Threshold factorization of the Drell-Yan process at NLP

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XVIth annual workshop on Soft-Collinear Effective Theory
25-28 March 2019
UC San Diego

Threshold factorization of the Drell-Yan process at next-to-leading power

Martin Beneke, Alessandro Broggio, Sebastian Jaskiewicz and Leonardo Vernazza

To appear soon

Leading-logarithmic threshold resummation of the Drell-Yan process at next-to-leading power

Martin Beneke, Alessandro Broggio, Mathias Garny, Sebastian Jaskiewicz, Robert Szafron, Leonardo Vernazza and Jian Wang

JHEP, 2019(3):43 arXiv:1809.10631

Outline

- The Drell-Yan process - review of factorization at leading power within the position space SCET framework.
- The Drell-Yan process - new features at next-to-leading power
  - Accounting for power corrections
  - Appearance of collinear functions
  - Generalized soft functions
- Factorization formula at next-to-leading power

The Drell-Yan Process

$$A(p_A)B(p_B) \to \mathrm{DY}(Q) + X$$

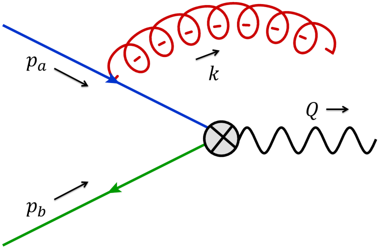
$$z = \frac{Q^2}{\hat{s}} \qquad \lambda = \sqrt{(1-z)}$$

$$\begin{aligned} p_c &= (n_+p_c, n_-p_c, p_{c\perp}) \sim Q(1, \lambda^2, \lambda) \\ p_{\bar{c}} &= (n_+p_{\bar{c}}, n_-p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2, 1, \lambda) \\ p_s &= (n_+p_s, n_-p_s, p_{s\perp}) \sim Q(\lambda^2, \lambda^2, \lambda^2) \end{aligned}$$

$$p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$$

$$\bar{\psi}\gamma_\mu\psi = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t})\, J_\mu^{A0}(t,\bar{t})$$

$$J_\rho^{A0}(t,\bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\chi_c(tn_+)$$



The Drell-Yan proccess - the leading power result

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, \hat{\sigma}_{ab}^{\rm LP}(z)$$

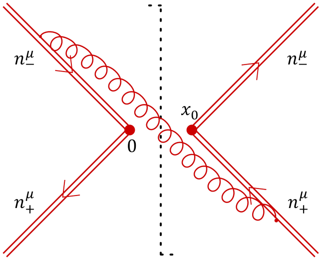
where

[G. P. Korchemsky *et al.*, 1993]

[T. Becher *et al.*, 0710.0680, S. Moch *et al.*, 0508265]

$$\hat{\sigma}^{\rm LP}(z) = |C(Q^2)|^2\; Q\; S_{\rm DY}(Q(1-z))$$

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}\, {\rm Tr}\, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$



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A complete calculation of the order α^2 scorrection to the Drell-Yan K factor

[R. Hamberg, W. van Neerven and T. Matsuura, 1991]

Dynamical Threshold Enhancement and Resummationin Drell-Yan Production

[T. Becher, M. Neubert, G. Xu , 0710.0680]

On next-to-leading power threshold corrections in Drell-Yan production at NNNLO

[N. Bahjat-Abbas, J. Sinninghe Damsté, L. Vernazza, C.D. White, 1807.09246]

On next-to-eikonal corrections to threshold resummation for the DY and DIS cross sections

[E. Laenen, L. Magnea, G. Stavenga, 0807.4412]

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NLP in other contexts:

Leading logarithmic result for the subleading power resummed thrust spectrum for $H \to gg$ in pure glue QCD.

[I. Moult, I.W. Stewart, G. Vita, H.X. Zhu, 1804.04665]

Power corrections for N-jettiness subtractions at $\mathcal{O}(\alpha_s)$

[M. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, 1807.10764]

Subleading power rapidity divergences and power corrections for q_T

[M. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, 1812.08189]

Helicity methods for high multiplicity subleading soft and collinear limits

[A. Bhattacharya, I. Moult, I.W. Stewart, G. Vita, 1812.06950]

Factorization formula at NLP

First a schematic formula:

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}^{\rm NLP}(z)$$

The $\hat{\sigma}_{ab}^{\rm NLP}(z)$ is given by

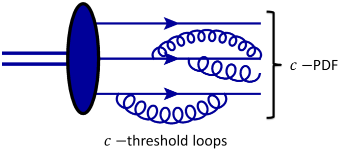
$$\hat{\sigma}^{\,\rm NLP} = \sum_{\rm terms} [C \otimes J \otimes \bar{J}\,]^2 \otimes S$$

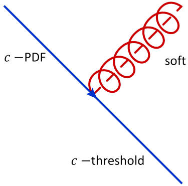
- C is the hard Wilson matching coefficient
- S is the *generalized* soft function
- J is the collinear function

Let us now motivate the emergence of this structure at next-to-leading power.

Collinear functions at LP and NLP

- There is no collinear function present at LP because of decoupling transformation [C. Bauer, D. Pirjol, and I. Stewart, 0109045]
- This is no longer true at NLP. Consider an example of subleading SCET Lagrangian: $\mathcal{L}_{2\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c z_\perp^\mu z_\perp^\rho \left[i\partial_\rho i n_- \partial \mathcal{B}_\mu^+(z_-) \right] \frac{\not{n}_+}{2} \chi_c, \;\; \mathcal{B}_\pm^\mu = Y_\pm^\dagger \left[i D_s^\mu Y_\pm \right]$
- Crucially, an insertion of a piece of a subleading lagrangian comes with an integral over its position, $\int d^4 z$





$$\left(J_{A0,2\xi}^{T2}(s,t) \right)^\mu = i \int d^4 x \, \mathbf{T} \left[J_{A0}^\mu(s,t) \, \mathcal{L}_{2\xi}^{(2)}(x) \right]$$

Collinear functions at NLP

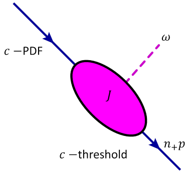
- PDF collinear modes *can* be radiated into the final state
  Modes: $p_c \sim Q(1,\lambda^2,\lambda)$ and $p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$

- Hence we define the matching equation which gives a SCET definition of what is known as the “radiative jet function”
  [D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza, C.D. White, 1503.05156]
  see also [D. Bonocore, E. Laenen, L. Magnea, L. Vernazza, C.D. White, 1610.06842]

$$i \int d^4z \, \mathbf{T}\Big[\chi_{c,\gamma f}(tn_+)\, \mathcal{L}^{(2)}(z)\Big]$$

$$= 2\pi \sum_i \int du \int \frac{d(n_+z)}{2}\, \tilde{J}_{i;\gamma\beta,\mu,fbd}\left(t,u;\frac{n_+z}{2}\right) \chi^{\mathrm{PDF}}_{c,\beta b}(un_+)\, \mathfrak{S}_{i;\mu,d}(z_-)$$

$$\mathfrak{S}_i(z_-) \in \left\{ \frac{\partial^\mu_\perp}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_-),\, \frac{\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}^+_{\nu_\perp]}(z_-),\, \frac{1}{(in_-\partial)}\left[\mathcal{B}^+_{\mu_\perp}(z_-), \mathcal{B}^+_{\nu_\perp}(z_-)\right], \dots \right\}$$



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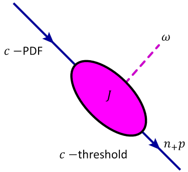

Equation of motion:

$$n_+\mathcal{B}^+(z_-) \;=\; -2\frac{i\partial^{\mu}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\mu_\perp}(z_-) \;-2\frac{\left[\mathcal{B}^{\mu}_{\perp}, \left[in_-\partial\mathcal{B}_{\mu_\perp}\right]\right]}{in_-\partial} + \ldots$$

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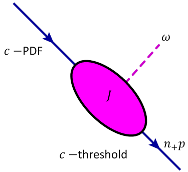

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- Note that the definition of the collinear function is at *amplitude* level.

