SCET for gravity

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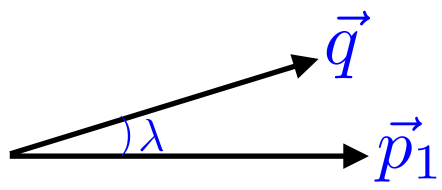
Example full theory and process:

$(c = \hbar = 8\pi G_\mathrm{N} = 1)$

Full theory: $\mathcal{L} = -\frac{R}{2} + \frac{g^{\mu\nu}}{2}\partial_\mu\phi\,\partial_\nu\phi - \frac{\kappa}{3!}\phi^3$ $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (graviton)

Process: $\phi(p_1) + \phi(p_2) \longrightarrow \phi(p_3) + \phi(p_4) + h_{\mu\nu}(q)$ with $\lambda \ll 1$



everything else "O(1)" to one another

Question: How does amplitude behave as a power series in λ?

Lightcone coordinates collinear to p_1:

$$\mathrm{d}s^2 = 2\mathrm{d}x^+\mathrm{d}x^- - \mathrm{d}x^i\mathrm{d}x^i \quad (i = 1, 2)$$

$$p_1^+ = p_{1-} \sim 1\,, \quad p_1^- = p_{1+} = 0\,, \quad p_1^i = -p_{1i} = 0$$

by definition

Our process characterized by

$$(q^+, q^-, q^i) = (q_-, q_+, -q_i) \sim (1, \lambda^2, \lambda) \longrightarrow q^2 \sim \lambda^2$$

$$p^\mu_{2,3,4} \sim 1 \quad (\mu = +, -, i)$$

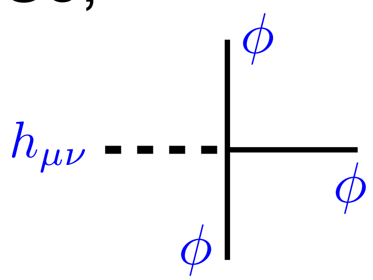
In covariant gauges, graviton must scale as $h_{\mu\nu}(q) \sim \lambda^a q_\mu q_\nu$.

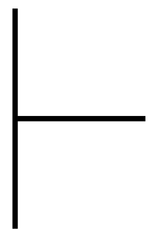
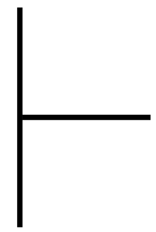
Then, from graviton kinetic term, we see

$$1 \sim \int \mathrm{d}^4x\,(\partial_* h_{**})^2 \sim \lambda^{-4}(q_*q_*q_*)^2\lambda^{2a} \sim \lambda^{-4}(q^2)^3\lambda^{2a} \sim \lambda^{-4+6+2a}$$

$\longrightarrow$ $h_{\mu\nu}(q) \sim \frac{q_\mu q_\nu}{\lambda}$ So, $h_{--} \sim \frac{1}{\lambda}$ $\quad h_{-i} \sim 1$ $\quad h_{-+} \sim \lambda$ etc.

So,

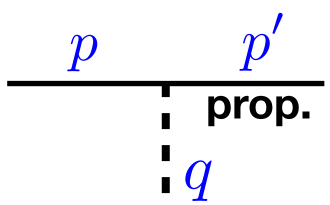


$= \frac{h}{2} \times$  $\sim \lambda \times$  because $h = h^\mu_\mu \sim \frac{q^2}{\lambda} \sim \lambda$

(from $\sqrt{-g} = 1 + h/2$)

$\parallel$ = "collinear to"

$\nparallel$ = "not collinear to"



