

Power Corrections to the Groomed Jet Mass

Aditya Pathak¹

in collaboration with

Andre Hoang¹, Sonny Mantry², Iain Stewart³

¹University of Vienna

²University of North Georgia

³Massachusetts Institute of Technology

University of California San Diego

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Outline

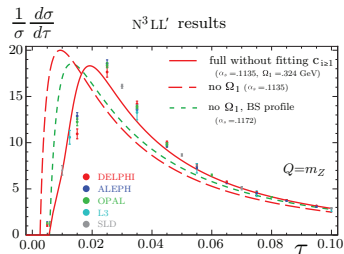
- 1 Introduction
- 2 Kinematics
- 3 Dynamics
- 4 Monte Carlo Studies
- 5 Conclusion

The main point of the talk is that simple shape function for groomed event shapes is wrong.

There are 3 universal nonperturbative parameters for the leading hadronization corrections.

Power corrections dictate the accuracy of precision measurements

α_s measurement from Thrust at LEP:



Abbate et al. 1006.3080

Power corrections can be modeled via a nonperturbative shape function

$$\frac{d\sigma}{d\tau} = \int dk \frac{d\sigma^{\text{pert}}}{d\tau} \left(\tau - \frac{k}{Q} \right) F(k - 2\bar{\Delta})$$

Only the first moment Ω_1 is relevant.

Hoang Stewart 0709.3519

One needs to fit for both α_s and Ω_1 :

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{exp}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}}$$

Field theory understanding of **hadronization** is important

SCET allows us to understand these aspects of the interface between NP function and the perturbative cross section. Consider the example of $pp \rightarrow H/Z + 1 \text{ jet}$:

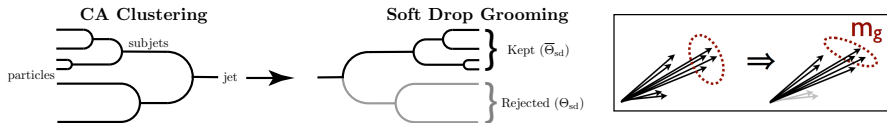
$$\frac{d\sigma}{dm_J^2 d\Phi_2} = \sum_{\kappa, a, b} H_{\kappa}(\Phi_2) \int dk_S dk_B (\mathcal{I}_{\kappa_a a} \mathcal{I}_{\kappa_b b} \otimes f_a f_b)(k_B) \times J_{\kappa J}(m_J^2 - 2p_T^J k_S) S_{\kappa}(k_S, p^{\text{cut}} - k_B, y_J, R)$$

$$\text{OPE region : } m_J^2 = (m_J^2)^{\text{pert}} + 2p_T^J \Omega_{\kappa}(R)$$

Stewart, Tackmann, Waalewijn. 1405.6722

Reduce hadronization corrections using Soft Drop

Studies of boosted objects at the LHC and the need to reduce contamination from the underlying event and pile-up led to development of **jet grooming**.



Soft drop grooming involves reclustering a jet with purely angular measure (CA clustering) and selectively throwing away the softer branches.

Soft Drop criteria:

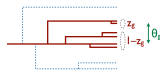
$$\frac{\min[p_{Ti}, p_{Tj}]}{(p_{Ti} + p_{Tj})} > z_{\text{cut}} \left(\frac{R_{ij}}{R_0} \right)^\beta$$

Larkoski, Marzani, Soyez, Thaler 2014

Groomed jet



Groomed Clustering tree



More Grooming

$\beta \rightarrow -\infty$

$\beta < 0$

$\beta = 0$

$\beta > 0$

Less Grooming

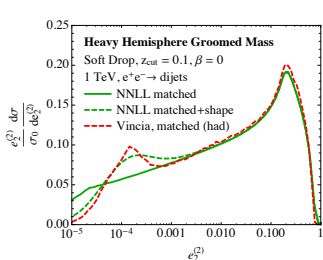
$\beta \rightarrow \infty$

The criteria is IR safe for $\beta > 0$ and Sudakov safe for $\beta = 0$ (calculable after performing resummation)

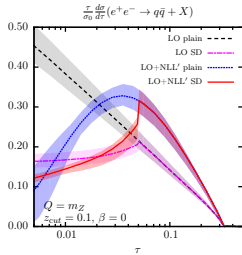
Larkoski, Thaler 2013

How well do we understand the groomed spectrum in theory?

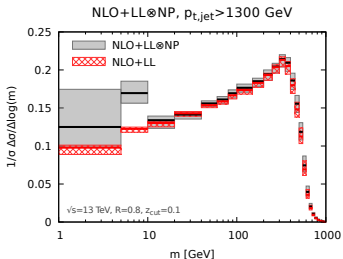
Partonic resummation of groomed jet mass is well understood:



Frye, Larkoski, Schwartz, Yan 2016



Baron et al. 1803.04719



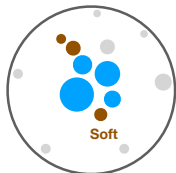
Marzani et al., JHEP07(2017)132

The fixed order corrections have also been evaluated at NNLO. [Kardos et al. 1807.11472].

See also groomed D_2 [Larkoski, Moul, Neill 2017], groomed jet mass for b quark jets [Lee, Shrivastava, Vaidya 2019]. [See talks by Yannis Makris, Kyle Lee]

What about the power corrections in the groomed spectrum?

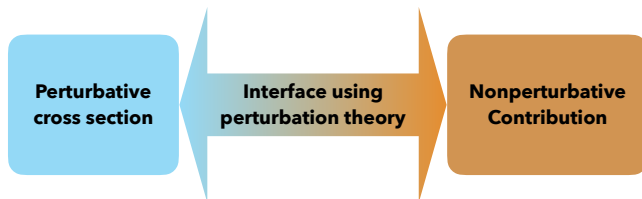
Although the hadronization effects are suppressed for groomed jet mass, in order to achieve the required accuracy of α_s we need to account for the left over soft particles.



Power corrections to groomed jet mass are intricate

In what way is the groomed jet mass different?

- **C/A clustering:** NP corrections could depend on perturbative branching history. Not even obvious if a nonperturbative factorization is possible!
- **NP catchment area:** no longer determined by the jet radius, no fixed geometric region.
- **Universality:** dependence on z_{cut} ? β ? R ? Q ? ...



Goal of this work is to deepen our understanding of the interface for groomed jet mass via field theory calculations

Power corrections for the groomed case need a closer look

Strategies for including nonperturbative corrections:

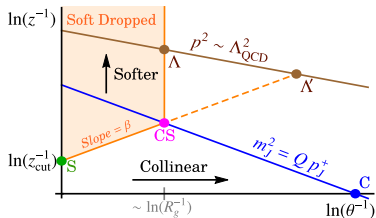
- Treat the nonperturbative corrections via a normalized **analytic shape function**:

$$\frac{d\sigma^{\text{had}}}{de_2^{(2)}} = \int d\epsilon \frac{d\sigma^{\text{pert}} \left(e_2^{(2)} - \left(\frac{\epsilon}{z_{\text{cut}} Q} \right)^{\frac{1}{1+\beta}} \frac{\epsilon}{Q} \right)}{de_2^{(2)}} F_{\text{shape}}(\epsilon), \quad e_2^{(2)} = \frac{m_J^2}{E_J^2}$$

[Frye, Larkoski, Schwartz, Yan 2016], used by [Larkoski, Moult, Neill 2017], [Lee, Shrivastava, Vaidya 2019], [Hoang, Mantry, AP, Stewart 2017 (v1)]

- Describe the power corrections via **Monte Carlo**: Use the difference between partonic and hadronic event shape to correct for nonperturbative effects.

