The Energy-Energy Correlator in the Forward Limit

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with Lance Dixon and Ian Moult, to appear

SCET 2019, March 26, San Diego

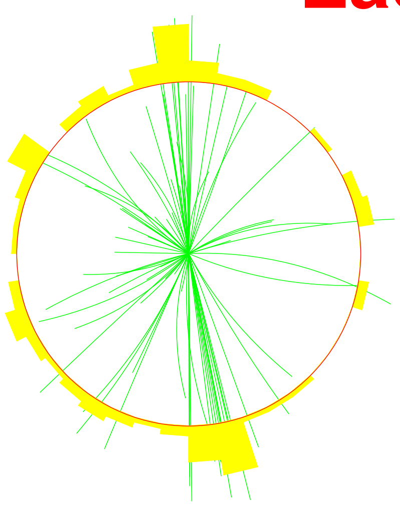
The Energy-Energy Correlator

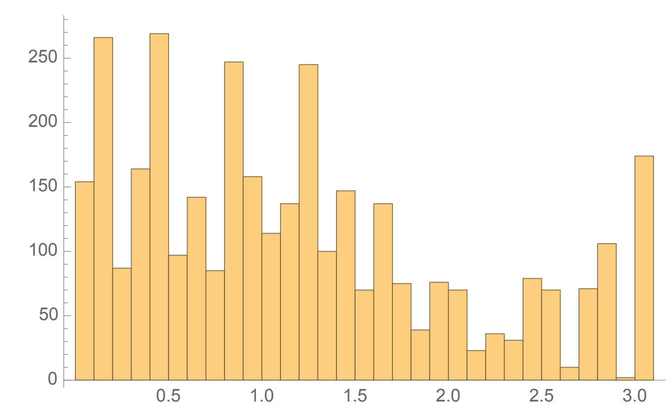
- Energy correlation of two calorimeter detector with angle χ, and sum over orientation

Basham, Brown, Ellis, Love, 1978

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos\chi) d\sigma \qquad z = \frac{1-\cos\chi}{2}$$

Each event gives a distribution!

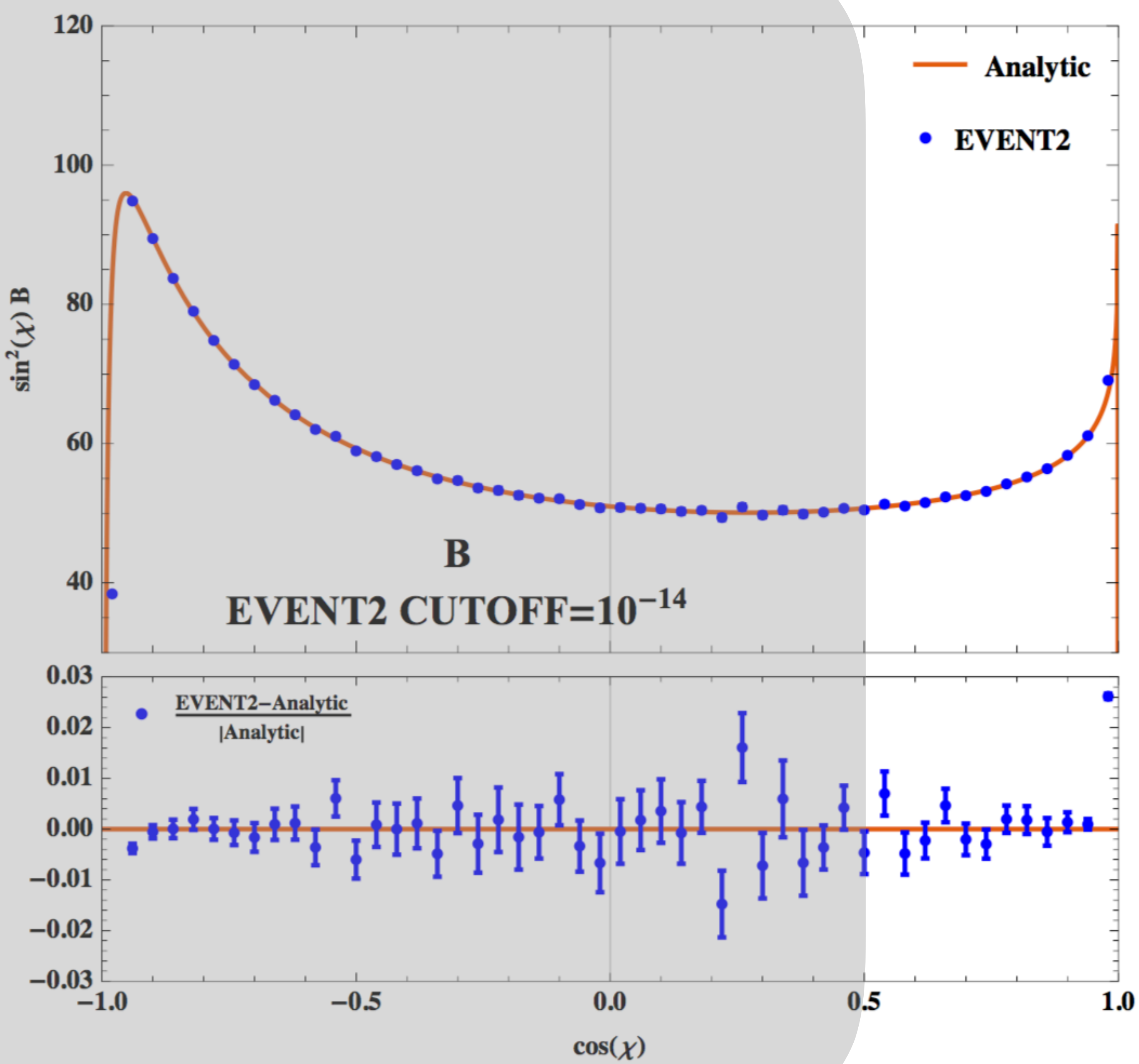




- Large QCD corrections, numerically hard
- N.P. unsurpassed even in the 3 jet region

