Angularities and the groomed jet radius using soft drop

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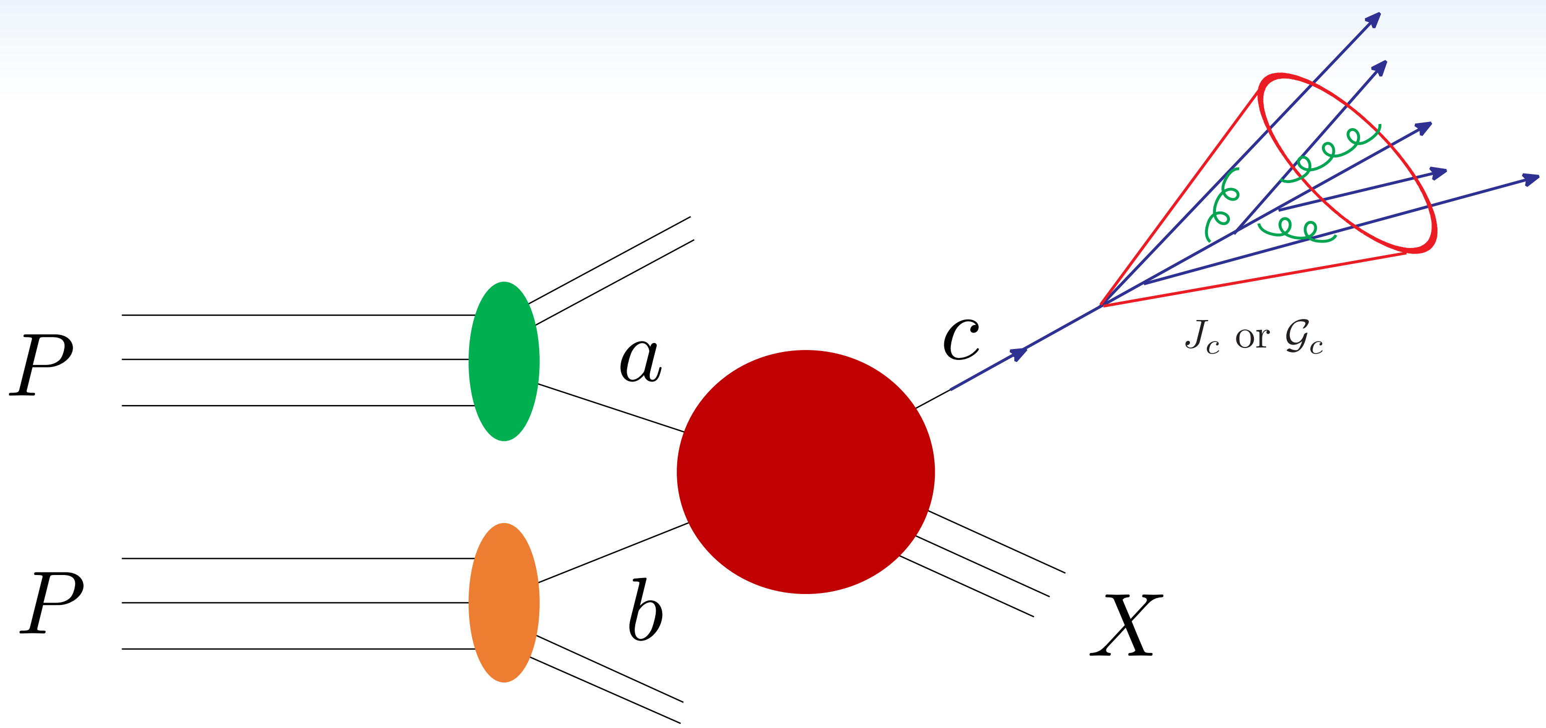
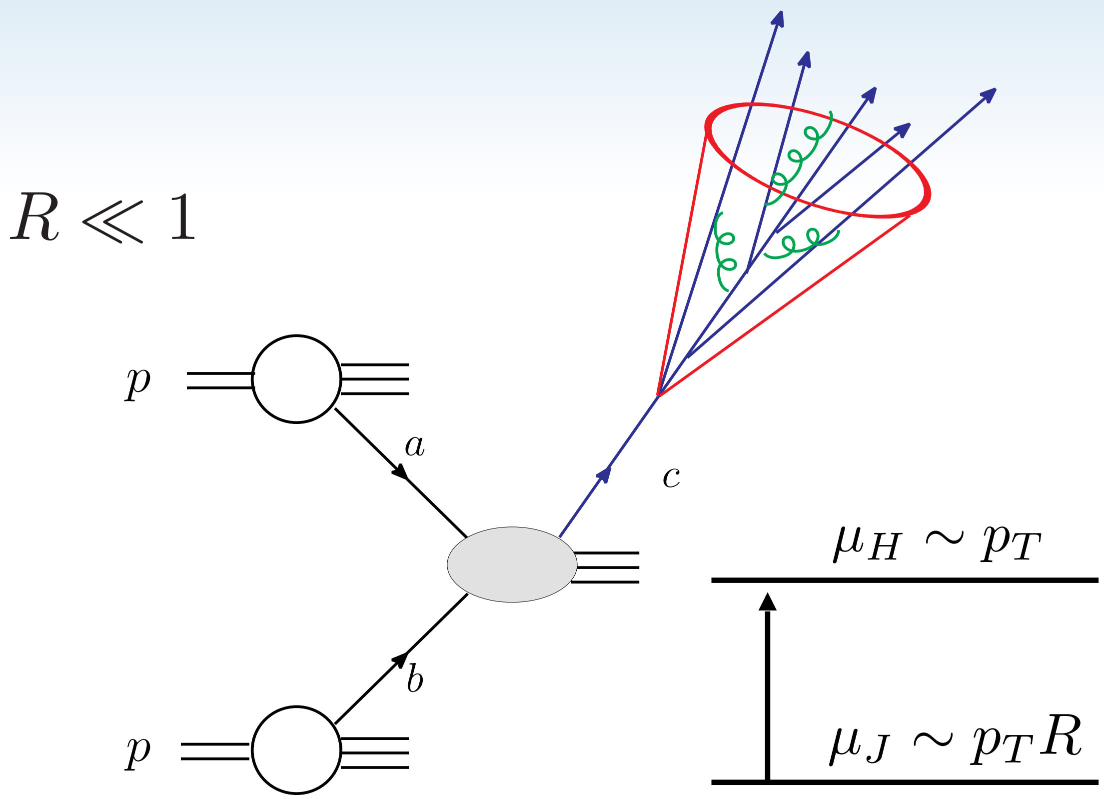
Stony Brook University

Processes of Interest

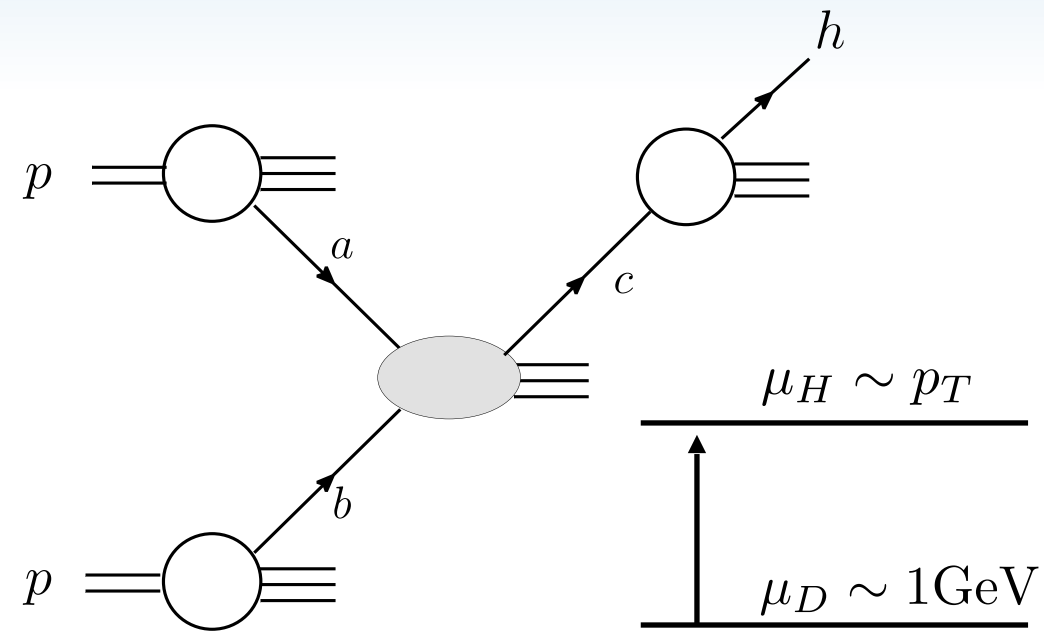


- **We want to study semi-inclusive jet production**
  p + p $\longrightarrow$ Jet((with/without) substructure) + X

Comparison with the inclusive hadron production case



Factorization



Evolution

Inclusive Jet

$$\frac{d\sigma^{pp\to \text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c + \mathcal{O}(R^2)$$

$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

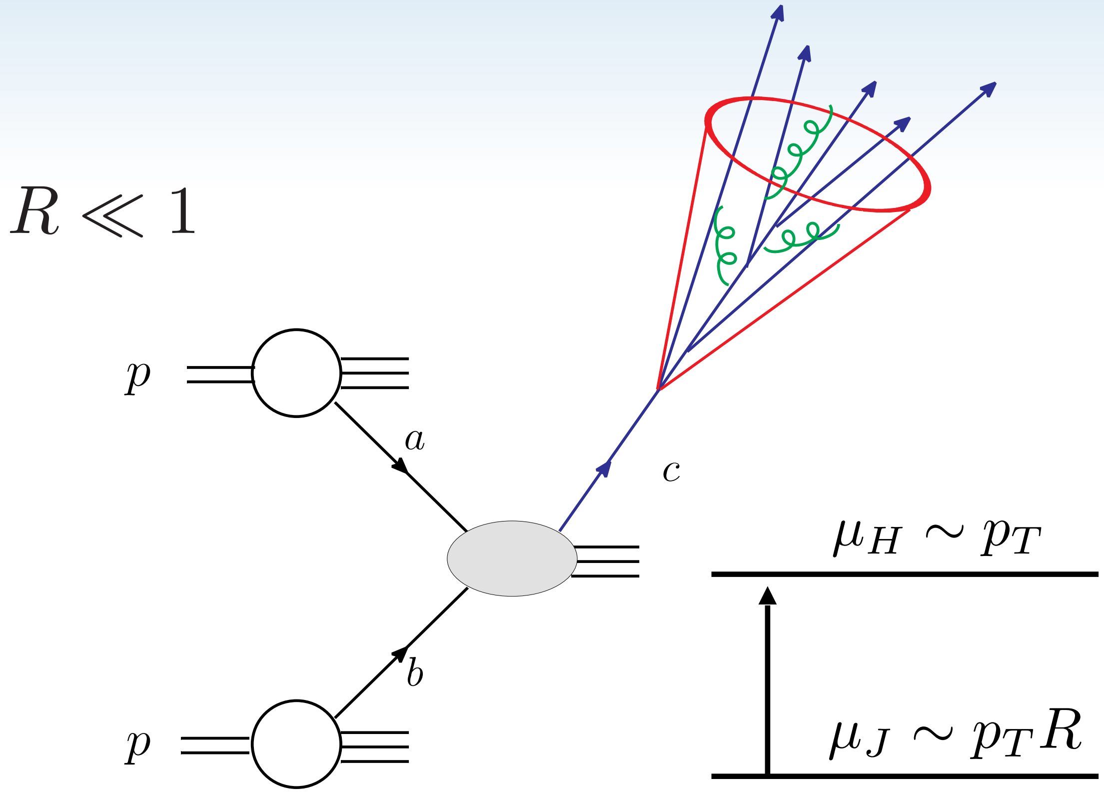
Hadron

$$\frac{d\sigma^{pp\to hX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes D_c^h$$