[Marzani et al., JHEP07(2017)132]



- Above formula came out of a scaling analysis and adding shape function. No field theoretic treatment so far.
- The most important mode is not Λ' but Λ .
- The goal of the talk is to rectify this

Leading Nonperturbative Mode for Groomed Jet Mass

We identify the relevant region for our analysis by considering the EFT modes for groomed jet mass measurement

- Turning on soft drop removes emissions in the shaded region.
- CS denotes the emission at widest angle that satisfies the soft drop condition.
- Leading non-perturbative corrections have the largest plus component - hence the same angle as the CS modes.

$$p_{CS}^+ \ll p_{\Lambda}^+$$

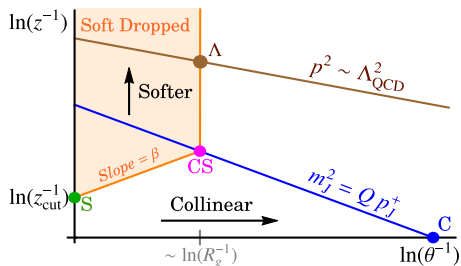
$$p_{\Lambda}^{\mu} \sim \Lambda_{\text{QCD}} \left(\zeta, \frac{1}{\zeta}, 1 \right)$$

$$z > z_{\text{cut}} \theta^{\beta}$$

$$p_{CS}^{\mu} \sim \frac{m_J^2}{Q \zeta} \left(\zeta, \frac{1}{\zeta}, 1 \right) \quad p_C^{\mu} \sim \left(\frac{m_J^2}{Q}, Q, m_J \right)$$

$$\zeta \equiv \left(\frac{m_J^2}{Q Q_{\text{cut}}} \right)^{\frac{1}{2+\beta}} \quad Q_{\text{cut}} \equiv 2^{\beta} Q z_{\text{cut}}$$

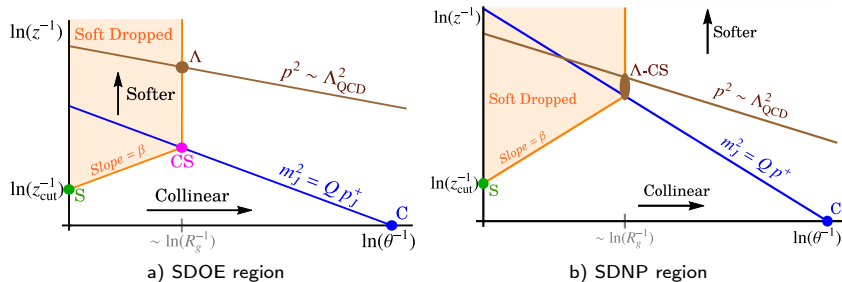
$$\theta_{CS}/2 \sim \zeta$$



Soft Drop Nonperturbative and Resummation Region

Distinguish two regions of the groomed jet mass spectrum:

- a) soft drop operator expansion (SDOE) region, $p_{cs}^+ \gg p_\Lambda^+$: $\frac{Q\Lambda_{\text{QCD}}}{m_J^2} \left(\frac{m_J^2}{Q Q_{\text{cut}}} \right)^{\frac{1}{2+\beta}} \ll 1$,
- b) soft drop nonperturbative (SDNP) region, $p_{cs}^+ \sim p_\Lambda^+$: $m_J^2 \lesssim Q\Lambda_{\text{QCD}} \left(\frac{\Lambda_{\text{QCD}}}{Q_{\text{cut}}} \right)^{\frac{1}{1+\beta}}$.



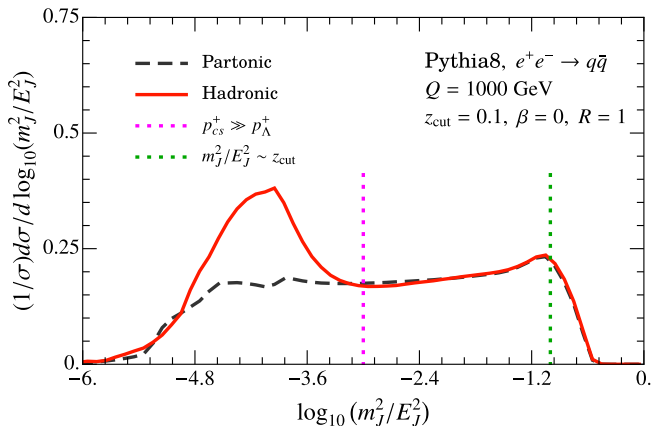
- In the SDNP region the Λ and CS mode come parametrically close merging into a single mode, Λ -CS.
- The nonperturbative corrections to the jet mass spectrum are $\mathcal{O}(1)$ in SDNP region.

Soft Drop Nonperturbative and Resummation Region

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b) soft drop nonperturbative (SDNP) region, $p_{cs}^+ \sim p_\Lambda^+$: $m_J^2 \lesssim Q\Lambda_{\text{QCD}} \left(\frac{\Lambda_{\text{QCD}}}{Q_{\text{cut}}} \right)^{\frac{1}{1+\beta}}.$



We identify two types of power corrections to the groomed spectrum

Start with **the measurement operator** for the plain jet mass:

$$\frac{d\hat{\sigma}^{\kappa}}{dm_J^2} = \sum_X H_{\mathcal{I}\mathcal{J}}^{\kappa} \langle 0 | \mathcal{O}_{\mathcal{J}}^{\kappa} \hat{\delta} | X \rangle \langle X | \mathcal{O}_{\mathcal{I}}^{\kappa \dagger} | 0 \rangle, \quad \hat{\delta} = \delta(m_J^2 - Q \hat{p}^+)$$

Define for soft drop:

$$\hat{\delta}_{\text{sd}} = \delta(m_J^2 - Q \hat{p}_{\text{sd}}^+(X)), \quad \hat{p}_{\text{sd}}^{\mu}(X; z_{\text{cut}}, \beta) \equiv \overline{\Theta}_{\text{sd}}(p_i^{\mu}, \{p_j^{\mu}; j \in X\}; z_{\text{cut}}, \beta) p_i^{\mu} | i \rangle$$

$\hat{p}_{\text{sd}}^{\mu}$ first projects to the final kept particles and then measures the momentum

$$\hat{p}_{\text{sd}}^{\mu}(X) | \{i_1, i_2, \dots, i_n\} \rangle = \left(\sum_{\alpha=1}^n \overline{\Theta}_{\text{sd}}(p_{i_{\alpha}}^{\mu}, \{p_j^{\mu}; j \in X\}) p_{i_{\alpha}}^{\mu} \right) | \{i_1, i_2, \dots, i_n\} \rangle$$

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Include NP emissions $|X_{\Lambda}\rangle$ in the jet of perturbative emissions $|X\rangle$:

- **“shift” correction:** Contribution of NP particles to the jet mass:

$$Q \hat{p}_{\text{sd}}^+(X, X_{\Lambda}) | X_{\Lambda} \rangle = Q p_{\Lambda \text{sd}}^+ | X_{\Lambda} \rangle$$

- **“boundary” correction:** modification of the soft drop test for a perturbative subjet in presence of NP radiation: $X \rightarrow X \cup X_{\Lambda}$

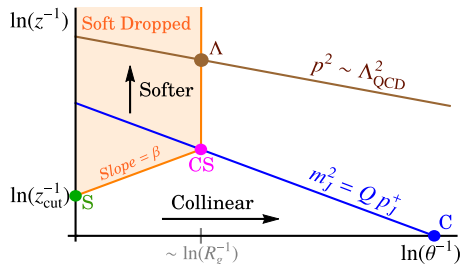
$$\hat{p}_{\text{sd}}^{\mu}(X, X_{\Lambda}) = [\hat{p}^{\mu} \bar{\Theta}_{\text{sd}}(\hat{p}^{\mu}, \{p_j^{\mu}; j \in X \cup X_{\Lambda}\})]$$

Both the corrections modify the shape of the spectrum.

