The Transverse Energy-Energy correlations in dijet limit at hadron colliders

Haitao Li
Los Alamos National Laboratory

Based on the work arXiv:1901.04497
with Anjie Gao, Ian Moult and Huaxing Zhu

SCET workshop, UCSD
03-26-2019

TEEC at hadronic colliders

TRANSVERSE ENERGY–ENERGY CORRELATIONS:
A TEST OF PERTURBATIVE QCD FOR THE PROTON–ANTIPROTON COLLIDER

A. ALI [1], E. PIETARINEN [2] and W.J. STIRLING
CERN, Geneva, Switzerland

Received 28 February 1984

electron-positron collider: Basham et al 1978
hadronic collider: Ali et al 1984

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- **weighted cross section**
- **the soft radiation does not contribute directly to the observable at leading power**
- **soft gluon contributes only via recoil**

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The particle level TEEC exhibits a remarkable perturbative simplicity in the dijet limit

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TEEC at hadronic colliders

- **the flow of radiation in a scattering event, as other event shape observables**
- **precision measurements of QCD parameters, such as the strong coupling constant**
- **non-trivial color evolution and amplitude level factorization violation occur for dijet event shapes**

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EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)

Submitted to: Phys. Lett. B

CERN-PH-EP-2015-177
21st October 2015

Measurement of transverse energy–energy correlations in multi-jet events in pp collisions at $\sqrt{s} = 7$ TeV using the ATLAS detector and determination of the strong coupling constant $\alpha_s(m_Z)$

The ATLAS Collaboration

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2017-093
23rd January 2018

Determination of the strong coupling constant α_s from transverse energy–energy correlations in multijet events at $\sqrt{s} = 8$ TeV using the ATLAS detector

TEEC at hadronic colliders

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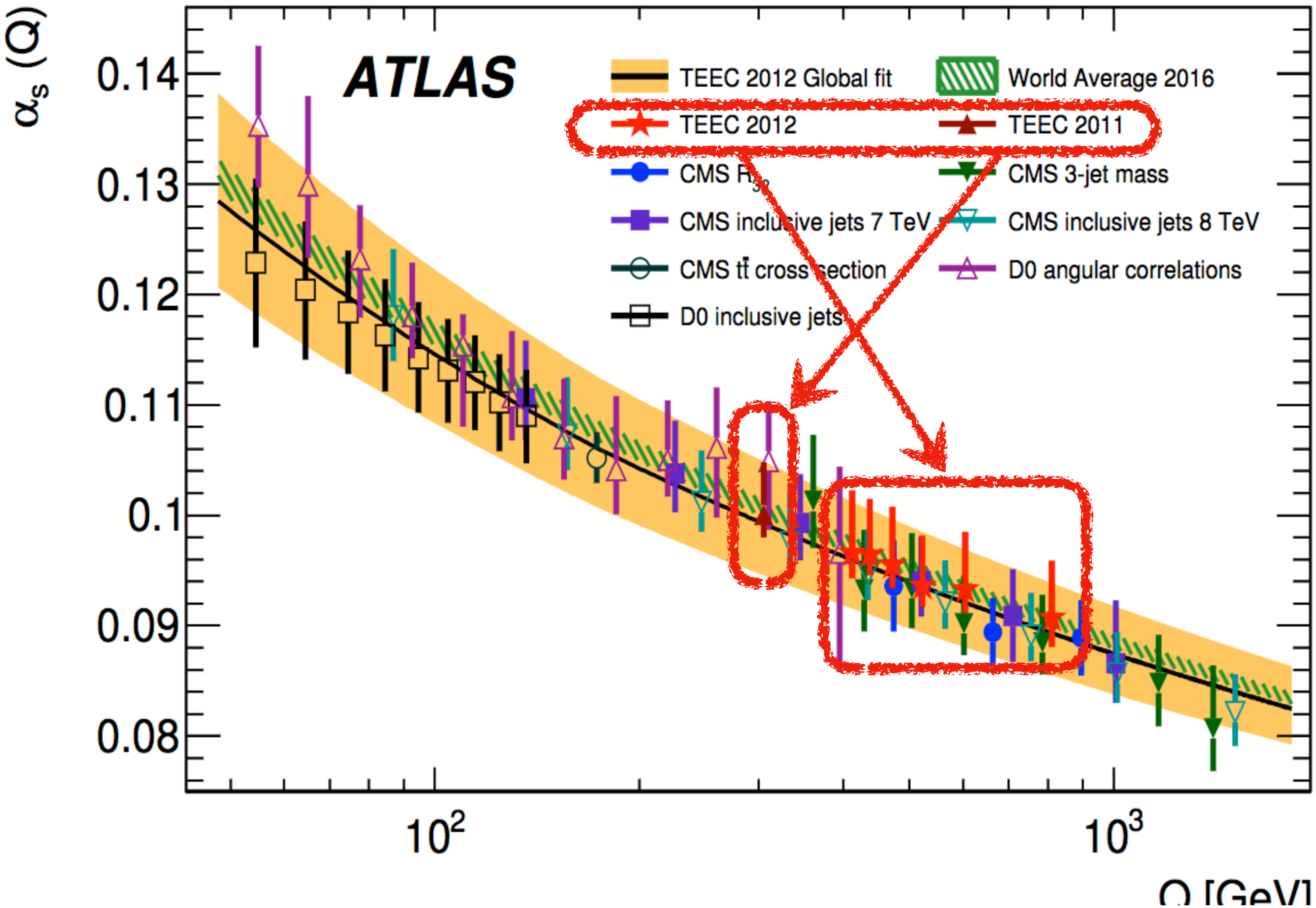
-EP-2017-093
January 2018



Measure ... ant α_s
events ... ns in
and ... TLAS

$$\alpha_s(m_Z) = 0.1162 \pm 0.0011\ (\text{exp.})\ {}^{+0.0076}_{-0.0061}\ (\text{scale}) \pm 0.0018\ (\text{PDF}) \pm 0.0003\ (\text{NP}),$$

Recent progresses

EEC predictions

Analytical NLO	*Dixon et al arXiv:1801.03219*
NNLO	*Vittorio Del Duca et al arXiv:1606.03453*
NNLL+NLO	*Florian, Grazzini arXiv:hep-ph/0407241*
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3-loop soft function	*Moult, Zhu arXiv:1801.02627*
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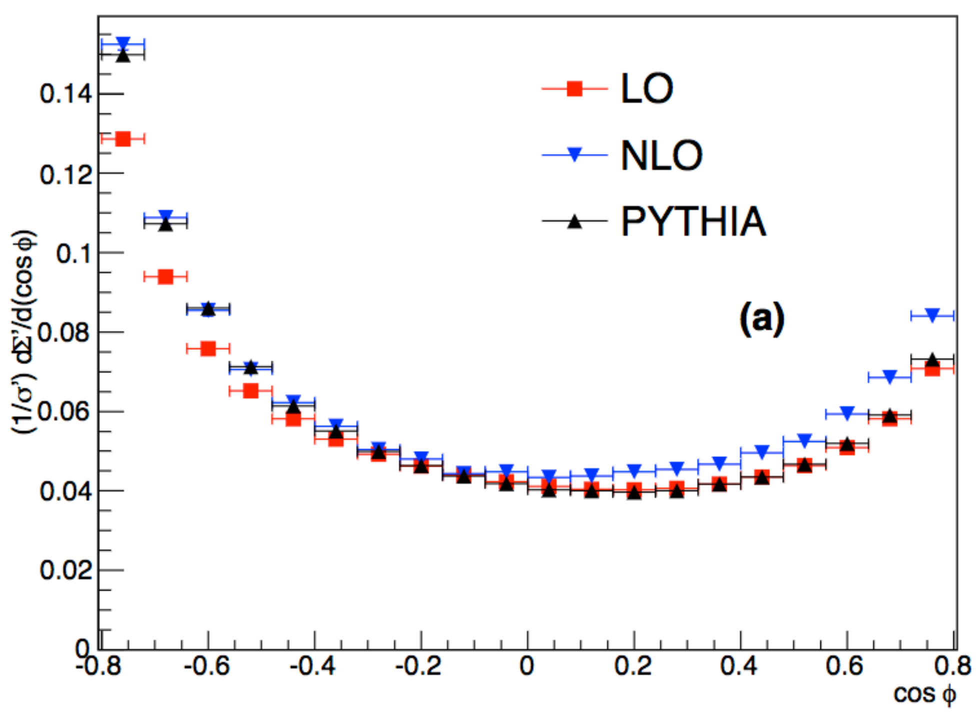
NLO QCD corrections to the TEEC for jets



Ali et al arXiv:1205.1689

"Transverse EEC distributions in hadronic collisions, on the other hand, are ***handicapped*** *due to the absence of the NLO perturbative QCD corrections."*

Kinematics

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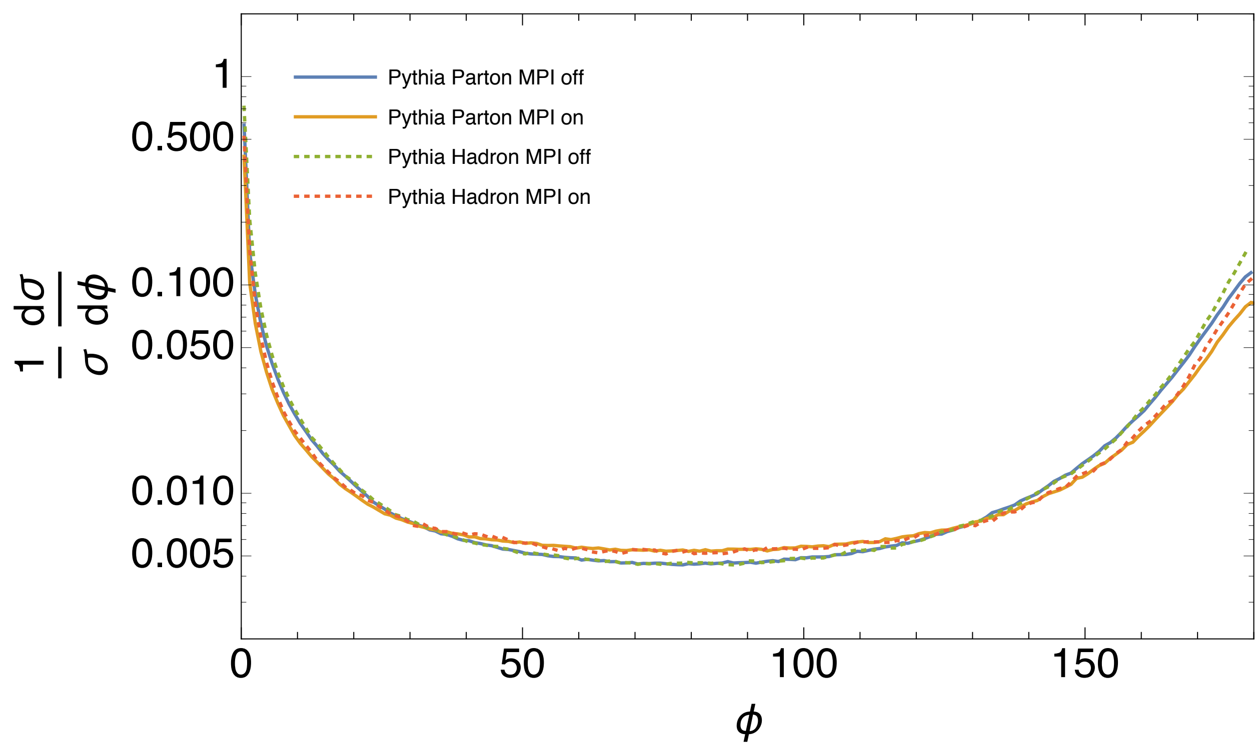

smaller corrections from the non-perturbative effects compared to other event shape observables

