SCET 2019 San Diego
March 26th, 2019

The Jet Shape at NLL′

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arXiv:1901.06389

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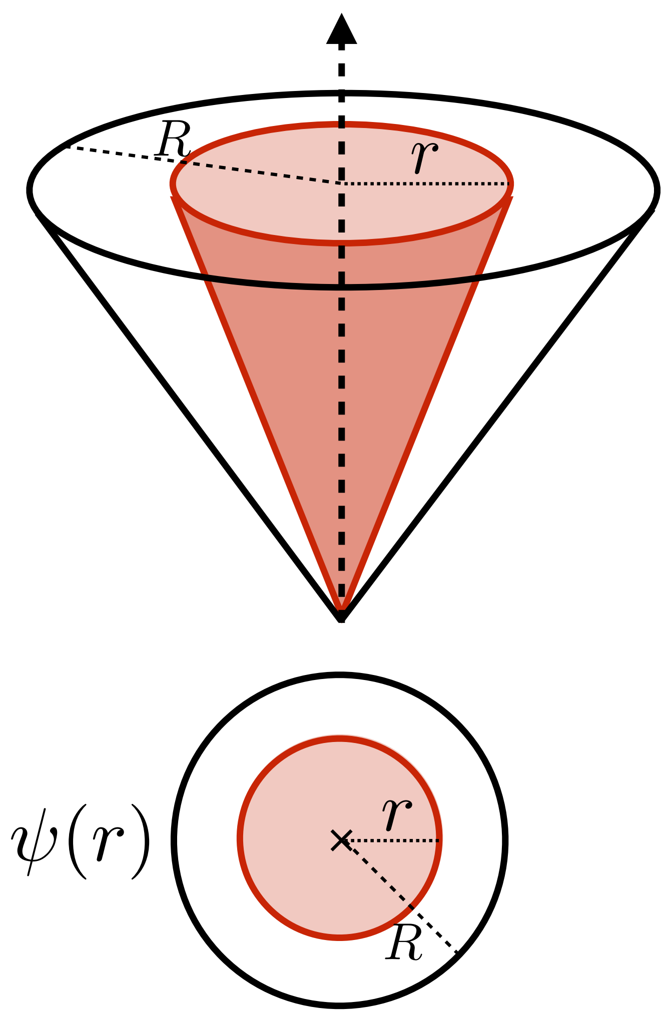
Nikhef

Jet shape

Integrated

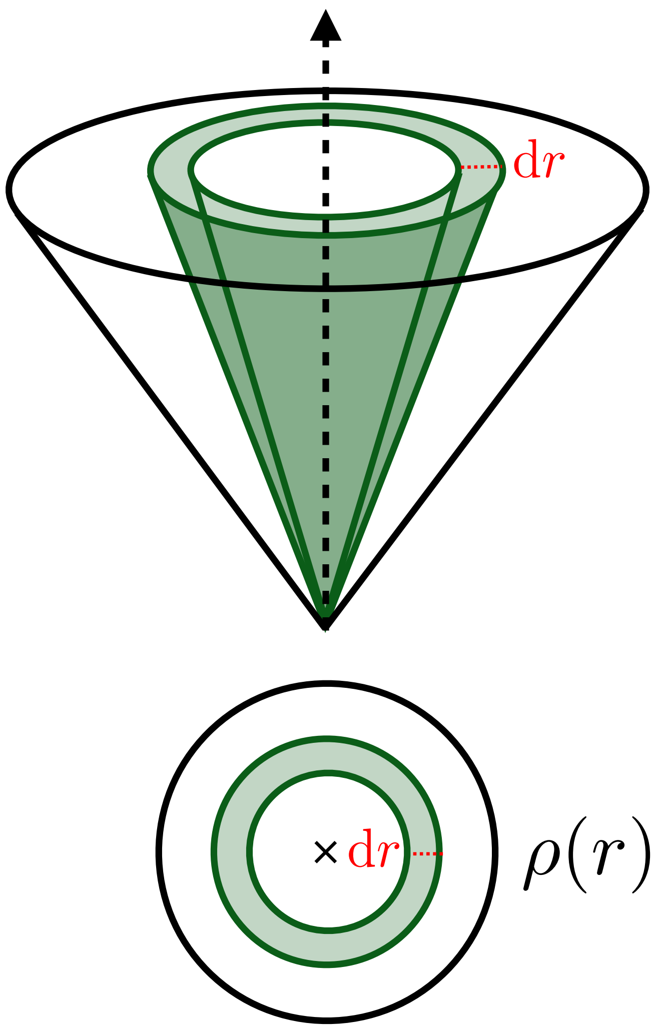


$$R = \text{Jet radius}$$

$$r = \text{Subjet radius}$$

$$z_r = \frac{p_T^{\text{subjet}}}{p_T^{\text{jet}}}$$

Differential

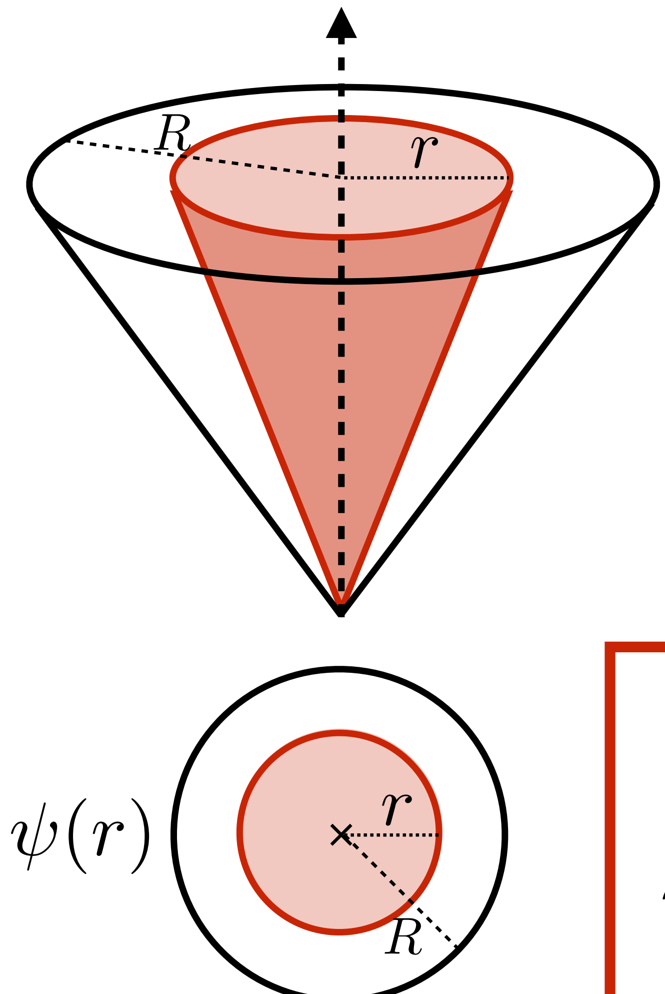


$$\psi(r) = \frac{\sum_{r_i<r} p_{Ti}}{\sum_{r_i<R} p_{Ti}}$$

$$\rho(r) = \frac{\mathrm{d}\psi(r)}{\mathrm{d}r}$$

Jet shape

Integrated



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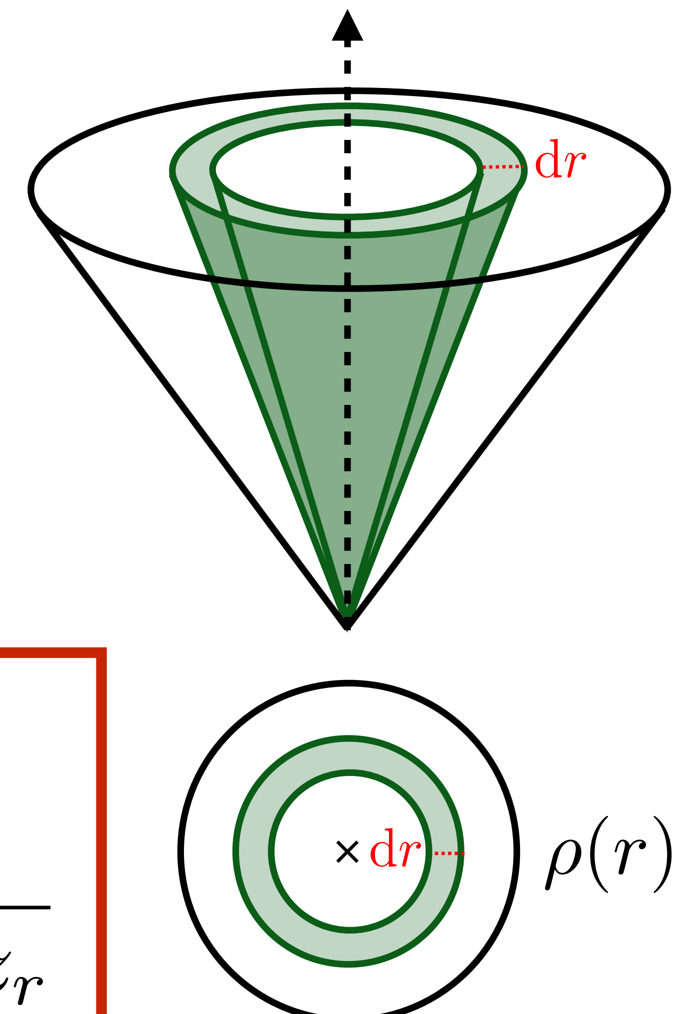


Theory:

$$\psi(r) = \int_0^1 \mathrm{d}z_r\, z_r \frac{\mathrm{d}\sigma}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}z_r}$$

$$\psi(r) = \frac{\sum_{r_i < r} p_{Ti}}{\sum_{r_i < R} p_{Ti}}$$

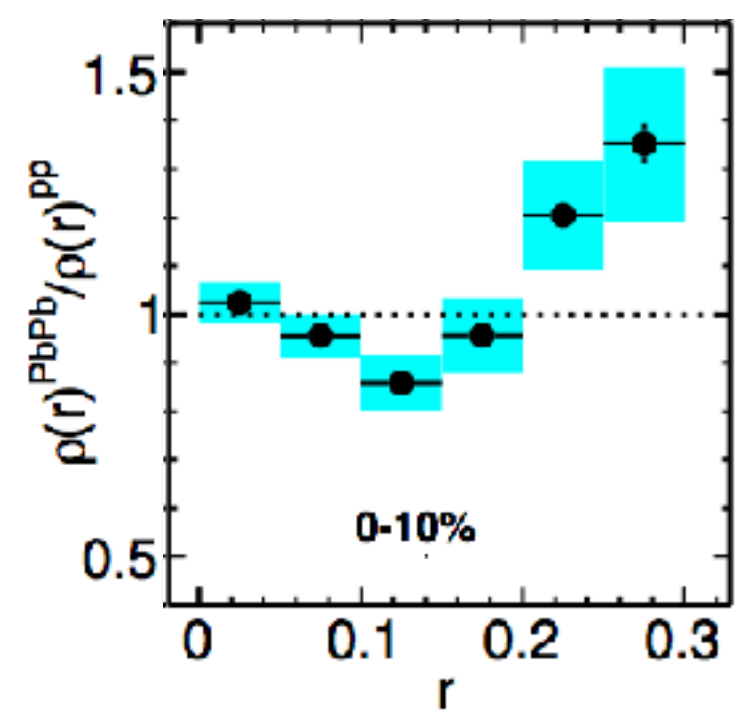
$$\rho(r) = \frac{\mathrm{d}\psi(r)}{\mathrm{d}r}$$

Jet shape

- Why the jet shape?

Ellis, Kunzt, Soper ’92
Seymour ’98
Li, Li, Yuan ’11
Chien, Vitev ’14

- Classic jet substructure observable
- Existing data sets include pp, $p\bar{p}$, ep, e^+e^- and heavy-ion
- Constrain parton shower event generators (had. models, underlying event contributions)
- Quark/Gluon jet discrimination
- Probe the QGP in AA collisions
- Measured for heavy-flavour jets



CMS, PLB 730 (2014) 243

Small R factorization

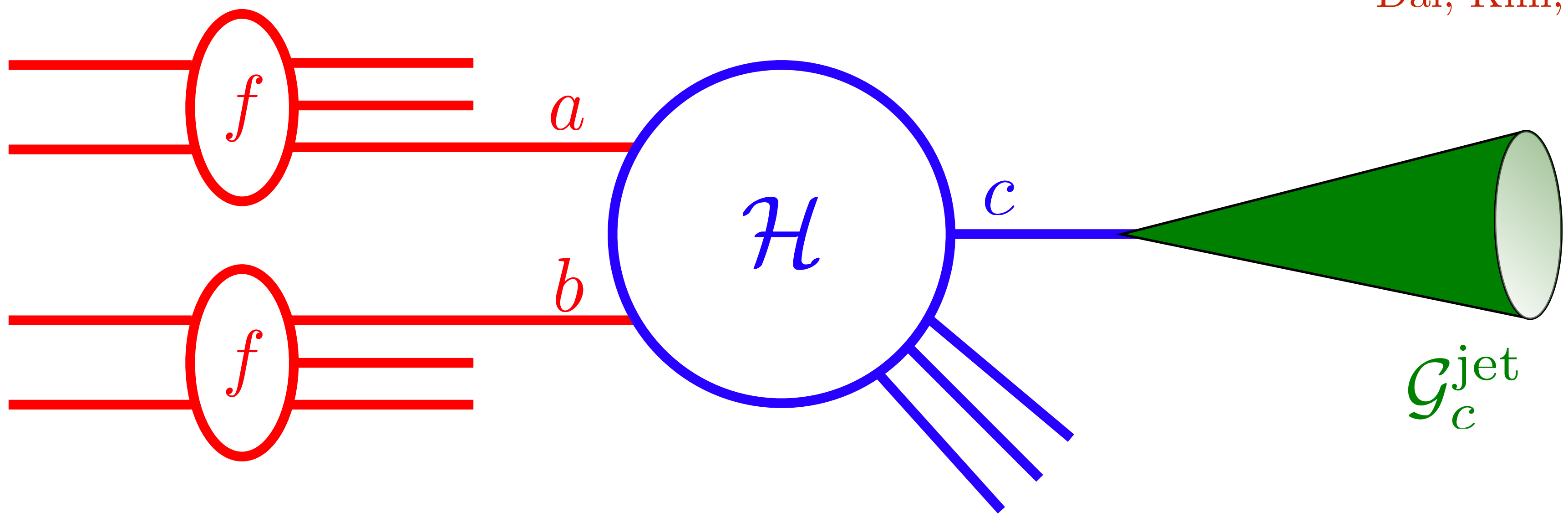
- $R \ll 1$ ⟶ Factorize jet from remaining cross section

$$\frac{d\sigma}{dp_T d\eta dz_r} = f_a \otimes f_b \otimes \mathcal{H}_{ab}^c \otimes \mathcal{G}_c^{\text{jet}} \left[1 + \mathcal{O}(R^2)\right]$$

