Subleading Power Rapidity Divergences and Power Corrections for q_T

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Based on: [Ebert, Moult, Stewart, Tackmann, GV, Zhu] [1812.08189]

[Chang, Stewart, GV] [to appear]

Power expansion for generic $\mathcal{O}$ observable

- A large class of observables $\mathcal{O}$ (q_T, threshold, event shapes, etc.) exhibit singularities in perturbation theory as $\mathcal{O} \to 0$.
- Standard factorization theorems describe only leading power term.
- More generally, we can consider expanding an observable $\mathcal{O}$

$$\frac{d\sigma}{d\mathcal{O}} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \mathcal{O}}{\mathcal{O}}\right)_+$$

Leading Power (LP)

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$$+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(1)} \log^m \mathcal{O} \qquad \text{Next to Leading Power (NLP)}$$

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$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \mathcal{O}}{\mathcal{O}}\right)_+ \\
&+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(1)} \log^m \mathcal{O} \\
&+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \mathcal{O} \log^m \mathcal{O} \\
&+ \cdots \\
&= \frac{d\sigma^{(0)}}{\mathrm{d}\mathcal{O}} + \frac{d\sigma^{(1)}}{\mathrm{d}\mathcal{O}} + \frac{d\sigma^{(2)}}{\mathrm{d}\mathcal{O}} + \cdots
\end{aligned}
$$

Leading Power (LP)

Next to Leading Power (NLP)

- Why do we want to understand power corrections?

NLP field theoretical motivations

- Various interesting field theoretical questions to answer at subleading power
- What is the structure of factorization theorems at each power?

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\mathcal{O}} = \sum_j H_j^{(n_{Hj})} \otimes J_j^{(n_{Jj})} \otimes S_j^{(n_{Sj})}$$

- What is the degree of universality?
- Appearance of universal structures, e.g. $\Gamma_{\mathrm{cusp}}(\alpha_s)$?
- Appearance of new structures, functions, objects, etc

Application: Fixed Order Subtractions

- IR divergences in fixed order calculations can be regulated using slicing parameter.

[Catani, Grazzini]

$$\boxed{\sigma(X) = \int_0 dq_T \frac{d\sigma(X)}{dq_T} = \int_0^{q_T^{\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T} + \int_{q_T^{\text{cut}}} dq_T \frac{d\sigma(X)}{dq_T}}$$

- q_T subtraction has been applied to many processes in pp at **NNLO**: $pp \to Z$, $pp \to W$, $pp \to H$, $pp \to \gamma\gamma$, $pp \to Z\gamma$, $pp \to W\gamma$, $pp \to ZZ$, $pp \to WW$, $pp \to WZ$

[Matrix collaboration]

- Error, $\Delta\sigma(q_T^{\text{cut}})$, (or computing time) can be exponentially improved by analytically computing power corrections.

$$\Delta\sigma(q_T^{\text{cut}}) = \int_0^{q_T^{\text{cut}}} dq_T \left(\frac{d\sigma(X)}{dq_T} - \frac{d\sigma(X)^{\text{LP}}}{dq_T} \right) \equiv \sigma^{\text{non sing.}}(q_T^{\text{cut}})$$

- Understanding of power corrections crucial for applications to more complicated processes.

Bootstrap for observables

- Bootstrap approaches aim to completely reconstruct amplitudes or cross sections from limits.
- Intensively applied for amplitudes in $\mathcal{N} = 4$.
- Can the bootstrap be extended from amplitudes to event shape observables?
  (e.g. EEC? see Kai's talk)

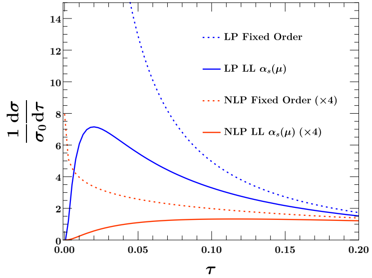
Remaining Parameters in Symbol of 6-Point MHV Remainder Function

LL All Powers →

NLP, NNLP →

Constraint	$L=2$	$L=3$	$L=4$
1. Integrability	75	643	5897
2. Total S_3 symmetry	20	151	1224
3. Parity invariance	18	120	874
4. Collinear vanishing (T^0)	4	59	622
5. OPE leading discontinuity	0	26	482
6. Final entry	0	2	113
7. Multi-Regge limit	0	2	80
8. Near-collinear OPE (T^1)	0	0	4
9. Near-collinear OPE (T^2)	0	0	0

[Dixon et al.] [Basso, Sever, Vieira]



[Moult, Stewart, Vita, Zhu]

Taming log divergence of NLP

- Fixed order power correction exhibits an integrable divergence for $\tau \to 0$
- If LP (singular) is resummed and NLP is not, the NLP (integrable) divergence dominates.

