Towards an extraction of $\alpha_s(m_Z)$ from e^+e^- angularities

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with G. Bell, C. Lee, Y. Makris, and H. Prager

hep-ph/1808.07867
& work-in-progress

25 March 2019 || SCET, UCSD || San Diego, CA, USA

Outline

[p] The global picture of $\alpha_s(m_Z)$

Why e^+e^- angularities?

Data and fit method

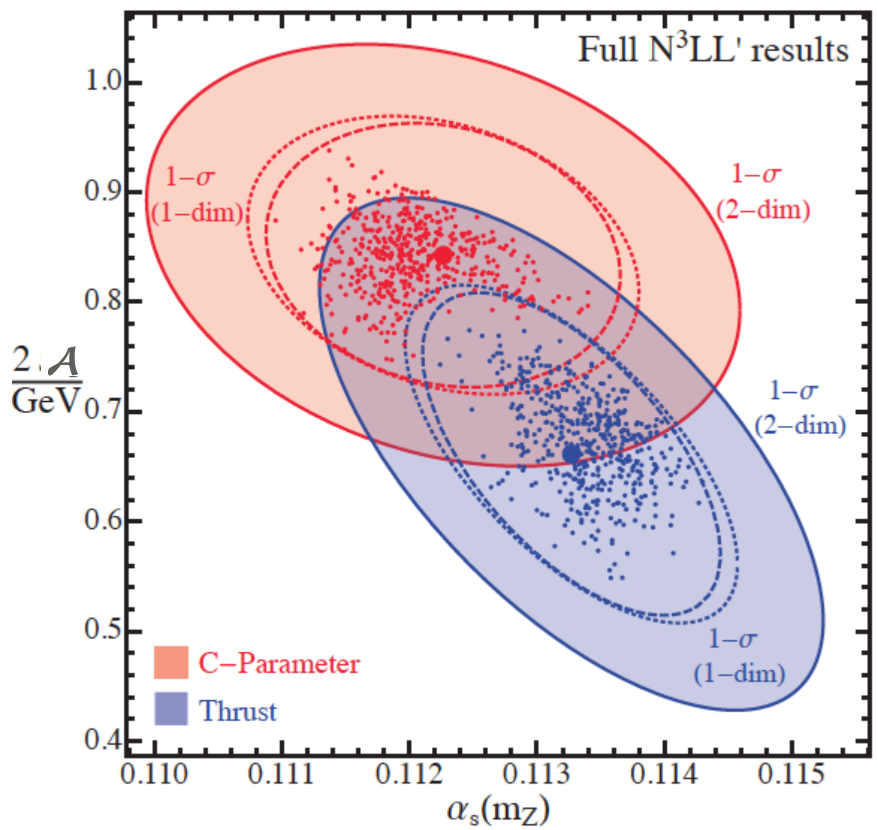
Preliminary results

SCET and the global picture of α_s

hep-ph/0803.0342 (BS)
hep-ph/1006.3080 (AFHMS)
hep-ph/1501.04111 (HKMS)

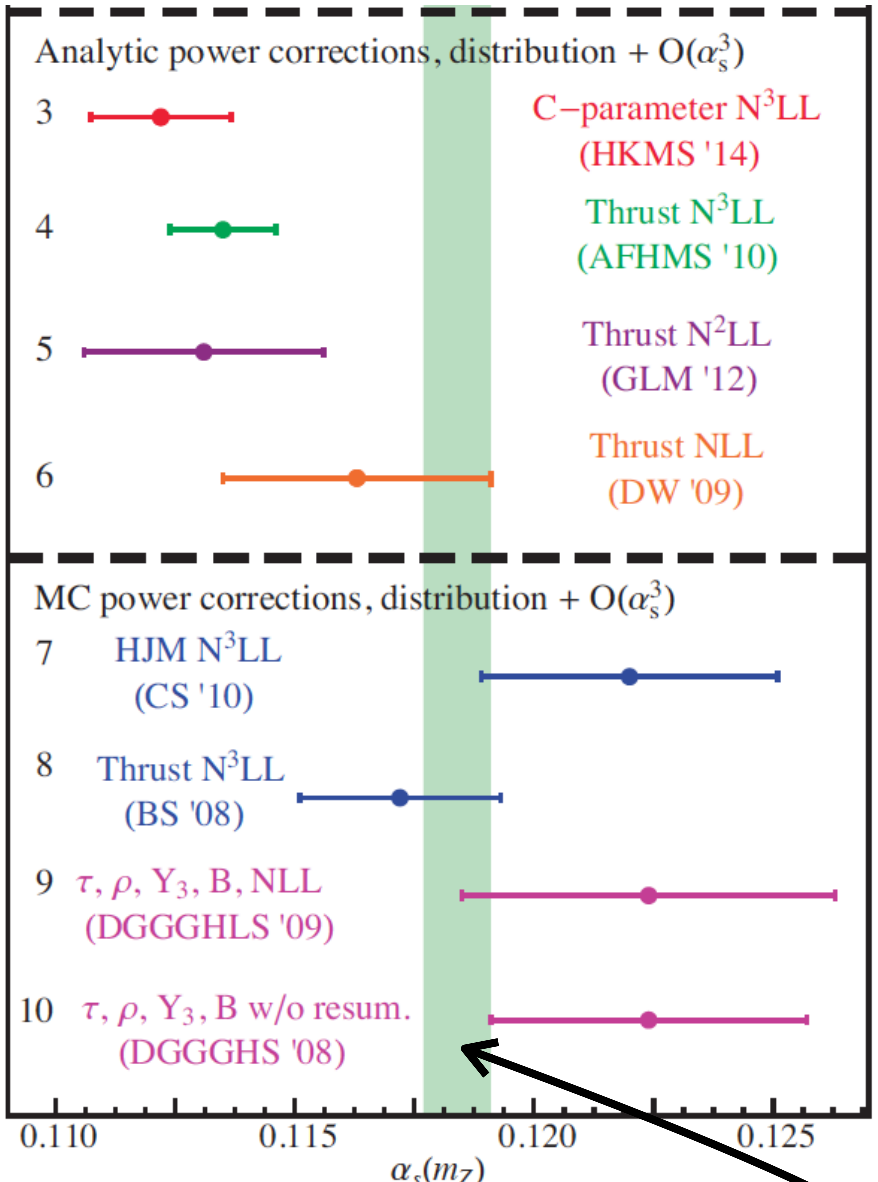
- Many groups have utilized high-precision event-shape results to extract a value for α_s. However, the value of α_s is highly correlated to non-perturbative physics.

C-parameter versus Thrust Tail Global Fit



2018 world average: .1181 +- .0011

see A. Hoang, 2015 workshop on precision α_s

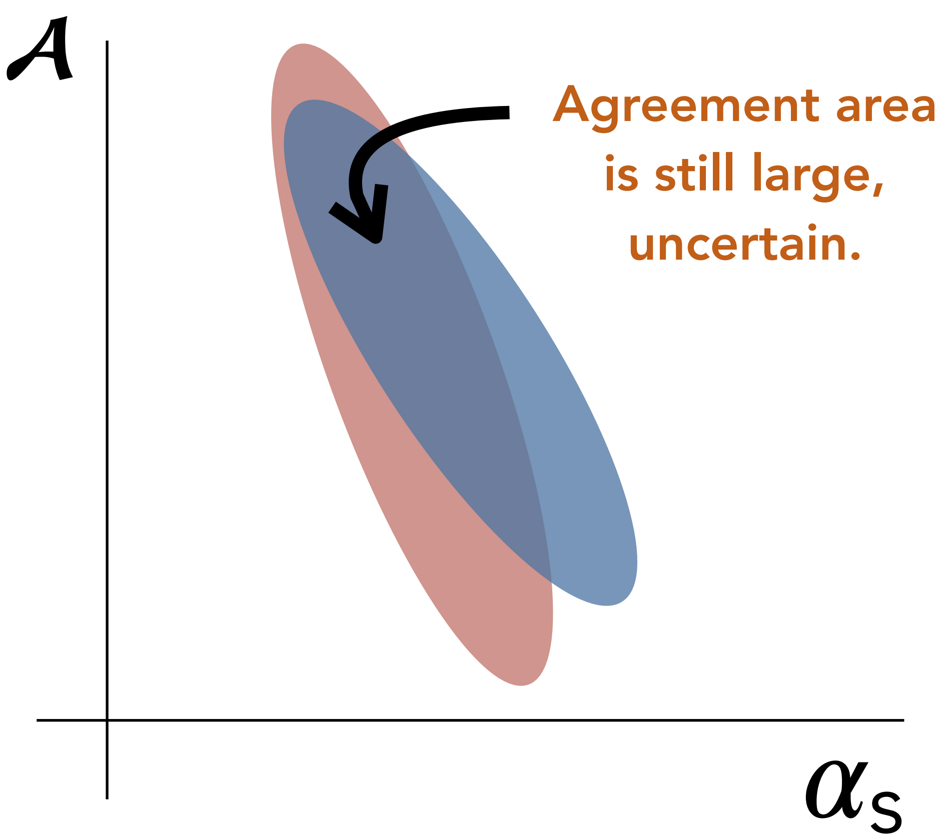


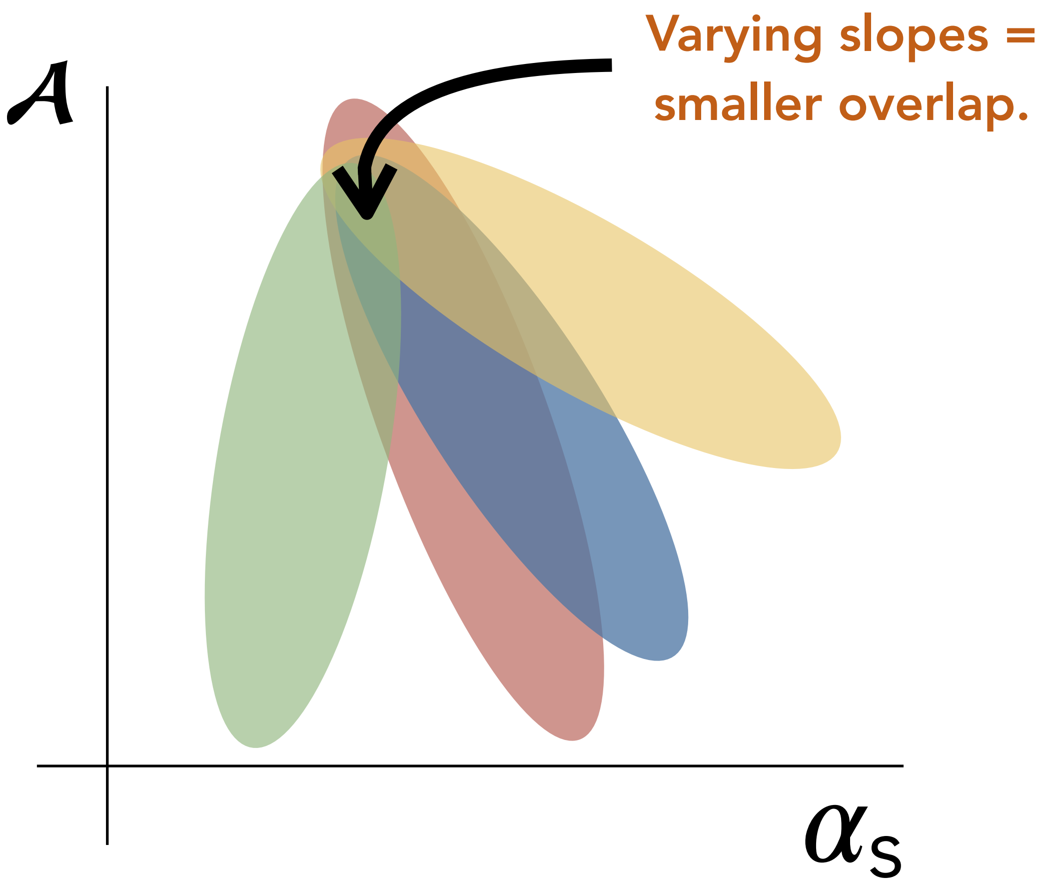
Note recent N^3LO + NNLL jet rate extraction: Verbytskyi et al., 1902.08158

