SCET 2019, UC SAN DIEGO, US, MARCH 25-28 2019

INVESTIGATING TRANSVERSE MOMENTUM DISTRIBUTIONS WITH JETS

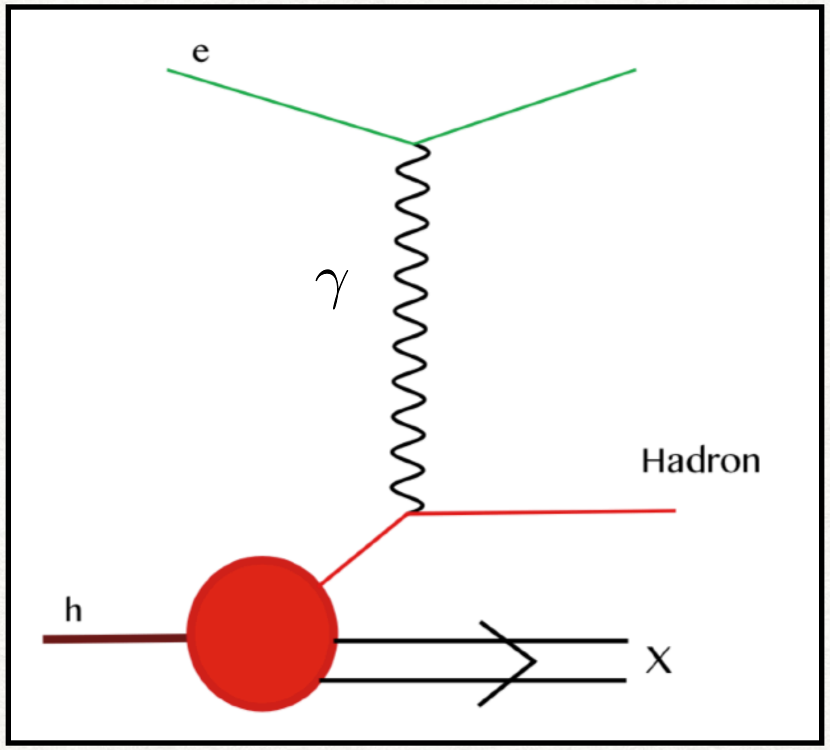
DANIEL GUTIÉRREZ REYES
UNIVERSIDAD COMPLUTENSE DE MADRID
(UCM AND IPARCOS)

WORK DONE IN COLLABORATION WITH

IGNAZIO SCIMEMI (UCM AND IPARCOS)
&
WOUTER J. WAALEWIJN (NIKHEF)
LORENZO ZOPPI (NIKHEF)

Based on:
PRL **121**, 162001(2018)
arXiv: 1807.07573
...and ongoing work

TMDS WITHOUT JETS

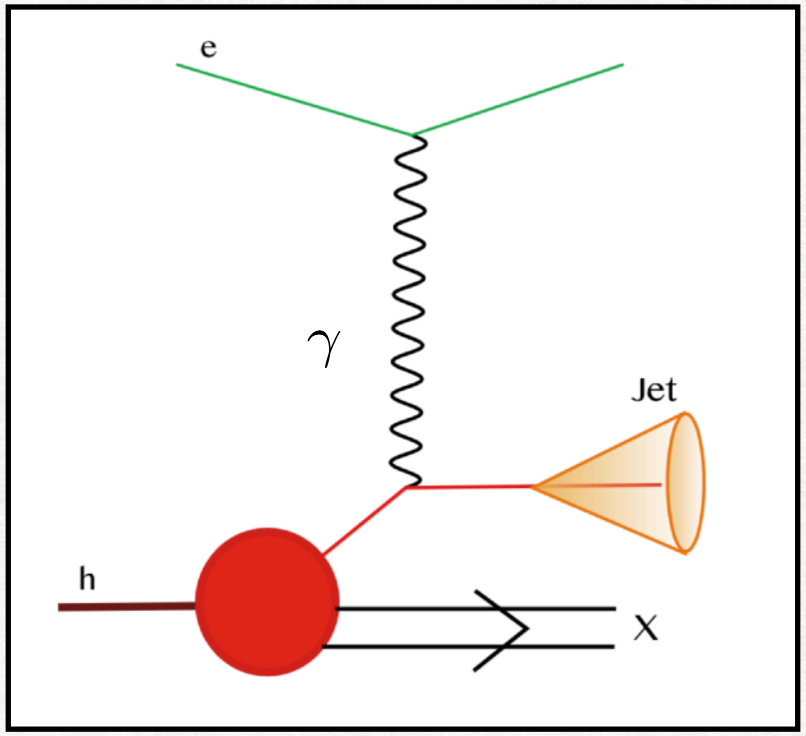


Factorization theorem for SIDIS

$$\frac{d\sigma_{eN\to eN'X}}{dQ^2dxdzd\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu)\int\frac{d\boldsymbol{b}}{(2\pi)^2}e^{-i\boldsymbol{b}\boldsymbol{q}}\underbrace{f_{a/N}(x,\boldsymbol{b},\mu,\zeta)}_{\text{TMDPDF}}\underbrace{d_{a/N'}(z,\boldsymbol{b},\mu,\zeta)}_{\text{TMDFF}}$$

TMDS WITH JETS

Can we obtain some information about TMDs in processes with jets in the final state?



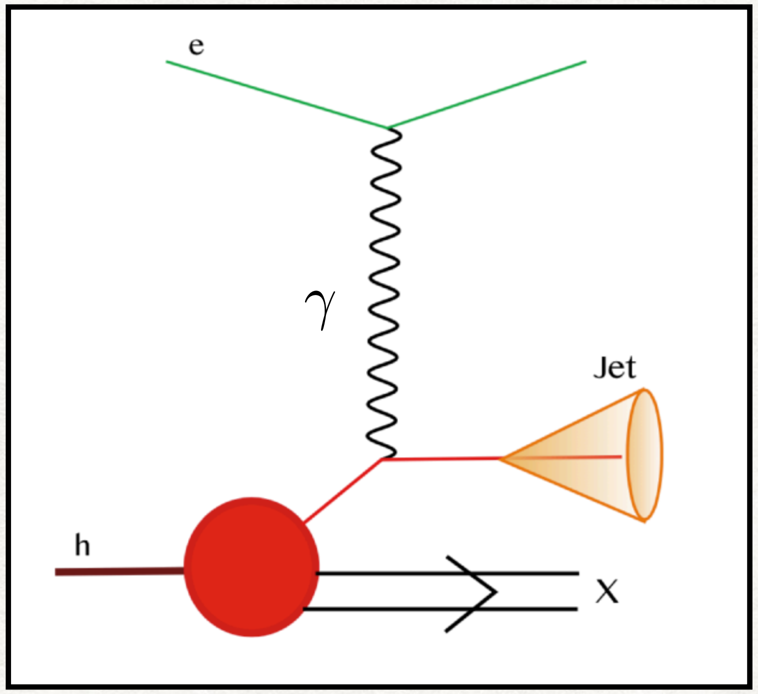
Factorization theorem for SIDIS with jet

$$\frac{d\sigma_{eN\to eJX}}{dQ^2dxdzd\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2}e^{-i\boldsymbol{bq}} f_{a/N}(x,\boldsymbol{b},\mu,\zeta) J_q^{\mathrm{axis}}(z,\boldsymbol{b},QR,\mu,\zeta)$$

TMDPDF — TMD jet function

TMDS WITH JETS

Can we obtain some information about TMDs in processes with jets in the final state?



Questions:

Can we write TMD factorization theorems for processes with jets?

Can we write factorization theorems for jets regardless of the size of the jet?

Factorization theorem for SIDIS with jet

$$\frac{d\sigma_{eN\to eJX}}{dQ^2dxdzd\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu)\int\frac{d\boldsymbol{b}}{(2\pi)^2}e^{-i\boldsymbol{bq}}f_{a/N}(x,\boldsymbol{b},\mu,\zeta)J_q^{\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta) \quad ?$$

TMDPDF — TMD jet function

Nonperturbative effects for jets are in principle more suppressed that for nuclear TMDs!
This would be a clean channel to measure nonperturbative effects in the initial nucleon

OUTLINE

- Building a jet
    - Radius and jet algorithms
    - Jet axis
- Factorization theorems for off-jet hadron measurements
    - TMD Factorization in a nutshell
    - Factorization for dijet decorrelation: regimes. The TMD semi inclusive jet function
    - Factorization for SIDIS + jet
    - Nonperturbative effects
- Phenomenology
- Conclusions and outlook

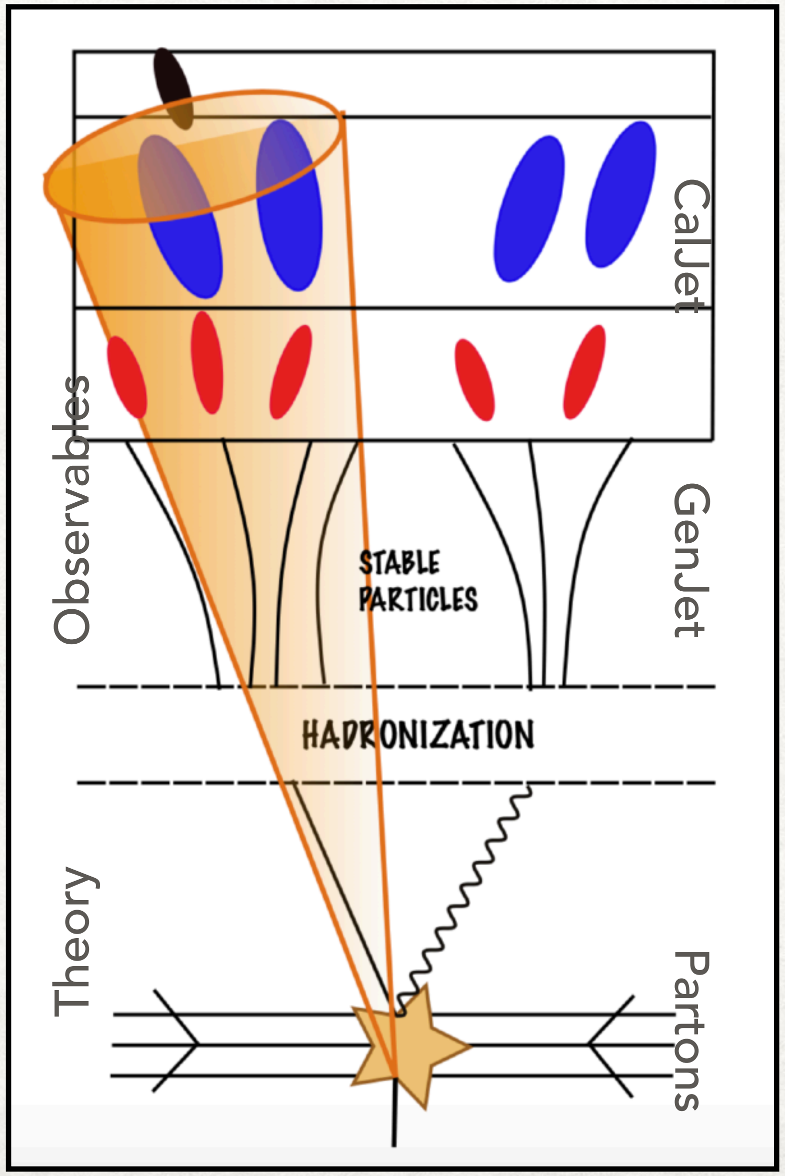
BUILDING A JET

RADIUS AND JET ALGORITHMS

Standard k_t-type algorithms

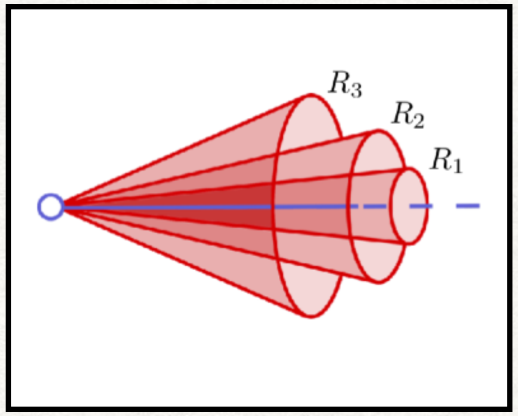
$$d_{ij} = \min\left(k_{T,1}^{2w}, k_{T,2}^{2w}\right) \frac{\Delta R_{ij}}{R}$$