General collinear functions

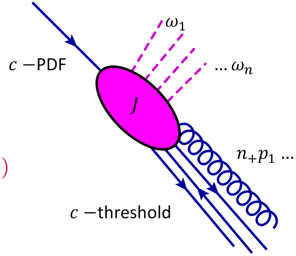
- The discussed construction is actually general at subleading powers, not only next-to-leading power
- There can be many Lagrangian insertions at various positions each with its own ω_i conjugate to the large component of threshold collinear momentum

We can separate the Lagrangian insertions

$$\mathcal{L}_V^{(n)}(z) = \mathcal{L}_c^{(n)}(z) \otimes \mathcal{L}_s^{(n)}(z_-)$$

$$i^n \left(\prod_{j=1}^{n} \int d^4 z_j \right) \times \mathbf{T}\left[\chi_c(tn_+) \times \mathcal{L}^{(n)}(z_1) \times \ldots \times \mathcal{L}^{(m)}(z_n)\right]$$

$$= 2\pi \sum_i \int du \left(\prod_{j=1}^{n} \int dz_{j-} \right) \tilde{J}_i\left(t, u; z_{1-}, \ldots, z_{n-}\right) \times \chi_c^{\mathrm{PDF}}(tn_+)\, \mathfrak{S}_i(z_{1-}, \ldots, z_{n-})$$



Generalized soft functions

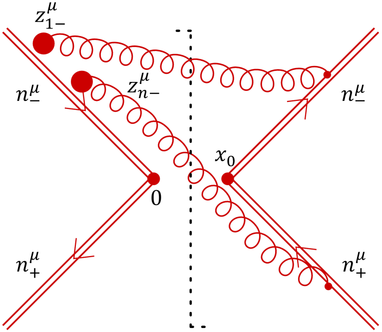
The necessary presence of the time-ordered products at NLP gives rise to the concept of generalized soft functions with explicit gauge fields. Schematically we have

$$S(\Omega,\omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \left(\prod_{j=1}^{n} \int \frac{d(n_+ z_j)}{4\pi}\, e^{-i\omega_j (n_+ z_j)/2} \right)$$

$$\times \, \mathrm{Tr}\, \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \times \mathcal{L}_s^{(n)}(z_{1-}) \times \ldots \times \mathcal{L}_s^{(n)}(z_{n-}) \right] |0\rangle$$

$\mathcal{L}_s^{(n)}(z_{j-})$ contains $\mathcal{B}^+_{\perp\nu}(z_{j-})$ fields, not only Wilson lines

[M. Beneke , F. Campanario, T. Mannel, B.D. Pecjak, 0411395]



Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.

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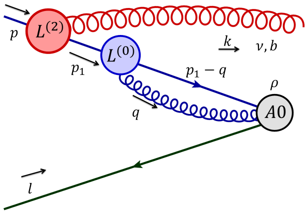

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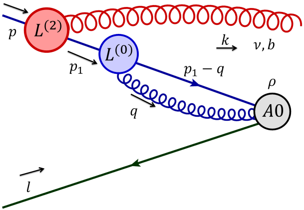

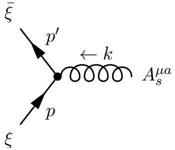

$$
ig_s t^a \begin{cases} \dfrac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \dfrac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p') \dfrac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}
$$

$$
S^{\rho\nu}(k,p,p') \equiv \frac{1}{2} \left[(n_- X) n_+^\rho n_-^\nu \; + \; (k X_\perp) X_\perp^\rho n_-^\nu \; + \; X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p} \right) \right]
$$

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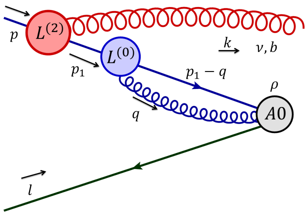
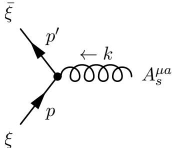


$$\mathcal{L}_\xi^{(2)} = \frac{1}{2}\bar{\chi}_c i\ (n_- x)\ n_+^\mu \left[i n_- \partial \mathcal{B}_\mu^+ (x_-) \right] \frac{\not{n}_+}{2}\chi_c + \ldots$$

[M. Beneke and Th. Feldmann, 0211358]

$$X^\alpha = -\frac{\partial}{\partial p_{1\alpha}} \left\{ (2\pi)^4 \delta^4 (p - k_+ - p_1) \right\}$$



$$ig_s t^a \begin{cases} \frac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p') \frac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2} \left[(n_- X) n_+^\rho n_-^\nu \ + \ (k X_\perp) X_\perp^\rho n_-^\nu \ + \ X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p} \right) \right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

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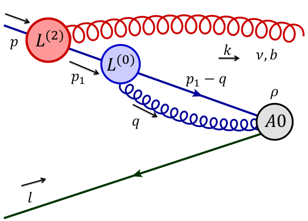
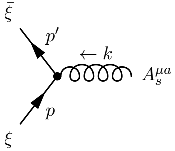


$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

$$\times\Big\{\ \left((n_+k)n_{-\nu} - (n_-k)n_{+\nu}\right)\ \left(\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)\ \left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]\ \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Big\}u_c(p)$$



$$i g_s t^a \begin{cases} \dfrac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \dfrac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

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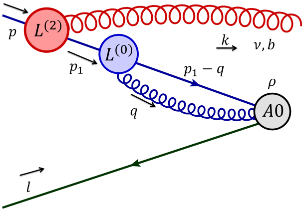

$$\bar{v}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

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$$+ \left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)\left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

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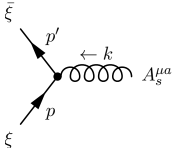

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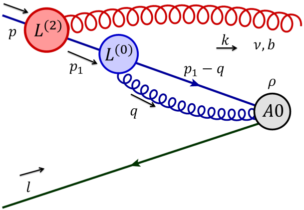
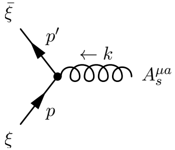


$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

$$\times\Bigg\{\ \left((n_+k)n_{-\nu} - (n_-k)n_{+\nu}\right)\ \left(\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+ \boxed{\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)}\left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+ \left[\gamma_{\perp\nu}\,,\ \not{k}_\perp\right]\ \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$



$$ig_s t^a \begin{cases} \dfrac{\not{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0) \\ \dfrac{\not{n}_+}{2}X_\perp^\rho n_-^\nu(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p')\dfrac{\not{n}_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[\ (n_-X)n_+^\rho n_-^\nu\ + \boxed{(kX_\perp)X_\perp^\rho n_-^\nu} + \ X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)\ \right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

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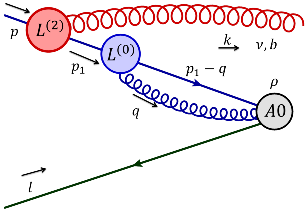
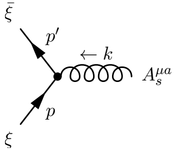


$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon} \frac{C_F t^b}{(n_+p)(n_-k)}$$

$$\times\Big\{ \;((n_+k)n_{-\nu} - (n_-k)n_{+\nu})\;\left(\frac{2}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+\;\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu} - k_{\perp\nu}\right)\;\left(\frac{2}{\epsilon^2} + \frac{4}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$

$$+\;\left[\gamma_{\perp\nu}\,,\,\not{k}_\perp\right]\;\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Big\} u_c(p)$$



$$i g_s t^a \begin{cases} \frac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p')\frac{\not{n}_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

