Resummation of the D-parameter

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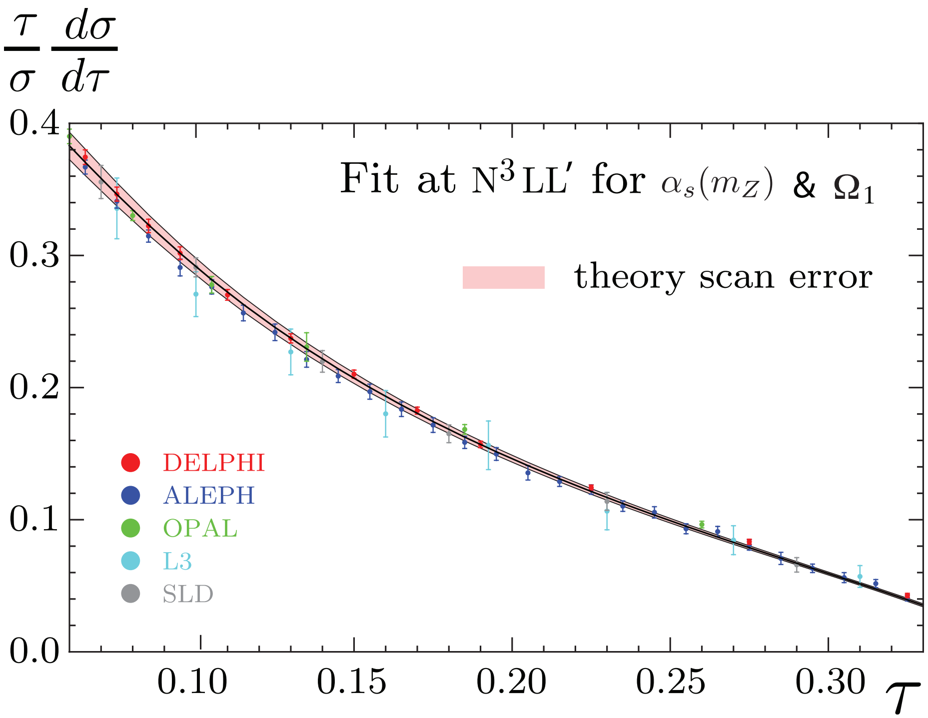
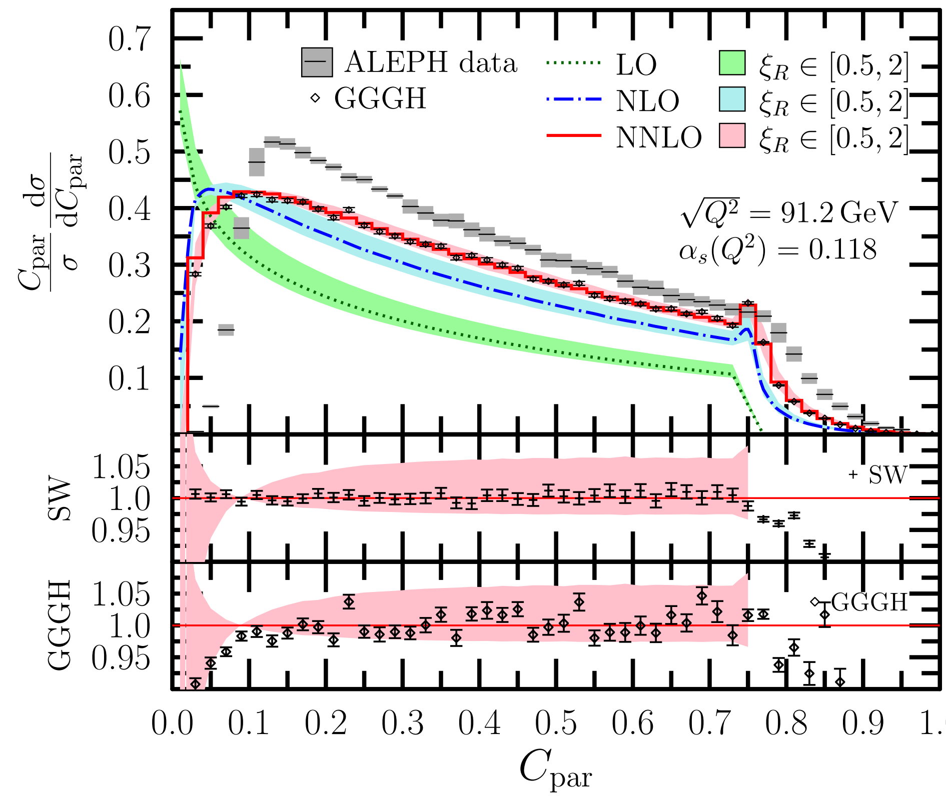
Legacy of LEP Precision Measurements

Thrust at N^{3}LL + NNLO



Abbate, Fickinger, Hoang, Mateu, Stewart 2010

C-parameter at NNLO



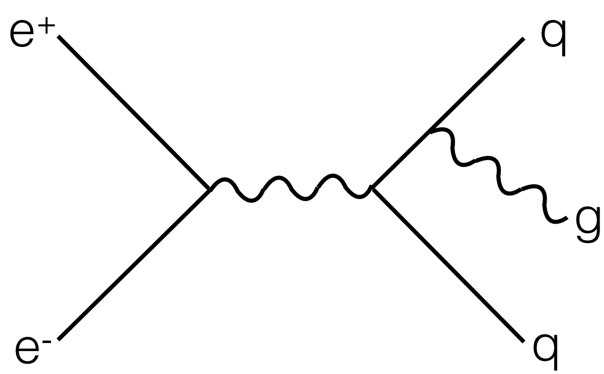
Del Duca, Duhr, Kardos, Somogyi, Szőr, Trócsányi, Tulipánt 2016

Precision data/theory comparisons enable α_s extraction

Legacy of LEP Precision Measurements

Why thrust and C-parameter?