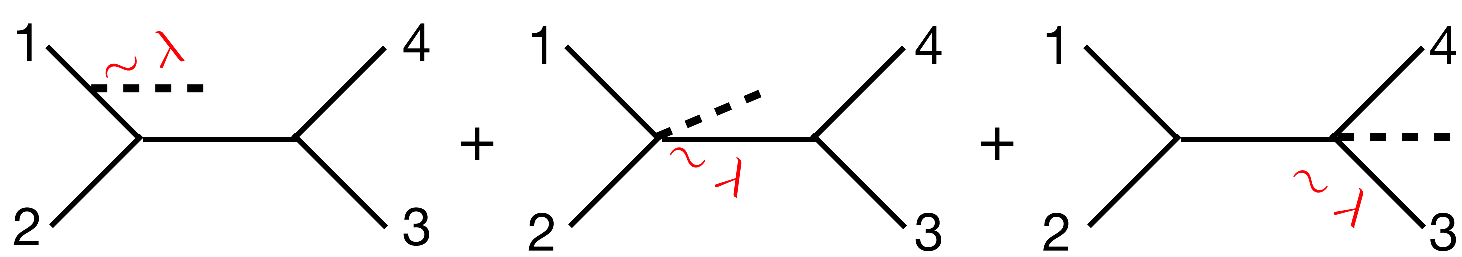
$$\propto \frac{p^\mu p'^\nu h_{\mu\nu} - (p\cdot p')h/2}{p'^2} \sim \frac{(p\cdot q)(p'\cdot q) + (p\cdot p')q^2}{\lambda p'^2} \begin{cases} \sim \frac{\lambda^2\lambda^2}{\lambda\lambda^2} \sim \lambda & \text{for } p \parallel q \\ \sim \frac{1\cdot 1}{\lambda \cdot 1} \sim \frac{1}{\lambda} & \text{for } p \nparallel q \end{cases}$$

(from phi kinetic term)

$h_{\mu\nu} \sim \frac{q_\mu q_\nu}{\lambda}$

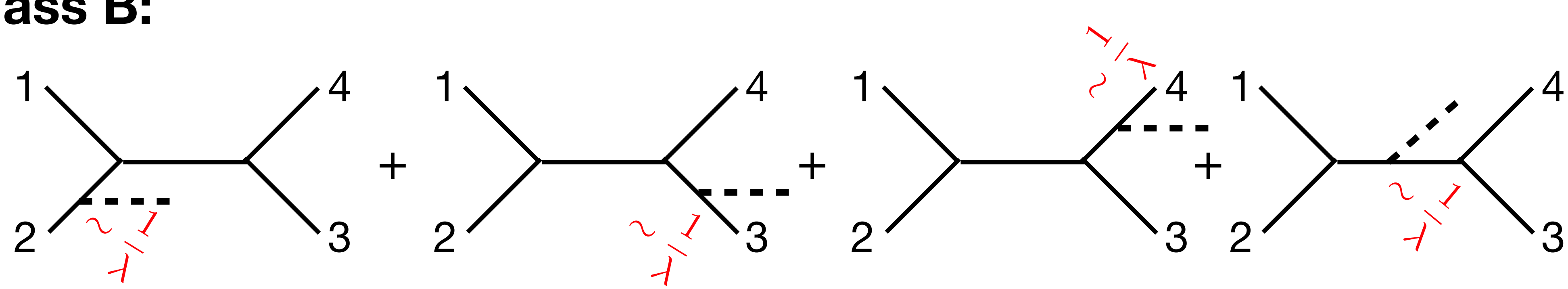
Full theory diagrams for $\phi(p_1) + \phi(p_2) \longrightarrow \phi(p_3) + \phi(p_4) + h_{\mu\nu}(q)$ with $q \parallel p_1$ and $q \nparallel p_{2,3,4}$:

Class A:



+ (t-channel) + (u-channel) $\sim \lambda$

Class B:



+ (t, u-channels) $\sim \frac{1}{\lambda}$ **?**

WRONG! The *grand total* of Class B diagrams is $\sim \lambda$!

$$\text{Each diagram} = \frac{\text{vertex}}{\text{propagator}} \sim \frac{\frac{1}{\lambda} + 1 + \lambda}{1 + \lambda + \lambda^2} \sim \frac{1}{\lambda} + 1 + \lambda$$

but

Sum of all diagrams $\sim \lambda$!

Cancellations of a large number of terms from a large number of diagrams!

Such cancellations first were hinted at within eikonal approximation by Weinberg in 1965.

Proven with a complete generality and rigor by Akhoury, Saotome and Sterman in 2011 via a typical "Sterman" style with power counting, combinatorics and Ward identities.