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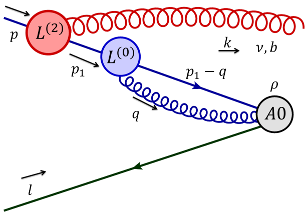

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$$+ \boxed{\left[\gamma_{\perp\nu}\,,\, \not{k}_\perp\right]} \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)\right)\Bigg\} u_c(p)$$

$$(n_+k)(n_-\epsilon^*) = 2\left(-\frac{(n_-k)(n_+\epsilon^*)}{2} - k_\perp\cdot\epsilon_\perp^*\right)$$

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[\boxed{(n_-X)n_+^\rho n_-^\nu} + \boxed{(kX_\perp)X_\perp^\rho n_-^\nu} + \boxed{X_\perp^\rho\left(\frac{\not{p}'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not{p}_\perp}{n_+p}\right)}\right]$$

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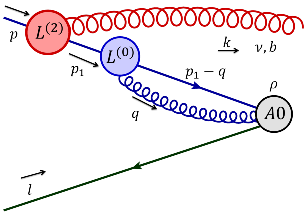

$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

$$\times\left\{\left(\frac{-2k_\perp^2}{n_-k}n_{-\nu}+2k_{\perp\nu}\right)\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\right.$$

$$+\left(\frac{k_\perp^2}{(n_-k)}n_{-\nu}-k_{\perp\nu}\right)\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$

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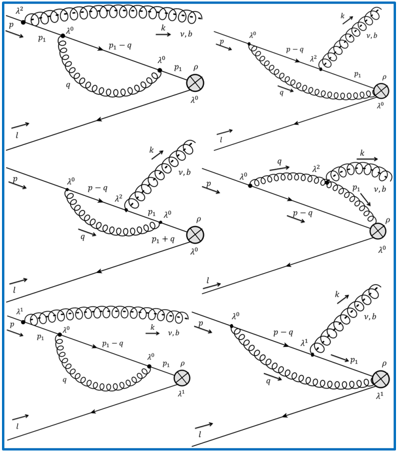
Amplitude calculation: 1-real emission

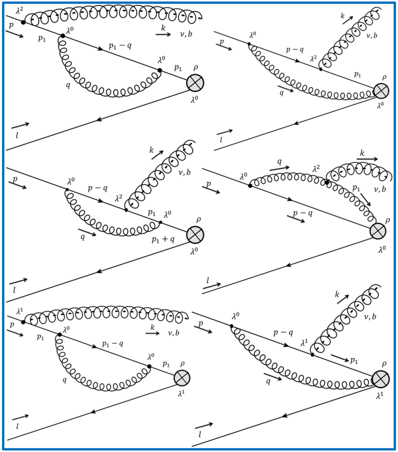
$$\mathcal{A} = C \otimes J_{2\xi} \otimes \langle X| \frac{\partial^{\mu}_{\perp}}{in_{-}\partial} \mathcal{B}^{+}_{\mu_\perp} |0\rangle + C \otimes J_{4\xi} \otimes \langle X| \frac{\partial_{[\mu_\perp}}{in_{-}\partial} \mathcal{B}^{+}_{\nu_\perp]} |0\rangle + C \otimes J_{\xi} \otimes \langle X| \mathcal{B}^{+}_{\mu_\perp} |0\rangle$$



1-loop collinear $\otimes$ 1-real soft emission

Amplitude calculation: 1-real emission

$$\mathcal{A} = C \otimes \boxed{J_{2\xi}} \otimes \langle X| \frac{\partial_\perp^\mu}{in_-\partial} \mathcal{B}^+_{\mu_\perp} |0\rangle + C \otimes \boxed{J_{4\xi}} \otimes \langle X| \frac{\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}^+_{\nu_\perp]} |0\rangle + C \otimes \boxed{J_\xi} \otimes \langle X| \mathcal{B}^+_{\mu_\perp} |0\rangle$$



1-loop collinear $\otimes$ 1-real soft emission

Extract 1-loop collinear functions

Amplitude calculation: 1-real emission

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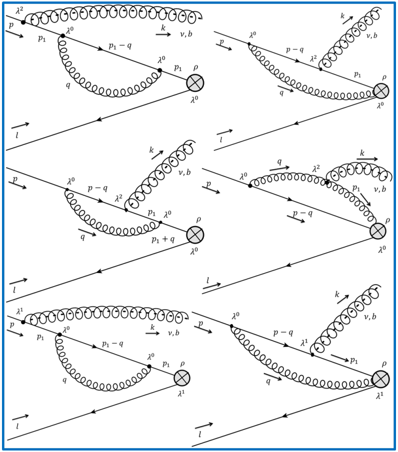

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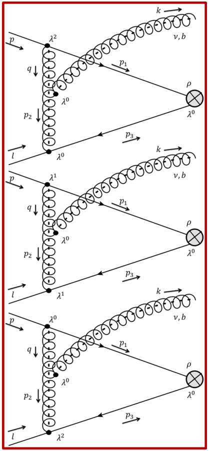
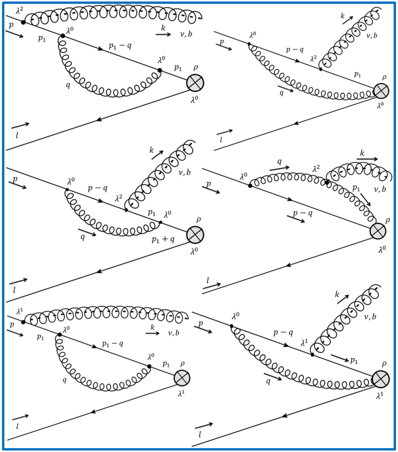
Extract 1-loop collinear functions



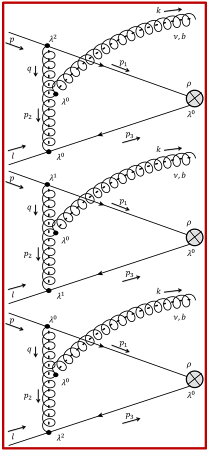
1-loop soft ⊗ 1-real soft emission

Amplitude calculation: 1-real emission

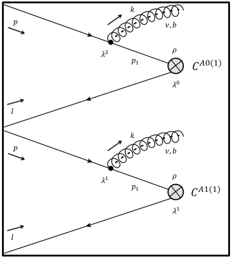
$$\mathcal{A} = \boxed{C} \otimes J_{2\xi} \otimes \langle X| \frac{\partial^{\mu}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\mu_\perp} |0\rangle + \boxed{C} \otimes J_{4\xi} \otimes \langle X| \frac{\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}^{+}_{\nu_\perp]} |0\rangle + \boxed{C} \otimes J_{\xi} \otimes \langle X| \mathcal{B}^{+}_{\mu_\perp} |0\rangle$$



1-loop collinear $\otimes$ 1-real soft emission
Extract 1-loop collinear functions



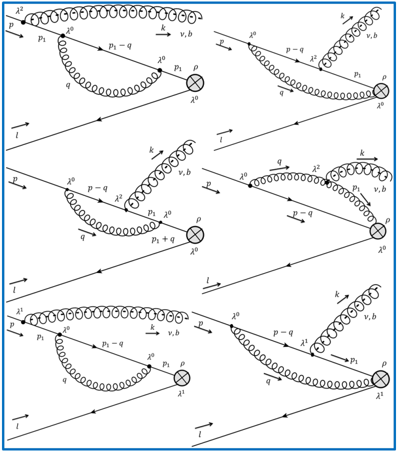
1-loop soft $\otimes$ 1-real soft emission



1-loop hard $\otimes$ 1-real soft emission

Amplitude calculation: 1-real emission

We find agreement with the method of regions expansion for the 1-real 1-virtual amplitude. For explicit results see back up slides.



