Extracting a Short Distance Top Mass with Light Grooming

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 Most precise top mass extractions come from direct reconstruction techniques

> $m_t^{\text{MC}} = 172.44(49) \text{GeV(CMS)}$ $m_t^{\text{MC}} = 172.84(70) \text{GeV(ATLAS)}$ $m_t^{\text{MC}} = 174.34(64) \text{GeV(Tevatron)}$

wivershat Additional 0.5 ~ 1 GeV uncertainty from relating Monte Carlo mass parameter m_t^{MC} to a well-defined top mass renormalizaton scheme.



Soft Drop

Soft Drop

Soft Drop

Larkoski, Marzani, Soyez, Thaler 2014

Larkoski, Marzani, Soyez, Thaler 2014

Grooms soft radiation from the jet

$$\frac{\min(p_{Ti}, p_{Tj})}{p_{Ti} + p_{Tj}} > z_{\text{cut}} \left(\frac{\Delta R_{ij}}{R_0}\right)^{\beta}$$

 $z > z_{\rm cut} \, \theta^{\beta}$

two grooming parameters

Groomed Jet







Soft Drop Factorization

Frye, Larkoski, Schwartz, Yan 2016







- Lessons learned:
 - Non-perturbative effects are tied to the perturbative branching history of the jet
 - Two types of non-perturbative effects: "shift correction" and "boundary correction"
- "Shift": contribution from the jet mass from NP radiation kept in the groomed jet
- "Boundary": Modification of the soft drop test on a perturbative subjet in the presence of NP radiation

NLL Factorization of Groomed Massless Jet Spectrum

• At NLL, at leading order in the OPE, the factorization for the groomed massless jet distribution can be written as:

$$\frac{d\sigma_{\kappa}^{\text{had}}}{dm_{J}^{2}} = \sum_{\kappa=q,g} D_{\kappa}(\Phi_{J}, z_{\text{cut}}, \beta, \mu) \int_{0}^{\infty} d\ell^{+} J_{\kappa}(m_{J}^{2} - Q \ell^{+}, \mu)$$

$$\frac{d\sigma_{\kappa}^{\text{had}}}{dm_{J}^{2}} = \sum_{\kappa=q,g} D_{\kappa}(\Phi_{J}, z_{\text{cut}}, \beta, \mu) \int_{0}^{\infty} d\ell^{+} J_{\kappa}(m_{J}^{2} - Q \ell^{+}, \mu)$$

$$\frac{\text{ollinear-Soft}}{\text{Function}} \leftarrow \times \int_{0}^{\infty} dk S_{c}^{\kappa} \Big[(\ell^{+} - C_{1}(m_{J}^{2}, Q) k) Q_{\text{cut}}^{\frac{1}{1+\beta}}, \beta, \mu \Big]$$

$$\times \Big(1 - Q k \frac{dC_{1}(m_{J}^{2}, Q)}{dm_{J}^{2}} + \frac{\Upsilon_{1}^{\kappa}(\beta)}{Q} C_{2}(m_{J}^{2}, Q) \Big) F_{\infty}^{\kappa}(k)$$

$$\underset{\text{Boundary}}{\text{shape function}} \underset{\text{(encode } \Omega_{1} \text{ here}}{\text{barefunction}}$$

)

• NP effects are not simply described by a shape function at this order.





physical width of the distribution in the peak region. Here k is the system width of the distribution of the second by the second by the second by the energy fraction relative to the jet energy. The energy fraction relative to the jet energy. $k_{jz}^{\mu} = k_{z}^{\mu} k_{z}^{\mu}, k_{z}^{\mu}$ $k_{z}^{\mu} = k_{z}^{\mu} k_{z}^{\mu}, k_{z}^{\mu}$ Soft Drop (1-z)Larkoski, Marzani, Soyez, Thaler 2014 (cf. Jesse Thaler's race Global Soft Non-perturbative rgy Q we hε comes Ultracollinear Decay products ¹C⁶We use a more restrictive light grooming region that simplifies the theoretical framework: $z > z_{
m cut} \, heta^{oldsymbol{eta}}$ $= z \begin{bmatrix} z & (1 - c) & \frac{\min(p_{Ti}, p_{Tj})}{\log treatment of decay products:} > z_{cut} \left(\frac{\Delta R_{ij}}{R_0}\right)^{\beta} \\ 1 \end{bmatrix}$ o grooming parameter the scale of f of η_J of η_J z_{cut} light groomed factorization invalid ultracollinear vetoed 10⁻¹ 0.1 ra-soft modes j soft modes j n of softgetto to the effects of n of softgetto to the ar radiation in Ī Znh?ľ 10⁻² allowed region wed region 2 $\beta = 2$ soft were to the soft wider-angler $\mathcal{A}^{2}_{\mathcal{W}^{-3}}$ 10^{-3}