- Admit a simple 4-pt wightman correlator representation
- Good analytically properties (fixed order and resummation)

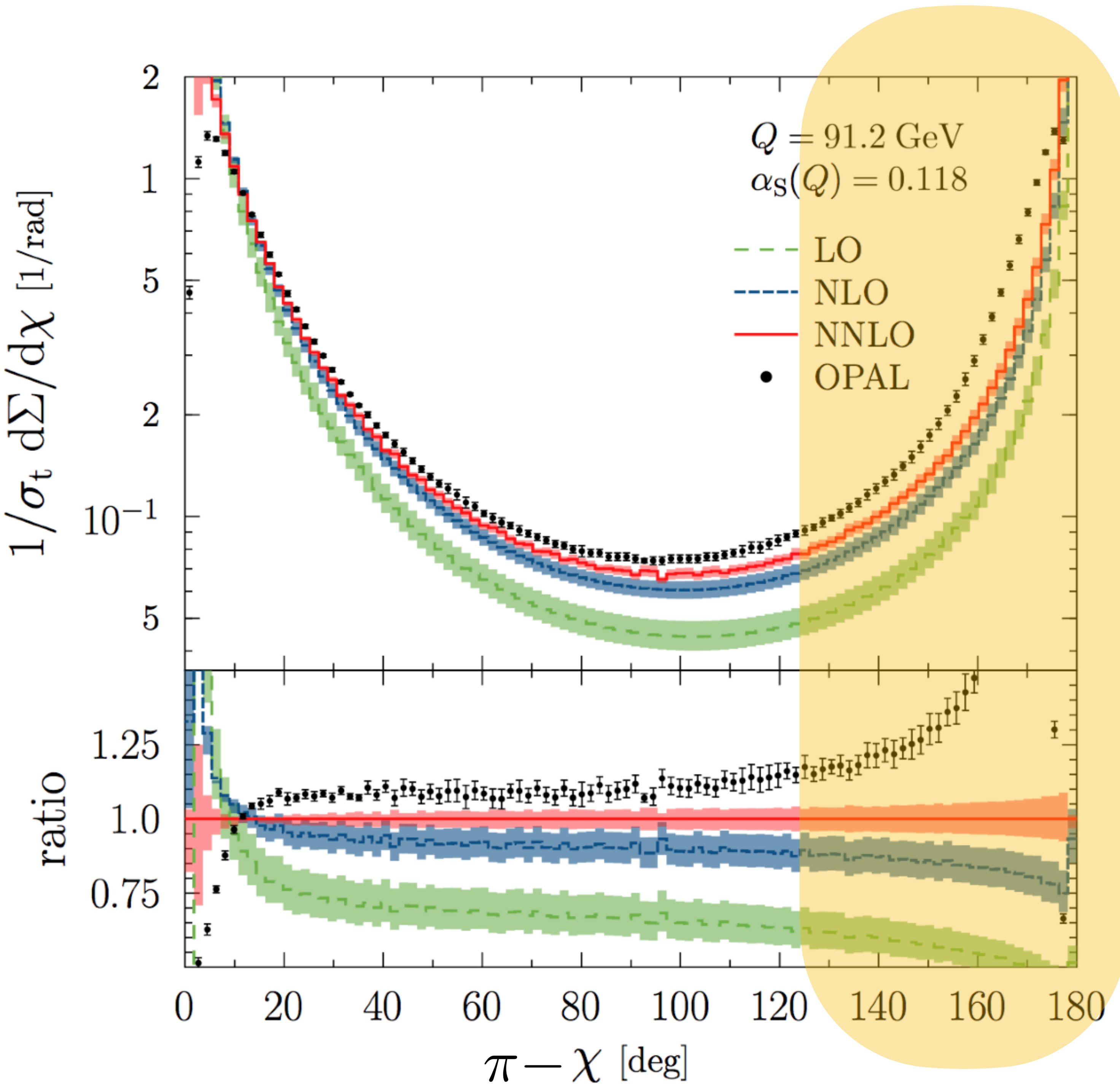
Back from SCET 2018, Amsterdam



- Analytic calculable in the bulk
  - NLO in QCD
  - NNLO in N=4 SYM

  See Kai Yan's talk
- N3LL TMD resummation

Challenge in the forward limit



- NNLO numerical calculation based on ColorfulNNLO indicate large QCD corrections
- Due to large perturbative and hadronization corrections, data for χ<60° is not used in fitting α_s
  Kardos, Kluth, Somogyi, Tulipant, 2018
- A resummation in the forward region desirable!

Leading Log resummation

- LL series resummed long time ago by jet calculus

Konishi, Ukawa, Veneziano, 1979

$$\sum_{a_1 a_2} D_{a_1 a_2, i}(1, 1; p_T^2, Q^2) = \frac{\alpha_s(4p_T^2)}{p_T^2\, 2\pi} \sum_{a,j} (-A_2)_{aj} \left[\frac{\alpha_s(4p_T^2)}{\alpha_s(Q^2)}\right]_{ji}^{A_2/2\pi b}, \qquad (5.17)$$

and the same with $4p_T^2 \to \delta^2 Q^2$.