Fixed order calculations at Subleading Power

- Various $\mathcal{O}(\alpha_s)$ fully differential fixed order results for perturbative power corrections have now appeared in the literature:
  - SCET$_{\mathrm{I}}$ with 2 collinear directions (τ_0 in color singlet production)

    [Moult, Stewart, Tackmann, Zhu + GV, Ebert, Rothen] 1612.00450, 1710.03227, 1807.10764

    (also resummed for $H \to gg$) [Moult, Stewart, GV, Zhu] 1804.04665, presented last year at SCET

    (results also for N-jet ops and threshold) [see M.Beneke's, R.Szafron's, and S.Jaskiewicz's talks]

	Color singlet		**H + 1 jet**
	SCET$_{\mathrm{I}}$ (τ_0)	SCET$_{\mathrm{II}}$ (p_T)	SCET$_{\mathrm{I}}$ ($\mathcal{T}_1$)
A^{LP}	✓		
A^{NLP}	✓		
Φ^{LP}	✓		
Φ^{NLP}	✓		

- A =Amplitude squared, Φ = Phase Space, LP = Leading Power, NLP = Subleading power

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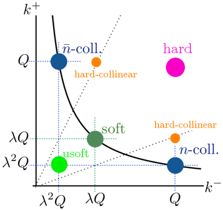
  - SCET$_{\rm II}$ with 2 collinear directions
    (q_T in color singlet production) [Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1812.08189
  - SCET$_{\rm I}$ with 3 collinear directions
    (τ_1 in Higgs production) [Bhattacharya, Moult, Stewart, GV] 1812.06950

	Color singlet		H + 1 jet
	SCET$_{\rm I}$ (τ_0)	SCET$_{\rm II}$ (p_T)	SCET$_{\rm I}$ $(\mathcal{T}_1)$
$A^{\rm LP}$	✓	✓	✓
$A^{\rm NLP}$	✓	✓	✓
$\Phi^{\rm LP}$	✓	✓	✓
$\Phi^{\rm NLP}$	✓	✓	?

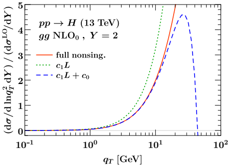
- A =Amplitude squared, Φ = Phase Space, LP = Leading Power, NLP = Subleading power

Outline

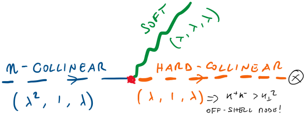
- Rapidity Divergences and Regularization at Subleading Power:
  - New Rapidity Divergences at Subleading Power
  - Rapidity Regularization at Subleading Power



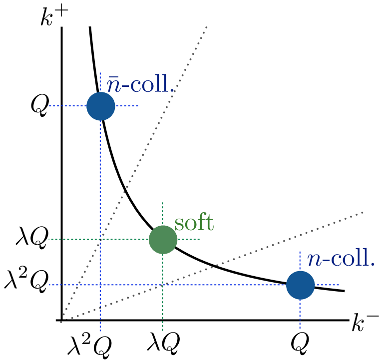
- Power Corrections for Color-Singlet q_T Spectra
  - Phase Space and Matrix Element Expansions
  - LL and NLL results



- Subleading Power Operators in SCET$_{\rm II}$
  - SCET$_{\rm II}$ Hard Scattering Operators
  - Fixed Order Comparison with SCET$_{\rm I}$



Rapidity Divergences and Regularization at Subleading Power



(Ebert, Moult, Stewart, Tackmann, GV, Zhu)

[1812.08189]

Mode setup in SCET

- Light cone coordinates: $k^\mu = \frac{\bar{n}^\mu}{2} k^+ + \frac{n^\mu}{2} k^- + k_\perp^\mu \equiv (k^+,\, k^-,\, k_\perp)$