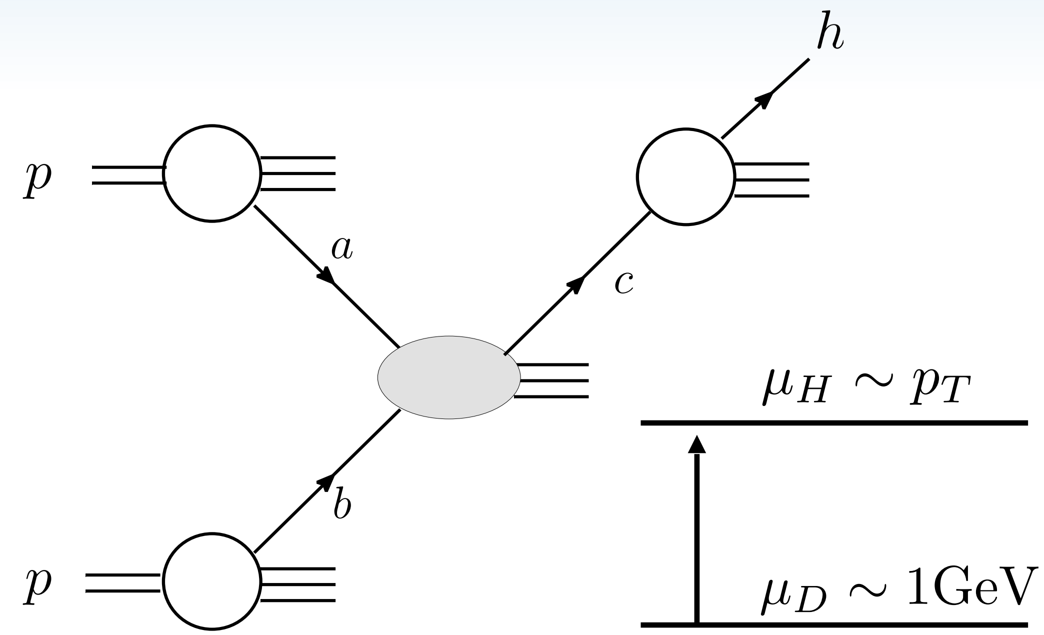
$$\mu \frac{d}{d\mu} D_i^h = \sum_j P_{ji} \otimes D_j^h$$

Kang, Ringer, Vitev `16

Comparison with the inclusive hadron production case



Factorization



Evolution

Inclusive Jet $$\frac{d\sigma^{pp\to \text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c + \mathcal{O}(R^2)$$

$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

- **Physically sensible for small R**
- **DGLAP demonstrated to 1-loop**
- **Dynamics that live inside the jet is separated when observable v is measured inside the jet, replace** $J_c(z, p_T R, \mu) \to \mathcal{G}_c(z, p_T R, v, \mu)$.

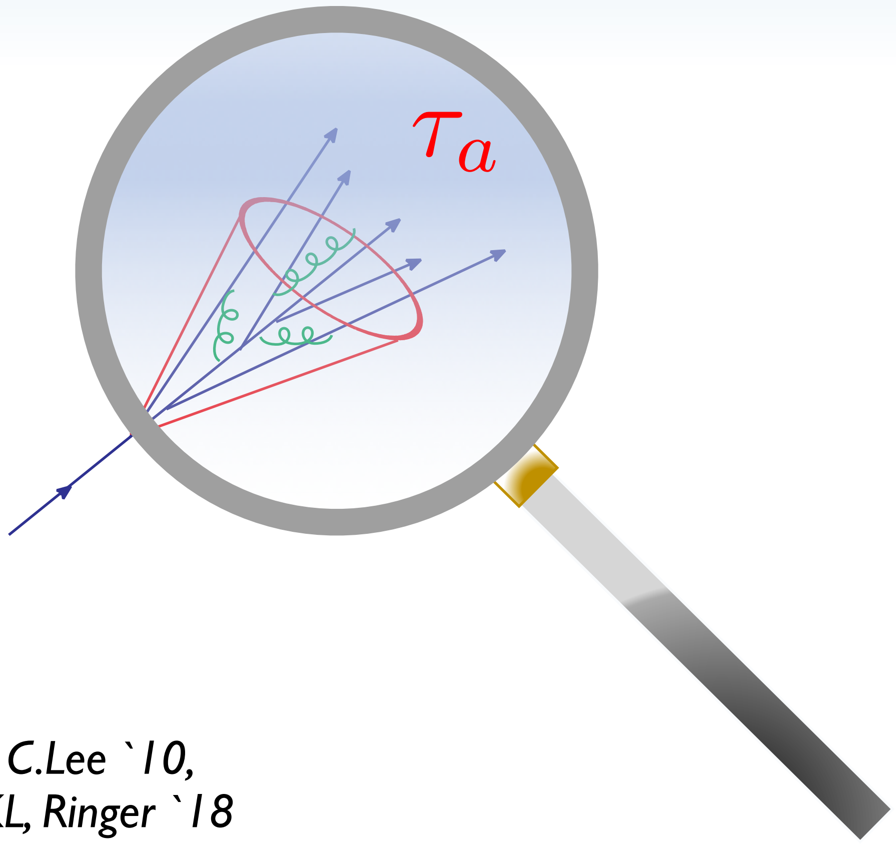
Kang, Ringer, Vitev `16

Jet angularity

- A generalized class of IR safe observables ($-\infty < a < 2$), angularity (applied to jet):

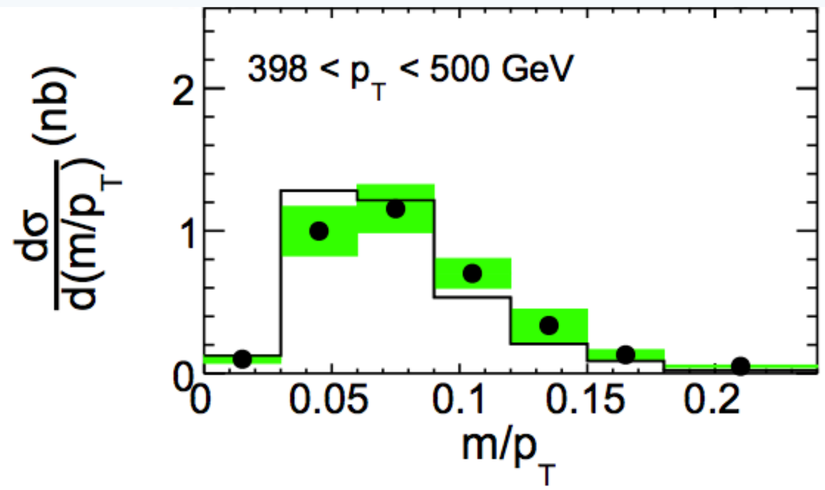
$$\tau_a^{pp} = \frac{1}{p_T} \sum_{i \in J} p_{T,i} (\Delta R_{iJ})^{2-a}$$

$$\tau_0^{pp} = \frac{m_J^2}{p_T^2} + \mathcal{O}((\tau_0^{pp})^2)$$

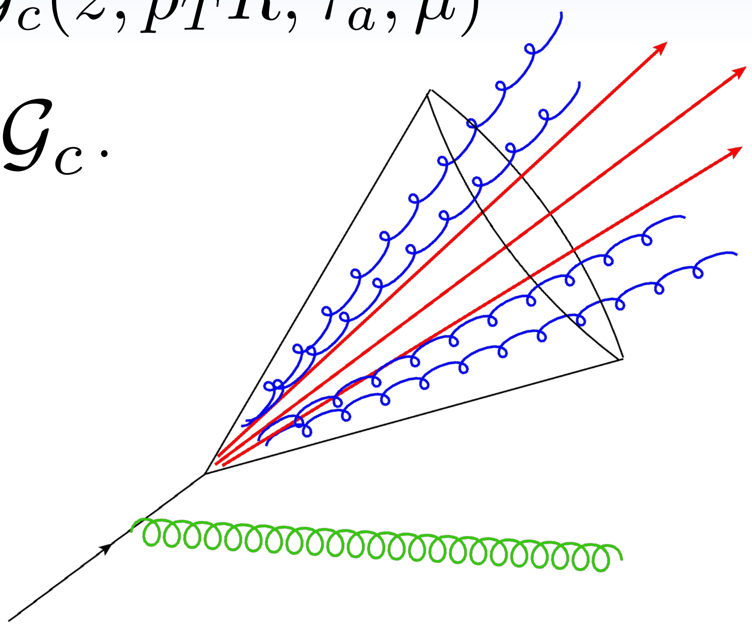


Sterman et al. `03, `08,
Hornig, C. Lee, Ovanesyan `09, Ellis, Vermilion, Walsh, Hornig, C.Lee `10,
Chien, Hornig, C. Lee `15, Hornig, Makris, Mehen `16, Kang, KL, Ringer `18

Factorization for the jet angularity



- Replace $J_c(z, p_T R, \mu) \to \mathcal{G}_c(z, p_T R, \tau_a, \mu)$
- When $\tau_a \ll R^{2-a}$, refactorize $\mathcal{G}_c$.



Relevant modes for $\tau_a \ll R^{2-a}$

$\tau_a \sim z\,\theta^{2-a}$

Collinear

$z_c \sim 1 \qquad \theta_c \sim \tau_a^{\frac{1}{2-a}} \qquad \mu_C \sim p_T \tau_a^{\frac{1}{2-a}}$

(Collinear-)soft

$\theta_s \sim R \qquad z_{cs} \sim \dfrac{\tau_a}{R^{2-a}} \qquad \mu_S \sim \dfrac{p_T \tau_a}{R^{1-a}}$

Hard-collinear

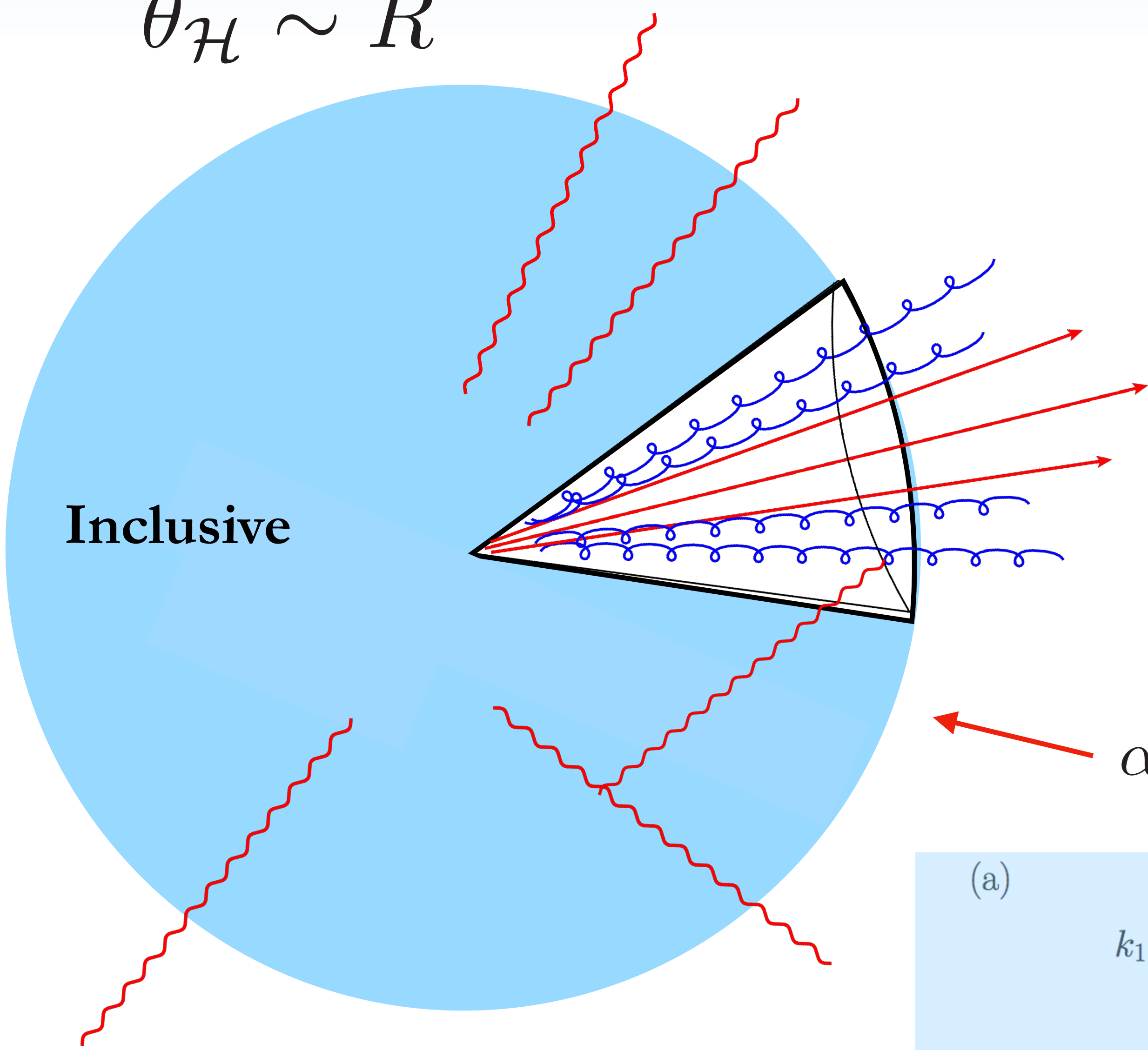
$\theta_{\mathcal{H}} \sim R \qquad z_{\mathcal{H}} \sim 1 \qquad \mu_{\mathcal{H}} \sim p_T R$