 $z_{\text{cut}}^{\frac{1}{2+\beta}} \gg \frac{1}{2} \left(\frac{\Gamma_t}{m_t} \frac{4m_t^2}{Q C} \frac{1}{a r r^3} \right)^{\frac{1}{2+\beta}} \text{ carry out } Carry \text{ out } Calculo 0 \text{ soft}_4 \text{ not groomed} \text{ soft}_4 \text{ soft}_4$

2000

physical width of the distribution in the peak region. Here k is the system width of the distribution of the second by the second by the second by the energy fraction relative to the jet energy. The energy fraction relative to the jet energy. $k_{jz}^{\mu} = k^{+}, k$ Larkoski, Marzani, Soyez, Thaler 2014 (cf. Jesse Thaler's race Global Soft Non-perturbative rgy Q we hε comes Ultracollinear Decay products ¹C⁶We use a more restrictive light grooming region that simplifies the theoretical framework: $\sum_{z \in \mathcal{I}} \left[\begin{array}{c} 1 - \mathbf{c} \\ -z \end{array} \right] \left[\begin{array}{c} \min(p_{Ti}, p_{Tj}) \\ \text{of decay products:} \end{array} \right] > z_{\text{cut}} \left(\frac{\Delta R_{ij}}{R_0} \right)^{\beta}$ $z > z_{\rm cut} \, \theta^{\beta}$ two grooming parameter the scale of f of η_J of η_J z_{cut} ultracollinear vetoed 0.1 ra-soft modes j soft modes in the effects of a of solution of solution in the solutin the solution in the solution in the solu $\sum_{\text{allowed region}} \int \int \beta = 2$ ±10⁻² $2m\Gamma_t$ cut softener soften wider-angler 10⁻³ ight Grooming $z_{\text{cut}}^{\frac{1}{2+\beta}} \gg \frac{1}{\rho} \left(\frac{\Gamma_t}{m} \frac{4m_t^2}{Q^2} \frac{1}{\cosh^2 \eta_J} \right)_{\text{carry out calculation}}^{\frac{1}{2+\beta}} \text{ of ultra-soft m}$ 10^{-4} ultrasoft not vetoed

10⁻⁵

500

1000

pT [GeV]

1500

distributions for pp and e^+e^- collisions are similar up to sensitivity to underlying event we reduced by soft drop.

3.1 Modes and Power Counting Analysis • Soft Drop (SD) and Peak 2 Region 2 constraints determinentine relevant 4923, UWThPh effective theory mode Following along the lines of derivation in Ref. [5]

e top quark mass m_t is one of the regest Engendary to a sintemation of the top quark a lard Model (SM) parameters. It significantly at ollinear solur method relies on deriving ner studies of the SM vacuum structure in the international elevents that enable the measurement eak precision observables $[2]_{d\sigma}$ The most precise top on a jet of radius $R \sim 1$ with light so Decay products measurements are based on kinematic (Φ_J) most precise Φ_{σ} , μ) adsorpt Φ_{σ} appled ℓ_{σ} appled ℓ_{σ} yielding results such as $m_t^{MCM} = 172.44(49) \text{ GeV}$ removes peripheral soft radiation b 5° B_{t} , $e_{m_{t}}$, $e_{m_{t}}$, $e_{m_{t}}$, h_{t} , = 174.34(64) GeV (Tevatranhetse The Freischtene physical) Wickhudf the Klistrik s $d\mathbf{n}(z)$ as a monte Carlo (MC) simulations and z is the energy fraction relative z mine the mass parameter $m_t^{(1)}$ of the MC generawhich depends on the shower dynamics and its ince with hadronization protentifying these wall of with 20 18 Die Djapid grangian top-mass scheme m_t induces an additional rame guity at the 0.5–1.0 GeV level [6, 7]. We propose torization approach to render dept to the state energy we **H**ius $t\bar{t}$ by constructing an observable that has high kine $s_{h}(z_{ti}) = m_t$ and m_t and m_t and m_t and Grooms soft on level predictions from QCD employing a short $\min(p_{Ti}, p_T)$ It also reque nental data, or to calibrate the parameter m_t^{MC} $p_{Ti} + p_{Ti}$