$$w \in \{-1, 0, 1\}$$

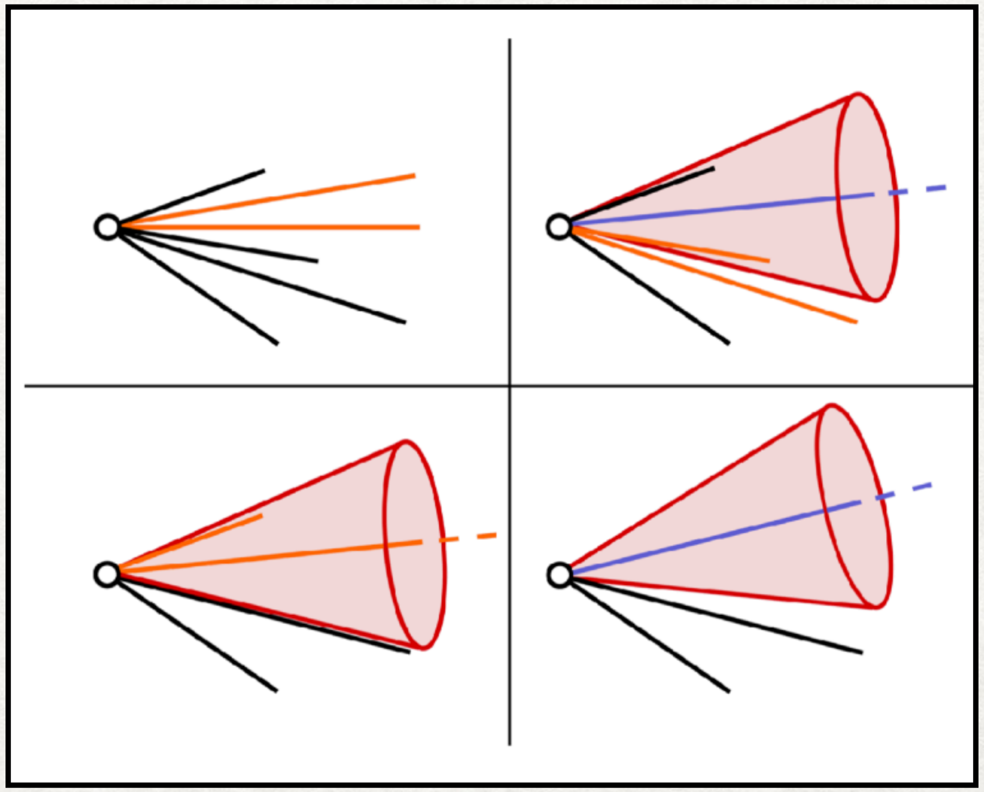


Building a jet…

Step 1: Set a size (Radius)



Step 2: Run a jet algorithm



JET AXIS

Larkoski, Neill, Thaler `14
arXiv: 1401.2158

Standard jet axis (SJA)

The sum of the momentum of collinear and soft particles is zero

Introduces soft-sensitivity to the axis definition. Important with unintegrated transverse momentum

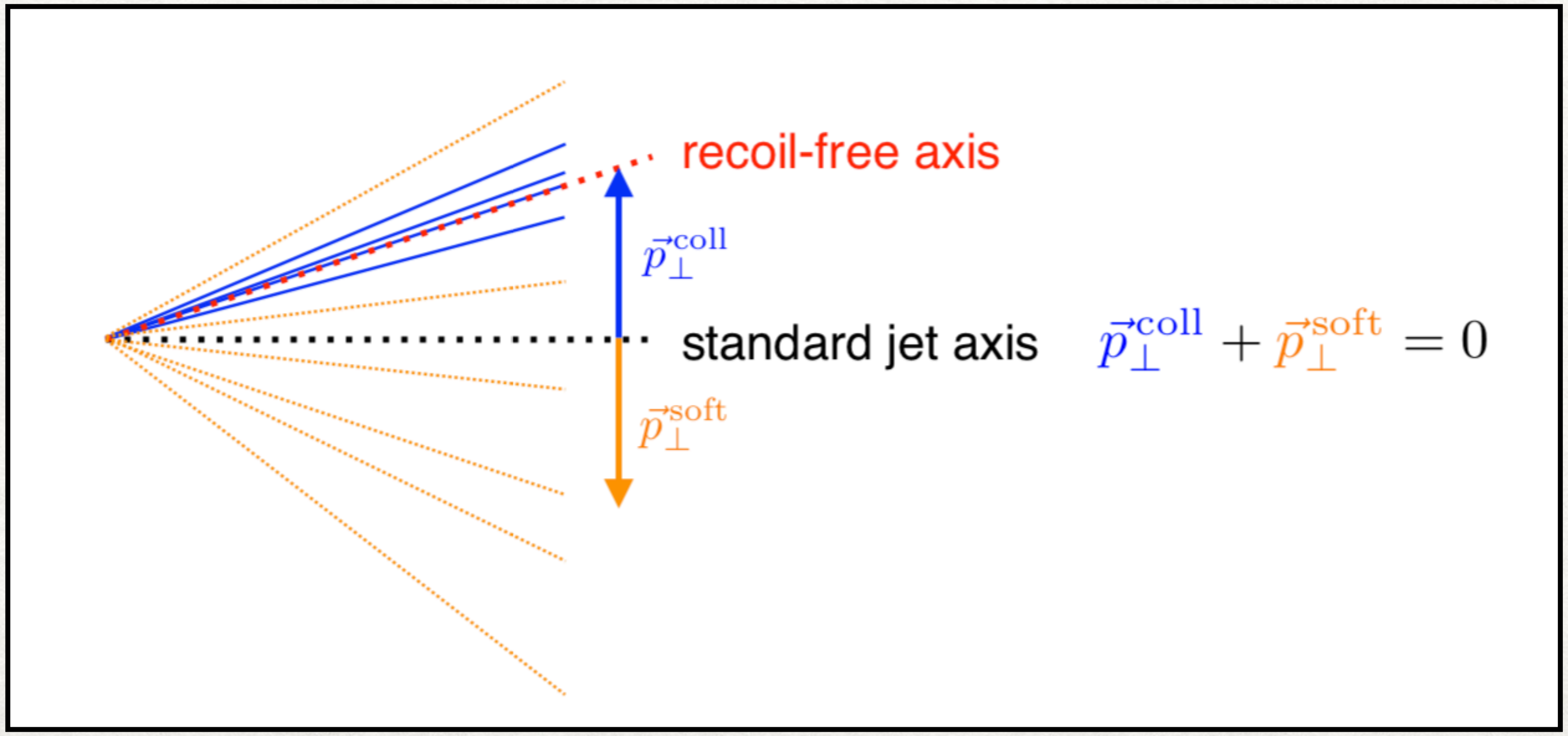
Invariant under splittings

Winner-take-all (WTA)

It always follows the direction of the most energetic particle

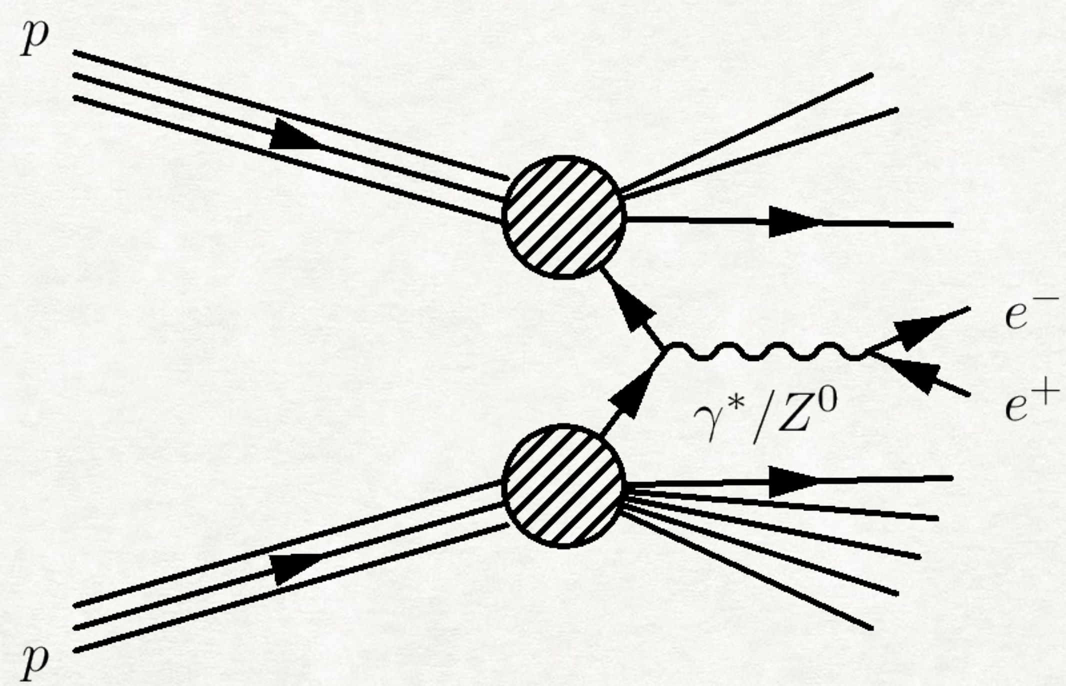
Recoil invariant. It is not sensitive to soft radiation

It varies its position under splittings



FACTORIZATION THEOREMS FOR OFF-JET HADRON MEASUREMENTS

TMD FACTORIZATION IN A NUTSHELL



Cross-section written as a product of two TMDs

Similar formulas are valid for SIDIS (EIC) and e+e-

TMDs have a double-scale evolution, associated to a particular kind of divergences: rapidity divergences.

See G. Vita's talk

We have new nonperturbative effects which cannot be included in PDFs.

Factorization theorems allow us to write cross sections as

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q) \sum_{ff'} z_{FF'}^{GG'} |C_V(q,\mu)|^2$$

$$\int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{b}\boldsymbol{q})} F_{f\leftarrow h_1}(x_1, \boldsymbol{b}; \mu, \zeta) F_{f'\leftarrow h_2}(x_2, \boldsymbol{b}; \mu, \zeta) + Y$$

Collins, Soper, Sterman `85
Nucl.Phys. B250, 199 (1985)

Collins `13
Foundations of perturbative QCD

Echevarría, Idilbi, Scimemi `11
arXiv:1111.4996

CRUCIAL ELEMENT FOR TMD FACTORIZATION: THE SOFT FUNCTION

$$\frac{d\sigma}{dQ^2dyd(q_T^2)} = H(Q^2,\mu)\int\frac{d^2\boldsymbol{b}}{4\pi}e^{i(\boldsymbol{bq})}F^{\text{BARE}}_{f\leftarrow h_1}(x_1,\boldsymbol{b};\mu,\delta^+)F^{\text{BARE}}_{f\leftarrow h_2}(x_2,\boldsymbol{b};\mu,\delta^-)S(\boldsymbol{b},\mu,\delta^+\delta^-)$$

Collinear modes + zero bin (overlap c/s)

Hard modes

ill defined! Rapidity divergences

Soft modes

The soft function renormalizes the rapidity divergences

$$S(\boldsymbol{b}) = \frac{\text{Tr}_{\text{color}}}{N_c}\langle 0|\left[S_n^{T\dagger}\tilde{S}_{\bar{n}}^T\right](\boldsymbol{b})\left[\tilde{S}_{\bar{n}}^{T\dagger}S_n^T\right](0)|0\rangle$$

