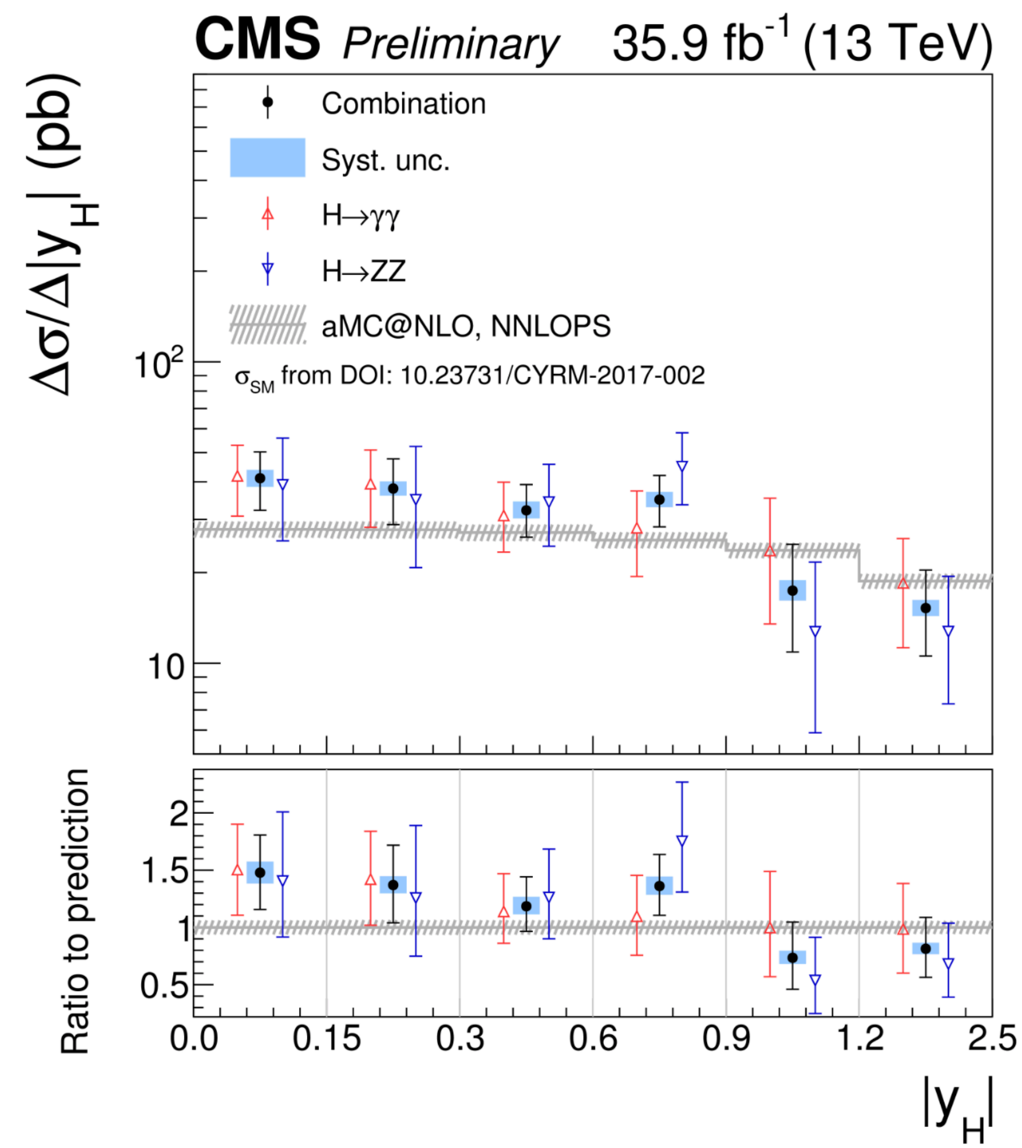


The Method: Predict & Compare.

Precision is key!