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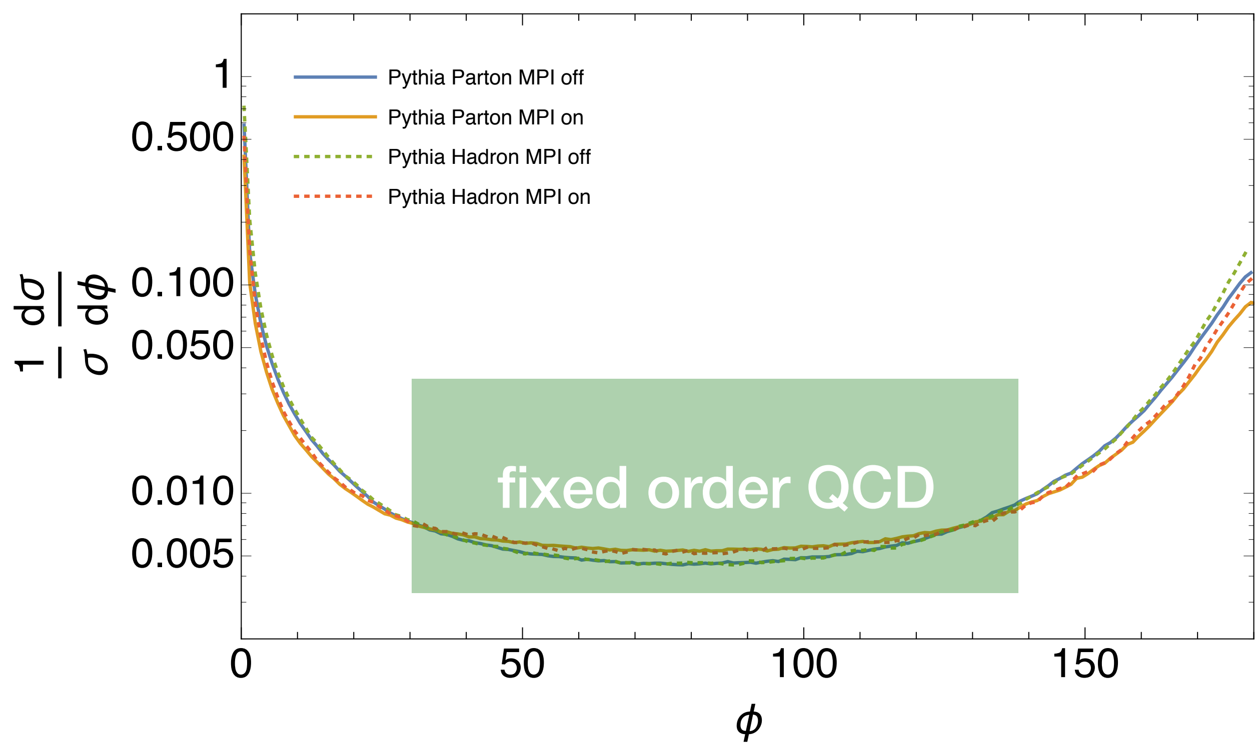

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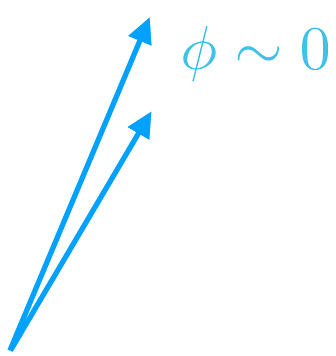
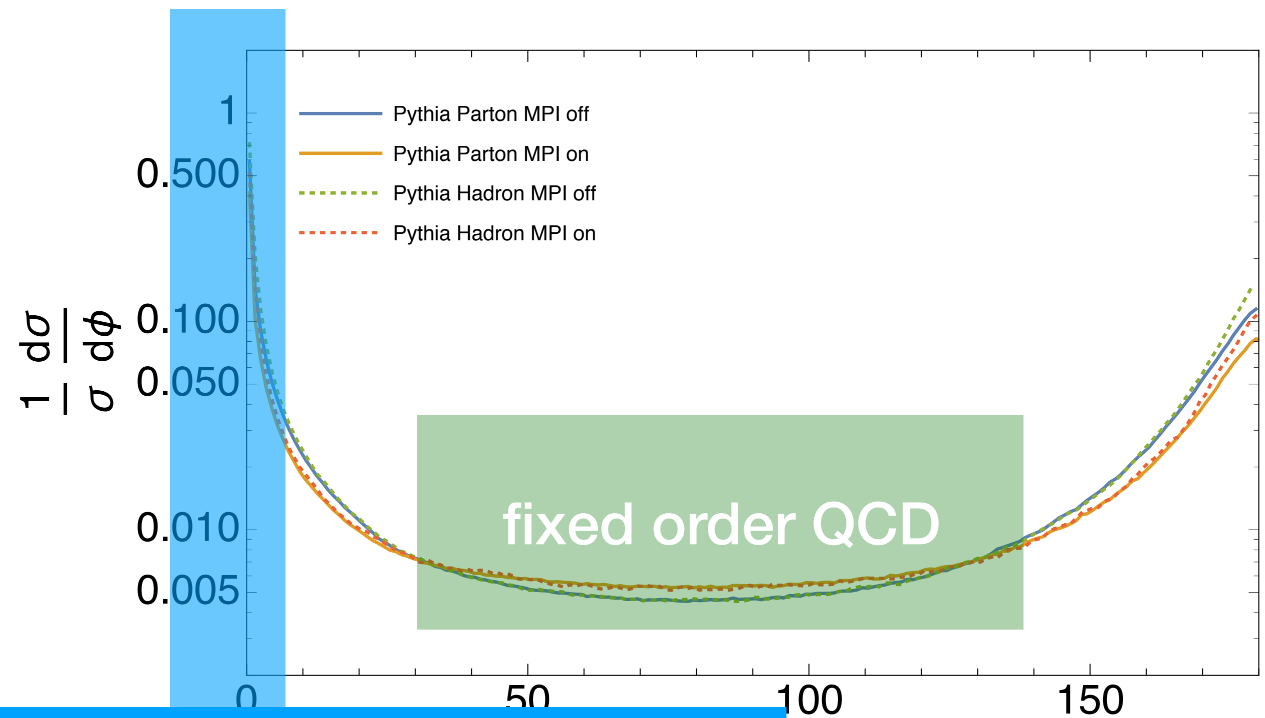
Collinear singularity

$$\cos\phi_{ab} \to 0$$





an extension of the jet calculus:
Konishi et al 1979
See Huaxing's talk

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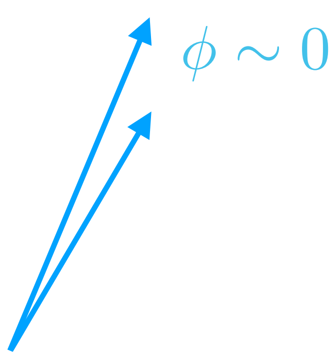
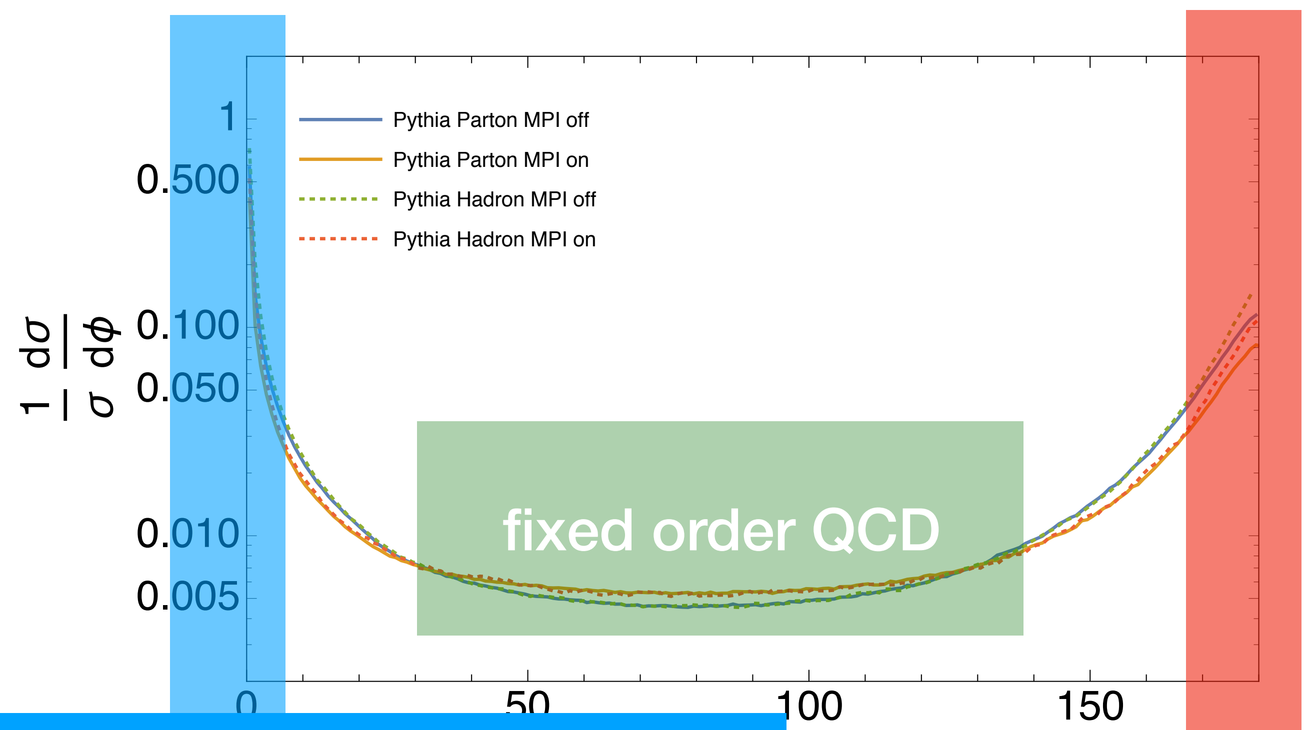
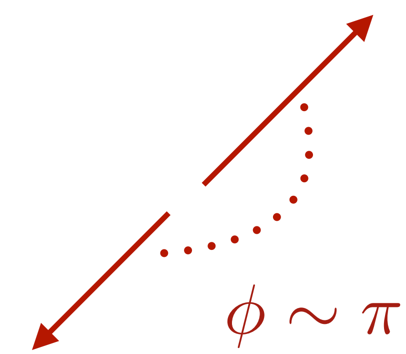
Collinear and soft singularity