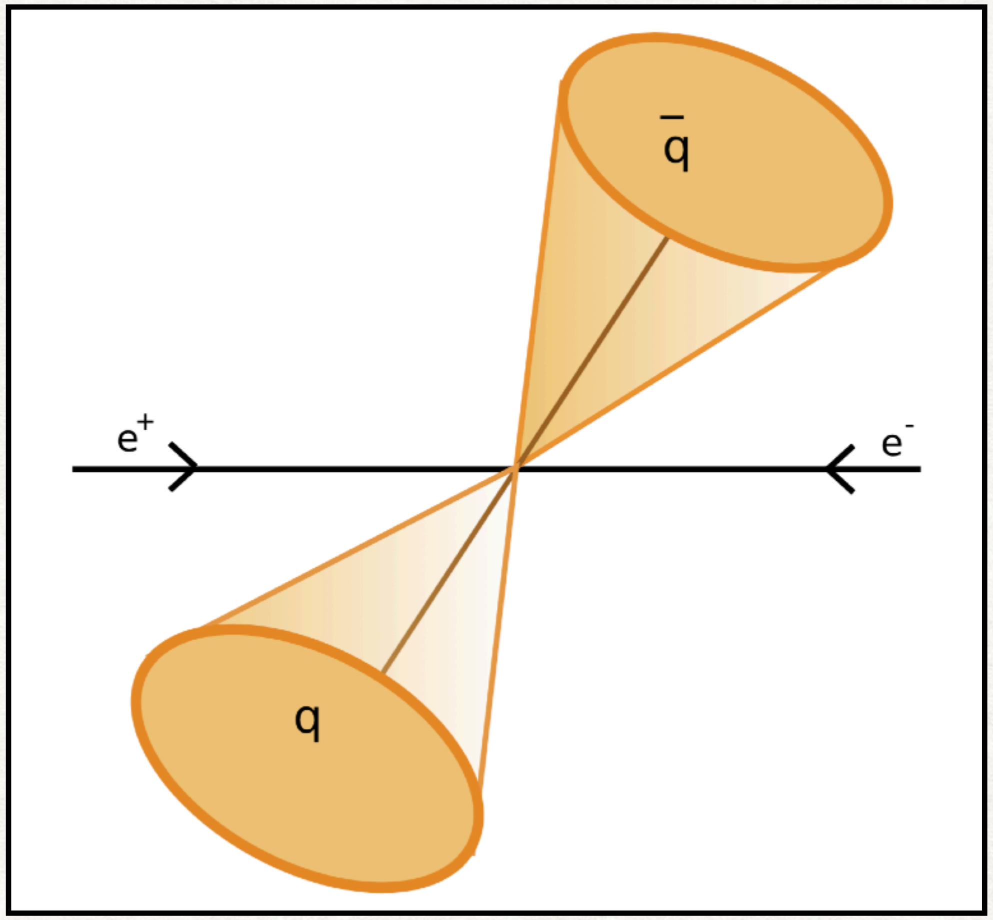
$$S(\boldsymbol{b}) = \sqrt{S(\boldsymbol{b},\zeta)}\sqrt{S(\boldsymbol{b},\zeta^{-1})}$$

Well defined TMDs!

FACTORIZATION FOR OFF-JET MEASUREMENTS

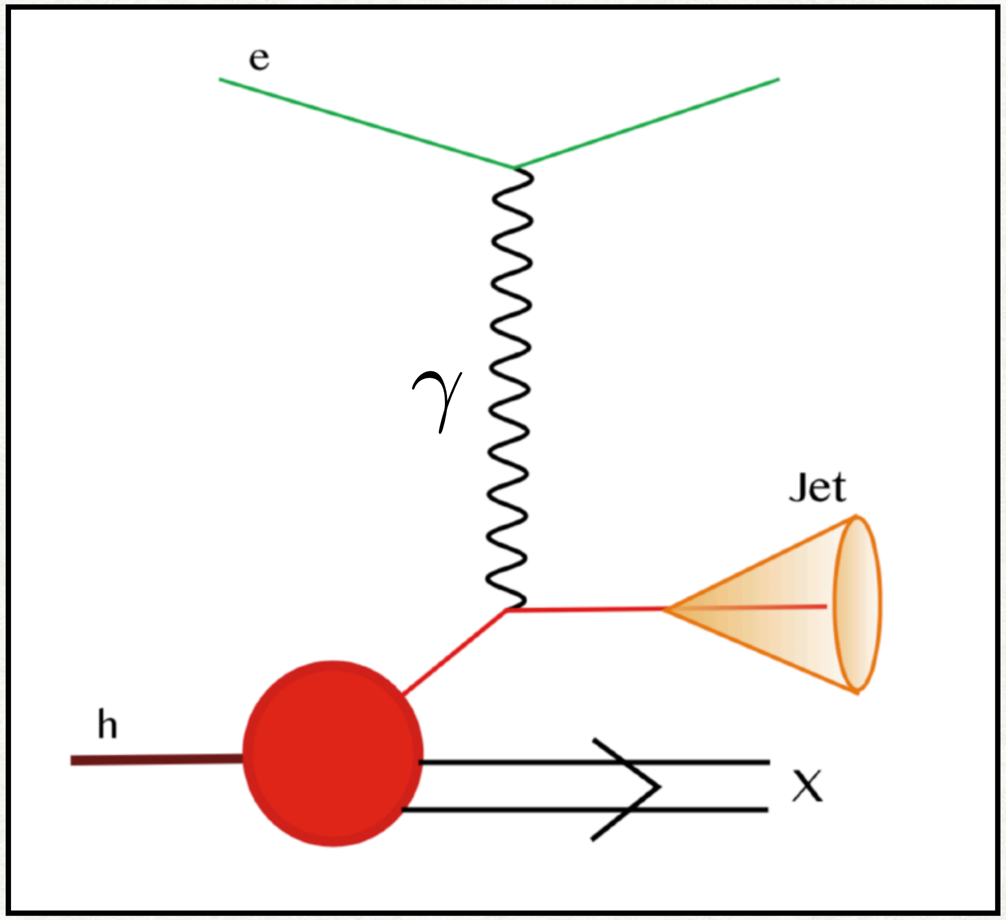
Interesting processes with jets in the final state

$e^+e^- \to \text{dijet} + X$



$$d\sigma \sim H \int d\boldsymbol{b} J_1 J_2 \ ?$$

SIDIS with jet



$$d\sigma \sim H \int d\boldsymbol{b} F J_1 \ ?$$

In which regimes do these cross-sections factorize?

FACTORIZATION FOR DIJET DECORRELATION

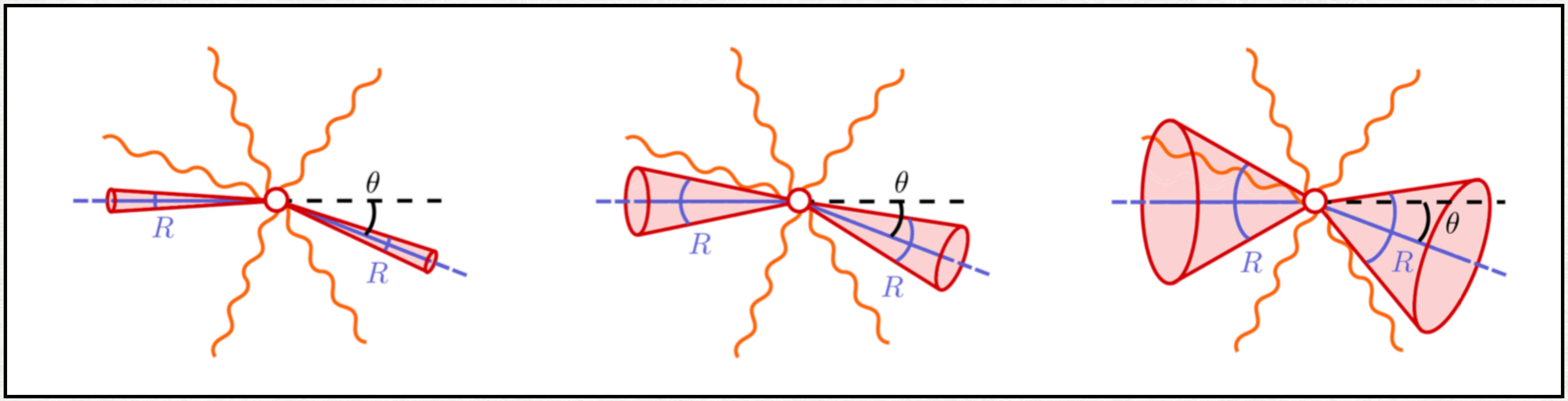
Transverse momentum decorrelation = $\dfrac{\text{Transverse momentum of the jet } (p_i)}{\text{Energy fraction of the jet } (z_i)}$

$$\boldsymbol{q} = \frac{\boldsymbol{p}_1}{z_1} + \frac{\boldsymbol{p}_2}{z_2}$$

$$\theta \approx \tan\theta = 2|\boldsymbol{q}|/Q$$

competes with R

In all cases $\theta \ll 1$



$\theta \gg R$ $\theta \sim R$ $\theta \ll R$

Dijet decorrelation

$e^+e^- \to \text{dijet} + X$

FACTORIZATION FOR DIJET DECORRELATION

Transverse momentum decorrelation = Transverse momentum of the jet (p_i) / Energy fraction of the jet (z_i)

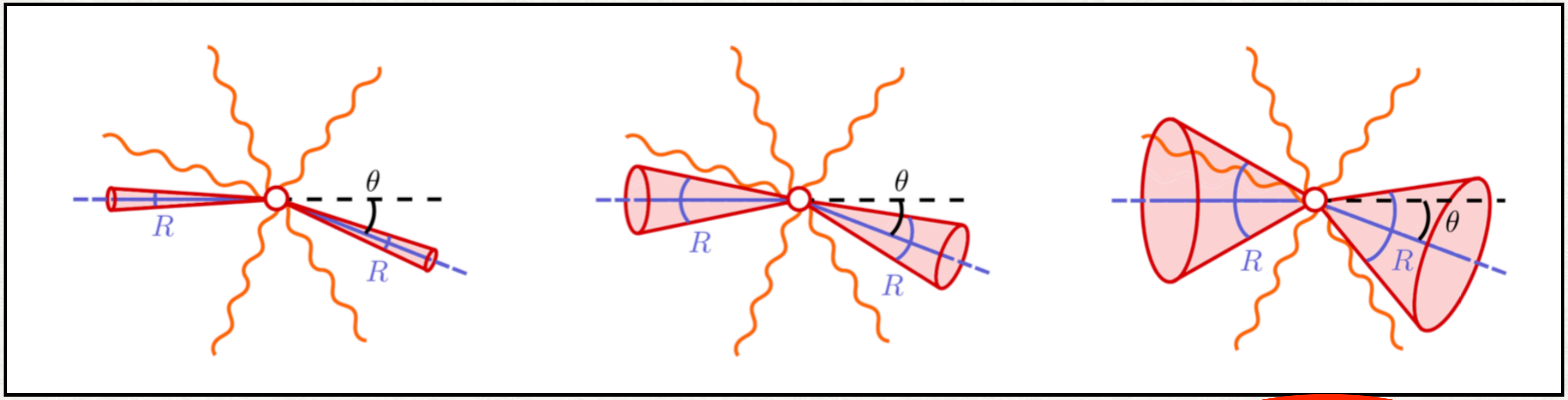
$$\boldsymbol{q} = \frac{\boldsymbol{p}_1}{z_1} + \frac{\boldsymbol{p}_2}{z_2}$$

$$\theta \approx \tan\theta = 2|\boldsymbol{q}|/Q$$

competes with R

In all cases

$$\theta \ll 1$$



$\theta \gg R$ | $\theta \sim R$ | $\theta \ll R$

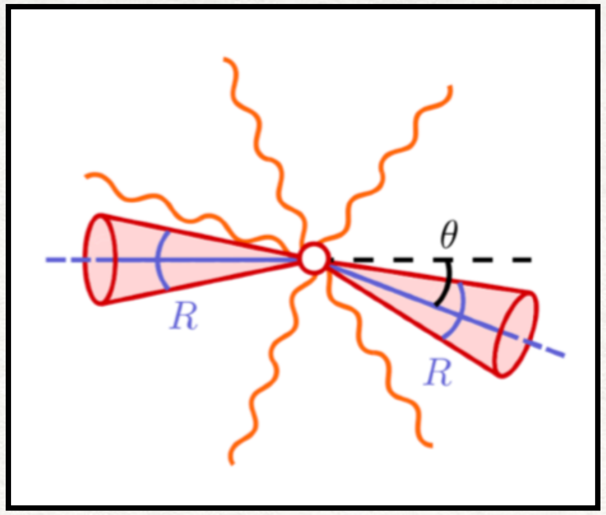
Dijet decorrelation

$$e^+e^- \to \text{dijet} + X$$

Most interesting case for current and future experiments!