1-loop collinear ⊗ 1-real soft emission
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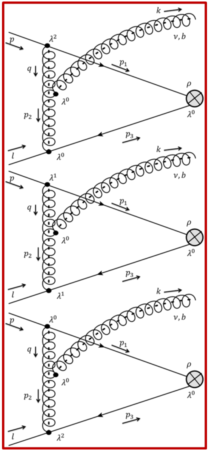
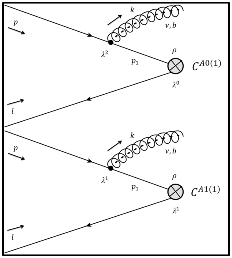


1-loop soft ⊗ 1-real soft emission



1-loop hard ⊗ 1-real soft emission

Cross section

Hadronic tensor is given by

$$W_{\mu\rho} = \int d^4x e^{-iq\cdot x} \langle A(p_A)B(p_B)|J_\mu^\dagger(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

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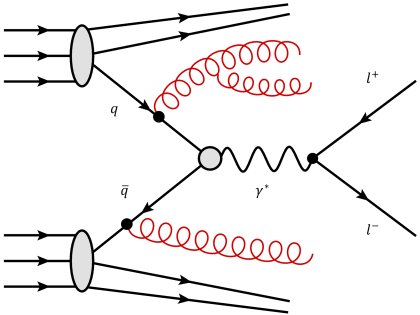
which is combined with the part from the lepton tensor

$$d\sigma = \frac{d^4q}{(2\pi)^4}\frac{4\pi\alpha^2}{3sq^2}\left(-g^{\mu\rho}W_{\mu\rho}\right)$$



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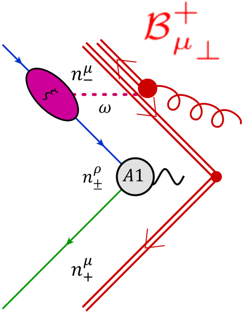

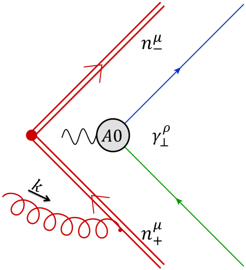

Contributions from power suppressed currents can start contributing at NNLP only!

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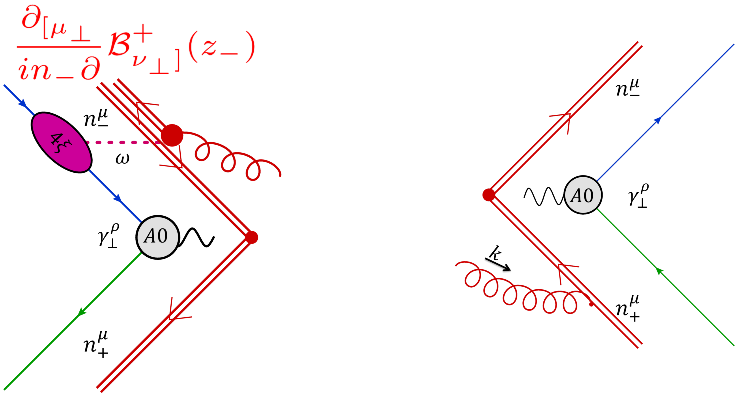

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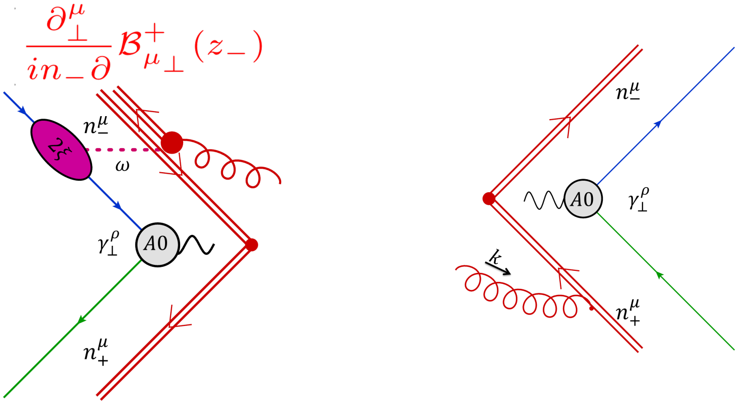

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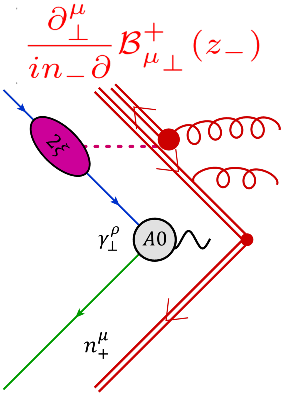

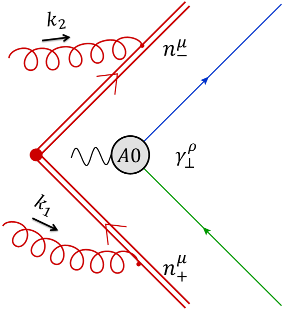

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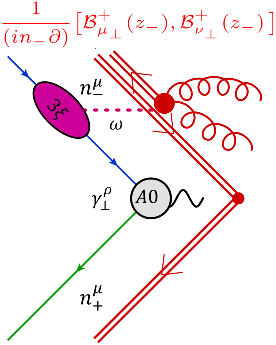
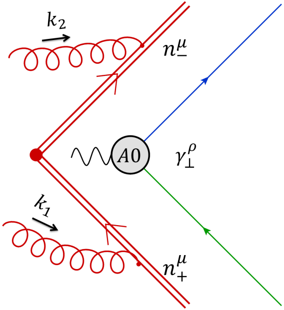




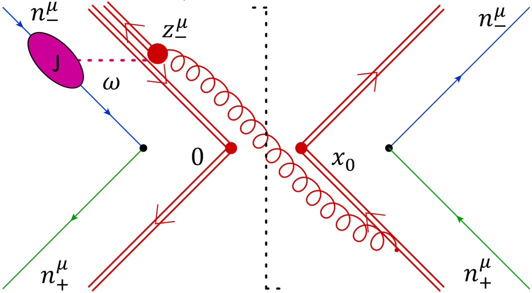
Final result

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4}\sum_{a,b}\int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\,\hat{\sigma}_{ab}(z)$$

After combination with leptonic part and stripping off the PDFs

$$\hat{\sigma}(z) \quad = \quad H(\hat{s})\times\, Q^2\int\frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^{\,2}}}\,\frac{1}{2\pi}\int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x}$$

$$\times\left\{\widetilde{S}_0(x) + \; 2\frac{1}{2}\int d\omega\, J_{2\xi}^{(O)}(x_a n_+ p_A;\omega)\,\widetilde{S}_{2\xi}(x,\omega) \; + \bar{c}\text{-term}\right\}$$



$$\mathfrak{S} = \frac{\partial_\perp^\mu}{in_-\partial}\mathcal{B}^+_{\mu_\perp}(z_-)$$