- Attempt to go beyond LL with limited success

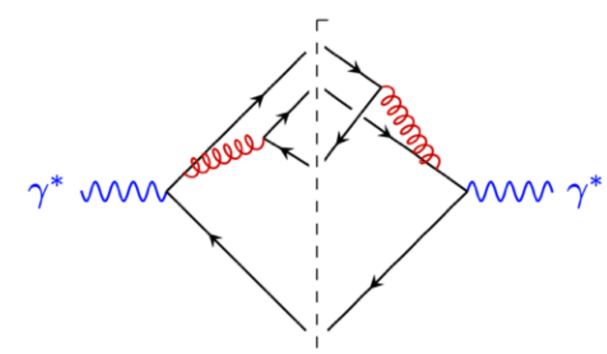
Kalinowski, Konishi, Scharbach, Taylor, 1980

$$E(x_1,x_2) = 2\left[-2 + \frac{1+x_1}{1-x_2} + \frac{1+x_2}{1-x_1} - \frac{2x_1}{(1-x_2)^2} - \frac{2x_2}{(1-x_1)^2} + \left(2 - \frac{1+x_1}{1-x_2} - \frac{1+x_2}{1-x_1} + \frac{2}{(1-x_1)(1-x_2)}\right)\log\left(\frac{(1-x_1)(1-x_2)}{1-x_1-x_2}\right)\right]$$

Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2018

$$\int_0^1 dx_1 \int_0^{1-x_2} dx_2\, E(x_1,x_2)\,[x_1 x_2 + (x_1+x_2)(1-x_1-x_2)] = -4\zeta_3 + \frac{43}{3}\zeta_2 - \frac{8011}{432}$$

1/z Coefficient of 4 quark interference



NLL resummation not available in general!

All order factorization in the forward limit

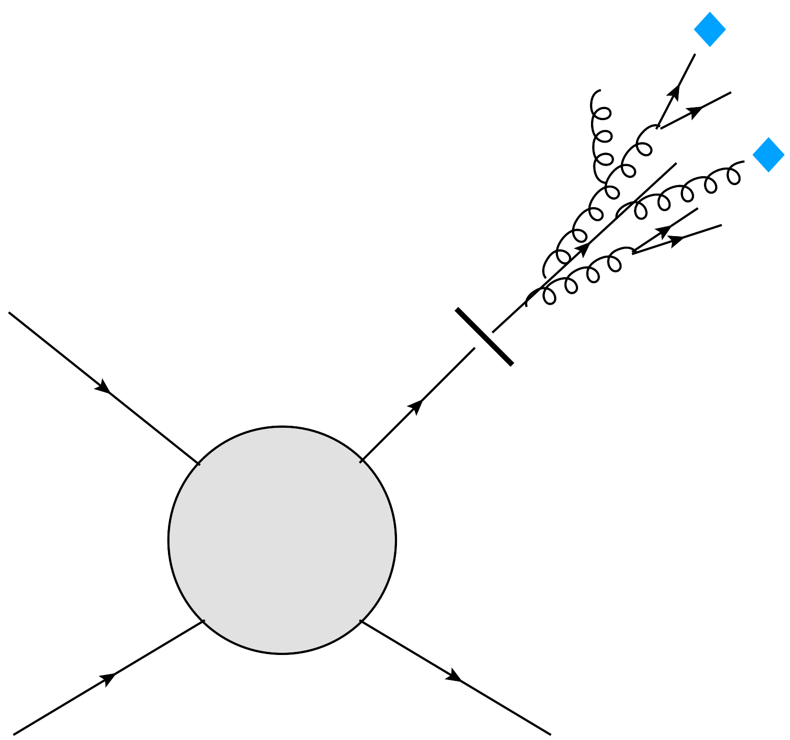
All-order factorization for z→0

- Cumulant $$\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^z dz'\, \Sigma(z', \ln\frac{Q^2}{\mu^2}, \mu)$$

$$\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}^T(\ln\frac{zx^2Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \ln\frac{Q^2}{\mu^2}, \mu)$$

$$z = \frac{q_T^2}{x^2 Q^2}$$

Fixed by kinematics and dimension analysis



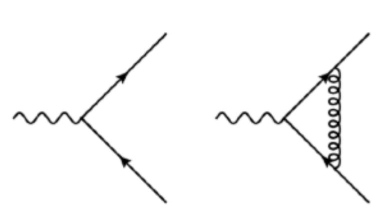
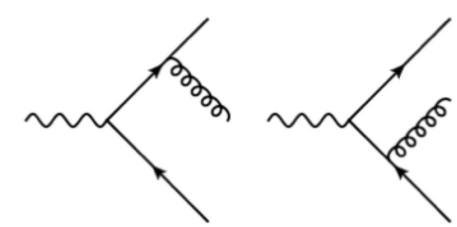
Full interference effects retained in H and J, separately

- Both jet and hard function are vector in flavor space
- H_q (H_g) : probability of finding a quark (gluon) with momentum fraction x
- J_q (J_g) : probability of finding two parton with momentum fraction y_1, y_2 and relative transverse momentum q_T in quark (gluon) initiated jet, weighted by $y_1 * y_2$

The hard function

- Process dependent information encoded in the hard func.

- For e+e- to jets at NLO

$\int PS_2 \Big|$  $\Big|^2 \delta(x_q - 1) \quad + \int PS_3 \Big|$  $\Big|^2 \delta(x_i - \frac{2p_i \cdot Q}{Q^2})$

- The hard function is simply the coefficient function for longitudinal fragmentation

- Know to NNLO for e+e- → q$\bar{\text{q}}$ and H → gg in HEFT

Mitov, Moch, 2006; Almasy, Moch, Vogt, 2011

RG equation

$$\frac{d\vec{H}(x, \ln\frac{Q^2}{\mu^2}, \mu)}{\ln \mu^2} = -\int_x^1 \frac{dy}{y} P_T(y, \mu) \cdot \vec{H}(\frac{x}{y}, \frac{Q^2}{\mu^2}, \mu)$$

singlet time-like splitting kernel

The jet function

- 4-pt Wightman correlation of collinear fields and energy flow operator

e.g., for quark jet $$J_q(z = \frac{1-\cos\chi}{2}) = \sum_{X_n} \sum_{i,j\in X_n} \langle 0|\overline{\chi}_n \hat{E}_i \hat{E}_j \Theta(\theta_{ij} < \chi)\chi_n|0\rangle$$

- Summation include self-correlation, i = j.