FACTORIZATION

$$\theta \sim R \ll 1$$



$$\frac{d\sigma_{ee\to JJX}}{dz_1 dz_2 d\boldsymbol{q}} = H(s,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} J_q^{\rm axis}\left(z_1,\boldsymbol{b},\frac{\sqrt{s}}{2}R,\mu,\zeta_1\right) J_q^{\rm axis}\left(z_2,\boldsymbol{b},\frac{\sqrt{s}}{2}R,\mu,\zeta_2\right)\left[1+\mathcal{O}\left(\frac{\boldsymbol{q}^2}{s}\right)\right]$$

The interplay between θ and R must be taken into account in the jet function

↓

$$J_q^{\rm axis} = \frac{z}{2N_c}\sum_X {\rm Tr}\left\{\frac{\bar{\not{n}}}{2}\langle 0|\delta\left(\bar{n}\cdot p_J/z - \bar{n}\cdot P\right) e^{-i\boldsymbol{b}\boldsymbol{P}_\perp}\chi_n(0)|J^{\rm axis}_{{\rm alg},R}X\rangle\langle J^{\rm axis}_{{\rm alg},R}X|\bar{\chi}_n(0)|0\rangle\right\}$$

TMD semi-inclusive Jet function

The wide angle soft radiation does not resolve individual collinear emissions in the jet

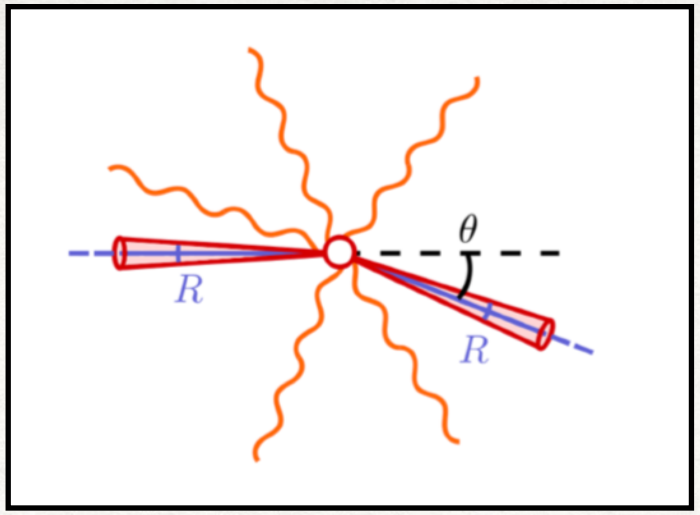
The soft function is the same That for TMD fragmentation

The TMD semi-inclusive jet function has the same RG Evolution that the TMDs!

It holds for both axis elections!

FACTORIZATION

$$R \ll \theta \ll 1$$



$$\frac{d\sigma_{ee\to JJX}}{dz_1 dz_2 d\boldsymbol{q}} = H(s,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} J_q^{\text{axis}}\left(z_1, \boldsymbol{b}, \frac{\sqrt{s}}{2}R, \mu, \zeta_1\right) J_q^{\text{axis}}\left(z_2, \boldsymbol{b}, \frac{\sqrt{s}}{2}R, \mu, \zeta_2\right)\left[1 + \mathcal{O}\left(\frac{\boldsymbol{q}^2}{s}\right)\right]$$

The jet function is refactorized

$$J_i^{\text{axis}} = \sum_j \int \frac{dz'}{z'} \left[(z')^2 \mathbb{C}(z', \boldsymbol{b}, \mu, \zeta)\right] \mathcal{J}_i\left(\frac{z}{z'}, \frac{\sqrt{s}}{2}R, \mu\right)\left[1 + \mathcal{O}\left(b^2 s^2 R^2/4\right)\right]$$

TMD semi-inclusive Jet function

TMDFFs Matching coefficients

$$D = C \otimes d$$

Echevarría, Scimemi, Vladimirov \`16
arXiv:1604.07869

$\otimes$

Collinear jet Function

Kang, Ringer, Vitev \`16
arXiv:1606.06732

→ All is independent on the axis election!

The wide angle soft radiation does not resolve individual collinear emissions in the jet

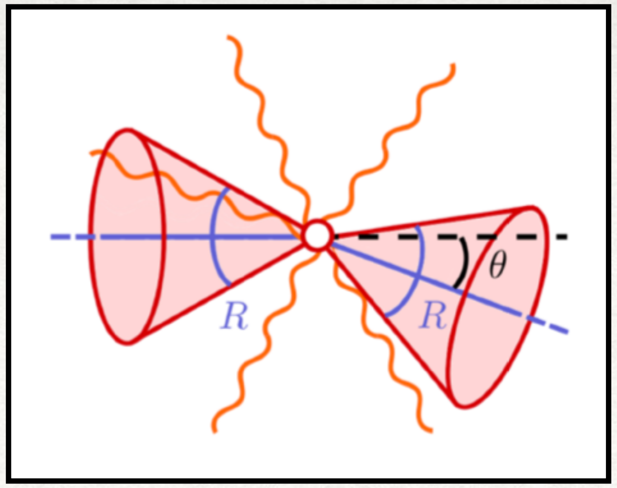
The soft function is the same that for TMD fragmentation

The TMD semi-inclusive jet function has the same RG Evolution that the TMDs!

FACTORIZATION

$$\theta \ll 1 \qquad \theta \ll R$$

It corresponds to the so called two-hemisphere jet function



Interesting case for Belle, BaBar. At low energies jets with big radius appear

Strong dependence on the axis election!

SJA

The SJA is aligned with the total momentum of the jet

↓

Hard splittings with typical angle R are allowed inside the jet, generating additional soft radiation

↓

Factorization broken!

$$\frac{d\sigma^{\text{SJA}}_{(ee\to JJX)}}{d\boldsymbol{q}} = \sum_{m=2}^{\infty} \text{Tr}_c \left[\mathcal{H}_m(\{n_i\}) \otimes \mathcal{S}_m(\boldsymbol{q}, \{n_i\})\right]$$

WTA

The soft radiation does resolve the jet boundary, but this fact does not affect to the position of the axis

↓

No distinction between soft radiation inside and outside the jet!

↓

TMD Soft function is conserved!
And factorization holds!

$$\frac{d\sigma^{\text{WTA}}_{(ee\to JJX)}}{dz_1 dz_2 d\boldsymbol{q}} = H \int \frac{d\boldsymbol{b}}{(2\pi)^2} J_q^{WTA}(z_1, \boldsymbol{b}) J_q^{WTA}(z_2, \boldsymbol{b})$$

Jet functions should be written in this limit…

TMD SEMI-INCLUSIVE JET FUNCTION AT NLO

$\theta \ll 1$

$\theta \ll R$

Collinear radiation of typical angle **θ** sees the jet boundary infinitely far away

↓

Independence of the radius of the jet!

The collinear radiation is mostly inside the jet

↓

The dependence on z is power suppressed!

$$J_i^{\mathrm{WTA}}(z, \boldsymbol{b}, ER, \mu, \zeta) = \delta(1-z)\,\mathscr{J}_i^{\mathrm{WTA}}(\boldsymbol{b}, \mu, \zeta)\left[1 + \mathcal{O}\left(\frac{1}{\boldsymbol{b}^2 E^2 R^2}\right)\right]$$

where

$$\mathscr{J}_i^{\mathrm{WTA}}(\boldsymbol{b}, \mu, \zeta) = \frac{1}{2N_c(\bar{n}\cdot p_J)}\mathrm{Tr}\left\{\frac{\not{\bar{n}}}{2}\langle 0|e^{-i\boldsymbol{b}\boldsymbol{P}_\perp}\chi_n(0)|J_{\mathrm{alg}}^{\mathrm{WTA}}\rangle\langle J_{\mathrm{alg}}^{\mathrm{WTA}}|\bar{\chi}_n(0)|0\rangle\right\}$$

$$\mathscr{J}_i^{[0]\mathrm{WTA}}(\boldsymbol{b}, \mu, \zeta) = 1$$

$$\mathscr{J}_i^{[1]\mathrm{WTA}}(\boldsymbol{b}, \mu, \zeta) = 2\left\{N_i + L_\mu\left[\mathcal{C}_i' + \mathcal{C}_i\left(\mathbf{l}_\zeta - \frac{1}{2}L_\mu\right)\right]\right\}$$

$$N_q = C_F\left(\frac{7}{2} - \frac{5\pi^2}{12} - 3\ln 2\right)$$

$$N_g = C_A\left(\frac{131}{36} - \frac{5\pi^2}{12}\right) - \frac{17}{18}n_f T_R - \beta_0 \ln 2$$

TMD SEMI-INCLUSIVE JET FUNCTION AT NNLO

We know the evolution of the TMD jet function and in this limit it does not depend on radius or z

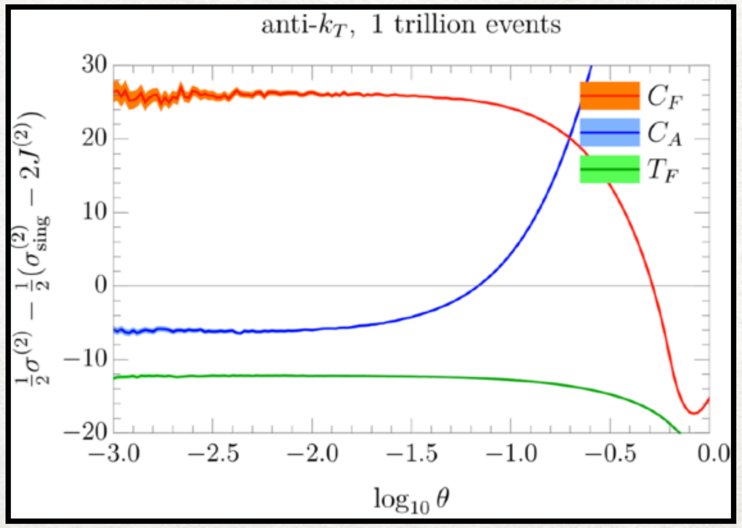
↓

We can predict the log behavior of the two-loop jet function solving renormalization group equations and the constant can be numerically calculated (EVENT2)

$$\theta \ll 1$$
$$\theta \ll R$$

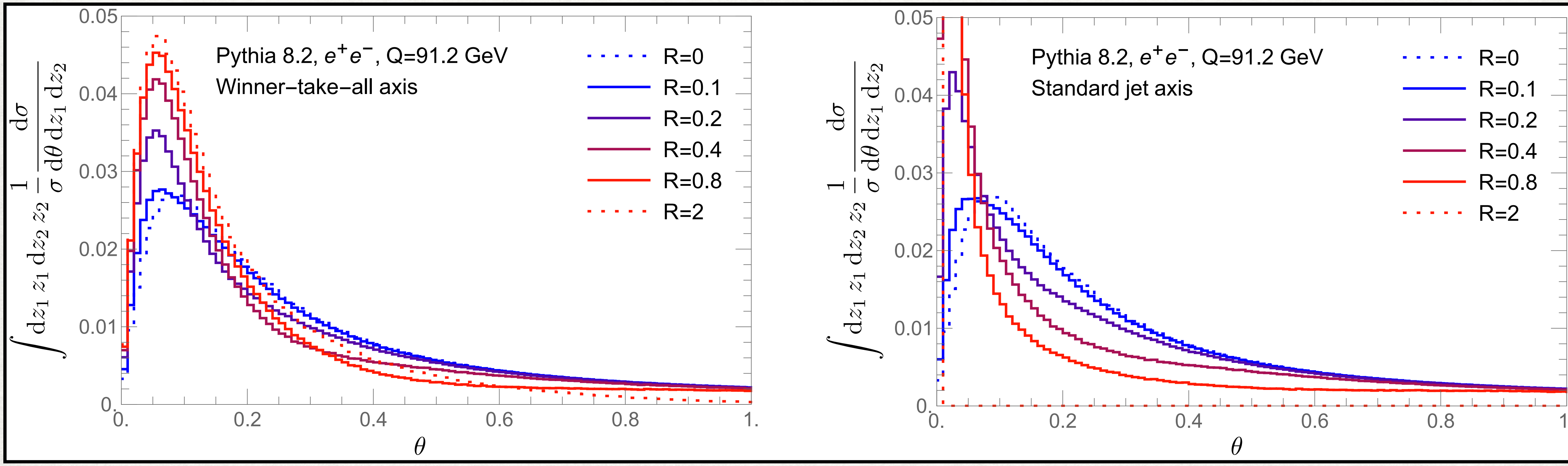
Predicted by RG equations ↑ C_{kl} — Predicted by EVENT2 ↑ C_0

$$\mathscr{J}^{[2]\mathrm{WTA}}(\boldsymbol{b},\mu,\zeta) = \sum_{k=1}^{4}\sum_{l=0}^{k} C_{kl} L_\mu^k \mathbf{l}_\zeta^l + C_0$$



$$C_0 = j_{C_F} + j_{C_A} + n_f j_{T_F}$$

CHECKING WITH PYTHIA 8.2.