EFT expansion: $\lambda \ll 1$

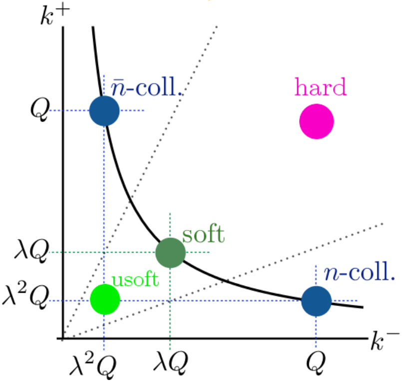
$(k^+, k^-, k_\perp)$

n-collinear: $k_n^\mu \sim Q(\lambda^2,\, 1,\, \lambda)$

$\bar{n}$-collinear: $k_{\bar{n}}^\mu \sim Q(1,\, \lambda^2,\, \lambda)$

SCET$_{\text{II}}$ $\rightarrow$ soft: $k_s^\mu \sim Q(\lambda,\, \lambda,\, \lambda)$

SCET$_{\text{I}}$ $\rightarrow$ usoft: $k_{us}^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$



hard scale: $k_{hard}^\mu \sim Q(1, 1, 1)$ (integrated out)

- Allows for a factorized description: Hard, Jet, Beam, Soft radiation

Rapidity Divergences

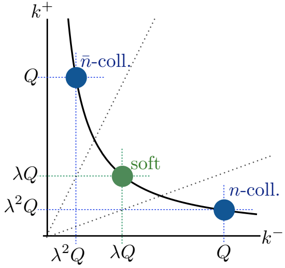
- Large class of observables e.g. $\vec{q}_T$, broadening, EEC, p_T^{veto}, ... belong to the class of SCET$_{\text{II}}$ observables
- SCET$_{\text{II}}$ calculations are affected by **Rapidity Divergences**
- Measurement fixes $\perp$ component of momentum, i.e. $k^+ k^- \sim k_\perp^2$ hyperbola

Light cone coordinates: $k^\mu = (k^+,\, k^-,\, \vec{k_\perp})$

n-collinear: $p_n \sim Q(\lambda^2,\, 1,\, \lambda)$

$\bar{n}$-collinear: $p_{\bar{n}} \sim Q(1,\, \lambda^2,\, \lambda)$

soft: $p_s \sim Q(\lambda,\, \lambda,\, \lambda)$



- Example of massless soft real emission with SCET$_{\text{II}}$ measurement:

$$\int \mathrm{d}^d k\, \delta_+(k^2)\delta^{(d-2)}(\vec{q}_\perp - \vec{k}_\perp) f(k^+, k^-, \vec{k}_\perp) = q_T^{-2\epsilon} \int_0^\infty \frac{\mathrm{d}k^-}{k^-} f(k^-, \vec{q}_\perp)$$

- Divergence when modes overlap

$$k^\pm \to 0\,, \qquad y = 1/2 \log(k^+/k^-) \to \pm\infty\,,$$

not regulated by dimensional regularization $\Longrightarrow$ need a rapidity regulator

Rapidity Divergences at Leading Power

- **Leading Power** (in $q_T^2 \ll Q^2$) representative rapidity divergent integral:

$$\frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}q_T^2} \sim \frac{1}{q_T^{2+2\epsilon}} \int_0^Q \frac{\mathrm{d}k^-}{k^-}$$

- **Log divergent**, from eikonal propagators from Wilson Lines. (typically...)
- It can be regulated in many ways:
  - Wilson lines off the light cone [Collins]
  - Analytic continuation of eikonal prop. [Beneke, Feldmann, Chiu, Manohar, ...]
  - Analytic regularization of real phase space $(k^+,\ k^0)$ [Becher, Bell] [Bell, Rahn, Talbert]
  - $|k_z|$ (or η, CMU), non-analytic regulator [Chiu, Jain, Neill, Rothstein] [Rothstein,Stewart]
  - δ regulator [Chiu, Fuhrer, Hoang, Kelley, Manohar], [Echevarria, Idilbi, Scimemi]
  - Exponential regulator [Li, Neill, Zhu]

  ⋮

For a recent review of the implementation of different rapidity regulators in the context of LP TMD factorization see App.B of [1901.03685] (Ebert, Stewart, Zhao)

Rapidity Divergences at Subleading Power

- At Subleading Power, much broader class of rapidity divergent integrals appearing
- Prototypical integrals take the form

soft sector: $$\mathcal{I}_s^{(\alpha)}[R] = \int_0^\infty \frac{\mathrm{d}k^-}{k^-}\left(\frac{k^-}{Q}\right)^\alpha R(k,\eta)$$

n-collinear sector: $$\mathcal{I}_n^{(\alpha)}[R] = \int_0^Q \frac{\mathrm{d}k^-}{k^-}\left(\frac{k^-}{Q}\right)^\alpha g_n(k^-)R(k,\eta)$$