$\cos\phi_{ab} \to -1$







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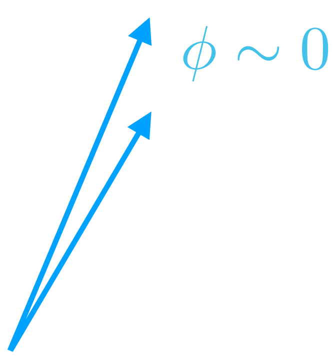

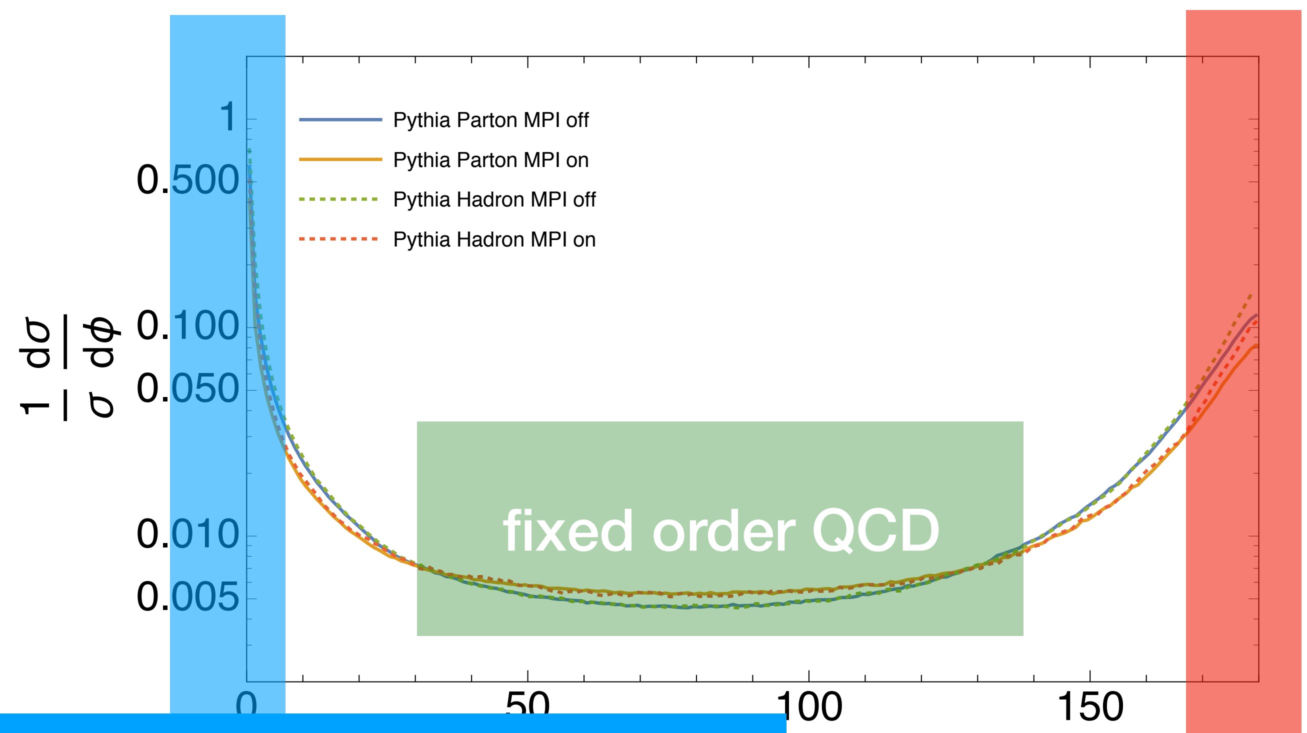

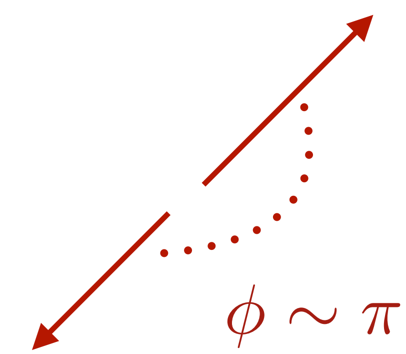

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The purpose of our work is using SCET to improve the predictions in the back-to-back limits

Back-to-back limit

Select dijet events $h_1 + h_2 \to J_1 + J_2 + x$

Define scattering plane: x-z

$$n_1 = (1,0,0,1), n_2 = (1,0,0,-1)$$
$$n_3 = (1, \sin\theta, 0, \cos\theta), n_4 = (1, -\sin\theta, 0, -\cos\theta)$$

The dijet limit is defined as

$$\tau = \frac{1+\cos(\phi)}{2} \to 0$$

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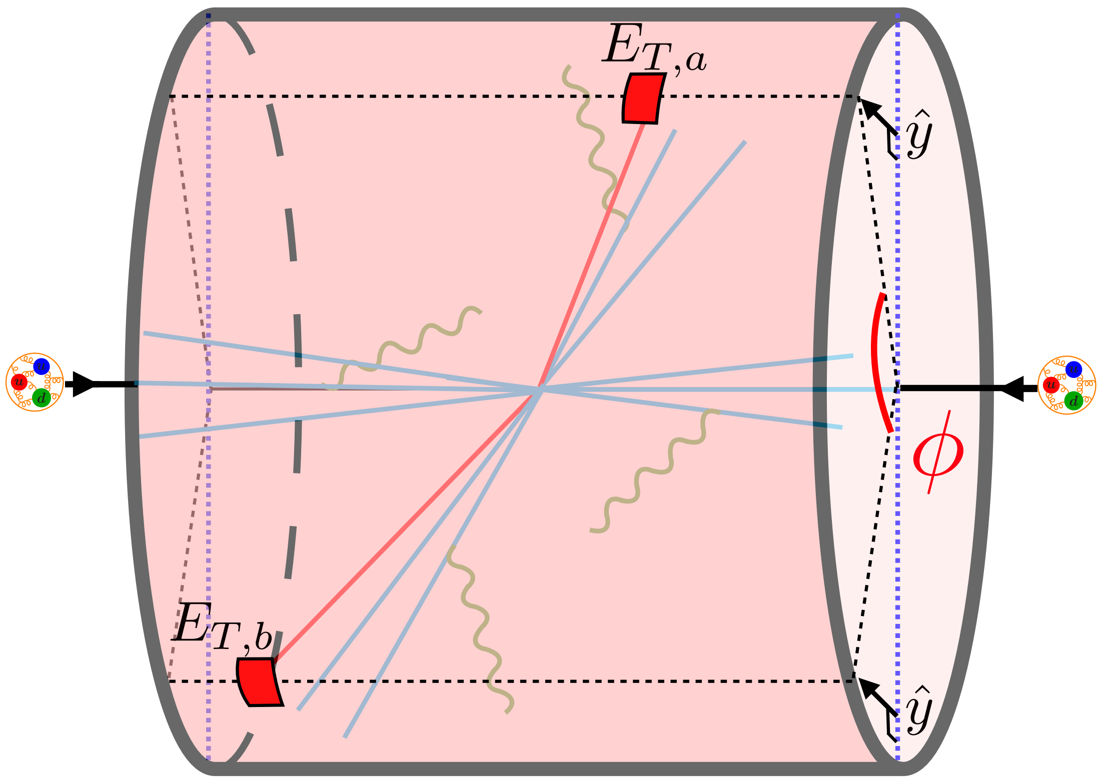

$$\frac{1+\cos\phi}{2} = \frac{\left(\frac{k_{3,y}}{\xi_3} + \frac{k_{4,y}}{\xi_4} + k_{1,y} + k_{2,y} - k_{s,y}\right)^2}{4P_T^2} + \ldots$$

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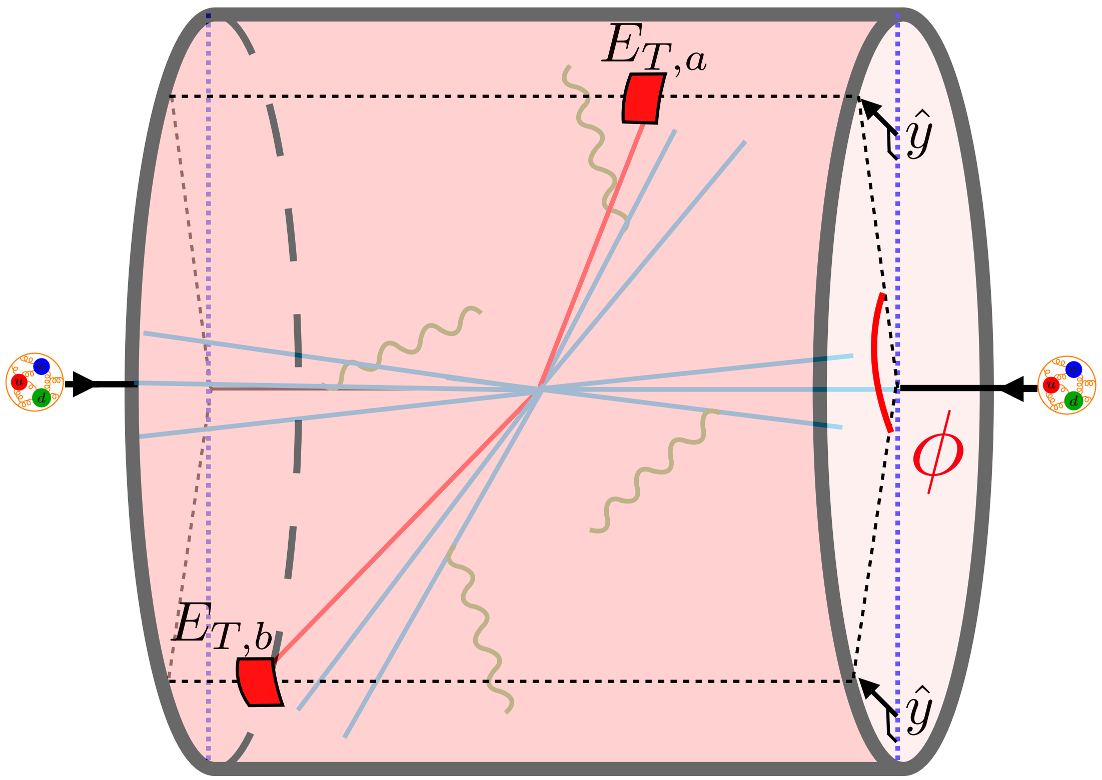