NNLO:
$2 \to 3$ at two-loops

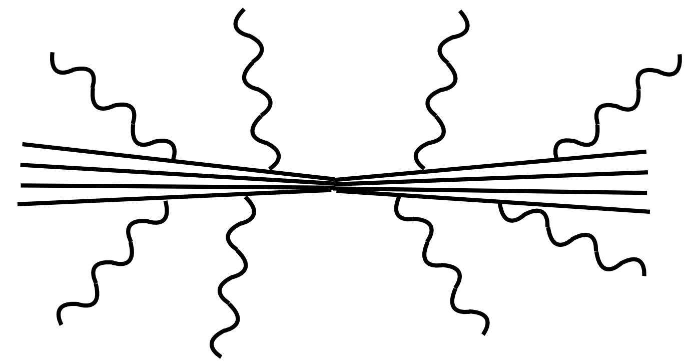


τ, C, B, first non-zero

Lots of tools!

EERAD3, CoLoRFulNNLO, etc.

NNLL/N^{3}LL:
Small values restrict radiation



τ, C, $B \ll 1$

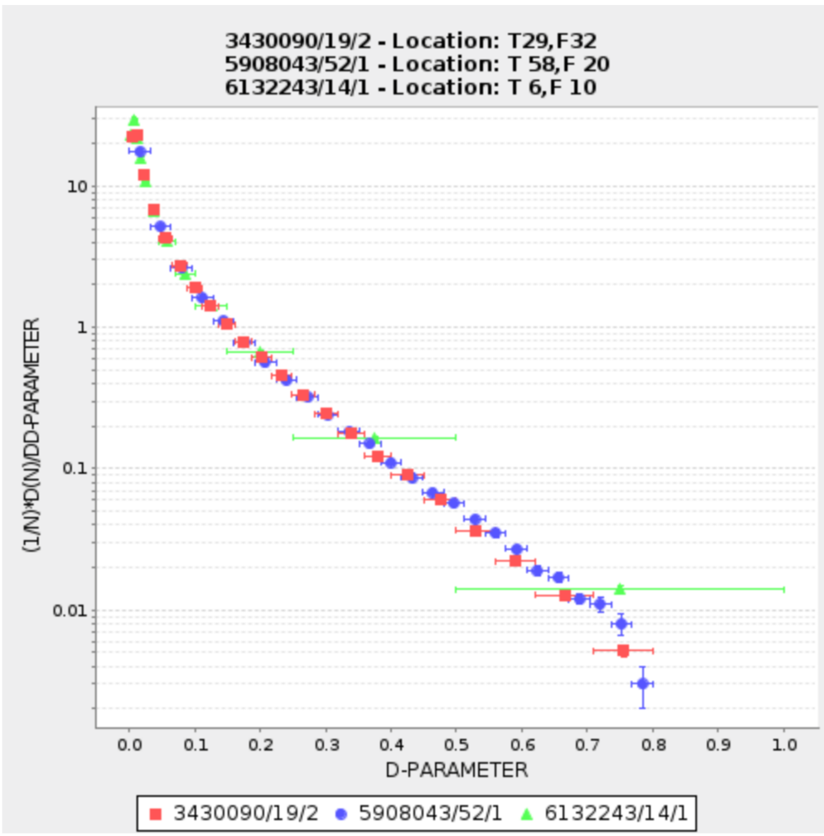
Cross section factorizes

Hard, Soft, Collinear functions

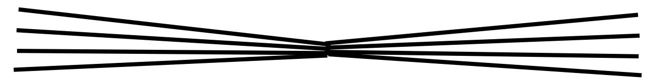
Legacy of LEP Precision Measurements

LEP measured more than just τ and C!