$$J_q^{(1)} = S_\epsilon \int_0^1 dx \int_0^{x(1-x)zQ^2} \frac{ds}{s^{1+\epsilon}} 2(x(1-x))^{1-\epsilon} P_{qq}(x)$$
$$= C_F \left(-\frac{3}{\epsilon_{\rm IR}} + 3\ln\frac{zQ^2}{\mu^2} - \frac{37}{3}\right) + \mathcal{O}(\epsilon)$$

$$S_\epsilon = e^{\epsilon\gamma_E}\mu^{2\epsilon}/\Gamma(1-\epsilon)$$

$$P_{qq}(x) = 2C_F[\frac{1+x^2}{1-x} - \epsilon(1-x)]$$

- Self-correlation contribution convert IR poles to UV poles

RGE from RG invariance of cross section

$$\frac{d\vec{J}^T(\ln\frac{zQ^2}{\mu^2},\mu)}{d\ln\mu^2} = \int_0^1 dy\, y^2 \vec{J}^T(\ln\frac{zy^2Q^2}{\mu^2},\mu)\cdot P_T(y,\mu)$$

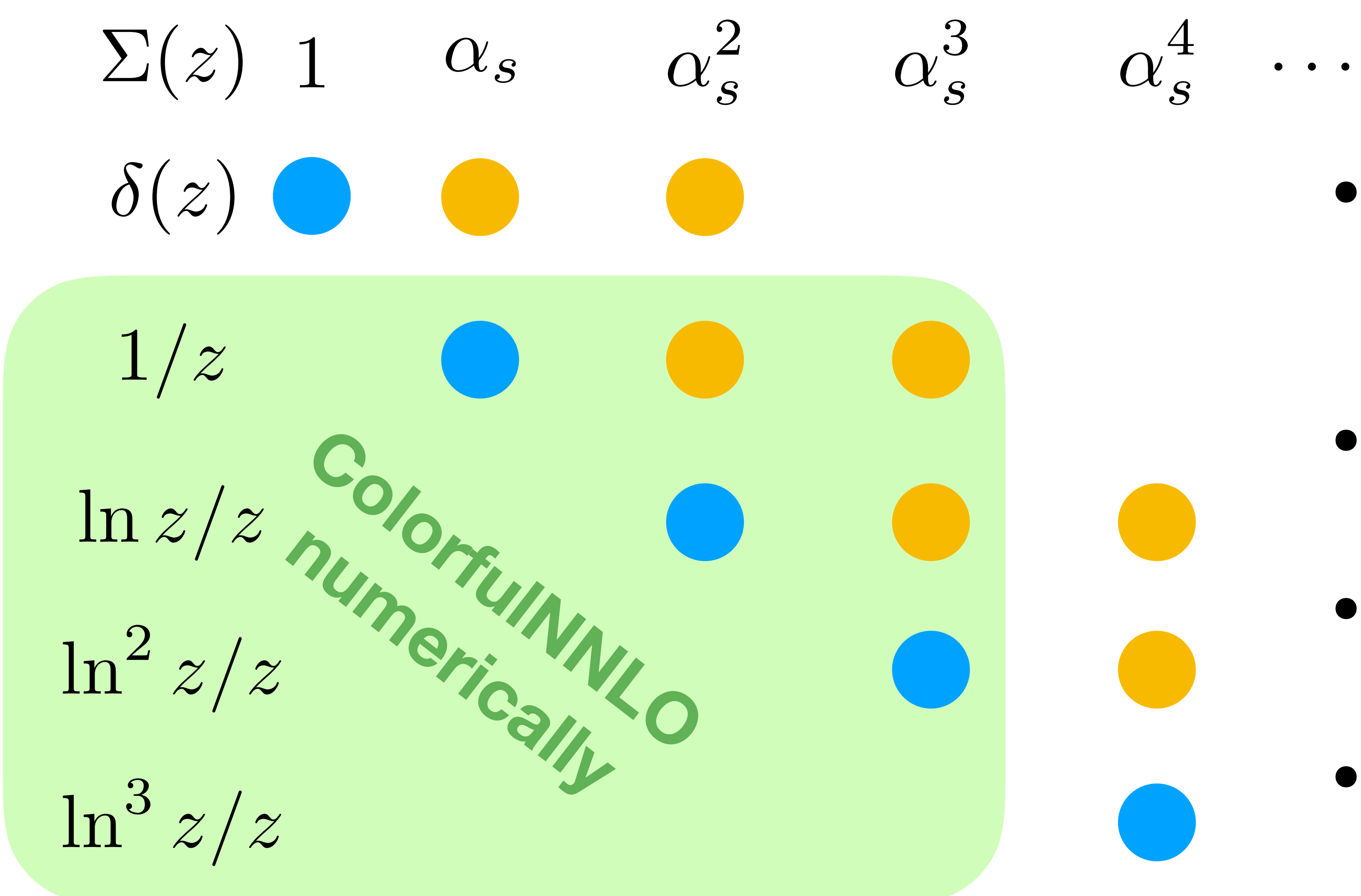
Solving for the evolution

$$\Omega(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}^T(\ln\frac{zx^2Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \ln\frac{Q^2}{\mu^2}, \mu)$$

- The canonical scale for the jet function is $\mu_J = x\sqrt{z}Q$
- Running the hard function from Q to μ_J, and perform the "x convolution"
- Evolution for hard function can be solved in Mellin space. Require analytic continuation in Mellin moment N to transform back
- Inside the x integral, integrand contain $\alpha_s(x\sqrt{z}Q)$. N.P. corrections unavoidable even for $\sqrt{z}Q > \Lambda_{QCD}$
- Instead of doing this, we truncate the perturbation series at $O(\alpha_s^9)$. Works for $z > 10^{-4}$

Counting the order

- LL — Konishi, Ukawa, Veneziano, 1979
- NLL + NNLL — Dixon, Moult, HXZ, 2019, This talk



- To get to NNLL require:

- NNLO splitting kernel
  Moch, Vermasseren, Vogt
- NNLO hard function
  Mitov, Moch, 2006; Almasy, Moch, Vogt, 2011
- NNLO jet function

Very challenging!

