Rapidity Logs and Overlap Subtractions in SCET

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Outline

Motivating SCET without modes

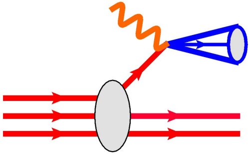
Old ideas from a new perspective

- Rapidity divergences and integration ambiguities
- Using ambiguities to sum rapidity logs

Defining overlap subtraction at subleading powers

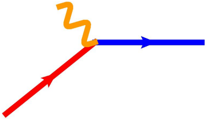
Recall: Endpoint DIS

In [Manohar, **hep-ph/0309176**] SCET was used to study DIS in the $x \to 1$ limit



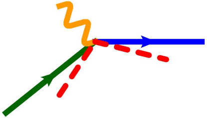
In Target Rest Frame, need 2 modes

(us) , (n)



In the Breit Frame, need 3 modes

($\bar{n}$) , (us) , (n)



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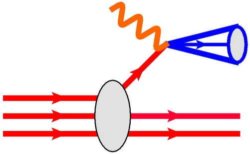

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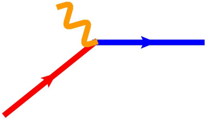

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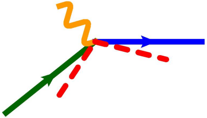
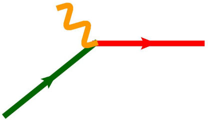


Boost invariance says 2 modes is sufficient in *all* frames!

Jet Rest Frame

In Jet Rest Frame, also need 2 modes

($\bar{n}$) , (us)



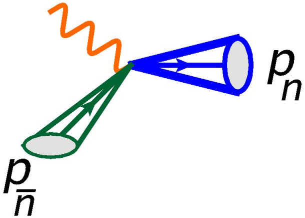
- Mode classification of incoming/outgoing states is frame-dependent
- Each collimated jet of particles is soft in its own frame → QCD
  [Bauer et al, 0809.1099.pdf], [Freedman and Luke, 1107.5823]

A Frame-Independent Formalism [Goerke and Luke, 1711.09136]

Sectors are defined based purely on invariant mass

2 sectors: $p_n^2 \ll Q^2$, $p_{\bar{n}}^2 \ll Q^2$

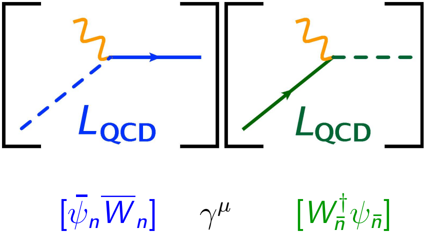
$$2p_n \cdot p_{\bar{n}} \sim Q^2$$



Each sector gets its own copy of QCD

Sectors are only coupled via the hard current, with expansion in inverse powers of the matching scale

$$\mathcal{J}^{\mu}_{\rm QCD} = e^{-iq\cdot x}\bar{\psi}\gamma^{\mu}\psi \to \sum_i \frac{C_2^i}{Q^{[i]}} O_2^{(i)}$$



Mode-Free Features

No λ-scaling

No explicit softs/ultrasofts/other modes
$\rightarrow$ No distinction between $\text{SCET}_{\text{I}}/\text{SCET}_{\text{II}}$ at the matching scale
$\rightarrow$ Further factorization of scales $\ll Q$ happens later
at low scale matching (Factorization = matching)

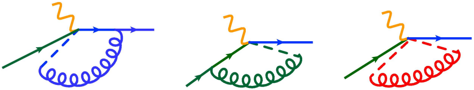
Structure of SCET at subleading powers is simpler
(no NLP collinear-soft interactions)

Subtlety: Overcounting

Just like the zero-bin subtraction of [Manohar and Stewart **hep-ph/0605001**], $n{+}\bar{n}$ sectors double-count some low energy degrees of freedom.