See: http://hepdata.cedar.ac.uk/review/shapes/

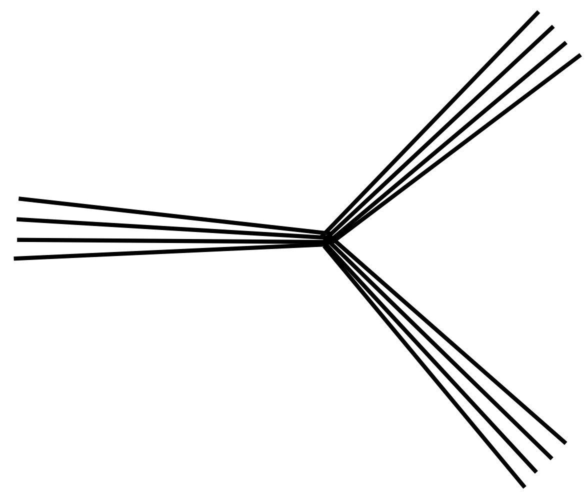


D parameter at DELPHI, L3, OPAL



$C \to 0$: linear event

codimension 2



$D \to 0$: planar event

codimension 1

Legacy of LEP Precision Measurements

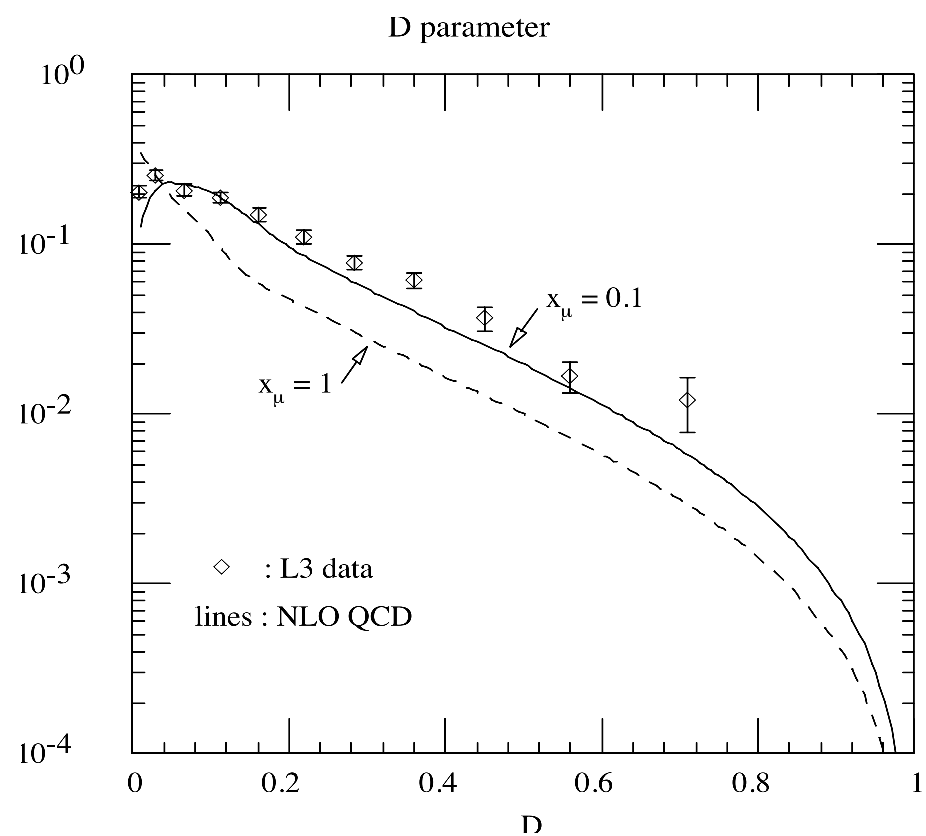
Challenges with D parameter: Fixed Order

NNLO:
$2 \to 4$ at two-loops

Just starting to be calculated

Badger, Brønnum-Hansen, Hartanto, Peraro 2017
Abreu, Febres Cordero, Ita, Page, Zeng 2017

State of the art NLO:
$2 \to 4$ at one-loop

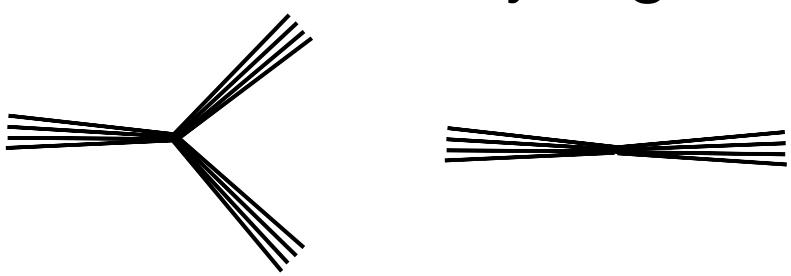


Nagy, Trócsányi 1997

Legacy of LEP Precision Measurements

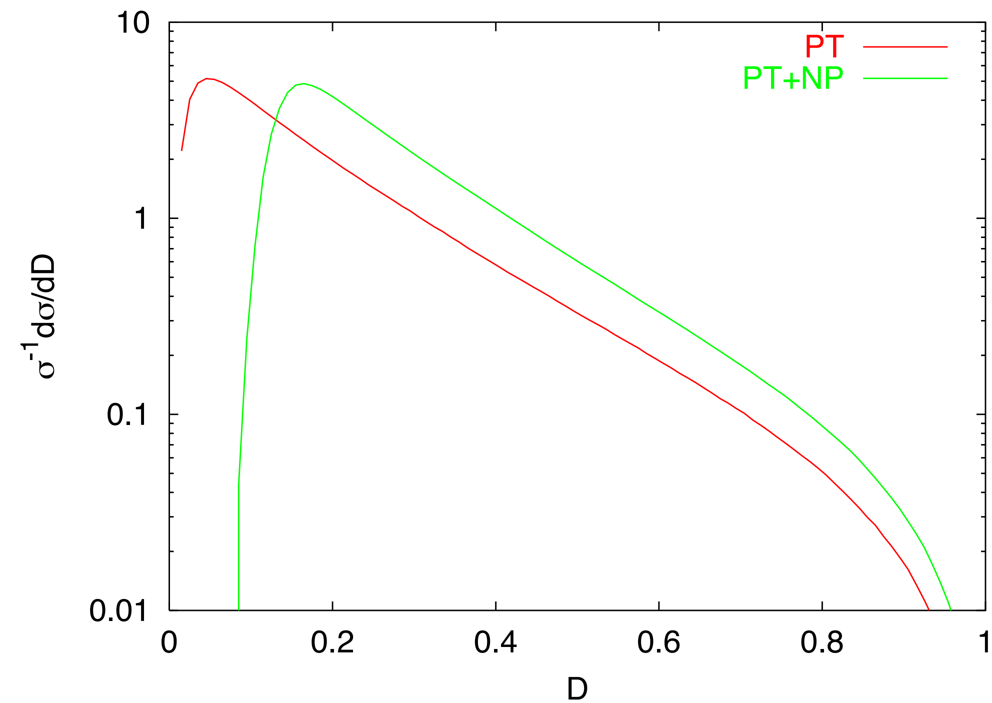
Challenges with D parameter: Resummation