Simplifications in the SDOE region

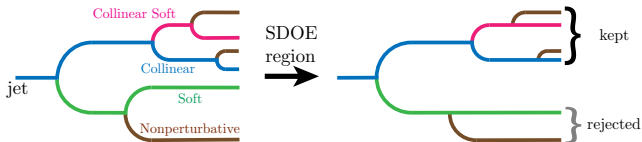
To capture an NP gluon one needs a perturbative emission:

- In the SDOE region at LL there is always a perturbative CS emission that stops soft drop.
- At LL emissions already exist but not accessible via tree level $S_c^{\text{tree}}(\ell^+, \mu) = \delta(\ell^+)$
- NP corrections require at least LL resummation to be even defined.



Simplifications in the SDOE region

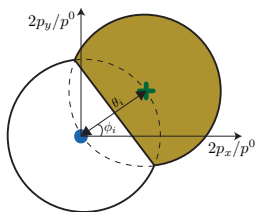
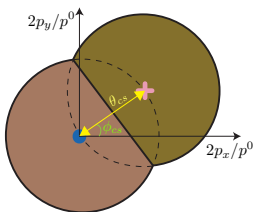
Changes in momenta of **perturbative CS emissions** due to **NP modes** in SDOE region are small:



At NLL perturbative emissions can be angularly ordered leading to a simple description of the shift and boundary corrections:

shift correction: CS emission sets the catchment area for the NP modes that are kept by soft drop

boundary correction: change in the soft drop test for CS or S mode on including an NP emission



Angular ordering simplifies the CA Clustering business

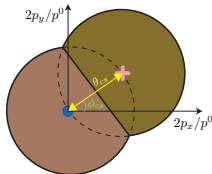
Shift Correction: Keep only the NP emissions clustered by **CS** or **C** subjet

$$Q \hat{p}_{\text{sd}}^+(X, X_\Lambda) | X_\Lambda \rangle \stackrel{\text{NLL}}{\sim} Q \hat{p}_\omega^+(\theta_{\text{CS}}, \phi_{\text{CS}}) | X_\Lambda \rangle = Q p_{\Lambda\text{sd}}^+ | X_\Lambda \rangle$$

$$\hat{p}_\omega^\mu(\theta_{\text{CS}}, \phi_{\text{CS}}) \equiv [\hat{p}^\mu \bar{\Theta}_{\text{NP}}^\omega(\hat{p}_\Lambda^\mu, \theta_{\text{CS}}, \phi_{\text{CS}})]$$

$\bar{\Theta}_{\text{NP}}^\omega = 1$: NP particle kept in the final soft dropped subjet

$$\bar{\Theta}_{\text{NP}}^\omega(\theta_\Lambda, \theta_{\text{CS}}, \Delta\phi) \equiv \Theta\left(|\Delta\phi| - \frac{\pi}{3}\right) \Theta\left(1 - \frac{\theta_\Lambda}{\theta_{\text{CS}}}\right) + \Theta\left(\frac{\pi}{3} - |\Delta\phi|\right) \Theta\left(2 \cos(\Delta\phi) - \frac{\theta_\Lambda}{\theta_{\text{CS}}}\right)$$



Angular ordering simplifies the CA Clustering business

Shift Correction: Keep only the NP emissions clustered by **CS** or **C** subject

$$Q \hat{p}_{\text{sd}}^+(X, X_\Lambda) | X_\Lambda \rangle \stackrel{\text{NLL}}{\sim} Q \hat{p}_\omega^+(\theta_{\text{CS}}, \phi_{\text{CS}}) | X_\Lambda \rangle = Q p_{\Lambda\text{sd}}^+ | X_\Lambda \rangle$$

$$\hat{p}_\omega^\mu(\theta_{\text{CS}}, \phi_{\text{CS}}) \equiv [\hat{p}^\mu \bar{\Theta}_{\text{NP}}^\omega(\hat{p}_\Lambda^\mu, \theta_{\text{CS}}, \phi_{\text{CS}})]$$

$\bar{\Theta}_{\text{NP}}^\omega = 1$: NP particle kept in the final soft dropped subset

$$\bar{\Theta}_{\text{NP}}^\omega(\theta_\Lambda, \theta_{\text{CS}}, \Delta\phi) \equiv \Theta\left(|\Delta\phi| - \frac{\pi}{3}\right) \Theta\left(1 - \frac{\theta_\Lambda}{\theta_{\text{CS}}}\right) + \Theta\left(\frac{\pi}{3} - |\Delta\phi|\right) \Theta\left(2 \cos(\Delta\phi) - \frac{\theta_\Lambda}{\theta_{\text{CS}}}\right)$$

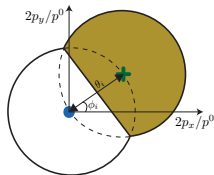
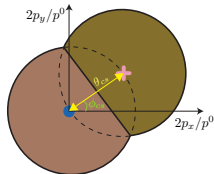
Boundary Correction: affects only softer subject

$$\begin{aligned} \bar{\Theta}_{\text{sd}}^{p_i+q_i} &= \Theta\left(\frac{p_i^- + q_i^-}{Q} - z_{\text{cut}} \left(\frac{1}{R_0} \frac{2|\vec{p}_{i,\perp} + \vec{q}_{i,\perp}|}{p_i^- + q_i^-}\right)^\beta\right) \\ &= \bar{\Theta}_{\text{sd}}^{p_i} + \delta\left(z_{p_i} - z_{\text{cut}} \left(\frac{\theta_i}{R_0}\right)^\beta\right) \frac{q_i^-}{Q} \left((1 + \beta) - \beta \frac{\theta_{q_i}}{\theta_i} \cos(\Delta\phi_i)\right) \end{aligned}$$

Relevant operator:

$$\hat{p}_\ominus(\theta_i, \phi_i, \beta) \equiv \left[\hat{p}(\theta_i, \phi_i, \beta) \left(\bar{\Theta}_{\text{NP}}^\ominus(\hat{p}^\mu, \theta_i, \phi_i) - \Theta_{\text{NP}}^\ominus(\hat{p}^\mu, \theta_i, \phi_i) \right) \right]$$

$$\hat{p}(\theta_i, \phi_i, \beta) \equiv \hat{p}^- + \beta \left(\hat{p}^\perp - \frac{\hat{p}^\perp}{\theta_i} \cos(\Delta\phi) \right)$$



- particle captured by the softer subject: $\bar{\Theta}_{\text{NP}}^\ominus(\hat{p}^\mu, \theta_i, \phi_i)$
- particle lost by the softer subject: $\Theta_{\text{NP}}^\ominus(\hat{p}^\mu, \theta_i, \phi_i)$

Angular ordering simplifies the CA Clustering business

Shift Correction: Keep only the NP emissions clustered by **CS** or **C** subject

$$Q \hat{p}_{\text{sd}}^+(X, X_\Lambda) | X_\Lambda \stackrel{\text{NLL}}{\sim} Q \hat{p}_\omega^+(\theta_{\text{CS}}, \phi_{\text{CS}}) | X_\Lambda = Q p_{\Lambda\text{sd}}^+ | X_\Lambda$$

$$\hat{p}_\omega^\mu(\theta_{\text{CS}}, \phi_{\text{CS}}) \equiv [\hat{p}_\Lambda^\omega(\theta_{\text{CS}}, \phi_{\text{CS}})]$$