Such behaviors of collinear amplitudes are completely obscure in full theory Lagrangian.

But, if constructed properly,

SCET Lagrangian should tell us that collinear graviton interactions are manifestly $\sim \lambda$!

The first terms ever allowed by symmetries of EFT must be $\sim \lambda$ from the outset.
No cancellations or manipulations of terms should be necessary.

Such EFT will lead to efficient calculations of collinear gravitons without large cancellations!

We were able to derive such SCET. What follows is a quick run-through of our construction.

Two types of graviton couplings:

(1) Those unrelated by gauge invariance to 0-graviton terms:

$$R_{\mu\nu\rho\sigma} \qquad R_{\mu\nu} \qquad R$$

(2) Those related by gauge invariance to 0-graviton terms:

$g_{\mu\nu}$ (and vierbein) $\qquad \nabla_\mu \qquad$ **Wilson lines!**

Type-1 objects are already $\sim \lambda$ or higher:

$$R_{\mu\nu\rho\sigma} \sim \partial_* \partial_* h_{**} \sim \frac{q_{[\mu} q_{\nu]} q_{[\rho} q_{\sigma]}}{\lambda}$$

Largest components = $R_{-i-j} \sim \dfrac{q_- q_i q_- q_j}{\lambda} \sim \dfrac{1 \cdot \lambda \cdot 1 \cdot \lambda}{\lambda} \sim \lambda$

$$R_{\mu\nu} \sim \frac{q_\mu q_\nu q^2}{\lambda} \sim \lambda q_\mu q_\nu$$

Largest components = $R_{--} \sim \lambda q_- q_- \sim \lambda$

$$R \sim \lambda q^2 \sim \lambda^3$$

So, any insertion of type-1 operators is manifestly $\sim \lambda$!

For type-2, we need to understand gauge symmetry structure:

Gauge group: $G = \mathrm{diff} \times \mathrm{Lorentz}$ (diffeomorphism and local Lorentz)

Vierbein ${e^a}_\mu$, graviton field $\varphi_{\mu\nu}$, its relation to old graviton field $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{ab}\, {e^a}_\mu {e^b}_\nu \qquad {e^a}_\mu = \delta^a_\mu + {\varphi^a}_\mu \qquad h_{\mu\nu} = \varphi_{\mu\nu} + \varphi_{\nu\mu} + O(\varphi^2)$$

Graviton transformation under G:

$$\delta\varphi_{\mu\nu} = \underbrace{\partial_\nu \xi_\mu}_{\text{diff}} + \underbrace{\omega_{\mu\nu}}_{\text{Lorentz}} + \cdots$$

Power counting:

$$\varphi_{\mu\nu} \sim h_{\mu\nu} \sim \frac{q_\mu q_\nu}{\lambda} \qquad \xi_\mu \sim \frac{q_\mu}{\lambda} \qquad \omega_{\mu\nu} \sim \frac{q_{[\mu} q_{\nu]}}{\lambda}$$

My convention: *Make everything a diff scalar by appropriately multiplying vierbeins.*

So, for collinear sector of our interest (i.e., collinear to p_1), collinear operators transform as

$$\delta\mathcal{O}_{\mathrm{c}} = \xi^\mu \partial_\mu \mathcal{O}_{\mathrm{c}} + \frac{1}{2}\omega_{\mu\nu} J^{\mu\nu} \mathcal{O}_{\mathrm{c}} \sim \frac{q^\mu q_\mu}{\lambda}\mathcal{O}_{\mathrm{c}} + \cdots \sim \lambda \mathcal{O}_{\mathrm{c}} \qquad \text{(e.g., } \mathcal{O}_{\mathrm{c}} = \phi_1\text{)}$$

Also, *in SCET, gauge transformations are also divided up into separate collinear sectors.*

So non-collinear operators do not transform under collinear G:

$$\delta\mathcal{O}_{\mathrm{nc}} = 0 \qquad \text{(e.g., } \mathcal{O}_{\mathrm{nc}} = \phi_{2,3,4}\text{)}$$

As $\lambda \to 0$, nothing transforms under collinear G, so type-2 collinear graviton couplings (i.e., those required by gauge invariance) vanish as $\lambda \to 0$!
SCET symmetry says that type-2 couplings must be $\sim \lambda$!