Kaufmann, Mukherjee, Vogelsang '15

Kang, Ringer, Vitev '16

Dai, Kim, Leibovich '16



- Resum $\alpha_s^n \ln^n R$ through DGLAP

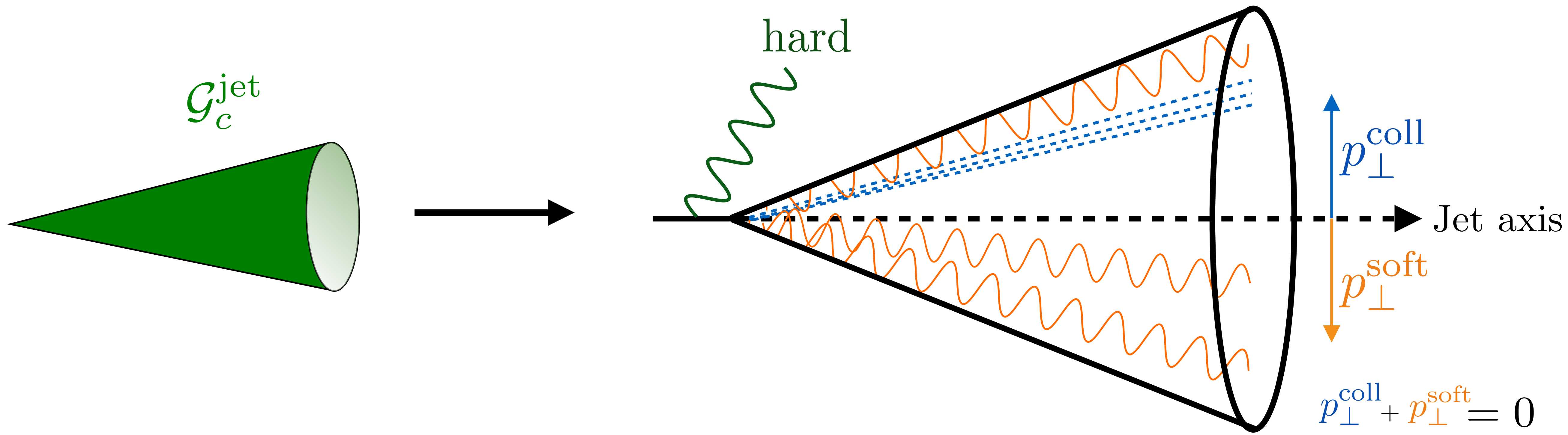
$$\mu \frac{d}{d\mu} \mathcal{G}_c^{\text{jet}}(z, z_r, p_T R, \mu) = \sum_j P_{ji}(z) \otimes \mathcal{G}_c^{\text{jet}}(z, z_r, p_T R, \mu)$$

Refactorization

- $r \ll R$ $\longrightarrow$ $\mathcal{G}_c^{\rm jet}$ contains large logs of $\dfrac{r}{R}$ which need to be resummed

- Perform factorization for $r \ll R$

$$\mathcal{G}_c^{\rm jet}(z, z_r, p_T R, r/R, \mu) = \sum_d H_{cd}\Big[C_d \otimes S_d\Big]\Big[1 + \mathcal{O}\Big(\frac{r}{R}\Big)\Big]$$

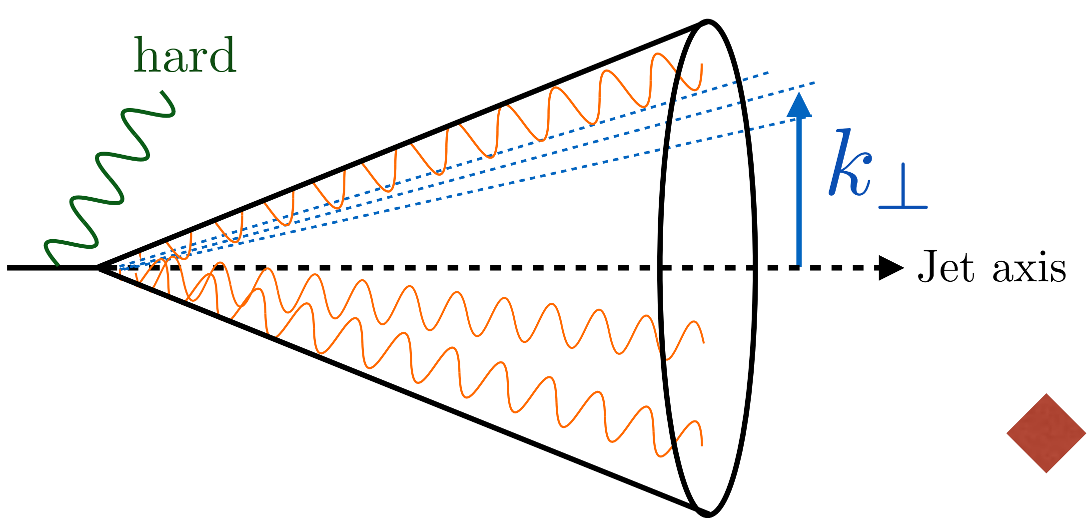


Jet shape at NLL′

		Fixed-order	β	γ_μ	γ_ν	NGLs
$\ln R$	LL	tree	1-loop	1-loop	-	-
→	NLL	1-loop	2-loop	2-loop	-	-
	NNLL	2-loop	3-loop	3-loop	-	-
$\ln(r/R)$	LL	tree	1-loop	1-loop	-	-
	NLL	tree	2-loop	2-loop	1-loop	LL
→	NLL′	1-loop	2-loop	2-loop	1-loop	LL
	NNLL	1-loop	3-loop	3-loop	2-loop	NLL

$$\mathcal{G}_c^{\rm jet}(z, z_r, p_T R, r/R, \mu) \overset{\rm NLL'}{=} \sum_d H_{cd}(z, p_T R, \mu) \int {\rm d}^2 k_\perp \, C_d(z_r, p_T r, k_\perp, \mu, \nu)$$
$$\times \, S_d^{\rm G}(k_\perp, \mu, \nu R) S_d^{\rm NG}\left(\frac{r}{R}\right)\left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right]$$

Global   Non Global

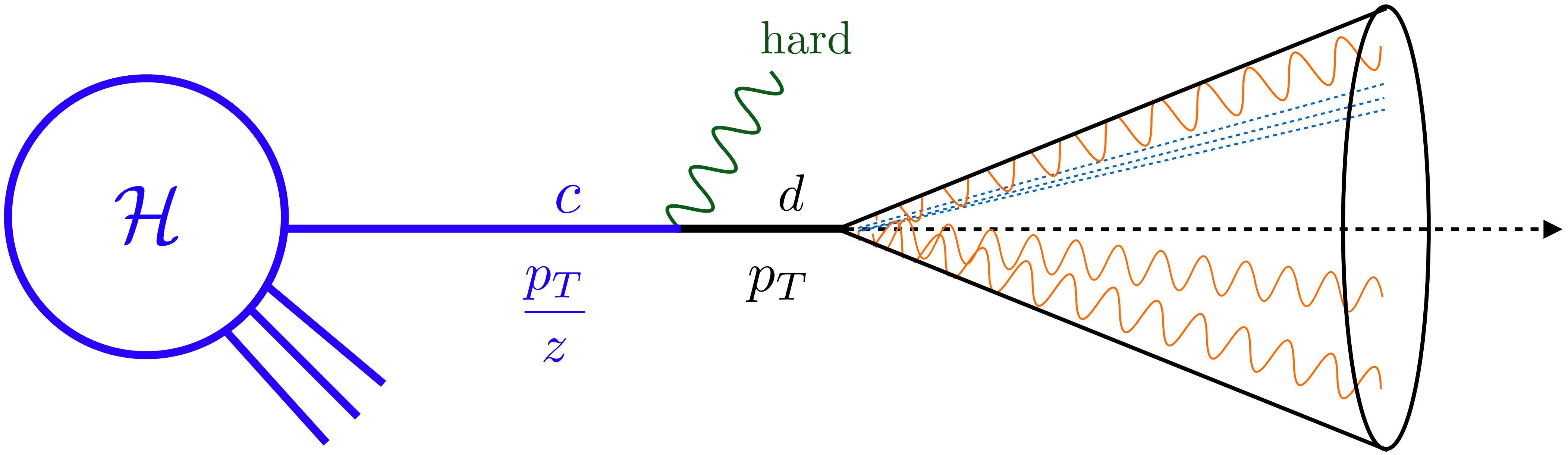


- Appearance of the ν (rapidity) scale

Jet shape at NLL′

1. Separate jet production from jet shape
2. Compute collinear function with recoil
3. Soft function and NGLs
4. Resummation and matching
5. Implementation
6. Nonperturbative effects
7. Results for the LHC

- This simplifies numerical implementation



$$d\sigma = f_a \otimes f_b \otimes \mathcal{H}_{ab}^{c}\left(\frac{p_T}{z}, \ldots\right) \otimes \mathcal{G}_c^{\text{jet}}\left(z, \ldots\right)$$

z appears in hard scattering and jet function

- Preferable to organize the calculation to separate jet production from jet shape

① Separate jet production and the jet shape

- Jet shape for $r \lesssim R$

$$\psi_{q,r\lesssim R}(r) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\frac{1}{2}L^2_{r/R} + \frac{3}{2}L_{r/R} - \frac{9}{2} + \frac{6r}{R} - \frac{3r^2}{2R^2}\right]$$

Kang, Ringer, Waalewijn '17

- Refactorization for $r \ll R$

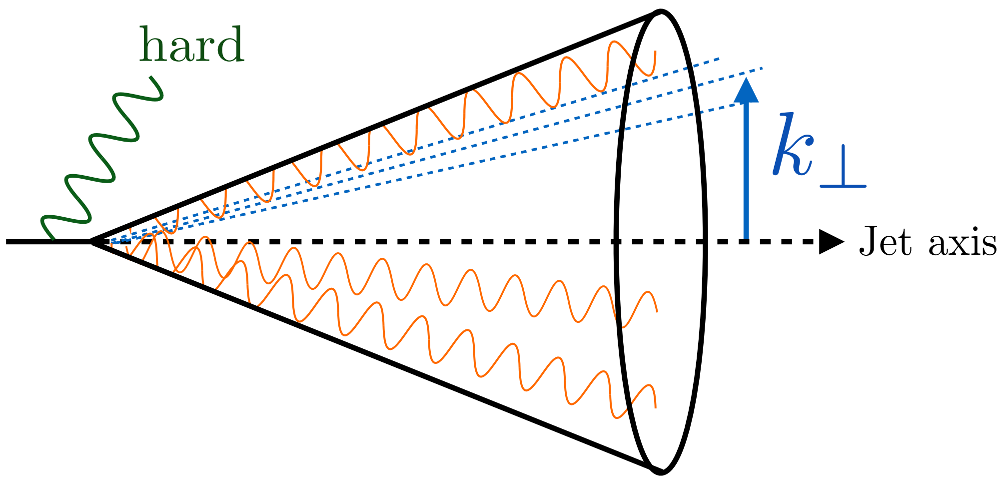
$$\psi_{d,r\ll R}(r) \overset{\mathrm{NLL}'}{=} \tilde{H}_d(p_T R,\mu)\int \mathrm{d}^2 k_\perp \int \mathrm{d}z_r\, z_r\, C_d(z_r, p_T r, k_\perp, \mu, \nu)$$
$$\times\, S_d^{\mathrm{G}}(k_\perp,\mu,\nu R)\, S_d^{\mathrm{NG}}\left(\frac{r}{R}\right)\left[1+\mathcal{O}\left(\frac{r}{R}\right)\right]$$

No z dependence

$$\tilde{H}_d(p_T R,\mu) = \int \mathrm{d}z \sum_e \left[H_{de}(z, p_T R,\mu) - J^{(1)}_{de}(z, p_T R,\mu)\right]$$

z dependence cancels out

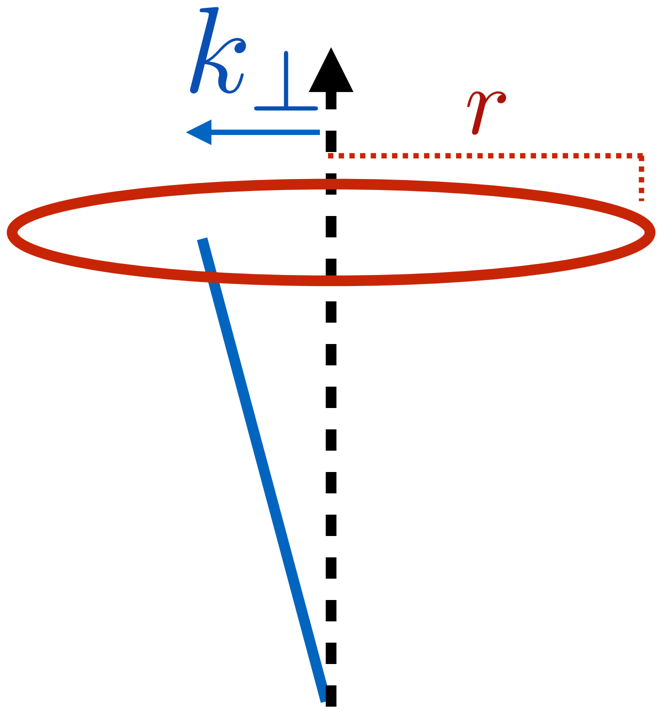
② Collinear function with recoil



$$\psi_{d,r\ll R}(r) \overset{\mathrm{NLL}'}{=} \tilde{H}_d(p_T R,\mu) \int \mathrm{d}^2 k_\perp \int \mathrm{d}z_r\, z_r\, \boxed{C_d(z_r, p_T r, k_\perp, \mu, \nu)} \times S_d^{\mathrm{G}}(k_\perp,\mu,\nu R)\, S_d^{\mathrm{NG}}\left(\frac{r}{R}\right)$$