- 2015 C-parameter result ~4σ away from lattice QCD / world average…
- What can break the degeneracy between $\mathcal{A}$ and α_s?

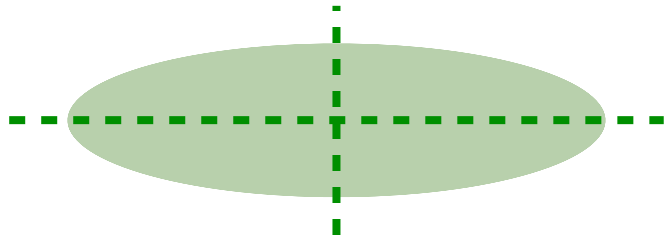
Visualizing disentanglement

- Thinking observable-by-observable, 'disentangling' $\mathcal{A}$ and α_s looks like a series of uncertainty ellipses with minimal overlap:





- The semi-major axis of an ellipse drawn in the $\mathcal{A}$-α_s plane can be generically written as:



$$\frac{\mathcal{A}}{(-slope)} = \alpha_s^{PT}(m_Z) - \alpha_s(m_Z)$$

- The slope of this line is **Q-dependent** for all event shapes, and also depends on multiplicative coefficients **c** (e.g. c = 2 for thrust). Can we gain analytic control over **c** for an entire class of observables?

Why e^+e^- angularities?

e$^+$e$^-$ angularities in SCET

hep-ph/0303051
hep-ph/0801.4569
hep-ph/0901.3780

- *Angularities* can be defined in terms of the of the rapidity and p_T of a final state particle 'i', with respect to the thrust axis:

IR safe for a ∈ {-∞, 2}!

$$\tau_a = \frac{1}{Q}\sum_i |\mathbf{p}_\perp^i|\, e^{-|\eta_i|(1-a)}$$

a = 0 <-> `Thrust'

a = 1 <-> `Jet Broadening'

- An all-order dijet factorization theorem for the observable is easily derived in SCET:

$$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S} \quad \overset{\text{RGE}}{\longleftrightarrow} \quad \frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln(\frac{Q^2}{\mu^2}) + 4\gamma_H(\alpha_s)\right] H(Q^2,\mu)$$

- Evolving all scales to/from their 'natural' settings, one arrives at the resummed cross section:

$$\frac{\sigma_{\text{sing}}(\tau_a)}{\sigma_0} = e^{K(\mu,\mu_H,\mu_J,\mu_S)}\left(\frac{\mu_H}{Q}\right)^{\omega_H(\mu,\mu_H)}\left(\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a}\right)^{2\omega_J(\mu,\mu_J)}\left(\frac{\mu_S}{Q\tau_a}\right)^{\omega_S(\mu,\mu_S)} H(Q^2,\mu_H)$$
$$\times\, \widetilde{J}\Big(\partial_\Omega + \ln\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a},\mu_J\Big)^2\, \widetilde{S}\Big(\partial_\Omega + \ln\frac{\mu_S}{Q\tau_a},\mu_S\Big) \times \begin{cases}\frac{1}{\tau_a}\mathcal{F}(\Omega) & \sigma = \frac{d\sigma}{d\tau_a}\\ \mathcal{G}(\Omega) & \sigma = \sigma_c\end{cases}$$

$$\mathcal{F}(\Omega) = \frac{e^{\gamma_E\Omega}}{\Gamma(-\Omega)} \qquad \mathcal{G}(\Omega) = \frac{e^{\gamma_E\Omega}}{\Gamma(1-\Omega)}$$

- This predicts the singular component of the cross section. One must then match to QCD:

$$\frac{\sigma_c(\tau_a)}{\sigma_0} - \frac{\sigma_{c,\text{sing}}(\tau_a)}{\sigma_0} = r_c(\tau_a) = \theta(\tau_a)\left\{\frac{\alpha_s(Q)}{2\pi}r_c^1(\tau_a) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 r_c^2(\tau_a)\right\} + \ldots$$

- Additionally, a treatment of non-perturbative effects is critical in e$^+$e$^-$ -> *hadrons*…

Non-pert. effects: parametric power

hep-ph/9504219
hep-ph/9806537
hep-ph/9902341
hep-ph/0611061

- When dominant power corrections come from the soft function, NP effects can be parameterized into a shape function f_{mod}:

'Gap' parameter accounting for parton -> hadron acceptance

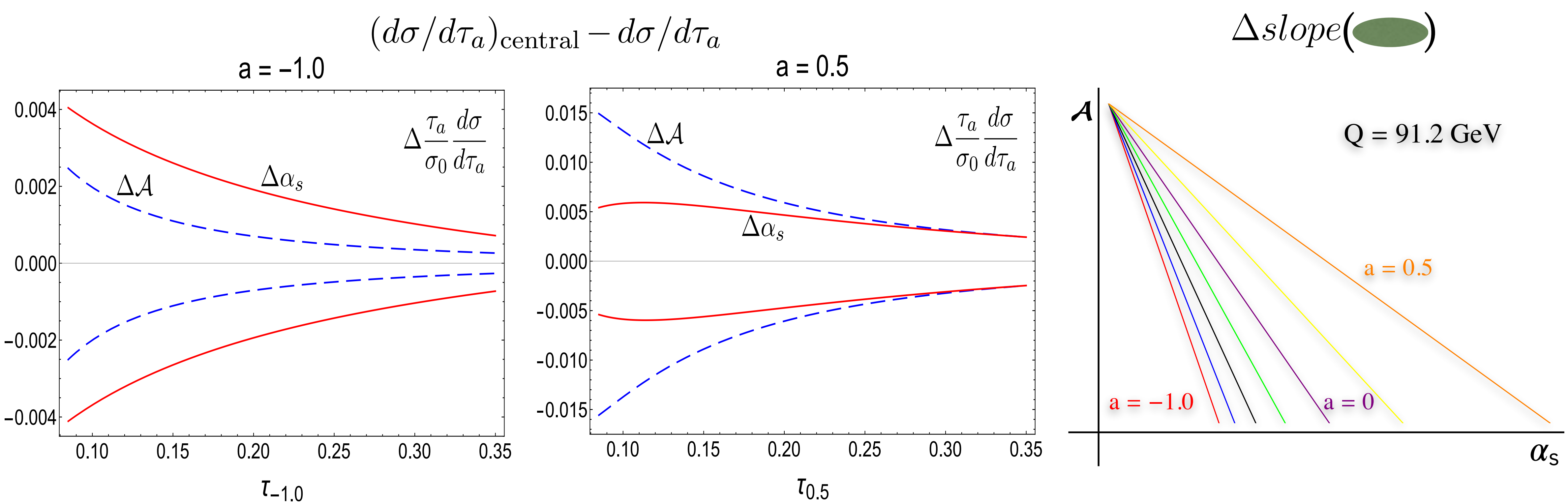
$$S(k,\mu) = \int dk'\, S_{\rm PT}(k-k',\mu)\, f_{\rm mod}(k'-2\overline{\Delta}_a) \qquad f_{\rm mod}(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} b_n\, f_n\left(\frac{k}{\lambda}\right)\right]^2$$

λ constrained by first moment of the shape function — complete orthonormal basis

- Convolution with f_{mod} reproduces leading NP distribution shift, derived from an OPE:

$$\frac{d\sigma}{d\tau_a}(\tau_a) \underset{\rm NP}{\longrightarrow} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\mathcal{A}}{Q}\right) \qquad c_{\tau_a} = \frac{2}{1-a} \qquad \mathcal{A} = \frac{1}{N_C}\,{\rm Tr}\,\langle 0|\,\overline{Y}_{\bar n}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(0) Y_n \overline{Y}_{\bar n}\,|0\rangle$$

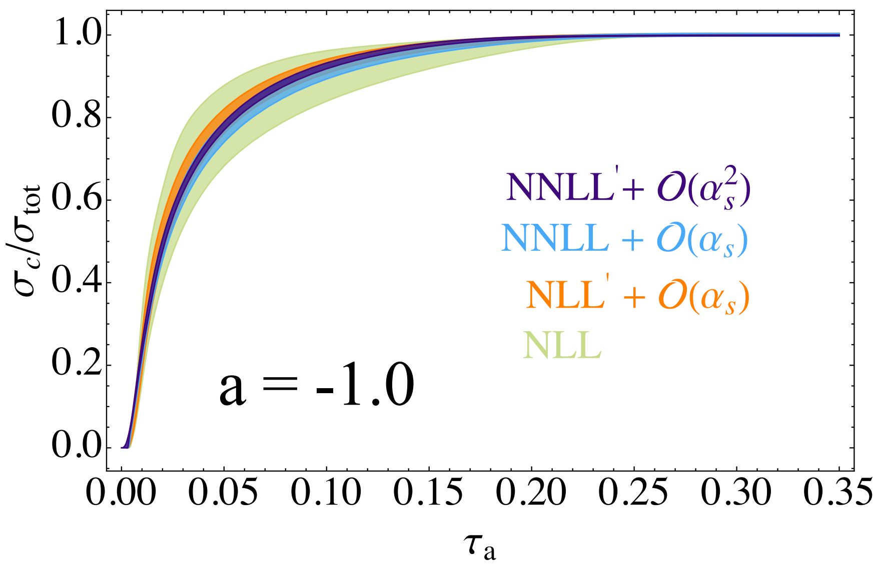
Note: this is only valid in the tail region!