$\bar{\Theta}_{\text{NP}}^\omega = 1$: NP particle kept in the final soft dropped subset

$$\bar{\Theta}_{\text{NP}}^\omega(\theta_\Lambda, \theta_{\text{CS}}, \Delta\phi) \equiv \Theta\left(|\Delta\phi| - \frac{\pi}{3}\right) \Theta\left(1 - \frac{\theta_\Lambda}{\theta_{\text{CS}}}\right) + \Theta\left(\frac{\pi}{3} - |\Delta\phi|\right) \Theta\left(2 \cos(\Delta\phi) - \frac{\theta_\Lambda}{\theta_{\text{CS}}}\right)$$

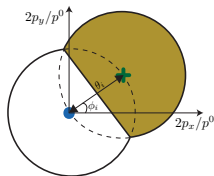
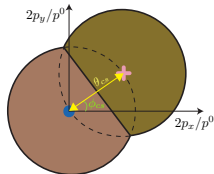
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$$\begin{aligned} \bar{\Theta}_{\text{sd}}^{p_i+q_i} &= \Theta\left(\frac{p_i^- + q_i^-}{Q} - z_{\text{cut}} \left(\frac{1}{R_0} \frac{2|\vec{p}_{i,\perp} + \vec{q}_{i,\perp}|}{p_i^- + q_i^-}\right)^\beta\right) \\ &= \bar{\Theta}_{\text{sd}}^{p_i} + \delta\left(z_{p_i} - z_{\text{cut}} \left(\frac{\theta_i}{R_0}\right)^\beta\right) \frac{q_i^-}{Q} \left((1 + \beta) - \beta \frac{\theta_{q_i}}{\theta_i} \cos(\Delta\phi_i)\right) \end{aligned}$$

Relevant Projection:

$$\bar{\Theta}_{\text{NP}}^\circ(\theta_\Lambda, \theta_i, \phi_i) = \Theta\left(\frac{\pi}{3} - |\Delta\phi_i|\right) \Theta\left(\frac{\theta_\Lambda}{\theta_i} - \frac{1}{2 \cos(\Delta\phi_i)}\right) \Theta\left(2 \cos(\Delta\phi_i) - \frac{\theta_\Lambda}{\theta_i}\right)$$

We have simplified the CA clustering issue but still need to disentangle the dependence on perturbative angles θ_i and θ_{CS} .



Angular ordering simplifies the CA Clustering business

Shift Correction: Keep only the NP emissions clustered by **CS** or **C** subjet

$$Q \hat{p}_{\text{sd}}^+(X, X_\Lambda) | X_\Lambda \rangle \stackrel{\text{NLL}}{\sim} Q \hat{p}_\omega^+(\theta_{\text{CS}}, \phi_{\text{CS}}) | X_\Lambda \rangle = Q p_{\Lambda\text{sd}}^+ | X_\Lambda \rangle$$

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Boundary Correction: Relevant Projection:

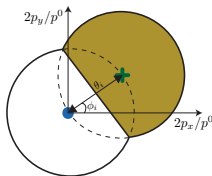
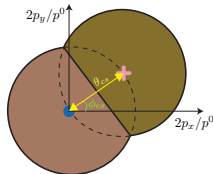
$$\bar{\Theta}_{\text{NP}}^\omega(\theta_\Lambda, \theta_i, \phi_i) = \Theta\left(\frac{\pi}{3} - |\Delta\phi_i|\right) \Theta\left(\frac{\theta_\Lambda}{\theta_i} - \frac{1}{2 \cos(\Delta\phi_i)}\right) \Theta\left(2 \cos(\Delta\phi_i) - \frac{\theta_\Lambda}{\theta_i}\right)$$

Rescale NP momenta: Make the following change of variables:

$$p_\Lambda^+ = \frac{\theta_X}{2} k_X^+ = \sqrt{\frac{p_X^+}{p_X^-}} k_X^+, \quad p_\Lambda^- = \frac{2}{\theta_X} k_X^- = \sqrt{\frac{p_X^-}{p_X^+}} k_X^-, \quad p_{\Lambda,\perp} = k_\perp$$

equivalent to a boost along the jet direction:

$$\hat{\Lambda}(\gamma) | (p^+, p^-, p_\perp) \rangle = |\Lambda_\nu^\mu(\gamma) p^\nu \rangle = \left| \left(\gamma p^+, \frac{1}{\gamma} p^-, p_\perp \right) \right\rangle$$



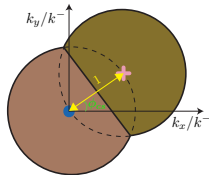
Rescaling helps factor out the perturbative dependence of power corrections

$$\hat{p}_{\infty}^{+}(\theta_{cs}, \phi_{cs}) = \frac{\theta_{cs}}{2} \left[\hat{\Lambda}\left(\frac{\theta_{cs}}{2}\right) \hat{k}_{\Lambda}^{+} \bar{\Theta}_{\text{NP}}^{\infty}\left(\frac{\hat{k}_{\Lambda, \perp}}{\hat{k}_{\Lambda}^{-}}, 2, \Delta\phi_{cs}\right) \hat{\Lambda}^{-1}\left(\frac{\theta_{cs}}{2}\right) \right],$$

$$\hat{p}_{\ominus}(\theta_i, \phi_i, \beta) = \frac{2}{\theta_i} \left[\hat{\Lambda}\left(\frac{\theta_i}{2}\right) \left(\hat{k}_{\Lambda}^{-} + \beta \left(\hat{k}_{\Lambda}^{-} - \hat{k}_{\Lambda, \perp} \cos(\Delta\phi_i) \right) \right) \right. \\ \left. \times \left(-\Theta_{\text{NP}}^{\ominus}\left(\frac{\hat{k}_{\Lambda, \perp}}{\hat{k}_{\Lambda}^{-}}, 2, \Delta\phi_i\right) + \bar{\Theta}_{\text{NP}}^{\ominus}\left(\frac{\hat{k}_{\Lambda, \perp}}{\hat{k}_{\Lambda}^{-}}, 2, \Delta\phi_i\right) \right) \hat{\Lambda}^{-1}\left(\frac{\theta_i}{2}\right) \right]$$

$$\bar{\Theta}_{\text{NP}}^{\infty}\left(\frac{k_{\perp}}{k^{-}}, 2, \Delta\phi\right) \equiv \Theta\left(|\Delta\phi| - \frac{\pi}{3}\right) \Theta\left(1 - \frac{k_{\perp}}{k^{-}}\right) + \Theta\left(\frac{\pi}{3} - |\Delta\phi|\right) \Theta\left(2 \cos(\Delta\phi) - \frac{k_{\perp}}{k^{-}}\right)$$

$$\bar{\Theta}_{\text{NP}}^{\ominus}\left(\frac{k_{\perp}}{k^{-}}, 2, \Delta\phi_i\right) \equiv \Theta\left(\frac{\pi}{3} - |\Delta\phi_i|\right) \Theta\left(\frac{k_{\perp}}{k^{-}} - \frac{1}{2 \cos(\Delta\phi_i)}\right) \Theta\left(2 \cos(\Delta\phi_i) - \frac{k_{\perp}}{k^{-}}\right)$$

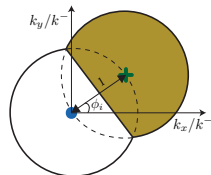


Rescale NP momenta: Make the following change of variables:

$$p_{\Lambda}^{+} = \frac{\theta_X}{2} k_X^{+} = \sqrt{\frac{p_X^{+}}{p_X^{-}}} k_X^{+}, \quad p_{\Lambda}^{-} = \frac{2}{\theta_X} k_X^{-} = \sqrt{\frac{p_X^{-}}{p_X^{+}}} k_X^{-}, \quad p_{\Lambda, \perp} = k_{\perp}$$

perturbative angular dependence now factors out

The circles now have radius = 1.