◆ LO Collinear function with recoil:

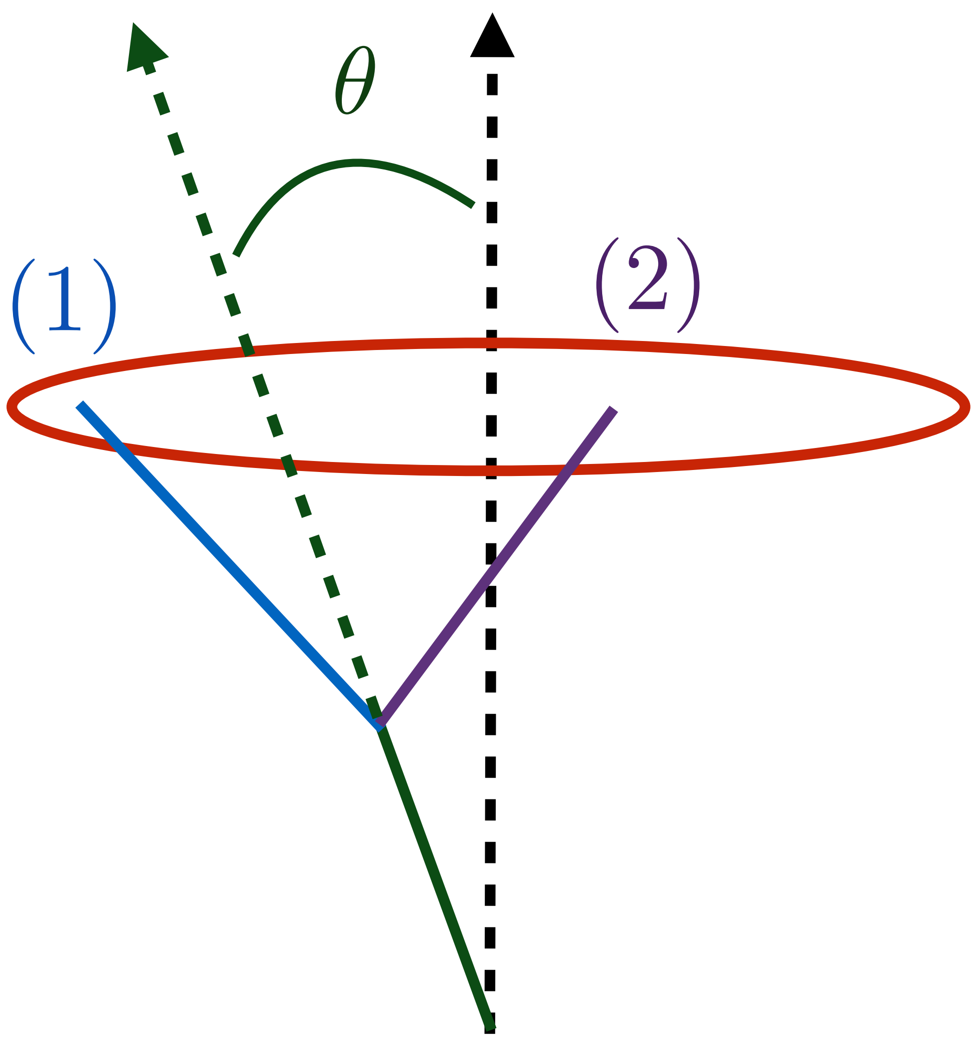
$$C_d^{(0)}(z_r, p_T r, k_\perp, \mu, \nu) = \delta(1-z_r)\,\Theta(k_\perp < p_T r)$$



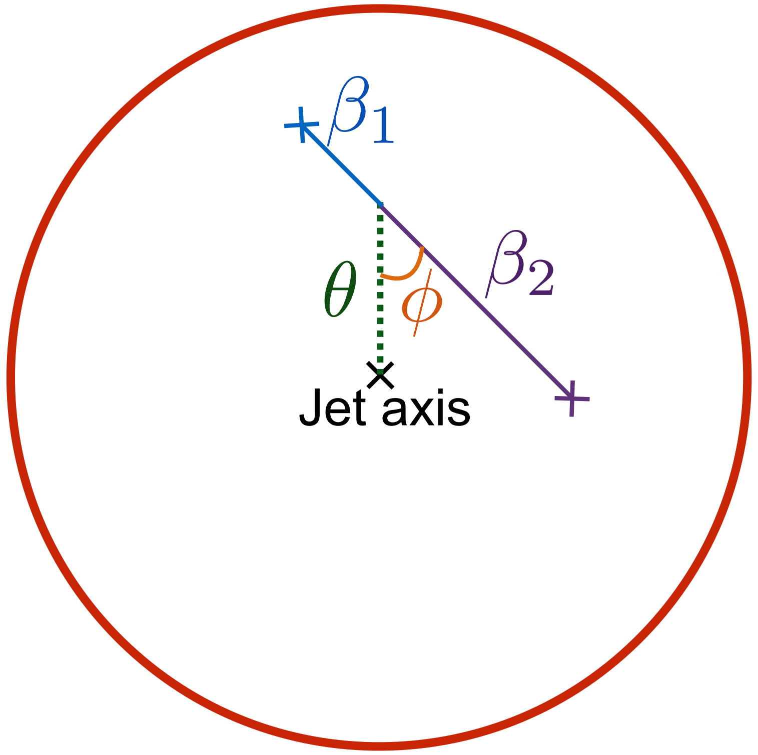
② Collinear function with recoil

- NLO Collinear function accounting for recoil is more involved
- One needs to consider all possible cases

Side view



Front view



$$\theta = \frac{k_\perp}{p_T}$$

② Collinear function with recoil

- First: separate cases with splitting inside and outside the subjet

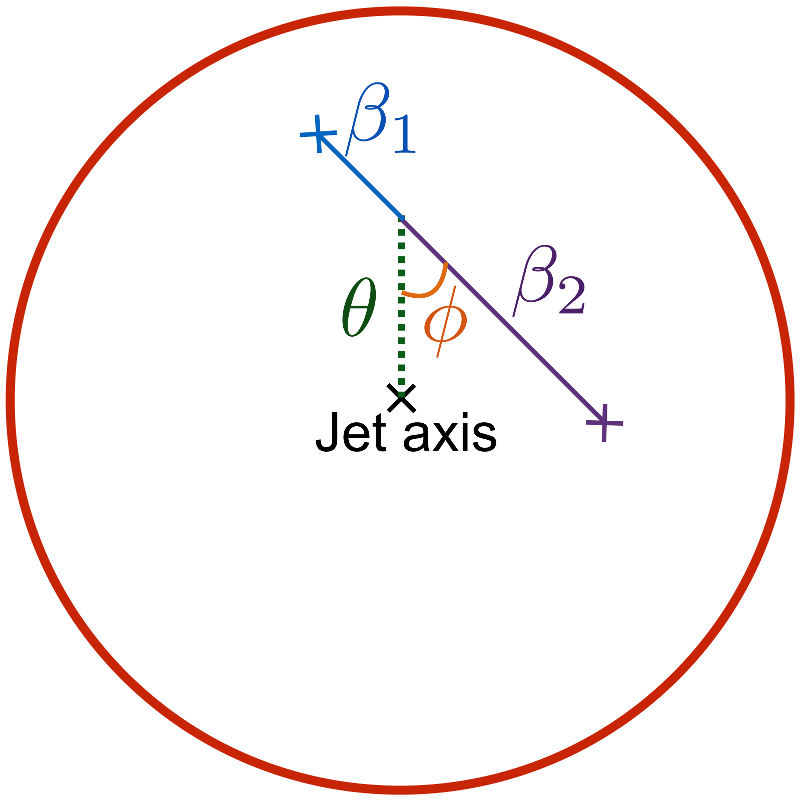
$$C_i^{(1)}(z_r, p_T r, k_\perp, \mu, \nu) = \Theta(k_\perp < p_T r)\, C_i^{(\theta<r)} + \Theta(k_\perp > p_T r)\, C_i^{(\theta>r)}$$

- Splitting inside the subjet:

(A) Both partons inside the subjet

(B) Parton (1) inside the subjet

(C) Parton (2) inside the subjet



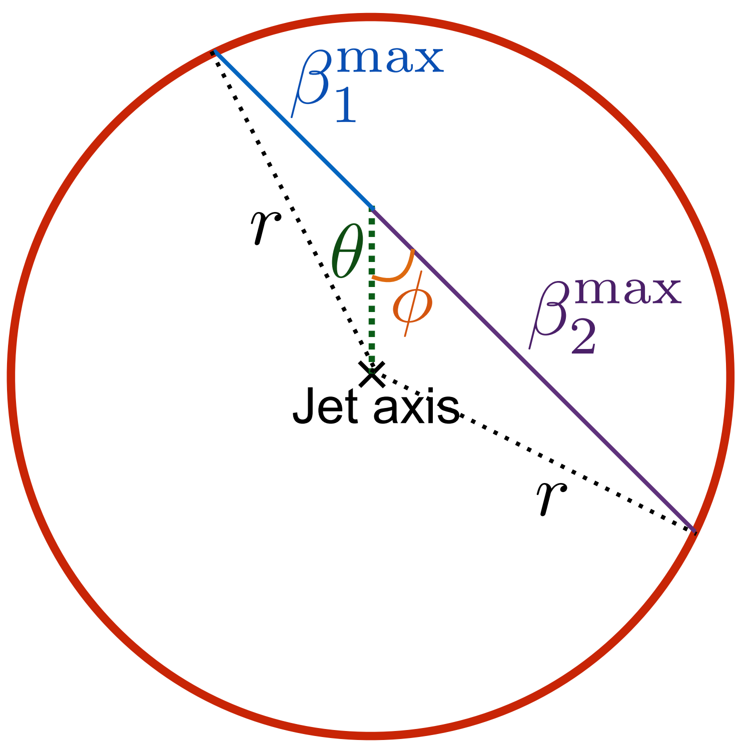
② Collinear function with recoil

- Appropriate collinear phase space restrictions

$$(\mathrm{A})_{\theta<\mathrm{r}} = \delta(1-z_r)\int \mathrm{d}\Phi_2\,\sigma^c_{2,q}\,\Theta(\beta_1<\beta_1^{\mathrm{max}})\Theta(\beta_2<\beta_2^{\mathrm{max}})$$

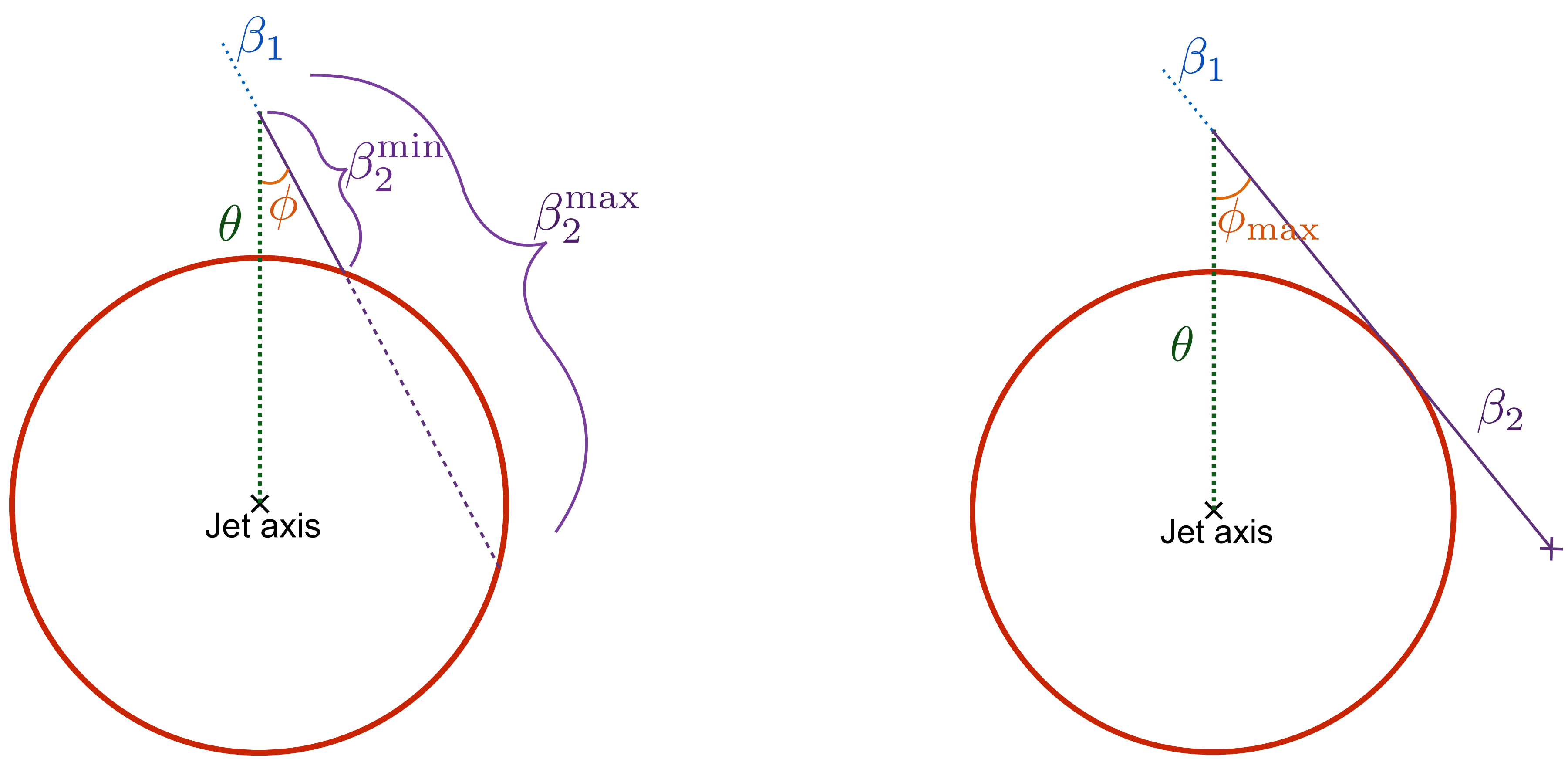
$$(\mathrm{B})_{\theta<\mathrm{r}} = \int \mathrm{d}\Phi_2\,\sigma^c_{2,q}\,\delta(x-z_r)\,\Theta(\beta_1<\beta_1^{\mathrm{max}})\Theta(\beta_2>\beta_2^{\mathrm{max}})$$

$$(\mathrm{C})_{\theta<\mathrm{r}} = \int \mathrm{d}\Phi_2\,\sigma^c_{2,q}\,\delta(1-x-z_r)\,\Theta(\beta_1>\beta_1^{max})\Theta(\beta_2<\beta_2^{max})$$