Kang, KL, Ringer, arXiv:1801.00790

Appearance of the NGLs

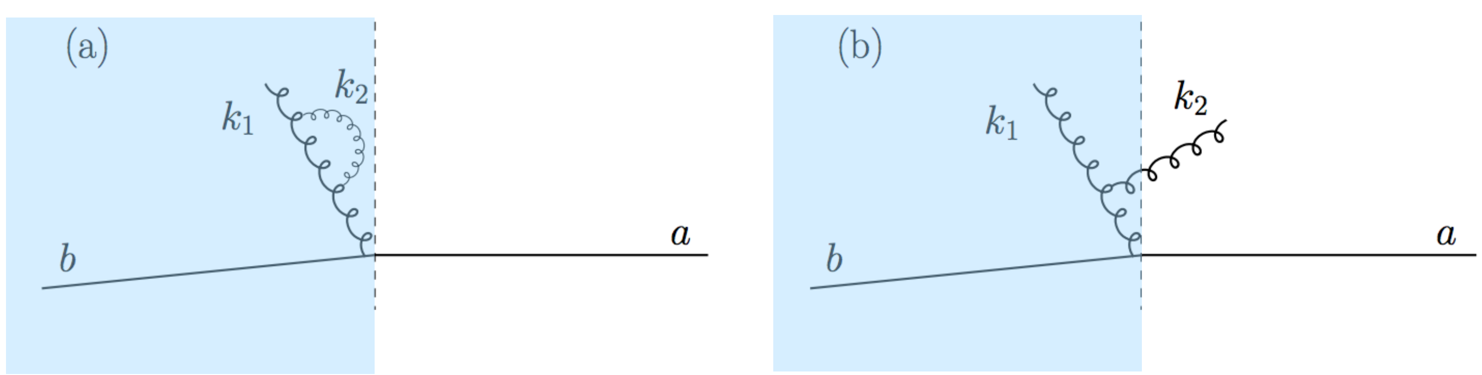
$$\theta_s \sim R$$
$$\theta_{\mathcal{H}} \sim R$$

Dasgupta, Salam `01
Banfi, Marchesini, Smye `02
Larkoski, Moult, Neill `15
Becher, Neubert, Rothen, Shao `15, `16 …



- **Non-global logarithms (NGLs): arises from the correlation between the in-jet and the out-of-jet radiation.**

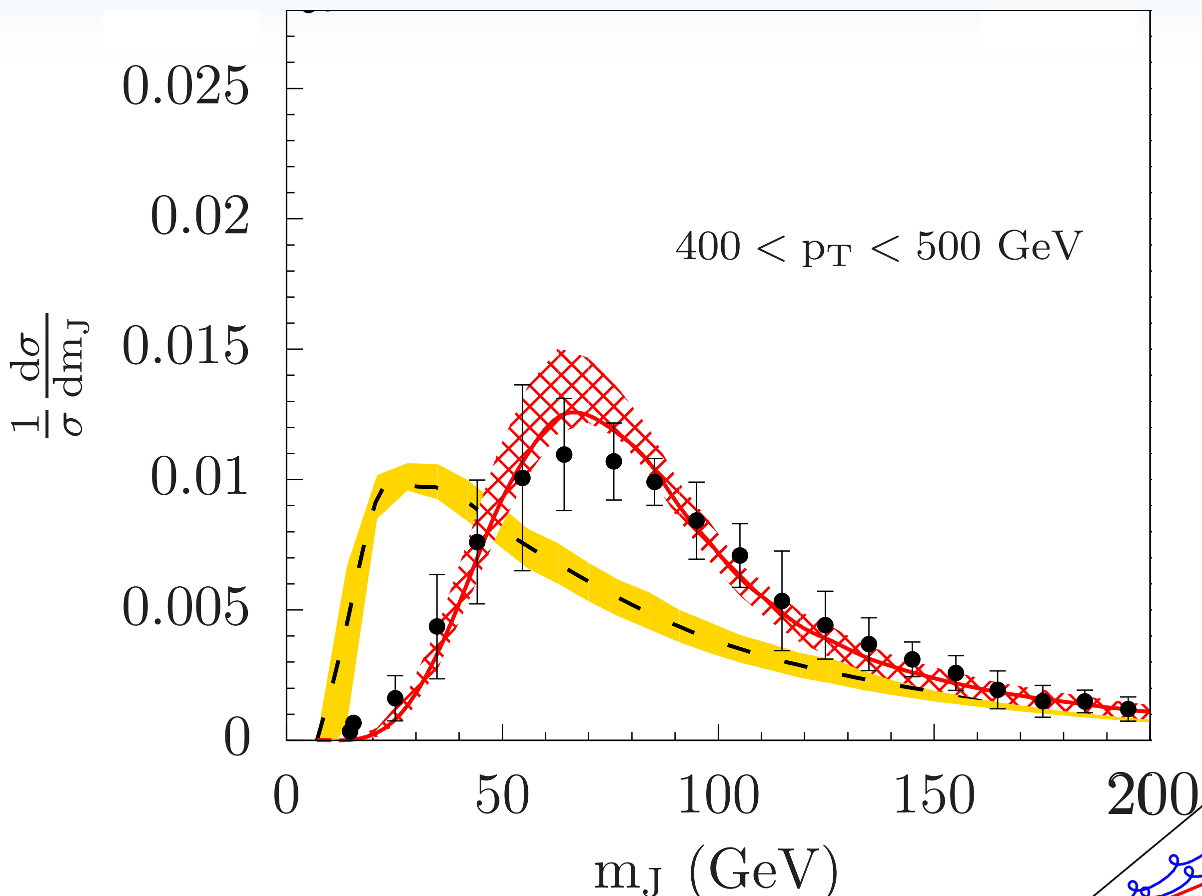
$$\alpha_s^n \ln^n(\tau_a/R^{2-a})$$



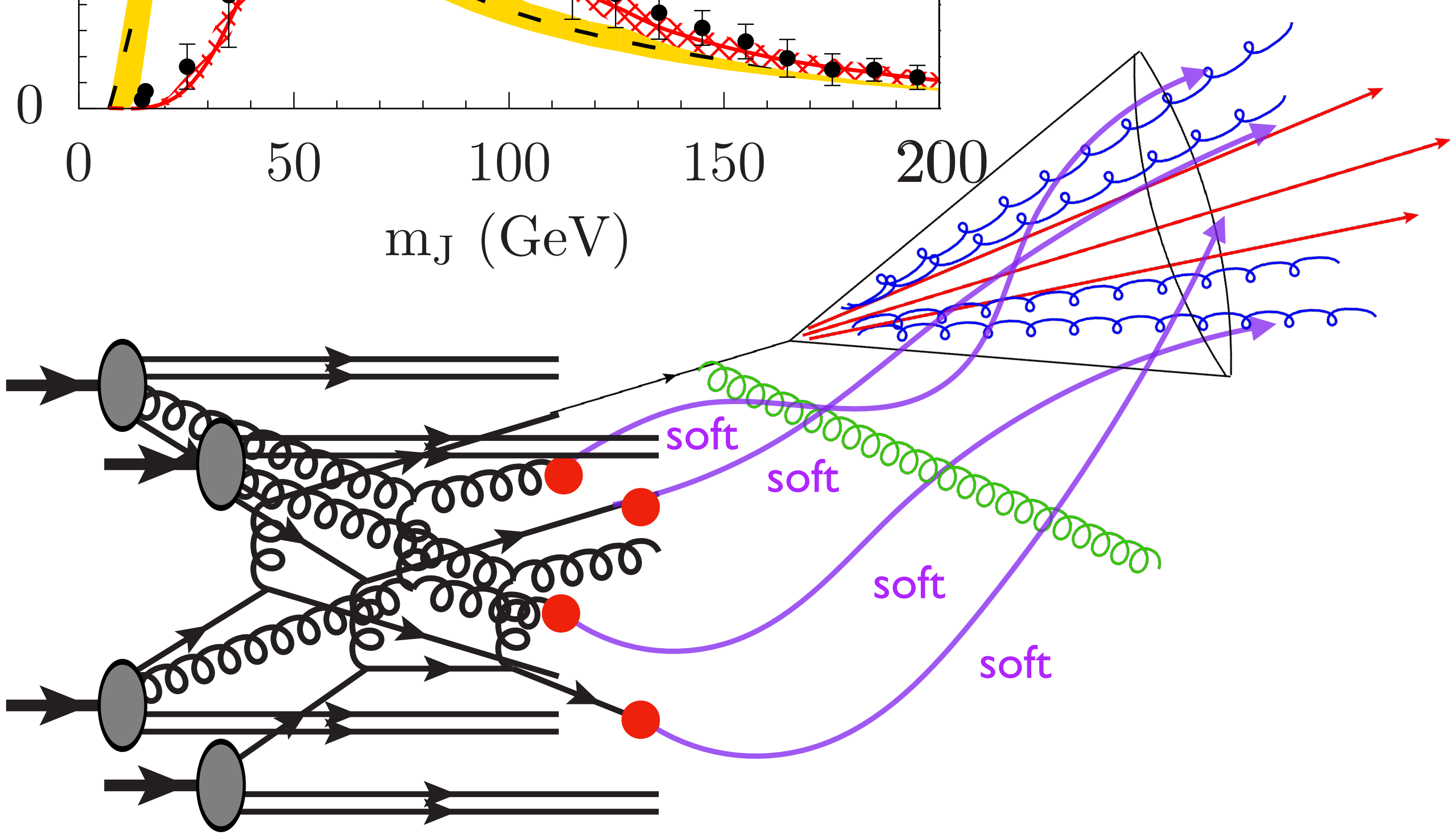
Dasgupta, Salam `01

Non-perturbative Effects

- **Non-perturbative effects:**



$$\mu_S \sim \frac{p_T \tau_a}{R^{1-a}}$$



- **Multi-Parton Interactions (MPI) (Underlying Events (UE))**
  Multiple secondary scatterings of partons within the protons may enter and contaminate jet.
- **Pileups**
  Secondary proton collisions in a bunch may enter and contaminate jet.

Soft Drop Grooming

- **Taming wide angle soft radiations, giving sensitivity to UE, PU, and NGLs directly changing distribution.**

Groom jets to reduce sensitivity to the wide-angle soft radiation.

- **Soft drop grooming algorithms:**

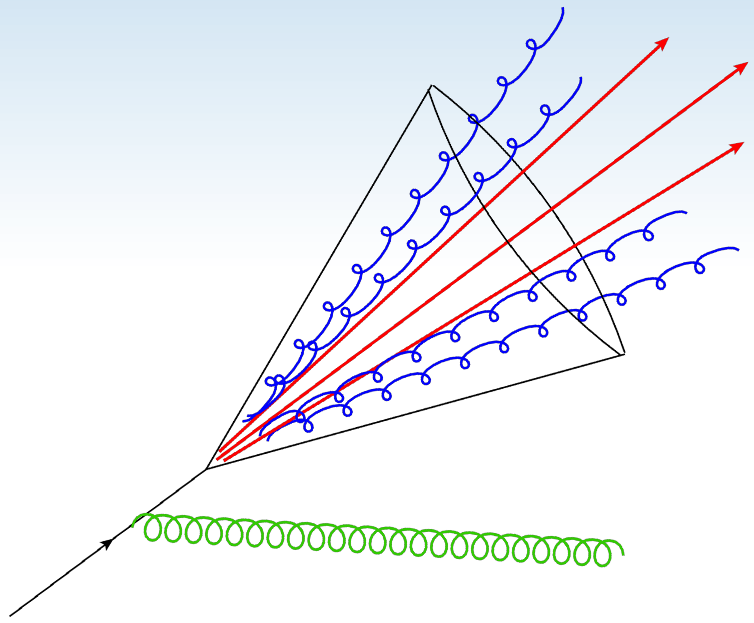
1. Reorder emissions in the identified jet according to their relative angle using C/A jet algorithm.