Dynamics of the NP radiation: An illustrative calculation

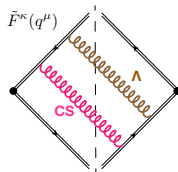
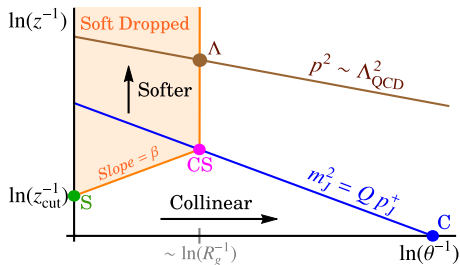
The partonic SCET factorization formula for groomed jet mass is given by

$$\frac{d\hat{\sigma}}{dm_J^2} = \sum_{\kappa=q,g} D_{\kappa}(\Phi_J, z_{\text{cut}}, \beta, \mu) Q_{\text{cut}}^{\frac{1}{1+\beta}} \int d\ell^+ J_{\kappa}(m_J^2 - Q\ell^+, \mu) S_c^{\kappa}[\ell^+ Q_{\text{cut}}^{\frac{1}{1+\beta}}, \beta, \mu], \quad \kappa = \{q, g\}$$

[Frye, Larkoski, Schwartz, Yan 2016]

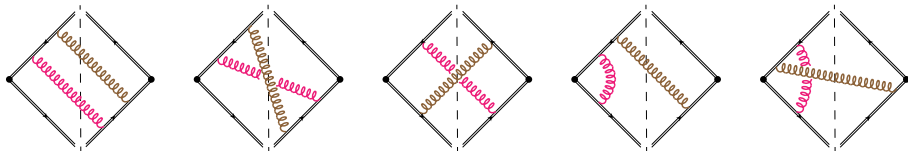
Unlike the ungroomed case we cannot simply modify the argument of the collinear-soft function S_c^{κ} to include NP corrections given the complex nature of shift and boundary corrections. Think about it like an OPE with two modes (following the approach of [Mateu, Stewart, Thaler 2013]):

- Consider the one-loop fixed order collinear soft function.
- Include an Eikonal NP emission q^{μ} sourced externally by $\tilde{F}^{\kappa}(q^{\mu})$.
- Source is agnostic about soft drop parameters and C/A Clustering: included explicitly via the catchment area selection operators $\hat{p}_{\ominus}^{\mu}(\theta_{cs}, \phi_{cs})$ and $\hat{p}_{\odot}^{\mu}(\theta_i, \phi_i)$.



Dashed line represents measurement with the soft drop cut: $\delta(\ell^+ - \hat{p}_{\text{sd}}^+)$

Start with Abelian diagrams



Expand the measurement operator in the SDOE region and keep $\mathcal{O}(\Lambda_{\text{QCD}})$ terms:

$$\begin{aligned}
 & \mathcal{M}^{p+q} \overset{\text{SDOE}}{\sim} \delta(\ell^+) + \bar{\Theta}_{\text{sd}}^p \left[\delta(\ell^+ - p^+) - \delta(\ell^+) \right] \\
 & - q^+ \bar{\Theta}_{\text{NP}}^\otimes(\theta_q, \theta_p, \Delta\phi) \bar{\Theta}_{\text{sd}}^p \frac{d}{d\ell^+} \delta(\ell^+ - p^+) \\
 & + \tilde{q}(\beta) \left(\bar{\Theta}_{\text{NP}}^\otimes(\theta_\Lambda, \theta_p, \Delta\phi) - \Theta_{\text{NP}}^\otimes(\theta_\Lambda, \theta_p, \Delta\phi) \right) \delta_{\text{sd}}^p \left[\delta(\ell^+ - p^+) - \delta(\ell^+) \right] \\
 & \mathcal{M}^q \overset{\text{SDOE}}{\sim} \delta(\ell^+) \quad (q^\mu \text{ alone cannot pass soft drop in SDOE region})
 \end{aligned}$$

On adding all the Abelian graphs the NP source factorizes:

$$\begin{aligned}
 S_c^{\text{had}}(\ell^+, \mu) &= \int \frac{d^d q}{(2\pi)^d} \left[\frac{4g^2 C_\kappa \ell^\epsilon \tilde{C}(q)}{q^+ q^-} \right] \left\{ S_c^{\text{pert}}(\ell^+, \mu) + \frac{d}{d\ell^+} \Delta \tilde{S}_c^\otimes(\ell^+, q^\mu, \mu) + \Delta \tilde{S}_c^\otimes(\ell^+, q^\mu, \mu) + \mathcal{O}(q^2) \right\} \\
 \tilde{F}^{\text{ab.}}(q^\mu) &\equiv \frac{4g^2 C_\kappa \ell^\epsilon \tilde{C}(q)}{q^+ q^-}
 \end{aligned}$$

Rescaling factorizes the power corrections

Apply the rescaling to decouple the q^μ dependence in the power corrections:

$$q^+ = \frac{\theta_p}{2} k^+ = \sqrt{\frac{p^+}{p^-}} k^+, \quad q^- = \frac{2}{\theta_p} k^- = \sqrt{\frac{p^-}{p^+}} k^-, \quad q_\perp = k_\perp$$

Note that the measure and the NP source are invariant under boosts

$$\theta_q = \frac{2 q_\perp}{q^-} = \theta_p \frac{k_\perp}{k^-}, \quad \tilde{F}^{\text{ab.}}(q^\mu) = \frac{4g^2 C_\kappa \ell^\epsilon \tilde{C}(q)}{q^+ q^-} = \tilde{F}^{\text{ab.}}(k^\mu), \quad d^d q = d^d k$$

$$S_c^{\text{had}}(\ell^+, \mu) = S_c^{\text{pert}}(\ell^+, \mu) - \Omega_1^{\text{ab.}} \frac{d}{d\ell^+} \Delta S_c^\otimes(\ell^+, \mu) + \frac{\Upsilon_1^{\text{ab.}}(\beta)}{Q} \Delta S_c^\otimes(\ell^+, \mu)$$

where

$$\Omega_1^{\text{ab.}} \equiv \int \frac{d^d k}{(2\pi)^d} k^+ \bar{\Theta}_{\text{NP}}^\otimes\left(\frac{k_\perp}{k^-}, 2, \phi_k\right) \tilde{F}^{\text{ab.}}(k^\mu),$$

$$\Upsilon_1^{\text{ab.}}(\beta) \equiv \int \frac{d^d k}{(2\pi)^d} \left(k^- + \beta(k^- - k_\perp \cos(\phi_k))\right) \left(\bar{\Theta}_{\text{NP}}^\otimes - \Theta_{\text{NP}}^\otimes\right) \tilde{F}^{\text{ab.}}(k^\mu),$$

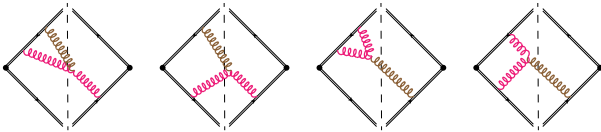
$$\Delta S_c^\otimes(\ell^+, \mu) = \frac{\alpha_s C_\kappa}{\pi} \frac{(\mu^2 e^{\gamma_E})^\epsilon}{\Gamma(1-\epsilon)} \int_0^\infty \frac{dp^+ dp^-}{(p^+ p^-)^{1+\epsilon}} \frac{\theta_p}{2} \delta(\ell^+ - p^+) \Theta\left(\frac{p^-}{Q} - z_{\text{cut}} \left(\frac{2}{R_0} \sqrt{\frac{p^+}{p^-}}\right)^\beta\right),$$

$$\Delta S_c^\otimes(\ell^+, \mu) = \frac{\alpha_s C_\kappa}{\pi} \frac{(\mu^2 e^{\gamma_E})^\epsilon}{\Gamma(1-\epsilon)} \int_0^\infty \frac{dp^+ dp^-}{(p^+ p^-)^{1+\epsilon}} \frac{2}{\theta_p} \delta(\ell^+ - p^+) \delta\left(\frac{p^-}{Q} - z_{\text{cut}} \left(\frac{2}{R_0} \sqrt{\frac{p^+}{p^-}}\right)^\beta\right).$$

The perturbative dependence of the power corrections factorizes!