② Collinear function with recoil

- Splitting occurring outside the subjet



$$(\mathrm{B})_{\theta>r} = \int \mathrm{d}\Phi_2\, \sigma^c_{2,q}\, \delta(x - z_r)\, \Theta(\pi - \phi_{\rm max} < \phi < \pi + \phi_{\rm max})\, \Theta(\beta_1^{\rm min} < \beta_1 < \beta_1^{\rm max})$$

$$(\mathrm{C})_{\theta>r} = \int \mathrm{d}\Phi_2\, \sigma^c_{2,q}\, \delta(1 - x - z_r)\, \Theta(-\phi_{\rm max} < \phi < \phi_{\rm max})\, \Theta(\beta_2^{\rm min} < \beta_2 < \beta_2^{\rm max})$$

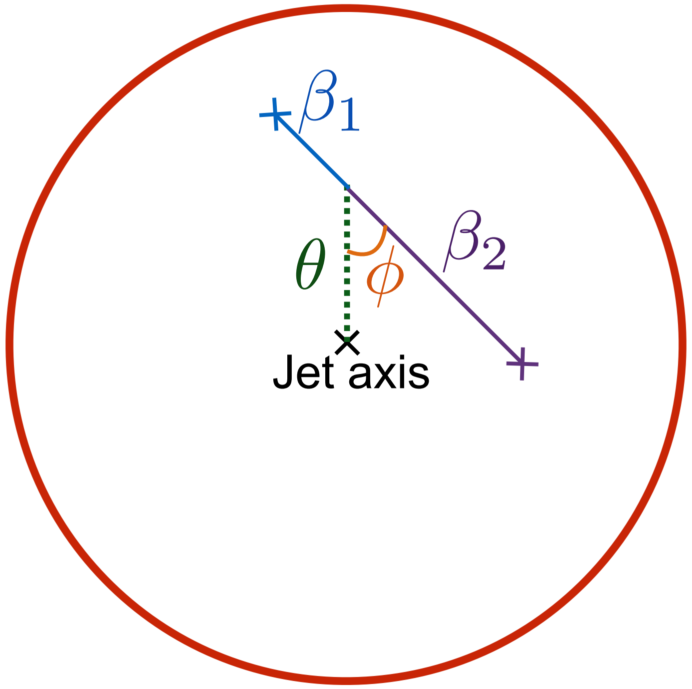
② Collinear function with recoil

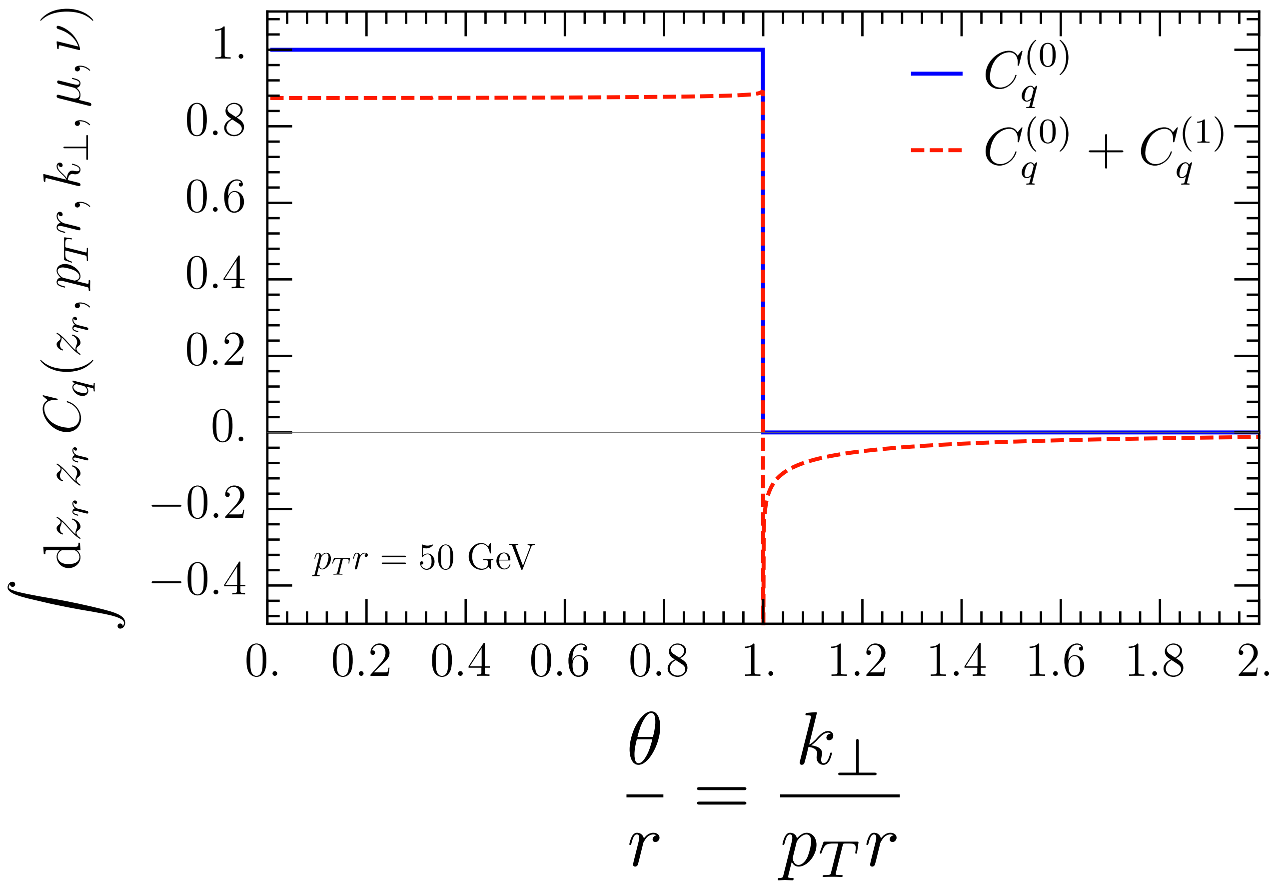
- Add it all up

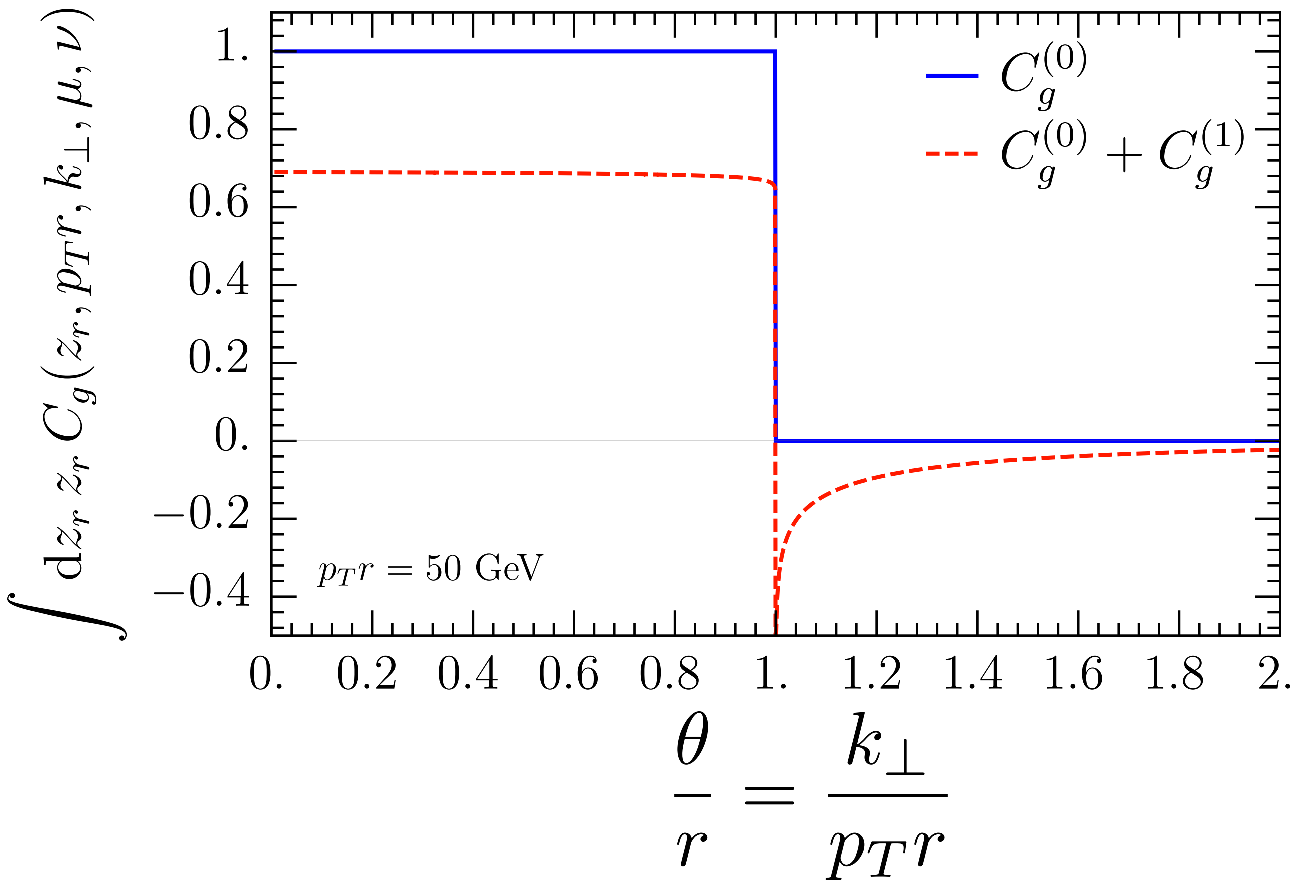
$$C_q^{(\theta<r)} = \frac{\alpha_s C_F}{2\pi^2}\int_0^{2\pi} \mathrm{d}\phi \left\{ \delta(1-z_r)\left[\frac{1}{\eta}\left(\frac{1}{\epsilon}+L_1\right)+\frac{1}{\epsilon}\left(L_\nu+\frac{3}{4}\right)+L_\nu L_1+\frac{3L_1}{4} \right.\right.$$
$$\left. - \ln^2(1-\widetilde{\beta}) + 2\ln\widetilde{\beta}\ln(1-\widetilde{\beta}) - \frac{3}{2}\ln\widetilde{\beta} + 2\mathrm{Li}_2(1-\widetilde{\beta}) - \frac{\widetilde{\beta}}{2} - \frac{\pi^2}{3} + 2\right]$$
$$+ \Theta(z_r > \widetilde{\beta})\left[-(1+z_r^2)\left(\frac{\ln(1-z_r)}{1-z_r}\right)_+ + \ln\left(\frac{z_r(1-\widetilde{\beta})}{\widetilde{\beta}}\right)\frac{1+z_r^2}{(1-z_r)}_+\right]$$
$$\left.+ \Theta(z_r > 1-\widetilde{\beta})\left[\frac{1+(1-z_r)^2}{z_r}\ln\left(\frac{z_r\widetilde{\beta}}{(1-z_r)(1-\widetilde{\beta})}\right)\right]\right\},$$
$$C_q^{(\theta>r)} = \frac{\alpha_s C_F}{2\pi^2}\left[\delta(1-z_r)\left(\frac{2}{\eta}+2L_\nu\right) - \frac{1+z_r^2}{(1-z_r)}_+ - \frac{1+(1-z_r)^2}{z_r}\right]\int_{-\phi_{\max}}^{\phi_{\max}} \mathrm{d}\phi \ln\left(\frac{\beta_2^{\min}}{\beta_2^{\max}}\right)$$

- Cannot preform ϕ integral analytically
- Can extract divergences analytically

② Collinear function with recoil





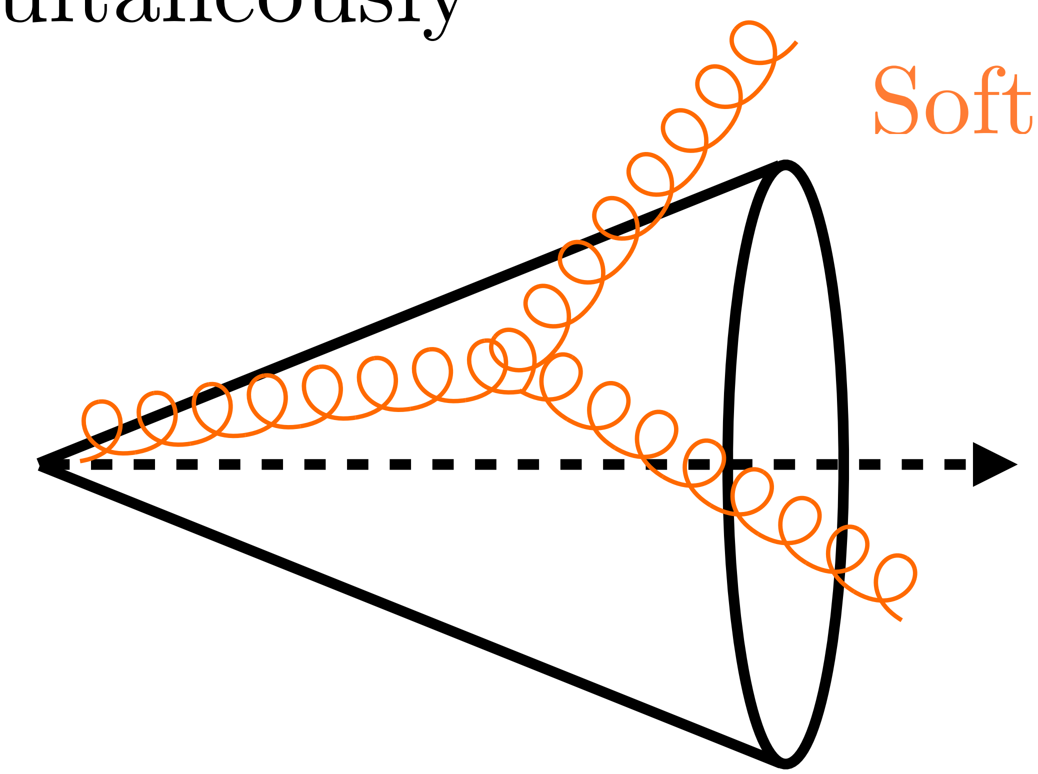


③ Soft function and Non Global Logs

- Global Soft function at one loop

$$S_q^{\rm G}(k_\perp,\mu,\nu R) = \delta^2(k_\perp) + \frac{\alpha_s C_F}{2\pi^2}\left[-\frac{1}{\mu^2}\left(\frac{\ln(k_\perp^2/\mu^2)}{k_\perp^2/\mu^2}\right)_+ + \frac{1}{\mu^2}\frac{1}{(k_\perp^2/\mu^2)}_+ \ln\frac{\nu^2 R^2}{4\mu^2} - \frac{\pi^2}{12}\delta(\vec{k}_\perp^{\,2})\right]$$