Constraint from sum rule

- Total cross section reproduced by integrating z in [0,1]

$$\int_0^1 dz\, \Sigma(z, \ln\frac{Q^2}{\mu^2}, \mu) = \sigma_{\rm tot}$$

At NLO

$$\frac{1}{\sigma_0}\Sigma^{(1)} = C_F\big[(j_1^q + h_1^q)\delta(z)$$

forward jet function and hard function

$$\frac{3}{2}\frac{1}{[z]_+} - 2\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{[1-z]_+} + \frac{1}{2z^5}\left(-9z^4 - 6z^3 - 42z^2\right.$$
$$\left. +36z + 4\left(-z^4 - z^3 + 3z^2 - 15z + 9\right)\ln(1-z)\right)\big]$$

LO bulk fixed order

$$+(-2\zeta_2 - 4)\delta(1-z)$$

back-to-back TMD factorization

- h_1^q known, j_1^q fixed by $\frac{\alpha_s}{4\pi}\int_0^1 dz\Sigma^{(1)} = \sigma_0\frac{\alpha_s}{\pi}$

- Above procedure can be applied at NNLO to determine jet constant
  - NLO Bulk known — **e+e-: Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, Higgs: M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2019**
  - Back-to-back part determined by brute force calculation (**Tong-Zhi Yang, 2019**), checked by TMD factorization (**Moult, HXZ, 2018**)

$$
\begin{aligned}
z\Sigma(z) = {} & \frac{\alpha_s}{4\pi}\frac{3C_F}{2} \\
& + \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg[C_A C_F \left(-\frac{50\zeta_2}{3} + 4\zeta_3 - \frac{107\log(z)}{15} + \frac{35366}{675} \right) + C_F n_f \left(\frac{53\log(z)}{60} - \frac{4913}{900} \right) \\
& + C_F^2 \left(\frac{86\zeta_2}{3} - 8\zeta_3 + \frac{25\log(z)}{4} - \frac{8263}{216} \right) \Bigg] \\
& + \left(\frac{\alpha_s}{4\pi}\right)^3 \Bigg[C_A C_F n_f \left(\frac{379579\zeta_2}{5400} + \frac{3679\zeta_3}{30} - \frac{118\zeta_4}{3} + \left(-\frac{108\zeta_2}{5} + \frac{16\zeta_3}{3} + \frac{6644267}{54000} \right) \log(z) \right. \\
& \left. - \frac{16259\log^2(z)}{1800} - \frac{1025118113}{2160000} \right) \\
& + C_A C_F^2 \left(-\frac{400\zeta_2^2}{3} + \frac{137305\zeta_2}{216} - 72\zeta_2\zeta_3 + \frac{10604\zeta_3}{15} + \frac{4541\zeta_4}{6} - 216\zeta_5 \right. \\
& \left. + \left(-\frac{1100\zeta_2}{3} + \frac{262\zeta_3}{3} + \frac{105425}{144} \right) \log(z) - \frac{340}{9}\log^2(z) - \frac{105395741}{51840} \right) \\
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& \left. + \frac{8059\log^2(z)}{300} + \frac{964892417}{540000} \right) \\
& + C_F^2 n_f \left(-\frac{15161\zeta_2}{120} - \frac{7994\zeta_3}{45} + \frac{236\zeta_4}{3} + \left(\frac{416\zeta_2}{9} - \frac{32\zeta_3}{3} - \frac{6760183}{64800} \right) \log(z) + \frac{4619\log^2(z)}{720} \right. \\
& \left. + \frac{164829499}{486000} \right) + C_F n_f^2 \left(\frac{6\zeta_2}{5} + \frac{23\log^2(z)}{45} - \frac{8867\log(z)}{1350} + \frac{88031}{4500} \right) \\
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Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 2018

This talk

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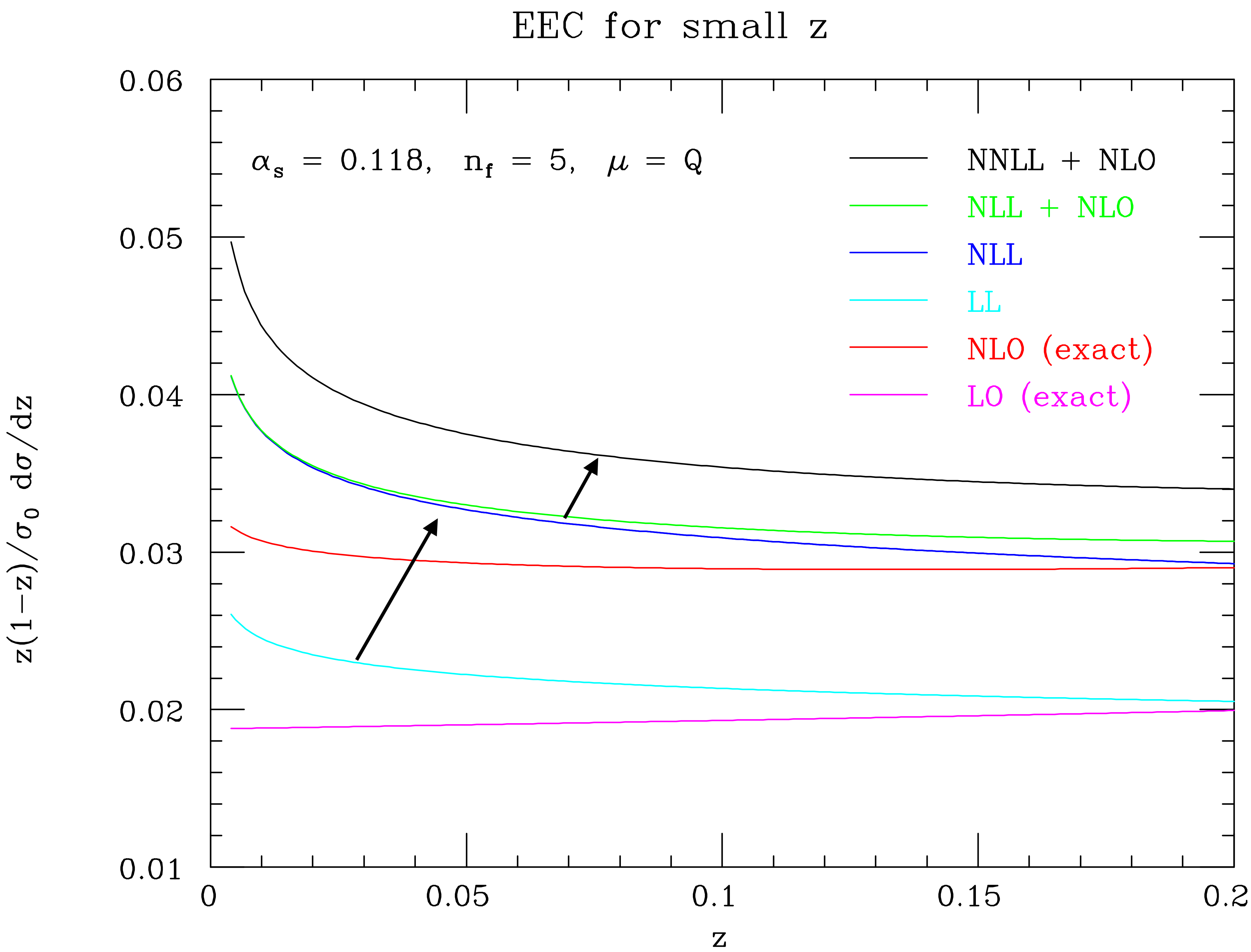
\end{aligned}
$$