E.g. $2p \cdot p_n \ll Q^2$ and $2p \cdot p_{\bar{n}} \ll Q^2 \rightarrow$ Overlap subtraction

$$I_n \quad + \quad I_{\bar{n}} \quad - \quad I_s$$

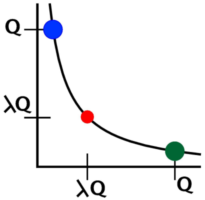


Overlap prescription we use is similar to the LO QCD factorization of [Feige and Schwartz **1403.6472**] or the soft subtraction of [Idilbi and Mehen **hep-ph/0702022**]

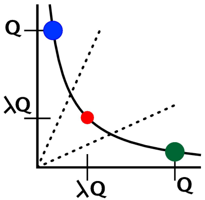
$$\langle p_f |\, O_2^{(0)} \,| p_i \rangle_{subtracted} = \frac{\langle p_f | O_2^{(0)} | p_i \rangle}{\frac{1}{N_c} \mathrm{Tr} \langle 0 | W_n^\dagger W_{\bar{n}} | 0 \rangle} = \frac{\langle p_f^n | \bar{\chi}_n | p_i^n \rangle \gamma^\mu \left\langle p_f^{\bar{n}} \middle| \chi_{\bar{n}} \middle| p_i^{\bar{n}} \right\rangle}{\frac{1}{N_c} \mathrm{Tr} \langle 0 | W_n^\dagger W_{\bar{n}} | 0 \rangle}$$

SCET$_{\mathrm{II}}$ and RRG without Explicit Softs?

Usual description of SCET$_{\mathrm{II}}$ processes is $n/\bar{n}/s$ lying on same hyperbola.



Ambiguous separation of modes necessitates rapidity cutoffs to distinguish between modes [Chiu et al. **1202.0814**]

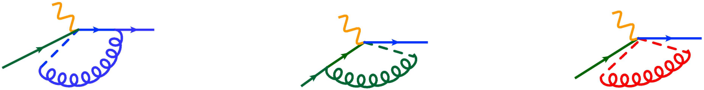


Independence of cutoffs gives Rapidity Renormalization Group

How does this arise in a formalism without explicit softs?

Hidden Scheme Dependence

Take O_2 renormalization with gluon mass IR regulator. Well-known that individual diagrams are unregulated, but well-defined in the sum
[Chiu et al, **0901.1332**]



$(I_n \sim \int \frac{dk^-}{k^-}(1-\frac{k^-}{p_1^-})^{(1-\epsilon)}) + (I_{\bar{n}} \sim \int \frac{dk^+}{k^+}(1-\frac{k^+}{p_2^+})^{(1-\epsilon)})$ - $(I_s \sim \int \frac{dk^-}{k^-})$
Each graph is individually rapidity divergent.

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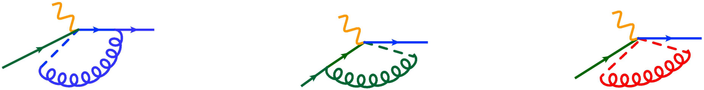

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Each graph is individually rapidity divergent.

$\rightarrow$ To get the usual result, do integrals in the same order

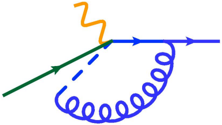
$(I_n \sim \int \frac{dk^-}{k^-}(1-\frac{k^-}{p_1^-})^{(1-\epsilon)}) + (I_{\bar{n}} \sim \int \frac{p_2^+ dk^-}{M^2}(1-\frac{p_2^+ k^-}{M^2})^{-1})$ - $(I_s \sim \int \frac{dk^-}{k^-})$

$$\sim \frac{2}{\epsilon^2} + \frac{3-2\log\frac{Q^2}{\mu^2}}{\epsilon} - \log^2\frac{M^2}{\mu^2} + 2\log\frac{Q^2}{\mu^2}\log\frac{M^2}{\mu^2} - 3\log\frac{M^2}{\mu^2}$$

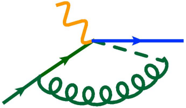
Careful! Adding 3 infinite quantities isn't well-defined.