Cross-section of angular decorrelation for different values of the radii of the jets

For small values of R the cross-section for both axis elections agrees!

For big values of R the cross section in SJA is inconsistent!

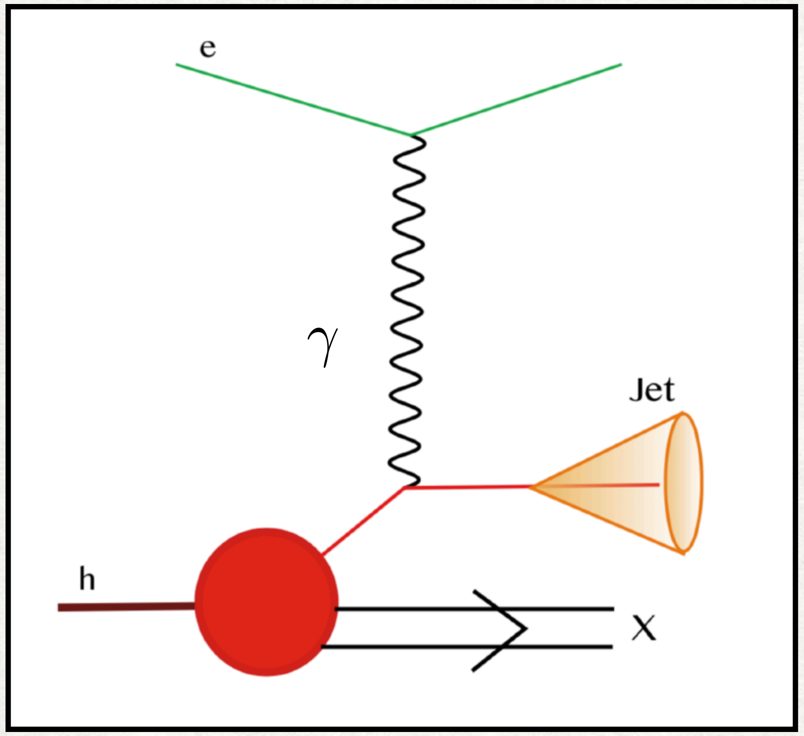
Factorization is broken

WTA axis solves the problems!

MEASURING TMDS WITH JETS - SIDIS CASE

With WTA axis factorization theorems hold in all cases

Let us apply it to SIDIS with jet!



Factorization theorem for SIDIS

$$\frac{d\sigma_{eN\to eN'X}}{dQ^2 dx dz d\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} f_{a/N}(x,\boldsymbol{b},\mu,\zeta) d_{a/N'}(z,\boldsymbol{b},\mu,\zeta)$$

TMDPDF   TMDFF

It has no dependence on the jet radius. Usual TMDs TMDFFs have nonperturbative effects important and poorly studied

Factorization theorem for SIDIS with jet

$$\frac{d\sigma_{eN\to eJX}}{dQ^2 dx dz d\boldsymbol{q}} = \sum_a \mathcal{H}_a(Q^2,\mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} f_{a/N}(x,\boldsymbol{b},\mu,\zeta) J_q^{\text{axis}}(z,\boldsymbol{b},QR/2,\mu,\zeta)$$

TMDPDF   TMD jet function

It has dependence on the jet radius. For all R we use WTA axis

Nonperturbative effects for jets are in principle suppressed! Clean channel to measure nonperturbative effects in the initial nucleon! Next slide…

NONPERTURBATIVE EFFECTS

Jets are perturbatively calculable objects

They eliminate final state hadronization NP-effects that appear (and are poorly studied) in TMDFFs

Jets obtain nonperturbative effects

Non perturbative effects of factorization formulas at low momentum

Uncertainties in the measurement of the jet axis position.

Hadronization effects can be reduce with Groomed Jets See Y. Makris' talk

But we still have some advantages and something to do…

Jets have no matching with NP-collinear objects

↓

We eliminate this source of uncertainty

These effects can be quantified: Fits to NP simple models in e+e- and applied to SIDIS

Last words in this aspect

Comparison with data!

NONPERTURBATIVE EFFECTS

Jets are perturbatively calculable objects

They eliminate final state hadronization NP-effects that appear (and are poorly studied) in TMDFFs

TOO NAIVE!

Jets obtain nonperturbative effects

Non perturbative effects of factorization formulas at low momentum

Uncertainties in the measurement of the jet axis position.

Hadronization effects can be reduce with Groomed Jets See Y. Makris' talk

But we still have some advantages and something to do…

Jets have no matching with NP-collinear objects

↓

We eliminate this source of uncertainty

These effects can be quantified: Fits to NP simple models in e+e- and applied to SIDIS

Last words in this aspect

Comparison with data!

PHENOMENOLOGY

DOUBLE-SCALE RENORMALIZATION GROUP EVOLUTION

Same RG evolution for hadronic TMDs and for TMD jet functions!

TMDs

$$\mu^2 \frac{d}{d\mu^2} D_i(z, \boldsymbol{b}, \mu, \zeta) = \frac{1}{2}\gamma_F^i(\mu, \zeta) D_i(z, \boldsymbol{b}, \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} D_i(z, \boldsymbol{b}, \mu, \zeta) = -\mathcal{D}^i(\mu, \boldsymbol{b}) D_i(z, \boldsymbol{b}, \mu, \zeta)$$

Jets

$$\mu^2 \frac{d}{d\mu^2} J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta) = \frac{1}{2}\gamma_F^i(\mu, \zeta) J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta) = -\mathcal{D}^i(\mu, \boldsymbol{b}) J_i^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta)$$

DOUBLE-SCALE RENORMALIZATION GROUP EVOLUTION

They have a common evolution factor

TMDs

$$D_i(z,\boldsymbol{b},\mu_f,\zeta_f) = \exp\left[\int_{(\mu_i,\zeta_i)}^{(\mu_f,\zeta_f)} \left(\gamma_F^i(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^i(\mu,\boldsymbol{b})\frac{d\zeta}{\zeta}\right)\right] D_i(z,\boldsymbol{b},\mu_i,\zeta_i)$$

Jets

$$J_i^{\text{axis}}(z,\boldsymbol{b},QR,\mu_f,\zeta_f) = \exp\left[\int_{(\mu_i,\zeta_i)}^{(\mu_f,\zeta_f)} \left(\gamma_F^i(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}^i(\mu,\boldsymbol{b})\frac{d\zeta}{\zeta}\right)\right] J_i^{\text{axis}}(z,\boldsymbol{b},QR,\mu_i,\zeta_i)$$

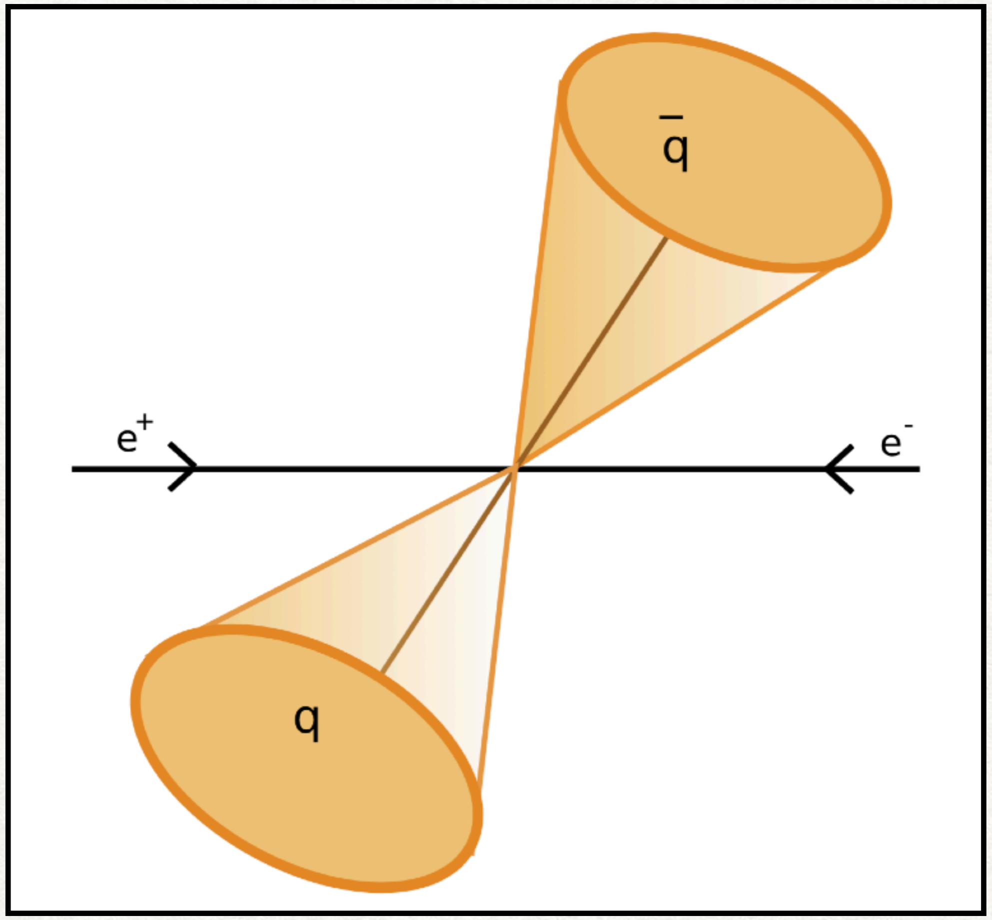
This fact makes phenomenological analysis simpler!