- NGLs arise from soft emissions probing inside and outside of jet simultaneously



Dasgupta, Salam 01'

③ Soft function and Non Global Logs

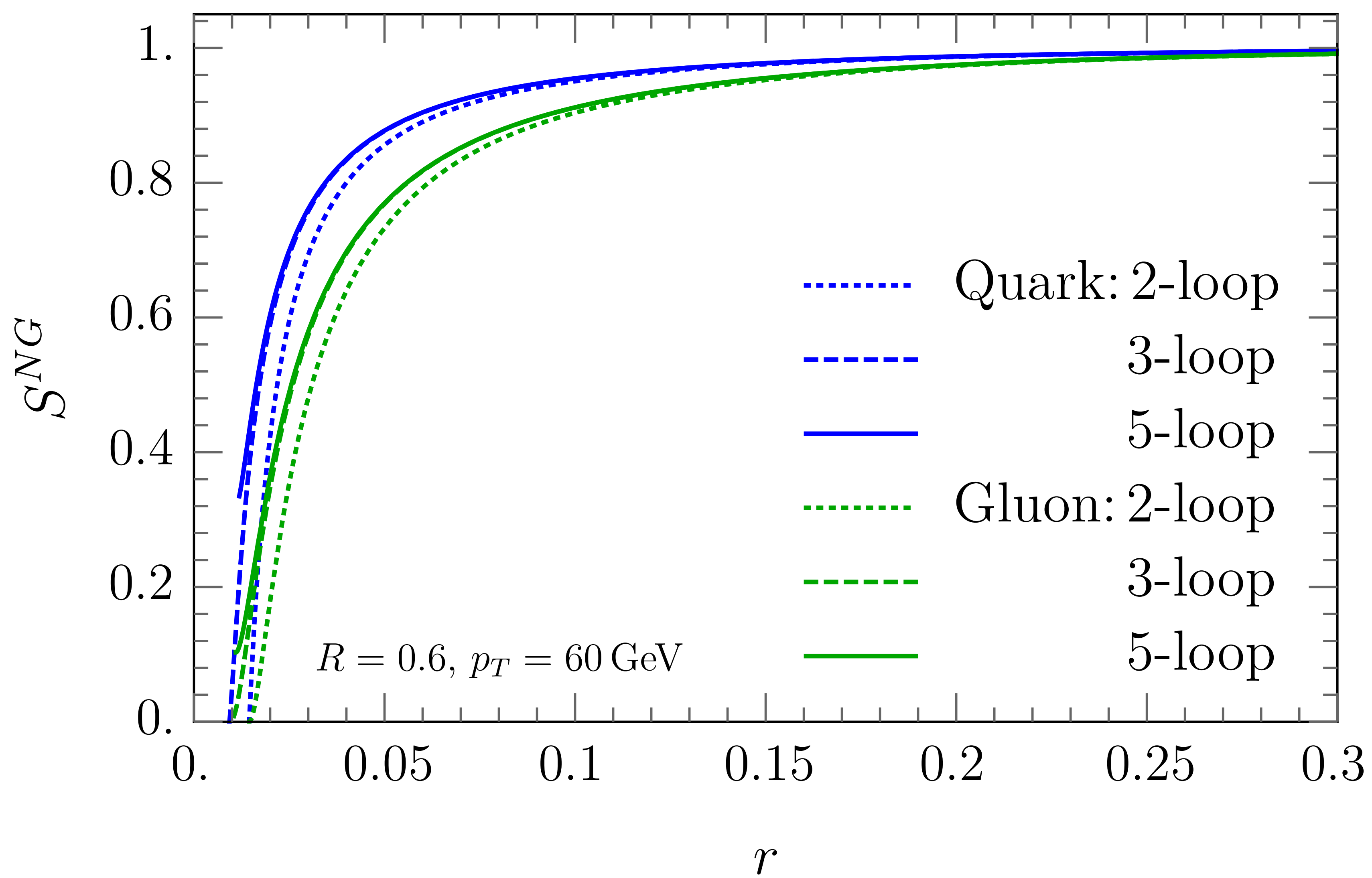
- For $S_i^{\rm NG}$: solution to BMS equation up to five-loop order

$$S_q^{\rm NG}(\widehat{L}) = 1 - \frac{\pi^2}{24}\widehat{L}^2 + \frac{\zeta_3}{12}\widehat{L}^3 + \frac{\pi^4}{34560}\widehat{L}^4 + \left(-\frac{\pi^2\zeta_3}{360} + \frac{17\zeta_5}{480}\right)\widehat{L}^5 + \mathcal{O}(L^6)$$

$$\widehat{L} = \frac{\alpha_s N_c}{\pi}\ln\frac{R}{r} \qquad S_g^{\rm NG} = \left(S_q^{\rm NG}\right)^2$$

Banfi, Marchesini, Smye '02

Schwartz, Zhu '14



④ Resummation and Matching

- We have all the ingredients for resummation
  - Hard functions
  - Global Soft function
  - Non Global Soft function

  ... and now also:
  - Collinear function with recoil
- Scales for the $r \lesssim R$ regime

Hard scale

$$\mu_{\mathcal{H}} = p_T$$

Jet scale

$$\mu_{\mathcal{G}} = p_T R$$

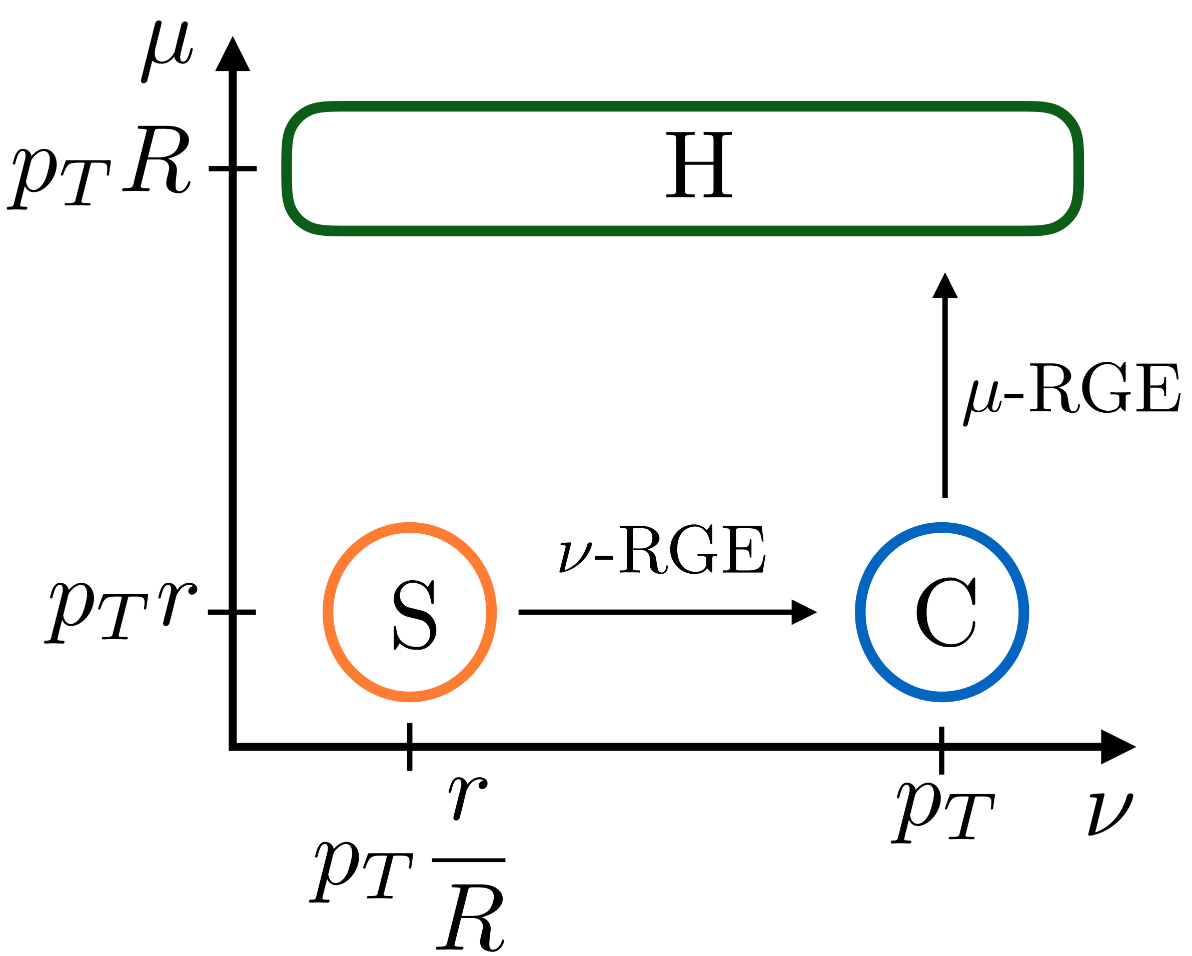
④ Resummation and Matching

$$\psi_{d,r\ll R}(r) \overset{\text{NLL}'}{=} \tilde{H}_d(p_T R,\mu) \int \mathrm{d}^2 k_\perp \int \mathrm{d}z_r\, z_r\, C_d(z_r, p_T r, k_\perp, \mu, \nu) \times S_d^{\text{G}}(k_\perp, \mu, \nu R)\, S_d^{\text{NG}}\left(\frac{r}{R}\right)\left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right]$$

- Recall refactorization led to ν scale
- Including $r \ll R$ regime, we take as central scales

$$\mu_{\mathcal{H}} = p_T\,, \qquad \mu_H = p_T R\,, \qquad \mu_C = p_T r\,, \qquad \mu_S = p_T r\,,$$