$\bar{n}$-collinear sector: $$\mathcal{I}_{\bar{n}}^{(\alpha)}[R] = \int_0^Q \frac{\mathrm{d}k^+}{k^+}\left(\frac{k^+}{Q}\right)^\alpha g_{\bar{n}}(k^+)R(k,\eta)$$

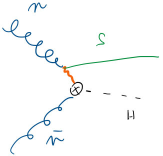
$\alpha = 0, \pm 1, \pm 2, \dots$ $g_n(k)$, $g_{\bar{n}}(k)$ regular as $k \to 0$ $R(k,\eta)$ is *some* rapidity regulator

Comments:

- α can be negative, hence not only log divergences

$$\int \frac{\mathrm{d}k^-}{(k^-)^2}, \quad \int \frac{\mathrm{d}k^-}{(k^-)^3} \quad \Longrightarrow \quad \textbf{Power Law Rapidity Divergences}$$

- **Regulating only Wilson lines is not sufficient.**
  Note that this is also true at LP for Glaubers, see [Rothstein, Stewart]
- Divergences also from soft-quark emissions, hard-collinear propagators, phase space expansion.



Rapidity Regularization at Subleading Power

Hence, at Subleading Power:

- Regularization should conveniently treat power law rapidity divergent integrals
- Common simplifications always used at Leading Power **no longer true**

Example: analytic reg. with energy k^0 (or η regulator with $|k_z|$). At LP it looks

$$\boxed{n: \int_0^Q \frac{\mathrm{d}k^-}{k^-} g_n(k^-) \left(\frac{k^-}{\nu}\right)^{-\eta}} \quad \boxed{\text{soft}: \int_0^\infty \frac{\mathrm{d}k^-}{k^-} \left(\frac{2k_0}{\nu}\right)^{-\eta}} \quad \boxed{\bar{n}: \int_0^Q \frac{\mathrm{d}k^+}{k^+} g_{\bar{n}}(k^+) \left(\frac{k^+}{\nu}\right)^{-\eta}}$$

but it truly is $\boxed{2k_0^{-\eta} = (k^- + k^+)^{-\eta}}$ in **all sectors** $\Longrightarrow$ regulator generates power corrections!

n collinear regulator: $\left(\frac{k_n^- + k_n^+}{\nu}\right)^{-\eta} = \nu^\eta \left(k_n^- + \frac{k_T^2}{k_n^-}\right)^{-\eta} = \left(\frac{k_n^-}{\nu}\right)^{-\eta} \left[1 - \eta \frac{k_T^2}{(k_n^-)^2} + \mathcal{O}(\lambda^4)\right]$

$$\mathcal{I}_n^{(0)} = \underbrace{\nu^\eta \int_0^Q \mathrm{d}k^- \frac{g_n(k^-/Q)}{(k^-)^{1+\eta}}}_{\text{LP collinear integral}} - \underbrace{k_T^2 \nu^\eta \int_0^Q \mathrm{d}k^- \eta \frac{g_n(k^-/Q)}{(k^-)^{3+\eta}}}_{\text{NLP integral induced by the regulator}} + \mathcal{O}(\lambda^4)$$

We'll see that the NLP integral induced by the regulator is $\frac{1}{\eta}$ divergent $\Longrightarrow$ the η prefactor cancels out and the term does NOT vanish for $\eta \to 0$

Pure rapidity regularization

At Subleading power it is **convenient** to use **homogeneous** rapidity regulators.

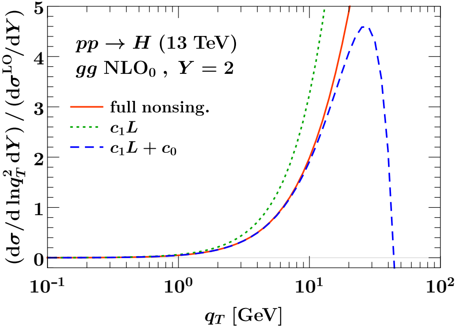
Introduce the *pure rapidity regulator*

$$\int \mathrm{d}^d k \to \int \mathrm{d}^d k\, \omega^2 \upsilon^\eta \left| \frac{\bar{n}\cdot k}{n\cdot k} \right|^{-\eta/2} = \int \mathrm{d}^d k\, \omega^2 \upsilon^\eta e^{-y_k \eta}$$

with bookkeeping and regulator parameters ω, η as in the η regulator.