2. Recursively remove soft branches until soft drop condition is met:

$$\frac{\min[p_{T,1}, p_{T,2}]}{p_{T,1} + p_{T,2}} > z_{\text{cut}} \left(\frac{\Delta R_{12}}{R} \right)^{\beta}$$

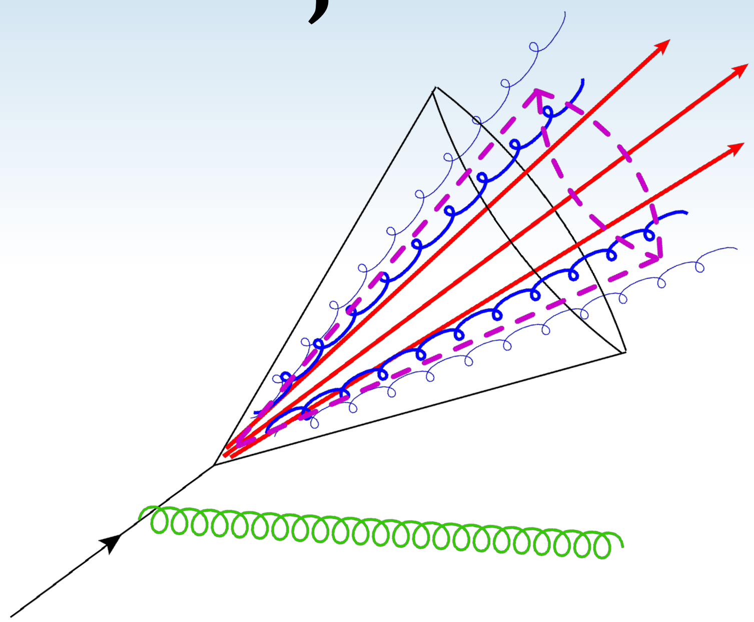
Larkoski, Marzani, Soyez, Thaler `14
Frye, Larkoski, Schwartz, Yan `16

Relevant modes in the groomed jet



$$\tau_a \sim z\,\theta^{2-a}$$

$$z > z_{\rm cut}(\theta/R)^\beta$$



- **The ungroomed case ($\tau_a \ll R^{2-a}$)**

Hard-collinear

$$\theta_{\mathcal{H}} \sim R \qquad z_{\mathcal{H}} \sim 1$$

Collinear

$$z_c \sim 1 \qquad \theta_c \sim \tau_a^{\frac{1}{2-a}}$$

(Collinear-)soft

$$\theta_s \sim R \qquad z_{cs} \sim \frac{\tau_a}{R^{2-a}}$$

- **The groomed case ($\tau_{a,gr}/R^{2-a} \ll z_{\rm cut} \ll 1$)**

Hard-collinear

$$\theta_{\mathcal{H}} \sim R \qquad z_{\mathcal{H}} \sim 1$$

Collinear

$$z_c \sim 1 \qquad \theta_c \sim \tau_a^{\frac{1}{2-a}}$$

$\notin$ *gr* soft

$$\theta_{\notin {\rm gr}} \sim R \qquad z_{\notin {\rm gr}} \sim z_{\rm cut}\left(\frac{\theta}{R}\right)^\beta = z_{\rm cut}$$

$\in$ *gr* soft (collinear-soft)

$$z_{\in {\rm gr}} \sim z_{\rm cut}\left(\frac{\theta}{R}\right)^\beta = z_{\rm cut}^{\frac{2-a}{2-a+\beta}}\left(\frac{\tau_a}{R^{2-a}}\right)^{\frac{\beta}{2-a+\beta}} \qquad \theta_{\in {\rm gr}} \sim \left(\frac{\tau_a R^\beta}{z_{\rm cut}}\right)^{\frac{1}{2-a+\beta}}$$

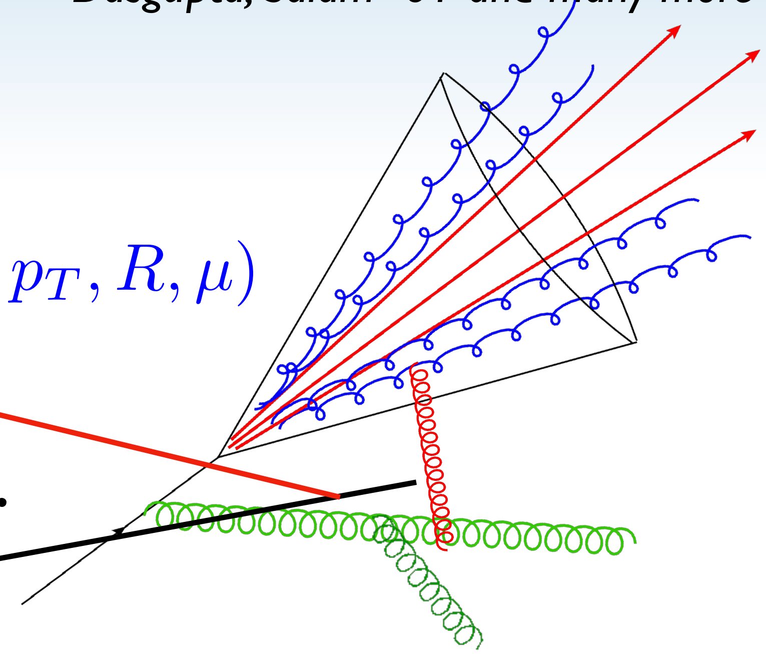
Non-global Logarithms

Dasgupta, Salam `01 and many more

- **The ungroomed case ($\tau_a \ll R^{2-a}$)**

$$\theta_{\mathcal{H}} \sim R \qquad \theta_s \sim R$$

$$\mathcal{G}_i(z, p_T R, \tau_a, \mu) = \sum_j \mathcal{H}_{i\to j}(z, p_T R, \mu) C_j(\tau_a, p_T, \mu) \otimes S_j(\tau_a, p_T, R, \mu)$$



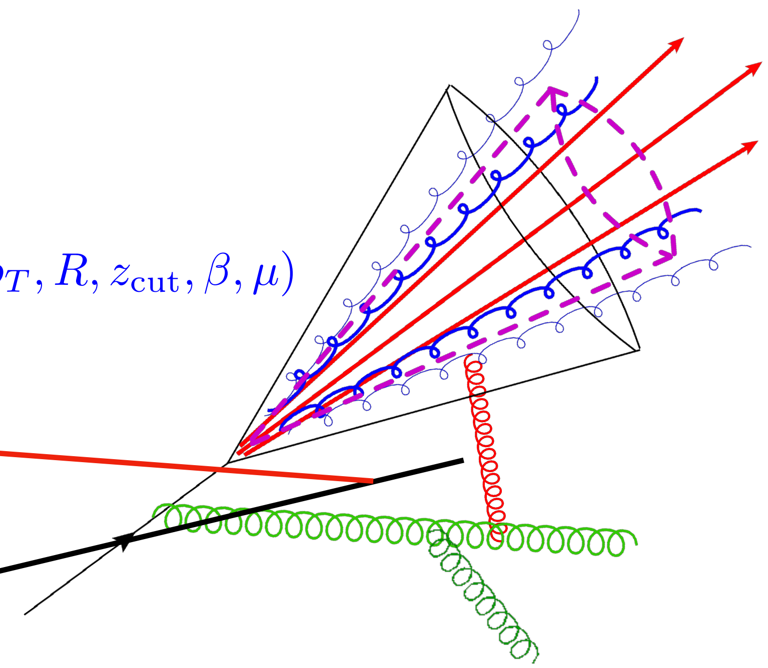
- **Non-global logs directly affect the jet angularity spectrum.**

$$\alpha_s^n \ln^n(\tau_a/R^{2-a}) \qquad n \geq 2$$

- **The groomed case ($\tau_{a,gr}/R^{2-a} \ll z_{\rm cut} \ll 1$)**

$$\theta_{\mathcal{H}} \sim R \qquad \theta_{\notin \rm gr} \sim R$$

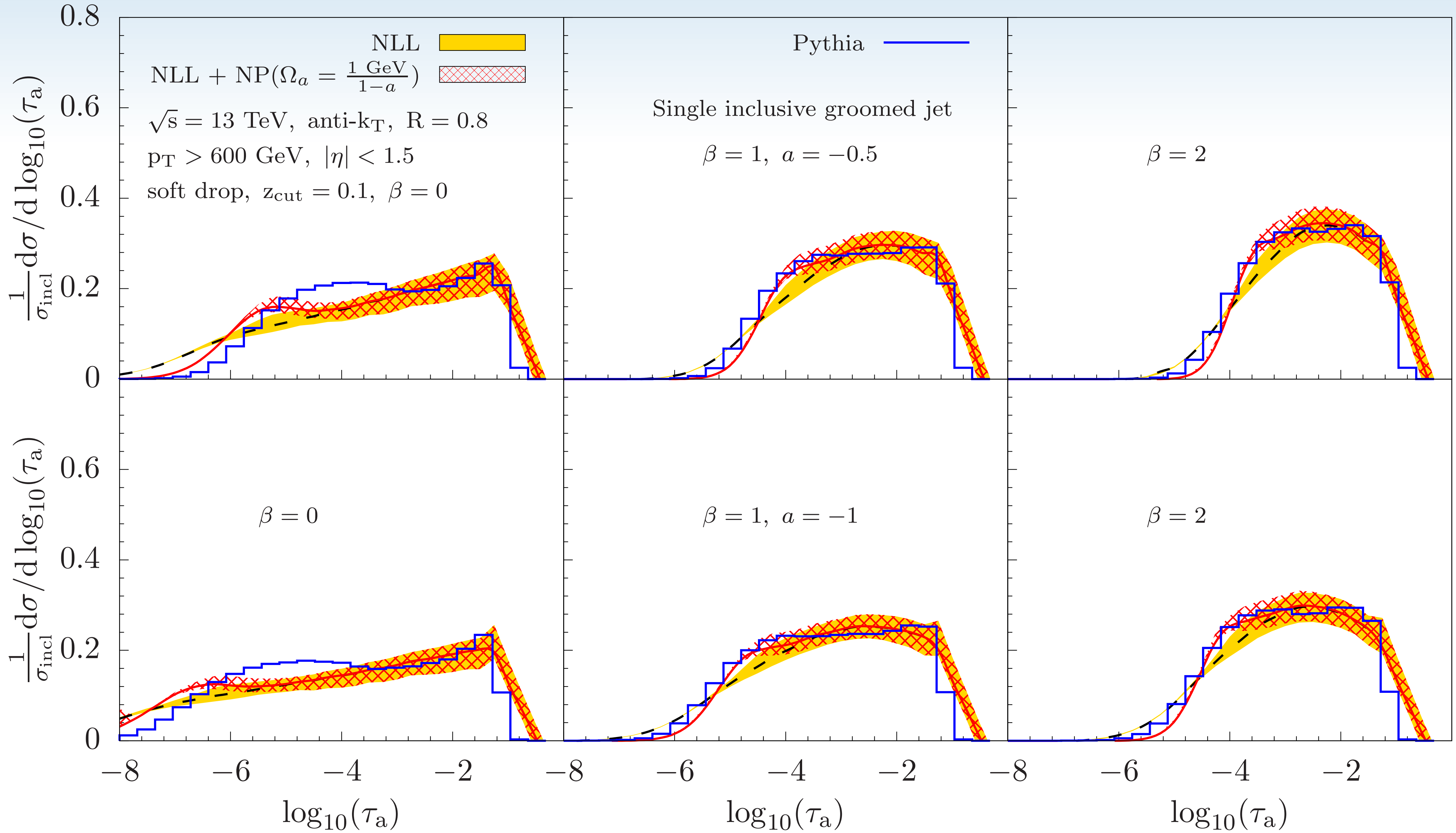
$$\mathcal{G}_i(z, p_T R, \tau_a, z_{\rm cut}, \beta, \mu) = \sum_j \mathcal{H}_{i\to j}(z, p_T R, \mu) S_j^{\notin \rm gr}(p_T, R, z_{\rm cut}, \beta, \mu) C_j(\tau_a, p_T, \mu) \otimes S_j^{\in \rm gr}(\tau_a, p_T, R, z_{\rm cut}, \beta, \mu)$$