CURRENT STATUS





Physics at 10 % level

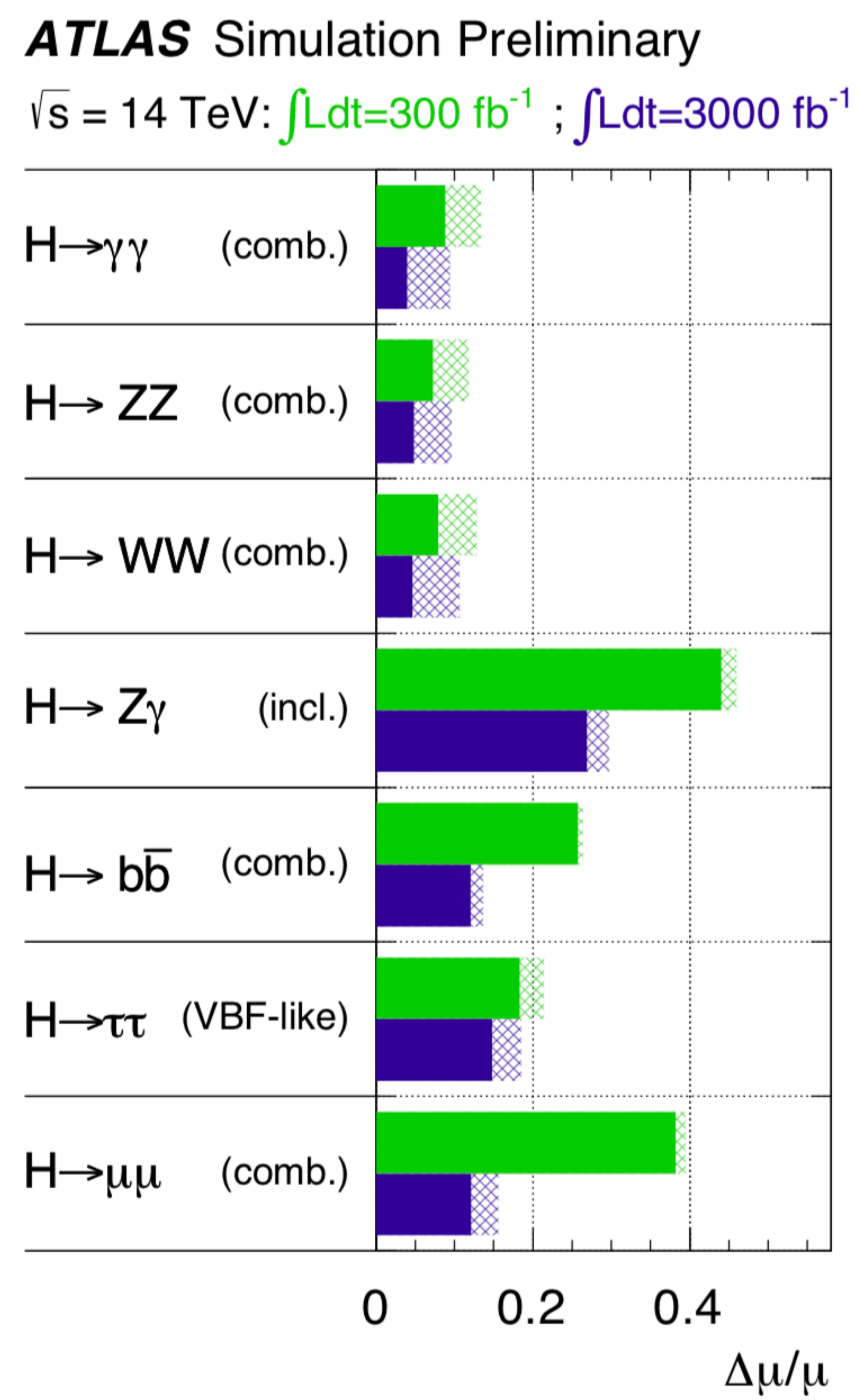
THE FUTURE – 3000 FB^-1

Inclusive signal strength

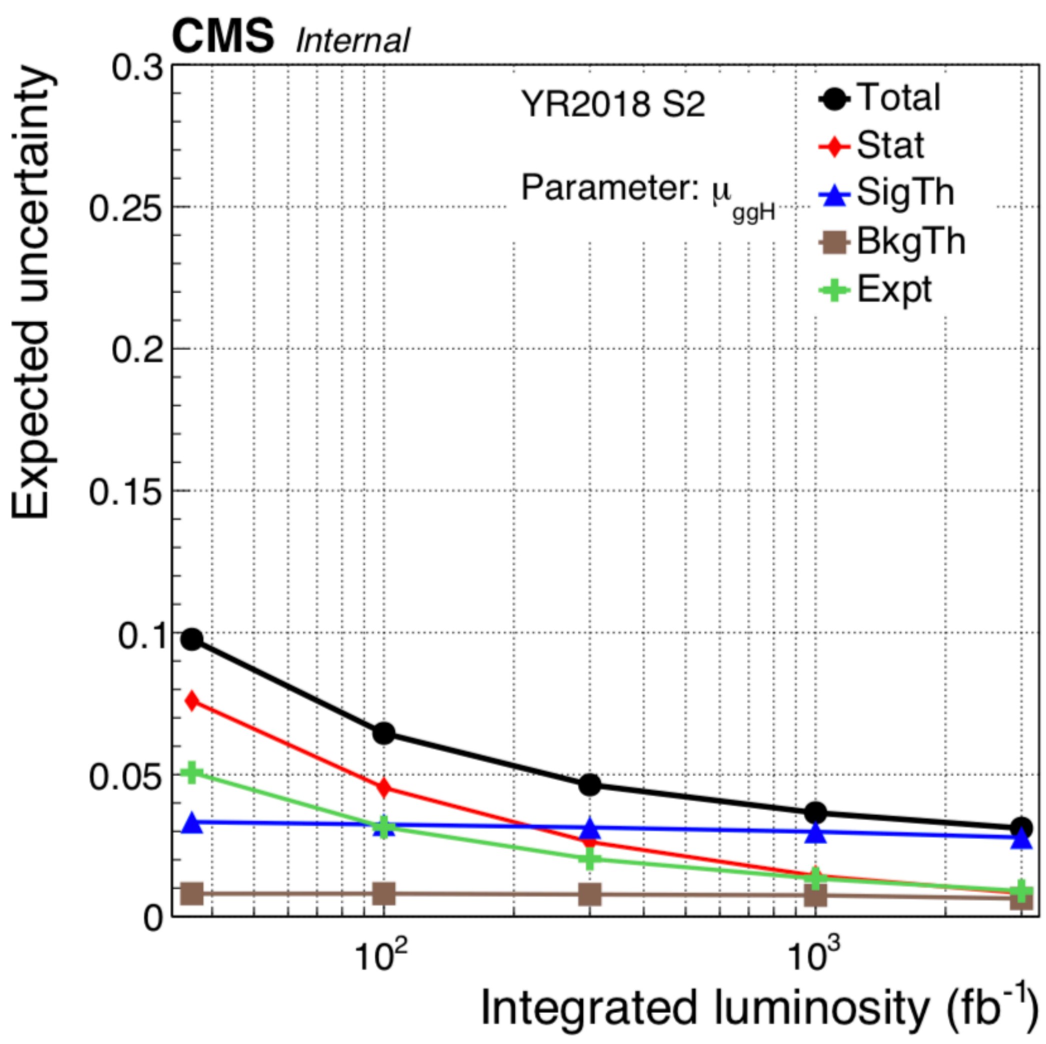
Projections

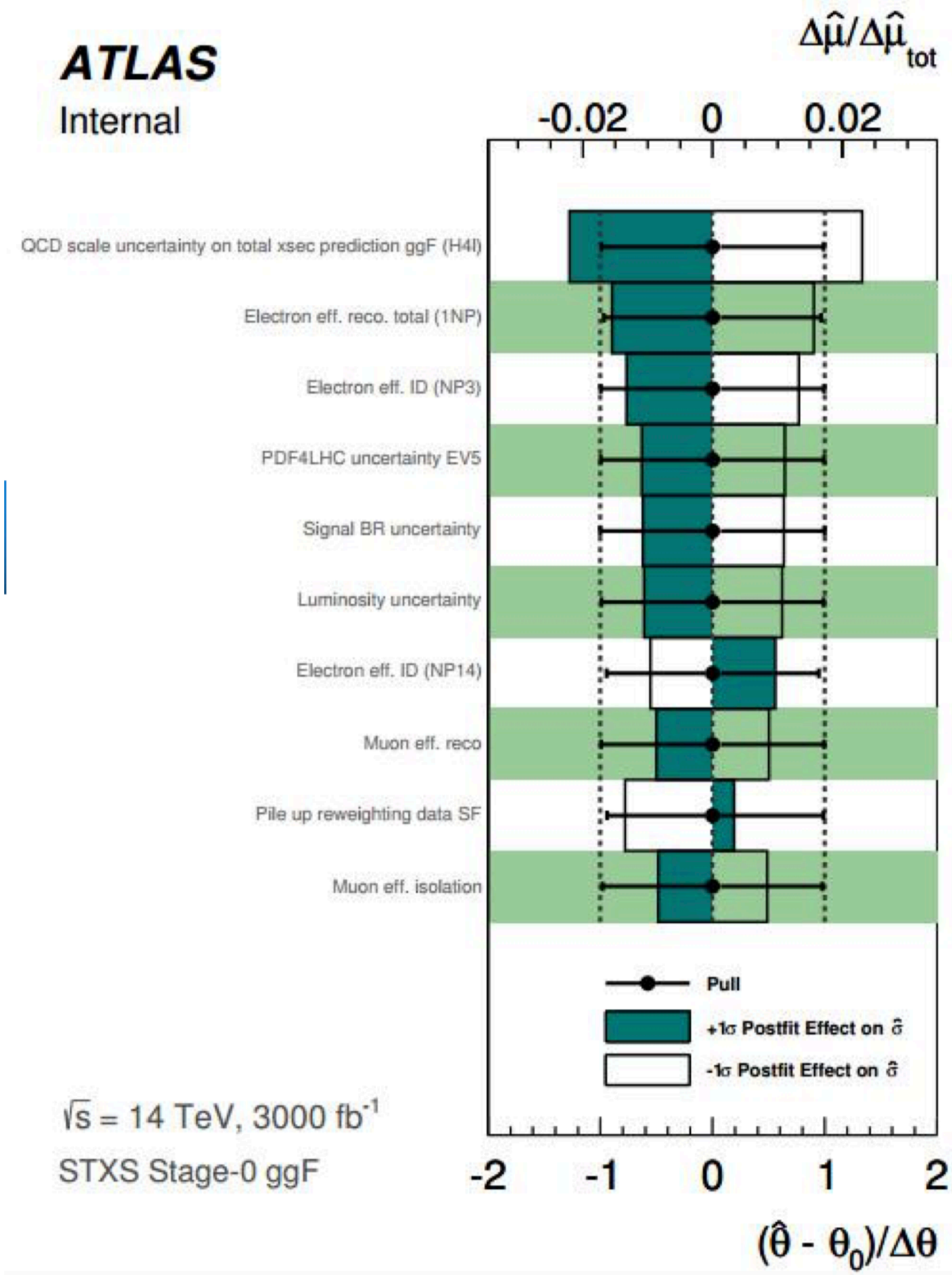
Relative uncertainty	Total	Stat	Exp.
S1	3.5%	0.6%	1.6%
S2	2.4%	0.6%	1.3%

- Luminosity at 1 %
- Couplings better then 5%
- Differential Cross Sections get precise



THE FUTURE – 3000 FB^-1

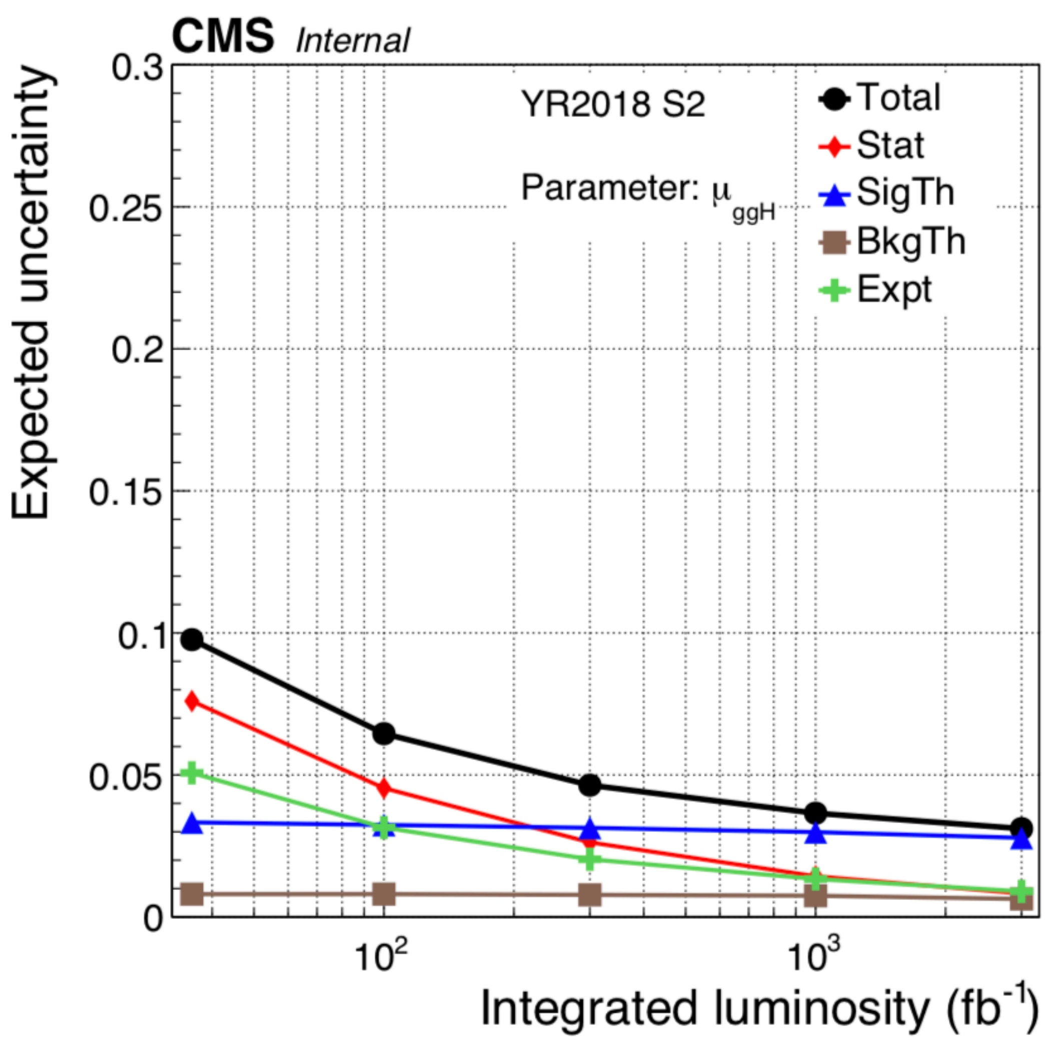


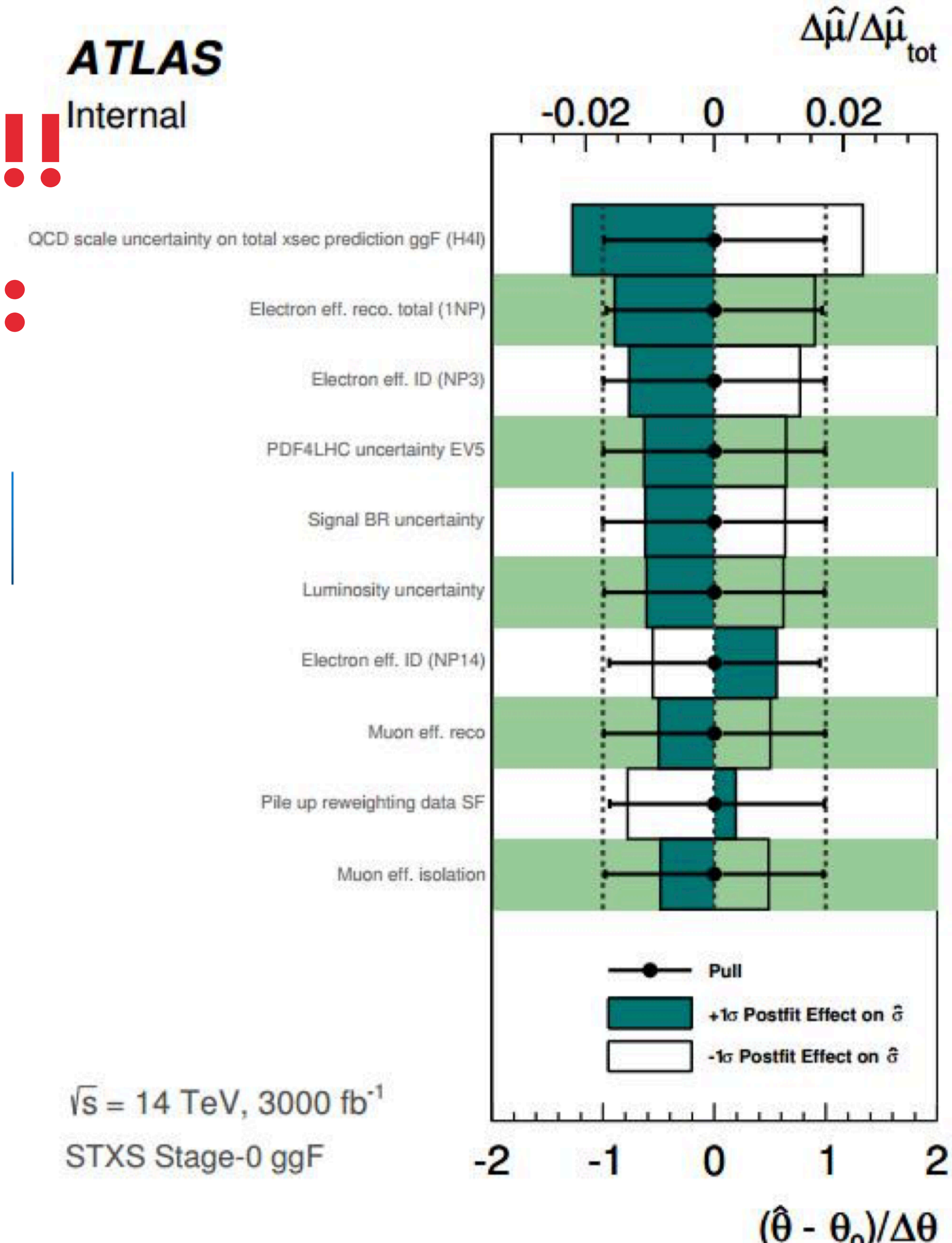


THE FUTURE – 3000 FB^-1

Theory uncertainties!!!