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final state collinear radiation

Jet Functions

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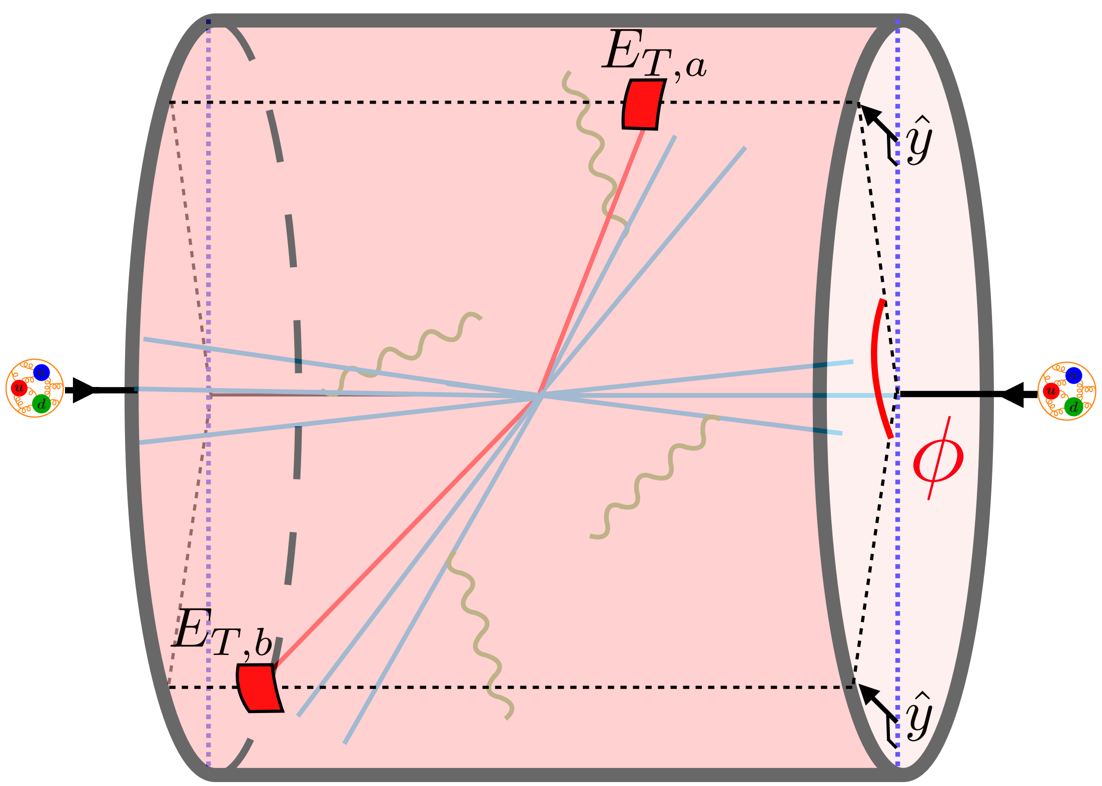

Beam Functions

initial state collinear radiation

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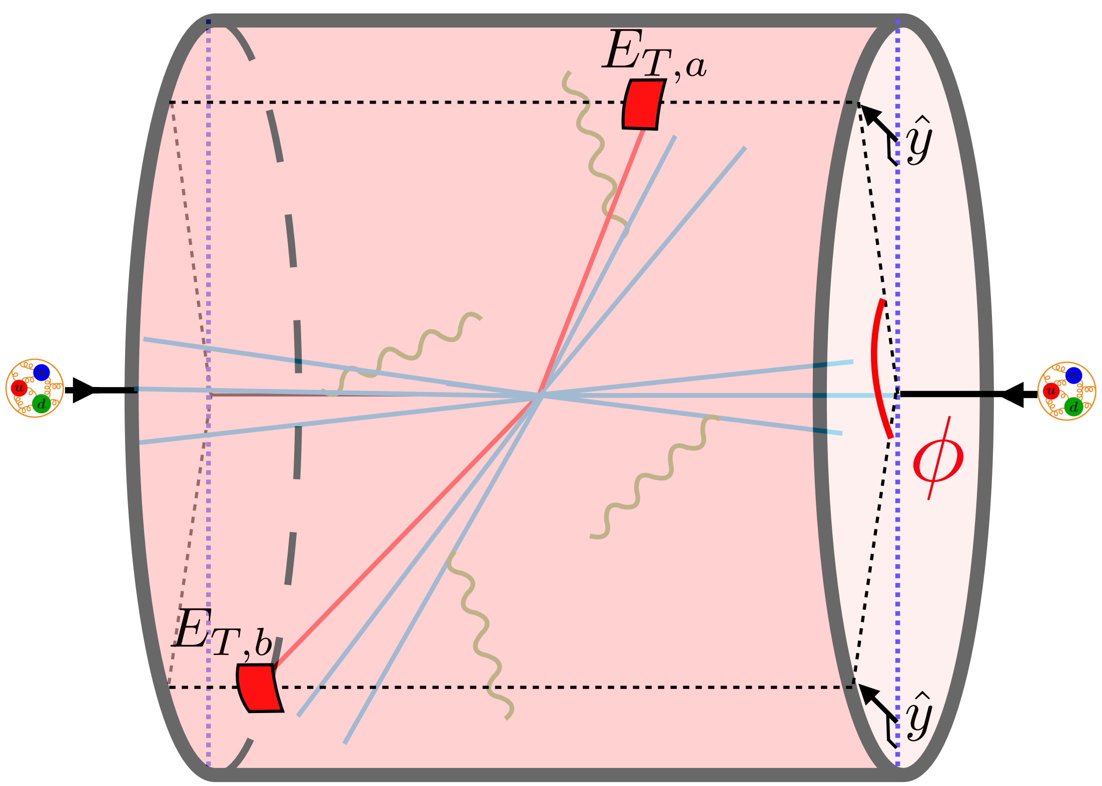

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final state collinear radiation

soft-recoil

Jet Functions

Soft Functions

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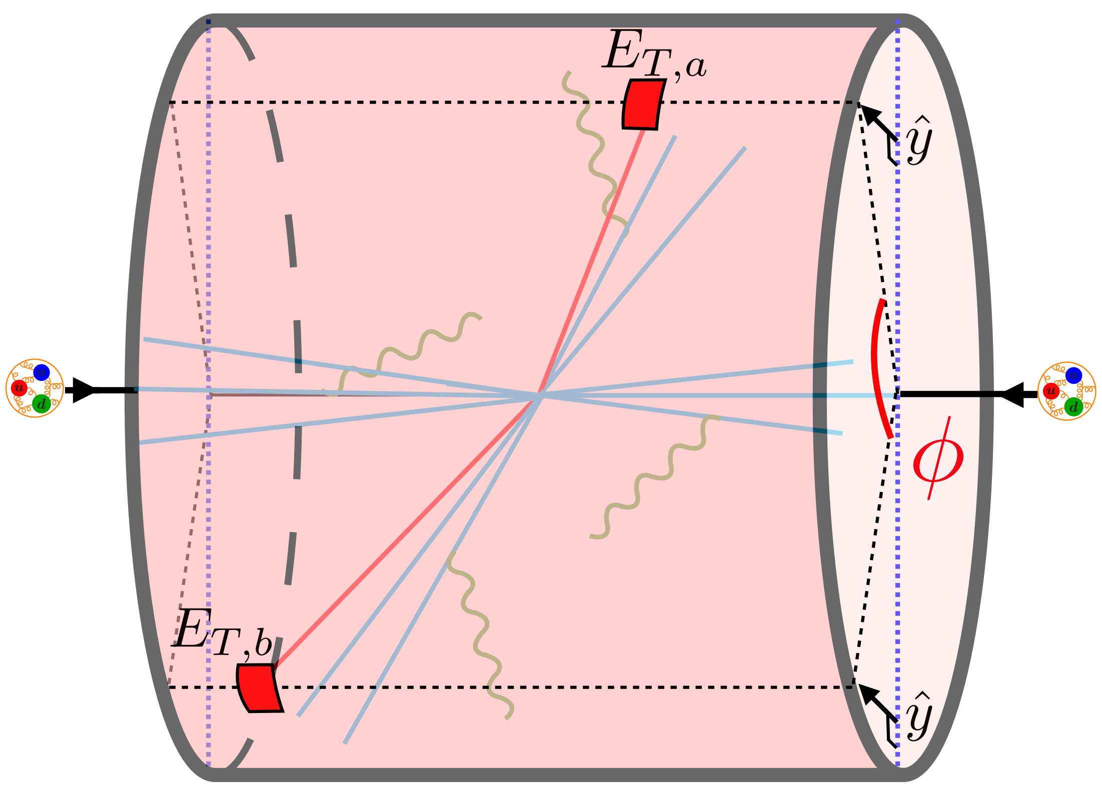

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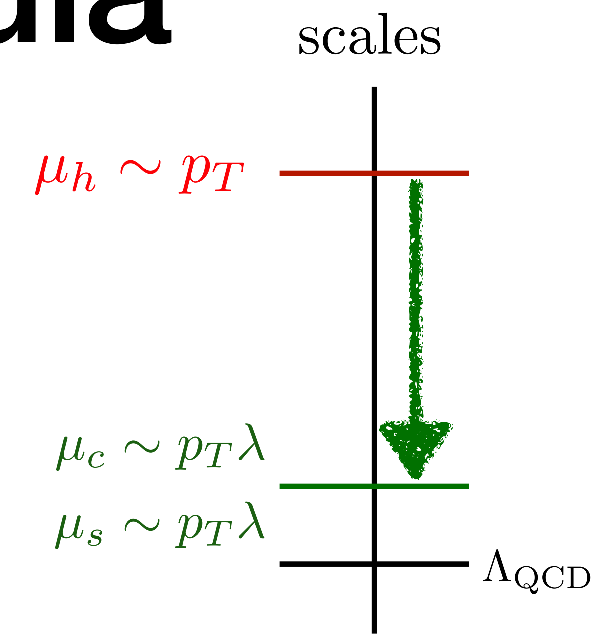
Jet Functions

Soft Functions

It is similar to the 1-dimensional TMD factorization

factorization formula

Soft and collinear momenta $\lambda = \frac{q_y}{p_T}$



$$p_s \sim Q(\lambda, \lambda, \lambda, \lambda) \qquad\qquad p_c \sim Q(1, \lambda, 1, 1)$$