Effective Theory Modes: Momentum Scalings



• Expansion parameters:

$$\lambda = rac{m}{Q}$$
 , $\eta \equiv \left[\left(rac{2m}{Q}
ight)^{eta} z_{ ext{cut}} rac{m}{\Gamma_t}
ight]^{rac{1}{eta+2}}$

Hierarchy of expansion parameters

 $1 \gg \eta \gg \lambda$

Factorization with Soft Drop (SD)

• Two possibilities for the termination of soft drop:



Global Soft Collinear Soft Non-perturbative Ultracollinear Decay products

- SD could terminate on a collinear-soft (CS) subjet
- SD could terminate on a subjet containing top decay products



Generalized Collinear-Soft function



Collinear-Soft function must compare the angles of the CS and decay subjets:

$$S_{C}^{(d)}(\ell,\beta,\theta_{d},\mu) = S_{C}^{q}(\ell,\beta,\mu)$$

$$-\frac{\alpha_{s}(\mu)C_{F}}{(\beta+2)\pi} \frac{2^{\beta+3}}{Q_{\text{cut}}\theta_{d}^{\beta+2}} \mathcal{L}_{1}\left(\frac{\ell}{Q_{\text{cut}}}\frac{2^{\beta+2}}{\theta_{d}^{\beta+2}}\right) \Theta\left[\frac{Q_{\text{cut}}\theta_{d}^{\beta+2}}{2^{\beta+2}} - \ell\right]$$
Extra term from
$$\theta_{cs} \text{ vs } \theta_{d} \text{ comparison}$$
No large logs in the light
grooming region

Generalized Collinear-Soft function



• At NLL, the same perturbative collinear-soft function can be used as in the mass groomed jet case:

$$S_C^{(d)}(\ell,\beta,\theta_d,\mu)\Big|_{\rm NLL} = S_C^q(\ell,\beta,\mu)\Big|_{\rm NLL}$$

 However, we will see that the NP part of the collinear-soft function must still know information about the decay products.

Combining "High-pT" & "Decay" Factorization



Combining "High-pT" & "Decay" Factorization



Top BHQET Jet function

• The unstable BHQET top jet function can be defined in terms of a convolution of the stable top BHQET function and a Breit Wigner:

$$J_B(\hat{s}_t, \Gamma_t, \delta m, \mu) = \int_{-\infty}^{\hat{s}_t} d\hat{s}' J_B(\hat{s}_t - \hat{s}', \delta m, \mu) \frac{\Gamma_t}{\pi(\hat{s}'^2 + \Gamma_t^2)}$$
Unstable top jet function
$$Stable top jet function$$
Breit
Wigner
$$\frac{t}{\sqrt{1 + \frac{1}{2}}} \cdots \frac{t}{\sqrt{1 + \frac{1}{2}}}$$

Differential Top BHQET Jet function

• Similarly, we define a stable BHQET top jet function, that is fully differential in the top decay phase space by cutting across one of the "bubbles":



Differential Top Jet function

• The unstable BHQET top jet function can be defined in terms of a convolution of the stable top BHQET function and