- It doesn't introduce power corrections
- It breaks boost symmetry in the most minimal way.
- Includes **dimensionless** (pure) rapidity scale υ (**upsilon**)
- Dimensional regularization-like $\frac{1}{\eta}$ poles for rapidity divergences.
- $\overline{\mathrm{MS}}$-analog scheme is pure rapidity renormalization.
- Symmetry in $n \leftrightarrow \bar{n}$: for a class of measurements (like q_T) results for $\bar{n}$ sector are trivially obtained from n-sector via $\eta \to -\eta\,, \upsilon \to \frac{1}{\upsilon} \quad \Longrightarrow$
  - Soft and zero-bin integrals are **scaleless**
  - Order by order poles have opposite sign (cancellation of poles is trivial).
  - Finite terms are **identical**!
- It shares features of both the k_z and k^+ analytic regulator

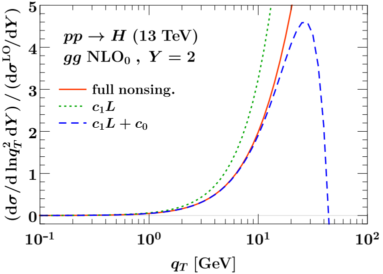
Power Corrections for Color-Singlet q_T Spectra



(Ebert, Moult, Stewart, Tackmann, GV, Zhu) [1812.08189]

Power corrections at FO: General Setup

- Want the **fully differential cross section** $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2}$ for color singlet production including $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(q_T^2/Q^2)$ corrections.



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} = \frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} + \frac{\mathrm{d}\sigma^{\mathrm{NLP}}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$
$$= \frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} + \frac{\alpha_s}{4\pi}\left(c_1(Y)\ln\frac{Q^2}{q_T^2} + c_0(Y)\right) + \mathcal{O}\left(\frac{q_T^2}{Q^2}, \alpha_s^2\right)$$

- Power corrections in $\lambda \sim q_T/Q \ll 1$:
  - **Perturbative**
  - **NOT** higher twist PDFs/non-perturbative power corrections.

Power corrections at FO: General Setup

- Get q_T/Q corrections by expanding the full QCD phase space and matrix element squared in soft and collinear limit:
  - **Phase space:** $\Phi = \Phi^{(0)} + \frac{q_T}{Q}\Phi^{(1)} + \frac{q_T^2}{Q^2}\Phi^{(2)} + \mathcal{O}(q_T^3/Q^3)$

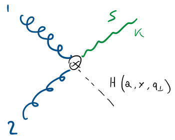
    Example: (soft limit)

    momentum fraction: $\zeta_a(k) = x_a\left[1 + \frac{k^- e^{-Y}}{Q} + \frac{q_T^2}{2Q^2} + \mathcal{O}(\lambda^3)\right]$ $\leftarrow$ (fixed by Y, Q, q_T measurement)

    Expansion of PDFs: $f_a(\zeta_a(k)) = f_a(x_a) + \frac{k^- e^{-Y}}{Q} f_a'(x_a) + \ldots$

  - **Matrix element squared:** $|\mathcal{M}|^2 = A^{(0)} + \frac{q_T}{Q}A^{(1)} + \frac{q_T^2}{Q^2}A^{(2)} + \mathcal{O}(q_T^3/Q^3)$

    Example: (soft limit $ggHg$)



$$\left|\;\cdots\;\right|^2 = A^{\rm LO}_{gg\to H}(Q) \times \frac{8\pi\alpha_s C_A}{Q^4}\left[\frac{Q^8 + s_{12}^4 + s_{1k}^4 + s_{2k}^4}{s_{12}s_{1k}s_{2k}}\right]$$

$$= A^{\rm LO}_{gg\to H}(Q) \times \frac{16\pi\alpha_s C_A}{q_T^2} + A^{(1)} + A^{(2)} + \mathcal{O}(q_T^3/Q^3)$$

- No corrections at $\mathcal{O}(q_T/Q)$ for the cross section

Schematically: $$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2} \sim \int \frac{\mathrm{d}z}{z}\left[A^{(0)}\Phi^{(0)} + \frac{q_T^2}{Q^2}\left(A^{(0)}\Phi^{(2)} + A^{(2)}\Phi^{(0)} + A^{(1)}\Phi^{(1)}\right)\right] + \mathcal{O}(q_T^3/Q^3)$$