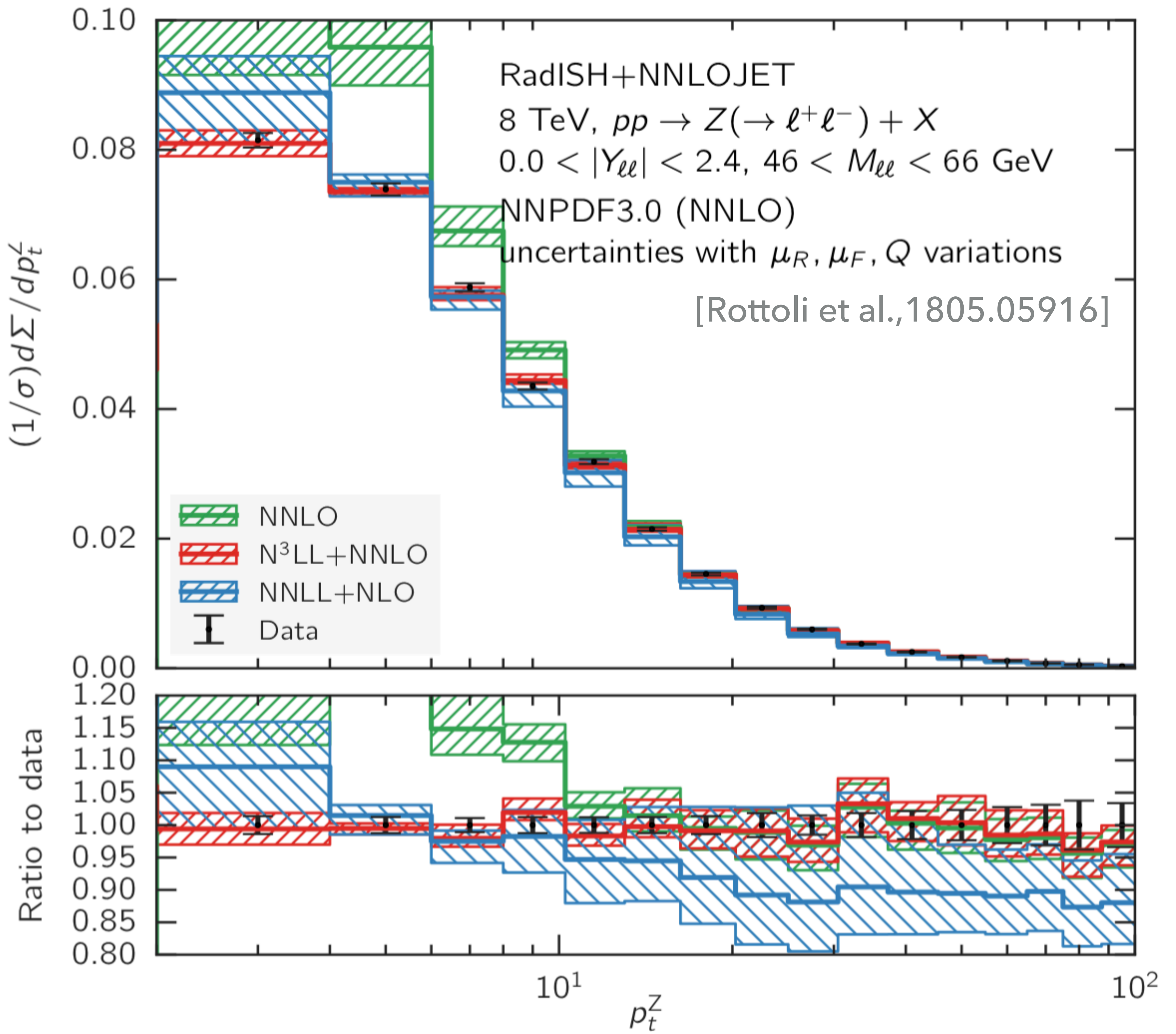
OPTIMISTIC Scenario:





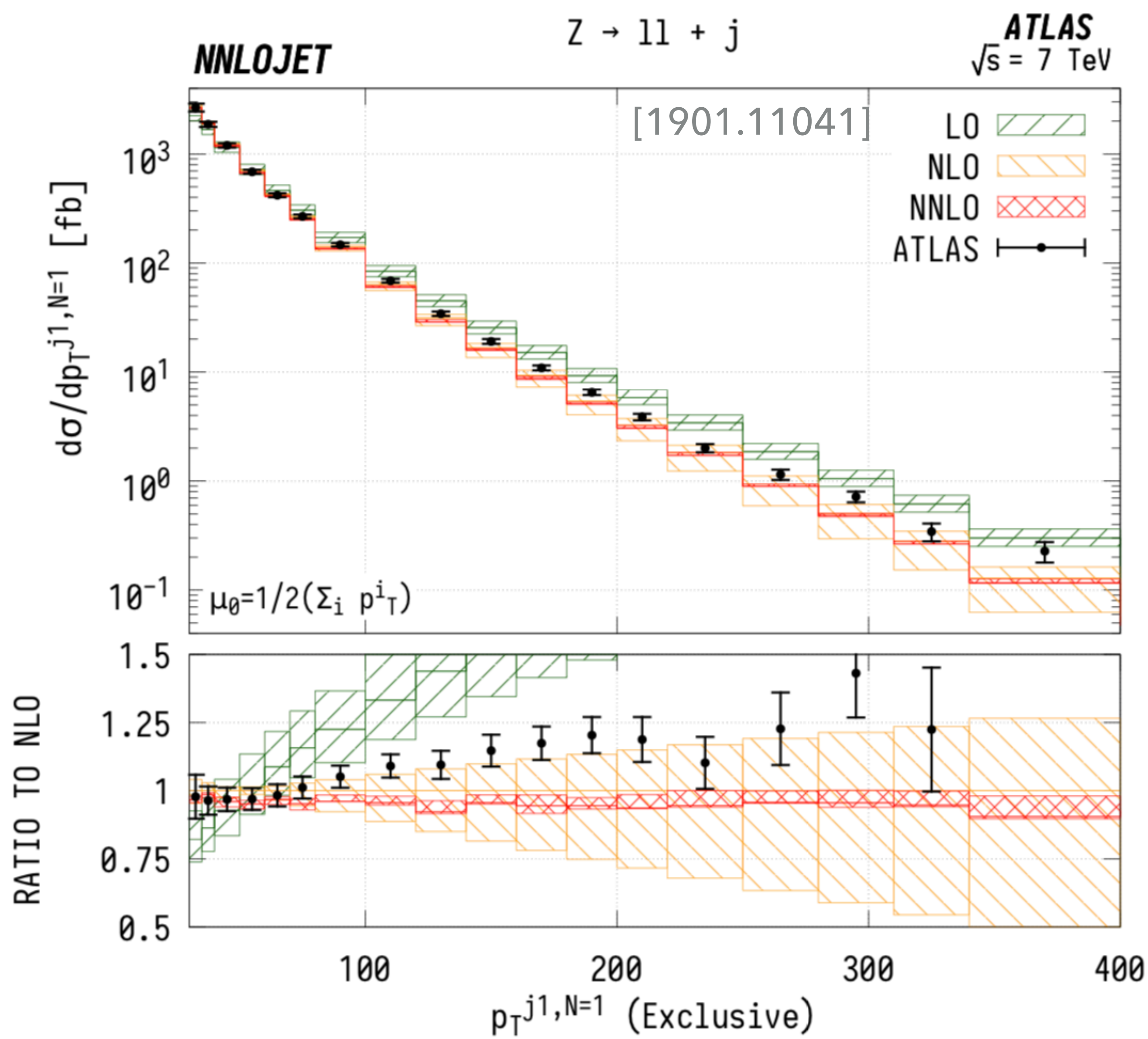
CURRENT STATUS – PRECISION DY

- Z - pT: One of the most precise LHC observables.
- Comparable uncertainties at 8 TeV.



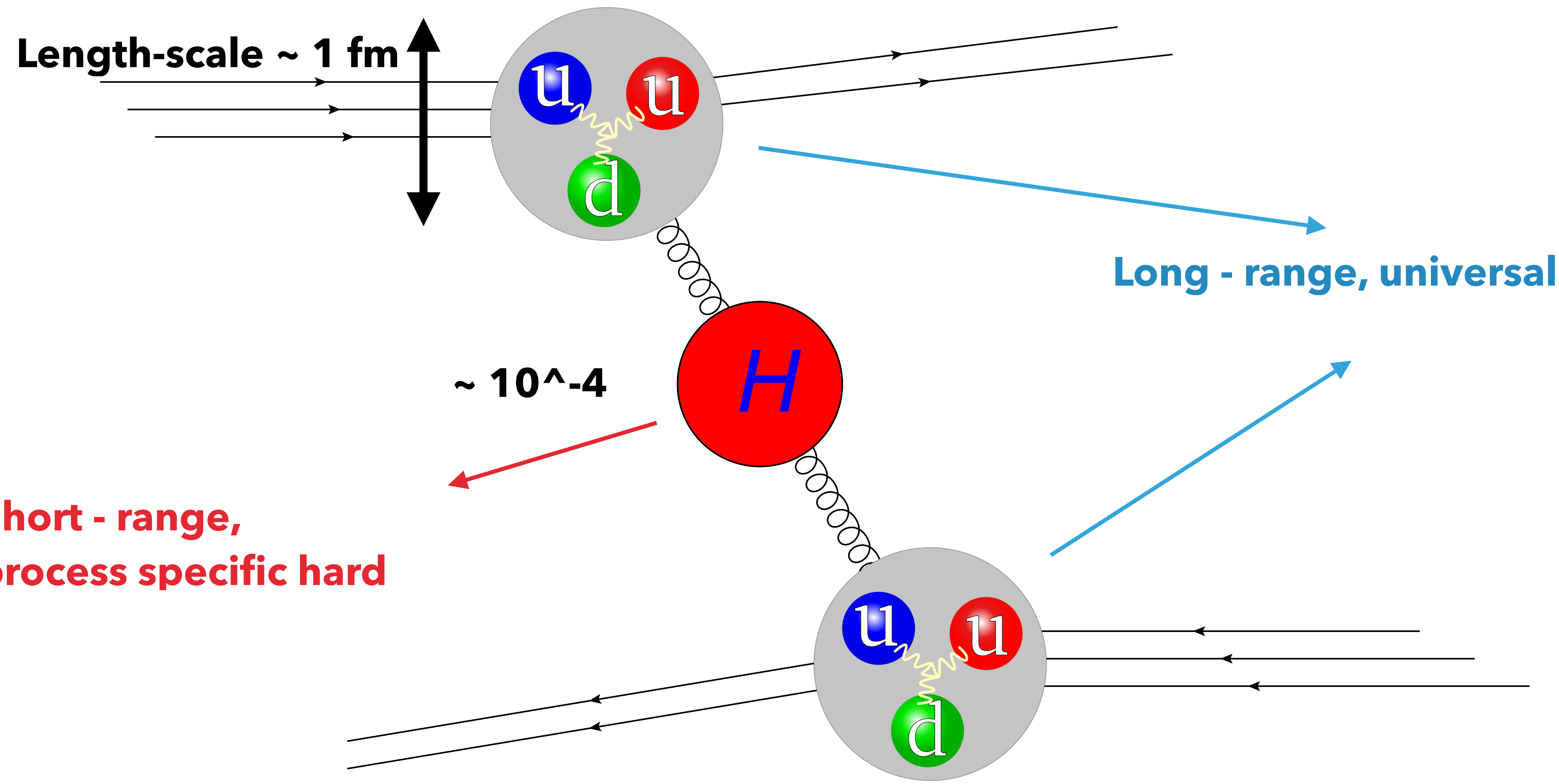
CURRENT STATUS - PRECISION DY

- Leading Jet pT
- NNLO sees slight increase of data above prediction. Calibration??
- Precision for SM process is **essential**.
- Predict as close as possible to the experimental measurement - **Fiducial XS**



TOWARDS PREDICTIONS

THE WAY TO PRECISION LHC PREDICTIONS



FACTORISATION

$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$ [Iain's talk]

- Intrinsic limitation = level of target precision?