NLL:
$D \to 0$ has many regions



Can have the same value of D

State of the art NLL:
Restrict to near-planar region



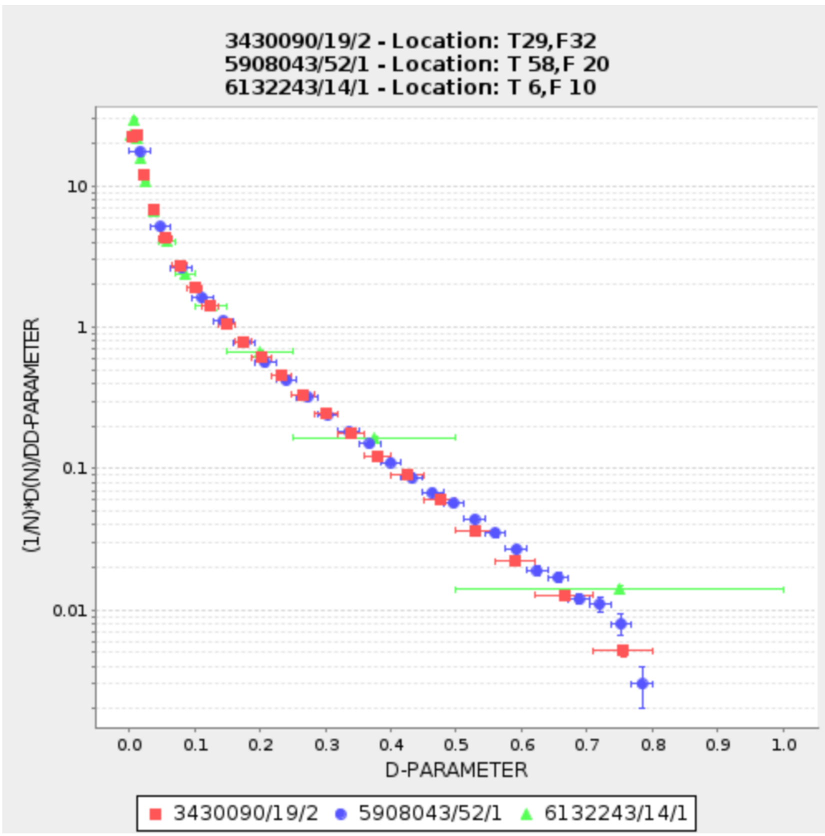
Banfi, Dokshitzer, Marchesini, Zanderighi 2001

$y_3 > 0.1$

Not inclusive over final state

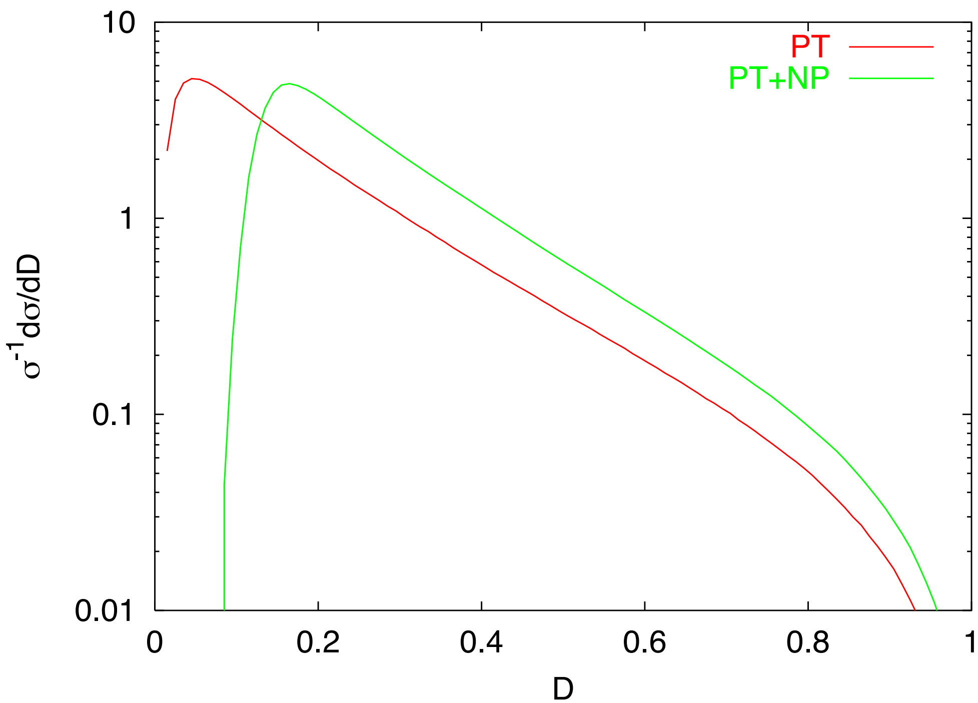
Legacy of LEP Precision Measurements

Challenges with D parameter: Resummation



Inclusive over everything but *D*

$\neq$



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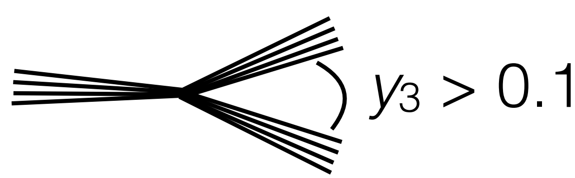