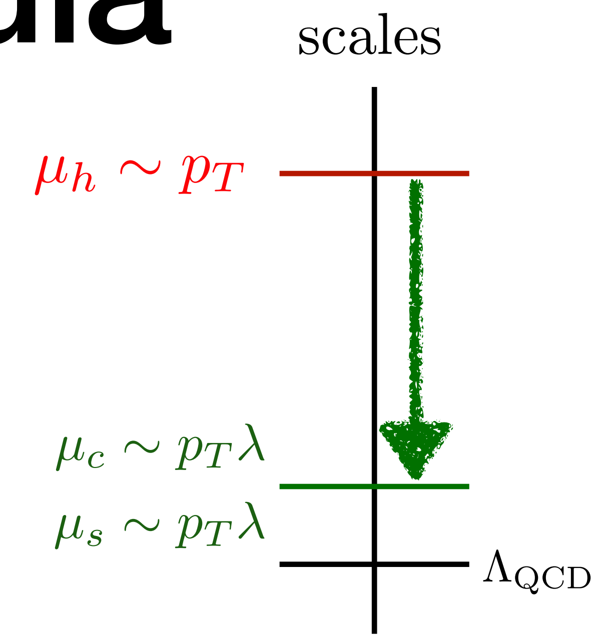
$$p_s^2 \sim Q^2\lambda^2 \qquad p_c^2 \sim Q^2\lambda^2$$

The exponential regulator was used to deal with the rapidity divergences

Li, Neill, Zhu, 2016

$$\int d^d k\theta(k^0)\delta(k^2) \to \int d^d k\theta(k^0)\delta(k^2)e^{-2k^0\hat{\tau}e^{\gamma_E}} \qquad\qquad \nu = \frac{1}{\hat{\tau}}$$

factorization formula



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The factorization formula in the back-to-back limit is

$$\frac{d\sigma^{(0)}}{d\tau} = \frac{p_T}{16\pi s^2(1+\delta_{f_3 f_4})\sqrt{\tau}} \sum_{\rm channels} \frac{1}{N_{\rm init}} \int \frac{dy_3 dy_4 dp_T^2}{\xi_1\xi_2} \int_{-\infty}^{\infty} \frac{db}{2\pi} e^{-2ib\sqrt{\tau}p_T} {\rm tr}\left[\mathbf{H}^{f_1 f_2 \to f_3 f_4}(p_T, y^*, \mu)\mathbf{S}(b, y^*, \mu, \nu)\right.$$
$$\cdot B_{f_1/N_1}(b,\, \xi_1,\, \mu,\, \nu)\, B_{f_2/N_2}(b,\, \xi_2,\, \mu,\, \nu) J_{f_3}(b, \mu, \nu)\, J_{f_4}(b, \mu, \nu)\,.$$

The azimuthal angle distribution

$$1 + \cos(\phi) \approx 2\tau \qquad \frac{d\sigma^{(0)}}{d\phi} = \frac{d\sigma^{(0)}}{d\tau}\frac{d\tau}{d\phi} + \mathcal{O}(\pi - \phi) \rightarrow \mathcal{O}\left(\frac{q_y}{p_T}\right)$$

Factorization formula

The hard functions for all 2->2 process in massless QCD are known up to NNLO

Broggio,Ferroglia,Pecjak,Zhang 2014

For example LO, gg->gg

$$\boldsymbol{H}^{(0)} = \left(\begin{array}{cccccc|c} a & b & c & c & b & a & \\ b & d & e & e & d & b & \\ c & e & f & f & e & c & \mathbf{0}_{3\times 6} \\ c & e & f & f & e & c & \\ b & d & e & e & d & b & \\ a & b & c & c & b & a & \\ \hline & & & \mathbf{0}_{6\times 3} & & & \mathbf{0}_{3\times 3} \end{array}\right)$$

Beam function: are identical to TMD beam functions $b^\mu = b(0,0,1,0)$

Gehrmann et al 2012 and 2014, Lubbert et al 2016, Echevarria et al 2016

Jet functions: $J_i = \sum_j \int_0^1 dx\ x\, \mathcal{I}_{ij}(\frac{b}{x}, x)$

Matching coefficients of TMD factorization function known up to NNLO

Echevarria et al 2016

Soft function

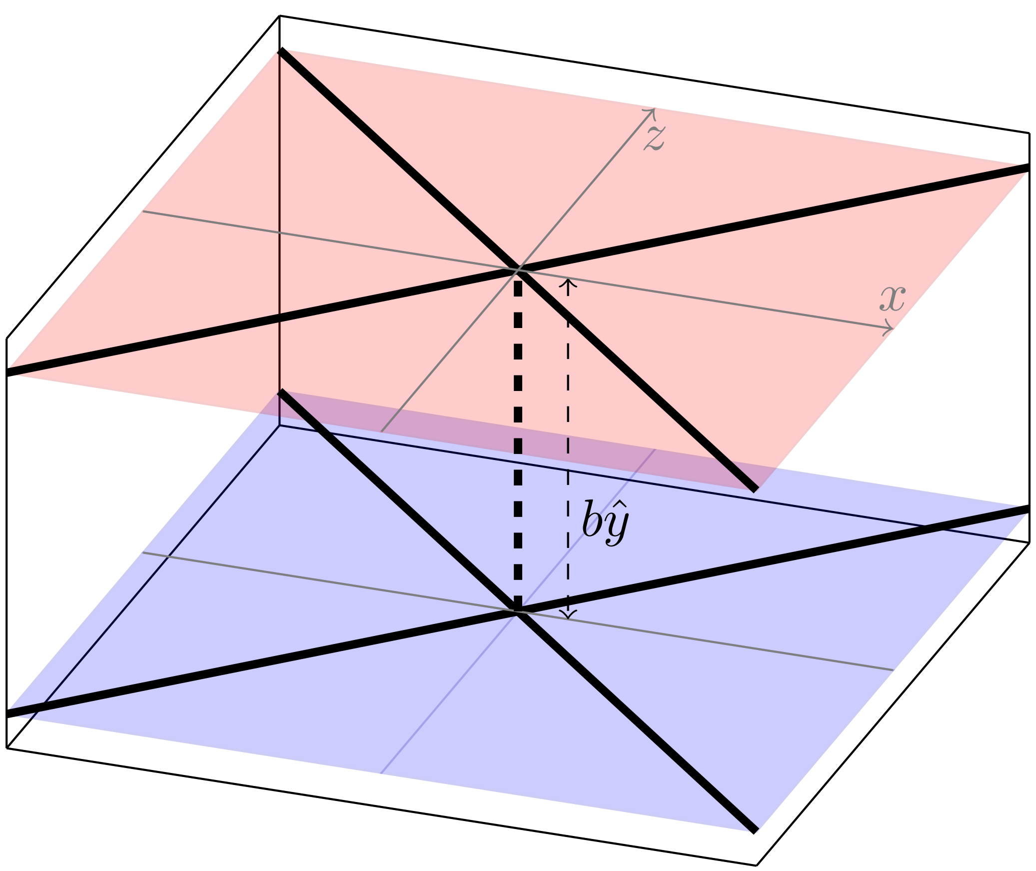
$$\mathbf{S}(b, y^*) = \langle 0 | T[\boldsymbol{O}_{n_1 n_2 n_3 n_4}(0^\mu)] \overline{T}[\boldsymbol{O}^\dagger_{n_1 n_2 n_3 n_4}(b^\mu)] | 0 \rangle$$

$$\boldsymbol{O}_{n_1 n_2 n_3 n_4}(x) = \boldsymbol{Y}_{n_1} \boldsymbol{Y}_{n_2} \boldsymbol{Y}_{n_3} \boldsymbol{Y}_{n_4}(x)$$

One-dimensional Fourier transformation $b^\mu = (0, 0, b, 0)$

scattering plane: x-z plane $n_i \cdot b = 0$

$$\mathbf{S}(b, y^*, \mu, \nu) = \mathbf{1} + \frac{\alpha_s}{4\pi} \mathbf{S}^{(1)}(y^*, L_b, L_\nu) + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{S}^{(2)}(y^*, L_b, L_\nu) + \mathcal{O}(\alpha_s^3)$$



$$\mathbf{S}^{(1)}(y^*, L_b, L_\nu) = -\sum_{i<j} (\mathbf{T}_i \cdot \mathbf{T}_j)\, S_\perp^{(1)}\left(L_b, L_\nu + \ln \frac{n_i \cdot n_j}{2}\right)$$

TMD soft function for color-singlet production at hadron colliders

When $n_i \cdot n_j = 2$ the soft integral gives

$$S_\perp^{(1)}(L_b, L_\nu) = 2L_b^2 - 4L_b L_\nu - 2\zeta_2$$

Soft function

NLO soft function $$\mathbf{S}^{(1)}(y^*, L_b, L_\nu) = -\sum_{i<j} (\mathbf{T}_i \cdot \mathbf{T}_j)\, S_\perp^{(1)} \left(L_b, L_\nu + \ln \frac{n_i \cdot n_j}{2}\right)$$

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NNLO soft function

Double soft gluon emission limit

Catani and Grazzini 1999

Catani and Grazzini 2000

$$|\mathcal{M}_{g,g,a_1,\dots,a_n}(q_1, q_2, p_1, \dots, p_n)|^2 \simeq \left(4\pi\alpha_S \mu^{2\epsilon}\right)^2$$
$$\cdot \left[\frac{1}{2} \sum_{i,j,k,l=1}^{n} \mathcal{S}_{ij}(q_1)\, \mathcal{S}_{kl}(q_2)\, |\mathcal{M}^{(i,j)(k,l)}_{a_1,\dots,a_n}(p_1, \dots, p_n)|^2 - C_A \sum_{i,j=1}^{n} \mathcal{S}_{ij}(q_1, q_2) |\mathcal{M}^{(i,j)}_{a_1,\dots,a_n}|^2\right]$$

convolution of the NLO soft integrals

non-abelian contribution

In the soft limit, the 2-loop soft integral has the dipole form

It is also true for the soft qqbar radiation and virtual real contributions
See AnJie's talk this afternoon

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See AnJie's talk this afternoon

$$\mathbf{S}^{(2)}(y^*, L_b, L_\nu) = \frac{1}{2!}\left(\mathbf{S}^{(1)}(y^*, L_b, L_\nu)\right)^2 - \sum_{i<j} (\mathbf{T}_i \cdot \mathbf{T}_j)\, S_\perp^{(2)}\left(L_b, L_\nu + \ln\frac{n_i \cdot n_j}{2}\right)$$

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Soft integral

For EEC $(n \cdot \bar{n} = 2)$

$$\int d^d l \; \delta^+(l^2) \; e^{ib_\perp \cdot l_\perp - 2l_0 \hat{\tau}} = \frac{1}{2} \int dl^+ dl^- d^{d-2} l_\perp \; \delta(l^+ l^- - l_\perp^2) \; e^{-(l^+ + l^-)\hat{\tau} + ib_\perp \cdot l_\perp}$$

For TEEC $(n \cdot \bar{n} = 1 - \cos\theta)$

$$l^\mu = \frac{1}{n \cdot \bar{n}} (l^- n_\mu + l^+ \bar{n}^\mu) + l_\perp^\mu \qquad\qquad l_\perp^\mu = l_y n_y^\mu + l_{\hat{x}} v_\perp^\mu$$

$$\int d^d l \; \delta^+(l^2) \; e^{ib_\perp \cdot l_\perp - 2l_0 \hat{\tau}} = \frac{1}{n \cdot \bar{n}} \int dl^+ dl^- d^{d-2} l_\perp \; \delta(\frac{2}{n \cdot \bar{n}} l^+ l^- - l_\perp^2) \; e^{-\frac{2}{n \cdot \bar{n}}(l^+ + l^-)\hat{\tau} + ib_\perp \cdot l_\perp - 2\hat{\tau} v_\perp^0}$$