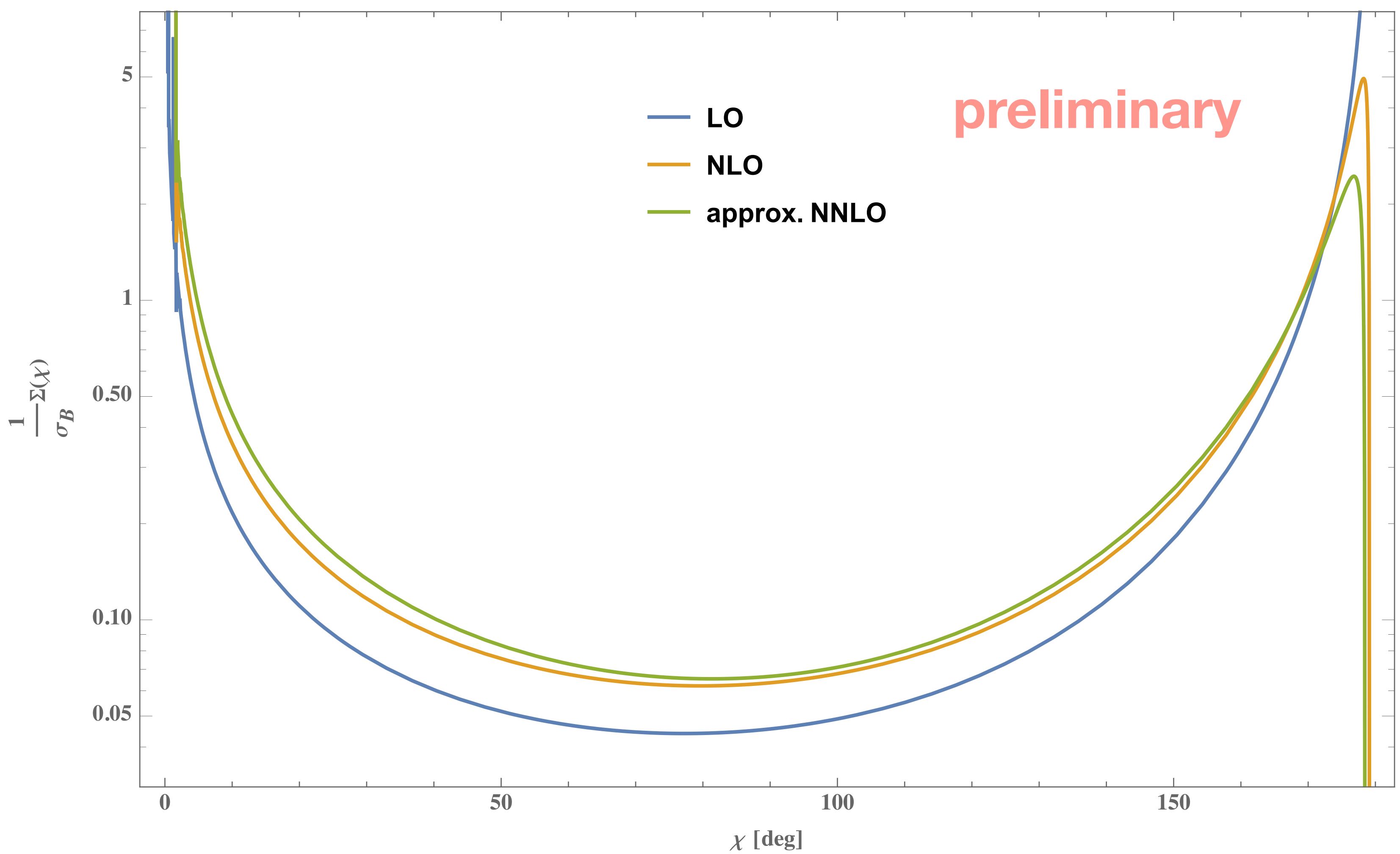
- The results obey leading transcendental principle: setting CF=CA, the leading transcendental series agree with N=4 SYM.

- Furthermore, this leading transcendental series is simply AD of twist-two spin 1 anomalous dimension in N=4 SYM $\gamma_{\text{uni}}(1)$. Maybe the first all order prediction in QCD cross section for the leading transcendental part!