But do building blocks of theory say they are *manifestly* $O(\lambda)$?

Two types of type-2 couplings:

(2A) Collinear graviton couplings to purely collinear operators.

Arise from $g_{\mu\nu}$, $e^{a}{}_{\mu}$, and ∇_μ inside the operators.

(2B) Collinear graviton couplings to hard (= collinear + non-collinear) interactions.

Some arise from $g_{\mu\nu}$, $e^{a}{}_{\mu}$, and ∇_μ inside the operators.

Others arise from Wilson lines acting on collinear field inside hard interaction.

Are type-2A couplings manifestly $\sim \lambda$?

Yes!

Take a term in $\mathcal{L}$ containing a collinear $h_{\mu\nu}$. It's divided by $M_{\rm Pl}$.

So, amplitude is $\sim E/M_{\rm Pl}$. How does E scale with λ?

Boost frame such that $(p^+, p^-, p^i) \sim (1, \lambda^2, \lambda) \longrightarrow (\lambda, \lambda, \lambda)$.

If term is purely collinear, everybody's boosted same way. All energy scales are $\sim \lambda$.

Therefore, $E \sim \lambda$. Term is a scalar so $\sim \lambda$ in all frames.

Are type-2B couplings manifestly $\sim \lambda$?

Power counting shows those from $g_{\mu\nu}$, $e^{a}{}_{\mu}$, and ∇_μ are $\sim \lambda$. (No time for showing this.)

For Wilson lines, let's construct it and see.

Diff collinear Wilson lines:

A collinear operator inside a hard interaction transforms under diff as

$$\delta\mathcal{O}_{\rm c} = \xi^\mu \partial_\mu \mathcal{O}_{\rm c}$$ (Recall everything is made a diff scalar.)

The rest of hard interaction is non-collinear so doesn't transform: $\delta\mathcal{O}_{\rm nc} = 0$.

So, we must cancel right-hand side above for gauge invariance. Since

$$\delta\Gamma^\mu_{--} = \partial_-^2 \xi^\mu$$ (Christoffel connection)

we see that this Wilson line,

$$W_{\rm d} = 1 - \left(\frac{1}{\partial_-^2}\Gamma^\mu_{--}\right)\partial_\mu \qquad\qquad \frac{1}{\partial_-} f(x^-) = \int_{-\infty}^{x^-} {\rm d}s\, f(s)$$

does the job:

$$\delta W_{\rm d} = -\left(\frac{1}{\partial_-^2}\partial_-^2\xi^\mu\right)\partial_\mu = -\xi^\mu\partial_\mu \longrightarrow \delta(W_{\rm d}\mathcal{O}_{\rm c}) = 0 \text{ to } O(\lambda^2)$$

And this coupling should be $\sim \lambda$. Let's check:

$$\frac{1}{\partial_-^2}\Gamma^\mu_{--}\partial_\mu \sim \frac{1}{q_-^2}\frac{q^\mu q_- q_-}{\lambda} q_\mu \sim \frac{\lambda^2}{\lambda} \sim \lambda$$

Indeed!

Lorentz collinear Wilson lines:

Local Lorentz group doesn't act on coordinates so direct analogy with spin-1 case holds:

$$W_{\rm L} = \hat{\mathcal{P}} \exp\left[-\frac{1}{2\partial_-}\gamma_{-\alpha\beta}J^{\alpha\beta}\right]$$

$\gamma_{\mu\alpha\beta}$ = spin connection

$J^{\alpha\beta}$ = Lorentz generator

$\nabla_\mu = \partial_\mu + \frac{1}{2}\gamma_{\mu ab}J^{ab}$

Let's check that this coupling is $\sim \lambda$:

$$\frac{1}{\partial_-}\gamma_{-\alpha\beta}J^{\alpha\beta}\mathcal{O}_{\rm c} \sim \frac{1}{q_-}\frac{q_- q_{[\alpha}q_{\beta]}}{\lambda}J^{\alpha\beta}\mathcal{O}_{\rm c} \sim \frac{q_{[\alpha}q_{\beta]}}{\lambda}J^{\alpha\beta}\mathcal{O}_{\rm c} \longrightarrow \frac{\lambda\lambda}{\lambda}\mathcal{O}_{\rm c} \sim \lambda\mathcal{O}_{\rm c}$$

Boost to $(\lambda, \lambda, \lambda)$ frame

Indeed!