Soft integral

For EEC $(n\cdot\bar{n}=2)$

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For TEEC $(n\cdot\bar{n}=1-\cos\theta)$

$$\nu^2\to\nu^2\frac{n\cdot\bar{n}}{2}$$

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vanishes when we take the limit $\hat{\tau}\to 0$

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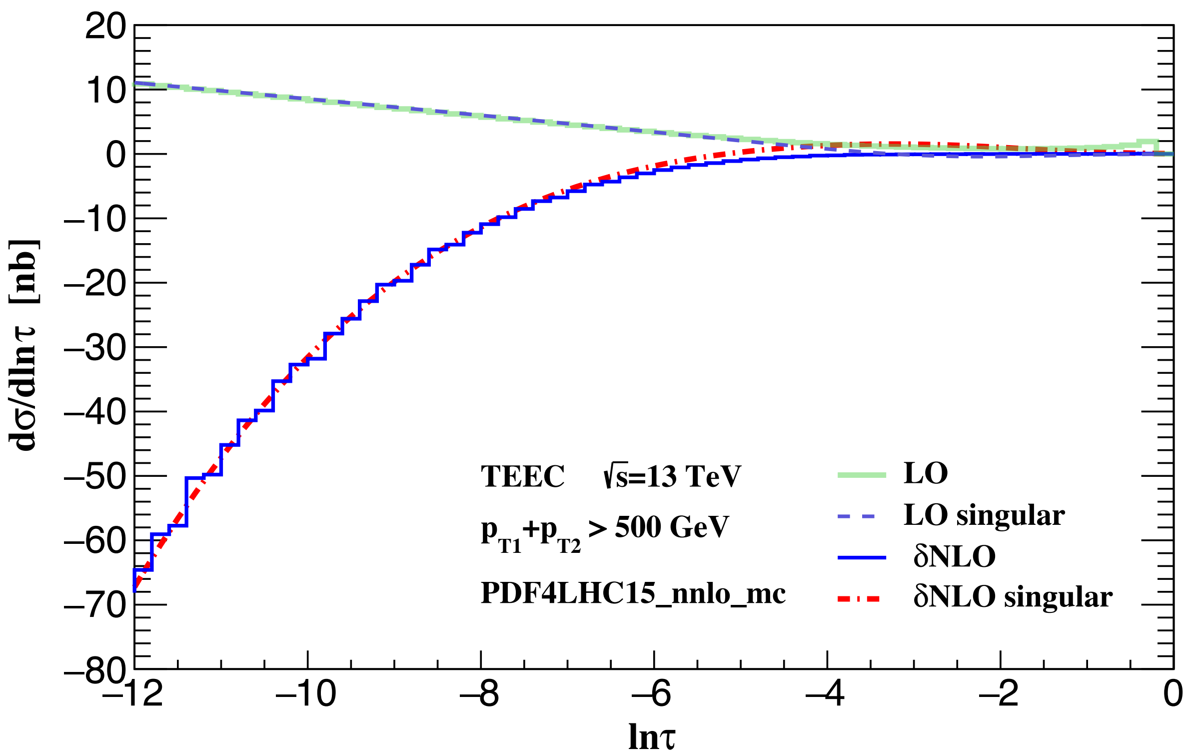
NNLO soft function

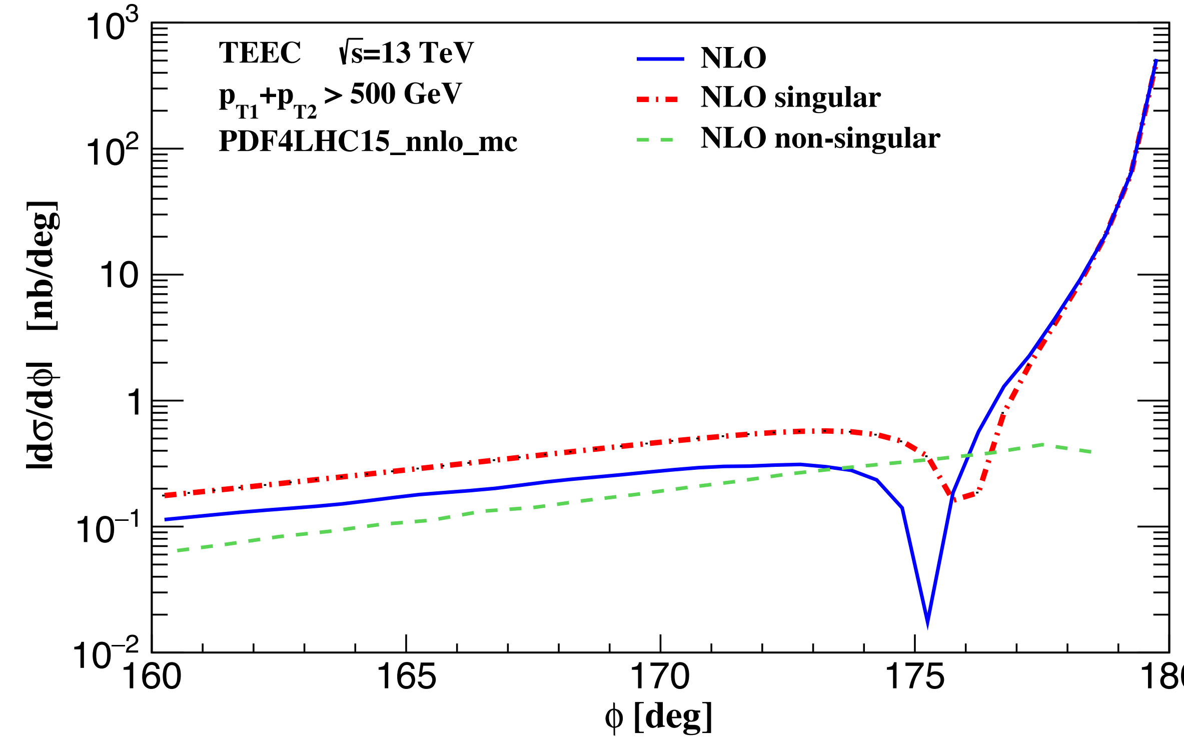
$$\mathbf{S}^{(2)}(y^*,L_b,L_\nu)=\frac{1}{2!}\left(\mathbf{S}^{(1)}(y^*,L_b,L_\nu)\right)^2-\sum_{i<j}(\mathbf{T}_i\cdot\mathbf{T}_j)\,S_\perp^{(2)}\left(L_b,L_\nu+\ln\frac{n_i\cdot n_j}{2}\right)$$

The first analytical soft function for dijet production

For N-jettiness soft function for dijet numerically,
see Bell et al 2018 (package SoftSERVE), Jin and Liu 2019

Numerical results





Fixed order results are calculated using NLOJET++

This is the first time that the singular behavior for a dijet differential distribution is under full control at this order.

It seems that the power corrections do not go to zero $\mathcal{O}(k_y/p_T) \sim \mathcal{O}(180-\phi)$

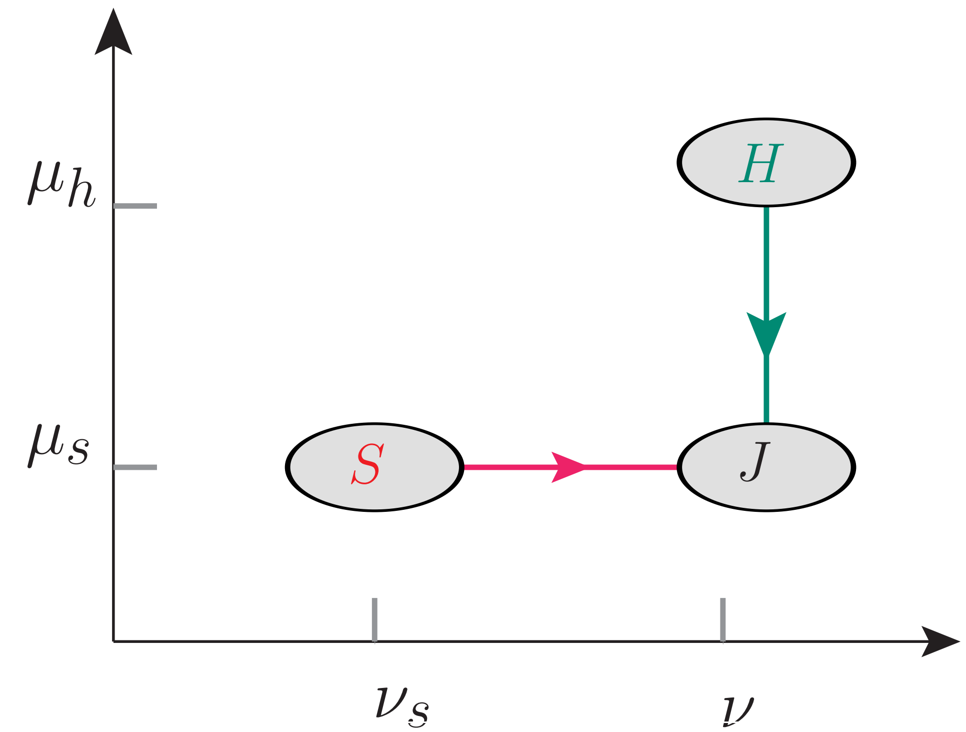
require more computation power to reach the limit

Resummation

To deal with Landau pole

$$b^* = \frac{b}{\sqrt{1 + b^2/b_{\max}^2}}$$

$$\mu_s = \mu_I = \nu_s = b_0/b^* \qquad \mu_h = \nu = p_T$$



The resummation is achieved by

- running the hard function from the hard scale to soft scale
- running the soft function from the soft rapidity scale to collinear rapidity scale.