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$$J_{D_{t}}\left(\hat{s}_{t}, \Phi_{d}, \frac{m}{Q}, \delta m, \mu\right) = \int_{-\infty}^{\hat{s}_{t}} d\hat{s}'_{t} J_{B}^{\Gamma_{t}=0}(\hat{s}_{t} - \hat{s}'_{t}, \delta m, \mu) D_{t}\left(\hat{s}'_{t}, \Phi_{d}, \frac{m}{Q}\right)$$

$$= \frac{\Gamma_{t}}{\pi(\hat{s}'^{2} + \Gamma_{t}^{2})} d_{t}\left(\Phi_{d}, \frac{m_{t}}{Q}\right)$$

$$= \frac{1}{\pi(\hat{s}'^{2} + \Gamma_{t}^{2})} d_{t}\left(\Phi_{d}, \frac{m_{t}}{Q}\right)$$

$$d_{t}\left(\Phi_{d}, \frac{m_{t}}{Q}\right) = \frac{1}{\Gamma_{t \to bqq'}} \frac{d\Gamma_{t \to bqq'}}{d\Phi_{d}}$$
Passes decay subjet angle information to the collinear-soft function for comparison

Differential Top Jet function

$$J_{D_{t}}\left(\hat{s}_{t}, \Phi_{d}, \frac{m}{Q}, \delta m, \mu\right) = \int_{-\infty}^{\hat{s}_{t}} \mathrm{d}\hat{s}_{t}' J_{B}^{\Gamma_{t}=0}(\hat{s}_{t} - \hat{s}_{t}', \delta m, \mu) D_{t}\left(\hat{s}_{t}', \Phi_{d}, \frac{m}{Q}\right)$$

Stable Differential Stable top jet function

Stable Differential top jet function

• Recover the standard inclusive unstable top BHQET function:

$$J_B(\hat{s}_t, \Gamma_t, \delta m, \mu) = \int \mathrm{d}\Phi_d J_{D_t}\left(\hat{s}_t, \Phi_d, \frac{m}{Q}, \delta m, \mu\right)$$

Top Decay: Phase Space & Subjet Angle



Decay subjet angle:

$$\tilde{\theta}_{xy} = \min\left(\tilde{\theta}_{q\bar{q}'}, \tilde{\theta}_{qb}, \tilde{\theta}_{\bar{q}'b}\right)$$
$$\theta_d \equiv \max\left(\tilde{\theta}_{(xy)t}, \tilde{\theta}_{zt}\right)$$

• Top decay subjet momentum scaling:

$$p_d^{\mu} \sim m(\frac{hm}{Q}, \frac{Q}{hm}, 1) \longrightarrow h(\theta_d) = \frac{Q}{m}\sqrt{\frac{1-\cos\theta_d}{1+\cos\theta_d}} = \frac{Q}{m}\tan\frac{\theta_d}{2}$$

Top Decay Subjet Angle



• Top decay subjet momentum scaling:

$$p_d^{\mu} \sim m\left(\frac{hm}{Q}, \frac{Q}{hm}, 1\right) \longrightarrow h(\theta_d) = \frac{Q}{m}\sqrt{\frac{1-\cos\theta_d}{1+\cos\theta_d}} = \frac{Q}{m}\tan\frac{\theta_d}{2}$$