Non Abelian graphs are tricky

Now consider the nonabelian graphs:



$$S_c^{\text{had, n.a.}} = \frac{\alpha_s C_\kappa (\mu^2 e^{\gamma_E})^\epsilon}{\pi \Gamma(1-\epsilon)} \int_0^\infty \frac{dp^+ dp^-}{(p^+ p^-)^{1+\epsilon}} \int \frac{d^d q}{(2\pi)^d} \frac{2g^2 C_A \iota^\epsilon \tilde{C}(q)}{q^+ q^-} \\ \times [\mathcal{M}^{p+q} - \mathcal{M}^q] \frac{q^+ p^- + p^+ q^-}{p^+ q^- + q^+ p^- - 2\sqrt{p^+ p^-} |\vec{q}_\perp| \cos(\phi_q)}.$$

The matrix element for emission of q^μ off the CS gluon p^μ is NOT invariant under boost of q^μ alone along the jet direction. Nonetheless go ahead and make the rescaling:

$$q^+ = \frac{\theta_p}{2} k^+ = \sqrt{\frac{p^+}{p^-}} k^+, \quad q^- = \frac{2}{\theta_p} k^- = \sqrt{\frac{p^-}{p^+}} k^-, \quad q_\perp = k_\perp$$

as a result of which the nonperturbative and the perturbative factors completely decouple:

$$\frac{q^+ p^- + p^+ q^-}{p^+ q^- + q^+ p^- - 2\sqrt{p^+ p^-} |\vec{q}_\perp| \cos(\phi_q)} = \frac{k^+ + k^-}{k^+ + k^- - 2|\vec{k}_\perp| \cos(\phi_k)},$$

Leading Power corrections to the groomed cross section

Thus we see how the shift and the boundary power corrections factorize:

$$S_c^{\text{had}} = S_c^{\text{pert}}(\ell^+, \mu) - \Omega_{\mathbf{1}}^{\otimes} \frac{d}{d\ell^+} \Delta S_c^{\otimes}(\ell^+, \mu) + \frac{\Upsilon_{\mathbf{1}}^{\otimes}(\beta)}{Q} \Delta S_c^{\otimes}(\ell^+, \mu)$$

Leading power corrections for the full cross section can be parameterized as

$$\frac{d\sigma_{\kappa}^{\text{had}}}{dm_J^2} = \frac{d\hat{\sigma}_{\kappa}}{dm_J^2} - Q \Omega_{\mathbf{1}}^{\otimes} \frac{d}{dm_J^2} \left(C_1(m_J^2, Q, z_{\text{cut}}, \beta) \frac{d\hat{\sigma}_{\kappa}}{dm_J^2} \right) + \frac{\Upsilon_{\mathbf{1}}^{\otimes}(\beta)}{Q} C_2(m_J^2, Q; z_{\text{cut}}, \beta) \frac{d\hat{\sigma}_{\kappa}}{dm_J^2}$$

Linear in β , hence two parameters:

$$\Upsilon_{\mathbf{1}}^{\otimes}(\beta) = \Upsilon_{\mathbf{1},\mathbf{0}}^{\otimes} + \beta \Upsilon_{\mathbf{1},\mathbf{1}}^{\otimes}$$

The Wilson coefficients $C_1(m_J^2, Q, z_{\text{cut}}, \beta)$ and $C_2(m_J^2, Q, z_{\text{cut}}, \beta)$ are not constants along the spectrum depend on both the grooming parameters, **but the hadronic power corrections themselves are universal:**

Three parameters total, **only depending on Λ_{QCD} :**

$$\Omega_{\mathbf{1}}^{\otimes} \equiv \int \frac{d^d k}{(2\pi)^d} k^+ \bar{\Theta}_{\text{NP}}^{\otimes} \tilde{F}(k^\mu)$$

$$\Upsilon_{\mathbf{1},\mathbf{0}}^{\otimes} \equiv \int \frac{d^d k}{(2\pi)^d} k^- \left(\bar{\Theta}_{\text{NP}}^{\otimes} - \Theta_{\text{NP}}^{\otimes} \right) \tilde{F}(k^\mu)$$

$$\Upsilon_{\mathbf{1},\mathbf{1}}^{\otimes} \equiv \int \frac{d^d k}{(2\pi)^d} (k^- - k_{\perp} \cos(\phi_k)) \left(\bar{\Theta}_{\text{NP}}^{\otimes} - \Theta_{\text{NP}}^{\otimes} \right) \tilde{F}(k^\mu)$$

We see that power corrections have nontrivial modifications of the entire spectrum.

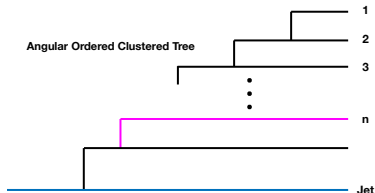
Use Coherent Branching Formalism to resum NP matching coefficients.

The calculation of $C_1(m_J^2, Q^2)$ and $C_2(m_J^2, Q^2)$ is easily carried out in the Coherent Branching formalism at NLL where the resummation is implemented via sum over real emissions.

[Catani et al. Nucl.Phys. B407 (1993) 3-42]

Partonic resummation formula in coherent branching

- Consider a series of angularly ordered emissions off an energetic quark, with the first emission being at the widest angle.
- At every stage of unclustering we will recover the emissions in the order they were emitted.
- The stopping pair is found after n unclusterings

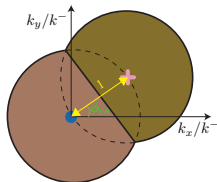
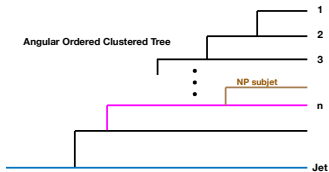


Partonic cross section ($\bar{\Theta}_{\text{sd}} = 1$: pass soft drop), [Dasgupta et al. JHEP09(2013)029]

$$\begin{aligned}
 \frac{1}{\hat{\sigma}} \frac{d\hat{\sigma}}{dm_J^2} &= \delta(m_J^2) + \sum_{n=1}^{\infty} \int_0^1 dz_n \int_0^R \frac{d\theta_n^2}{\theta_n^2} \frac{\alpha_s(z_n \theta_n Q/2) C_F}{\pi} p_{gq}(z_n) \\
 &\times \left\{ \bar{\Theta}_{\text{sd}}^n \delta\left(m_J^2 - \frac{1}{4} z_n \theta_n^2 Q^2\right) + \Theta_{\text{sd}}^n \delta(m_J^2) - \delta(m_J^2) \right\} \Theta(\theta_{n-1} - \theta_n) \\
 &\times \prod_{i=1}^{n-1} \int_0^1 dz_i \int_0^R \frac{d\theta_i^2}{\theta_i^2} \frac{\alpha_s(z_i \theta_i Q/2) C_F}{\pi} p_{gq}(z_i) \{ \Theta_{\text{sd}}^i - 1 \} \Theta(\theta_{i-1} - \theta_i),
 \end{aligned}$$

Including power correction in the partonic cross section is now easy

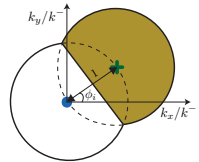
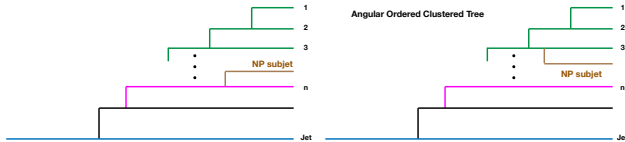
The analysis can be easily generalized to calculate the shift and the boundary power corrections:



Do the rescaling and perform the measurement on the **NP subject** in the boosted frame ($m_J^2 > 0$):

$$\begin{aligned}
 \frac{1}{\hat{\sigma}} \frac{d\hat{\sigma}^{\text{shift}}}{dm_J^2} &= \int \frac{d^d k}{(2\pi)^d} \tilde{F}(k^\mu) \sum_{n=1}^{\infty} \int d^2 \omega_n \Theta(\theta_{n-1} - \theta_n) \\
 &\times \left\{ \bar{\Theta}_{\text{sd}}^n \delta\left(m_J^2 - \frac{1}{4} z_n \theta_n^2 Q^2 - \frac{\theta_n}{2} \bar{\Theta}_{\text{NP}}^\infty\left(\frac{k_\perp}{k^-}, 2, \Delta\phi\right) Q k^+\right) + \Theta_{\text{sd}}^n \delta(m_J^2) - \delta(m_J^2) \right\} \\
 &\times \prod_{i=1}^{n-1} \int d^2 \omega_i \{ \Theta_{\text{sd}}^i - 1 \} \Theta(\tilde{\theta}_{i-1} - \tilde{\theta}_i) \\
 \int d^2 \omega_i &= \int_0^1 dz_i \int_0^1 \frac{d\tilde{\theta}_i^2}{\tilde{\theta}_i^2} \frac{\alpha_s(z_i \theta_i; RE_J) C_F}{\pi} p_{gq}(z_i)
 \end{aligned}$$

Boundary term comes from both passing and failing subjets



Rescale relative to each of the subjets

$$\begin{aligned}
 \frac{1}{\hat{\sigma}} \frac{d\hat{\sigma}^{\text{bdry}}}{dm_j^2} &= 1 + \int \frac{d^d k}{(2\pi)^d} \tilde{F}(k^\mu) \sum_{n=1}^{\infty} \int d^2 \omega_n \delta\left(m_j^2 - \frac{1}{4} z_n \theta_n^2 Q^2\right) \Theta(\theta_{n-1} - \theta_n) \\
 &\quad \times \left[\bar{\Theta}_{\text{sd}}^n + \frac{2}{\theta_n} \frac{\tilde{k}(\beta)}{Q} \delta_{\text{sd}}^n \right] \prod_{i=1}^{n-1} \int d^2 \omega_i \Theta(\tilde{\theta}_{i-1} - \tilde{\theta}_i) [-\bar{\Theta}_{\text{sd}}^i] \\
 &+ \int \frac{d^d k}{(2\pi)^d} \tilde{F}(k^\mu) \sum_{n=1}^{\infty} \int d^2 \omega_n \delta\left(m_j^2 - \frac{1}{4} z_n \theta_n^2 Q^2\right) \Theta(\tilde{\theta}_{n-1} - \tilde{\theta}_n) \bar{\Theta}_{\text{sd}}^n \\
 &\quad \times \sum_{j=1}^{n-1} \int d^2 \omega_j \Theta(\tilde{\theta}_{j-1} - \tilde{\theta}_j) \left[-\bar{\Theta}_{\text{sd}}^j - \frac{2}{\theta_j} \frac{\tilde{k}(\beta)}{Q} \delta_{\text{sd}}^j \right] \prod_{\substack{i=1 \\ i \neq j}}^{n-1} \int d^2 \omega_i \Theta(\tilde{\theta}_{i-1} - \tilde{\theta}_i) [-\bar{\Theta}_{\text{sd}}^i]
 \end{aligned}$$

$$\tilde{k}(\beta) = \left(k^- + \beta (k^- - k_\perp \cos(\phi_k)) \right) \left(\bar{\Theta}_{\text{NP}}^\ominus - \Theta_{\text{NP}}^\ominus \right), \quad \delta_{\text{sd}}^i = \delta\left(z_i - z_{\text{cut}} \left(\frac{\theta_i}{R_0} \right)^\beta \right)$$

NLL Resummed Formulae for the Matching Coefficients

Taylor expand in the OPE region:

$$\frac{d\sigma^{\text{had}}}{dm_J^2} = \frac{d\hat{\sigma}}{dm_J^2} - Q \Omega_1^\omega \frac{d}{dm_J^2} \left[C_1(m_J^2, z_{\text{cut}}, \beta, Q) \frac{d\hat{\sigma}}{dm_J^2} \right] + \frac{\Upsilon_1(\beta)}{Q} C_2(m_J^2, z_{\text{cut}}, \beta, Q) \frac{d\hat{\sigma}}{dm_J^2},$$

$$C_1(m_J^2, z_{\text{cut}}, \beta, Q) \equiv \frac{1}{C_0(m_J^2, z_{\text{cut}}, \beta, Q)} \int_0^1 dz \frac{\theta_{cs}(m_J, E_J)}{2} \frac{\alpha_s(m_J \sqrt{z}) C_F}{\pi} \rho_{gq}(z) \\ \times \Theta \left(z - \max \left\{ z^* \left(\frac{m_J^2}{Q} \right), \frac{m_J^2}{R^2 E_J^2} \right\} \right) \exp \left[-\mathcal{R}_q \left(\frac{m_J}{R E_J \sqrt{z}} \right) \right],$$

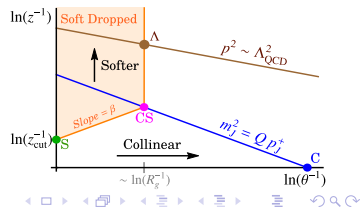
$$\frac{1}{\hat{\sigma}} \frac{d\hat{\sigma}}{dm_J^2} = \frac{1}{m_J^2} C_0(m_J^2, z_{\text{cut}}, \beta, Q), \quad \frac{\theta_{cs}(m_J, E_J)}{2} = \frac{m_J}{Q \sqrt{z}}, \quad z^* \left(\frac{m_J^2}{Q} \right) = \frac{Q_{\text{cut}}}{Q} \left(\frac{m_J^2}{Q Q_{\text{cut}} R_0^2} \right)^{\frac{\beta}{2+\beta}}$$

(A more complicated expression for C_2 not shown)

To a rough approximation:

$$C_1(m_J^2, z_{\text{cut}}, \beta, Q) \simeq 0.6 \left(\frac{m_J^2}{Q Q_{\text{cut}}} \right)^{\frac{1}{2+\beta}} \simeq 0.6 \times \frac{1}{2} \theta_{cs}^{\text{EFT guess}}$$

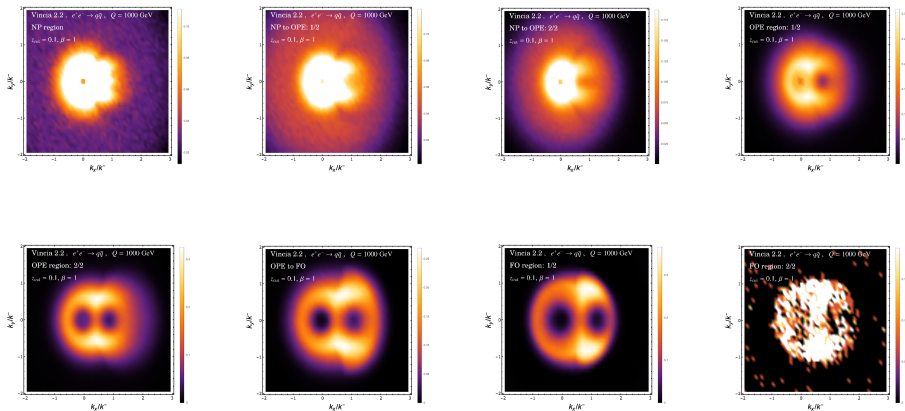
$$p_{cs}^\mu \sim \frac{m_J^2}{Q \zeta_0} \left(\zeta_{cs}, \frac{1}{\zeta_{cs}}, 1 \right), \quad \zeta_{cs} \equiv \left(\frac{m_J^2}{Q Q_{\text{cut}}} \right)^{\frac{1}{2+\beta}}$$