So, exponential form not appropriate; it should be expanded:

$$W_{\rm L} = 1 - \frac{1}{2\partial_-}\gamma_{-\alpha\beta}J^{\alpha\beta}$$

Summary of collinear Wilson lines:

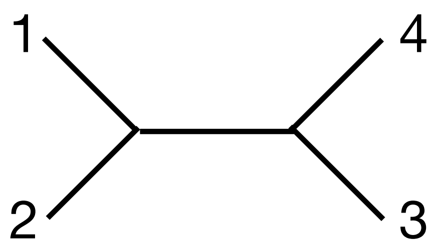
To make hard interaction invariant under collinear gauge group in question, do

$$\mathcal{O}_{\rm c} \longrightarrow W_{\rm d}W_{\rm L}^{-1}\mathcal{O}_{\rm c} \qquad\qquad \mathcal{O}_{\rm nc} \longrightarrow \mathcal{O}_{\rm nc}$$

More generally, multiply each collinear sector by its own Wilson lines (and soft Wilson lines).
The graviton couplings in these Wilson lines begin at $\sim \lambda$!

Go back to our example process $\phi(p_1) + \phi(p_2) \longrightarrow \phi(p_3) + \phi(p_4) + h_{\mu\nu}(q)$**:**

Matching at $O(\lambda^0)$ (no collinear graviton at all):



+ (t-channel) + (u-channel) $\longrightarrow$ $\mathcal{L}_{\text{hard-0}} = C\phi_1\phi_2\phi_3^*\phi_4^*$

with $C = \frac{\kappa^2}{2}\frac{1}{{}^1\partial_-}\left(\frac{1}{{}^2\partial_+} + \frac{1}{{}^3\partial_+} + \frac{1}{{}^4\partial_+}\right)$

(${}^i\partial_\mu$ acts only on $\mathcal{O}_i$)

At $O(\lambda)$ there are three operators:

$$\mathcal{L}_{\text{hard-0}} \longrightarrow \mathcal{L}_{\text{hard-1}} = C({}^1W_{\text{d}}\phi_1)\phi_2\phi_3^*\phi_4^*$$ (Fixed by gauge invariance. No parameters.)

$$\mathcal{L}_{\text{hard-2}} = \left(\frac{1}{\partial_-^3}{}^1R_{-i-j}\right)C\left(\frac{{}^2\partial^i\,{}^2\partial^j}{{}^2\partial_+} + \frac{{}^3\partial^i\,{}^3\partial^j}{{}^3\partial_+} + \frac{{}^4\partial^i\,{}^4\partial^j}{{}^4\partial_+}\right)\phi_1\phi_2\phi_3^*\phi_4^*$$

$$\mathcal{L}_{\text{hard-3}} = \frac{1}{2}\left(\frac{1}{\partial_-^2}{}^1R_{--}\right)C\phi_1\phi_2\phi_3^*\phi_4^*$$

To get $\mathcal{L}_{\text{hard-2,3}}$, just match h_{-+} or h_{ij} , which are *already* $O(\lambda)$ so no cancellations occur!

You only need a sheet of paper to get them and reproduce your student's full theory calculation (~ 20 pages with 19.5 pages of cancellations of $O(1/\lambda)$ **&** $O(\lambda^0)$ **terms and half a page of answer)!!!**

Future directions:

* Implications of RPI (ongoing work with S. Chakraborty and A. Yunesi)

  - Preliminary results:

    + No way to obtain $O(\lambda)$ from $O(\lambda^0)$.

    + Some constraints/relations among $O(\lambda)$ operators.

    + Some constraints/relations between $O(\lambda)$ and $O(\lambda^2)$ operators.