THE WAY TO PRECISION LHC PREDICTIONS

$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

- Perturbative approach to computing partonic cross sections.
- QCD perturbation theory is dominant $\alpha_S = 0.118$
- Naively:

LO **NLO** **NNLO** **N3LO**

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(0)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \ldots$$

10% 1% 0.1%

- Resum and match where fixed order breaks down

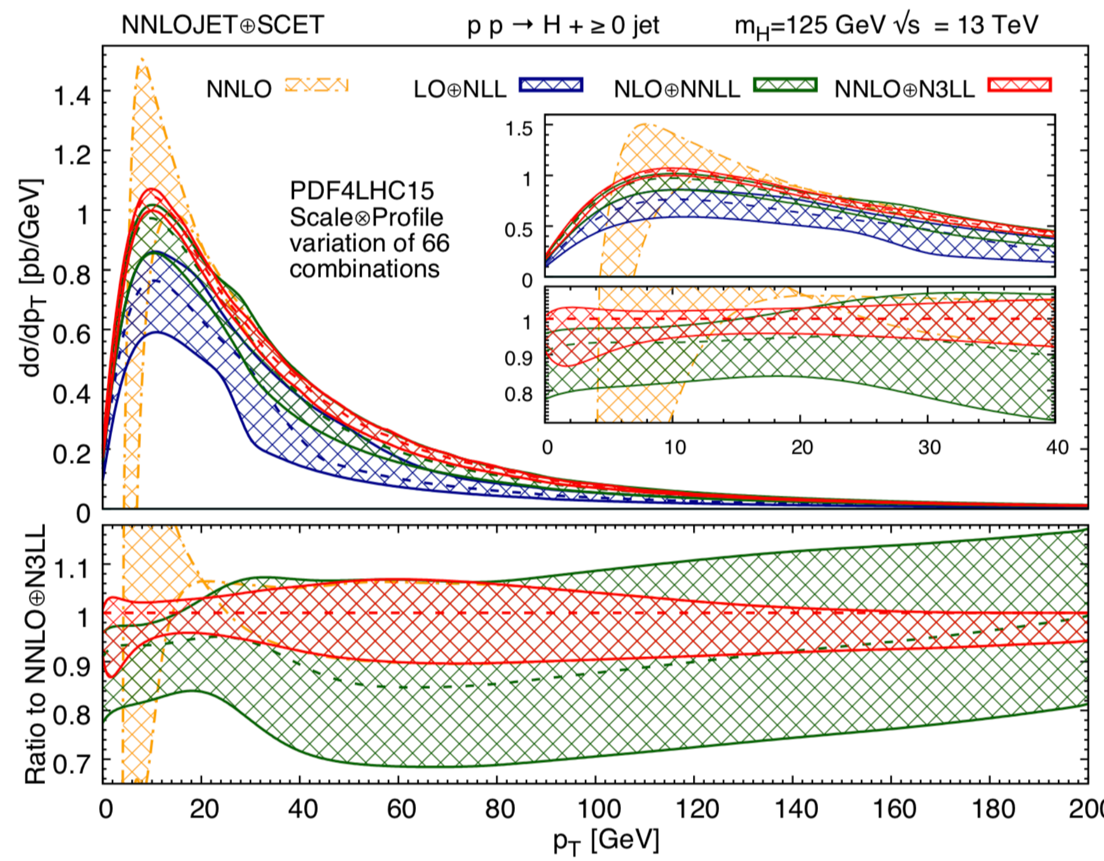
REQUIRED INGREDIENTS FOR PREDICTIONS

Resummation!

- Fixed order perturbation theory breaks in kinematic edges of phase - space: Re-order the series!
- Lots of progress!

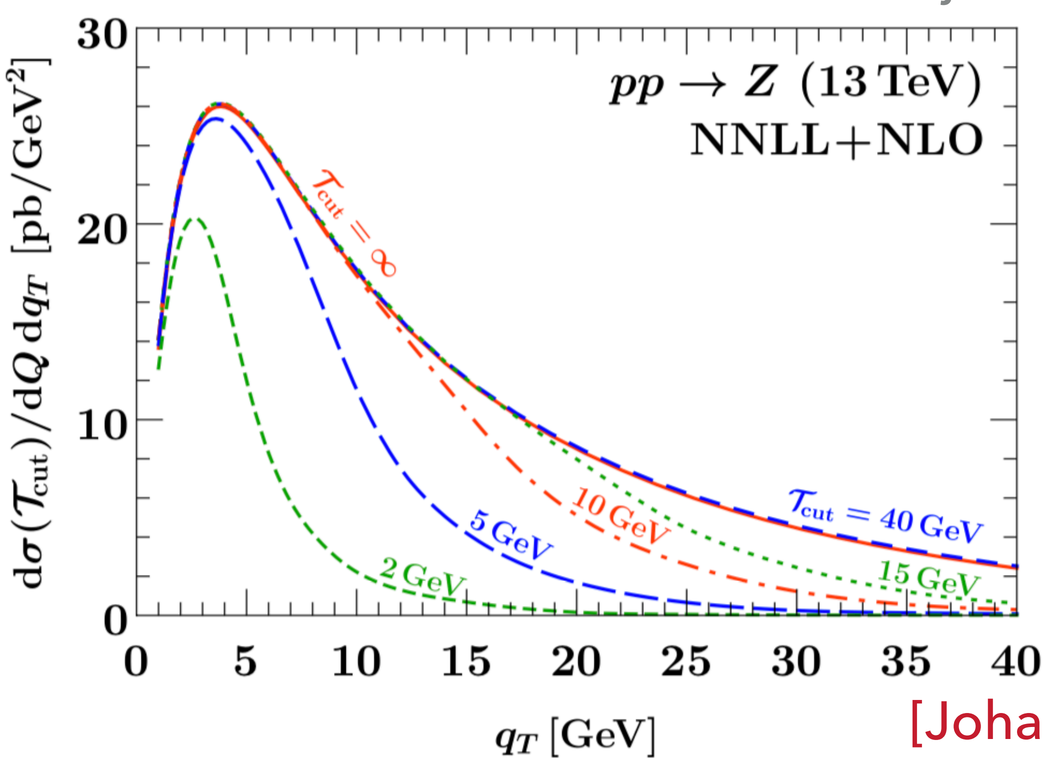
N3LL+NNLO Higgs pT

[Neill, Stewart, Zhu et al., 1805.00736]



NNLL double resum: Tau + pT

[Lustermans , Michel, Tackmann, Waalewijn, 1901.03331]

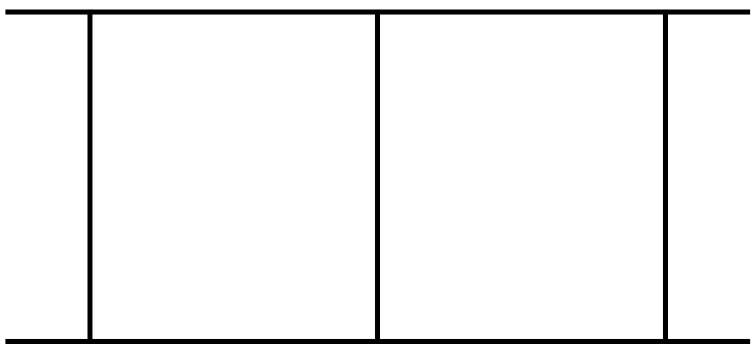


[Johannes' talk]

REQUIRED INGREDIENTS FOR PREDICTIONS

Virtual Corrections!

$$gg \to \gamma\gamma$$



$$\begin{aligned}
G^{\mathrm{L}}_{-+-+} = &\left(1-2\frac{x}{y^2}\right)\Big[4\mathrm{Li}_4(-y/x)+4\mathrm{Li}_4(-y)+(3X-2Y+i\pi)\mathrm{Li}_3(-y/x)\\
&\quad -(X+2Y+3i\pi)\mathrm{Li}_3(-y)+((X-Y)^2+\pi^2)\mathrm{Li}_2(-y/x)\\
&\quad +((Y+i\pi)^2+\pi^2)\mathrm{Li}_2(-y)+\frac{1}{8}X^2(X-2Y)^2\\
&\quad -i\frac{\pi}{6}X\Big((X+i\pi)^2-3XY\Big)\Big]\\
&-\frac{1}{2}\left(1+6\frac{x}{y^2}\right)\Big[\mathrm{Li}_3(-x)-\zeta_3-(X+i\pi)\Big(\mathrm{Li}_2(-x)-\frac{\pi^2}{6}\Big)\\
&\quad -\frac{1}{6}X(X^2+4\pi^2)+\frac{1}{2}(X-2Y-i\pi)((X+i\pi)^2+\pi^2)\Big]\\
&-\frac{1}{12}\left(5-2\frac{x}{y}\right)(X+i\pi)((X+i\pi)^2+3\pi^2)+(X-Y)((X+i\pi)^2+\pi^2)\\
&+\pi^2(X+i\pi)+\frac{1}{8}\left(14\frac{x-1}{y}-8y+9y^2\right)((X-Y)^2+\pi^2)\\
&+\frac{1}{8}\left(14\frac{1-x}{y}-8\frac{y}{x}+9\frac{y^2}{x^2}\right)((Y+i\pi)^2+\pi^2)\\
&+\frac{1}{8}\left(38\frac{x}{y^2}-13\right)((X+i\pi)^2+\pi^2)\\
&-\frac{\pi^2}{6}-\frac{9}{4}\Big[\left(y+2\frac{x}{y}\right)(X-Y)-\left(\frac{y}{x}+\frac{2}{y}\right)(Y+i\pi)\Big]+\frac{1}{2}\,,
\end{aligned}\qquad (5.6)$$

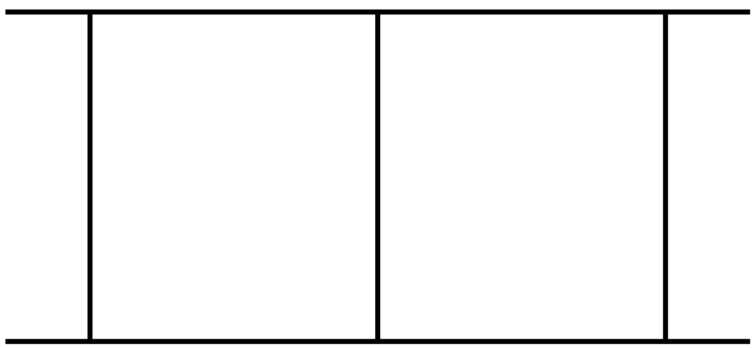
[Bern,Dixon,Freitas]

REQUIRED INGREDIENTS FOR PREDICTIONS

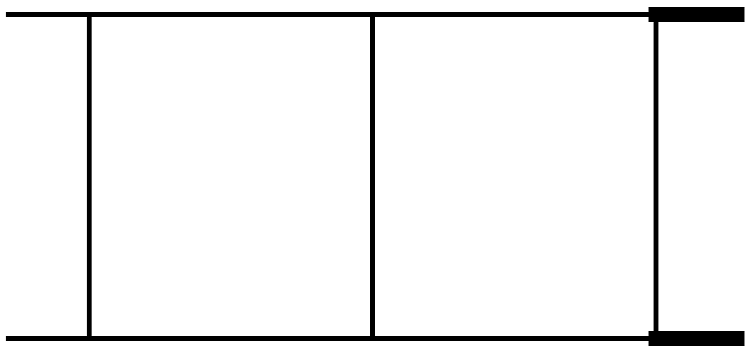
Virtual Corrections!

Rapid rise in complexity

$gg \to \gamma\gamma$



$q\bar{q} \to WW$



$$
\begin{aligned}
G^{\mathrm{L}}_{-+-+} = {} & \left(1 - 2\frac{x}{y^2}\right)\Big[4\mathrm{Li}_4(-y/x) + 4\mathrm{Li}_4(-y) + (3X - 2Y + i\pi)\mathrm{Li}_3(-y/x) \\
& \quad -(X + 2Y + 3i\pi)\mathrm{Li}_3(-y) + ((X - Y)^2 + \pi^2)\mathrm{Li}_2(-y/x) \\
& \quad +((Y + i\pi)^2 + \pi^2)\mathrm{Li}_2(-y) + \frac{1}{8}X^2(X - 2Y)^2 \\
& \quad -i\frac{\pi}{6}X\Big((X + i\pi)^2 - 3XY\Big)\Big] \\
& -\frac{1}{2}\left(1 + 6\frac{x}{y^2}\right)\Big[\mathrm{Li}_3(-x) - \zeta_3 - (X + i\pi)\Big(\mathrm{Li}_2(-x) - \frac{\pi^2}{6}\Big) \\
& \quad -\frac{1}{6}X(X^2 + 4\pi^2) + \frac{1}{2}(X - 2Y - i\pi)((X + i\pi)^2 + \pi^2)\Big] \\
& -\frac{1}{12}\left(5 - 2\frac{x}{y}\right)(X + i\pi)((X + i\pi)^2 + 3\pi^2) + (X - Y)((X + i\pi)^2 + \pi^2) \\
& +\pi^2(X + i\pi) + \frac{1}{8}\left(14\frac{x-1}{y} - 8y + 9y^2\right)((X - Y)^2 + \pi^2) \\
& +\frac{1}{8}\left(14\frac{1-x}{y} - 8\frac{y}{x} + 9\frac{y^2}{x^2}\right)((Y + i\pi)^2 + \pi^2) \\
& +\frac{1}{8}\left(38\frac{x}{y^2} - 13\right)((X + i\pi)^2 + \pi^2) \\
& -\frac{\pi^2}{6} - \frac{9}{4}\Big[\Big(y + 2\frac{x}{y}\Big)(X - Y) - \Big(\frac{y}{x} + \frac{2}{y}\Big)(Y + i\pi)\Big] + \frac{1}{2}, \qquad (5.6)
\end{aligned}
$$