arTeMiDe

Scimemi, Vladimirov `17

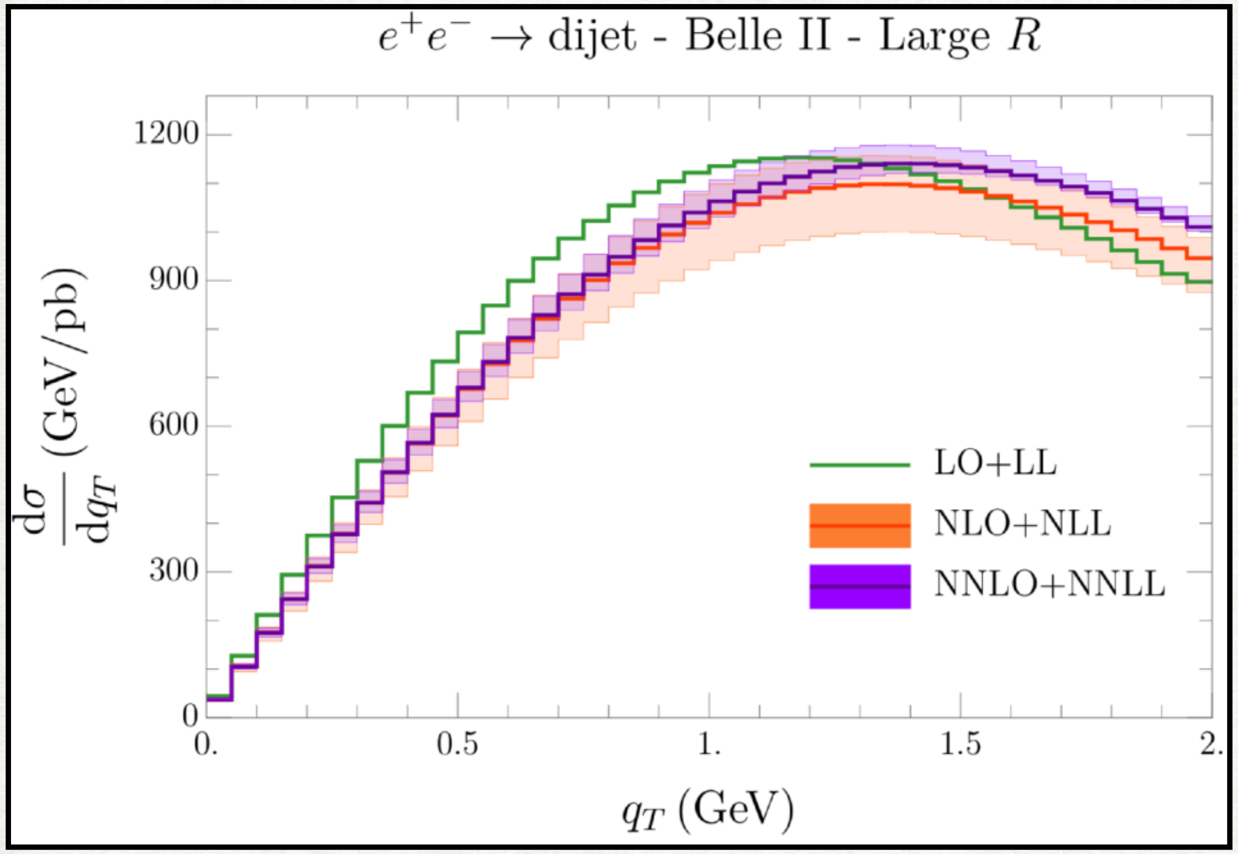
https://teorica.fis.ucm.es/artemide/

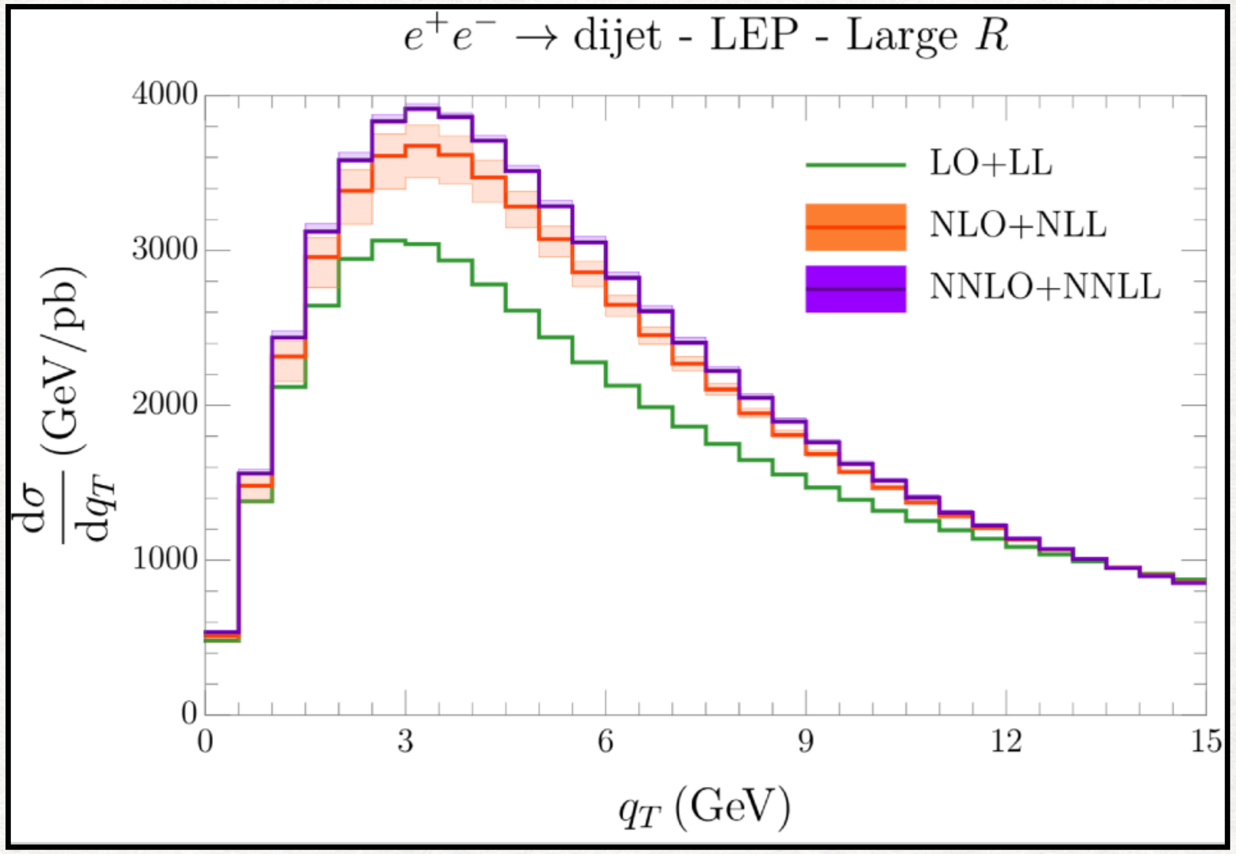
$$e^+e^- \to \text{dijet} + X$$



$$d\sigma \sim H \int d\boldsymbol{b} J_1 J_2$$

PERTURBATIVE CONVERGENCE: LARGE RADIUS





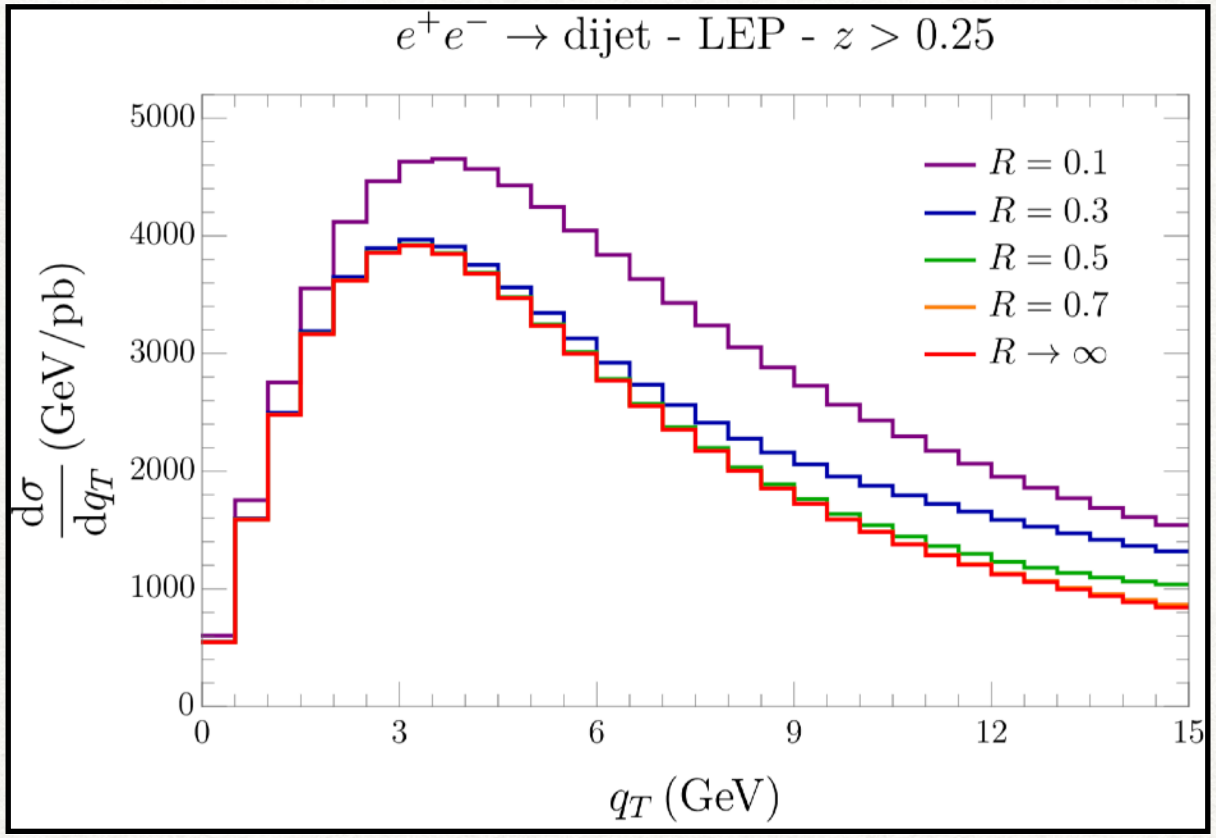
As the jet function does not depend on radius or z, we can predict the two loop jet function by RG + Numerical constant

We have NNLO+NNLL!

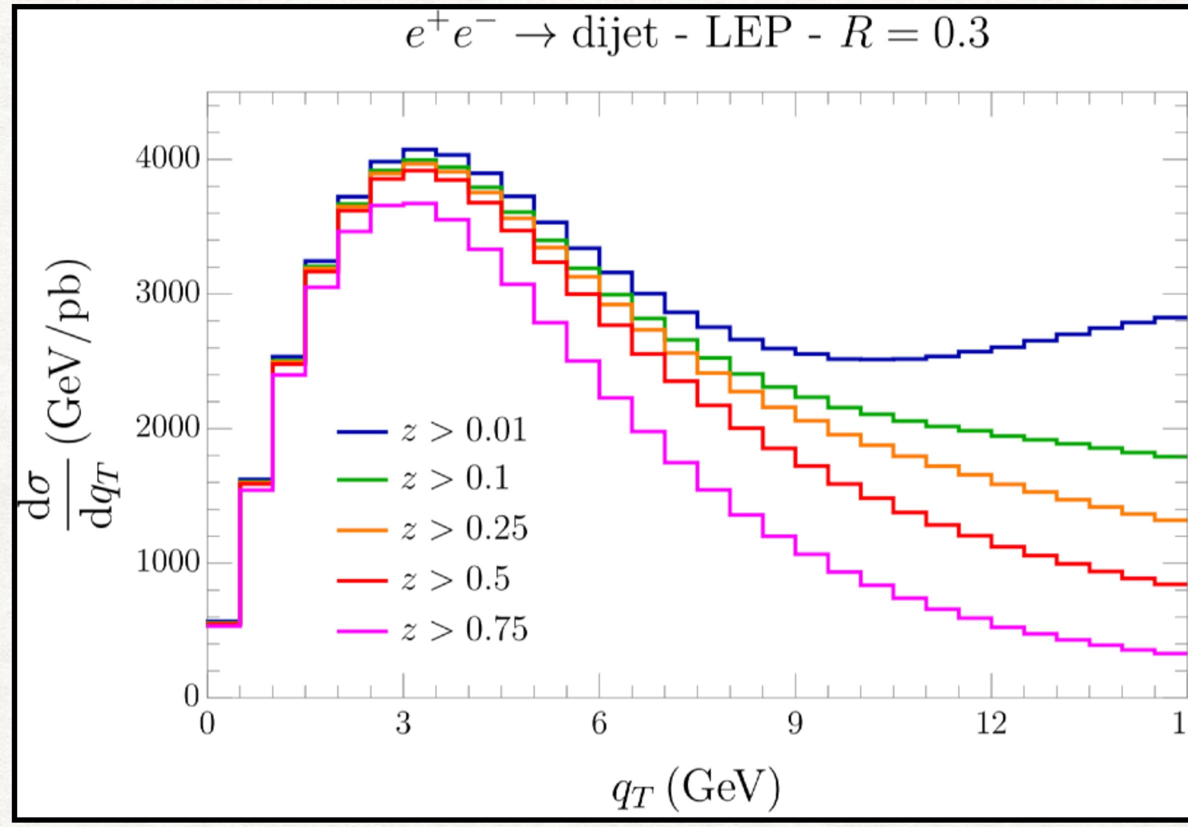
Theoretical errors are reduced when the perturbative order is increased!