Spherocity Tensor

C and *D* are defined from the eigenvalues of the spherocity tensor

$$\Theta_{\alpha\beta} = \frac{1}{Q}\sum_i \frac{p_{i\alpha}p_{i\beta}}{E_i}$$

$$C = 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) = \frac{3}{Q^2}\sum_{i<j} E_i E_j \sin^2\theta_{ij}$$

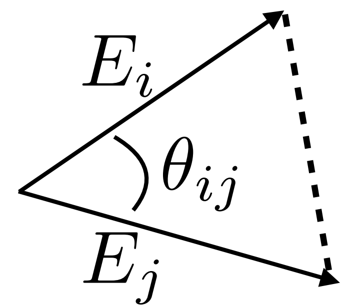
$$D = 27\lambda_1\lambda_2\lambda_3 = \frac{27}{Q^3}\sum_{i<j<k} \frac{|\mathbf{p}_i \cdot (\mathbf{p}_j \times \mathbf{p}_k)|^2}{E_i E_j E_k}$$

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \qquad\qquad \lambda_1 + \lambda_2 + \lambda_3 = 1$$

Spherocity Tensor

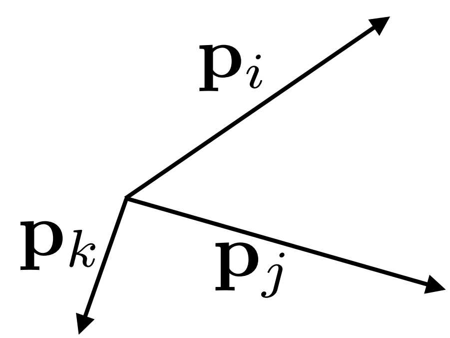
$$C = 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) = \frac{3}{Q^2}\sum_{i<j} E_i E_j \sin^2\theta_{ij}$$

$C \approx$ (area of triangle)^2



$$D = 27\lambda_1\lambda_2\lambda_3 = \frac{27}{Q^3}\sum_{i<j<k}\frac{|\mathbf{p}_i\cdot(\mathbf{p}_j\times\mathbf{p}_k)|^2}{E_i E_j E_k}$$

$D \approx$ (volume of parallelpiped)^2



Spherocity Tensor

Simultaneous measurement of C and D isolate configurations!

Three parametric relations in $D \rightarrow 0$ limit:

- Region 1: Large Areas, Small Volumes

$$\lambda_3 \ll \lambda_2 \sim \lambda_1 \sim 1 \qquad\qquad D \ll C^2 \sim 1$$

- Region 2: Small Areas, Smaller Volumes

$$\lambda_3 \ll \lambda_2 \ll \lambda_1 \sim 1 \qquad\qquad D \ll C^2 \ll 1$$

- Region 3: Small Areas, Small Volumes

$$\lambda_3 \sim \lambda_2 \ll \lambda_1 \sim 1 \qquad\qquad D \sim C^2 \ll 1$$

Spherocity Tensor

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Banfi, Dokshitzer, Marchesini, Zanderighi 2001

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Spherocity Tensor

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- Region 1: Large Areas, Small Vol

Banfi, Dokshitzer, Marchesini, Zanderighi 2001

$$D \ll C^2 \sim 1$$

- Region 2: Small Areas, Smaller V

New; described by SCET+ (Bauer, Tackmann, Walsh, Zuberi 2011)

$$D \ll C^2 \ll 1$$

- Region 3: Small Areas, Small Volumes

$$\lambda_3 \sim \lambda_2 \ll \lambda_1 \sim 1 \qquad\qquad D \sim C^2 \ll 1$$

Spherocity Tensor

Simultaneous measurement of *C* and *D* isolate configurations!

Three parametric relations in *D* → 0 limit:

- Region 1: Large Areas, Small Vol...

  Banfi, Dokshitzer, Marchesini, Zanderighi 2001

  $D \ll C^2 \sim 1$

- Region 2: Small Areas, Smaller V...

  New; described by SCET$_+$ (Bauer, Tackmann, Walsh, Zuberi 2011)

  $D \ll C^2 \ll 1$

- Region 3: Small Areas, Small V...

  New; no known method for resummation (focus of this talk)

  $\lambda_2 \ll \lambda_1 \sim 1$ $\qquad D \sim C^2 \ll 1$

Aside: Requirements for Resummation

(Almost all) factorization theorems are for additive observables

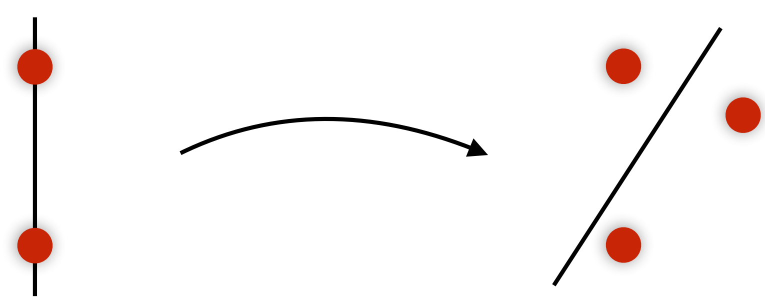
$$\frac{d\sigma}{d\tau} = H(Q^2)J(\tau) \otimes S(\tau)$$

Multiplicative factorization in conjugate space

D-parameter is not additive in Region 3!