[Bern,Dixon,Freitas]

```
Aj_A-1.0.inc
Aj_B-1.0.inc
Aj_C-1.0.inc
```

Aj_A-1.0.inc	39 MB
Aj_B-1.0.inc	39 MB
Aj_C-1.0.inc	27 MB

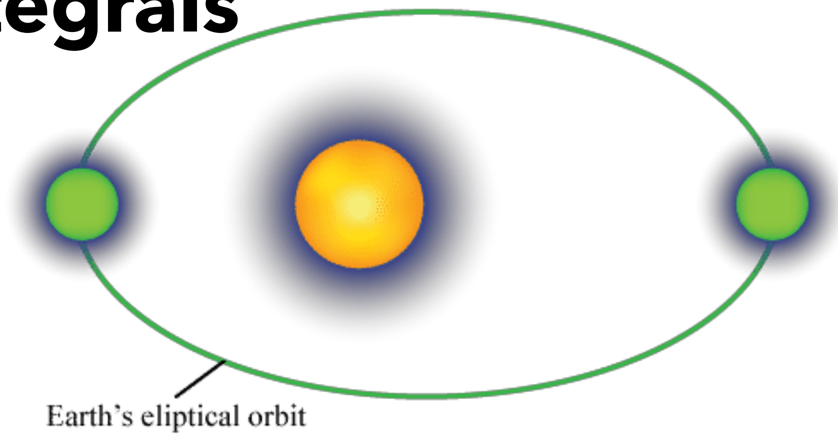
[Gehrmann, Manteuffel,Tancredi]

[Caola,Henn,Melnikov,Smirnov,Smirnov]

REQUIRED INGREDIENTS FOR PREDICTIONS

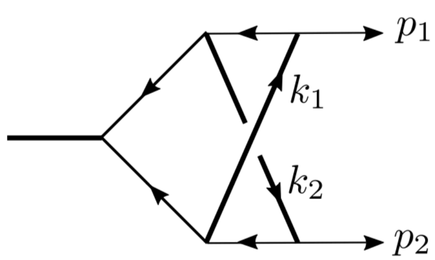
Virtual Corrections!

- Higher Orders = New Functions to be understood!
- Huge progress in understanding **Elliptic Integrals**
  - ✳ Key to just be able to compute.
  - ✳ Insights on the structure of scattering theory (cuts, thresholds, etc.)
  - ✳ Ties to string theory (propagator) and pure math.



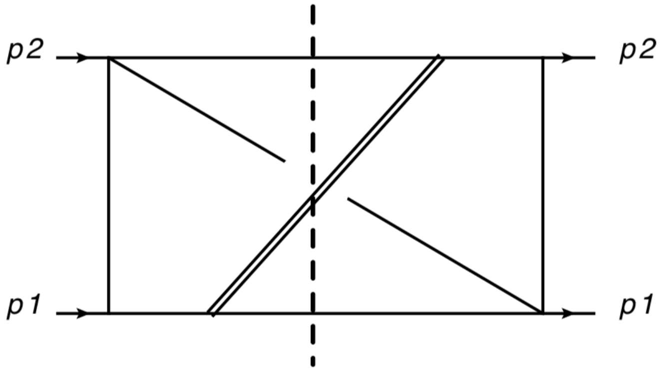
- Analytic results for phenomenology:

EWK Form Factor



[1902.09971]

Higgs Production at N3LO



[BM, 1802.00833]

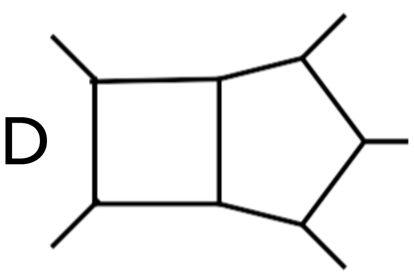
REQUIRED INGREDIENTS FOR PREDICTIONS

Virtual Corrections!

- Highly technical

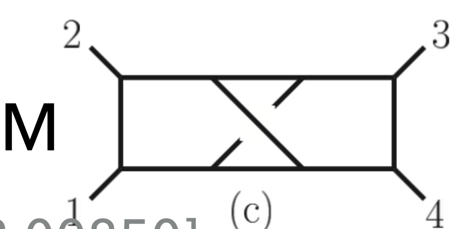
- Analytic tools are growing more powerful!
  (Laporta, Differential Equations, etc.)
- New technology is being developed
  (Finite Field arithmethics, geometric IBPs, etc.)
- Numerical techniques are on the rise.
  (SecDec, Local Subtraction, TayInt)

✳ 2-loop, 5-point planar QCD
N=4 / 8 SYM



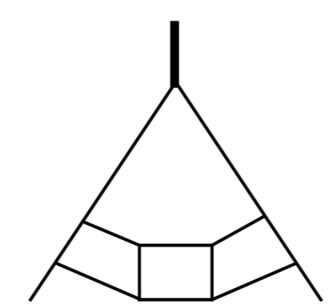
e.g. [1811.11699, 1812.04586, 1812.11057, ...]

✳ 3-loop, 4-point N=4 / 8 SYM



[Henn, BM: 1902.07221, 1608.00850]

✳ 4-loop, 3-point, planar+
QCD



e.g. [1901.02898, 1903.06171, 1612.04389]

REQUIRED INGREDIENTS FOR PREDICTIONS

Real Corrections!

- When integrating over final state parton momenta we encounter soft and collinear singularities. That causes problems - we need to regulate.

NNLO:

Inclusive DY **~ CPU seconds**

REQUIRED INGREDIENTS FOR PREDICTIONS

Real Corrections!

- When integrating over final state parton momenta we encounter soft and collinear singularities. That causes problems - we need to regulate.

NNLO:

Inclusive DY ~ **CPU seconds**

Differential DY (pT, Y, etc.) ~ **10-100 CPU hours**

REQUIRED INGREDIENTS FOR PREDICTIONS

Real Corrections!

- When integrating over final state parton momenta we encounter soft and collinear singularities. That causes problems - we need to regulate.

NNLO:

Inclusive DY	**~ CPU seconds**
Differential DY (pT, Y, etc.)	**~ 10-100 CPU hours**
Differential DY+Jet (pT-Z)	**~ 100000 CPU hours**

REQUIRED INGREDIENTS FOR PREDICTIONS

Real Corrections!

- Current state of the art: $2 \to 2$ @ NNLO QCD
- Extension to one higher order or one more leg is very complicated.
- Requires improved understanding of structure of real singularities.
- Many developments in the past couple of years:

Subtraction

- Antenna
- STRIPPER
- FKS+
- Nonlinear Mappings
- Colourful
- Projection To Born
- Geometric Subtraction
- Physical Sector Decomposition
- …

Slicing

- qT
- N-Jettiness

Power corrections: [Gherardo's talk]

REQUIRED INGREDIENTS FOR PREDICTIONS

All the Rest!

- Parton Distribution Functions
  - N3LO?, Theory Uncertainties, Small-x, Threshold, Non-pert., Flavour Thresholds, ….
- Parton Showers [Christian's talk?]
  - Higher Log accuracy, hadronization, formal accuracy, matching to FO, merging, …
- Electro-Weak Corrections
  - Combination with FO, final state definition, interference, large logs, ….
- Mass Effects
  - Mass definitions, small mass expansions, resummation of small mass effects, ….
- Uncertainty Estimates [Frank's talk]
  - Theory definition, What beyond scale variation, bin-to-bin correlation, statistical basis?
- Perturbative Convergence?
  - ?
- …

HIGGS BOSON

$$\frac{\partial \sigma}{\partial Y}$$

HIGGS BOSON RAPIDITY DISTRIBUTION

With Andrea Pelloni and Falko Dular

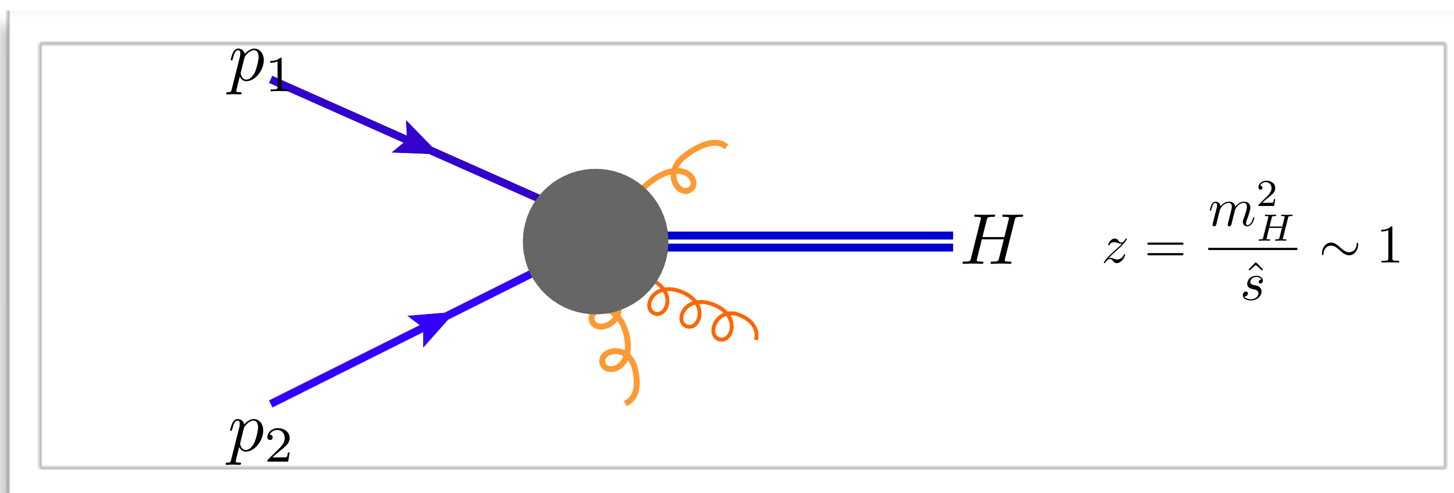
$$Y = \frac{1}{2}\log\left(\frac{2P_1 p_h}{2P_2 p_h}\right)$$

- Compute the rapidity distribution of the Higgs Boson at the LHC.
- Gluon Fusion - dominant production mechanism.
- Heavy top quark EFT.
- Compute N3LO corrections.
- Get fully analytic results for the partonic cross sections.