Order	$H, B, S, \mathscr{S}, f$	γ_X^i	Γ_{cusp}^i	β
LL	LO		1-loop	1-loop
NLL	LO	1-loop	2-loop	2-loop
NNLL	NLO	2-loop	3-loop	3-loop
NNNLL	NNLO	3-loop	4-loop	4-loop

RG equations

RG Evolution

$$\frac{d\mathbf{H}}{d\ln\mu^2} = \frac{1}{2}\left(\boldsymbol{\Gamma}_H \cdot \mathbf{H} + \mathbf{H}\cdot\boldsymbol{\Gamma}_H^\dagger\right) \qquad \frac{d\mathbf{S}}{d\ln\mu^2} = \frac{1}{2}\left(\boldsymbol{\Gamma}_S^\dagger \cdot \mathbf{S} + \mathbf{S}\cdot\boldsymbol{\Gamma}_S\right)$$

$$\boldsymbol{\Gamma}_H = -\sum_{i<j}\mathbf{T}_i\cdot\mathbf{T}_j\gamma_{\rm cusp}\ln\frac{\sigma_{ij}\hat{s}_{ij}+i0}{\mu^2} + \sum_i \gamma_i \mathbf{1} + \gamma_{\rm quad}$$

3-loop soft anomalous dimensions
Almelid et al 2015 and 2017

$$\boldsymbol{\Gamma}_S = \sum_{i<j}\mathbf{T}_i\cdot\mathbf{T}_j\gamma_{\rm cusp}\ln\frac{\nu^2\, n_i\cdot n_j}{2\mu^2} - \sum_i \frac{c_i}{2}\gamma_s\mathbf{1} - \gamma_{\rm quad}$$

$$\frac{dG_i}{d\ln\mu^2} = \left(-\frac{1}{2}c_i\gamma_{\rm cusp}\ln\frac{4(p_i^0)^2}{\nu^2} + \gamma_{G,i}\right)G_i$$

Beam and jet

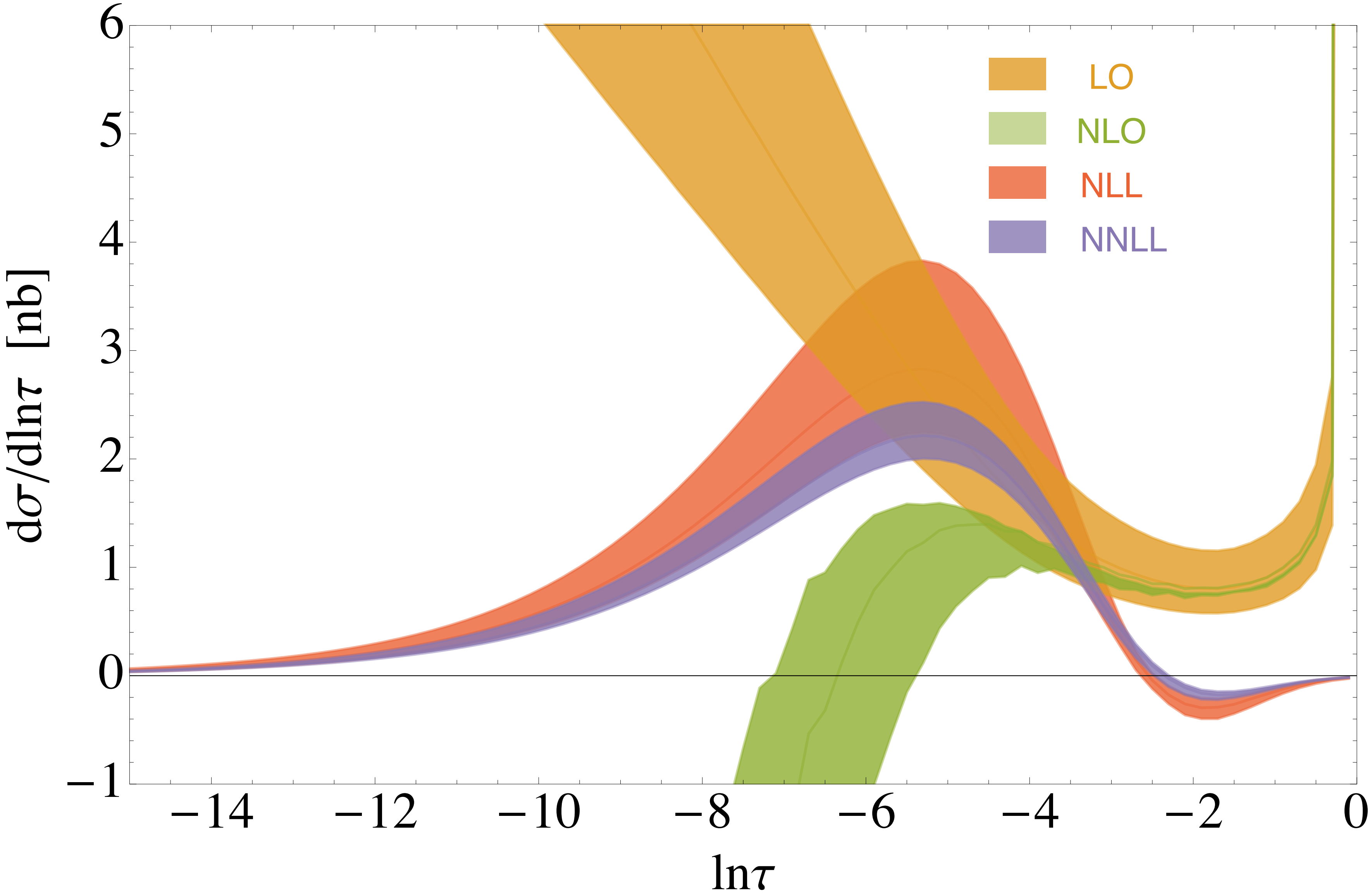
Evolution associated with the rapidity scale

$$\frac{d\mathbf{S}}{d\ln\nu^2} = \frac{1}{2}\left(\boldsymbol{\Gamma}_y^\dagger\cdot\mathbf{S} + \mathbf{S}\cdot\boldsymbol{\Gamma}_y\right) \qquad \frac{dG_i}{d\ln\nu^2} = \frac{c_i}{2}\left(\int_{b_0^2/b^2}^{\mu^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\gamma_{\rm cusp}[\alpha_s(\bar{\mu})] - \gamma_r[\alpha_s(b_0/b)]\right)G_i$$

$$\boldsymbol{\Gamma}_y = \left(\int_{\mu^2}^{b_0^2/b^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\gamma_{\rm cusp}[\alpha_s(\bar{\mu})] + \gamma_r[\alpha_s(b_0/b)]\right)\sum_i c_i\mathbf{1} + \gamma_X[y^*,\alpha_s(b_0/b)]$$

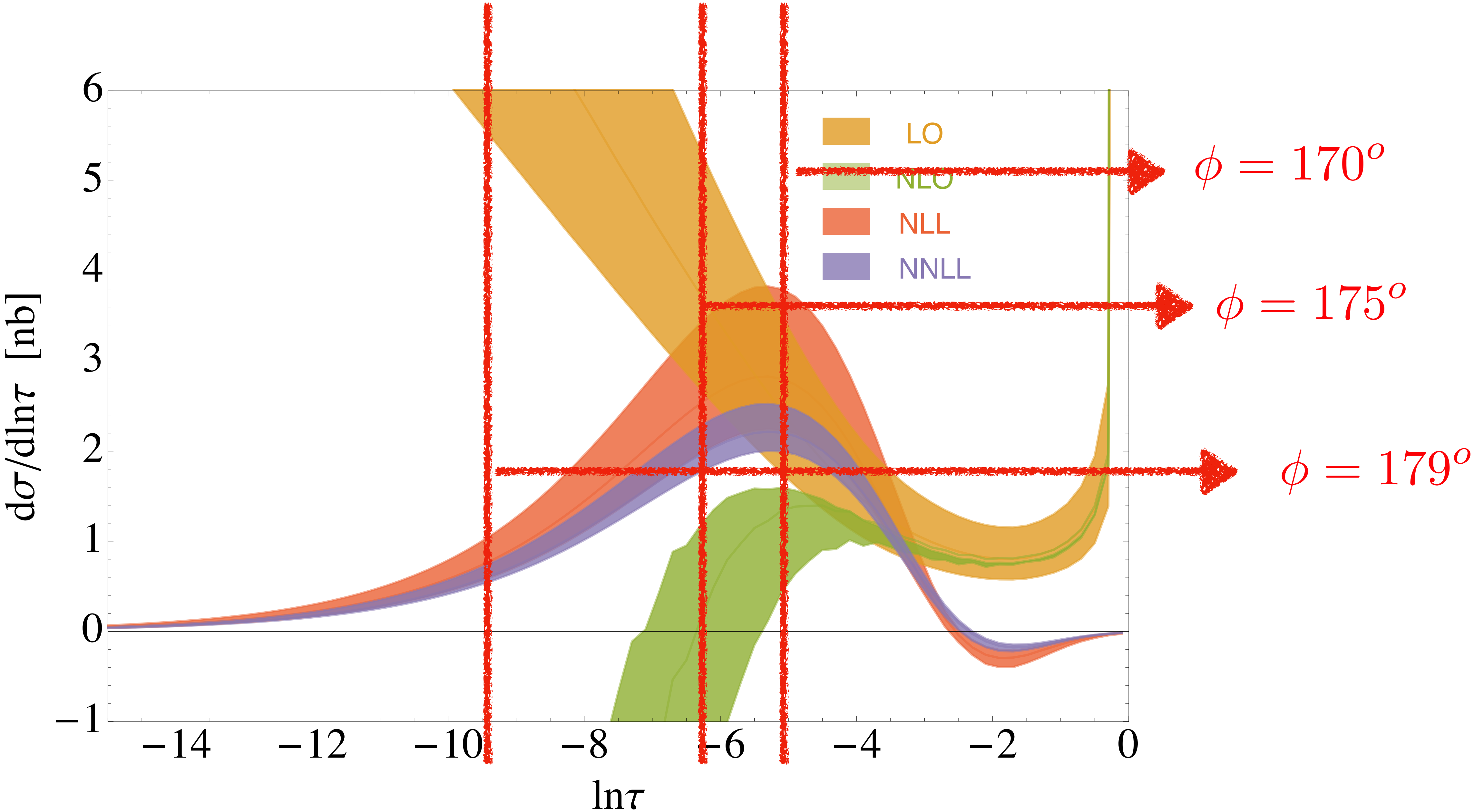
possible factorization violation

Resummed—lnτ



- It is divergent for fixed order calculations
- The resummation cures the divergences
- There is a reduction of scale uncertainties from NLL to NNLL