Scheme Dependence in Detail

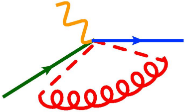
$$\langle p_f | \, O_2^{(0)} \, | p_i \rangle_{subtracted} = \frac{\left\langle p_f^n \middle| \bar{\chi}_n \middle| p_i^n \right\rangle \gamma^\mu \left\langle p_f^{\bar{n}} \middle| \chi_{\bar{n}} \middle| p_i^{\bar{n}} \right\rangle}{\frac{1}{N_c} \mathrm{Tr} \langle 0 | W_n^\dagger W_{\bar{n}} | 0 \rangle}$$



$$I_n \sim \int_0^{p_1^-} \frac{dk^-}{-k^-} \left(1 - \frac{k^-}{p_1^-}\right)^{1-\epsilon}$$



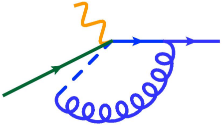
$$I_{\bar{n}} \sim A + \int_0^\infty \frac{p_2^+ d\ell^-}{M^2} \frac{1}{1 - \frac{p_2^+ \ell^-}{M^2}}$$



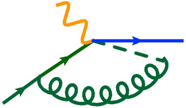
$$I_{sub} \sim \int_0^\infty \frac{dq^-}{-q^-}$$

Scheme Dependence in Detail - Rescaling

$$\langle p_f | \, O_2^{(0)} \, | p_i \rangle_{subtracted} = \frac{\langle p_f^n | \bar{\chi}_n | p_i^n \rangle \gamma^\mu \langle p_f^{\bar{n}} | \chi_{\bar{n}} | p_i^{\bar{n}} \rangle}{\frac{1}{N_c} \mathrm{Tr} \langle 0 | W_n^\dagger W_{\bar{n}} | 0 \rangle}$$

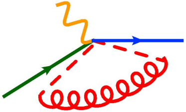


$$I_n \sim \int_0^{p_1^-} \frac{dk^-}{-k^-} \left(1 - \frac{k^-}{p_1^-}\right)^{1-\epsilon} = \int_0^1 \frac{dx}{-x} (1-x)^{1-\epsilon}$$



$$p_2^+ d\ell^- \rightarrow \zeta^2 y$$

$$I_{\bar{n}} \sim A + \int_0^\infty \frac{p_2^+ d\ell^-}{M^2} \frac{1}{1 - \frac{p_2^+ \ell^-}{M^2}} = A + \int_0^\infty \frac{\zeta^2 dx}{M^2} \frac{1}{1 - \frac{\zeta^2 x}{M^2}}$$



$$I_{sub} \sim \int_0^\infty \frac{dq^-}{-q^-} = \int_0^\infty \frac{dx}{-x}$$

The only scale in the calculation, ζ, is arbitrary!

Hidden Scheme Dependence

$$I_n + I_{\bar{n}} - I_{sub} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{3 - 2\log\frac{\zeta^2}{\mu^2}}{\epsilon} - \log^2\frac{M^2}{\mu^2} + 2\log\frac{\zeta^2}{\mu^2}\log\frac{M^2}{\mu^2} - 3\log\frac{M^2}{\mu^2}\right]$$

Previous calculations find $\log\frac{Q^2}{\mu^2}$ but, here we find $\log\frac{\zeta^2}{\mu^2}$

Interpretation: **rapidity logarithms are related to scheme dependence of overlap subtraction**

Matching from QCD then fixes the scheme parameter ζ:

$$C_2(\mu,\zeta) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\log\frac{Q^2}{\mu^2} + 3\log\frac{Q^2}{\mu^2} + 2\log\frac{Q^2}{\zeta^2}\log\frac{M^2}{\mu^2}\right)$$

Matching at hard scale Q fixes $\zeta = Q$
(required if no IR dependence in matching condition)

Scheme Dependence with a Delta Regulator

Can formalize scheme dependence with δ-regulator [Chiu et al. **0901.1332**]

$$\mathrm{n-sector}: \frac{1}{-k^{-}+i0^{+}} \to \frac{1}{-k^{-}-\delta_n+i0^{+}}$$

$$\bar{\mathrm{n}}\mathrm{-sector}: \frac{1}{-k^{+}+i0^{+}} \to \frac{1}{-k^{+}-\delta_{\bar{n}}+i0^{+}}$$

$$\mathrm{Subtraction}: \frac{1}{-k^{-}+i0^{+}} \to \frac{1}{-k^{-}-\delta_{sub}+i0^{+}}$$

$$C_2(\mu,\nu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\log\frac{Q^2}{\mu^2} + 3\log\frac{Q^2}{\mu^2} + 2\log\frac{\delta_n\delta_{\bar{n}}}{\delta_{sub}^2}\log\frac{M^2}{\mu^2}\right)$$