//EXPAND

Simplifications:

- Perform expansion around kinematic limit: **Production Threshold**

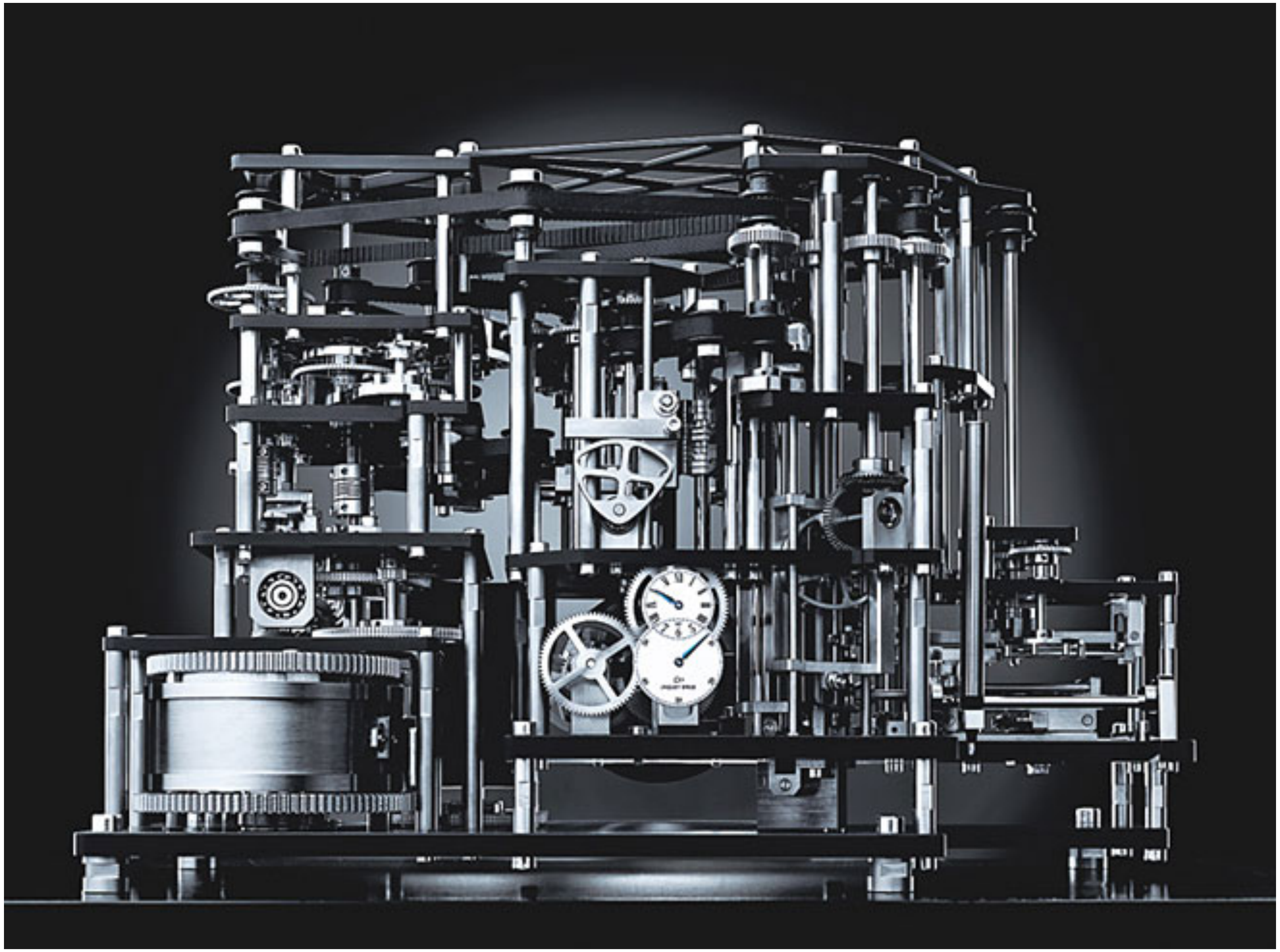


$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

- Expand to sufficiently high order to ensure stable results.

- Remarkably successful for inclusive N3LO.

SO WE USE OUR CROSS SECTION MACHINE AND COMPUTE!



OUR PARTONIC COEFFICIENT FUNCTION – SUMMARY

$$\frac{\partial\sigma}{\partial Y}$$

Ingredients:

- ★ **Six** terms in the expansion around the partonic threshold.
- ★ Integrates to the exact N3LO cross section.
- ★ Contains a bunch of logarithms exactly.

OUR PARTONIC COEFFICIENT FUNCTION – SUMMARY

$$\frac{\partial\hat{\sigma}^{(3)}}{\partial Y}(x_a, x_b) =$$

[Terms with two distributions](xa, xb) Exact! Soft limit

$$+\sum_{i=0}^{5}\left[\frac{\log^i(1-x_a)}{1-x_a}\right]_+ f_+^{(i)}(x_b)$$

Exact! Consistency Relations [Johannes' talk]

$$+\sum_{i=2}^{5}\log^i(1-x_a) f_L^{(i)}(x_a, x_b)$$

Exact! Consistency Relations

$$+(x_a \leftrightarrow x_b)$$

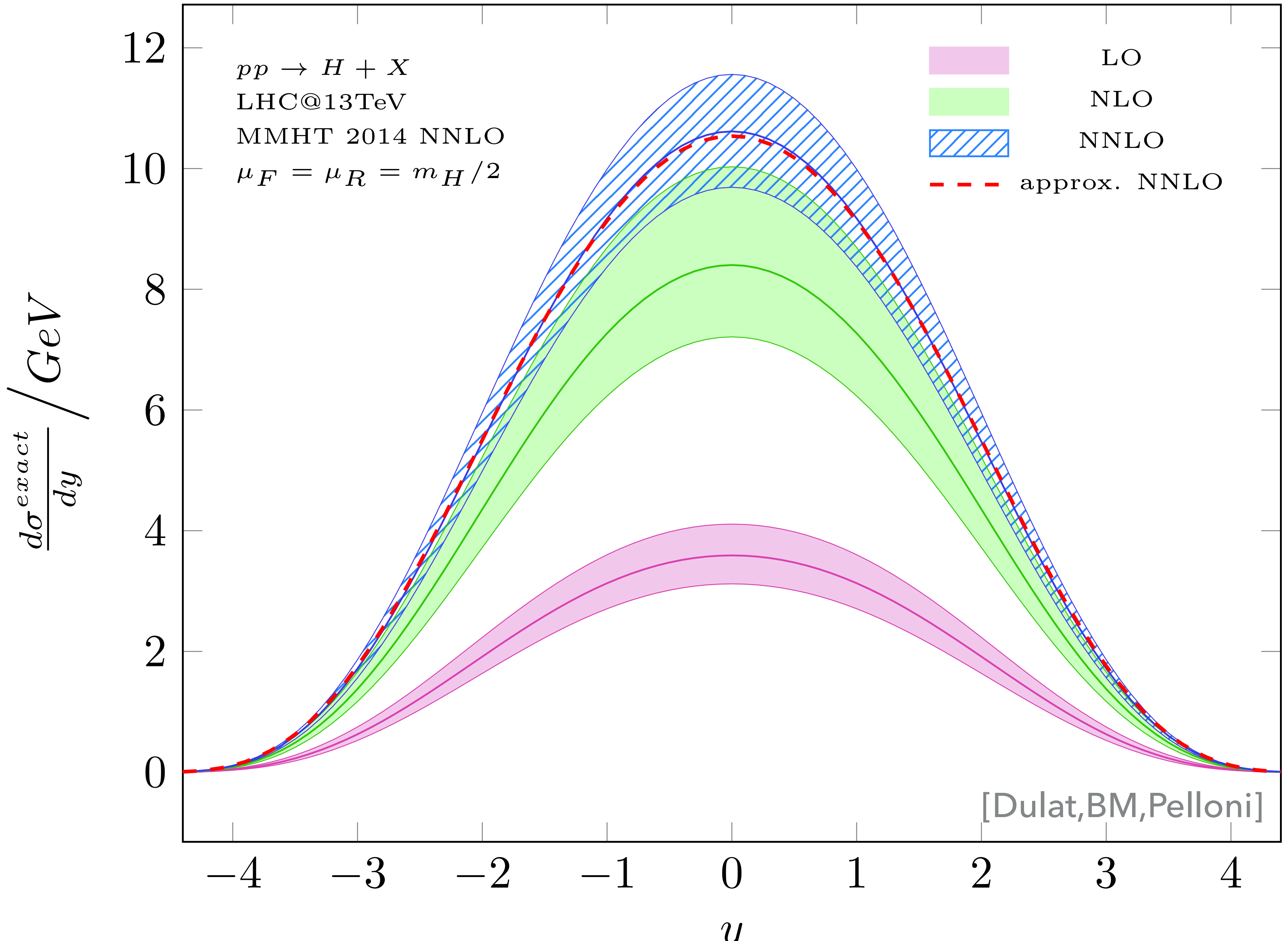
Threshold Expansion

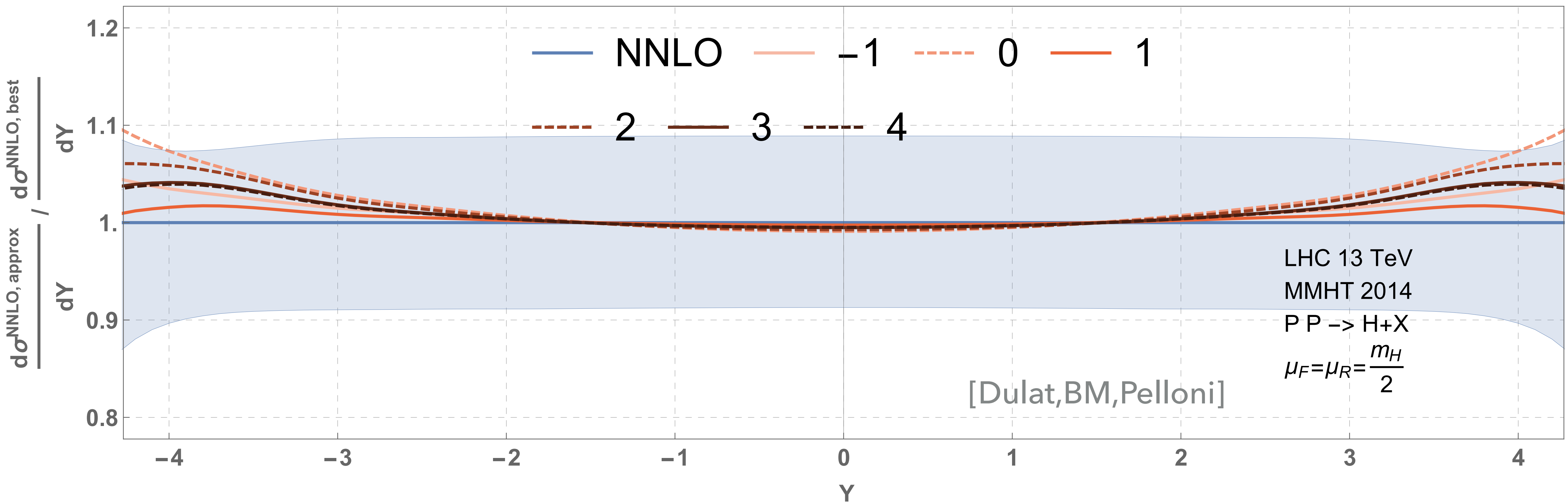
$$+f_{\text{rest}}(x_a, x_b)+\delta(1-x_a)f_\delta(x_b)+\delta(1-x_b)f_\delta(x_a)$$