Numerical impact of NNLL



Current knowledge of EEC



- Full tower of logs at χ → 0 and 180
- Intermediate χ known analytically at NLO, numerically at NNLO
- Straightforward to resum logs at χ → 0 and 180

Some applications

EEC and jet substructure

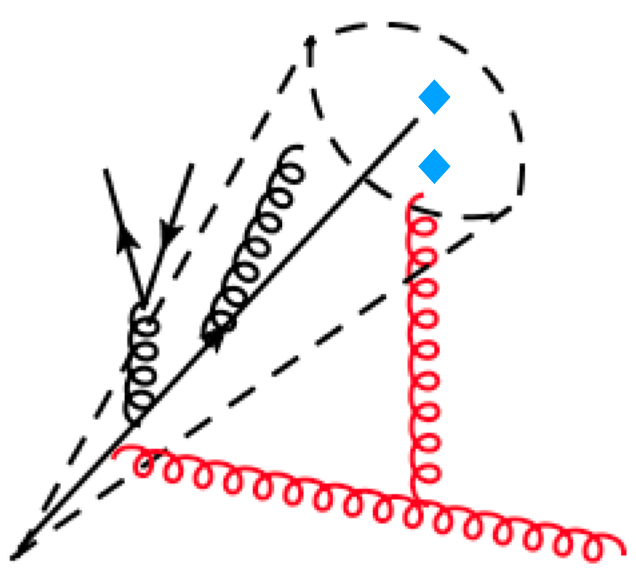
- Our formula can be applied to jet production at hadron collider

$$\Omega(z, \ln \frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}^T (\ln \frac{z x^2 Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \ln \frac{Q^2}{\mu^2}, \mu)$$

- The hard function is process dependent, and is *not* collinear safe. NNLO Dipole subtraction can deal with identified final state parton. NNLO subtraction with fragmentation not yet available

- Out of jet cone soft radiation can contribute to EEC, but is weighted by the soft gluon energy

- Non-global logarithms are naturally power suppressed for EEC



EEC and Casimir scaling

- Many jet substructure observable exhibit casimir scaling at LL, which limited their power in quark/gluon discrimination

Frye, Larkoski, Thaler, K. Zhou, 2017

- At LL level, EEC is controlled by exponential of splitting kernel

$$\vec{J}^T_{LL} = (J_{q,LL}, J_{g,LL}) \exp\left(\frac{\gamma(3)}{2\beta_0} \ln \frac{\alpha_s(\sqrt{z}Q)}{\alpha_s(Q)}\right)$$

$$\gamma(3) = \begin{pmatrix} -\frac{25}{6}C_F & \frac{7}{15}n_f \\ \frac{7}{6}C_F & -\frac{14}{5}C_A - \frac{2}{3}n_f \end{pmatrix}$$

quark jet **gluon jet**

- Manifest violation of Casimir scaling at LL level!

N=1 Supersymmetry

- Setting $C_F = n_f = C_A$ gives EEC in N=1 SYM

$$
\begin{aligned}
z\Sigma^{\mathcal{N}=1}(z)\Big|_{\rm NLL} =& \frac{\alpha_s}{4\pi}\frac{3C_A}{2} \\
&+ \left(\frac{\alpha_s}{4\pi}\right)^2 \left(12\zeta_2 - 4\zeta_3 + \frac{625}{72} + 0\,\log(z)\right) C_A^2 \\
&+ \left(\frac{\alpha_s}{4\pi}\right)^3 C_A^3 \left((33 - 36\zeta_2)\log(z) + 0\,\log^2(z)\right) \\
&+ \mathcal{O}(\alpha_s^4) \\
=& \frac{3}{2} C_A \frac{\alpha_s}{4\pi} + (-4\zeta_3 + \frac{1417}{72}) C_A^2 \left(\frac{\alpha_s}{4\pi}\right)^2 + \frac{(12\zeta_2 - 11) C_A^2 \left(\frac{\alpha_s}{4\pi}\right)^2}{\left(1 + 3C_A \frac{\alpha_s}{4\pi}\ln z\right)} \,.
\end{aligned}
$$

- Note that the beta function in N=1 SYM is $\beta_0 = 11/3\ C_A - 2/3\ C_A = 3$

- Manifest Laudau pole in the perturbative series

N=4 Supersymmetry

- In N=4 SYM, naively more complicated flavor structure

$$\vec{J}^{T}_{\mathcal{N}=4} = (J_\phi, J_\lambda, J_g) \qquad P_T(z) = \begin{pmatrix} P_{\phi\phi} & P_{\phi\lambda} & P_{\phi g} \\ P_{\lambda\phi} & P_{\lambda\lambda} & P_{\lambda g} \\ P_{g\phi} & P_{g\lambda} & P_{gg} \end{pmatrix}$$

- But N=4 SUSY implies

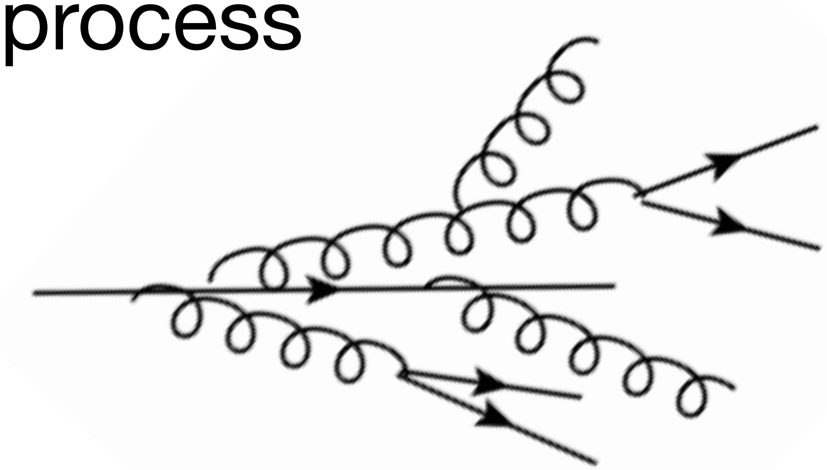
$$J_\phi = J_\lambda = J_g \qquad \sum_{j=\phi,\lambda,g} P_{j\phi} = \sum_{j=\phi,\lambda,g} P_{j\lambda} = \sum_{j=\phi,\lambda,g} P_{jg} = P_{uni}(z)$$