fects studies of the SM vacuum stability [1] and the elecorems that enable the measurement of the jet mass M_{J} troweak precision observables $[2]_{d\sigma}$ The most precise top on a jet of radius $R \approx 1$ with light soft drop grooming in mass measurements are based on kinemati V_t (49) GeV tion, yielding results such as $w_t^{MOV} = 172.44(49)$ GeV haliodete Dtop, tample I Thessoft idrop algorithm [12, 13] remotes perpheral soft radiation $b_{\mathcal{Y}}^m$ comparing subse-(CMS) [3], $m_t^{\rm MC} = 172.84(70)$ GeV (ATLAS) [4] and m_k quent it constituents i, j in an angular ordered cluster $m_t^{\text{MC}} = 174.34(64) \text{ GeV} (\text{Tevatron}) [5].$ These measures $\frac{m}{O}$ in $\tilde{h}(\Phi anti)) (2^{\beta}Qz_{\text{cut}})^{\frac{1}{1+\beta}}, \beta, \mu | F_C(\vec{k}, 1)$ (3.29)ments are based on Monte Carlo (MC) simulations and $\min[p_{Ti}, p_{Tj}]/(p_{Ti} + p_{Tj}) > z_{\rm cut}(R_{ij}/R)^{\beta},$ determine the mass parameter m_t^{MC} of the MC genera-(1)tor, which depends on the shower dynamics and its interface with hadronization. Identifying Φ_J with a shower dynamics and its interface with hadronization. the angular distance in the rapidity ϕ plane, and z_{cut} and β are fixed soft drop paa Lagrangian top-mass scheme m_t induces an \mathfrak{A}_{tional} rameters. When Eq. (1) is satisfied all subsequent conambiguity at the 0.5-1.0 GeV level [6, 7]. We propose stituents on the Rear and Appt, 1 thus setting a new jet rafactorization approach to remove this uncertainty hin Vius Rg that for the groupen jet. This Theainse strong $pp \xrightarrow{\alpha_{j}} t\bar{t}$ by constructing an observable that has high kine-matic sensitivity to m_t and at the same time allows for $P_t \rightarrow$ And the second the sec had on level predictions from QCD employing a short Kienalieuse of the second seco distance top-mass. It can be used to extract m_t from exd, perimental data, or to calibrate the parameter $0.\mathfrak{B}_{t}^{\mathrm{MC}}$ as was done for 2-Jettiness in e^+e^- collision [8]. to derive peak region tactorization formulae for the cross- $\beta \in \mathfrak{g}$ if \mathfrak{g} if We consider boosted tops whose decay products are collimated in a single jet region, enabling a simultaneous theoretical description of both the top production and de $d_t \left(\Phi_d, \frac{m_t}{Q} \right) = \frac{1}{\Gamma_t \rightarrow b_{q}^{\text{transform}} \text{ and } \Phi_{ein}^{\text{transform}} \text{ being the jet's transverse momentum and } \prod_{i=1}^{n} \left[\left(\ell - \frac{m_t}{D} \right) \left(\frac{m_t}{D} \right) \left$ top-decay top ducts and discuss $\Gamma_t/m_t (Q/2m_t)^{\beta} \gg z_{sut}$ seudo-rapidity, respectively. Recently an experimental so that of possicin a line of the second in the second of analysis along these lines was carried out by $PCMS' = 10\overline{5}$ associated with the top quark is not modified. For $e^+e^- \to t\bar{t}$ a hadron level factorization theorem for a distribution with high in the metal in the second of th $P\left(\frac{\tilde{h}}{Q}, \frac{m_t}{Q}\right) = \int_{\mathcal{T}} d\Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \text{a distribution with high the particular flucture for the stance m_t} \\ \Phi_d \, d_t \left(\begin{array}{c} \Phi_d \, d_t \left(\begin{array}{c} \Phi_d \, d_t \,$ the scheme for m_t through $\delta m = m_{configuration}$ in the configuration then the straint is significantly stronger than that needed theretain the decay products. $(Q/2m_1)^{\beta} \gg z_{cut}$. The second configuration that straint ensures that wide angle soft: radiation $(V-2m_1)^{\beta}$ complications in controlling external radiation, parameters like the jet radius $\frac{3.8}{R}$, and solv containing Logarithms and • Top decay subjet momentum scaling single which the straint of th EIG. 4. Relevant SCET modes for soft drop jet mass for a top jets containing top decay products carryⁱⁿallfho $p_d^{\mu} \sim m(\frac{hm}{O}, \frac{1}{P})$ the jet-1600 Snorpu. Hence at leading did r in the et. The dashed vertical line corresponds to the angle of the furthest top-decay product from the parage which determines h, shown with two different values in (a) and (b). This leads top dependence on h if the **Eactorization** formulas for rights parameters $(p_t, \eta_J, z_{cut}, \beta)$ are held fixed. 4.1.1 Tree-level Cross Section counting they are in the same paper as the tot the mentage we der and the angle between the to product subjets is $\theta_d + \theta'_d$. Then this sum of the subject of compared with the angle θ_{sc} of the first C_{A}^{can} as δ_{mod}^{can} the pair $xy = q\bar{q}', qb$, Her \bar{q}' we have its closes the initial grade of the fact of the fact of the transmission of t

large enough p_T and R_{ii} to stop the softAdrop ig





$$\begin{aligned} \theta_{\text{stop}} &= 2 \int d\tilde{h} P\left(\tilde{h}, \frac{m_t}{Q}\right) \\ &\times \max\left\{\arctan\left(\frac{m_t}{Q}\tilde{h}\right), C_1\left(M_J^2 - m_t^2, Q, z_{\text{cut}}, \beta\right)\right\} \\ & \downarrow \\ \theta_d & \theta_{cs} \longrightarrow \text{(See talk by A. Pathak)} \end{aligned}$$