$$\lambda_3 \sim \lambda_2 \ll \lambda_1 \sim 1 \qquad\qquad D \sim C^2 \ll 1$$

Event plane responds to each new emission



A Way Forward: A Trick

Factorization = Breaking into simpler pieces

$$p(C,D) = p(C)p(D|C)$$

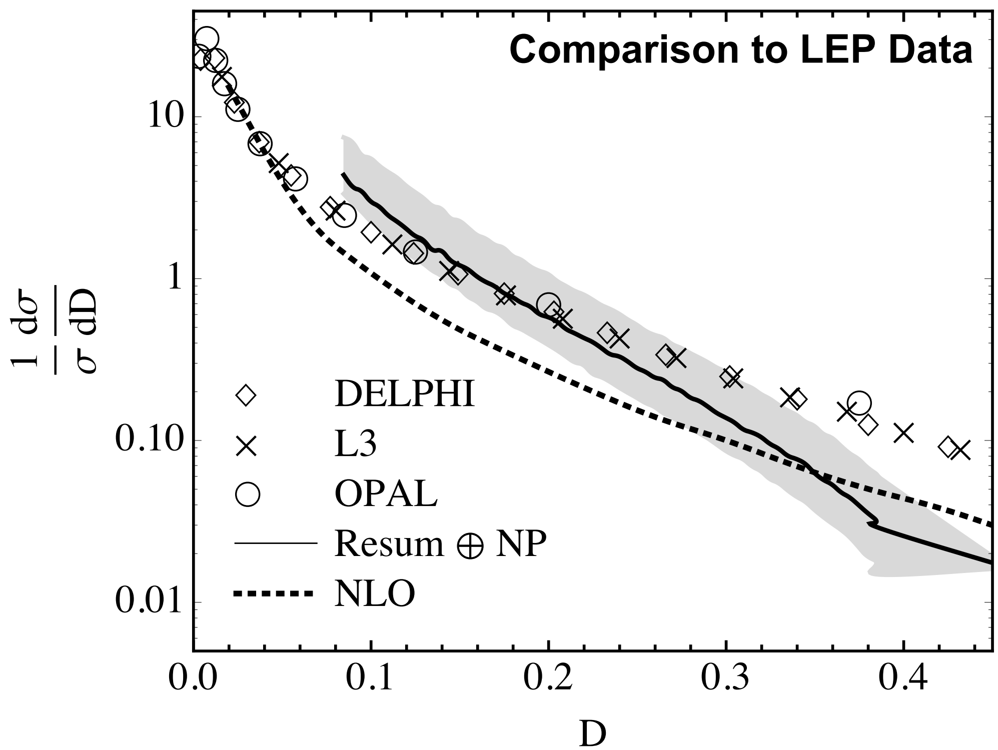
$$p_{\mathrm{resum}}(C,D) \simeq p_{\mathrm{resum}}(C)p_{\mathrm{fo}}(D|C)$$

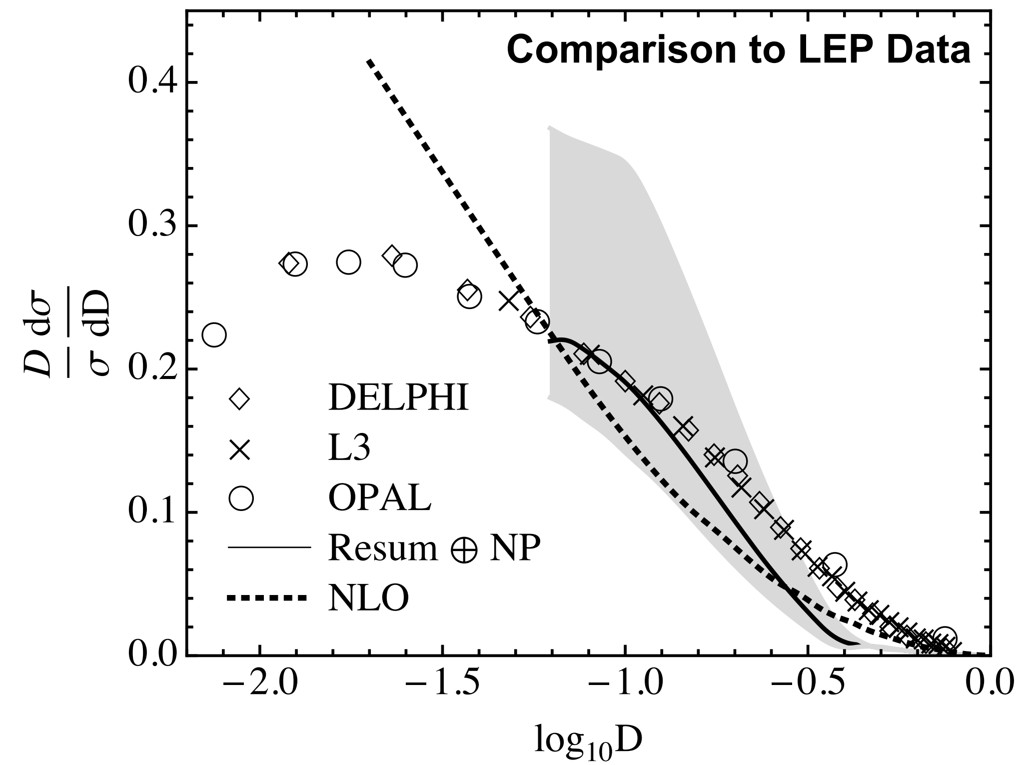
C-parameter can be resummed to $\mathrm{N}^3\mathrm{LL}$