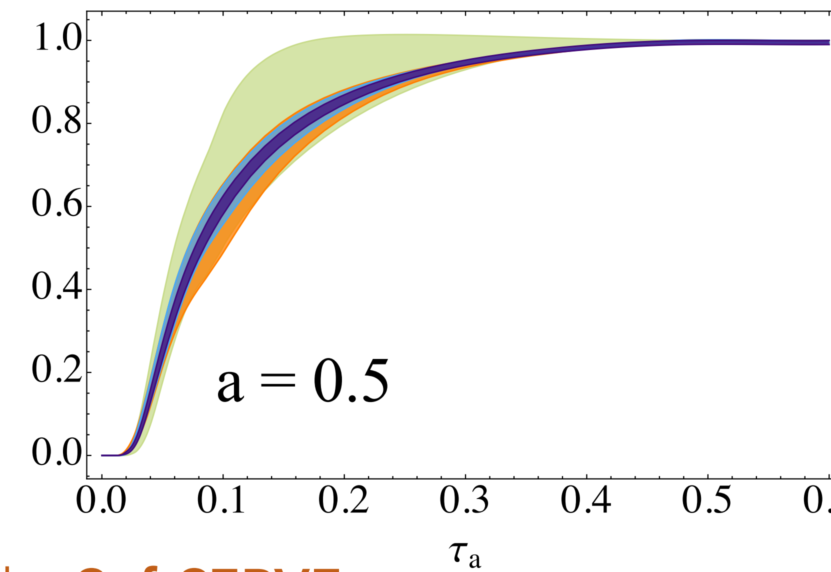
- Varying **Q** between 35 and 207 GeV generates same difference as varying a ∈ {-2.0, 0.5} (~6)!!

Recent progress: NLL′ to NNLL′

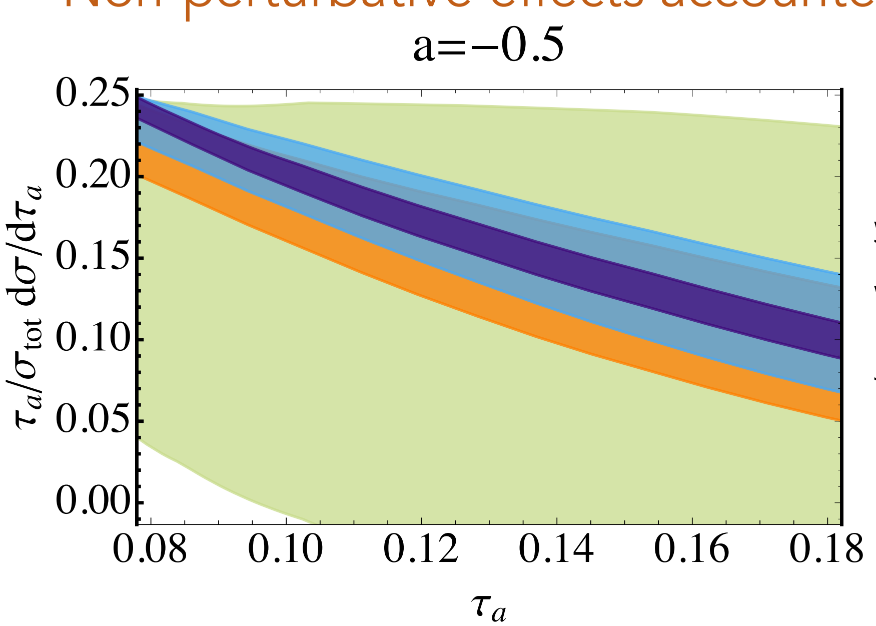
hep-ph/0901.3780
hep-ph/1805.12414
hep-ph/1808.07867
hep-ph/1812.08690

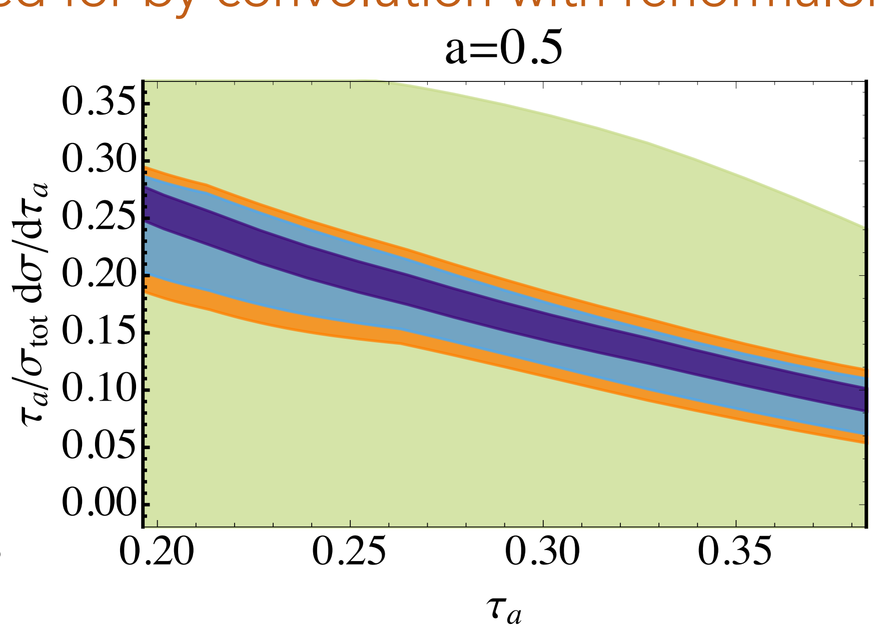


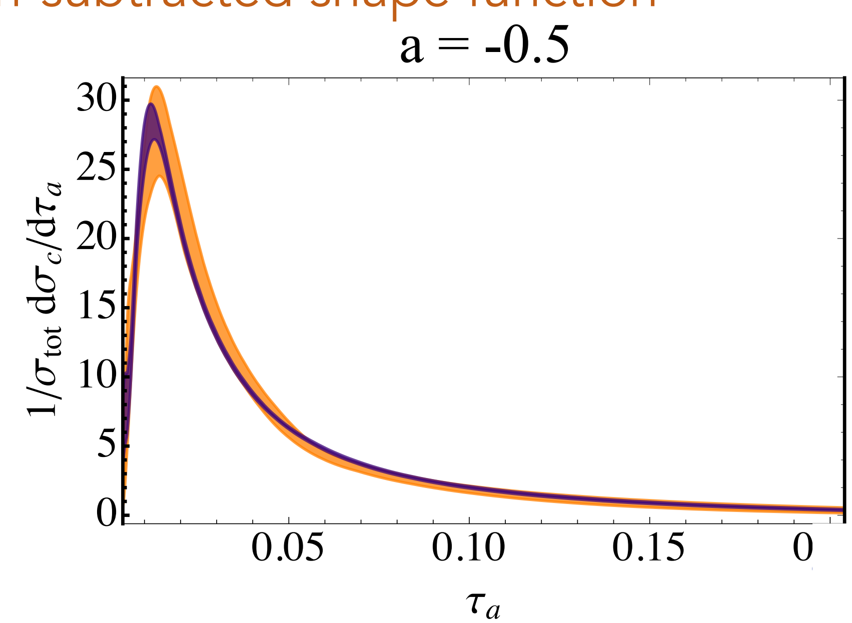
softserve.hepforge.org



- Two-loop soft anomalous dimensions and singular constants provided by **SoftSERVE**
- Two-loop jet anomalous dimension obtained from consistency relations
- Two-loop singular jet constants extracted from **EVENT2**
- Matching to QCD at $\mathcal{O}(\alpha_s^2)$ extracted from **EVENT2**
- Includes set of H,J,S, & non-sing. profile scales, tuned for a-dependence, and varied with a random scan over parameters
- Non-perturbative effects accounted for by convolution with renormalon-subtracted shape function







Data and fit method

The (only) dataset

Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0\,\text{GeV}$

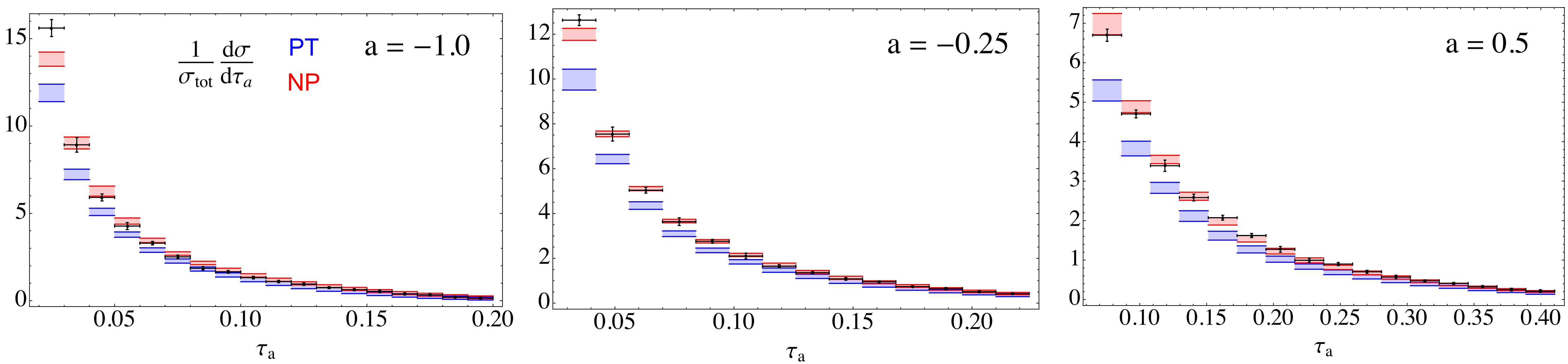
L3 Collaboration

JHEP 10 (2011) 143

Received: *May 12, 2009*
Revised: *May 3, 2011*
Accepted: *August 24, 2011*
Published: *October 31, 2011*

Also see thesis by Pratima Jindal, Panjab University, Chandigarh

- Data for a = {**-1.0**, **-0.75**. **-0.5**, **-0.25**, **0.0**, **0.25**, **0.5**, 0.75} at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ Q = 91.2 GeV
- Compare to 404 bins **included** in 2015 C-Parameter fit (across all Q considered)…
- Early theory predictions look good against the data, but what does this translate to for $\mathcal{A}$ and α_s?



BLUE: NNLL′ + $O(\alpha_s^2)$ RED: NNLL′ + $O(\alpha_s^2)$ + NP

Fit goals and methodology

hep-ph/1006.3080
hep-ph/1501.04111

EARLY GOALS

1) Gauge the quality of the available data and resulting fits, given our best theory predictions and independent extraction codes…Do we need better data or better theory at the moment?
2) Determine if the expected benefit of using angularities (parametric NP behavior) is roughly observed.
3) Gauge whether our (early) results are consistent with prior SCET analyses…Still tension with PDG?