$$\nu_C = p_T\,, \qquad \nu_S = \frac{1}{b_\perp R}$$

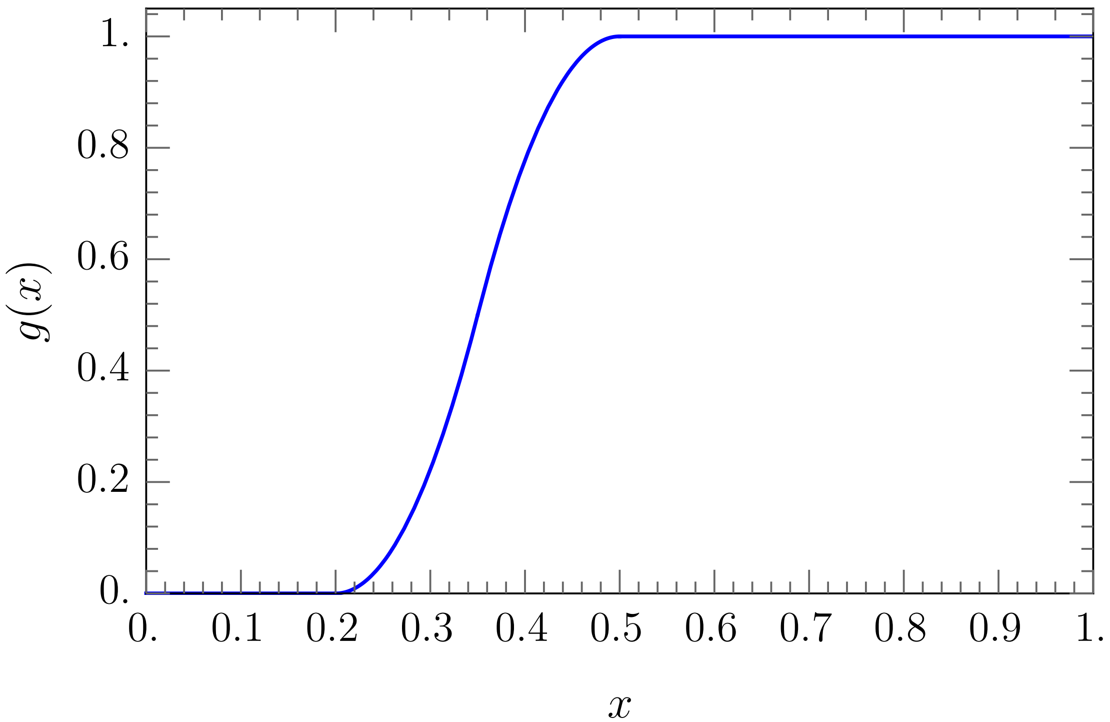


- Use simple interpolation to go from resummation to FO region

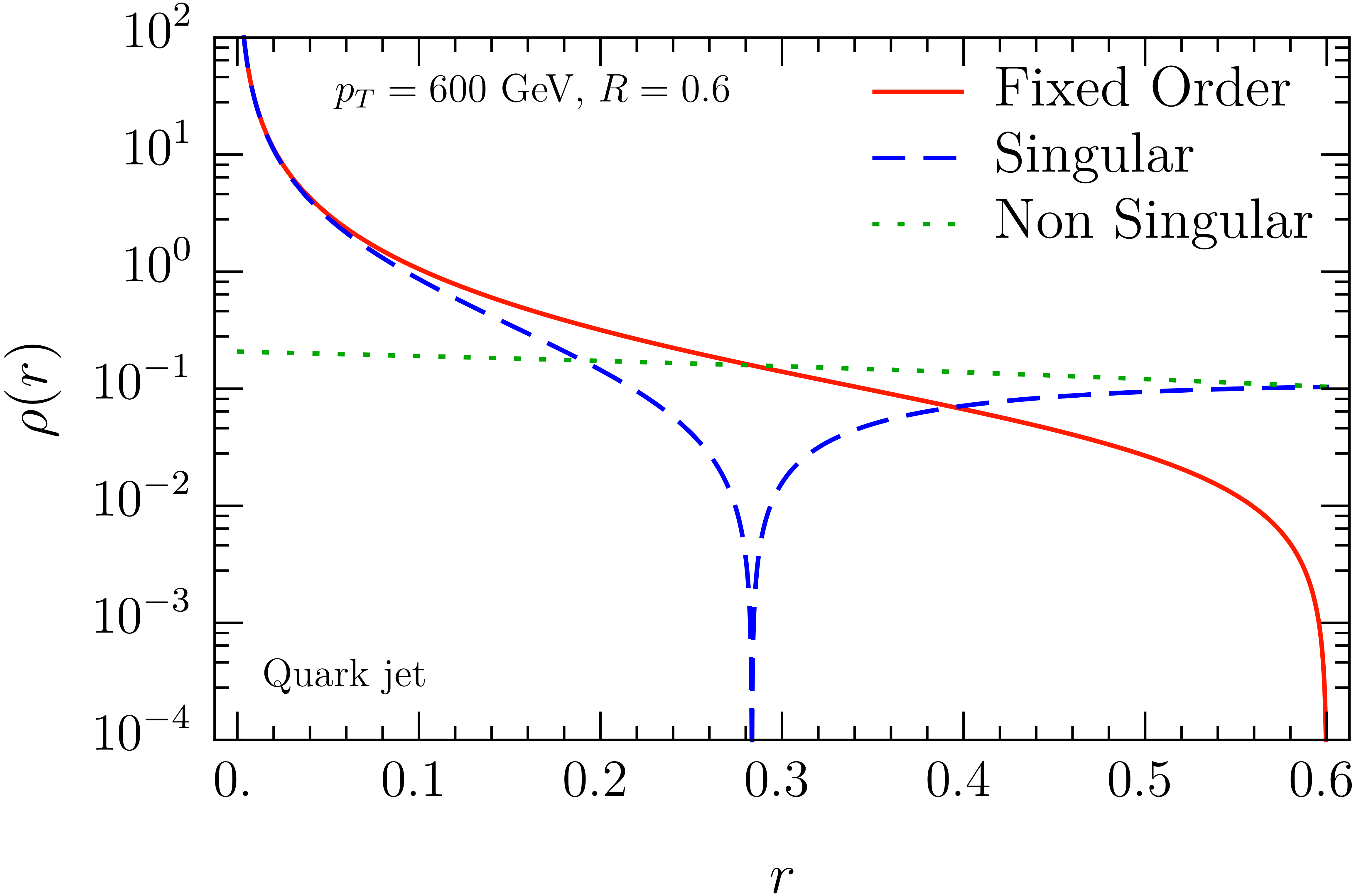
$$\psi(r) = \left[1 - g\left(\frac{r}{R}\right)\right]\psi_{r\ll R}(r) + g\left(\frac{r}{R}\right)\psi_{r\lesssim R}(r)$$

Smooth function going from 0 to 1

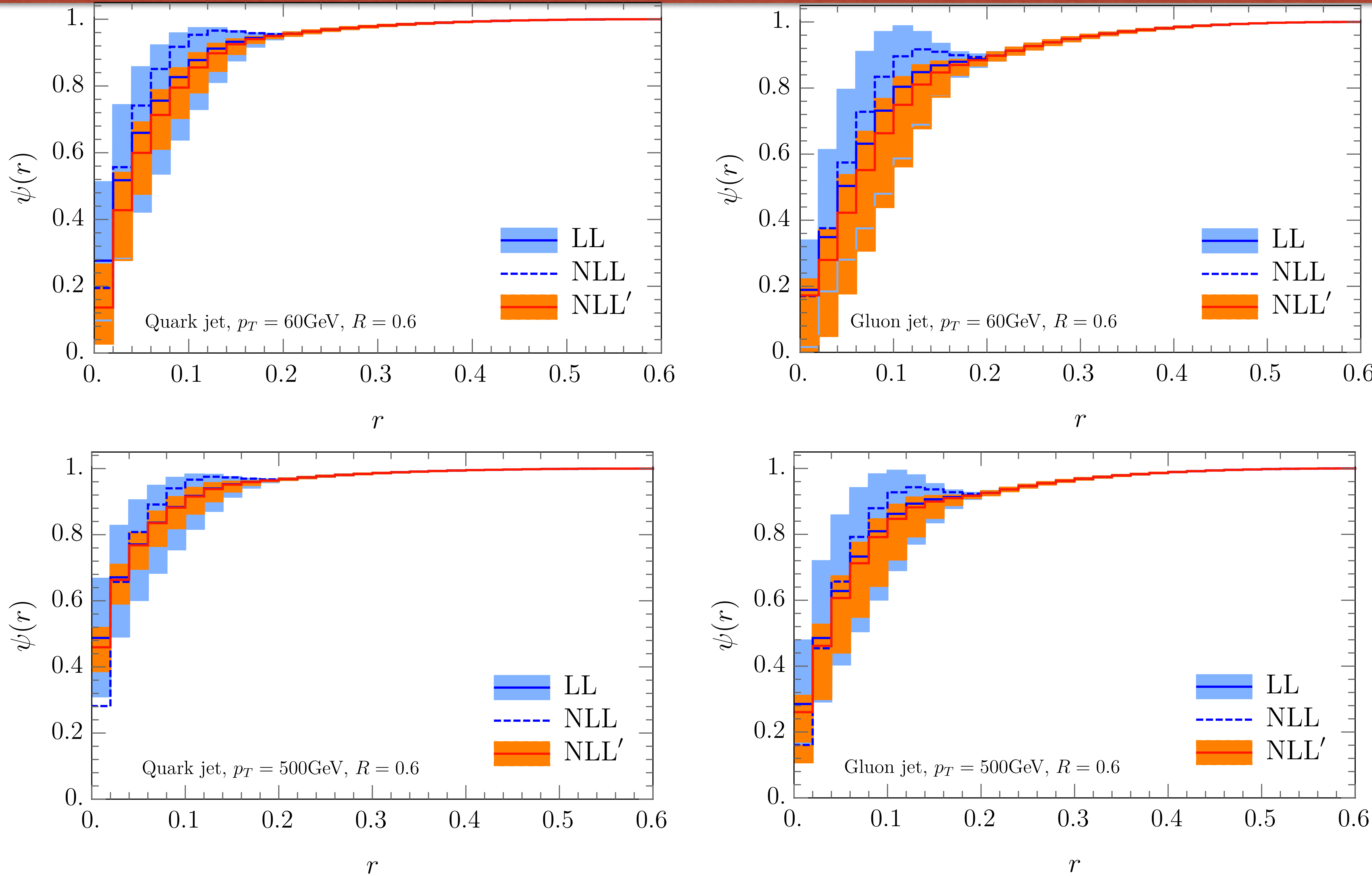
$$g(x) = \begin{cases} 0 & x \le x_1 \\ \frac{(x-x_1)^2}{(x_2-x_1)(x_3-x_1)} & x_1 \le x \le x_2 \\ 1 - \frac{(x-x_3)^2}{(x_3-x_1)(x_3-x_2)} & x_2 \le x \le x_3 \\ 1 & x_3 \le x \end{cases}$$



- Checking refactorization and choosing transition points



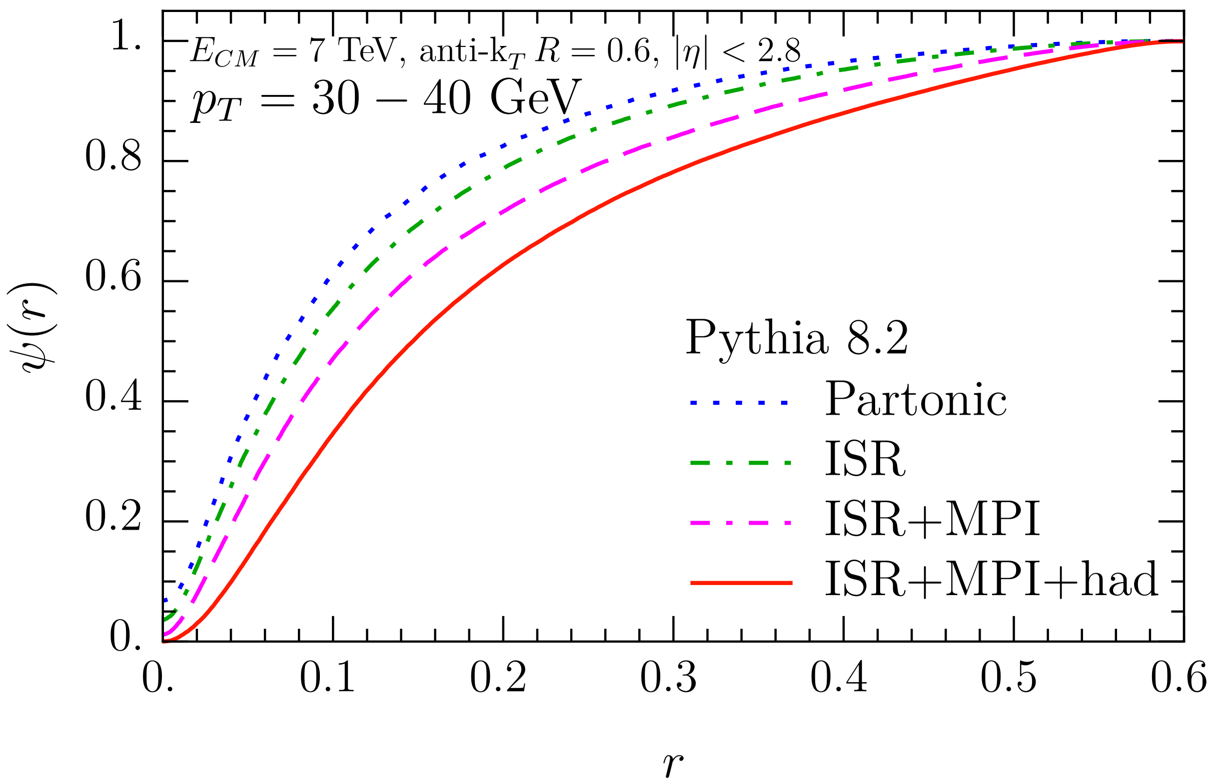
◇5 Implementation

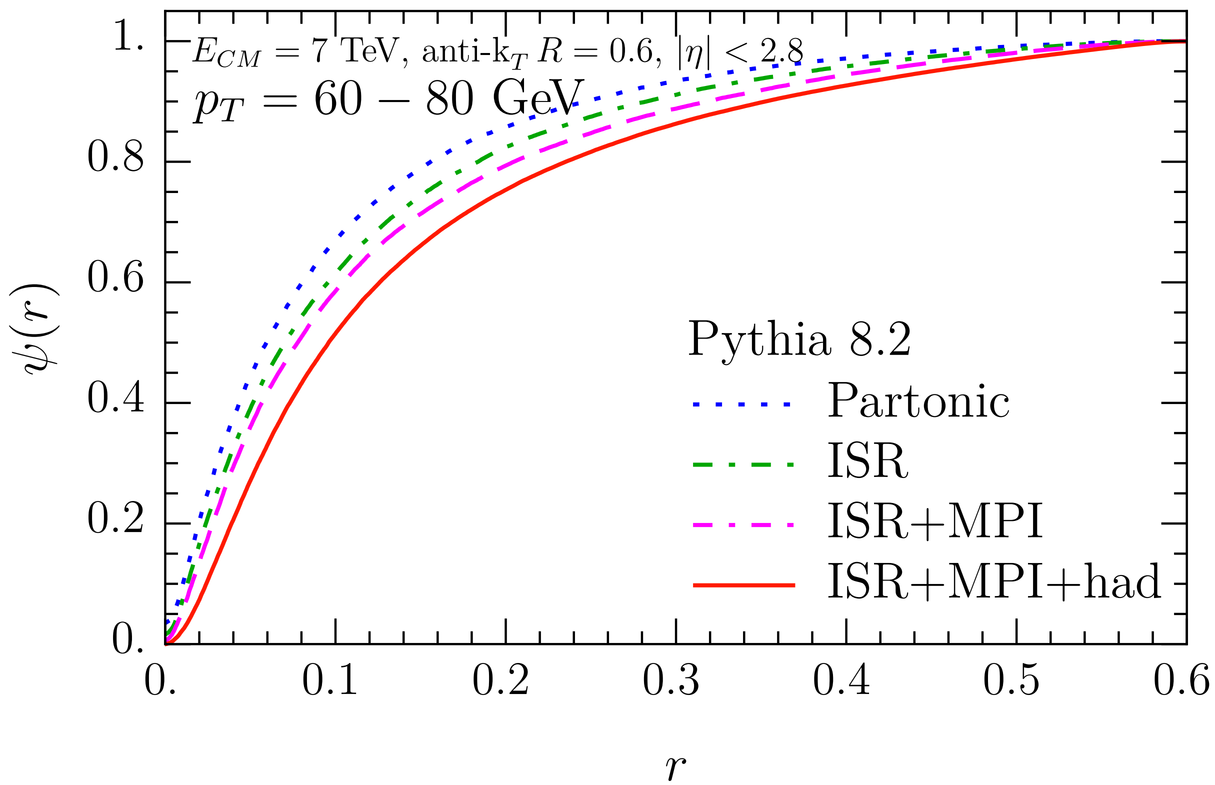


- Good convergence
- All matched to NLO

⑥ Nonperturbative effects

- Study nonperturbative (ISR, MPI and hadronization) effects on the jet shape using Pythia