Non-Perturbative Effects

Non-Perturbative Effects



 Non-perturbative effects for the groomed jet mass spectrum depend on the perturbative branching history.



Dominant non-perturbative modes:

$$p_{\Lambda}^{\mu} \sim \frac{2\Lambda_{\rm QCD}}{\theta_{\rm CS}} (\frac{\theta_{\rm CS}^2}{4}, 1, \frac{\theta_{CS}}{2})$$

$$p_{\Lambda}^{\mu} \sim \frac{2\Lambda_{\text{QCD}}}{\theta_d} (\frac{\theta_d^2}{4}, 1, \frac{\theta_d}{2})$$

Non-Perturbative "Shift" Correction

• "Shift": contribution from the jet mass from NP radiation kept in the groomed jet

• Catchment area for NP radiation clustered in with the final soft drop stopping pair of subjets:



Non-Perturbative "Boundary" Correction



• "Boundary correction" ONLY affects the "high-pT" case. Due to the much larger energy of the "decay" subjet, the NP effect on soft drop is suppressed.

Non-Perturbative Effects in the Collinear-Soft function

$$\frac{d\sigma_{\kappa}^{\text{had}}}{dm_{J}^{2}} = \frac{d\hat{\sigma}_{\kappa}}{dm_{J}^{2}} - Q \,\Omega_{1\kappa}^{\infty} \,\frac{d}{dm_{J}^{2}} \left(C_{1}(m_{J}^{2}, Q, z_{\text{cut}}, \beta) \,\frac{d\hat{\sigma}_{\kappa}}{dm_{J}^{2}} \right) + \frac{\Upsilon_{1}^{\kappa}(\beta)}{Q} C_{2}(m_{J}^{2}, Q; z_{\text{cut}}, \beta) \,\frac{d\hat{\sigma}_{\kappa}}{dm_{J}^{2}}$$
Boundary term only present

 The top jet mass spectrum in the peak region is dominated by the "decay" contribution. Thus, boundary term effect can be neglected!

for "High-pT" case





• Can still encode leading power correction in a more sophisticated shape function, unlike for massless jets.

Plots

(Hoang, SM, Pathak, Stewart arXiv: 1708.02586)

zcut Dependence



• Light grooming region effective in removing contamination

• For more aggressive grooming, SD stops on decay subjects. Thus, peak position remains stable since decay products are always kept (unlike massless case).

Jet Radius Dependence



• Groomed top jet mass spectrum is independent of the original top jet radius for $R \gtrsim 0.9$

pT-Veto Dependence



• Groomed top jet mass spectrum is independent of jet veto on additional jets for $p_T^{
m veto}\gtrsim 50\,{
m GeV}$



• MPI effects are well-described through a shift in the shape function parameters, just like in the ungroomed case:

$$\Omega_n^{\rm od} \to \Omega_n^{\rm odMPI}$$

Conclusions

• New factorization framework for the groomed top jet mass distribution in the peak region that allows for the a short distance top mass extraction.

• Soft Drop can terminate on top decay products, requiring a more careful treatment of top decay dynamics compared to the ungroomed case.

• The new factorization framework provides a unified treatment of the two possibilities: (i) soft drop terminates on a collinear-soft subjet (ii) soft drop terminates on subjet containing top decay products

• Novel dependence in the factorization formula on a perturbative decay distribution P(h).

• Unlike the case of massless groomed jets, non-perturbative effects can be treated with a standard shape function analysis.

• MPI effects are well described through a shift in the shape function moment, just like in the ungroomed case.