- We perform a χ^2/d.o.f. analysis, accounting for stat. + (correlated) syst. experimental uncertainties and theory uncertainties as determined by all relevant variations in 1808.07867.
- Correlations amongst data bins accounted for with *Minimal Overlap Model*.
- Experimental uncertainty ellipse determined via $\Delta\chi^2$=1, using central values of profile parameters. Correlation matrices (also for theory and total uncertainty) defined by:

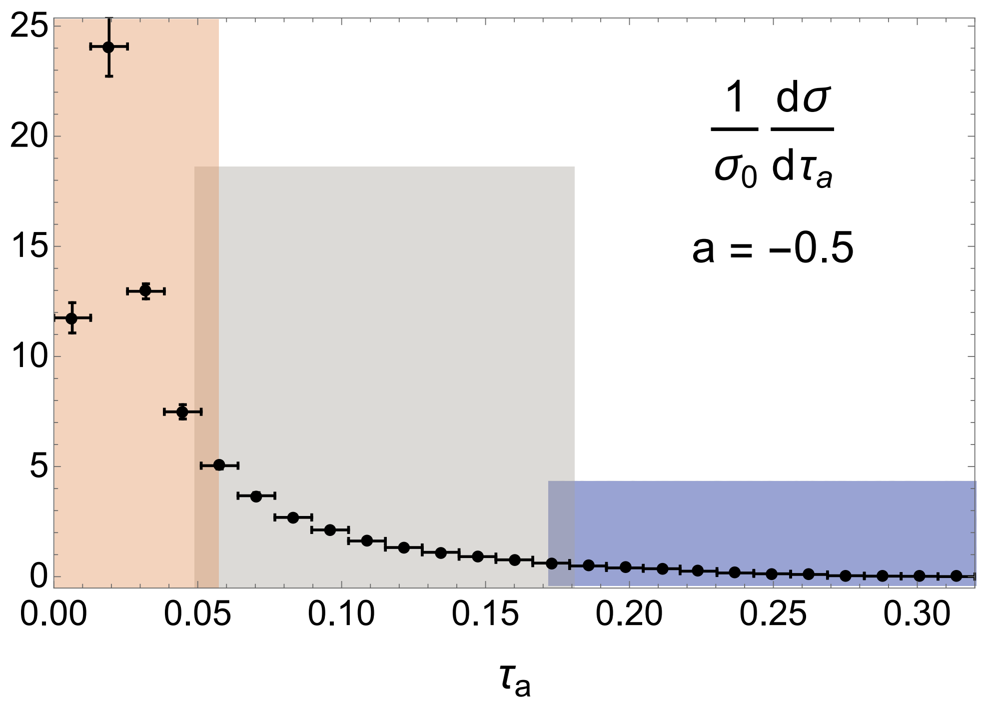
$$V_{ij}^{corr.} \equiv \begin{pmatrix} \sigma_{\alpha_s}^2 & \sigma_{\alpha_s}\,\sigma_{\mathcal{A}}\,\rho_{\alpha\,\mathcal{A}} \\ \sigma_{\alpha_s}\,\sigma_{\mathcal{A}}\,\rho_{\alpha\,\mathcal{A}} & \sigma_{\mathcal{A}}^2 \end{pmatrix} \qquad V_{ij}^{total} \equiv V_{ij}^{exp.} + V_{ij}^{theory}$$

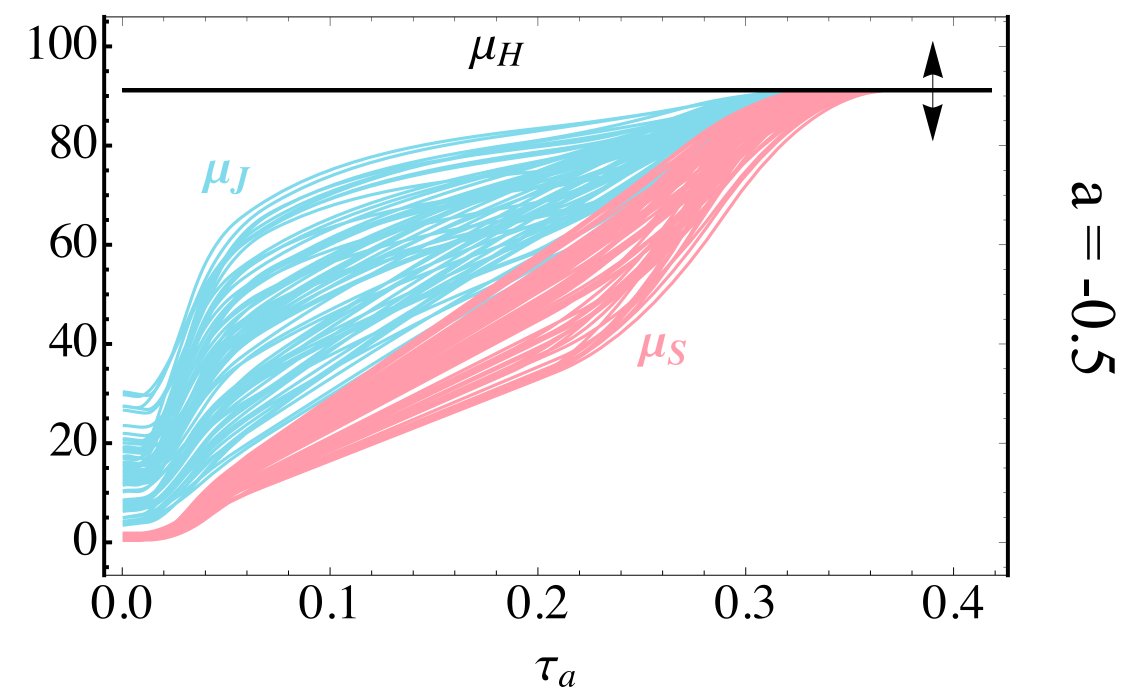
- Theory predictions only include (for now) leading non-pert. shift: $\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}(\tau_a - c_{\tau_a}\frac{\mathcal{A}}{Q})$
- Theory uncertainty ellipse determined as envelope of all best fit points, after 500 random draws of theory parameters in pre-defined ranges, found in 1808.07867.
- Fits performed for each angularity individually, and globally for all available *a*, once a fit window is chosen. We only use the Q = 91.2 GeV data in our fits.

Profiling a fit window

hep-ph/1808.07867

- How can we identify a region sensitive to $\mathcal{A}$ and α_s, and for which our best theory curves are reliable? Look to the profiles!





- Profiles trace scale hierarchies through different regimes of a given distribution:

Peak	$\mu_H \gg \mu_J \gg \mu_S \sim \Lambda_{QCD}$
Tail	$\mu_H \gg \mu_J \gg \mu_S \gg \Lambda_{QCD}$
Far Tail	$\mu_H = \mu_J = \mu_S \gg \Lambda_{QCD}$

Tracks the peak

$$t_0 = \frac{n_0}{Q}\,3^a$$

Turns off resummation

$$t_2 = n_2 \times 0.295^{1-0.637\,a}$$

$$t_1 = \frac{n_1}{Q}\,3^a$$

Transitions between NP and PT physics

$$t_3 = n_3\,\tau_a^{\rm sph}$$

Reverts to fixed-order perturbation theory

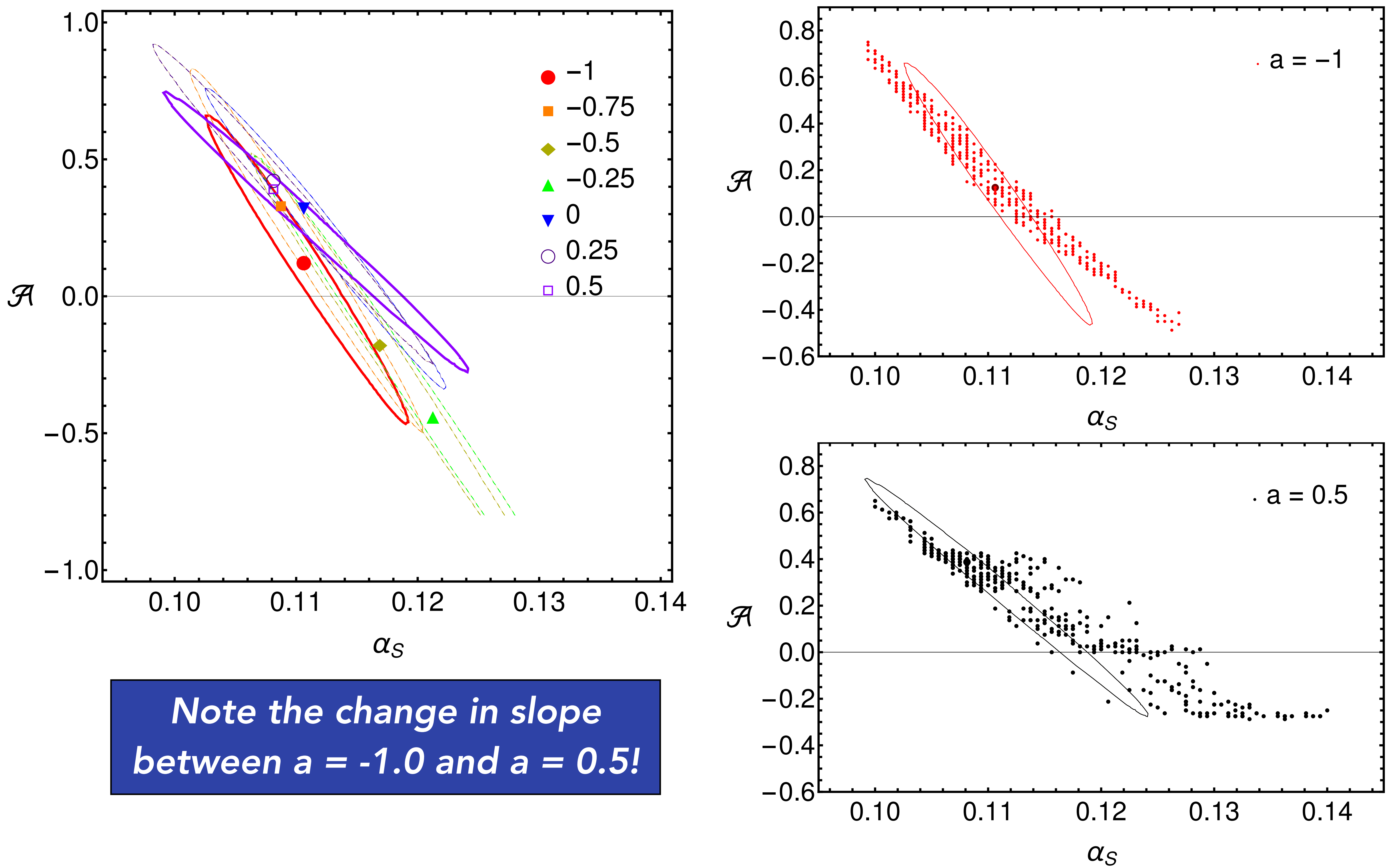
- Our default fit window will be between **$t_{1'}$ and t_2**, which roughly tracks the tail (former) and far-tail (latter) of the distribution.* *

Preliminary results

Default fits: individual observables

Preliminary!