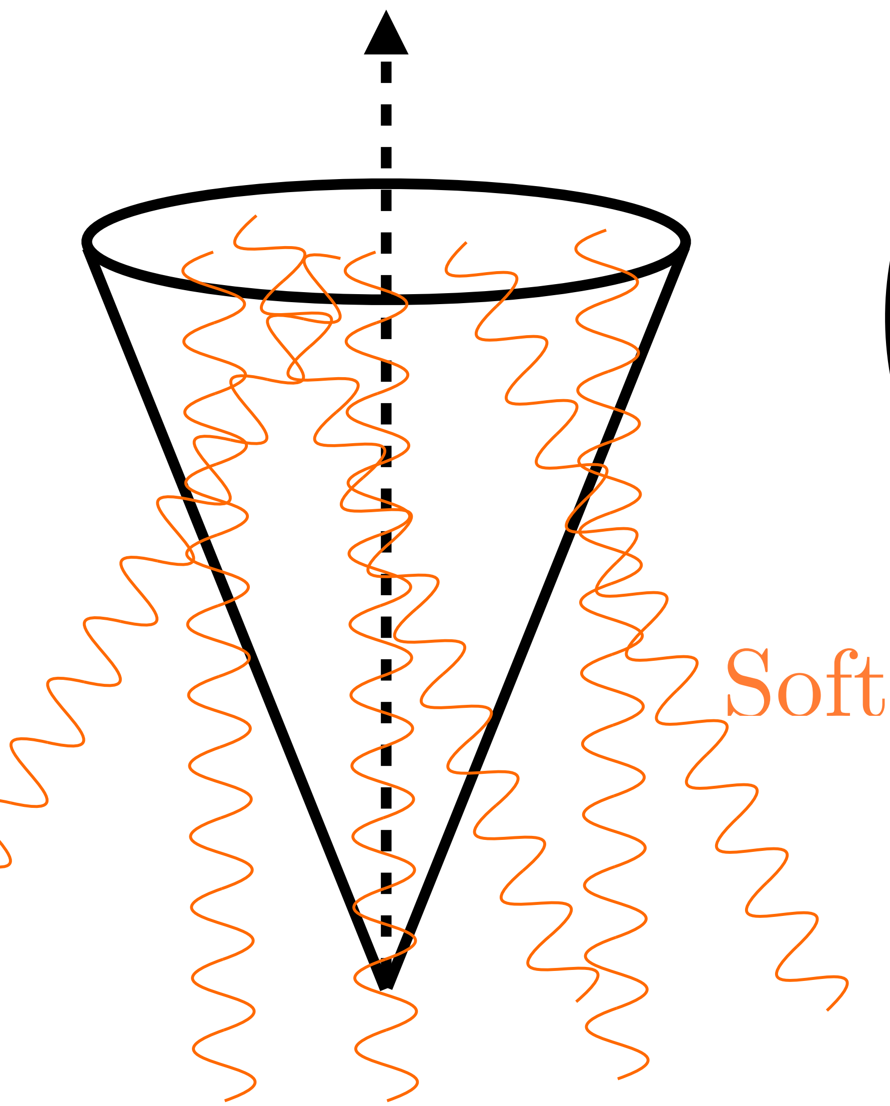
Significant nonperturbative effects!

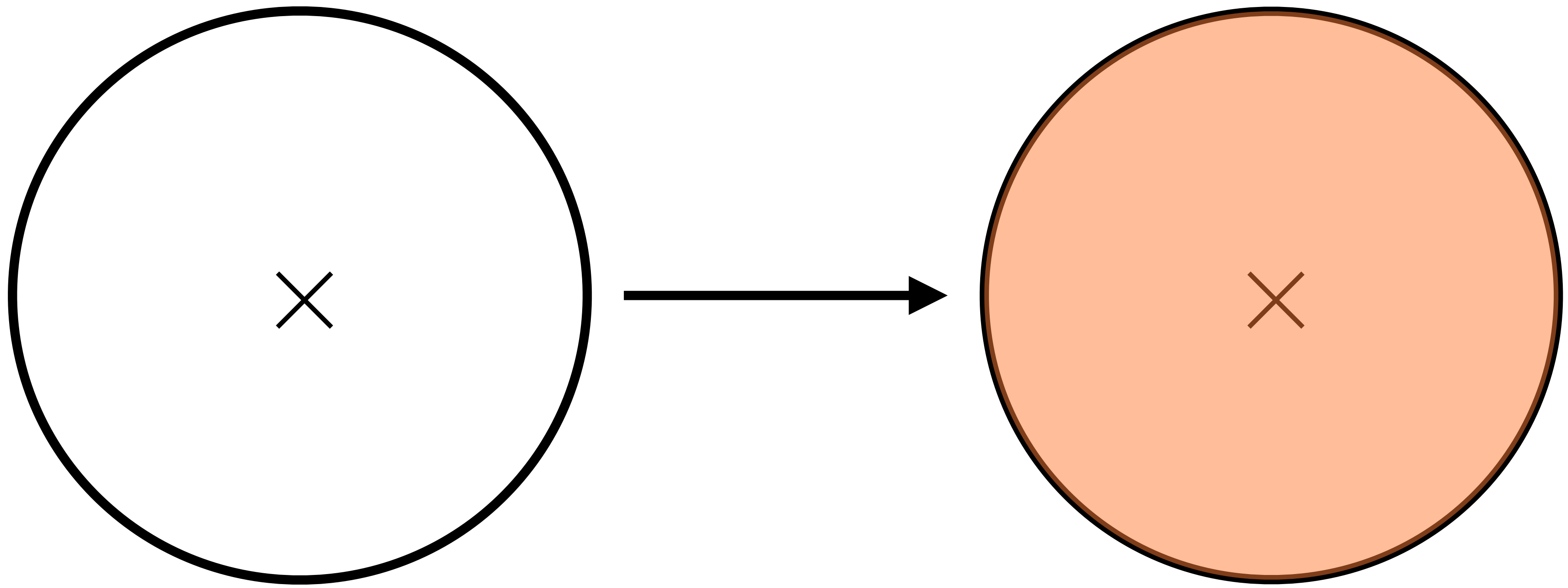
- Need model to describe nonperturbative effects

⑥ Nonperturbative effects

Model 1:

Uniform contamination





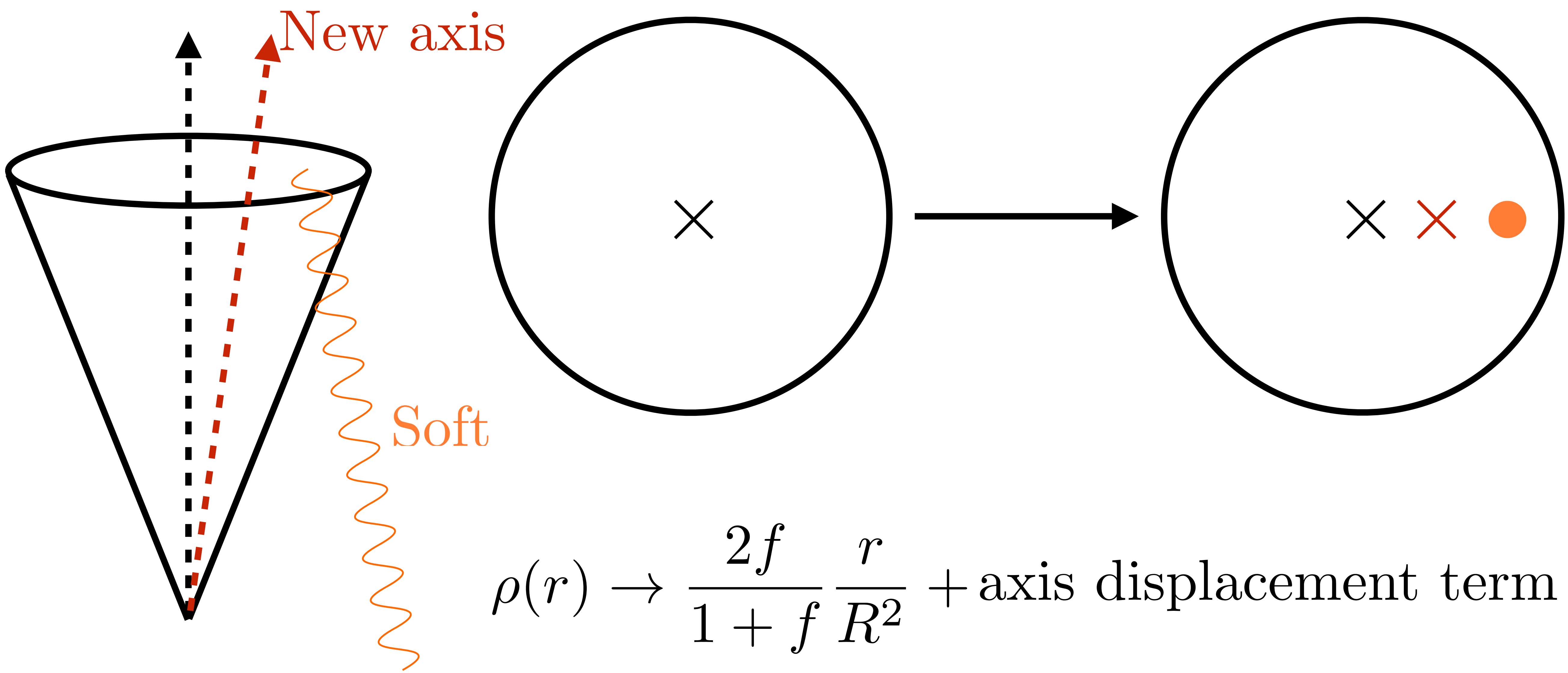
$$\rho(r) \to \frac{2f}{1+f}\frac{r}{R^2} + \frac{1}{1+f}\rho(r)$$

with $f = \frac{p_T^{\text{soft}}}{p_T^{\text{jet}}}$

⑥ Nonperturbative effects

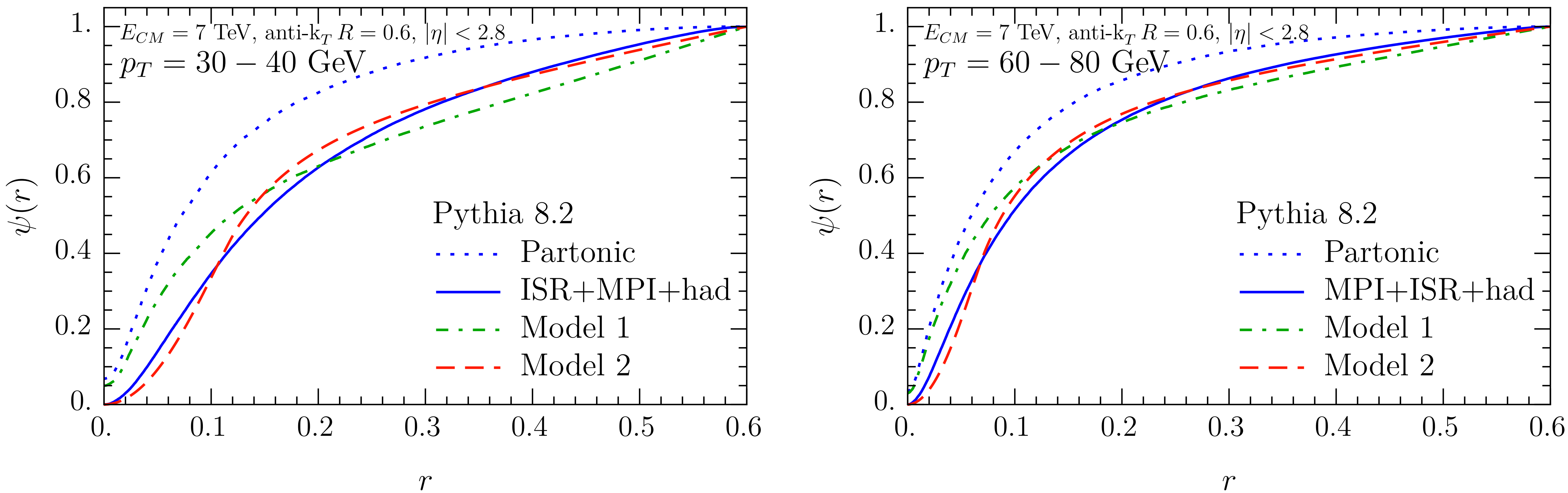
Model 2:

1 particle contamination



$$\rho(r) \to \frac{2f}{1+f}\frac{r}{R^2} + \text{axis displacement term}$$

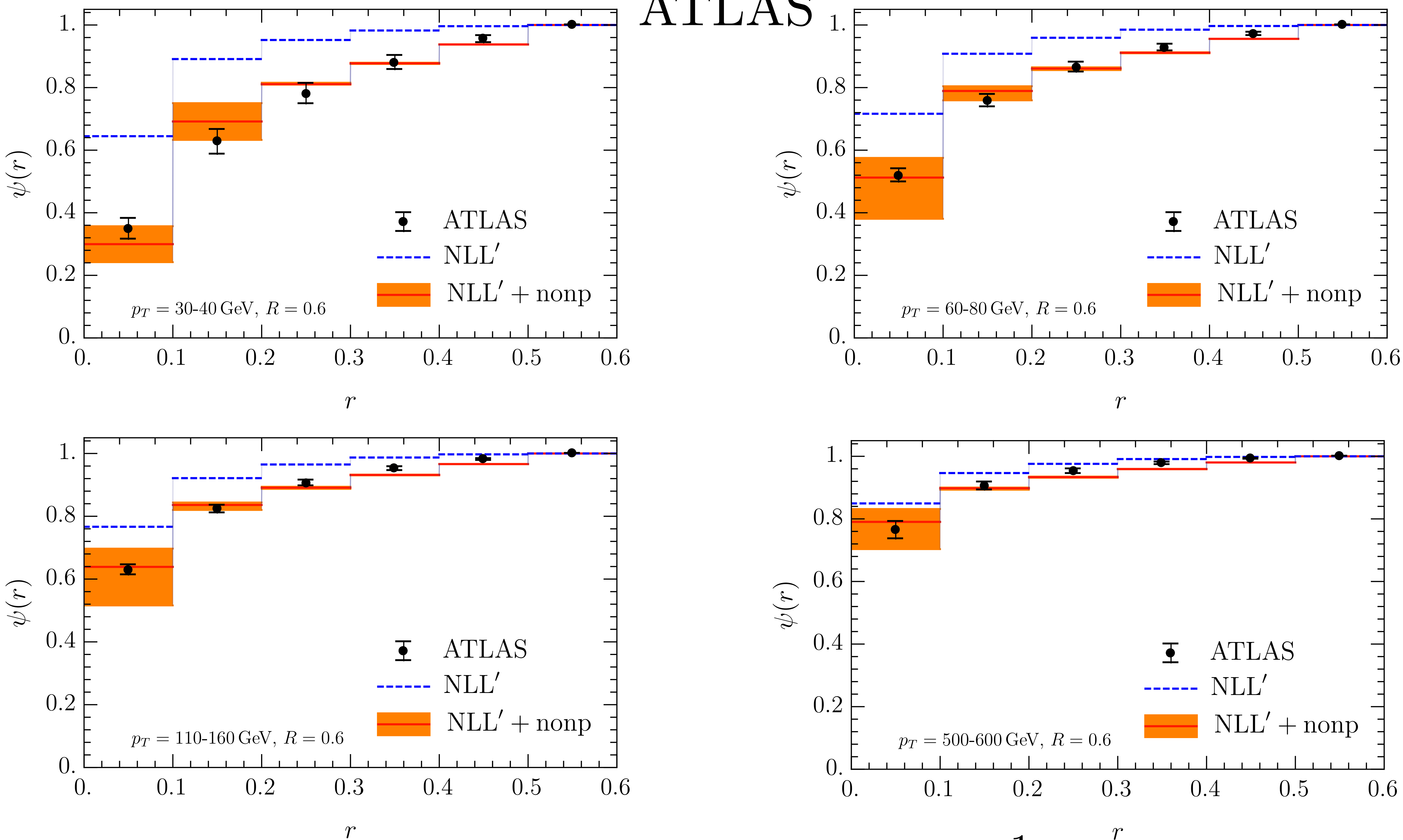
⑥ Nonperturbative effects



- Both models reasonably capture nonperturbative effects
- Model 2 better than model 1, used for LHC data