Direction of $\{\delta_n, \delta_{\bar{n}}, \delta_{sub}\} \to 0$ determines scheme:

$$\frac{\delta_n\delta_{\bar{n}}}{\delta_{sub}^2} = \frac{Q^2}{\nu^2}$$

Scheme freedom at $\mu \sim M$

After running from $\mu = Q$ down to $\mu = M$, the scheme parameter ν becomes free. To see this, match from SCET with $\nu = Q$ onto SCET with ν arbitrary.

$$\langle p_2 | \, O_2(\mu, Q) \, | p_1 \rangle \sim -\log^2 \frac{M^2}{\mu^2} + 2 \log \frac{Q^2}{\mu^2} \log \frac{M^2}{\mu^2} - 3 \log \frac{M^2}{\mu^2}$$
$$\langle p_2 | \, O_2(\mu, \nu) \, | p_1 \rangle \sim -\log^2 \frac{M^2}{\mu^2} + 2 \log \frac{\nu^2}{\mu^2} \log \frac{M^2}{\mu^2} - 3 \log \frac{M^2}{\mu^2}$$

$$C_{scheme \;\; matching} \sim \log \frac{Q^2}{\nu^2} \log \frac{M^2}{\mu^2}$$

All logs are minimized in C_{sm} by taking $\mu = M$ and $\nu = Q$. Building up many of these small matching procedures gives the renormalization group!

Resummation using Scheme Parameter

Interpret previous O_2 loops as massive form factor calculation

$$\begin{aligned}F(Q^2, M^2) &= \langle p_1| J^\mu_{QCD} |p_2\rangle \\ &= C_2(\mu, Q) \langle p_1| O_2(\mu, Q) |p_2\rangle \\ &= C_2(M, Q) C_{sm}(M, \nu) \langle p_1| O_2(M, \nu) |p_2\rangle \\ &= C_2(M, Q)[C_{sm}(\tfrac{\nu}{Q}) D(\tfrac{\nu}{M})]_{\mu=M}\end{aligned}$$

Now everything looks similar to the factorization of $F(M^2, Q^2)$ by [Chiu et al. **1202.0814**], where resummation is achieved through the Rapidity Renormalization Group

Since $\frac{dF(M^2,Q^2)}{d\log\nu} = 0$, can derive

$$\begin{aligned}\frac{d}{d\log\nu} C_{sm} &= \left(- D^{-1} \frac{d}{d\log\nu} D \right) C_{sm} \\ &= \left(-\frac{\alpha_s C_F}{\pi} \log\frac{M^2}{\mu^2}\right) C_{sm} = \gamma^\nu_{sm} C_{sm}\end{aligned}$$

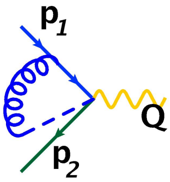
$\rightarrow$ Reproduces the standard result

More SCET_{II} – Drell-Yan at Small Q_T

Same techniques can be applied to DY with $Q^2 \gg Q_T^2 \gg \Lambda_{QCD}^2$. Can reproduce momentum space dQ_T^2 results of [Ebert and Tackmann, 1611.08610]

$\mu = Q$
$\nu = Q$

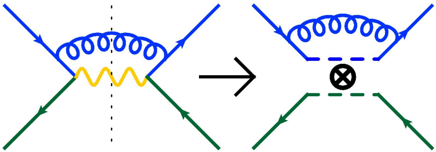
QCD→SCET



$\downarrow$

$\mu = Q_T$
ν free

SCET→PDFs



$\uparrow$

Large $\log \frac{\nu^2}{Q_T^2} \log \frac{Q_T^2}{\mu^2}$ in matching

Identifying Overcounting at Next-to-Leading Power

Easiest to study Q_T^2/Q^2 corrections are the subleading contributions to the cross-section from the multipole expansion, e.g.