- We perform fits at individual *a*, to see if we observe the NP shift (theory at NNLL′ + $O(\alpha_s^2)$ + NP):

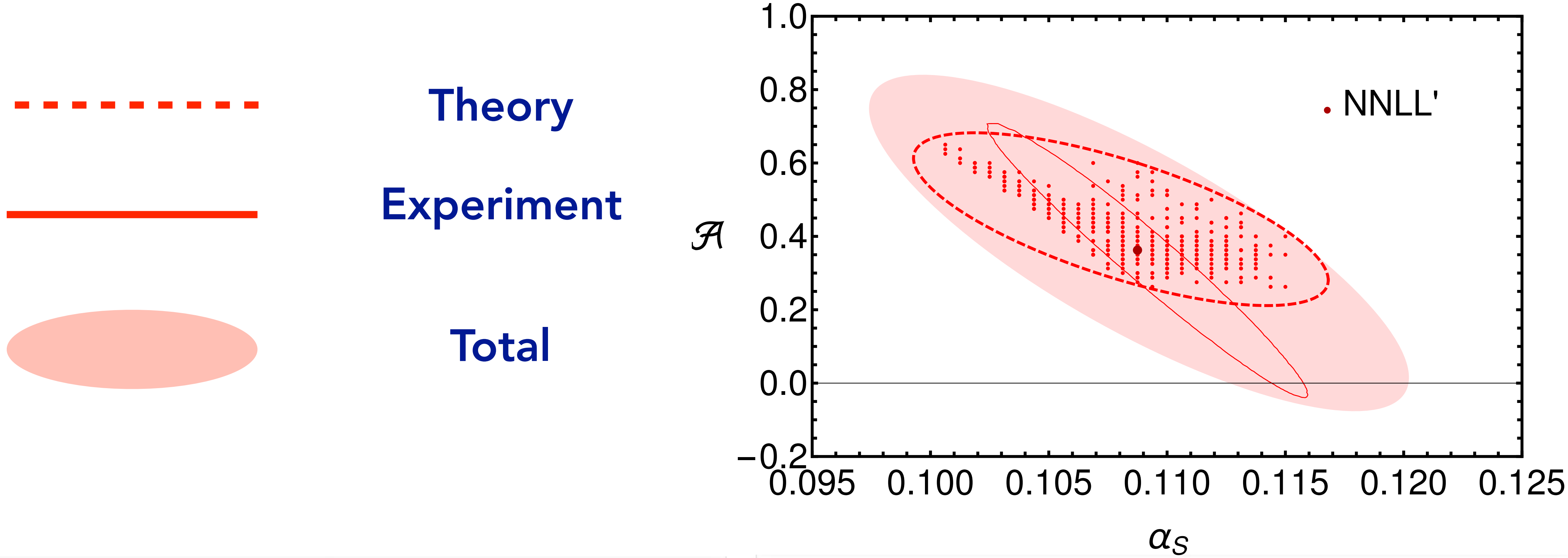


Note the change in slope between a = -1.0 and a = 0.5!

Default fits: global analysis

Preliminary!

- If we instead perform a fit to all available observables/bins simultaneously, we obtain:



$$\alpha_s(m_Z)\big|_{\rm NNLL'} = 0.109 \pm 0.007_{\rm exp} \pm 0.007_{\rm th}$$

$$\mathcal{A}\big|_{\rm NNLL'} = 0.36 \pm 0.37_{\rm exp} \pm 0.19_{\rm th} \quad ({\rm GeV})$$

- Compare the central results to 2015 C-parameter results in 1501.04111:

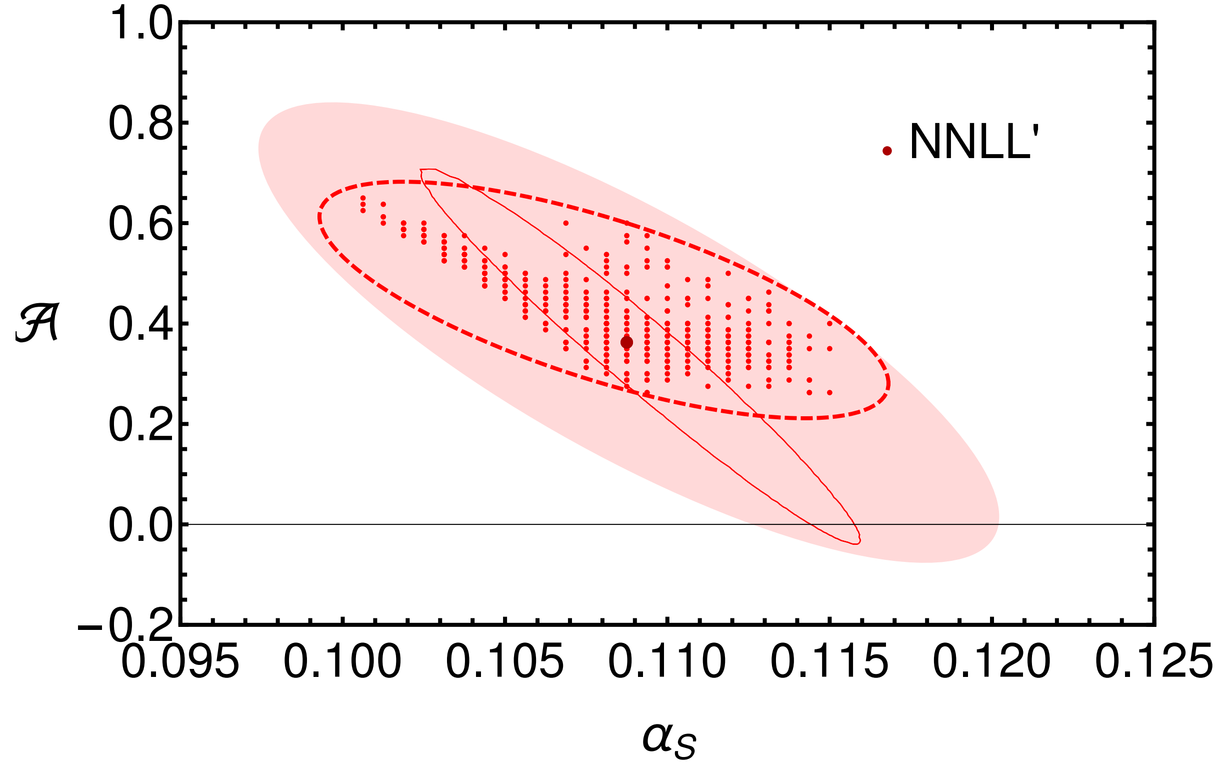
order	$\alpha_s(m_Z)$ (with $\mathcal{A}$)	$\alpha_s(m_Z)$ (with $\mathcal{A}(R_\Delta,\mu_\Delta)$)
NLL′	0.1071(60)(05)	0.1059(62)(05)
N^{2}LL′	0.1102(32)(06)	0.1100(33)(06)
N^{3}LL′ (full)	0.1117(16)(06)	**0.1123(14)(06)**

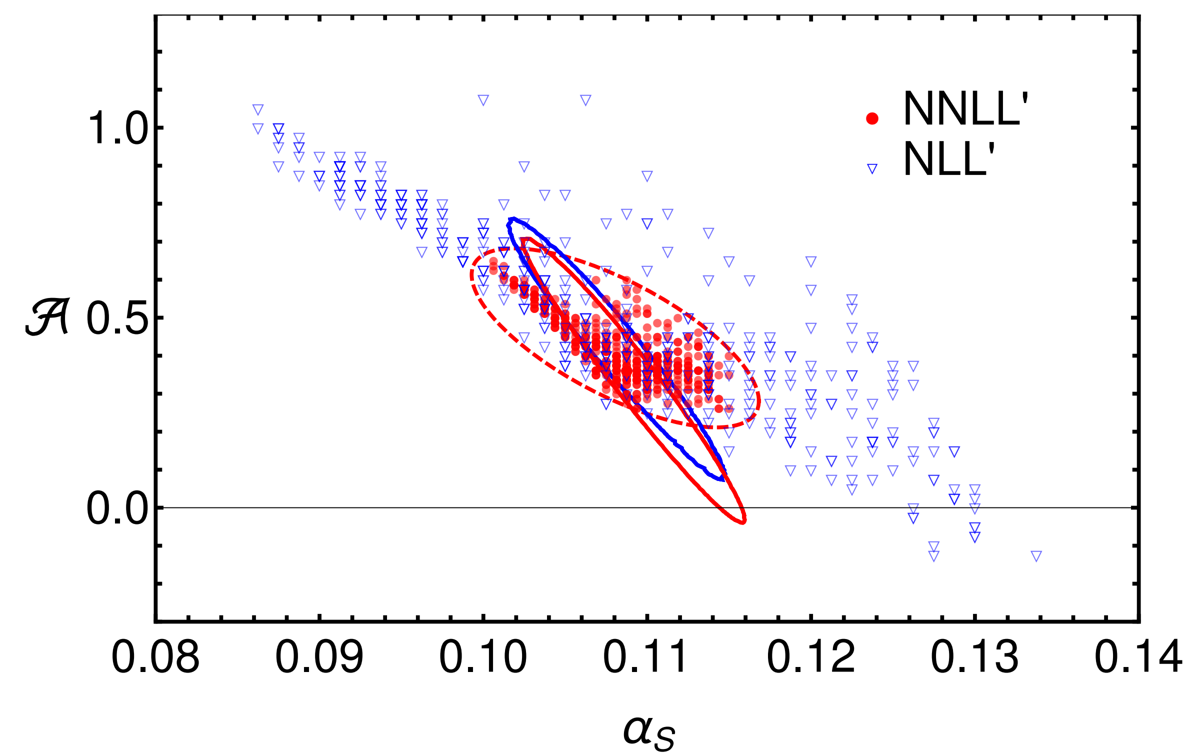
order	$\mathcal{A}$ [GeV]	$\mathcal{A}(R_\Delta,\mu_\Delta)$ [GeV]
NLL′	0.533(154)(18)	0.582(134)(16)
N^{2}LL′	0.443(119)(19)	0.457(83)(19)
N^{3}LL′ (full)	0.384(91)(20)	**0.421(60)(20)**