- Evolution dramatically simplified

$$\frac{dJ_{\mathcal{N}=4}(\ln \frac{zQ^2}{\mu^2})}{d\ln \mu^2} = \int_0^1 dy\, y^2 J_{\mathcal{N}=4}(\ln \frac{zy^2Q^2}{\mu^2}) P_{uni}(y)$$

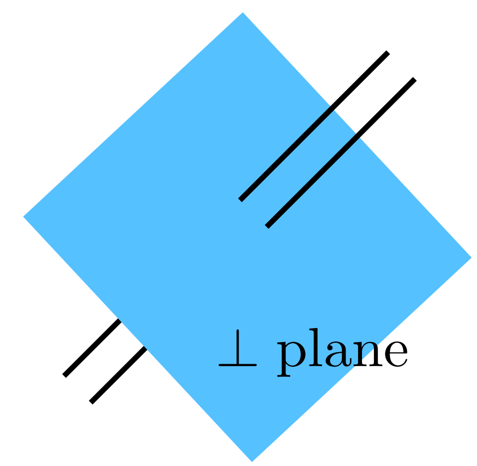
The puzzling space-like time-like connection

- The evolution equation of the jet function describe a time-like showering process



$t \rightarrow$

- Alternately, EEC admits a four point correlation representation **Hofman, Maldacena, 2008**



$$\text{EEC} \sim \langle 0 | O^\dagger \mathcal{E}(\vec{y}_\perp) \mathcal{E}(\vec{0}_\perp) O | 0 \rangle \qquad |\vec{y}_\perp|^2 \sim z$$

$$\mathcal{E}(\vec{y}_\perp)\mathcal{E}(\vec{0}_\perp) \sim \int dy^- T_{--}(y^-, y^+ = 0, \vec{y}_\perp) \int dy'^- T_{--}(y'^-, y^+ = 0, \vec{0}_\perp) \sim \sum_n |\vec{y}_\perp|^{\tau_n - 4} \mathcal{U}_{j-1,n}\Big|_{j=3}$$

Light-ray operator OPE

Simmons-Duffin, Zhiboedov et al.

- To reconcile the two pictures requires reciprocity **Caron-Huot, 2018**

Time-like space-like connection from evolution equation

- The use of reciprocity naturally arise in our framework

- Power-law ansatz for the jet function

$$J(zQ^2,\mu) = C_J(\alpha_s)\left(\frac{zQ^2}{\mu^2}\right)^{\gamma^{\mathcal{N}=4}(\alpha_s)} \qquad \frac{dJ_{\mathcal{N}=4}(\ln\frac{zQ^2}{\mu^2})}{d\ln\mu^2} = \int_0^1 dy\, y^2 J_{\mathcal{N}=4}(\ln\frac{zy^2Q^2}{\mu^2})P_{uni}(y)$$

$$\begin{aligned} 2\gamma^{\mathcal{N}=4}(\alpha_s) &= -2\int_0^1 dy\, y^{2+2\gamma^{\mathcal{N}=4}(\alpha_s)} P_{T,uni.}(x,\alpha_s) \\ &= 2\gamma_T^{\mathcal{N}=4}(3+2\gamma^{\mathcal{N}=4},\alpha_s) \end{aligned}$$

- Reciprocity $2\gamma_S^{\mathcal{N}=4}(N,\alpha_s) = 2\gamma_T^{\mathcal{N}=4}(N+2\gamma_S^{\mathcal{N}=4}(N,\alpha_s),\alpha_s)$ **Basso, Korchemsky, 2006**

$$\Rightarrow \qquad \gamma^{\mathcal{N}=4}(\alpha_s) = \gamma_S^{\mathcal{N}=4}(3,\alpha_s)$$

- We can also derive a reciprocity relation in pure Yang-Mills

$$2\gamma_S^{\text{pure YM}}(N,\alpha_s(\mu)) = 2\gamma_T^{\text{pure YM}}\left(N + \frac{2\gamma_S^{\text{pure YM}}(N,\alpha_s(\mu))}{1+\frac{\alpha_s(Q)}{4\pi}\beta_0\ln(z)},\alpha_s(\mu)\right)$$

NNLL for N=4 SYM

- The small z asymptotic in N=4 SYM has power-law form

$$\Omega(z) = \frac{1}{2} C(\alpha_s) \, z^{\gamma_{S,uni}^{\mathcal{N}=4}(1,\alpha_s)}$$

Known even non-perturbatively at large Nc

Gromov, Kazakov et al.

$$C(\alpha_s) \;=\; 1 - \frac{C_A \alpha_s}{\pi} + \left(\frac{11}{4}\zeta_4 - 3\zeta_2 + 7\right)\left(\frac{C_A\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)$$

$$\gamma_S^{\mathcal{N}=4}(1,\alpha_s) = \frac{C_A\alpha_s}{\pi} + \left(-\frac{\zeta_3}{2} + \zeta_2 - 2\right)\left(\frac{C_A\alpha_s}{\pi}\right)^2 + \left(\frac{3}{2}\zeta_5 + \frac{3}{8}\zeta_4 - \frac{3}{2}\zeta_3 - 4\zeta_2 + 8\right)\left(\frac{C_A\alpha_s}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

- Expanded to O(α_s^3), agree perfectly with a very recent NNLO calculation **Henn, Sokatchev, K. Yan, Zhiboedov, 2019**

Conclusion

fixed order, world of elliptic integrals
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Jet substructure
PP event shape, factorization violation or not
precision alpha_s
EEC and its generalization
light-ray operator and OPE
TMD rapidity AD
leading transcendentality, integrability
Moment of time-like splitting kernel
reciprocity, AD of local operator