  - Goals:

    + To generally understand where and how RPI is useful.

    + To see if it makes sense to generalize "RPI" to include gravitons in transformations.

* Sub-leading soft theorems by extending our construction to $O(\lambda^2)$.

* Construct gravity SCET for Regge region (= Gravity analog of Rothstein and Stewart (2016))

  - Foreseeable challenges:

    + Glauber (or Newtonian?) modes

    + Rapidity divergences

Thank you!

Backup slides

Soft gravitons:

It has long been known since Weinberg (1965) that soft gravitons couple at $O(\lambda^0)$ via a soft Wilson line:

$$Y = \exp\left[\left(\frac{1}{2\partial_+} h_{++}\right)\partial_-\right]$$

In SCET, this should be re-derived as a consequence of effective soft gauge symmetry and BPS decoupling transformation. [Beneke, Kirilin (2012)] [TO, Yunesi (2017)]

First, power counting is modified for soft gravitons:

$$q_\mu \sim \lambda^2 \quad (\mu = +, -, i) \qquad 1 \sim \int \mathrm{d}^4x\, (\partial_* h_{**})^2 \sim \lambda^{-8} (\lambda^2 h_{**})^2 \longrightarrow h_{\mu\nu} \sim \lambda^2$$

which further implies

$$\varphi_{\mu\nu} \sim h_{\mu\nu} \sim \lambda^2 \quad \text{and} \quad \xi_\mu \sim 1 \quad \omega_{\mu\nu} \sim \lambda^2 \quad \text{in} \quad \delta\varphi_{\mu\nu} = \partial_\nu \xi_\mu + \omega_{\mu\nu}$$

So, collinear matter fields transform as

$$\delta\mathcal{O}_\mathrm{c} = \xi^\mu \partial_\mu \mathcal{O}_\mathrm{c} + \frac{1}{2}\omega_{\mu\nu} J^{\mu\nu} \mathcal{O}_\mathrm{c} = \underline{\xi_+ \partial_- \mathcal{O}_\mathrm{c} \ \text{at } O(\lambda^0)}$$

Soft gravitons do not care about spin ($J_{\mu\nu}$) at leading order!

So, the soft Wilson line in BPS decoupling must transform as

$$\delta Y = \xi_+ Y \partial_-$$

From $\delta h_{++} = 2\delta\varphi_{++} = 2\partial_+ \xi_+$ we see that the Y given above works and is the only way that works!

$O(\lambda)$ can be eliminated by RPI as in spin-1 case (Larkoski, Neill and Stewart, 2014).

Comparison with "Soft collinear gravity" by Beneke and Kirilin (2012)

Their treatment of collinear gravitons is completely different from ours!

What they did with collinear gravitons:

Expressed the $O(1/\lambda)$ couplings of h_{--} as a "Wilson line" $W_{\rm BK}$ acting on **non-collinear** fields, where

$$W_{\rm BK} = \exp\left[\left(\frac{1}{2\partial_+} h_{--}\right)\partial_+\right] \sim \mathrm{e}^{\frac{1}{\lambda}}$$

Unlike our Wilson lines, neither the form of $W_{\rm BK}$ or the rule for how it enters $\mathcal{L}$ is dictated by symmetry. Recall that in SCET a collinear Wilson line cannot act on a non-collinear field. Symmetries lead to $W_{\rm d}$ and $W_{\rm L}$, not $W_{\rm BK}$.

But $W_{\rm BK}$ does reproduces all the $O(1/\lambda^N)$ terms for N collinear graviton emissions that come from non-collinear external lines (but not those from internal lines).

They showed that $W_{\rm BK}$ can be eliminated by field redefinition, thereby demonstrating the $O(1/\lambda^N)$ contributions are actually absent. They also identified an explicit gauge transformation to go to gauge away h_{--}.

They don't show, nor implicitly imply, the cancellations of all $O(1/\lambda^p)$ terms with $N-1 \leq p \leq -N+1$ as that would involve components other than h_{--} .

In the SCET as described in our work, it is manifest that the amplitude is $O(\lambda^N)$.