Using large R approximation we have some predictions for low energy experiments!

VARY RADIUS AND Z DEPENDENCE: LARGE-R VS FINITE R



Constant z vary R



Constant R vary z

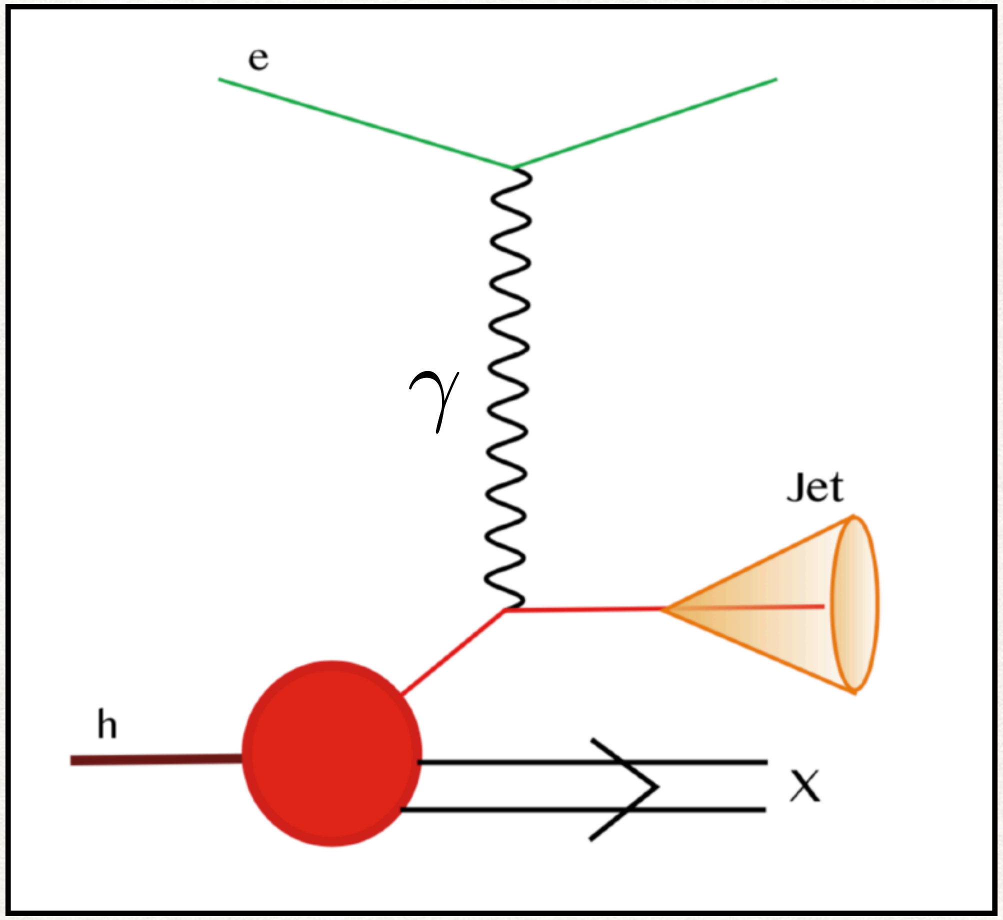
The large radius approximation is a very accurate approximation for jet functions with finite radius (but not so small)

This fact allow us to skip some of the technical complications of the finite radius jet function

The case with z > 0.5 gives the same result that the large R case

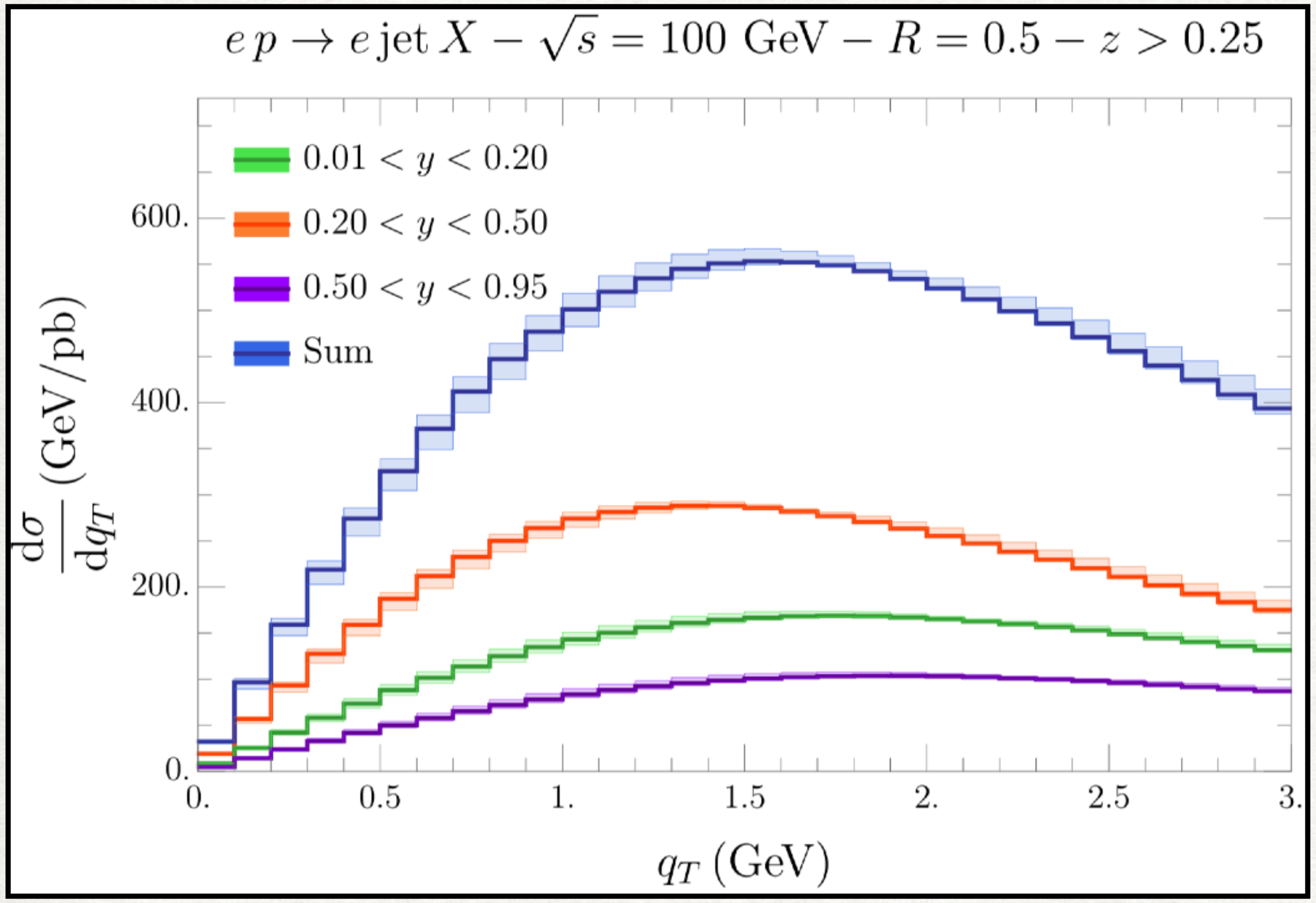
One loop effect!

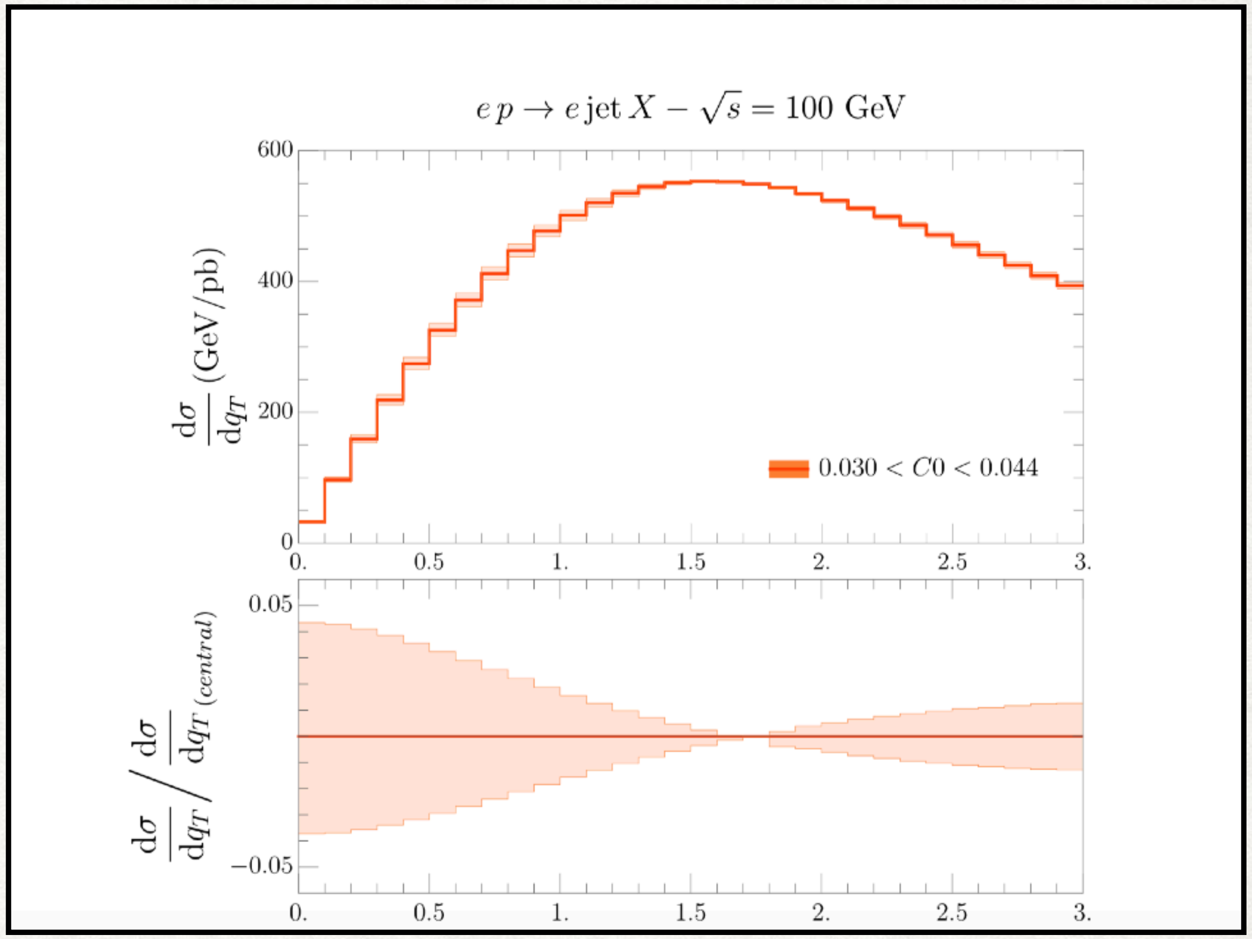
SIDIS with jet



$$d\sigma \sim H \int d\boldsymbol{b} F J_1$$

PHENO RESULTS FOR SIDIS





We include an effect of the two-loop jet function

Most of the cross section comes from low elasticity region

Theoretical errors from the Hard scale and the OPE scale are shown and are small

Non perturbative model for the TMDPDF taken from Bertone, Scimemi, Vladimirov \`19 arXiv:1902.08474

$$f_{NP}(x, \boldsymbol{b}) = \exp\left(-\frac{(\lambda_1(1-x) + \lambda_2 x + \lambda_3 x(1-x))\boldsymbol{b}^2}{\sqrt{1 + \lambda_4 x^{\lambda_5} \boldsymbol{b}^2}}\right)$$

But the cross section is dominated by nonperturbative parameter of evolution

$$\mathcal{D}(\mu, \boldsymbol{b}) = \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'}\Gamma + \mathcal{D}_{\text{pert}}(\mu_0, \boldsymbol{b}) + c_0 \boldsymbol{b}\boldsymbol{b}^*$$

CONCLUSIONS AND OUTLOOK

CONCLUSIONS AND OUTLOOK

To study jets we have to take into account some particularities that we have not for hadronic TMD structure functions

Choice of the axis | Choice of the algorithm | Radius

In general, the factorization theorems depend on the size of the radius of the jet

With WTA axis election the Soft function is the same that for hadronic TMD structure functions in all the cases

Hadronic TMDs and TMD jet functions share the same double-scale RG evolution

Phenomenological applications are simplified!
Same evolution factor that for TMDs
We have phenomenological results to be published very soon!

arTeMiDe

Nonperturbative effects in jets are more suppressed than in TMD fragmentation functions where they are poorly studied

Checks with experimental data will have the last word!