Visualizing the angular distribution of NP subjets

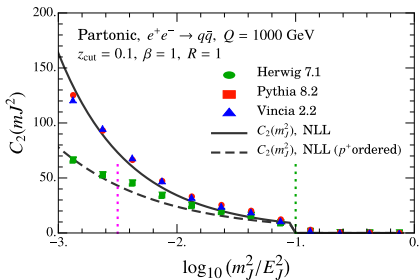
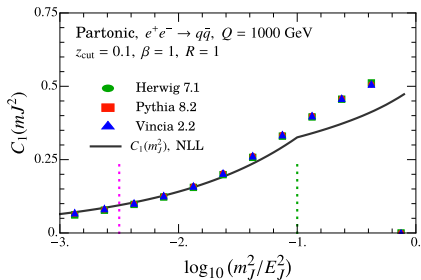
Tag an NP subjct with $E \lesssim 1$ GeV in the CA clustering tree of the groomed jet and apply the rescaling.

In the OPE region we find the expected geometry with $R \sim 1$



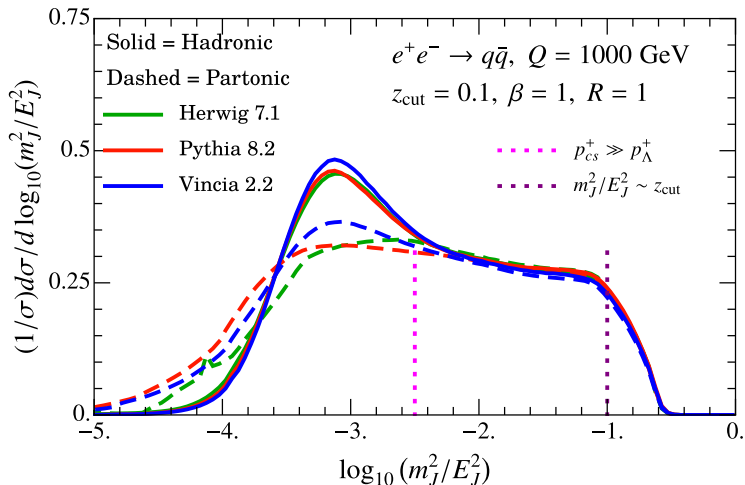
Comparing the NLL nonperturbative Wilson coefficients with Monte Carlos

NLL calculation of $C_1(m_J^2)$ and $C_2(m_J^2)$ agrees well with Monte Carlo.



Fitting for the power corrections in Monte Carlo

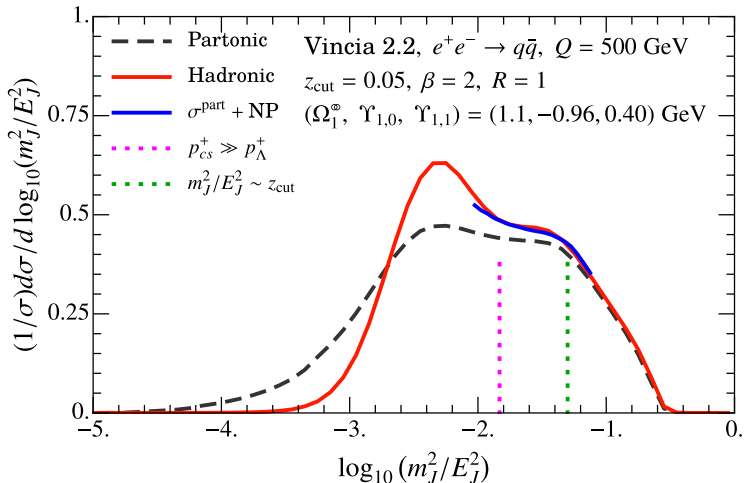
Monte Carlos have very different implementations of partonic resummation and hadronization model.



Fitting for the power corrections in Monte Carlo

Fit for the three hadronic parameters for the following grid in the SDOE region:

$$Q = 500, 1000 \text{ GeV}, z_{\text{cut}} = \{0.05, 0.1, 0.15, 0.2\}, \beta = \{0, 0.5, 1.0, 1.5, 2.0\}$$

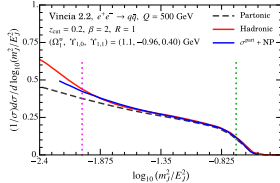
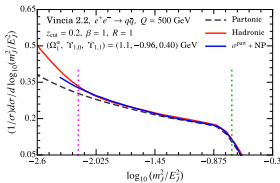
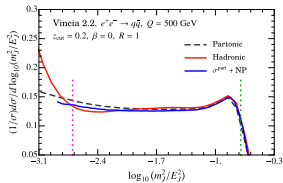
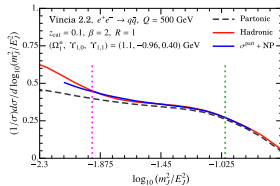
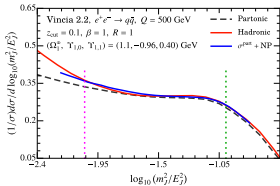
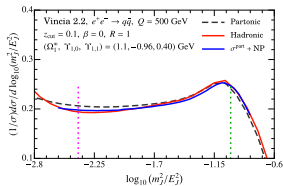


Fitting for the power corrections in Monte Carlo

Fit for the three hadronic parameters for the following grid in the SDOE region:

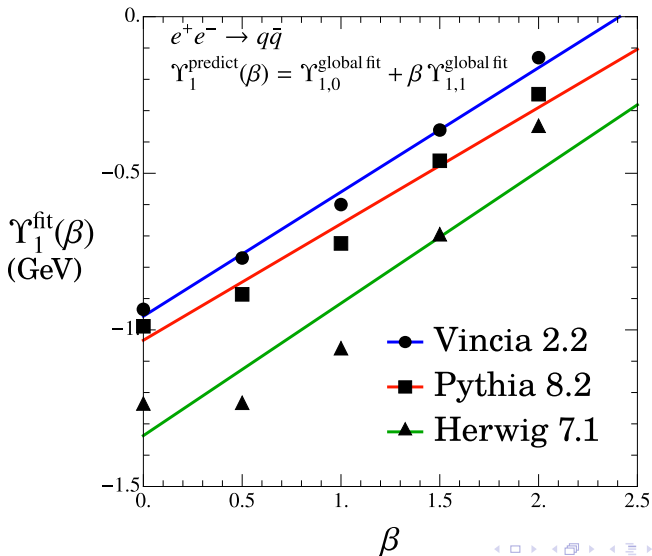
$$Q = 500, 1000 \text{ GeV}, z_{\text{cut}} = \{0.05, 0.1, 0.15, 0.2\}, \beta = \{0, 0.5, 1.0, 1.5, 2.0\}$$

3 universal parameters fit the whole grid well



Linear behavior of boundary correction

Fit for individual β 's using the Ω_1^\oplus from the global fit. Fits agree with prediction.



Conclusion

- QFT based treatment of power corrections to groomed observables. 3 Universal parameters
- Calculate the shape dependence of power corrections via Wilson Coefficients determined with Coherent Branching at NLL
- A unique way of characterizing the hadronization models of Monte Carlos
- A nontrivial cross check on Hadronization tunes that are based on simpler ungroomed observables.
- Enables precision measurements involving direct comparison of data with theory. [See Sonny Mantry's talk later today on top mass measurement]

Thank you.