Collinear Master Formula for q_T power corrections

$k^- \to 0$ rapidity divergences mapped to (standard) splitting variable $z \to 1$ via $\int_0^Q \frac{\mathrm{d}k^-}{(k^-)} \to \int_{x_a}^1 \frac{\mathrm{d}z_a}{z_a(1-z_a)}$

$$\frac{\mathrm{d}\sigma_n^{(2)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} \sim \nu^\eta \overbrace{\int_{x_a}^1 \frac{\mathrm{d}z_a}{z_a} \frac{z_a^{1+\eta}}{(1-z_a)^{1+\eta}} \Big\{ f_a\Big(\frac{x_a}{z_a}\Big) f_b(x_b)}^{\Phi^{(0)}(\text{LP phase space})} A_n^{(2)}(q_T; z_a)$$
$$+ \underbrace{A_n^{(0)}(q_T; z_a)}_{\text{LP } |\mathcal{M}|^2} \frac{q_T^2}{2Q^2} \Big[\frac{(1-z_a)^2 - 2}{1-z_a} f_a\Big(\frac{x_a}{z_a}\Big) f_b(x_b) + x_a f_a'\Big(\frac{x_a}{z_a}\Big) f_b(x_b)$$
$$+ \frac{1+z_a}{1-z_a} f_a\Big(\frac{x_a}{z_a}\Big) x_b f_b'(x_b)$$

$\uparrow$ $\leftarrow$ Subleading power expansion of phase space $\Phi^{(2)}$

Power correction from the expansion of the $|k_z|$ regulator. Not there in pure rapidity regularization $\to$

$$+ \frac{2\eta}{e^{2Y}} \frac{z_a^2}{(1-z_a)^2} f_a\Big(\frac{x_a}{z_a}\Big) f_b(x_b) \Big] \Big\}$$

- Combining rapidity divergent factors $(1-z_a)^{-1}$ we get
$$\frac{1}{(1-z_a)^{2+\eta}}, \quad \frac{1}{(1-z_a)^{3+\eta}} \quad \Longrightarrow \text{ power law rapidity divergences!}$$
- They appear in $\Phi^{(2)}$ $\Longrightarrow$ The fact that they appear is generic
- They can also be generated by $\Phi^{(0)} \times A^{(2)}$ when
$$A^{(2)}(z_a \to 1) \sim \frac{1}{(1-z_a)}$$
- Analogous master formula exists for the soft limit

How to treat power law divergences

- Consider rapidity divergent integral $\displaystyle\int_x^1 \mathrm{d}z \, \frac{g(z)}{(1-z)^{a+\eta}}$.
- When $g(z)$ is not known analytically (eg. when it involves PDFs), need to extract **pole** as $\eta \to 0$ without computing the integral.
- For $a = 1$, use standard distributional identity

$$\frac{1}{(1-z)^{1+\eta}} = -\frac{\delta(1-z)}{\eta} + \mathcal{L}_0(1-z) + \mathcal{O}(\eta)\,, \qquad \mathcal{L}_0(y) = [\theta(y)/y]_+\,,$$

- For $a > 1$, these distributions need to be generalized to higher-order plus distributions subtracting higher derivatives as well. For example, for $a = 2$ one obtains

$$\boxed{\frac{1}{(1-z)^{2+\eta}} = \frac{\delta'(1-z)}{\eta} - \delta(1-z) + \mathcal{L}_0^{++}(1-z) + \mathcal{O}(\eta)},$$

  where the second-order plus function $\mathcal{L}_0^{++}(1-z)$ acts on a test function $g(z)$ as a double subtraction.
- Power law divergences generate new PDF derivatives

$$\int_{x_a}^1 \mathrm{d}z_a \, \frac{f(x_a/z_a)f(x_b/z_b)}{(1-z_a)^{2+\eta}} = \frac{f'(x_a)f(x_b/z_b)}{\eta} + \mathcal{O}(\eta^0)$$

Leading-Logarthmic power corrections

- Compute power corrections in the n-collinear, $\bar{n}$-collinear and soft limits (soft is scaless for homogeneous regulators)
- Sum together results
- Rapidity divergences cancel between sectors, finite terms add up.