Final result

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$$\begin{aligned}J^{(O)}_{2\xi}(x_a n_+ p_A;\omega) \quad &= \quad -\frac{2}{x_a(n_+p_A)} - 2\frac{\partial}{x_a\partial(n_+p_A)} \\ &+ \frac{i\alpha}{4\pi}\frac{2}{x_a(n_+p_A)}\left[\frac{\omega(x_a n_+ p_A)}{\mu^2}\right]^{-\epsilon}\left(C_A\left(5+\mathcal{O}(\epsilon^1)\right) - C_F\left(\frac{4}{\epsilon}+5+\mathcal{O}(\epsilon^1)\right)\right)\end{aligned}$$

where the scalar collinear function in the factorization theorem is defined as:

$$J^A_{2\xi,\gamma\beta,fb}\left(n_+p, n_+p_a;\omega\right) = J^{(O)}_{2\xi}\left(n_+p_a;\omega\right)\mathbf{T}^A_{fb}\,\delta_{\gamma\beta}\delta\left(n_+p - n_+p_a\right)$$

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$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, \hat{\sigma}_{ab}(z)$$

After combination with leptonic part and stripping off the PDFs

$$\hat{\sigma}(z) = H(\hat{s}) \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2+\vec{q}^{\,2}}} \frac{1}{2\pi} \int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x}$$

$$\times \left\{ \widetilde{S}_0(x) + \boxed{2\frac{1}{2}\int d\omega\, J_{2\xi}^{(O)}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}(x,\omega)} + \bar{c}\text{-term} \right\}$$

$$J_{2\xi}^{(O)}(x_a n_+ p_A;\omega) = -\frac{2}{x_a(n_+p_A)} - 2\frac{\partial}{x_a \partial(n_+p_A)}$$

$$+\frac{i\alpha}{4\pi}\frac{2}{x_a(n_+p_A)}\left[\frac{\omega(x_a n_+ p_A)}{\mu^2}\right]^{-\epsilon}\left(C_A\left(5+\mathcal{O}(\epsilon^1)\right) - C_F\left(\frac{4}{\epsilon}+5+\mathcal{O}(\epsilon^1)\right)\right)$$

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Comments on the final result

$$\int d\omega\, J^{(O)}_{2\xi}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}(x,\omega)$$

$$\begin{aligned} J^{(O)}_{2\xi}(x_a n_+ p_A;\omega) \quad &= \quad -\frac{2}{x_a(n_+p_A)} - 2\frac{\partial}{x_a\partial(n_+p_A)} \\ &+\frac{i\alpha}{4\pi}\frac{2}{x_a(n_+p_A)}\left[\frac{\omega(x_a n_+ p_A)}{\mu^2}\right]^{-\epsilon}\left(C_A\left(5+\mathcal{O}(\epsilon^1)\right)-C_F\left(\frac{4}{\epsilon}+5+\mathcal{O}(\epsilon^1)\right)\right) \end{aligned}$$

$$S_{2\xi}\,(\Omega,\omega) = \frac{\alpha\, C_F}{2\pi}\frac{\mu^{2\epsilon}\, e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]}\,\frac{1}{\omega^{1+\epsilon}}\,\frac{1}{(\Omega-\omega)^{\epsilon}}\theta(\omega)\theta(\Omega-\omega)\, +\, \mathcal{O}(\alpha^2)$$

Comments on the final result

$$\int d\omega \, J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \, \widetilde{S}_{2\xi}(x, \omega)$$

For resummation, we treat the two objects independently, and expand in ϵ prior to performing the final convolution. However, there is a problem! At two loops:

$$J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \sim \alpha \log(\omega)$$

and

$$S_{2\xi}\,(\Omega, \omega) \sim \alpha \, \delta(\omega) + \mathcal{O}(\alpha^2)$$

Comments on the final result

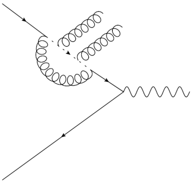
$$\int d\omega\, J^{(O)}_{2\xi}(x_a n_+ p_A;\omega)\, \widetilde{S}_{2\xi}(x,\omega)$$

The factorization formula is valid for unrenormalized objects. Performing the convolution in d - dimensions reproduces fixed NNLO result:

$$\frac{i\alpha^2}{(4\pi)^2}\frac{1}{Q}\left(C_A C_F\left(\frac{-10}{\epsilon}+30\log(1-z)+\mathcal{O}(\epsilon^1)\right)\right.$$
$$\left.-C_F^2\left(\frac{-8}{\epsilon^2}-\frac{10}{\epsilon}+\frac{24}{\epsilon}\log(1-z)+30\log(1-z)-36\log^2(1-z)+\mathcal{O}(\epsilon^1)\right)\right)$$

after we set the scale to hard.

New soft structures can appear at N^3LO: see C.White talk from NLP Corrections in Particle Physics workshop, Amsterdam 2018.



Summary

- Introduction of the collinear functions at *amplitude* level and explicit computation within SCET framework.
- Presented general factorization formula for DY threshold production at next-to-leading power, checked its validity up to fixed NNLO and identified a problem for resummation at NLL
- For leading logarithmic resummation - see Robert's talk

Thank you

Back up slides

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm(x) = \mathbf{P}\exp\left[ig_s\int_{-\infty}^{0} ds\, n_\mp A_s(x+sn_\mp)\right]$$

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The LP quark Lagrangian is

$$\mathcal{L}_{\rm LP} = \bar{\chi}\left(in_-D + i\not{D}_{\perp c}\frac{1}{in_+D_c}\,i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\chi$$

[M. Beneke and Th. Feldmann, 0211358]

where

$$in_-D = in_-\partial + g\,n_-A_c(x) + g\,n_-A_s(x_-)$$

and *after* the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)}\frac{\not{n}_+}{2}\left(n_-\mathcal{A}_c + n_-\partial\right)\chi^{(0)}(x)$$

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and *after* the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)}\frac{\not{n}_+}{2}\left(n_-\mathcal{A}_c + n_-\partial\right)\chi^{(0)}(x)$$

From now on we use decoupled fields. Leading power current becomes

$$J_\rho^{A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}^{(0)}(\bar{t}n_-)Y_-^\dagger(0)\gamma_{\perp\rho}Y_+(0)\chi_c^{(0)}(tn_+)$$

Matching to quark current at NLP

N-jet operators are built out of following relevant building blocks.

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742.]

(A1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i\partial\!\!\!/_\perp]\chi_c(tn_+),\; \bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho(-i)\overleftarrow{\partial\!\!\!/}_\perp]\chi_c(tn_+)$$

(B1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \mathcal{A}\!\!\!/_{c\perp}(t_2n_+)]\chi_c(t_1n_+),\; \bar{\chi}_{\bar{c}}(\bar{t}_1n_-)[n_\pm^\rho \mathcal{A}\!\!\!/_{\bar{c}\perp}(\bar{t}_2n_-)]\,\chi_c(tn_+)$$

With the the scaling

$$[n_\pm^\rho i\partial\!\!\!/_\perp]\,\chi_c(tn_+) \sim \lambda$$
$$[n_\pm^\rho \mathcal{A}\!\!\!/_{c\perp}(t_2n_+)]\,\chi_c(tn_+) \sim \lambda$$

relative to LP.