Fixed order calculation can be systematically improved

<u>Pros</u>	<u>Cons</u>
Produces a result	Not formally log accurate
Systematically improvable	Not factorizable? (Cf. NGLs)
Numerically small at LEP	

Results and Comparison to LEP





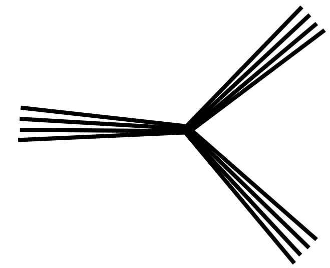
Much improved agreement with data at small D

Numerical impact of resummation of region 3 is small

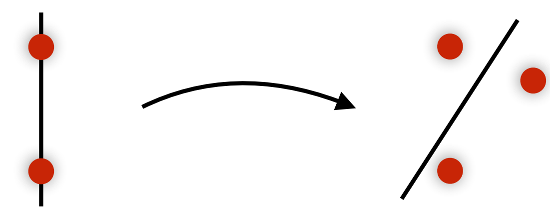
Include a shift of $D_{\rm NP} \sim \dfrac{\Lambda_{\rm QCD}}{m_Z}$

Conclusions

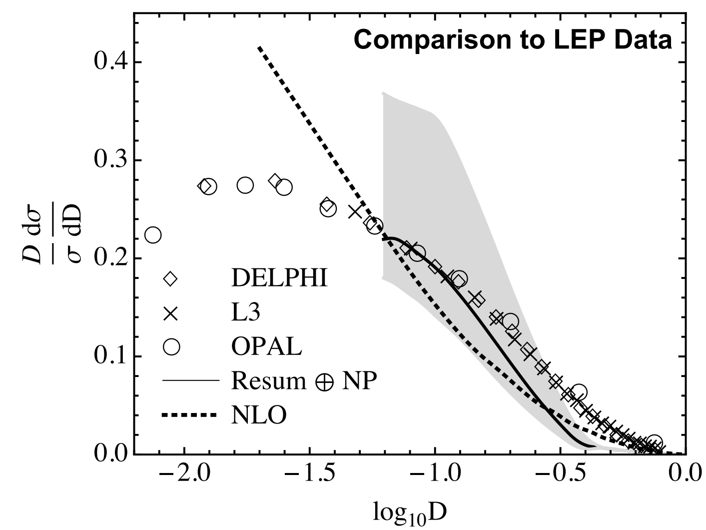
D-parameter probes aplanarity of events



Factorization through additional measurements



Comparison of resummed distribution to LEP

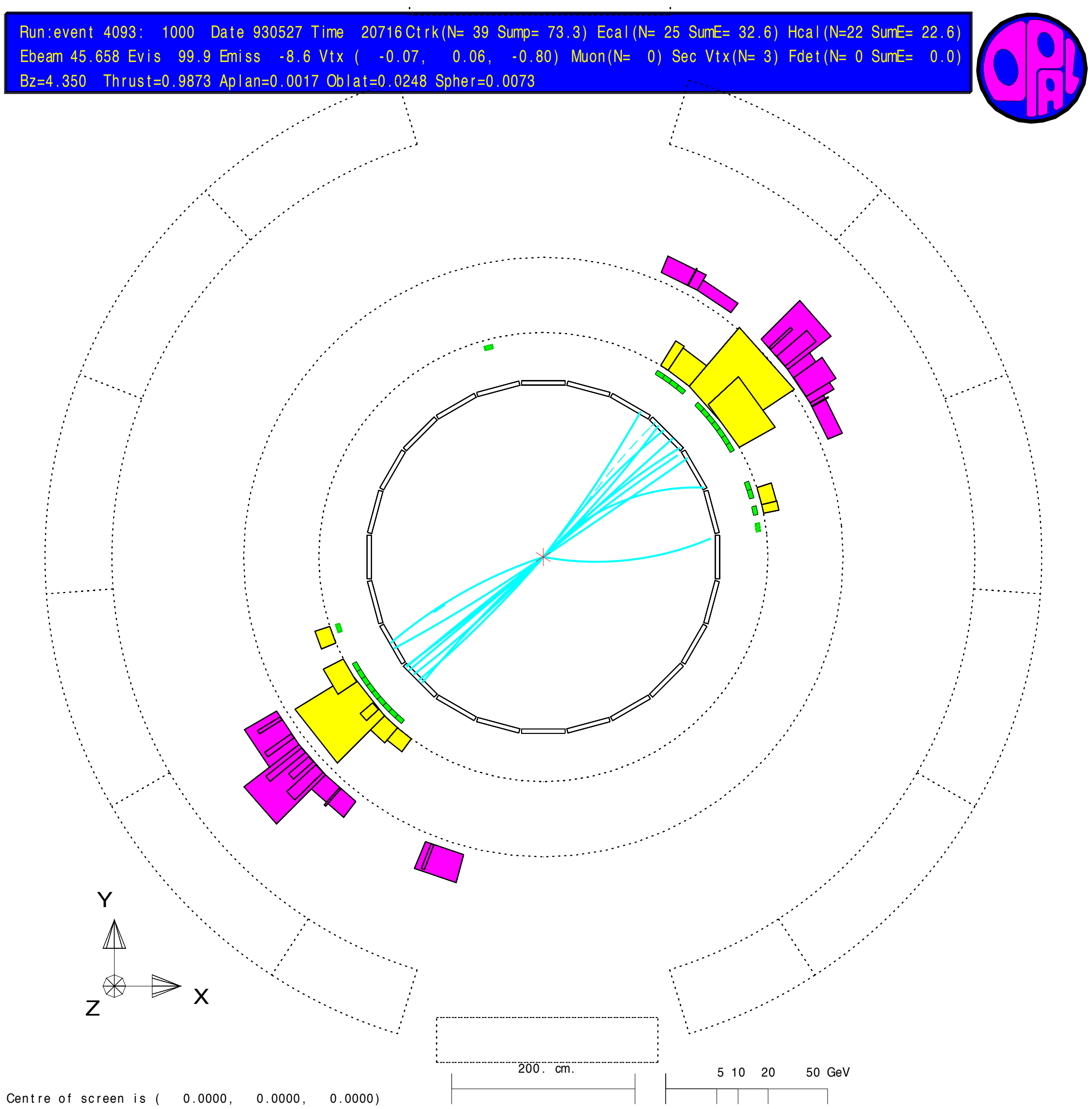