$$\delta(p_n^- + p_{\bar{n}}^- - Q^-) \to \delta(p_n^- - Q^-) + p_{\bar{n}}^- \frac{d}{dp_n^-}\delta(p_n^- - Q^-) \to O_2^{(0)} + O_2^{(2\delta^-)}$$

$$I_n^{(0)} \sim \int \frac{d\omega^-}{\omega^-}(2 - 2\omega^- + (\omega^-)^2)$$
$$I_n^{(2\delta^-)} \sim 0$$
$$I_n^{(2\delta^+)} \sim \int \frac{d\omega^-}{\omega^-}(-\omega^+)(2 - 2\omega^- + (\omega^-)^2)$$

$$\omega^- \equiv \frac{k^-}{p_1^-},\ \omega^+ \equiv \frac{Q_T^2}{\omega^- p_1^- p_2^+}$$

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Again, easy to see the double-counting

Next-to-Leading Power Subtraction – Identification

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$I_{\bar{n}}^{(2\delta^+)} \sim 0$

$I_{sub} \sim \int \frac{d\omega^-}{\omega^-}(2 - 2\omega^+ - 2\omega^-$

Again, easy to see the double-counting

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$$I_{sub} \sim \int \frac{d\omega^-}{\omega^-}(2 - 2\omega^+ - 2\omega^- + 2\omega^-\omega^+)$$

Again, easy to see the double-counting.

Next-to-Leading Power Subtraction – Prescription

$$I_{sub} \sim \int \frac{d\omega^-}{\omega^-}(2 + 2\omega^-\omega^+ - 2\omega^+ - 2\omega^-)$$

$$\frac{\langle p_1 p_2 | T\{O_2^{(0)}(x) O_2^{(0)\dagger}(0)\} | p_1 p_2\rangle}{\frac{1}{N_c}\mathrm{Tr}\langle 0 | T\{(1+\frac{D_T^2}{Q^2}) W^\dagger(x) W(x) \overline{W}^\dagger(0) \overline{W}(0)\} | 0\rangle} \supset \int \frac{d\omega^-}{\omega^-}(2 + 2\omega^-\omega^+)$$

$\rightarrow$ NLP subtraction of LP operators

$$\frac{\langle p_1 p_2 | T\{O_2^{(2\delta^+)}(x) O_2^{(0)\dagger}(0)\} | p_1 p_2\rangle}{\frac{1}{N_c}\mathrm{Tr}\langle 0 | T\{W^\dagger(x) W(x) \overline{W}^\dagger(0) \overline{W}(0)\} | 0\rangle} \supset \int \frac{d\omega^-}{\omega^-}(-2\omega^+)$$

$$\frac{\langle p_1 p_2 | T\{O_2^{(2\delta^-)}(x) O_2^{(0)\dagger}(0)\} | p_1 p_2\rangle}{\frac{1}{N_c}\mathrm{Tr}\langle 0 | T\{W^\dagger(x) W(x) \overline{W}^\dagger(0) \overline{W}(0)\} | 0\rangle} \supset \int \frac{d\omega^-}{\omega^-}(-2\omega^-)$$

$\rightarrow$ LP subtraction of NLP operators

Net Subleading Contribution

$$I_n^{(0)} + I_n^{(2\delta^-)} + I_n^{(+2\delta^+)} + I_{\bar{n}}^{0} + I_{\bar{n}}^{(2\delta^-)} + I_{\bar{n}}^{(+2\delta^+)} - I_{sub} \sim (1 + \frac{Q_T^2}{Q^2}) \log \frac{\zeta^2}{Q_T^2}$$

- Subtractions automatically tame all rapidity divergences
- Rescaling of different integrals again gives a scheme dependent result
- Subleading power subtractions required

Works in progress:

- Other subleading power operators need to be included in calculation
- As pointed out by [Ebert et al, 1812.08189], power-law divergences need better regulator than δ or η. Still shopping.

Conclusion

SCET without modes is useful to study next-to-leading power calculations

In this formalism, rapidity logs are related to ambiguities in summing diagrams

At low energies the choice of scheme becomes free, allowing for resummation of rapidity logs

Overlap subtraction at subleading powers requires additional denominator insertions