Time-ordered products

$$\left(J^{Tm}_{W,V}(s,t)\right)^{\mu} = i \int d^4x \, \mathbf{T}\left[J^{\mu}_{W}(s,t)\,\mathcal{L}^{(n)}_{V}(x)\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742]

The NLP soft-collinear SCET quark-gluon interaction Lagrangian written in terms of building blocks $\mathcal{B}^{\mu}_{\pm} = Y^{\dagger}_{\pm}\left[iD^{\mu}_{s}Y_{\pm}\right]$ and $q^{\pm}(x) = Y^{\dagger}_{\pm}(x)\, q_s(x)$ is

[M. Beneke, F. Campanario, T. Mannel, B.D. Pecjak , 0411395]

$$\begin{aligned}
\mathcal{L}^{(1)}_{\xi} &= \bar{\chi}_c i x^{\mu}_{\perp}\left[in_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{1\xi} &= \frac{1}{2}\bar{\chi}_c in_- x n^{\mu}_{+}\left[in_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{2\xi} &= \frac{1}{2}\bar{\chi}_c x^{\mu}_{\perp} x^{\rho}_{\perp}\left[i\partial_{\rho} in_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{3\xi} &= \frac{1}{2}\bar{\chi}_c x^{\mu}_{\perp} x^{\rho}_{\perp}\left[\mathcal{B}^{+}_{\rho}(x_-),\, in_-\partial\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{4\xi} &= \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial} i x^{\mu}_{\perp}\gamma^{\nu}_{\perp}\left[i\partial_{\nu}\mathcal{B}^{+}_{\mu}(x_-) - i\partial_{\mu}\mathcal{B}^{+}_{\nu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.} \\
\mathcal{L}^{(2)}_{5\xi} &= \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial} i x^{\mu}_{\perp}\gamma^{\nu}_{\perp}\left[\mathcal{B}^{+}_{\nu}(x_-),\mathcal{B}^{+}_{\mu}(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.} \\
\mathcal{L}^{(1)}_{\xi q} &= \bar{q}_{+}(x_-)\mathcal{A}_{c\perp}\chi_c + \text{h.c.}
\end{aligned}$$

based on [M. Beneke and Th. Feldmann, 0211358]

Definition of PDFs

$$\mathcal{A}_{\perp\mu} = Y_+^\dagger W_c^\dagger \left[i\, D_c\, W_c \right] Y_+$$

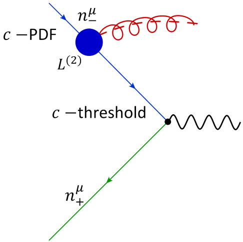
$$\langle A(p_A)|\bar{\chi}_{c,\alpha a}(x+u'n_+)\chi_{c,\beta b}(un_+)|A(p_A)\rangle = \frac{\delta_{ba}}{N_c}\left(\frac{\not{n}_-}{4}\right)_{\beta\alpha} n_+p_A \times \int_0^1 dx_a\, f_{a/A}(x_a)\, e^{i(x+u'n_+-u\,n_+)\cdot x_a p_A}$$

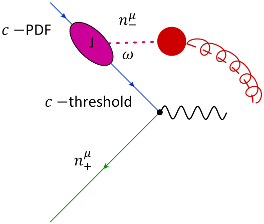
Collinear functions

Threshold collinear fields are matched to collinear-PDF fields

$$\int dt\, e^{i(n_+p)t}\, i \int d^4z\, e^{i\omega(n_+z)/2}\, \mathbf{T}\left[\chi_c(tn_+) \times \mathcal{L}_c^{(n)}(z)\right]$$

$$= \int d(n_+p') \int dt\, e^{i(n_+p')t}\, J(n_+p, n_+p'; \omega)\, \chi_c^{\mathrm{PDF}}(tn_+)$$





Computation of collinear function

Recall the operator matching equation. The short-distance coefficient can be extracted by computing the partonic matrix element $\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}(n_+q_a,\omega)|q(q)_q\rangle$. Running to collinear scale: only tree level collinear function is necessary.

$$\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}(n_+q_a,\omega)|q(q)_q\rangle = \int dt e^{i(n_+q_a)t}\, i\int d^4z\left[(in_-\partial_z)^2 e^{i\omega\frac{n_+z}{2}}\right] \times\frac{1}{2}z^\nu_\perp z^\rho_\perp\langle 0|\mathbf{T}\left[\chi_{c,\gamma f}(tn_+)\bar{\chi}_{c,e}(z)\frac{\not{n}_+}{2}\chi_{c,d}(z)\right]|q(q)_q\rangle$$

$$= i\omega^2\int dt e^{i(n_+q_a)t}\int d^4z\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(tn_+-z)}\frac{i(n_+k)}{(k)^2}\delta_{fe}e^{i\omega\frac{n_+z}{2}} \times\frac{1}{2}z^\nu_\perp z^\rho_\perp\langle 0|\left(\frac{\not{n}_-}{2}\frac{\not{n}_+}{2}\chi_{c,d}(z)\right)_\gamma|q(q)_q\rangle$$

Computation of collinear function - continued

$$\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}\left(n_+q_a,\omega\right)|q(q)_q\rangle = -\frac{1}{2}2i\omega^2\left(2\pi\right)\int d^4k\delta\left((n_+q_a)-(n_+k)\right) \times\left[\frac{\partial}{\partial k_{\perp\nu}}\frac{\partial}{\partial k_{\perp\rho}}\frac{i(n_+k)}{(k)^2}\right]\delta\left((n_+q)-(n_+k)\right) \times\delta\left(\omega+(n_-k)\right)\delta^2\left(k_\perp\right)\delta_{fe}\delta_{dq}\,u_{c,\gamma}$$

Then

$$\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}\left(n_+q_a,\omega\right)|q(q)_q\rangle = -(2\pi)\delta\left((n_+q_a)-(n_+q)\right)\left[\frac{g_\perp^{\nu\rho}}{(n_+q)}\right]\delta_{fe}\delta_{dq}\,u_{c,\gamma}$$

Matching to:

$$\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}\left(n_+q_a,\omega\right)|q(q)_q\rangle = \int d(n_+p_a)J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\left(n_+q_a,n_+p_a;\omega\right)\langle 0|\hat{\chi}^{\mathrm{PDF}}_{c,\beta b}(n_+p_a)|q(q)_q\rangle$$

$$\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}\left(n_+q_a,\omega\right)|q(q)_q\rangle = 2\pi J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\left(n_+q_a,(n_+q);\omega\right)\delta_{bq}u_{c,\beta}(q)$$

Collinear function:

$$J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\left(n_+q_a,(n_+q);\omega\right) = -\delta_{bd}\delta_{fe}\delta_{\beta\gamma}\delta\left((n_+q_a)-(n_+q)\right)\frac{g_\perp^{\nu\rho}}{(n_+q)}$$

More steps in derivation of collinear function

$$\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}\,(n_+q_a,\omega)\,|q(q)_q\rangle = i\omega^2\,(2\pi)\int\frac{d^4k}{(2\pi)^4}\delta\,((n_+q_a)-(n_+k))\,\int d^4z\,\frac{i(n_+k)}{(k)^2}\delta_{fe}$$
$$\times e^{i\omega\frac{n_+z}{2}}\,\frac{1}{2}\left[-\frac{\partial}{\partial k_{\perp\nu}}\,\frac{\partial}{\partial k_{\perp\rho}}e^{+ik\cdot z}\right]\,\langle 0|\chi_{c,\gamma d}\,(z)\,|q(q)_q\rangle$$

$$\chi_{c,\gamma d}(z)|q(q)_q\rangle = \delta_{dq}\,u_{c,\gamma}(q)e^{-iz\cdot q}|0\rangle$$

$$\hat{\chi}^{\mathrm{PDF}}_{c,\beta b}\,(n_+p_a) = \int du\,e^{i(n_+p_a)u}\,\chi^{\mathrm{PDF}}_{c,\beta b}\,(un_+)$$

Soft functions

We introduce the soft operator

$$\widetilde{\mathcal{S}}_{2\xi}\,(x, z_-) = \bar{\mathbf{T}}\left[Y_+^\dagger(x) Y_-(x)\right]\mathbf{T}\left[Y_-^\dagger(0) Y_+(0)\,\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}_{\perp\nu}^+(z_-)\right]$$

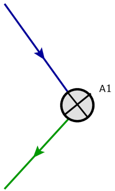
and the Fourier transform of its (colour-traced) vacuum matrix element

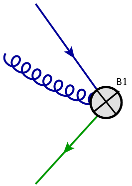
$$S_{2\xi}(\Omega,\omega) = \int\frac{dx^0}{4\pi}\int\frac{d(n_+z)}{4\pi}\,e^{ix^0\Omega/2 - i\omega(n_+z)/2}\,\frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\widetilde{\mathcal{S}}_{2\xi}(x^0, z_-)|0\rangle$$

Possible contributing structures

First we check whether subleading power contributions start at order λ.