Does factorization exist in Region 3?

Can the D-parameter alone be factorized?

Bonus Slides

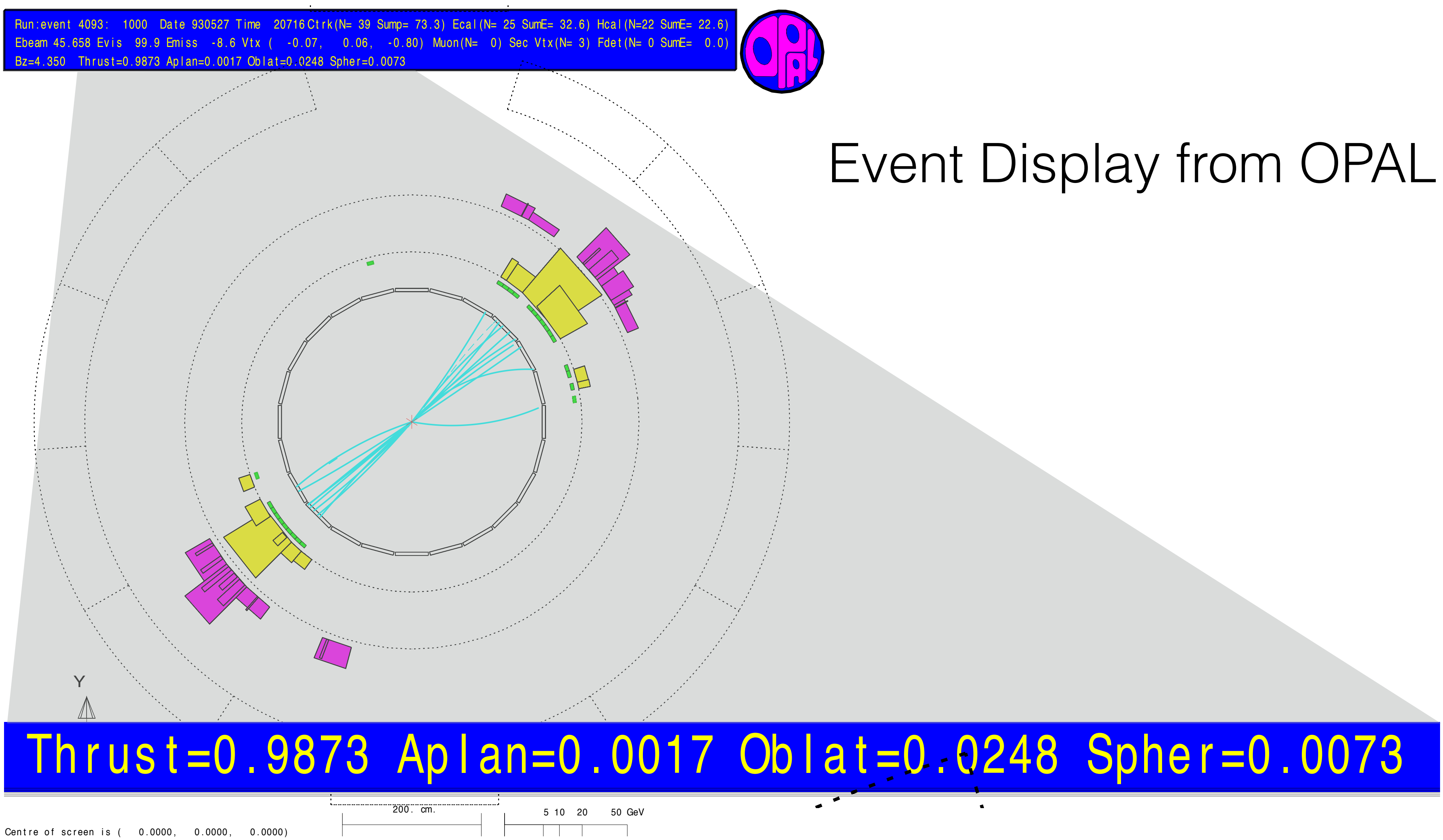
Legacy of LEP Precision Measurements



Event Display from OPAL

Legacy of LEP Precision Measurements

Event Display from OPAL



Event shape variables quantify energy flow distribution

Aside: Requirements for Resummation

1) Matrix element factorization:

$$|\mathcal{M}|^2 = |\mathcal{M}_H|^2 + |\mathcal{M}_J|^2 + |\mathcal{M}_S|^2$$

Essentially always assumed to exist

2) Most general statement of observable factorization:

$$\tau = f(\tau_J, \tau_S)$$

Typically just taken to mean the property of *additivity*:

$$\tau = \tau_J + \tau_S$$