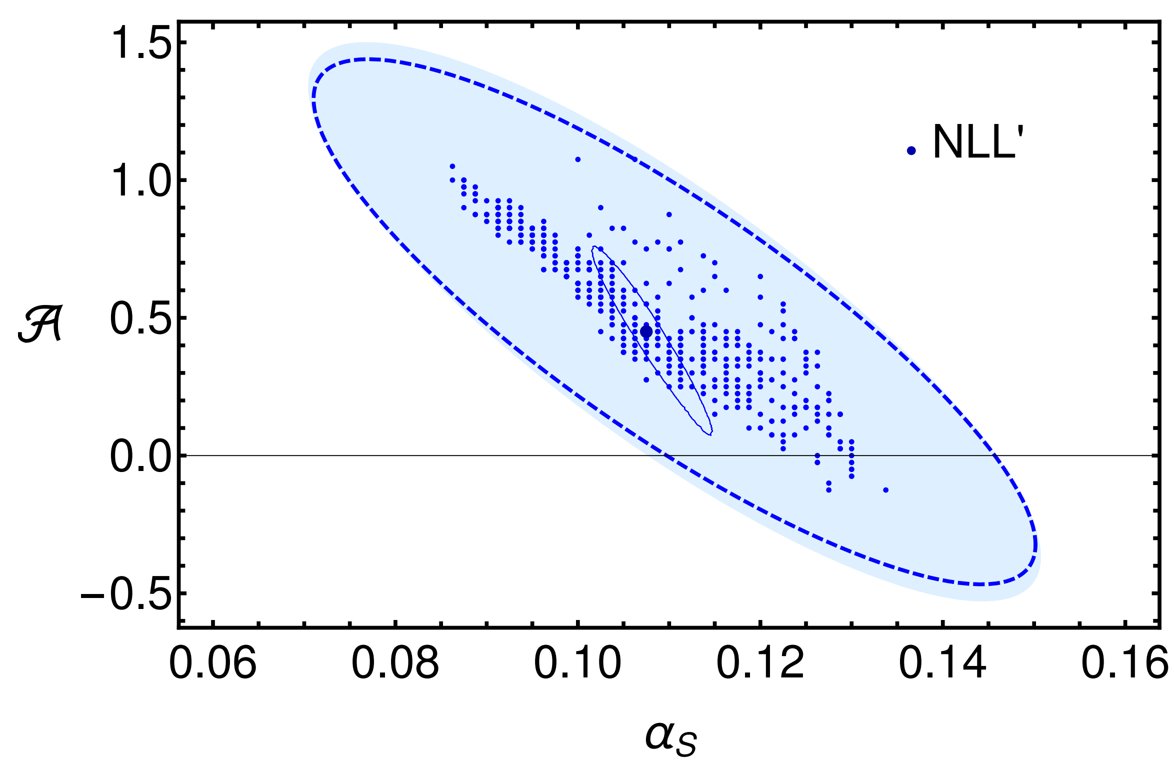
Default fits: convergence

Preliminary!

- The improvement from NLL′ to NNLL′ accuracy makes a substantial difference in the uncertainty ellipses generated:







NNLL′

$$\alpha_s(m_Z)|_{\mathrm{NNLL'}} = 0.109 \pm 0.007_{\mathrm{exp}} \pm 0.007_{\mathrm{th}}$$
$$\mathcal{A}|_{\mathrm{NNLL'}} = 0.36 \pm 0.37_{\mathrm{exp}} \pm 0.19_{\mathrm{th}} \quad (\mathrm{GeV})$$

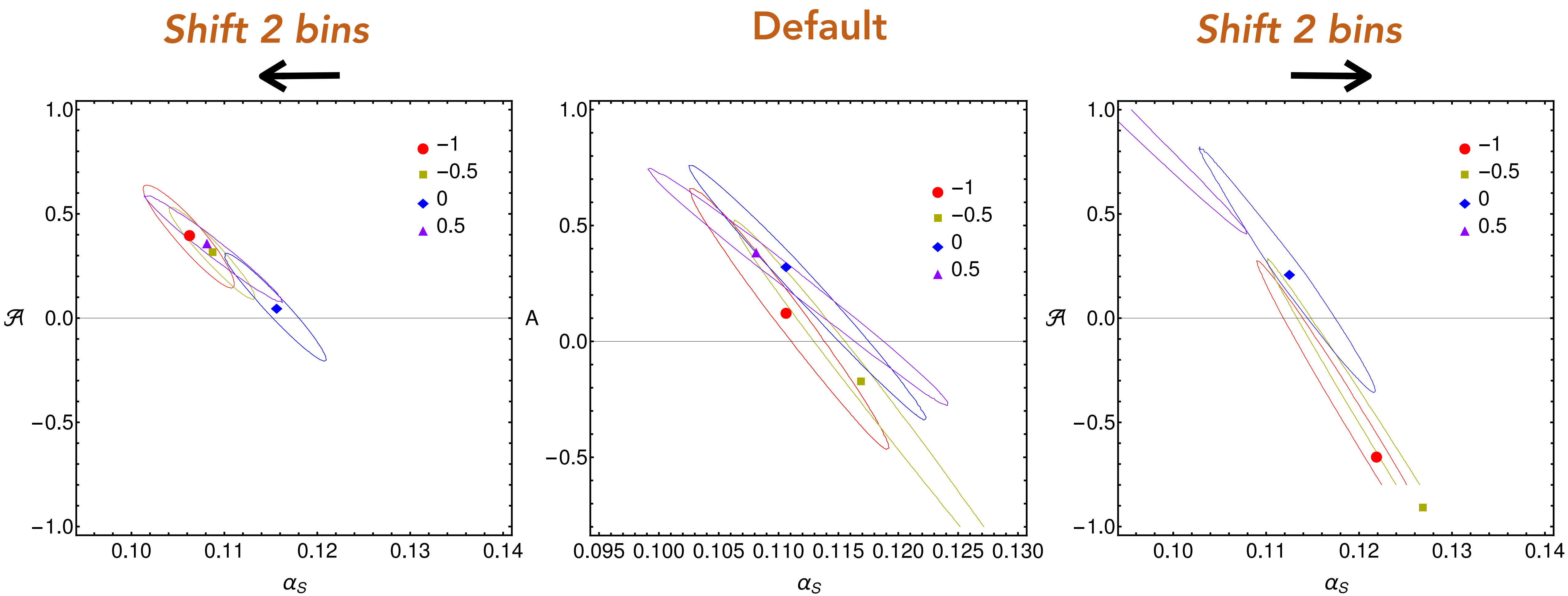
NLL′

$$\alpha_s(m_Z)|_{\mathrm{NLL'}} = 0.108 \pm 0.007_{\mathrm{exp}} \pm 0.02_{\mathrm{th}}$$
$$\mathcal{A}|_{\mathrm{NLL'}} = 0.45 \pm 0.34_{\mathrm{exp}} \pm 0.60_{\mathrm{th}} \quad (\mathrm{GeV})$$

Fit windows — a major systematic

Preliminary!

- Taking more of the peak leads to smaller experimental ellipses, whereas taking more of the far tail leads to larger experimental ellipses:



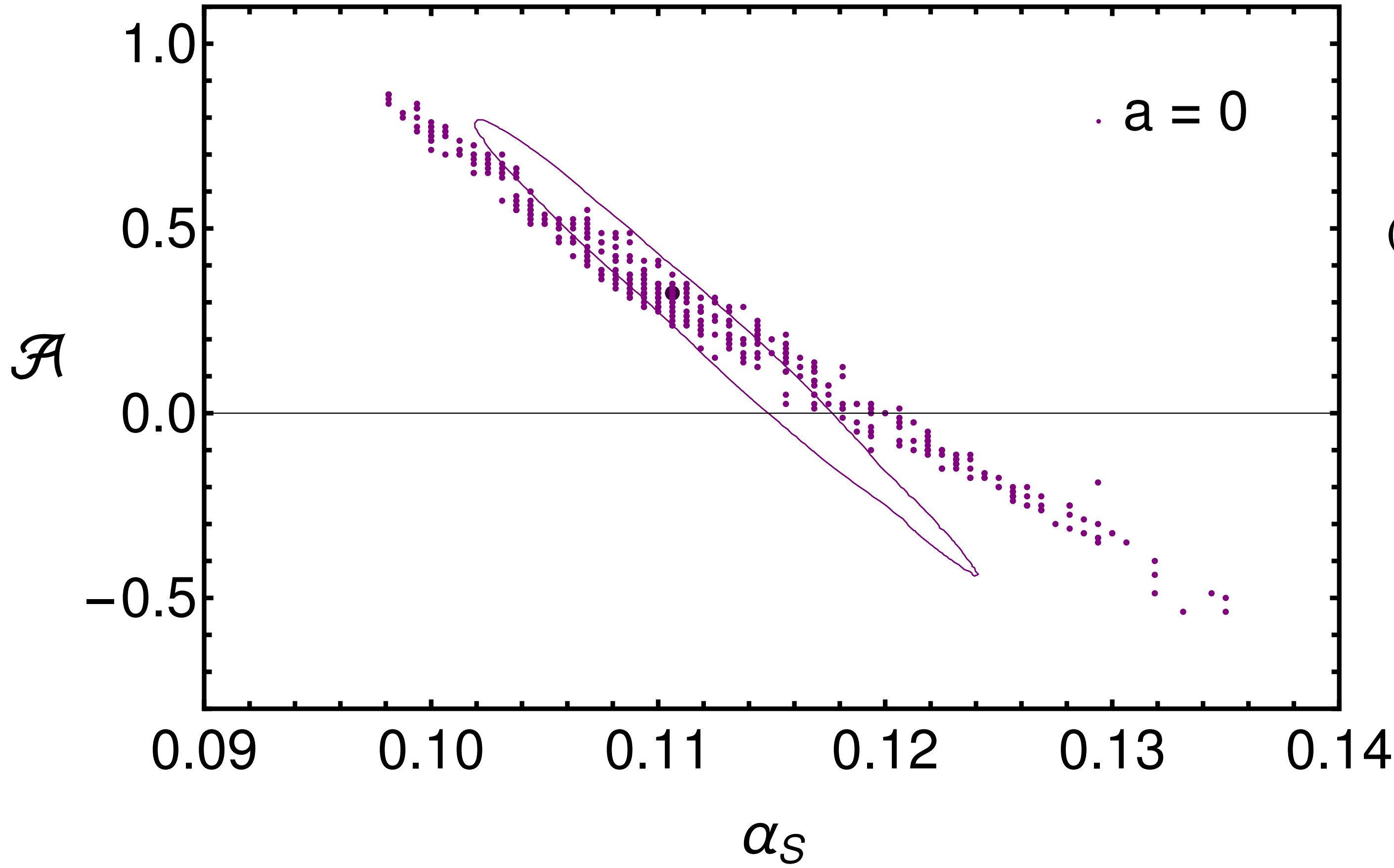
- But both effects will clearly generate different central values for $\mathcal{A}$ and α_s…
- This effect was already noted before, cf. Fig. 17 in 1006.3080. But can we really justify not taking more of the far-tail data? Would a significant tension survive if not?