$$+\Delta_{\text{inc.}}(x_a x_b)$$

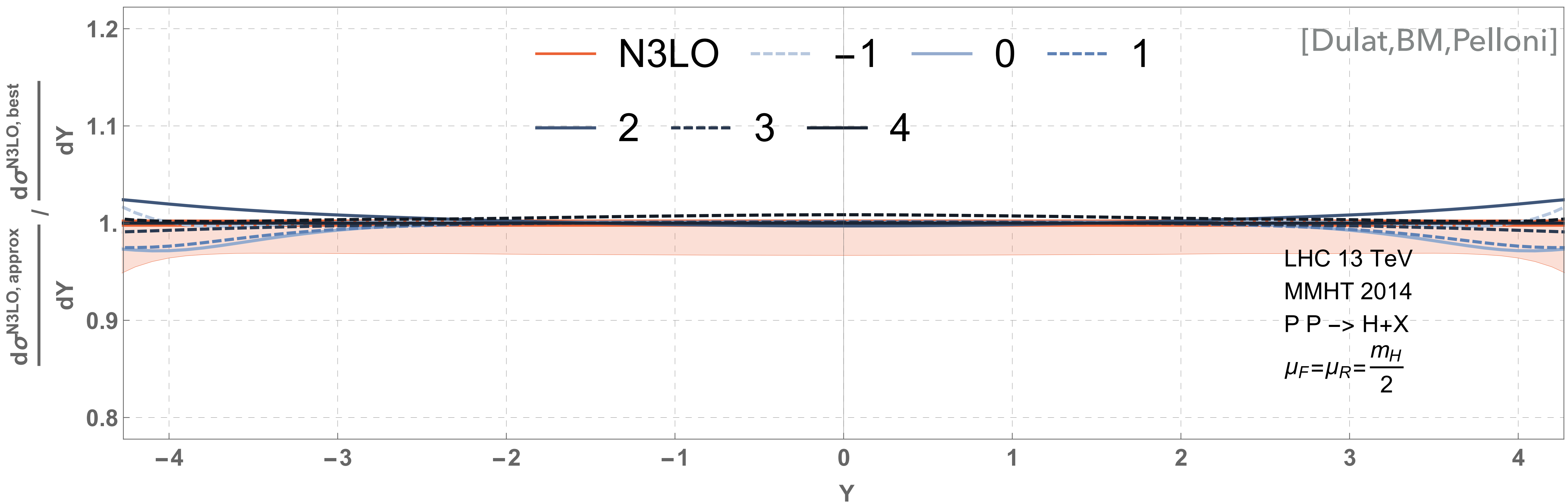
Ensure Inclusive Cross Section

HIGGS BOSON RAPIDITY - APPROX VS EXACT



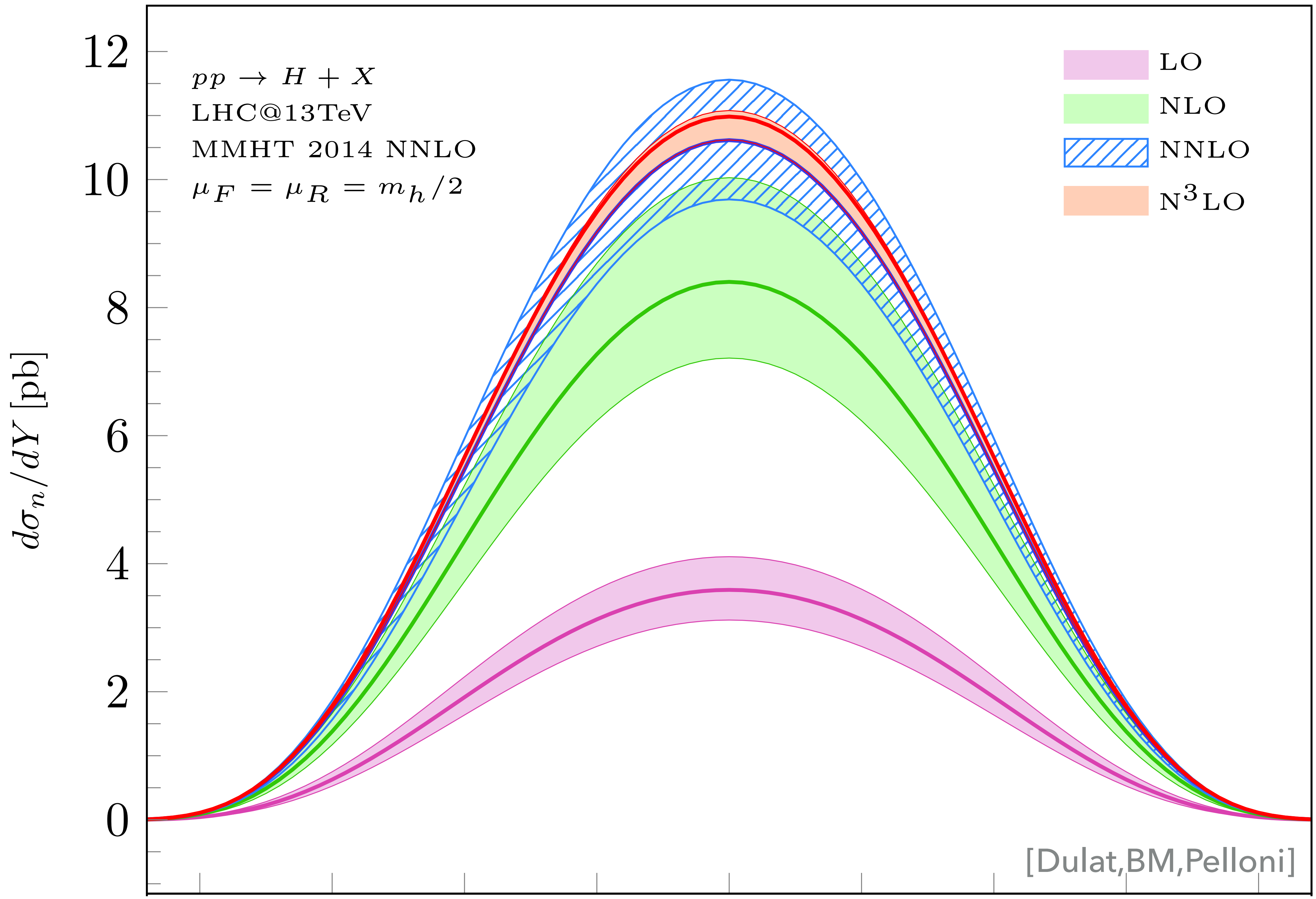


- Our approximation performs nicely!
- Especially for central rapidities |Y| < 3
  Larger Rapidities ~ More energetic final states = further from threshold
- After first couple of orders: Systematic improvement by including more terms in threshold expansion.
- To cover the remaining difference to exact NNLO other ingredients than threshold expansion are necessary.

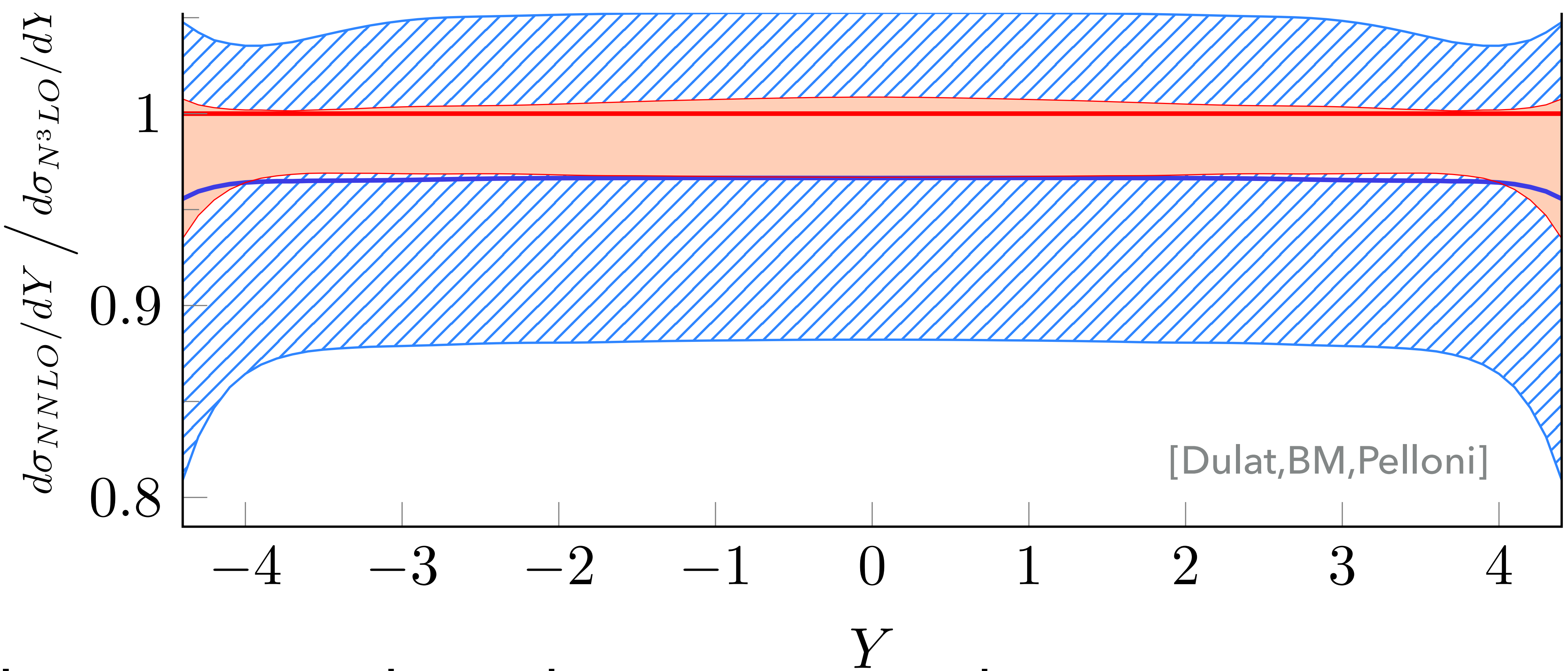


- Similar picture as at NNLO.
- Central rapidities very stable under adding more threshold terms.
- Larger rapidities: expansion varies more.
- High confidence in central rapidity region.

HIGGS BOSON RAPIDITY



HIGGS BOSON RAPIDITY – RATIO



- Flat correction throughout entire rapidity range.
- Significant reduction in scale uncertainty.
- Excellent agreement with other computation of **[Cieri,Chen,Gehrmann,Glover,Huss]**

FULLY DIFFERENTIAL CROSS SECTIONS

SUBTRACTION ALGORITHMS

Fully Differential Cross Sections

Typical treatment of singularities in fixed order perturbation theory:
Example:

$$\sigma \sim \int_0^{q^2} dp_\perp^2 \frac{1}{(p_\perp^2)^{1+\epsilon}} \left(M(p_\perp^2) - \tilde{M}(0)\right)$$

+Integrated Counter Term

Divergence

Local Counter Term

Matrix Element

SUBTRACTION ALGORITHMS

Fully Differential Cross Sections

Typical treatment of singularities in fixed order perturbation theory:
More general example:

Measurement Function

$$\sigma_J \sim \int d\phi_n \left(M(\phi_n, \phi_{\mathrm{Born}}) J(\phi_n, \phi_{\mathrm{Born}}) - \tilde{M}(\phi_n, \phi_{\mathrm{Born}}) J(0, \phi_{\mathrm{Born}})\right)$$

Integrate over n-partons

+Integrated Counter Term

Divergence

Local Counter Term

IRC - Safety: $J(\phi_n, \phi_{\mathrm{Born}}) \to J(0, \phi_{\mathrm{Born}})$ in singular limits

SUBTRACTION ALGORITHMS – PROJECTION TO BORN

[Cacciari,Dreyer,Karlberg,Salam,Zanderighi]

Fully Differential Cross Sections

The perfect subtraction algorithm!

$$\sigma_J \sim \int d\phi_n (M(\phi_n, \phi_{\text{Born}}) J(\phi_n, \phi_{\text{Born}}) - \tilde{M}(\phi_n, \phi_{\text{Born}}) J(0, \phi_{\text{Born}}))$$

+Integrated Counter Term

SUBTRACTION ALGORITHMS – PROJECTION TO BORN

[Cacciari,Dreyer,Karlberg,Salam,Zanderighi]

→ **Fully Differential Cross Sections**

The perfect subtraction algorithm!

$$\sigma_J \sim \int d\phi_n M(\phi_n, \phi_{\text{Born}}) \Big[J(\phi_n, \phi_{\text{Born}}) - J(0, \phi_{\text{Born}}) \Big]$$

+Integrated Counter Term

$$\tilde{M}(\phi_n, \phi_{\text{Born}}) = M(\phi_n, \phi_{\text{Born}})$$

SUBTRACTION ALGORITHMS - PROJECTION TO BORN

[Cacciari,Dreyer,Karlberg,Salam,Zanderighi]

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$$\tilde{M}(\phi_n, \phi_{\text{Born}}) = M(\phi_n, \phi_{\text{Born}})$$

- **Fully local subtraction**
- **No large numerical discrepancy between local CT and matrix element possible**
- **Successfully used in DIS - like processes (VBF H / HH @ N3LO, differential DIS)**
- **Need to know Integrated Counter Term exactly. Hard!!**