⑦ Results for the LHC

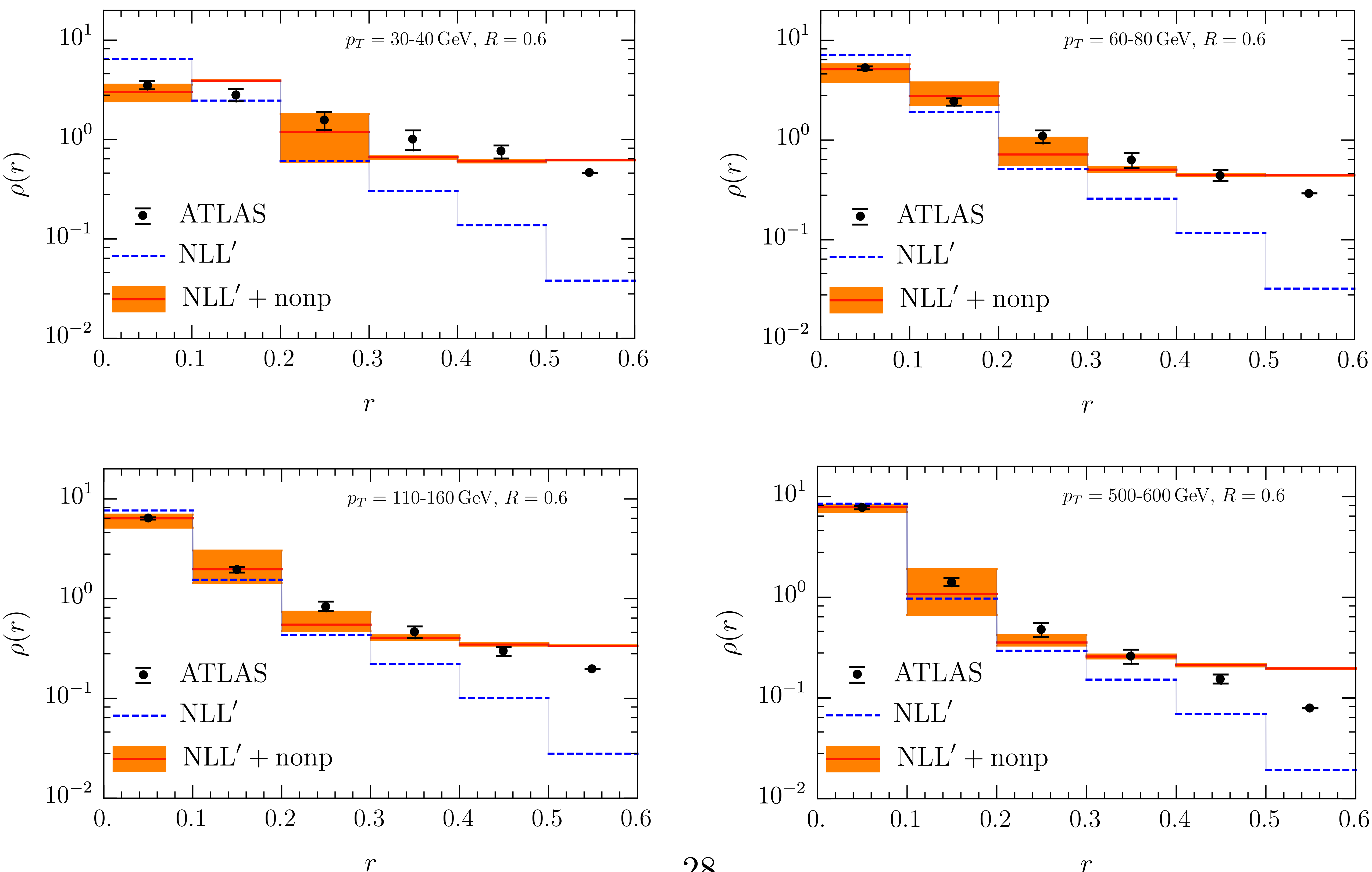
ATLAS



- Fit parameter f for central curves: $f \propto \dfrac{1}{p_T}$

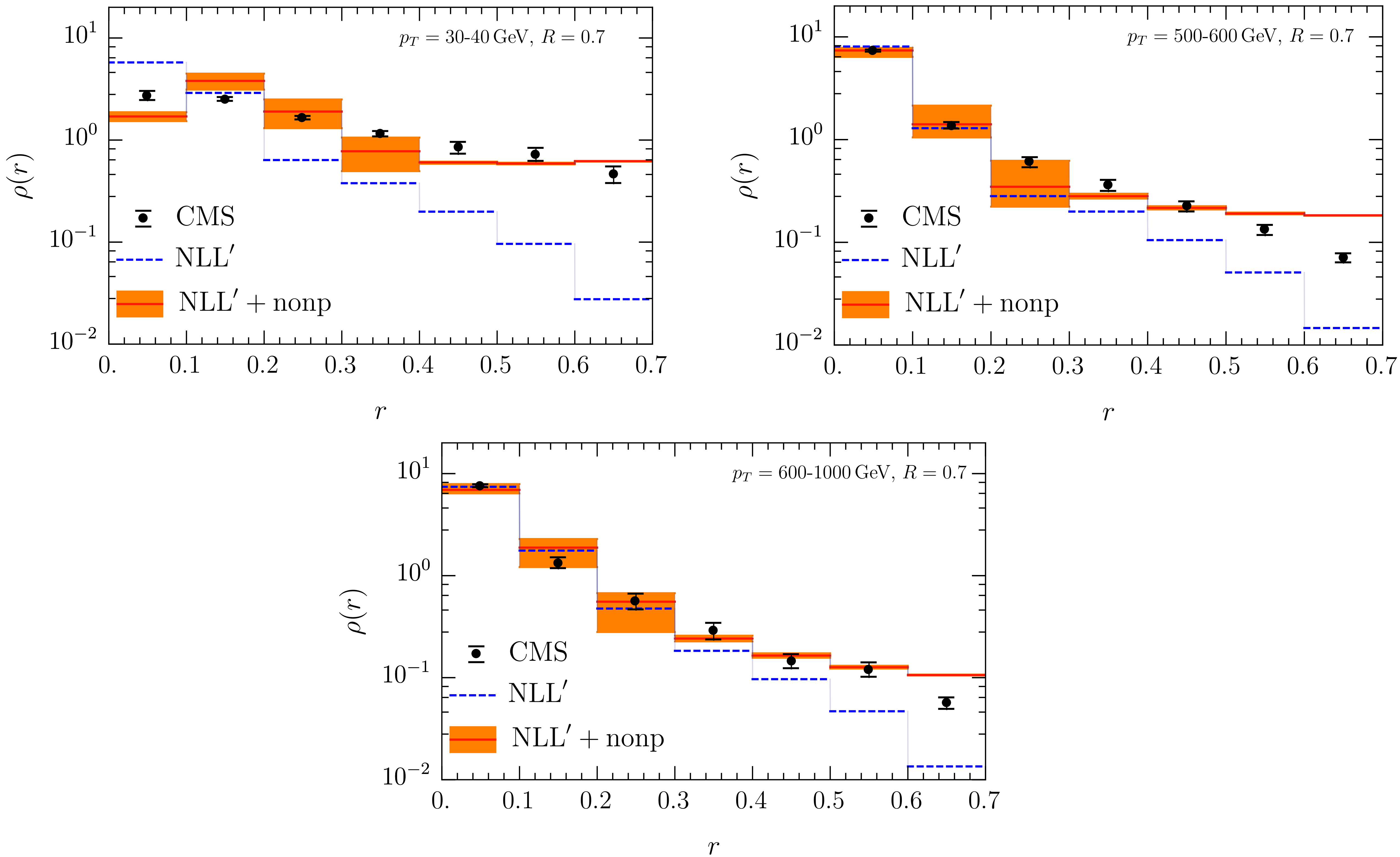
⟨7⟩ Results for the LHC

ATLAS



7 Results for the LHC

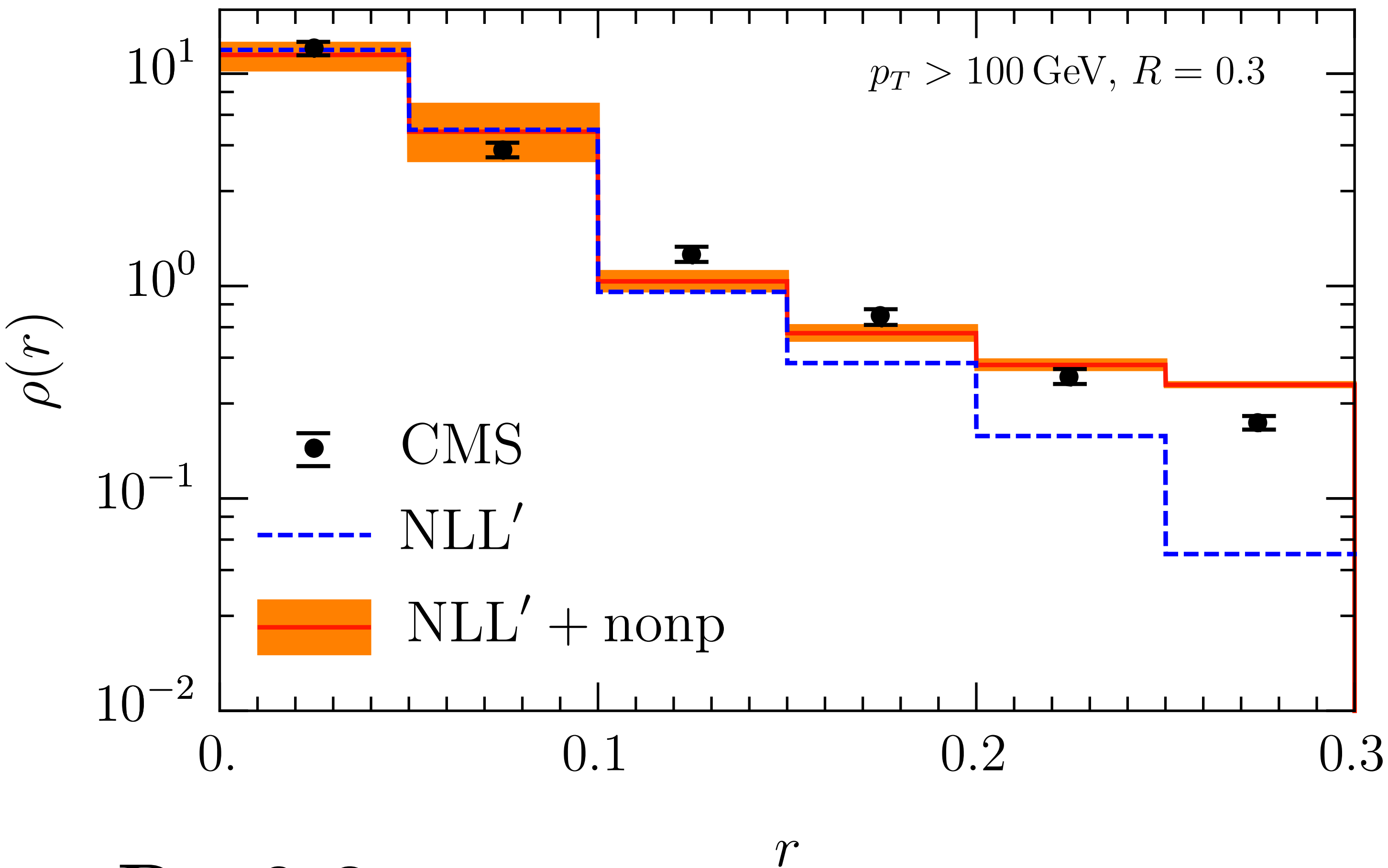
CMS



⑦ Results for the LHC

CMS: Heavy Ion

$\sqrt{s} = 2.76$ TeV



- Jet radius R=0.3
- Data closer to purely perturbative results
- Lower $\sqrt{s} \longrightarrow$ smaller non-perturbative parameter f

Conclusions

- First beyond leading log calculation of the jet shape
- Higher order corrections reduce uncertainty bands
- Non perturbative effects are significant for LHC, one parameter models lead to good agreement
- Future directions:
  - Compare to LEP or HERA data
  - Jet shape with grooming for the LHC to reduce contamination

Back up slides

Back up slides

⑤ Implementation