CONCLUSIONS AND OUTLOOK

To study jets we have to take into account some particularities that we have not for hadronic TMD structure functions

Choice of the axis — Choice of the algorithm — Radius

In general, the factorization theorems depend on the size of the radius of the jet

With WTA axis election the Soft function is the same that for hadronic TMD structure functions in all the cases

Hadronic TMDs and TMD jet functions share the same double-scale RG evolution

STAY TUNED

Phenomenological applications are simplified!
Same evolution factor that for TMDs
We have phenomenological results to be published very soon!

arTeMiDe

Nonperturbative effects in jets are more suppressed than in TMD fragmentation functions where they are poorly studied

Checks with experimental data will have the last word!

Thanks!!!

BACKUP SLIDES

DEPENDENCE ON THE RADIUS. EVOLUTION

Jets depend explicitly on the radius

The RG evolution of the jet function the same that for the hadronic fragmentation functions!

For small radii, large logarithms ln R appear, but can be resummed with evolution equations

JET: $$\mu^2 \frac{d}{d\mu^2} \mathcal{J}_i(z, Q, R, \mu) = \sum_j \gamma_{ji}^{\mathcal{J}}(z, \mu) \otimes \mathcal{J}_i(z, Q, R, \mu)$$

$$\gamma_{ji}^{\mathcal{J}}(z, \mu) = \frac{\alpha_s}{\pi} P_{ji}(z, \mu)$$

Same AD that for usual FFs!

FFs: $$\mu^2 \frac{d}{d\mu^2} D_i^h(z, \mu) = \frac{\alpha_s}{\pi} \sum_j P_{ji}(z, \mu) \otimes D_j^h(z, \mu)$$

DELTA REGULARIZATION

$$W_n = P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)e^{-\delta\sigma x}\right)$$

$$S_n = P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)\right) \to P\exp\left(-ig\int_0^\infty d\sigma (n\cdot A)(n\sigma)e^{-\delta\sigma}\right)$$

At diagram level → Eikonal propagators

$$\frac{1}{(k_1^+ + i0)(k_1^+ + k_2^+ + i0)...(k_1^+ + ... + k_n^+ + i0)} \to \frac{1}{(k_1^+ + i\delta)(k_1^+ + k_2^+ + 2i\delta)...(k_1^+ + ... + k_n^+ + ni\delta)}$$

Non-abelian exponentiation satisfied at all orders!

δ-regularization violates gauge properties of WL by power suppressed in δ terms

Only calculation at $\delta \to 0$ is legitimate!

This regularization makes zero-bin equal to soft factor

R-factor is scheme dependent!

$$R = \frac{\sqrt{S(\boldsymbol{b})}}{\text{zero-bin}} \xrightarrow{\delta-\text{reg.}} R_{\delta-\text{reg.}} = \frac{1}{\sqrt{S(\boldsymbol{b})}}$$

TMD SEMI-INCLUSIVE JET FUNCTION AT NLO

The Soft function is the same that for TMDs in some cases for SJA and in ALL cases for WTA

The Semi-inclusive jet function is renormalized as a TMD

$$J_q^{\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta) = Z_{UV}(\mu,\epsilon)R_q(\delta,\zeta,\epsilon)J_q^{\text{axis},B}(z,\boldsymbol{b},QR,\mu,\delta)$$

$$\theta \ll 1$$
$$\theta \sim R$$

The definition of the operator is the usual one

$$J_i^{\text{axis}} = \sum_{n=0}^{\infty} a_s^n J_i^{[n]\text{axis}}$$

$$J_i^{[0]\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta) = \delta(1-z)$$

$$J_i^{[1]\text{axis}}(z,\boldsymbol{b},QR,\mu,\zeta) = 2\left(\sum_j c_{ji}p_{ji}\right)\left[L_R - L_\mu - 2\ln(1-z) + \frac{1}{4}|\boldsymbol{b}|^2Q^2R^2(1-z)^2\right.$$
$$\left.\times {}_2F_3\left(\{1,1\},\{2,2,2\};-\frac{1}{4}|\boldsymbol{b}|^2Q^2R^2(1-z)^2\right)\right] + \delta(1-z)\left[2\mathcal{C}_i' L_R - \mathcal{C}_i L_\mu^2 + 2\mathcal{C}_i L_\mu \mathbf{l}_\zeta + 2\tilde{d}_i^{\text{axis}}(\boldsymbol{b}QR)\right]$$

The dependence on the axis is only here!

TMD SEMI-INCLUSIVE JET FUNCTION AT NLO

The Soft function is the same that for TMDs in some cases for SJA and in ALL cases for WTA

The Semi-inclusive jet function is renormalized as a TMD

$$J_q^{\text{axis}}(z, \boldsymbol{b}, QR, \mu, \zeta) = Z_{UV}(\mu, \epsilon) R_q(\delta, \zeta, \epsilon) J_q^{\text{axis},B}(z, \boldsymbol{b}, QR, \mu, \delta)$$

$$\theta \ll 1$$
$$\theta \ll R$$

We have a new definition of the operator only valid for WTA axis!

Collinear radiation of typical angle **θ** sees the jet boundary infinitely far away

→ Independence of the radius of the jet!

The collinear radiation is mostly inside the jet

→ The dependence on z is power suppressed!

$$J_i^{\text{WTA}}(z, \boldsymbol{b}, QR, \mu, \zeta) = \delta(1-z)\, \mathscr{J}_i^{\text{WTA}}(\boldsymbol{b}, \mu, \zeta) \left[1 + \mathcal{O}\left(\frac{1}{\boldsymbol{b}^2 Q^2 R^2}\right)\right]$$

where

$$\mathscr{J}_i^{\text{WTA}}(\boldsymbol{b}, \mu, \zeta) = \frac{1}{2N_c(\bar{n}\cdot p_J)} \text{Tr}\left\{ \frac{\not{\bar{n}}}{2} \langle 0| e^{-i\boldsymbol{b}\boldsymbol{P}_\perp} \chi_n(0) | J_{\text{alg}}^{\text{WTA}} \rangle \langle J_{\text{alg}}^{\text{WTA}} | \bar{\chi}_n(0) | 0 \rangle \right\}$$

NUMERICAL RESULTS

Ingredients to build cross-section

$$\frac{d\sigma_{ee\to JJX}}{dz_1 dz_2 d\boldsymbol{q}} = H(Q^2,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} J_q^{\text{axis}}(z_1,\boldsymbol{b},QR,\mu,\zeta) J_q^{\text{axis}}(z_2,\boldsymbol{b},QR,\mu,\zeta) R^2[\boldsymbol{b};(\mu_i,\zeta_i)\to(\mu_f,\zeta_f)]$$

Hard factor: Same that for DY. Known and introduced in arTeMiDe up to 2-loops.

Evolution kernel: Same that for TMDs. Known and introduced in arTeMiDe up to 3-loops.

TMD jet functions: Calculated at 1-loop. New arTeMiDe module built.

NUMERICAL RESULTS: LARGE RADIUS

The cross-section is simplified! →

The jet functions do not depend on the radius size

The dependence in z is power suppressed (cross-section is less differential)

In the case of big radius factorization is only held for WTA axis!

$$\frac{d\sigma_{ee\to JJX}}{d\boldsymbol{q}} = H(Q^2,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}}\, \mathscr{J}_q^{\mathrm{WTA}}(\boldsymbol{b},\mu,\zeta)\, \mathscr{J}_q^{\mathrm{WTA}}(\boldsymbol{b},\mu,\zeta)\, R^2[\boldsymbol{b};(\mu_i,\zeta_i)\to(\mu_f,\zeta_f)]$$

CHOOSING SCALES AND ζ-PRESCRIPTION

The election of the final scales is dictated by the hard scales of the process

$$\mu_f = Q \qquad \zeta_f = Q^2$$

Some of the logs in the jet function (or in the coefficient function) in TMDs are dangerous!

$$L_\mu^2, L_\mu \mathrm{l}_\zeta$$

Related to TMD evolution.

They make the cross-section blowing up!

These logs are cancelled by a particular choice of $\zeta = \zeta_\mu$ **ζ-prescription**

$$\mu^2 \frac{dJ(z, \boldsymbol{b}, \mu, \zeta_\mu)}{d\mu^2} = 0 \rightarrow \frac{\gamma_F(\mu, \zeta_\mu(\boldsymbol{b}))}{2\mathcal{D}(\mu, \boldsymbol{b})} = \frac{\mu^2}{\zeta_\mu(\boldsymbol{b})} \frac{d\zeta_\mu(\boldsymbol{b})}{d\mu^2} \rightarrow \mathrm{l}_{\zeta_\mu} = \frac{L_\mu}{2} - \frac{3}{2} + \mathcal{O}(a_s)$$

THE CROSS SECTION

Ingredients to build cross-section

$$\frac{d\sigma_{ee\to JJX}}{dz_1 dz_2 d\boldsymbol{q}} = H_{ee\to q\bar{q}}(s,\mu) \int \frac{d\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\boldsymbol{q}} J_q\left(z, \boldsymbol{b}, \frac{\sqrt{s}R}{2}, \mu, \zeta\right) J_q\left(z, \boldsymbol{b}, \frac{\sqrt{s}R}{2}, \mu, \zeta\right) R^2\left[\boldsymbol{b}; (\mu_i, \zeta_i) \to (\mu_f, \zeta_f)\right]$$

Hard factor: Same that for DY. Known and introduced in arTeMiDe up to 2-loops.

Evolution kernel: Same that for TMDs. Known and introduced in arTeMiDe up to 3-loops.

TMD jet functions: Calculated at 1-loop. New arTeMiDe module built.

CHOOSING SCALES AND ζ-PRESCRIPTION

The election of the final scales is dictated by the hard scales of the process

$$\mu_f = Q \qquad \zeta_f = Q^2$$

Some of the logs in the jet function (or in the coefficient function) in TMDs are dangerous!

$$L_\mu^2, L_\mu \mathrm{l}_\zeta$$

Related to TMD evolution.
They make the cross-section blowing up!

These logs are cancelled by a particular choice of $\zeta = \zeta_\mu$ **ζ-prescription**

$$\mu^2 \frac{dJ(z, \boldsymbol{b}, \mu, \zeta_\mu)}{d\mu^2} = 0 \rightarrow \frac{\gamma_F(\mu, \zeta_\mu(\boldsymbol{b}))}{2\mathcal{D}(\mu, \boldsymbol{b})} = \frac{\mu^2}{\zeta_\mu(\boldsymbol{b})} \frac{d\zeta_\mu(\boldsymbol{b})}{d\mu^2} \rightarrow \mathrm{l}_{\zeta_\mu} = \frac{L_\mu}{2} - \frac{3}{2} + \mathcal{O}(a_s)$$

THE CROSS SECTION

Ingredients to build cross-section

$$\frac{d\sigma_{ep\to eJX}}{dQ^2dxdzd\boldsymbol{q}} = \sigma_0 H_{eq\to eq}(Q^2,\mu)\int \frac{d\boldsymbol{b}}{(2\pi)^2}e^{-i\boldsymbol{b}\boldsymbol{q}}F_q(x,\boldsymbol{b},\mu,\zeta)J_q\left(z,\boldsymbol{b},\frac{QR}{2},\mu,\zeta\right)R^2\left[\boldsymbol{b};(\mu_i,\zeta_i)\to(\mu_f,\zeta_f)\right]$$

Hard factor: Same that for DY. Known and introduced in arTeMiDe up to 2-loops.

Evolution kernel: Same that for TMDs. Known and introduced in arTeMiDe up to 3-loops.

TMDPDFs: Calculated and introduced in arTeMiDe up to 2-loop

TMD jet functions: Calculated at 1-loop. New arTeMiDe module built.