- **Non-global logs only indirectly affects the jet angularity spectrum through normalization.**

$$\alpha_s^n \ln^n(z_{\rm cut}) \qquad n \geq 2$$

Phenomenology

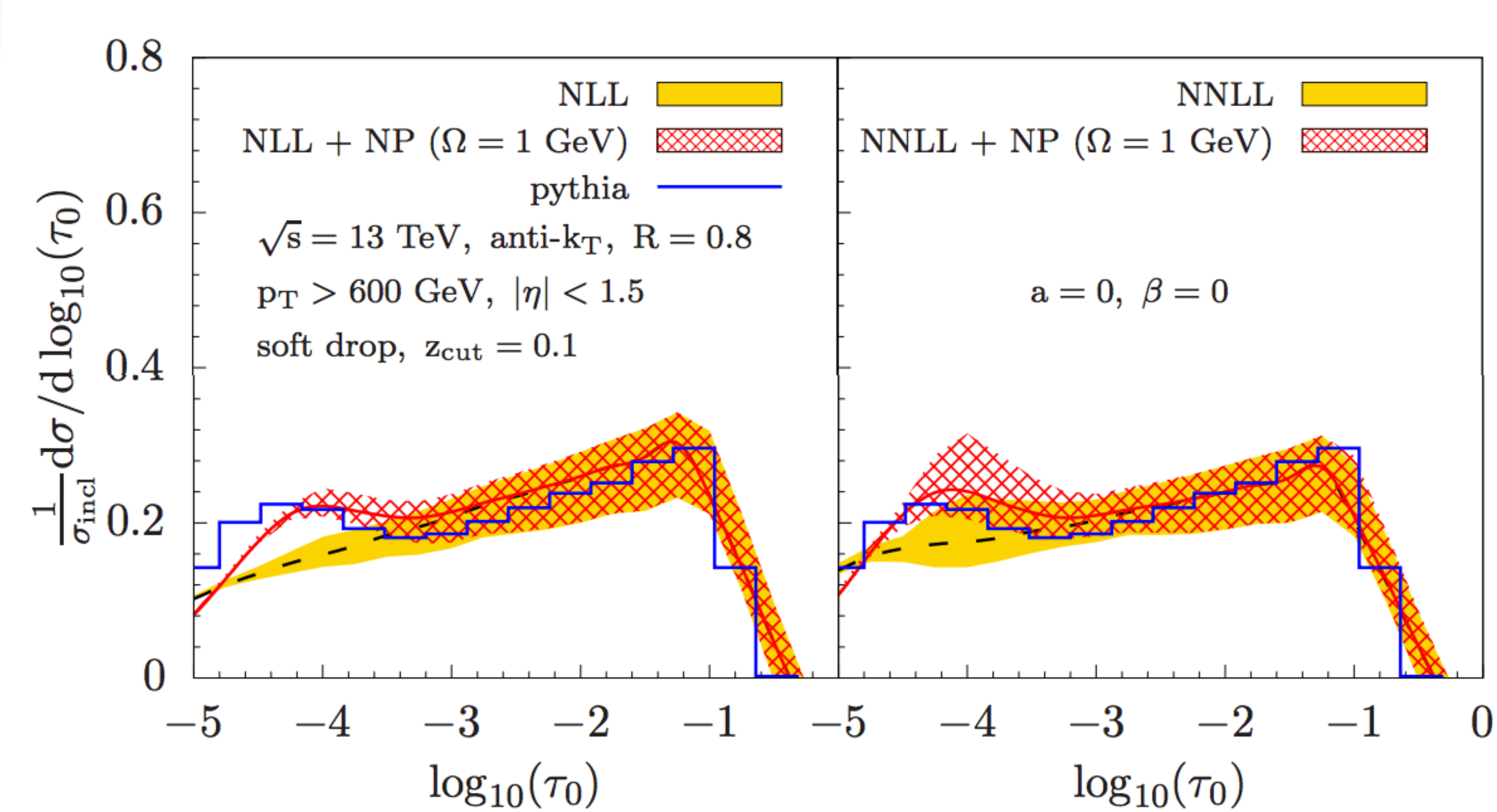


- General angularities show decent agreement with Pythia with reduced contamination from UE/PU.
- Observe $\Omega_a = \dfrac{\Omega_0}{1-a}$ shift does well.

See Jim Talbert and Aditya Pathak's talk

Kang, KL, Liu, Ringer, arXiv:1811.06983

NNLL for $a = 0$ (jet mass)



- NNLL calculated for jet mass.
- Analytically derived non-cusp anomalous dimensions in 2-loop, and further improves agreement

Kang, KL, Liu, Ringer, arXiv:1811.06983

α_s extraction

- World Average with 1.1% total uncertainty

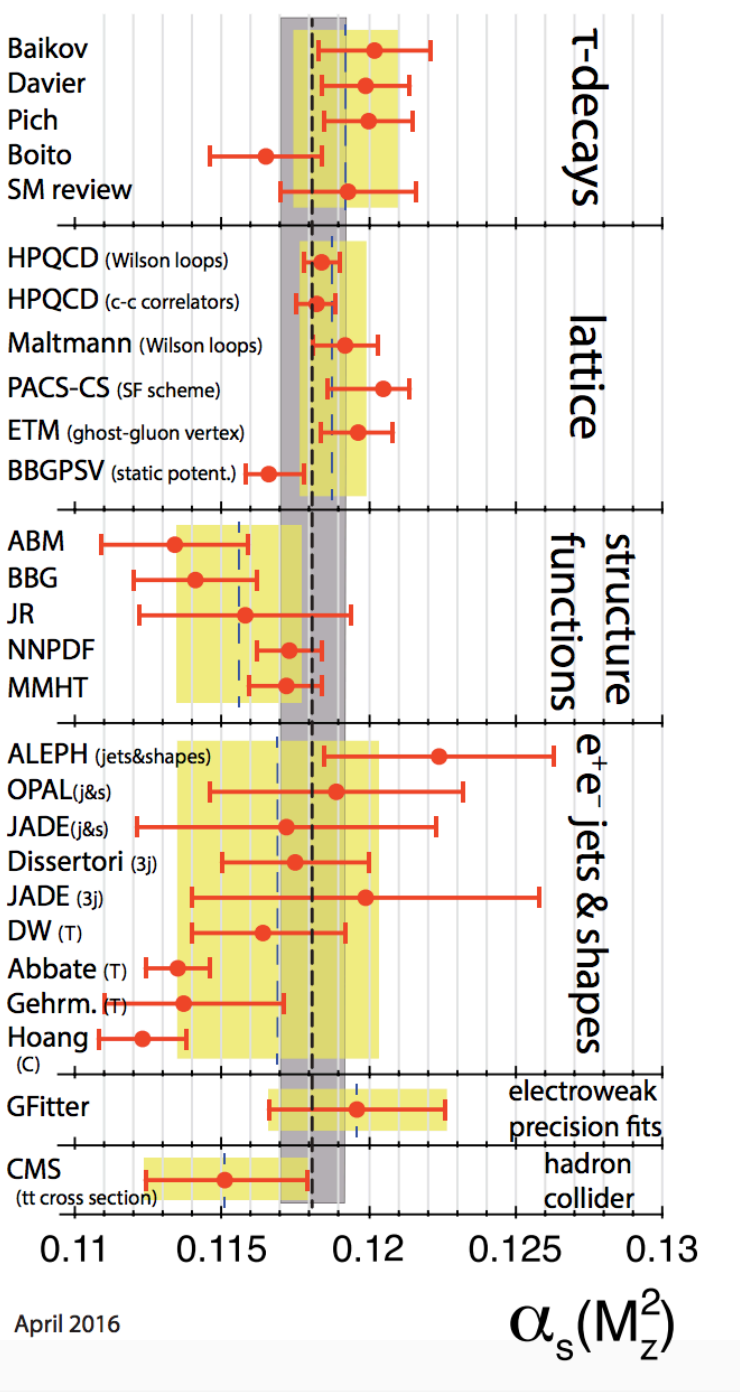
$$\alpha_s(m_Z) = 0.118 \pm 0.0013$$

- Most precise input: lattice has less than 1% uncertainty
- Next precise input: e^+e^- event shape determination: thrust and C-parameter.
    - $3-4\sigma$ tension with lattice.

Using pp-extractions:

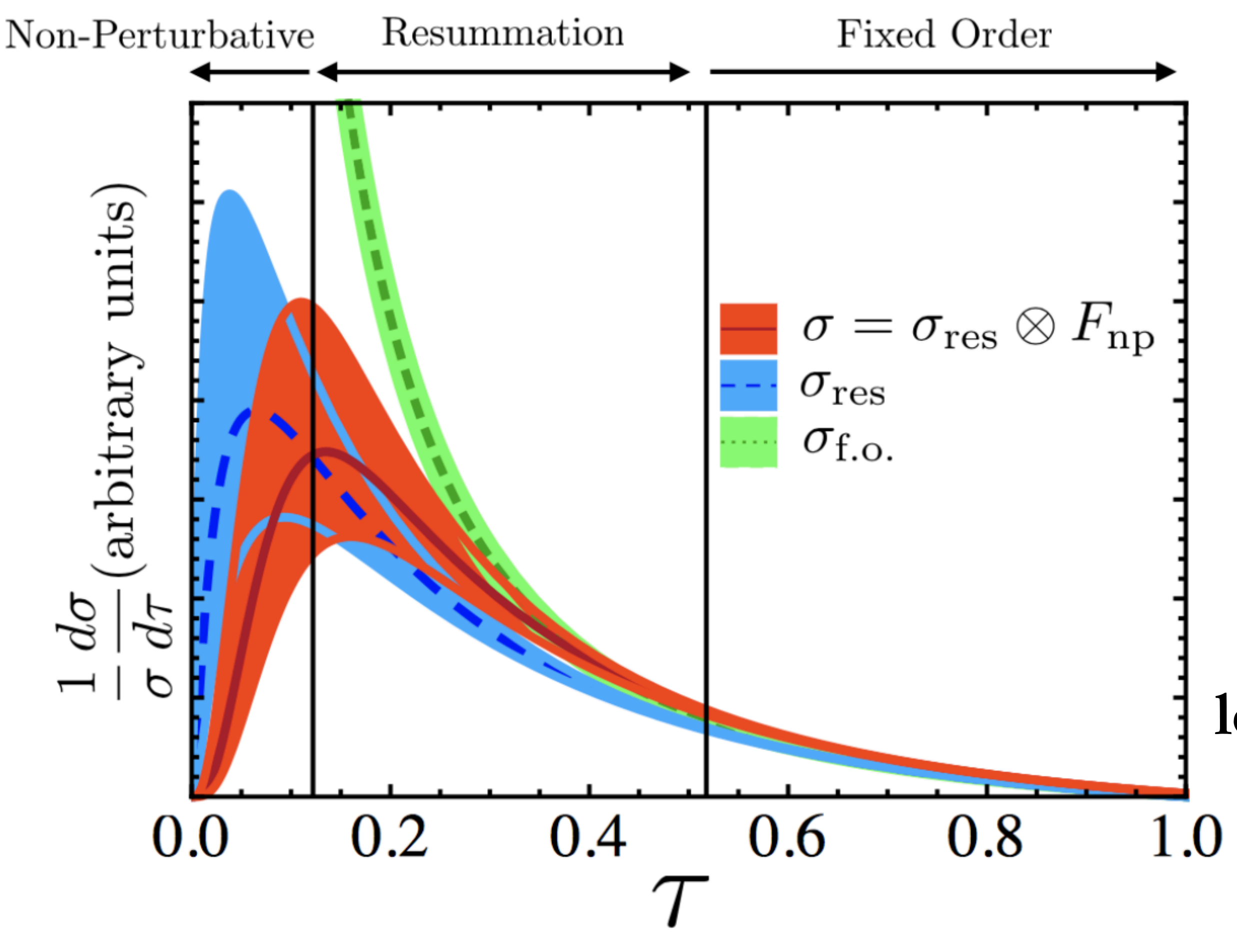
- High-quality of data pouring out of the LHC.
- Complimentary study to e^+e^- extractions.
- Currently feasible to determine with 10% uncertainty.

Les Houches 2017 I. Moult, B. Nachman, G. Soyez, J. Thaler (section coordinators)



α_s extraction

- Key challenges in α_s extraction is the degeneracy with non-perturbative effects.