Resummed—lnτ



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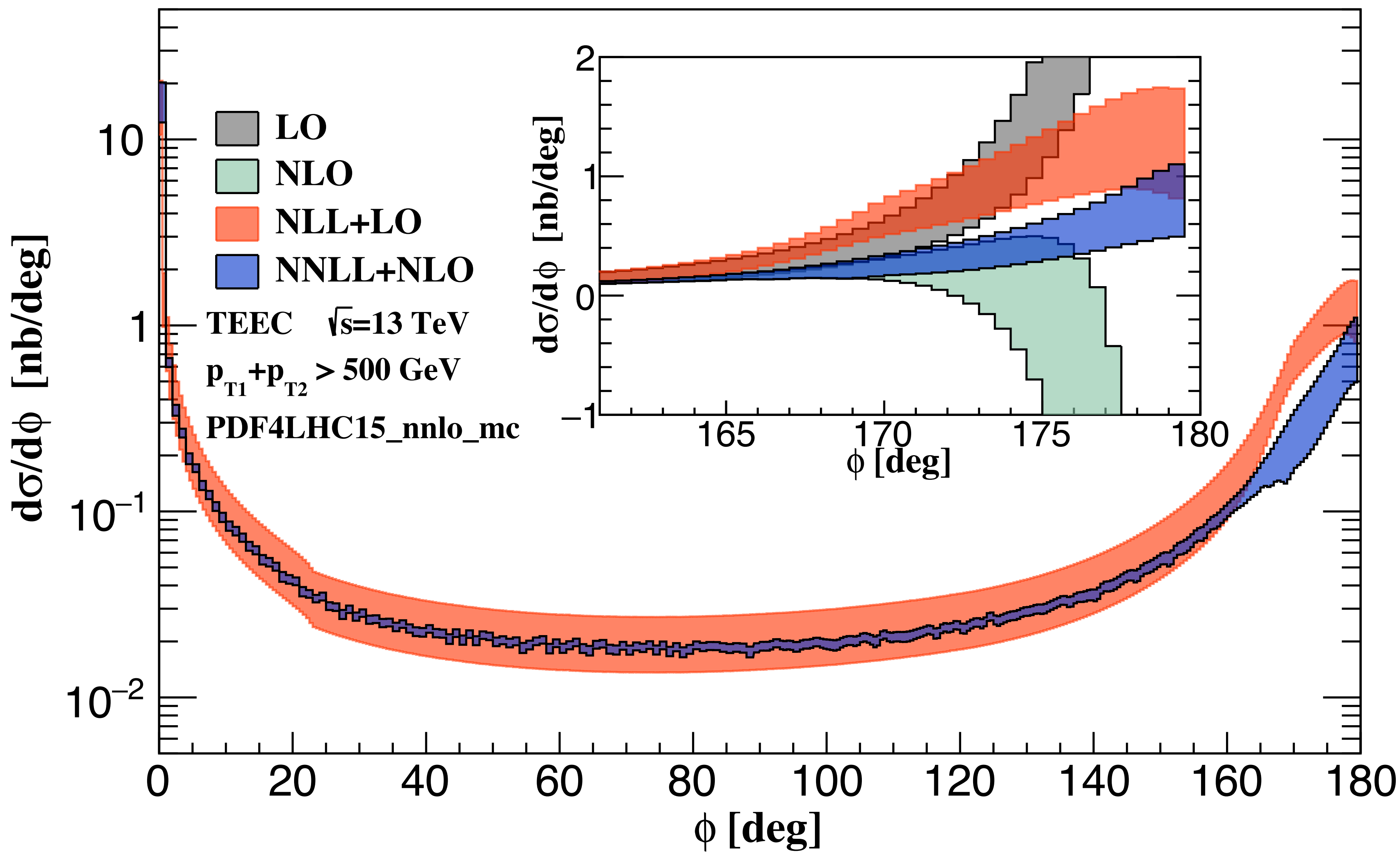
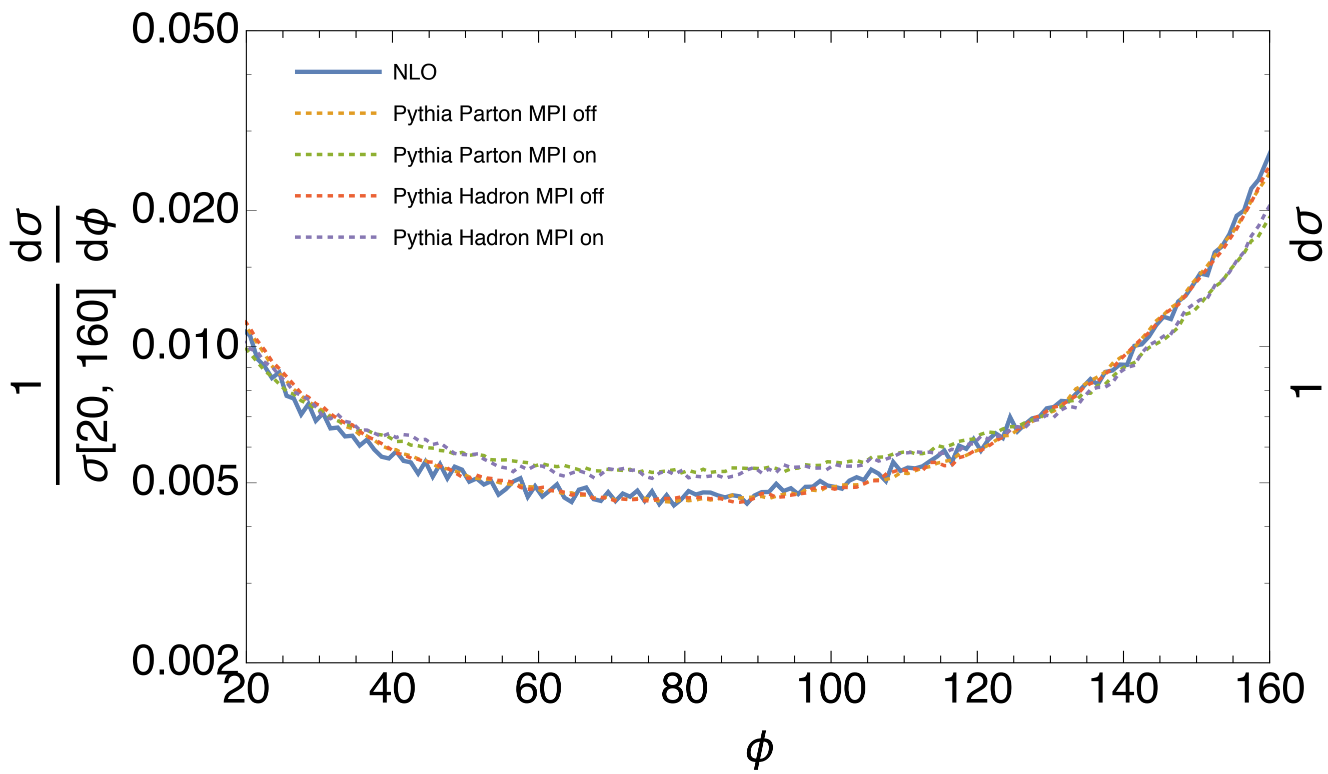
Resummed—ϕ



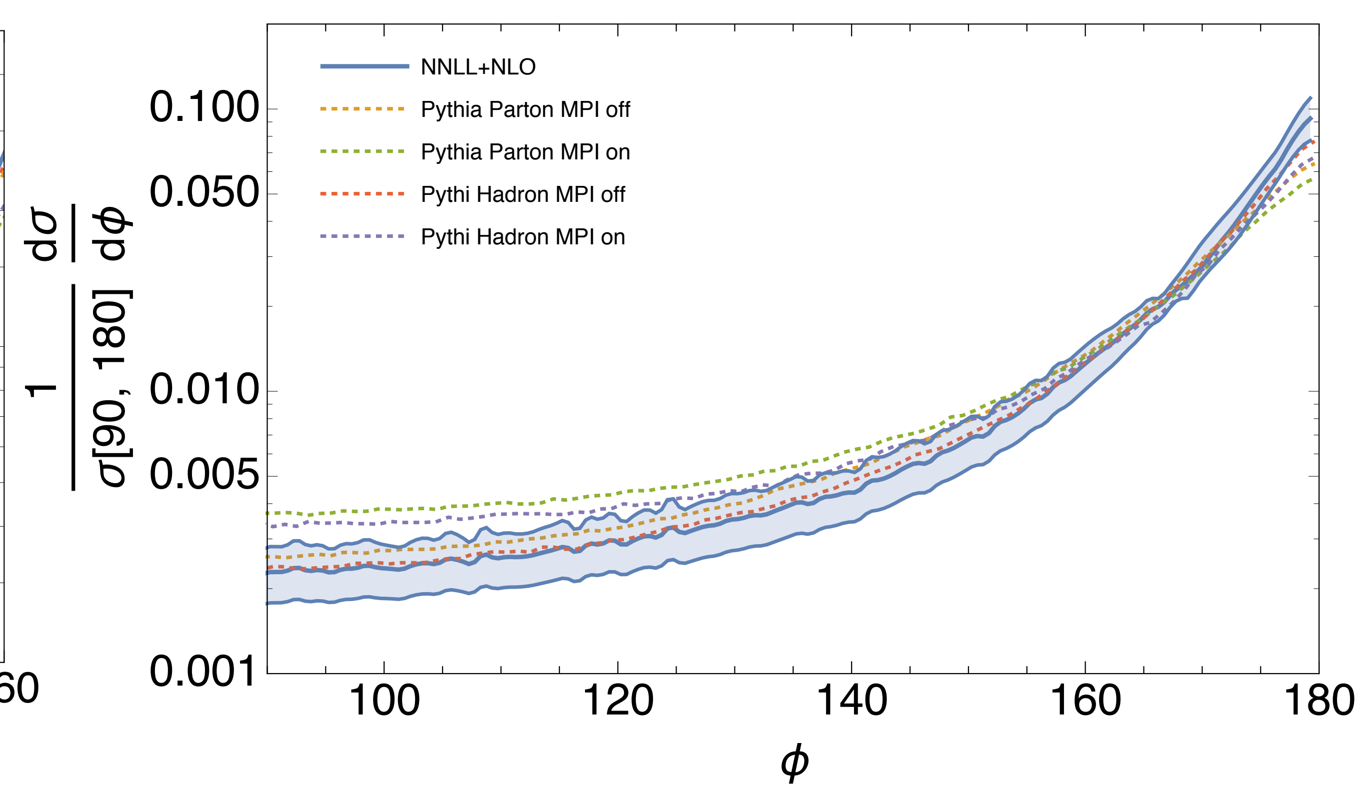
- The transition range is [155,170]
- It is divergent in the collinear limits
- There is no peak in the back-to-back limits for the resummed distributions

Compare with PYTHIA

Fixed order vs PYTHIA



NNLL+NLO vs PYTHIA



- The effects from MPI and Hadronization are small compared to other event shape observables
- Normalized distribution agrees with the PYTHIA predictions, especially when MPI is off

Conclusion

- studied the TEEC in the framework of SCET
- discussed the beam, hard, jet and soft functions
- calculated the singular distribution up to NLO
- present the NNLL+NLO angle distribution and compare with PYTHIA

There are many things to do

- include the non-perturbative corrections
- try to improve the results in the collinear limit
- predict NNLO singular contribution to TEEC using the 3-loop anomalous dimensions
- study TEEC in heavy ion collisions
- ……

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- ……

Thank you

polarized gluon beam function

The contribution from the traceless tensor structure vanishes

$$\int_{-\infty}^{\infty} dk_x dk_y e^{-ibk_y} \frac{k_y^{-2\epsilon}}{k_\perp^2} \left(\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \frac{g_\perp^{\mu\nu}}{2} \right) \to 0$$

We have $\mathcal{B}^{\mu\nu} \propto g_\perp^{\mu\nu}$