- Consider A1 and B1 type currents:
  A1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_{\pm}^{\rho} i\partial\!\!\!/_{\perp}]\chi_c(tn_+)$ B1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_{\pm}^{\rho} \mathcal{A}\!\!\!/_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$





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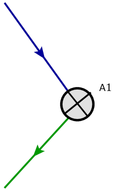
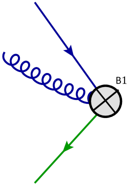
  A1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i\not{\partial}_\perp]\chi_c(tn_+)$ B1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \not{\mathcal{A}}_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$

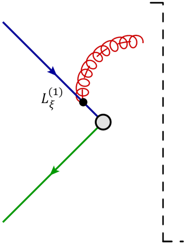




- Another possibility is a single power suppressed time-ordered product of the form $\left(J^{T1}_{A0,\xi}(s,t)\right)^\mu = i\int d^4x\,\mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(1)}_\xi(x)\right]$

Only one possibility

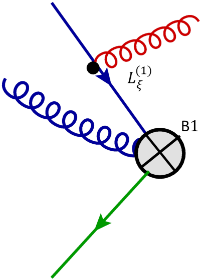
$$\mathcal{L}^{(1)}_\xi \quad = \quad \bar{\chi}_c i x_\perp^\mu \left[in_-\partial\mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c$$

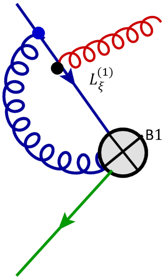


Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

- B1-type current, $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \not{\mathcal{A}}_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$, with Lagrangian insertion $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\not{n}_+}{2} \chi_c$

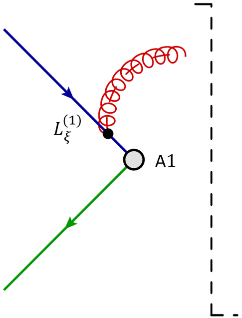




Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

- A1-type current with $\mathcal{L}_{\xi}^{(1)} = \bar{\chi}_c i x_{\perp}^{\mu} \left[i n_- \partial \mathcal{B}_{\mu}^{+} \right] \frac{\not{n}_+}{2} \chi_c$ insertion



Feynman rule for emission of a soft gluon from $\mathcal{B}_{\mu}^{+}$ is

$$g\, T^A \left[-\frac{k_{\perp}^{\mu} n_{-\nu}}{(n_- k)} + g_{\perp}^{\mu\nu} \right] \epsilon_{\nu}^{*}\, e^{+ik\cdot z_-}$$

Possible contributing structures

Update (???)

- B1-type current with $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\not{n}_+}{2} \chi_c$ insertion does not contribute to NLP
- A1-type current with $\mathcal{L}_\xi^{(1)}$ insertion vanishes.

Power suppression at LL accuracy must come from Lagrangian insertions.

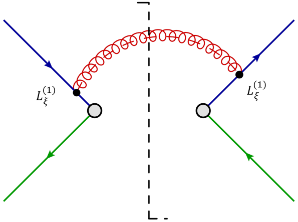
$$\left(J_{W,V}^{Tm}(s,t) \right)^\mu = i \int d^4x \, \mathbf{T} \left[J_W^\mu(s,t) \, \mathcal{L}_V^{(n)}(x) \right]$$

Considering the Lagrangian insertions

Previous arguments allow us also to drop following possible contributions

$$\left(J^{T1}_{A0,\xi}(s,t)\right)^{\mu} = i \int d^4x_1 \, \mathbf{T}\left[J^{\mu}_{A0}(s,t)\, \mathcal{L}^{(1)}_{\xi}(x_1)\right]$$
$$\left(\bar{J}^{T1}_{A0,\xi}(\bar{s},\bar{t})\right)^{\mu} = (-i) \int d^4x_2 \, \mathbf{T}\left[\bar{J}^{\mu}_{A0}(\bar{s},\bar{t})\, \mathcal{L}^{(1)}_{\xi}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi} \quad = \quad \bar{\chi}_c i x^{\mu}_{\perp}\left[i n_{-}\partial \mathcal{B}^{+}_{\mu}\right]\frac{\not{n}_{+}}{2}\chi_c$$

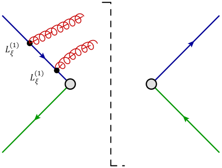


Considering the Lagrangian insertions

The following contributions start at $\mathcal{O}(\alpha^2)$

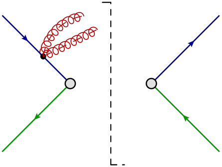
$$\left(J^{T2}_{A0,\xi}(s,t)\right)^\mu = i\int d^4x_1\, i\int d^4x_2\, \mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(1)}_\xi(x_1)\,\mathcal{L}^{(1)}_\xi(x_2)\right]$$

$$\mathcal{L}^{(1)}_\xi = \bar{\chi}_c i x^\mu_\perp \left[in_-\partial\mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c$$



$$\left(J^{T2}_{A0,V}(s,t)\right)^\mu = i\int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\,\mathcal{L}^{(2)}_V(x)\right]$$

$$\begin{aligned}
\mathcal{L}^{(2)}_{3\xi} &= \frac{1}{2}\bar{\chi}_c x^\mu_\perp x^\rho_\perp \left[\mathcal{B}^+_\rho,\, in_-\partial\mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c \\
\mathcal{L}^{(2)}_{5\xi} &= \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_\perp + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial} i x^\mu_\perp\gamma^\nu_\perp \\
&\quad \times\left[\mathcal{B}^+_\nu, \mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.}
\end{aligned}$$

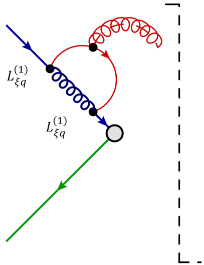


Considering the Lagrangian insertions

It is also possible to construct diagrams containing soft quarks

$$\left(J^{T2}_{A0,\xi q}(s,t)\right)^{\mu} = i\int d^4x_1 i\int d^4x_2\, \mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(1)}_{\xi q}(x_1)\,\mathcal{L}^{(1)}_{\xi q}(x_2)\right]$$

$$\mathcal{L}^{(1)}_{\xi q} \quad = \quad \bar{q}_{+}\mathcal{A}_{c\perp}\chi_c + \text{h.c.}$$

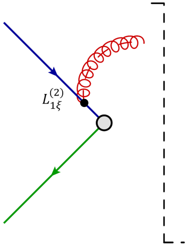


These contributions also start at $\mathcal{O}(\alpha^2)$

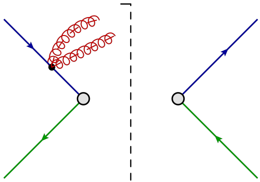
Considering the Lagrangian insertions

Two more possible contributions with following Lagrangian terms making up the time-ordered product

$$\mathcal{L}_{1\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c i n_- x n_+^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\not{n}_+}{2} \chi_c$$



$$\mathcal{L}_{4\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c \left(i\not{\partial}_\perp + \not{\mathcal{A}}_{c\perp} \right) \frac{1}{i n_+ \partial} i x_\perp^\mu \gamma_\perp^\nu \times \left[i\partial_{\nu\perp} \mathcal{B}_{\mu\perp}^+ - i\partial_{\mu\perp} \mathcal{B}_{\nu\perp}^+ \right] \frac{\not{n}_+}{2} \chi_c + \text{h.c.}$$