See Jim Talbert's talk

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\mathcal{A}}{Q}\right)$$

$$c_{\tau_a} = \frac{2}{1-a} \qquad \mathcal{A} = \frac{1}{N_C}\,\mathrm{Tr}\,\langle 0|\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(0) Y_n \overline{Y}_{\bar{n}}\,|0\rangle$$

leading shift of the first moment shown to be universal

α_s extraction

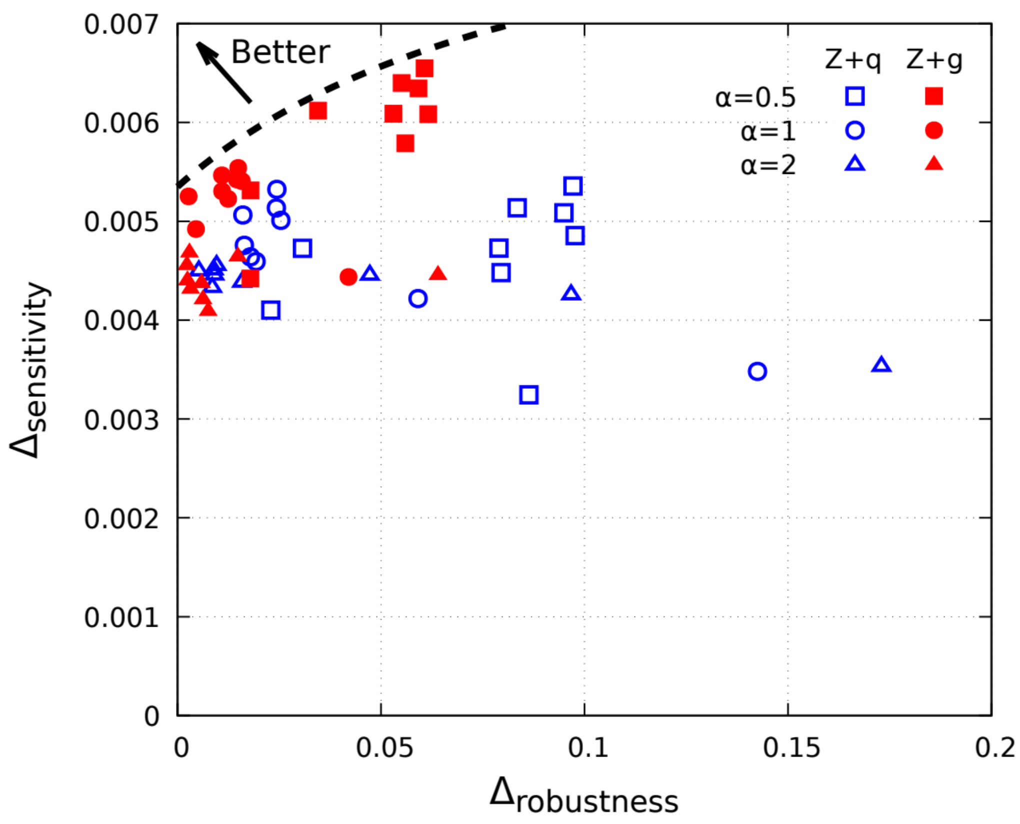
Ungroomed: $$\mu_S \sim \frac{p_T \tau}{R}$$

SD Groomed: $$\mu_S \sim \frac{p_T \tau}{R}\left(\frac{z_{\rm cut} R^2}{\tau^2}\right)^{\frac{1}{2+\beta}}$$

with $$\mu_S = \Lambda_{\rm QCD} \sim 1 \text{ GeV},$$

Onset of NP physics

$$\tau_{\rm gr} = \tau_{\rm ungr}\left(\frac{\Lambda_{QCD}}{p_T R z_{\rm cut}}\right)^{\frac{1}{1+\beta}}$$



- **Extend range of validity by two orders for 1 TeV jet.**
- **Reduced robustness to NP effects and increased sensitivity to** α_s
- **Groomed angularities or energy-energy correlations provide additional independent handles with `a'.**
- **Currently feasible to determine with 10% uncertainty.**

Les Houches 2017 I. Moult, B. Nachman, G. Soyez, J. Thaler (section coordinators)

Groomed Jet Radius, R_g

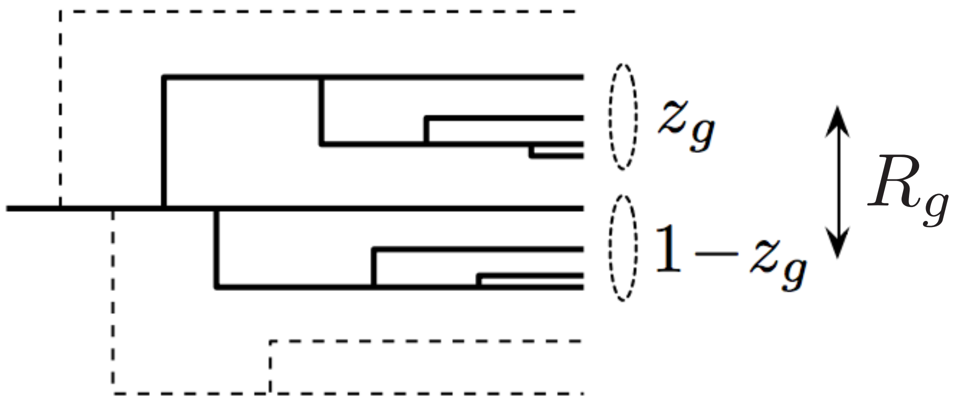


fig. from Tripathee, Xue, Larkoski, Marzani, Thaler`17

Larkoski, Marzani, Soyez, Thaler `14
Tripathee, Xue, Larkoski, Marzani, Thaler`17

- **Two characteristic variables that describe soft drop groomed jet:**

$$z_g = \frac{\min[p_{T,1}, p_{T,2}]}{p_{T,1} + p_{T,2}} \qquad R_g = \Delta R_{12}$$

when soft drop condition is met

$$\frac{\min[p_{T,1}, p_{T,2}]}{p_{T,1} + p_{T,2}} > z_{\mathrm{cut}} \left(\frac{\Delta R_{12}}{R}\right)^{\beta}$$

Kang, KL, Liu, Ringer, In Preparation

Refactorization

- Replace $J_c(z, p_T R, \mu) \to \mathcal{G}_c(z, p_T R, R_g, z_{\rm cut}, \beta, \mu)$
- Refactorize for resummation region $R_g \ll R$ and $z_{\rm cut} \ll 1$

Large logs of $\frac{R_g}{R}$ Grooming soft radiation

- **Two of the modes are immediate from the groomed angularities, and in general universal for $z_{\rm cut} \ll 1$ groomed jet observables:**

Hard-collinear

$$\theta_{\mathcal{H}} \sim R \qquad z_{\mathcal{H}} \sim 1$$

$\notin gr$ **soft**

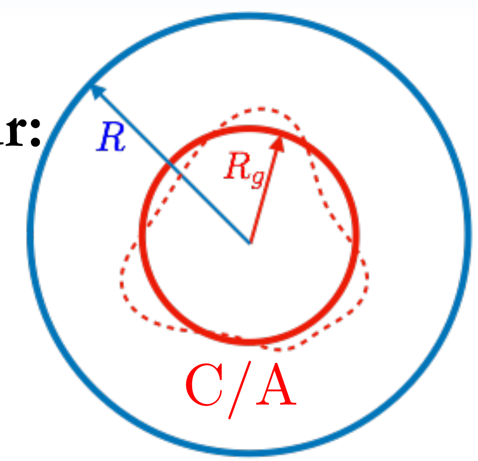
$$\theta_{\notin \rm gr} \sim R \qquad z_{\notin \rm gr} \sim z_{\rm cut} \left(\frac{\theta}{R}\right)^{\beta} = z_{\rm cut}$$

- **Independent of observables (i.e.** τ_a, R_g, $\ldots$ **)**

Groomed jet size

Observation 1: groomed jet is of size $\sim \mathcal{O}(\pi R_g^2)$

Cacciari, Salam, Soyez `08



Kelley, Walsh, Zuberi `12

- R **defines the maximal angle where a single clustering can occur:**
- **Consider** k_T **type clustering**

$$\rho_{ij} = \min[(p_T^i)^{2p}, (p_T^j)^{2p}]\frac{\Delta R_{ij}^2}{R^2}$$
$$\rho_i = (p_T^i)^{2p}$$

$$k_T = +1 \qquad \text{C/A} = 0 \qquad \text{anti-}k_T = -1$$

- $\min[\rho_i, \rho_j] > \rho_{ij} \implies R > \Delta R_{ij}$ **needs to be satisfied for a clustering to occur.**

- R_g **defines the maximal angle where a single clustering can occur.**

Groomed jet size

Observation 1: groomed jet is of size $\sim \mathcal{O}(\pi R_g^2)$

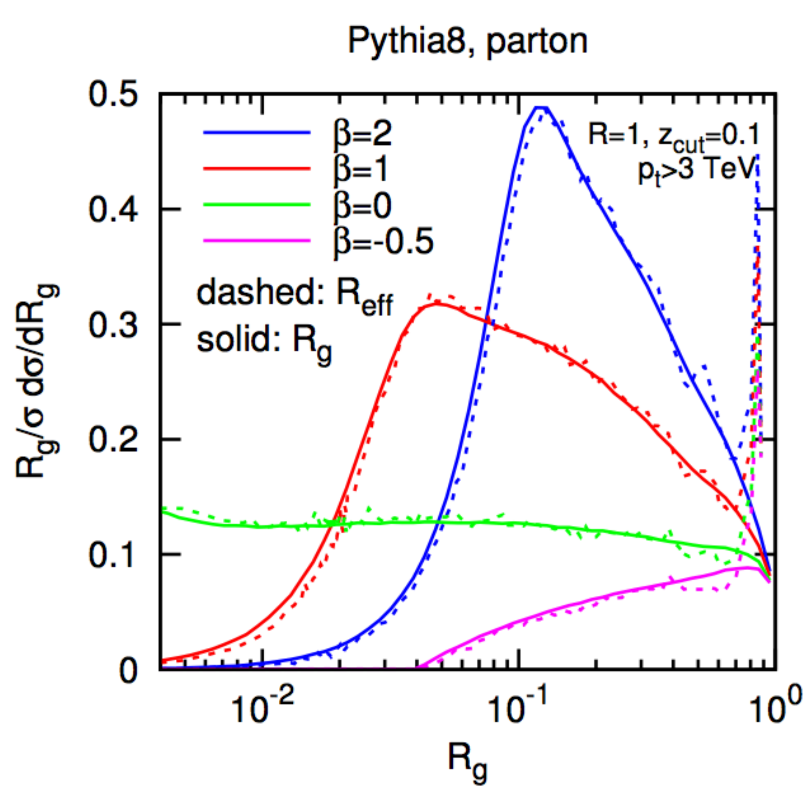


fig. from Larkoski, Marzani, Soyez, Thaler `14

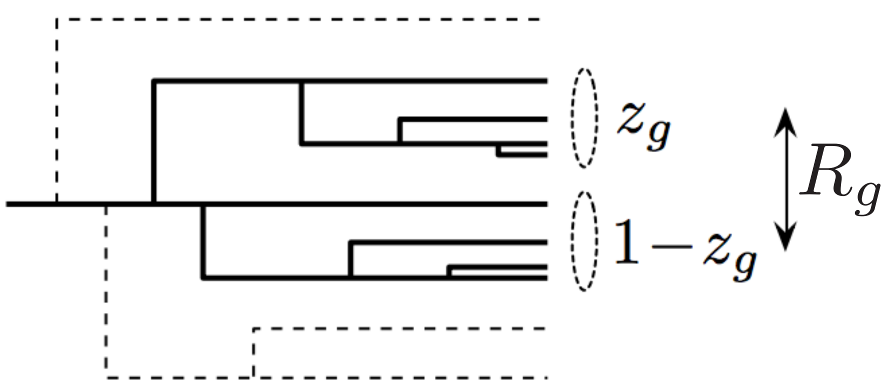
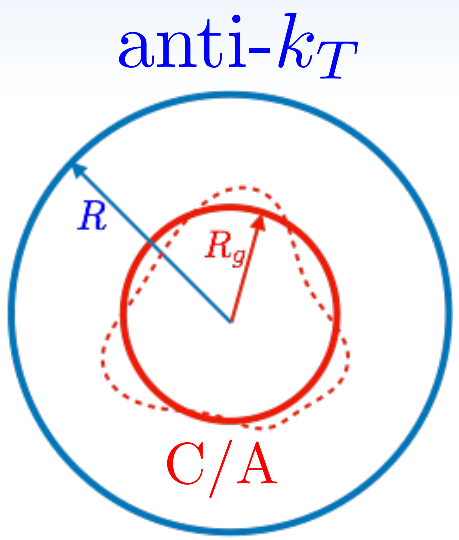


fig. from Tripathee, Xue, Larkoski, Marzani, Thaler` 17



Kelley, Walsh, Zuberi `12

• R_g **defines the maximal angle where a single clustering can occur.**

Modes sensitive to R_g

Observation 2: no measurement to angularly order radiations inside the groomed jet.

(for instance groomed angularities, $\tau_a \sim z\,\theta^{2-a}$)

Modes of the groomed jet have $\theta \sim R_g$

Collinear

$$z_c \sim 1 \qquad \theta_c \sim R_g$$

$\in gr$ soft (collinear-soft)

$$z_{\in\mathrm{gr}} \sim z_{\mathrm{cut}} \left(\frac{\theta}{R}\right)^{\beta} = z_{\mathrm{cut}} \left(\frac{R_g}{R}\right)^{\beta} \qquad \theta_{\in\mathrm{gr}} \sim R_g$$

Inclusive regions for R_g

Observation 3: collinear modes create inclusive region inside the jet and show double correlations.