SUBTRACTION ALGORITHMS - PROJECTION TO BORN

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+Integrated Counter Term

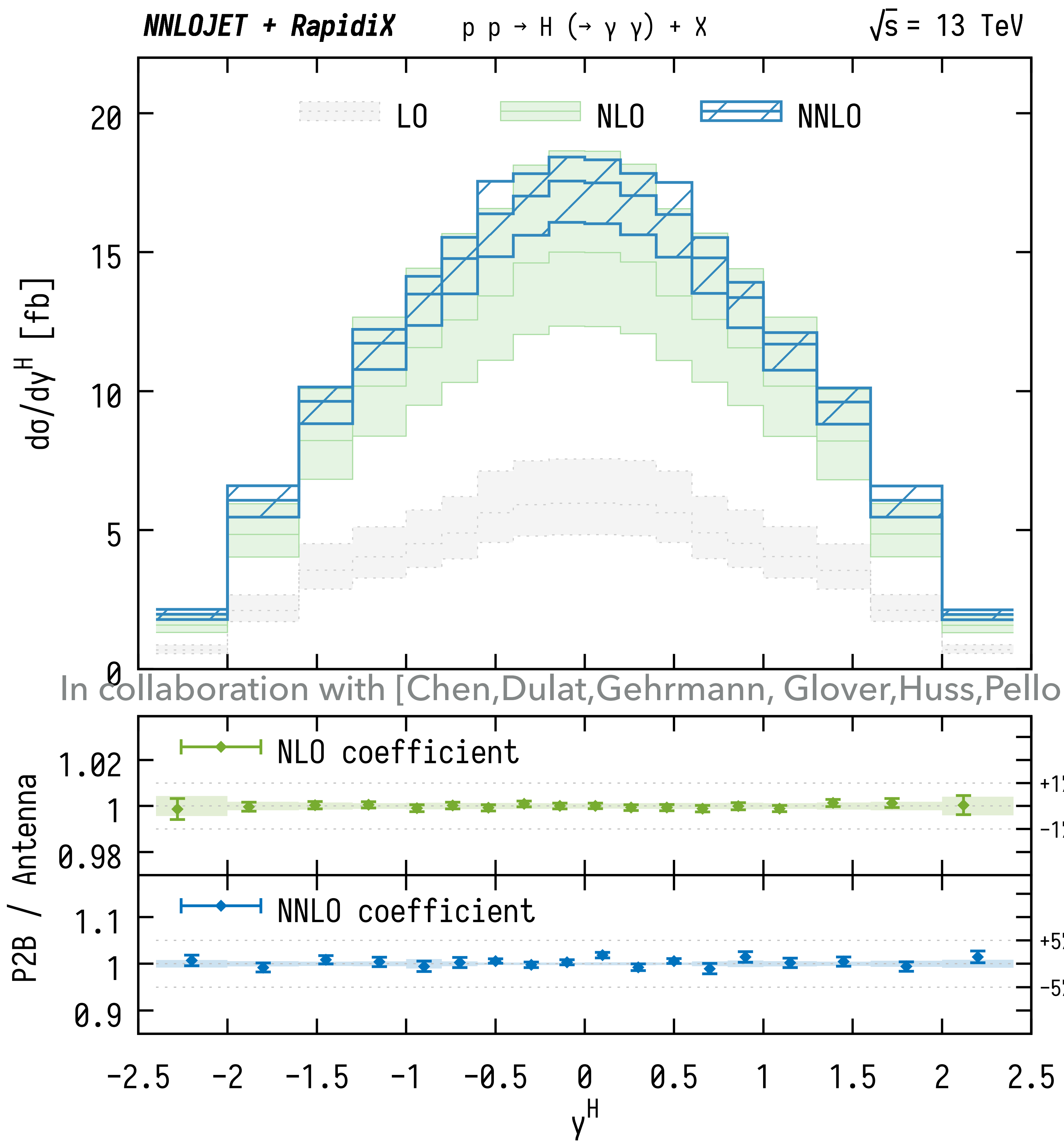
For Higgs Boson Production:

$$\phi_{\mathrm{Born}} = \{z, Y\}$$

Integrated Counter Term → $\dfrac{d\sigma}{dz\,dY}$

$$H \to \gamma\gamma$$

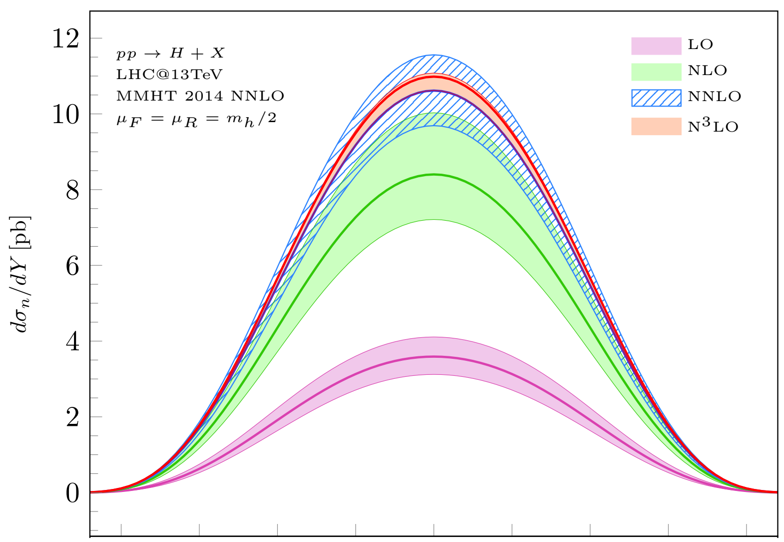
- Combination with H+J
- Validation at NNLO
- Fiducial Cross Sections for LHC Phenomenology!
- Extension to N3LO in progress



In collaboration with [Chen,Dulat,Gehrmann, Glover,Huss,Pelloni]

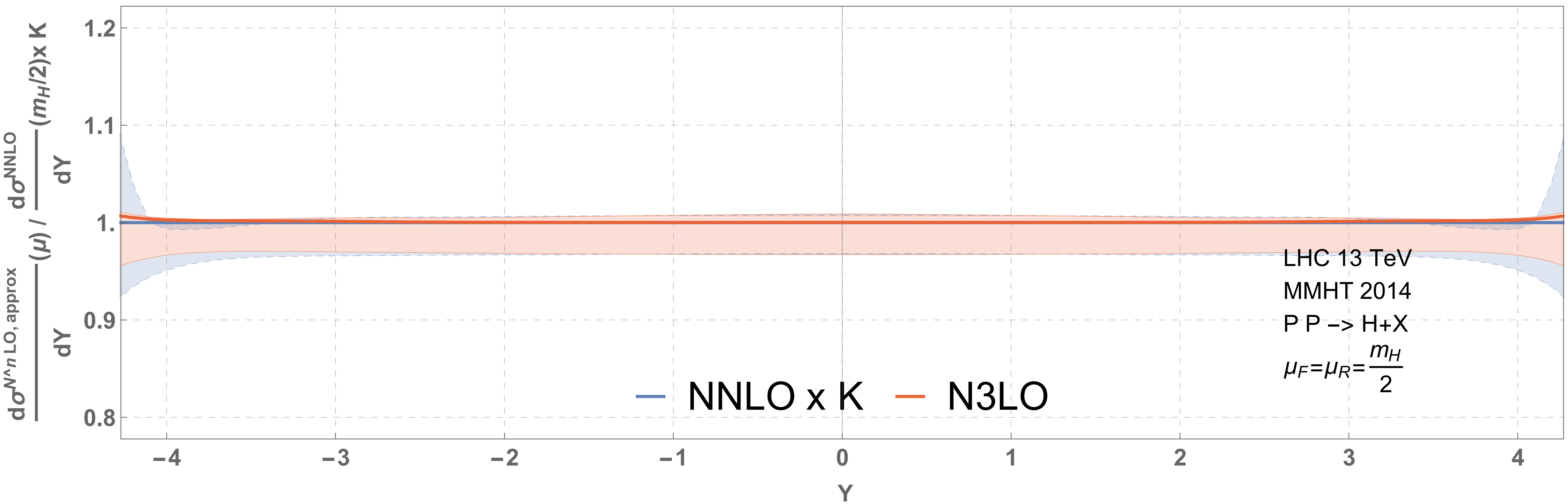
CONCLUSIONS

- The LHC provides a remarkable opportunity to study high energy physics.
- Many fascinating avenues have to be explored to match and surpass the experimental demand.
- We computed the Higgs Boson Rapidity Distribution at N3LO.
- Our result is the cornerstone for future fully differential predictions of Higgs boson phenomenology.



Thank you!

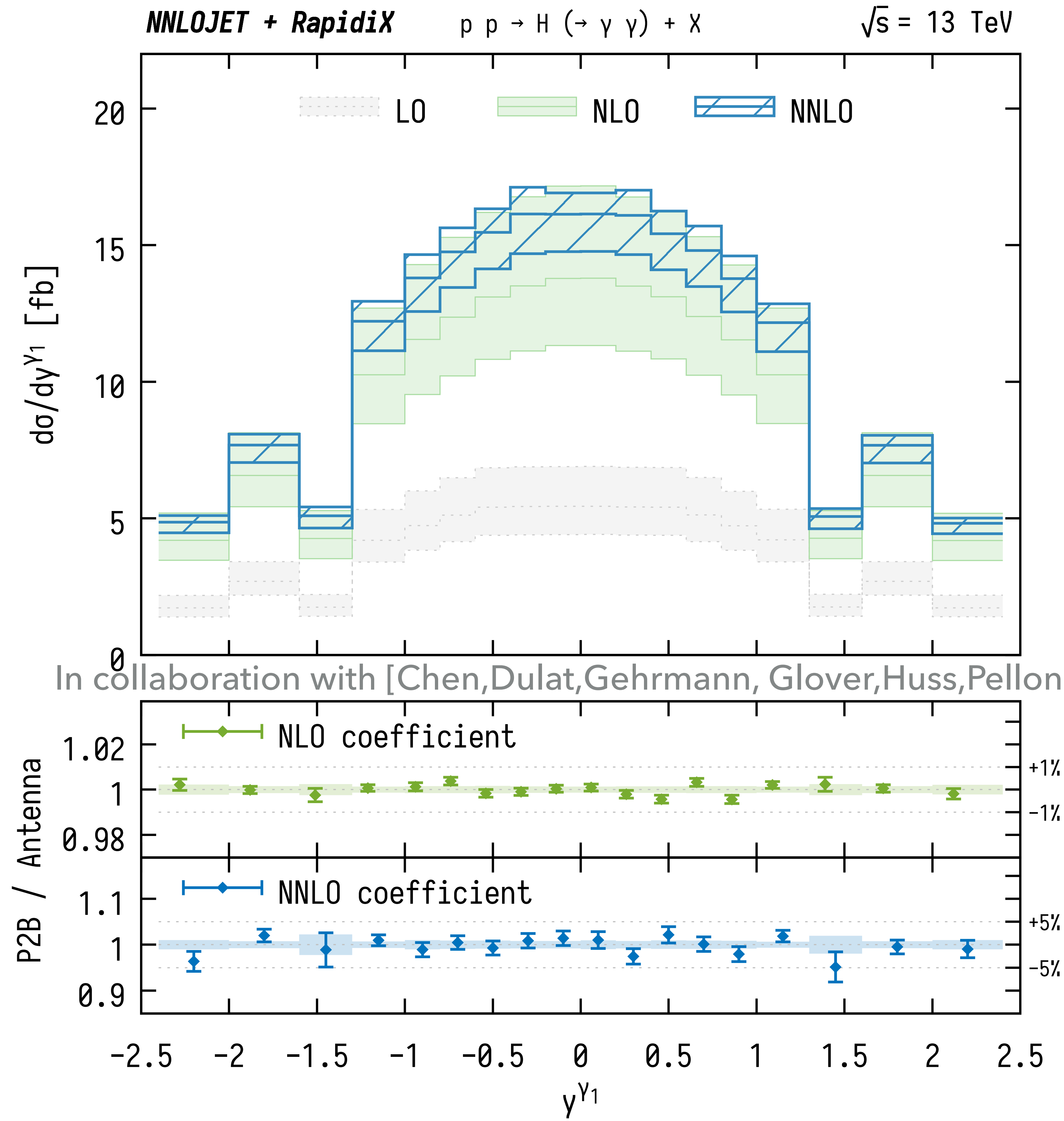
HIGGS BOSON RAPIDITY – RATIO



- Very compatible with rescaling of NNLO distribution
- Good news for current experimental usage!
  Re-weighted Parton-Shower MC.

$$H \to \gamma\gamma$$

- Leading Photon Y
- Extension to N3LO in progress
- Combination with other uncertainties required!



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EXPANDED VS. EXACT: INCLUSIVE