  (In rapidity regularization this is trivial since $g_n(\eta) = g_{\bar{n}}(-\eta)$)

At Leading Log the result is quite simple. Here a couple of examples:

- Drell Yan production ($q\bar{q} \to Vg$)

$$\frac{\mathrm{d}\sigma^{(2),\mathrm{LL}}_{q\bar{q}\to Vg}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} = \hat{\sigma}^{\mathrm{LO}}_{q\bar{q}\to V}(Q) \times \frac{\alpha_s C_F}{4\pi}\frac{2}{Q^2}\ln\frac{Q^2}{q_T^2}\left[f^{q\bar{q}}_{\mathrm{uni}}(x_a, x_b)\right],$$

- Gluon fusion Higgs production ($gg \to Hg$)

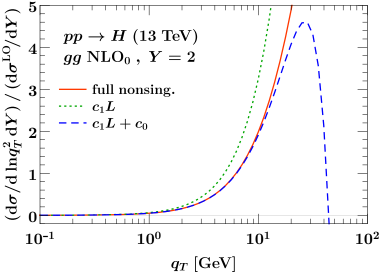
$$\frac{\mathrm{d}\sigma^{(2),\mathrm{LL}}_{gg\to Hg}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}q_T^2} = \hat{\sigma}^{\mathrm{LO}}_{gg\to H}(Q) \times \frac{\alpha_s C_A}{4\pi}\frac{2}{Q^2}\ln\frac{Q^2}{q_T^2}\left[8f_g(x_a)f_g(x_b) + f^{gg}_{\mathrm{uni}}(x_a, x_b)\right],$$

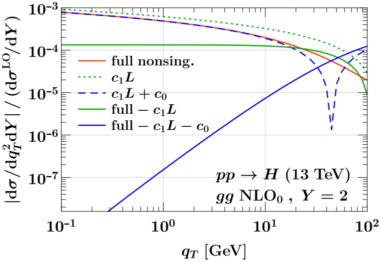
- Common factor

$$f^{ij}_{\mathrm{uni}}(x_a, x_b) = -x_a f_i'(x_a) f_j(x_b) - f_i(x_a)\, x_b f_j'(x_b) + 2x_a f_i'(x_a)\, x_b f_j'(x_b)$$

Next Leading-Logarithmic power corrections

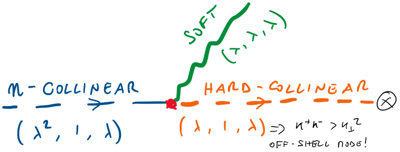
- We computed also the NLL kernels at $\mathcal{O}(\alpha_s)$ for all channels both in DY and ggH.
- z_a, z_b kernels pretty complicated. They involve $\mathcal{L}_0^{++}(1-z_a)$, etc.
- Remainder is q_T^2/Q^2 suppressed
- Describes q_T distribution up to 10 GeV

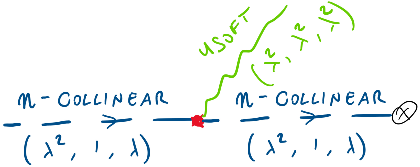




$$\frac{\mathrm{d}\sigma}{\mathrm{d}Y\mathrm{d}q_T^2} - \frac{\mathrm{d}\sigma^{\mathrm{LP}}}{\mathrm{d}Y\mathrm{d}q_T^2} = c_1(Y)\ln\frac{Q^2}{q_T^2} + c_0(Y) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

Subleading Power Operators in SCET$_{II}$





(Chang, Stewart, GV) [to appear]

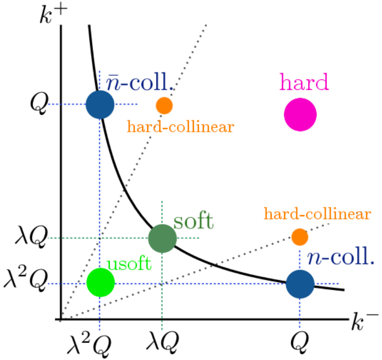
Subleading SCET$_{\mathrm{II}}$ Operators

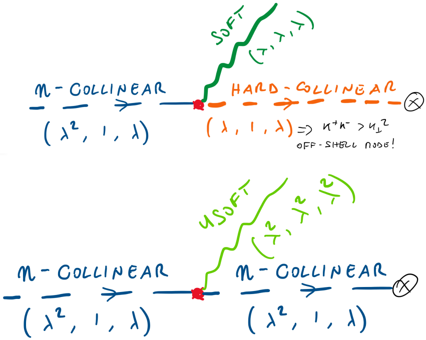
Structure of SCET$_{\mathrm{II}}$ at subleading power is much richer than SCET$_{\mathrm{I}}$:

- 2 classes of singularities and regularizations: UV and Rapidity (ϵ, η)

- New non-localities: **hard-collinear** modes

- **Hard-collinear** modes mediate interaction of soft and collinear fields (which can't happen at LP) $\Longrightarrow$ They are crucial at subleading powers





Hard scattering Operators

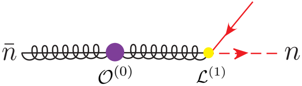
- Hard-collinear propagators **enhance** the power counting of the operators

$$\frac{1}{p_{hc}^2} = \frac{1}{\bar{n}\cdot p_n n\cdot p_s} \sim \frac{1}{\lambda^0\lambda} \sim \lambda^{-1}$$

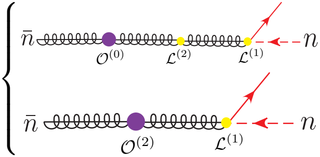
- For q_T first non vanishing power correction is at $\mathcal{O}(\lambda^2)$
- Therefore, need to consider more operators than in the $\mathrm{SCET_I}$ case.