- $z_{\rm cut} \ll 1$ and $R_g \ll R$

Hard-collinear

$$z_H \sim 1 \qquad \theta_H \sim R$$

$\notin gr$ soft

$$z_{\notin \rm gr} \sim z_{\rm cut}\left(\frac{\theta}{R}\right)^\beta = z_{\rm cut} \qquad \theta_{\notin \rm gr} \sim R$$

correlation

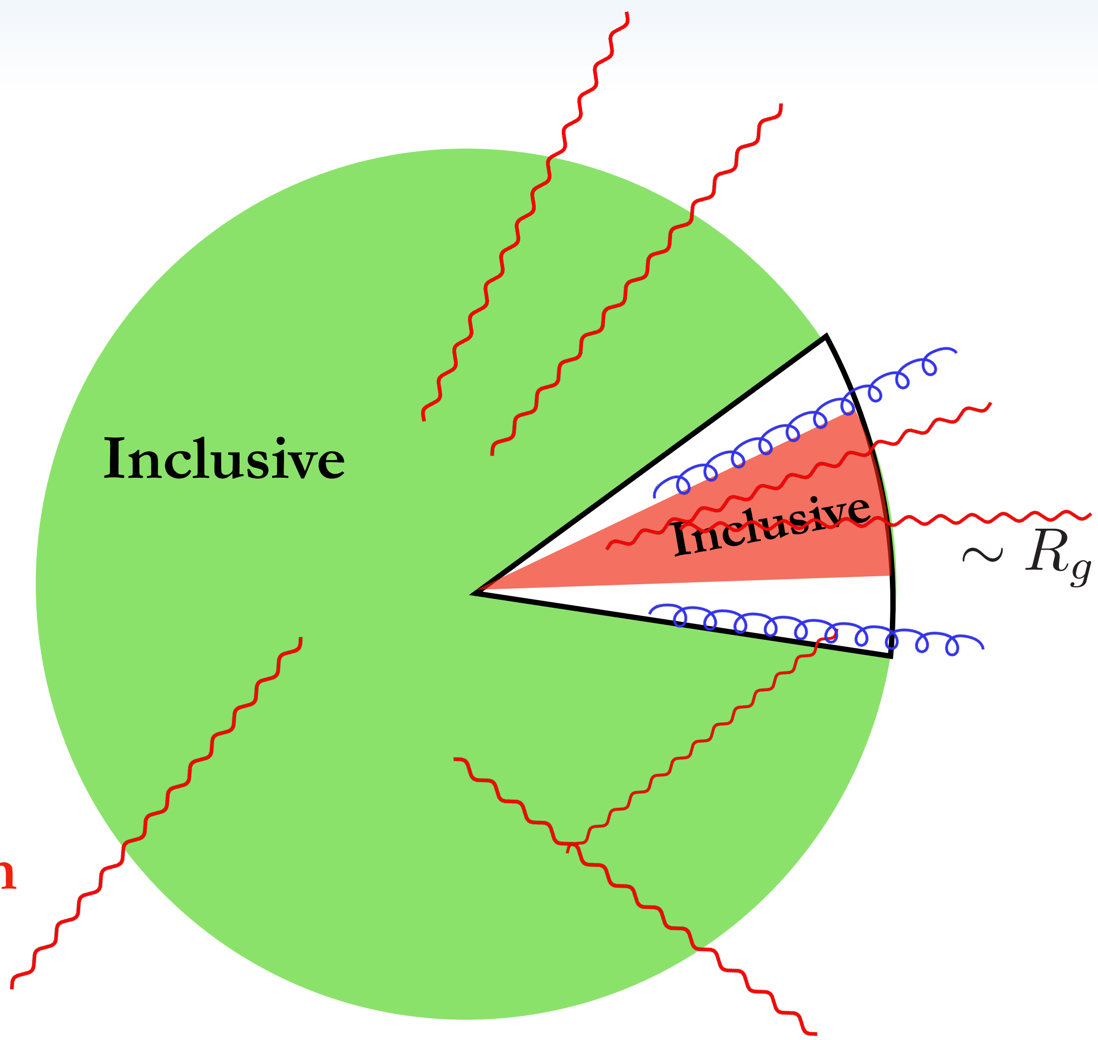
Collinear

$$z_c \sim 1 \qquad \theta_c \sim R_g$$

$\in gr$ soft (collinear-soft)

$$z_{\in \rm gr} \sim z_{\rm cut}\left(\frac{\theta}{R}\right)^\beta = z_{\rm cut}\left(\frac{R_g}{R}\right)^\beta \qquad \theta_{\in \rm gr} \sim R_g$$

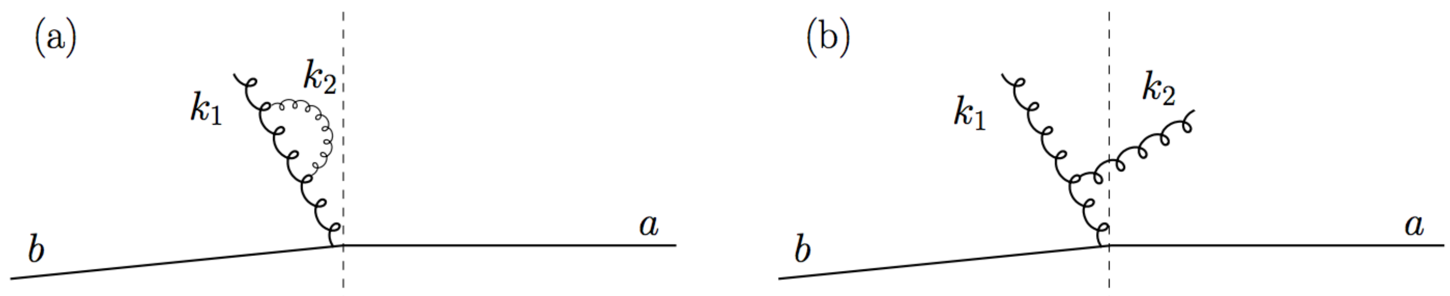
correlation



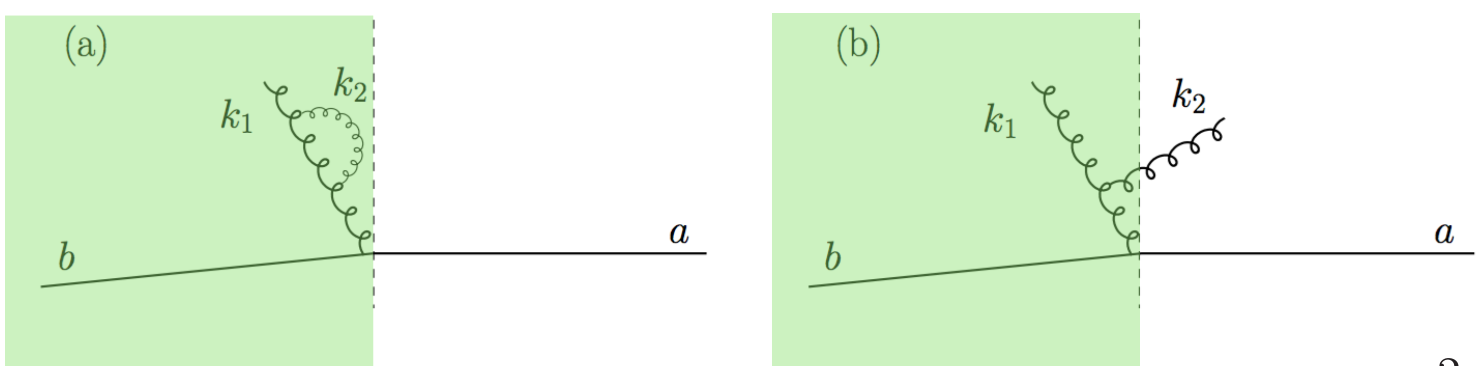
Kang, KL, Liu, Ringer, In Preparation

Inclusive regions for R_g

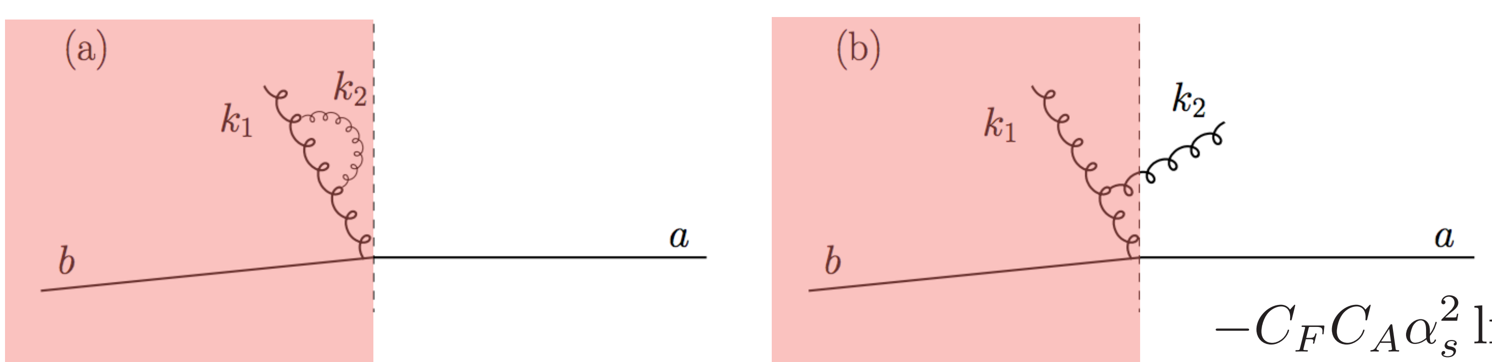
Observation 3: collinear modes create inclusive region inside the jet and show double correlations.



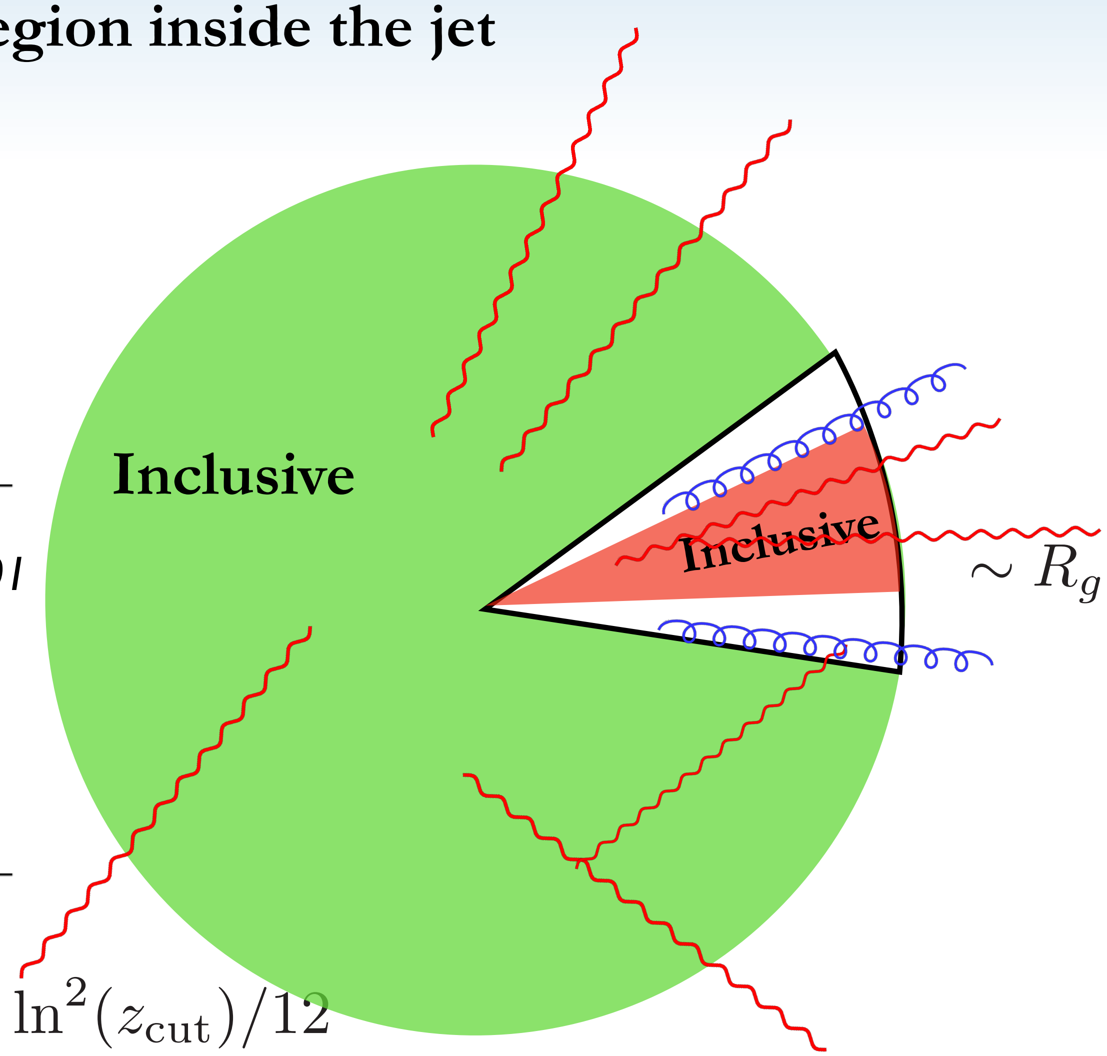
Dasgupta and Salam `01



$$-C_F C_A \alpha_s^2 \ln^2(z_{\text{cut}})/12$$



$$-C_F C_A \alpha_s^2 \ln^2(z_{\text{cut}}(R_g/R)^\beta)/12$$ *** up to clustering**