Projections: better data

Preliminary!

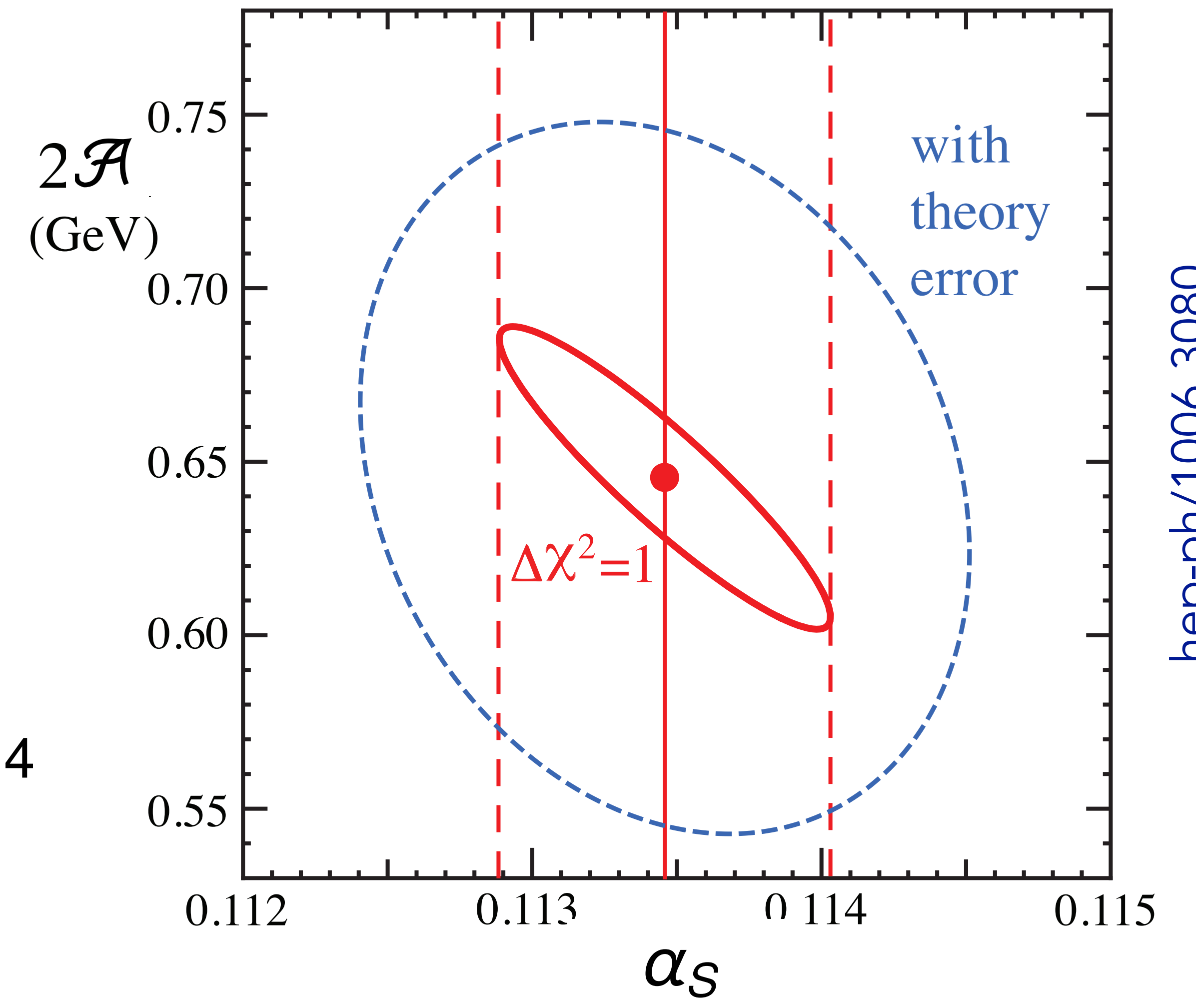
- Compare the relative theory vs. experimental ellipses in 2010 thrust paper to our own:

2019 N^2LL' Angularities (a = 0)
(Q = 91.2 GeV)



Note the difference in the y-axis!

2010 N^3LL' Thrust
(Q ∈ {35 - 207} GeV)



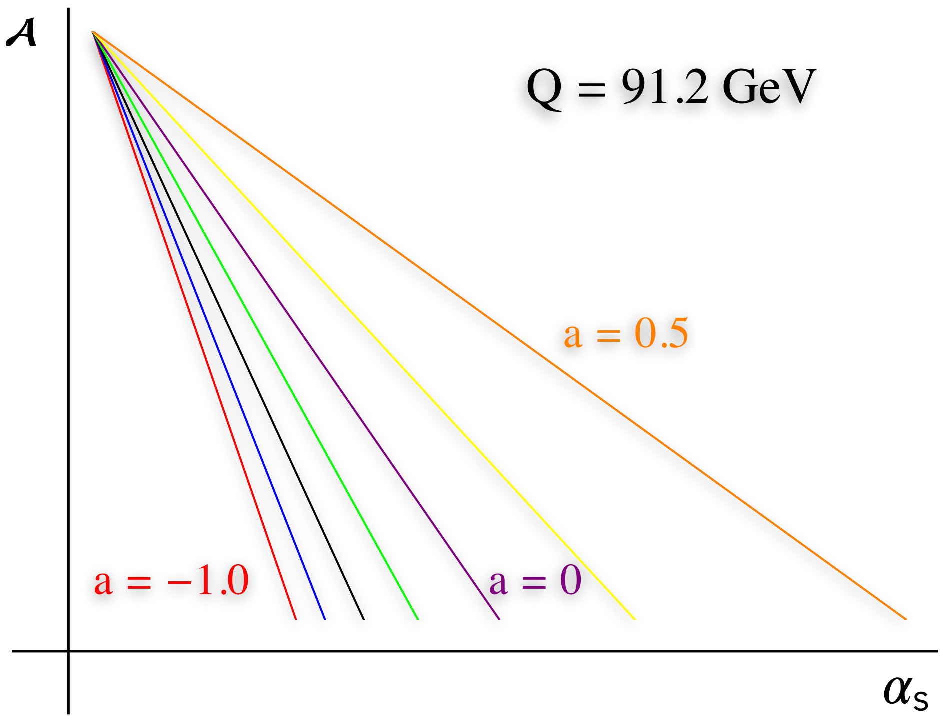
$$V_{ij}^{\rm exp}\Big|_{a=0}^{2019} \simeq \begin{pmatrix} 1.01\cdot 10^{-4} & -5.50\cdot 10^{-3}\ \text{GeV} \\ -5.50\cdot 10^{-3}\ \text{GeV} & 3.10\cdot 10^{-1}\ \text{GeV}^2 \end{pmatrix}$$

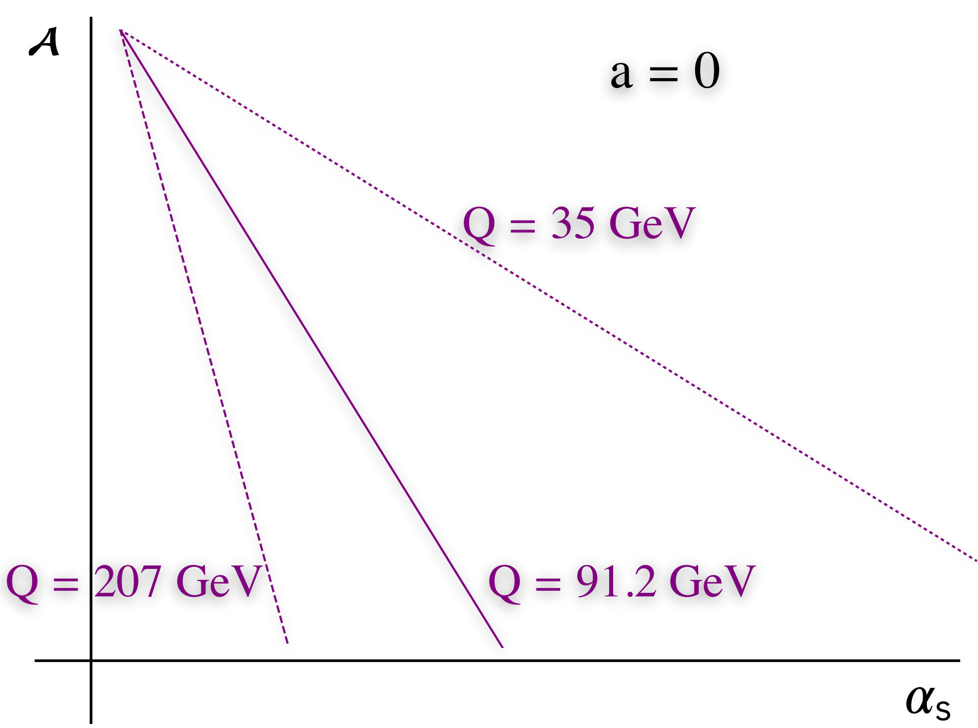
$$V_{ij}^{\rm exp}\Big|_{\rm thrust}^{2010} \simeq \begin{pmatrix} 3.29\cdot 10^{-7} & -2.30\cdot 10^{-5}\ \text{GeV} \\ -2.30\cdot 10^{-5}\ \text{GeV} & 1.90\cdot 10^{-3}\ \text{GeV}^2 \end{pmatrix}$$

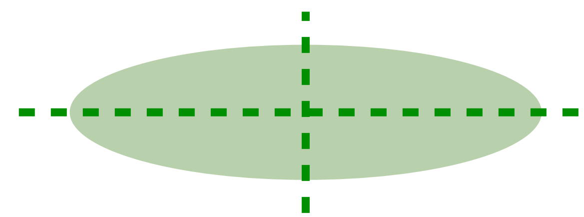
- Measurements at more c.o.m energies Q, for each angularity *a*, are clearly welcome!