Example: Hard scattering emission of a soft quark in $gq \to Hq$

$$\mathcal{O}_{qq1}^{(1/2)} \sim \frac{\omega_n\omega_{\bar{n}}}{\omega_n n\cdot p_s}\bar{\chi}_n(\omega_n) g \mathcal{B}_{\bar{n}}^{\perp}(\omega_{\bar{n}})\psi_S^{(n)}(p_s) H :$$



$$\mathcal{O}_{qq1}^{(3/2)} \sim \frac{\omega_n\omega_{\bar{n}}}{\omega_n n\cdot p_s}\bar{\chi}_n(\omega_n) g \mathcal{B}_{\bar{n}}^{\perp}(\omega_{\bar{n}}) \left[n\cdot \mathcal{P}_s\psi_S^{(n)}(p_s)\right] H :$$



- Using these operators we **reproduce** power correction for gq channel.

Comparison with FO calculations at Subleading Power

- Various $\mathcal{O}(\alpha_s)$ fully differential fixed order results for perturbative power corrections have now appeared in the literature:
  - SCET$_\mathrm{I}$ with 2 collinear directions
    (τ_0 in color singlet production) [Ebert,Moult,Stewart,Tackmann, GV, Zhu] 1807.10764
  - SCET$_\mathrm{II}$ with 2 collinear directions
    (q_T in color singlet production) [Ebert,Moult,Stewart,Tackmann, GV, Zhu] 1812.08189
  - SCET$_\mathrm{I}$ with 3 collinear directions
    (τ_1 in Higgs production) [Bhattacharya,Moult,Stewart, GV] 1812.06950

- Use them to compare $k^- \to 0$ behavior for Amplitude squared contributions $A^{(k)}$ and Phase Space $\Phi^{(k)}$, at Leading Power (LP) and subleading power (NLP) in different situations.

	Color singlet		**H + 1 jet**
	SCET$_\mathrm{I}$ (τ_0)	SCET$_\mathrm{II}$ (p_T)	SCET$_\mathrm{I}$ $(\mathcal{T}_1)$
$A^{\mathrm{LP}}(k^-)$	$\frac{1}{k^-}$	1	$\frac{1}{k^-}$
$A^{\mathrm{NLP}}(k^-)$	$\frac{1}{k^-}$	$\frac{1}{k^-}$	$\frac{1}{(k^-)^2}$
$\Phi^{\mathrm{LP}}(k^-)$	1	$\frac{1}{k^-}$	1
$\Phi^{\mathrm{NLP}}(k^-)$	1	$\frac{1}{(k^-)^2}$	?

Conclusions

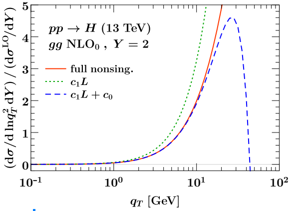
- Described new structure of rapidity divergences arising at subleading powers

$$\boxed{\int_x^1 \frac{\mathrm{d}z}{z} \frac{f_a(x/z)}{(1-z)^{(2+\eta)}}}$$

- Looked at how to implement rapidity regularization at subleading powers and proposed a new regulator *purely* based on rapidity

$$\boxed{\int \mathrm{d}^d k \to \int \mathrm{d}^d k \, \omega^2 \upsilon^\eta \left| \frac{\bar{n} \cdot k}{n \cdot k} \right|^{-\eta/2}}$$

- Computed full $\mathcal{O}(\alpha_s)$ power correction of q_T differential distribution for color singlet production



- Used $\mathrm{SCET_{II}}$ operators to reproduce FO result

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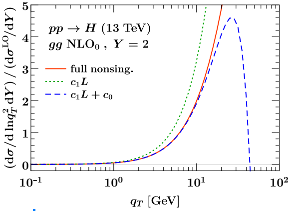

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THANK YOU!