Kang, KL, Liu, Ringer, In Preparation

Factorization of R_g

$$\mathcal{G}_c(z, p_T, R, R_g, \mu; z_{\rm cut}, \beta) = \sum_i \sum_n \mathcal{H}^n_{c\to i}(z, p_T R, \mu) \otimes_\Omega S^{\notin \rm gr}_{i,n}(z_{\rm cut} p_T R, \mu) \sum_m C_i^m (p_T R_g, \mu) \otimes_\Omega S^{\in \rm gr}_{i,m}(\theta_g z_{\rm cut} p_T R, \mu; \beta)$$

See also Becher, Neubert, Rothen, Shao `15, `16

$$\frac{d\Sigma(R_g)}{d\eta dp_T} = \sum_{abc} f_a(x_a, \mu) \otimes f_b(x_b, \mu) \otimes H^c_{ab}(x_a, x_b, \eta, p_T/z, \mu) \otimes \mathcal{G}_c(z, p_T, R_g, R, \mu; z_{\rm cut}, \beta)$$

$$\frac{d\sigma}{d\eta dp_T dR_g} = \frac{d}{dR_g} \frac{d\Sigma(R_g)}{d\eta dp_T}$$

- **Two Multi-Wilson line structures to account for NGLs.**
  (correlation in matrix elements is accounted for by tracking radiations in the inclusive regions through multi-wilson lines.)
- **Compared to the usual multi-wilson line consideration, modification of measurement functions due to clustering effects between** C_i^m and $S_{i,m}$.
- **Similarly,**

  Dasgupta, Salam `01, Banfi, Marchesini, Syme `02, Larkoski, Moult, Neill `15

  Monte Carlo resummation including clustering effects in *Larkoski, Moult, Neill `17*

- **For now, we neglect both effects.**

Consistency checks I

MLL (modified LL), considers any number of independent emissions:

Larkoski, Marzani, Soyez, Thaler `14

$$\Sigma(R_g) = f_q\ \Sigma_q(R_g) + f_g\ \Sigma_g(R_g)$$

where $\Sigma_i(R_g) \overset{\text{f.c.}}{\simeq} \exp\left[-\frac{\alpha_s}{\pi}C_i\left(\beta\ln^2\frac{R_g}{R} + 2\ln z_{\text{cut}}\ln\frac{R_g}{R} + 2\frac{\gamma_i}{C_i}\ln\frac{R_g}{R}\right)\right]$

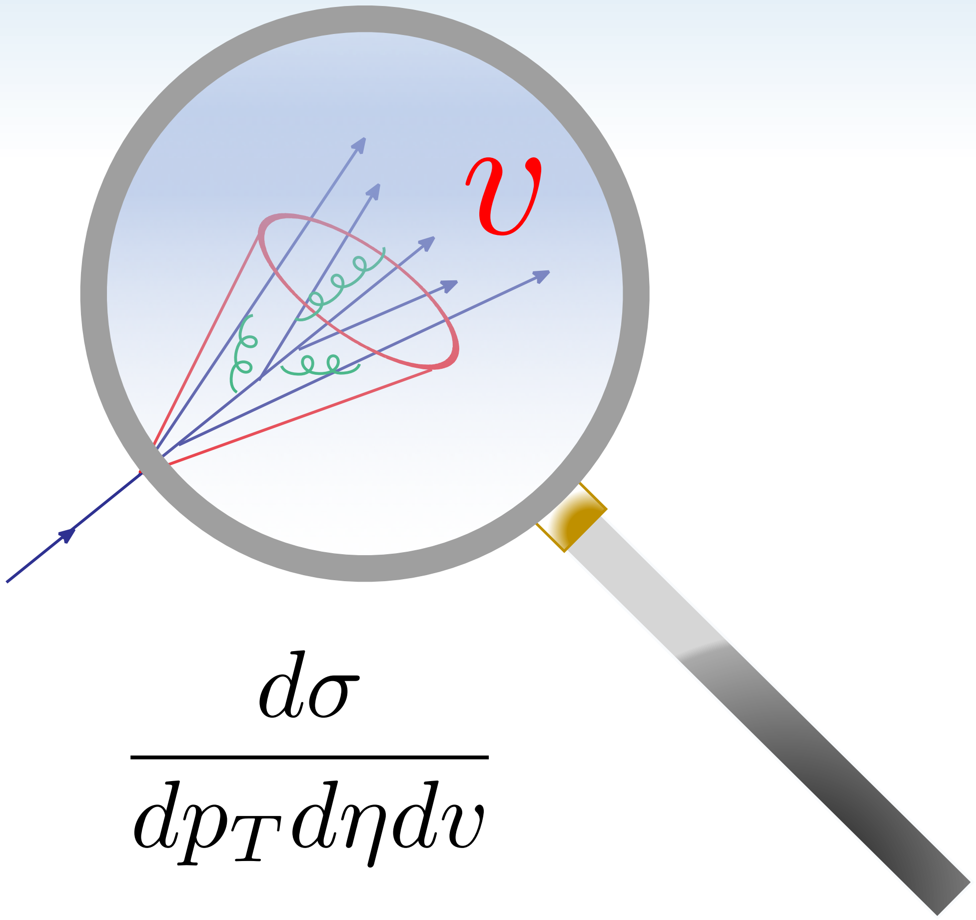
This (Sudakov) exponents are derived from evolution factors of the modes inside the jet.

$$U_{S_i}^{\notin\text{gr}}(\mu_S^{\notin\text{gr}},\mu_{\mathcal{H}})\,U_{C_i}(\mu_C,\mu_{\mathcal{H}})\,U_{S_i}^{\in\text{gr}}(\mu_S^{\in\text{gr}},\mu_{\mathcal{H}}) \overset{\text{f.c.}}{\simeq} \exp\left[\frac{\alpha_s C_i}{\pi}\frac{1}{1+\beta}\left(-\ln^2\frac{\mu_J}{\mu_S^{\in gr}} + \ln^2\frac{\mu_J}{\mu_S^{\notin gr}} + (1+\beta)\ln^2\frac{\mu_J}{\mu_C}\right) + \frac{\alpha_s\gamma_i}{\pi}\ln\frac{\mu_J}{\mu_C}\right]$$

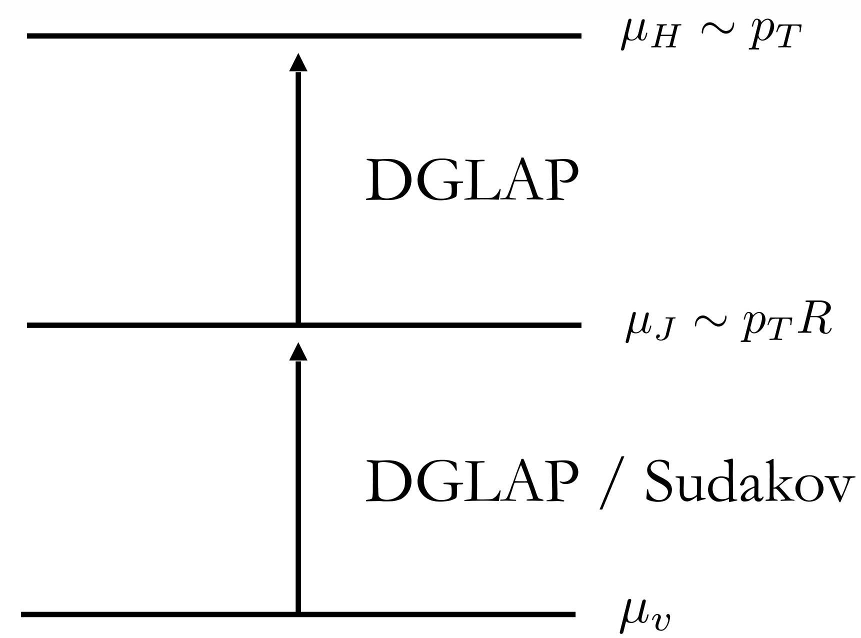
$$\underset{\text{canonical}}{\overset{\text{f.c.}}{\simeq}} \exp\left[-\frac{\alpha_s}{\pi}C_i\left(\beta\ln^2\frac{R_g}{R} + 2\ln z_{\text{cut}}\ln\frac{R_g}{R} + 2\frac{\gamma_i}{C_i}\ln\frac{R_g}{R}\right)\right]$$

where NLO anomalous dimensions are used in $U_F(\mu,\mu_F) = \exp\{\int_{\mu_F}^{\mu}\gamma_F(\alpha_s(\mu'),\ln\mu')\ d\ln\mu'\}$

Consistency checks II



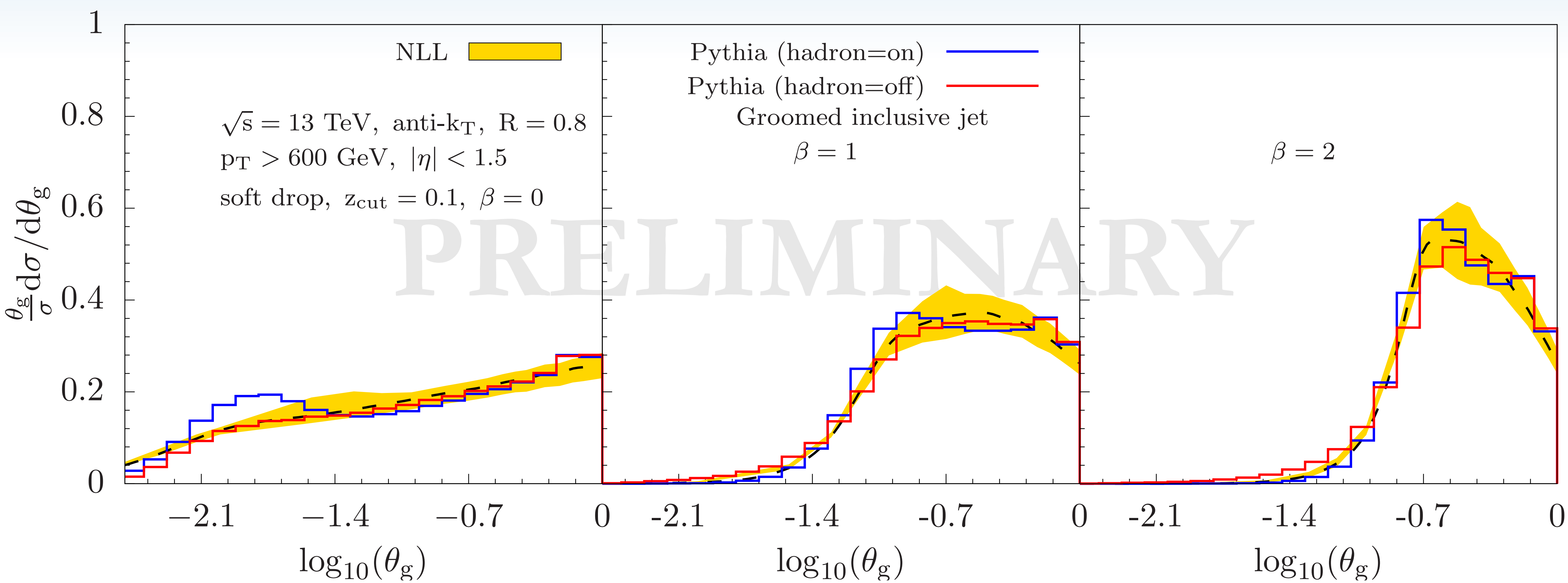
$$\frac{d\sigma}{dp_T\, d\eta\, dv}$$



- **When we measure a substructure v from the jet, once we evolve to μ_J (sometimes used $\mu_{\mathcal{H}}$) the remaining evolution to μ_H is given by DGLAP evolution.**

- **This is checked for our R_g refactorization.**

Phenomenology (groomed jet radius)



$$\theta_g = \frac{R_g}{R}$$

- Shows very good agreement without having to account for NGLs and clustering logs.
- At the LHC kinematics, small changes due to non-perturbative effects.

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Conclusions

- Formalisms for studying semi-inclusive jet production with and without a substructure measurement were introduced.
- Discussed soft drop and reduced sensitivity to NP effects and NGLs.
- Groomed jet angularity and its application to α_s extraction was discussed.
- Refactorized groomed jet radius in the context of SCET, which gave double correlation structures and clustering effects.
- Showed comparisons against Pythia for the groomed angularities and Rg.