Projections: more observables

- As are measurements at more values of *a*, for a given Q! Data across broad ranges in both promises intense probative power:

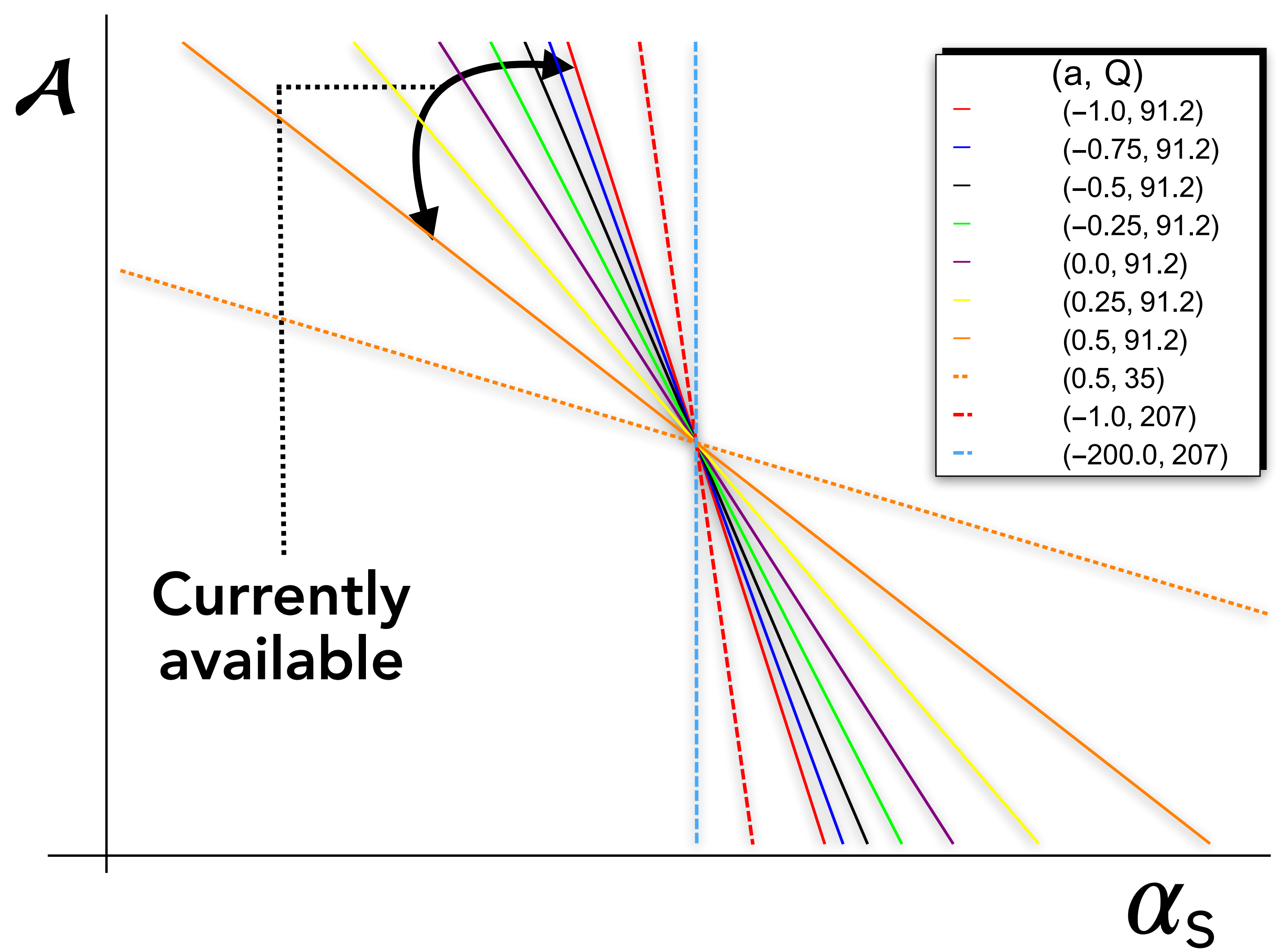






$$\frac{\mathcal{A}}{(-slope)} = \alpha_s^{PT}(m_Z) - \alpha_s(m_Z)$$

The 'angularity star'



Summary and outlook

- Due to the **parametric dependence of non-perturbative effects**, *angularity* distributions offer a unique opportunity to break the degeneracy in two-dimensional $\mathcal{A} - \alpha_s$ fits.
- Our recent improvement to **NNLL' + $O(\alpha_s^2)$ + NP accuracy** motivates such a fit.
- We have presented preliminary results using a simple correlation model. The central values we obtain from a global fit to all seven observables are:

$$\alpha_s(m_Z)\big|_{\mathrm{NNLL'}} = 0.109 \pm 0.007_{\mathrm{exp}} \pm 0.007_{\mathrm{th}}$$
$$\mathcal{A}\big|_{\mathrm{NNLL'}} = 0.36 \pm 0.37_{\mathrm{exp}} \pm 0.19_{\mathrm{th}} \quad (\mathrm{GeV})$$

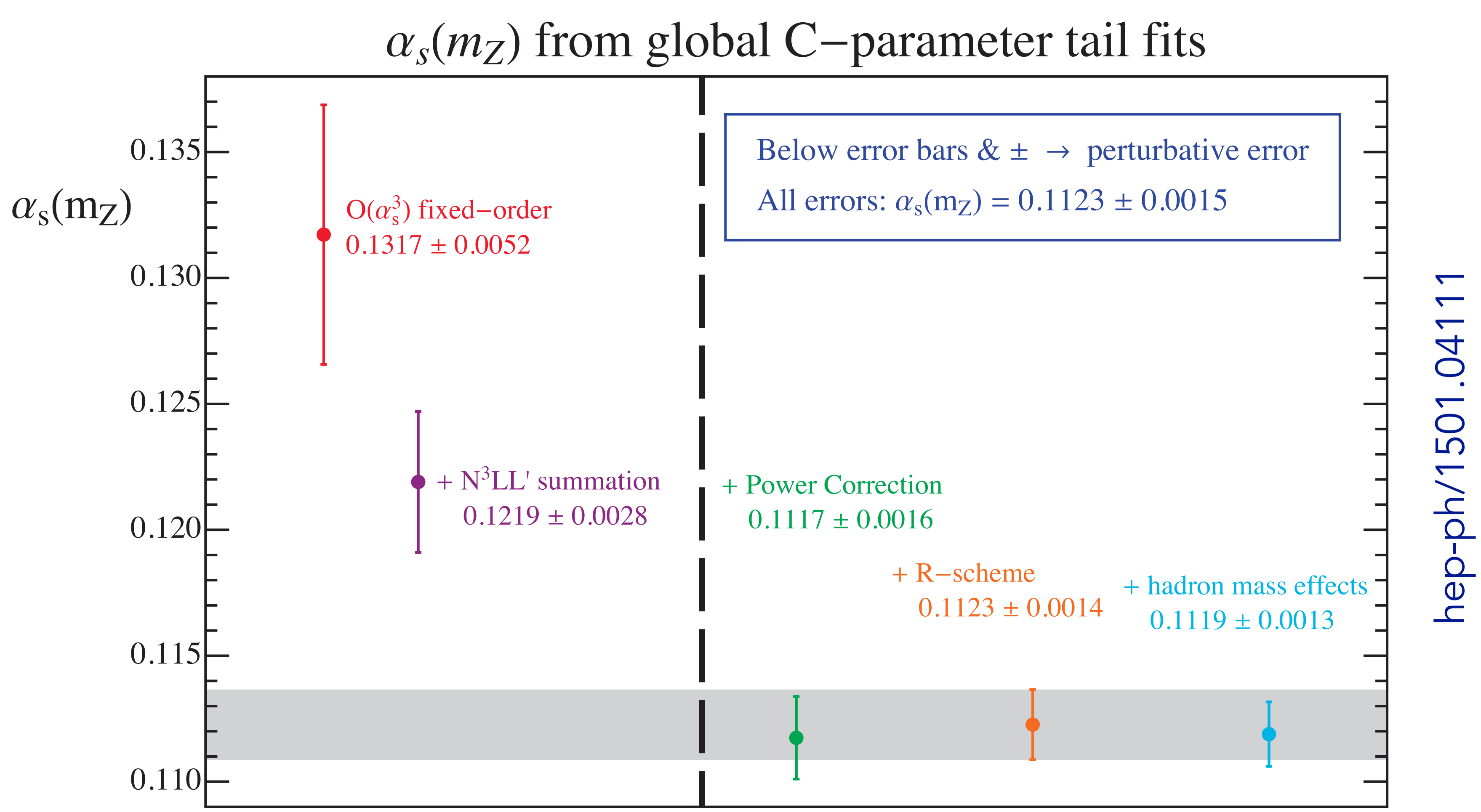
Preliminary!

- These values are **consistent with** prior **SCET extractions**, but are still well **below the world average**…additionally, central values appear **highly sensitive to the fit window** chosen.
- Results do not **yet** include a complete non-perturbative treatment (WIP). We are also validating our results with a second, **independent Python code**. More theory improvements possible.
- Only one dataset exists. **More data**, at more values of ***Q*** and ***a***, could permit an unambiguous **disentangling** of leading non-perturbative effects.
- Other **statistical models/methods** should also be explored.

Talk dedicated to A. Hornig

Backup: renormalon expectations

- Although we have not yet performed extractions with fully shape- and renormalon-corrected theory curves, we have naive estimates of their effects from prior analyses:



order	$\alpha_s(m_Z)$ (with $\overline{\Omega}_1$)	$\alpha_s(m_Z)$ (with $\Omega_1(R_\Delta,\mu_\Delta)$)
NLL′	0.1071(60)(05)	0.1059(62)(05)
N^{2}LL′	0.1102(32)(06)	0.1100(33)(06)
N^{3}LL′ (full)	0.1117(16)(06)	**0.1123(14)(06)**

order	$\overline{\Omega}_1$ [GeV]	$\Omega_1(R_\Delta,\mu_\Delta)$ [GeV]
NLL′	0.533(154)(18)	0.582(134)(16)
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N^{3}LL′ (full)	0.384(91)(20)	**0.421(60)(20)**

Backup: the PDG table on α_s

To be included in the PDG average, a fit must:

- be published in a peer-reviewed journal…
- include $O(\alpha_s^3)$ fixed-order perturbative results…
- include `reliable' error estimates, including NP effects…

2018 PDG world average:
.1181 +- .0011

Thrust at N^{3}LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

Riccardo Abbate,[1] Michael Fickinger,[2] André H. Hoang,[3] Vicent Mateu,[3] and Iain W. Stewart[1]

hep-ph/1006.3080

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert}$$

A Precise Determination of α_s from the C-parameter Distribution

André H. Hoang,[1,2] Daniel W. Kolodrubetz,[3] Vicent Mateu,[1] and Iain W. Stewart[3]

hep-ph/1501.04111

$$\alpha_s(m_Z) = 0.1123 \pm 0.0002_{\rm exp} \pm 0.0007_{\rm hadr} \pm 0.0014_{\rm pert}$$