Conclusion

We therefore find that for LL resummation at NLP in the quark-antiquark channel only the single time-ordered product contribution:

$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^{\mu} = i\int d^4x\, \mathbf{T}\left[J^{\mu}_{A0}(s,t)\,\mathcal{L}^{(2)}_{2\xi}(x)\right]$$

To NLP LL accuracy the matching equation is then extended to

$$\bar{\psi}\gamma^{\mu}\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\left[J^{\mu}_{A0}(t,\bar{t}\,) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}\,)\right)^{\mu} + \ \bar{c}\text{-term}\right]$$

Again we consider

$$\langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle$$

A power suppressed amplitude

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}) \left[J^\mu_{A0}(t,\bar{t}) + \boxed{i \int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\, \mathcal{L}^{(2)}_{2\xi}(x) \right]} + \bar{c}\text{-term} \right]$$

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi} \int dt\, d\bar{t}\, e^{it\, n_+p} e^{i\bar{t}n_-\bar{p}} C^{A0}(n_+p, n_-\bar{p})$$

$$\times \langle X|\mathbf{T}\Bigg[\underbrace{\bar{\chi}_{\bar{c}}(\bar{t}n_-) Y_-^\dagger(0)\gamma^\mu_\perp\, Y_+(0)\chi_c\,(tn_+)}_{J^\mu_{A0}(t,\bar{t})}\, i\int d^4z\, \bar{\chi}_{c,e}\,(z)\, \frac{1}{2} z^\nu_\perp z^\rho_\perp$$

$$\times \left[\boxed{\left(\frac{in_-\partial_z}{in_-\partial_z}\right)} (in_-\partial_z) i\partial^\rho_\perp \mathcal{B}^+_{\perp\nu,ed}\,(z_-) \right] \frac{\not{n}_+}{2}\chi_{c,d}\,(z) \Bigg] |A(p_A)B(p_B)\rangle$$

A power suppressed amplitude

The states factorize as at leading power: $\langle X| = \langle X_{\bar{c}}^{\mathrm{PDF}}| \, \langle X_c^{\mathrm{PDF}}| \, \langle X_s|$ as they are eigenstates of the LP Lagrangian

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int dt\, d\bar{t}\, e^{it\, n_+p}e^{i\bar{t}n_-\bar{p}}C^{A0}(n_+p, n_-\bar{p})$$
$$\times\langle X_{\bar{c}}^{\mathrm{PDF}}|\bar{\chi}_{\bar{c},\alpha a}(\bar{t}n_-)|B(p_B)\rangle\gamma^\mu_{\perp,\alpha\gamma}$$
$$\times i\int d^4z\langle X_c^{\mathrm{PDF}}|\frac{1}{2}z_\perp^\nu z_\perp^\rho(in_-\partial_z)^2\,\mathbf{T}\left[\chi_{c,\gamma f}\,(tn_+)\,\bar{\chi}_{c,e}\,(z)\,\frac{\not{n}_+}{2}\chi_{c,d}\,(z)\right]|A(p_A)\rangle$$
$$\times\langle X_s|\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]_{af}\frac{i\partial_\perp^\rho}{in_-\partial_z}\mathcal{B}^+_{\perp\nu,ed}\,(z_-)\right)|0\rangle$$

Amplitude with collinear function

$$\langle X|\bar{\psi}\gamma^{\mu}\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int d(n_+p_a)d(n_-p_b)$$
$$\delta\left(n_-\bar{p}+(n_-p_b)\right)\, C^{A0}(n_+p,n_-\bar{p})$$
$$\times\int\frac{d\omega}{2\pi}\, J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\,(n_+p,n_+p_a;\omega)\langle X^{\rm PDF}_{\bar{c}}|\hat{\bar{\chi}}^{\rm PDF}_{\bar{c},\alpha a}(n_-p_b)|B(p_B)\rangle$$
$$\times\gamma^{\mu}_{\perp,\alpha\gamma}\langle X^{\rm PDF}_{c}|\hat{\chi}^{\rm PDF}_{c,\beta b}(n_+p_a)|A(p_A)\rangle$$
$$\times\int\frac{dn_+z}{2}e^{-i\omega\frac{n_+z}{2}}\langle X_s|\mathbf{T}\left(\left[Y_-^{\dagger}(0)Y_+(0)\right]_{af}\frac{i\partial^{\rho}_{\perp}}{in_-\partial}\mathcal{B}^{+}_{\perp\nu,ed}\,(z_-)\right)|0\rangle$$

Computation of collinear function

The short-distance coefficient can be extracted by computing the partonic matrix element $\langle 0| \mathcal{J}^{\rho\nu}_{\gamma,fed}(n_+ q_a, \omega) |q(q)_q\rangle$. Running to collinear scale: only tree level collinear function is necessary.

Collinear function:

$$J^{\rho\nu}_{2\xi,\gamma\beta,fbed}(n_+ q_a, (n_+ q); \omega) = -\delta_{bd}\delta_{fe}\delta_{\beta\gamma}\delta\left((n_+ q_a) - (n_+ q)\right)\frac{g_\perp^{\nu\rho}}{(n_+ q)}$$

LP + NLP amplitude

We are considering the matching up to NLP

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,) \left[J^\mu_{A0}(t,\bar{t}\,) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}\,)\right)^\mu + \ \bar{c}\text{-term}\right]$$

For which we obtained

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{dn_+p_a}{2\pi}\frac{dn_-p_b}{2\pi}\, C^{A0}(n_+p_a, -n_-p_b)$$

$$\times \langle X_{\bar{c},\mathrm{PDF}}|\hat{\bar{\chi}}^{\mathrm{PDF}}_{\bar{c},\alpha a}(n_-p_b)|B(p_B)\rangle\, \gamma^\mu_{\perp\alpha\beta}\, \langle X_{c,\mathrm{PDF}}|\hat{\chi}^{\mathrm{PDF}}_{c,\beta b}(n_+p_a)|A(p_A)\rangle$$

$$\times \Bigg\{ \langle X_s|\mathbf{T}\left[Y^\dagger_-(0)Y_+(0)\right]_{ab}|0\rangle$$

$$+ \frac{1}{2}\int \frac{d\omega}{4\pi}\, J^{(O)}_{2\xi}(n_+p_a;\omega) \int d(n_+z)\, e^{-i\omega(n_+z)/2}$$

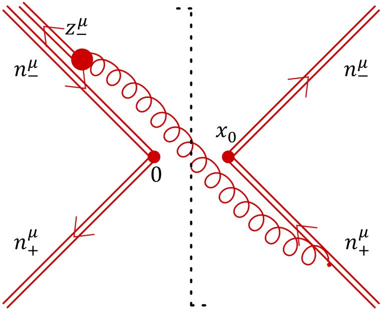
$$\times \langle X_s|\mathbf{T}\left(\left[Y^\dagger_-(0)Y_+(0)\right]_{af} \frac{i\partial^\nu_\perp}{in_-\partial_z}\mathcal{B}^+_{\perp\nu;fb}(z_-)\right)|0\rangle \Bigg\} + \bar{c}\text{-term}$$

Note that $J^{(\mathrm{O})}_{2\xi}(n_+p_a;\omega) = -\frac{2}{n_+p_a}$

Relevant soft function

The generalized soft function at cross section level here is

$$S_{2\xi}(\Omega,\omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+z)/2}$$
$$\times \frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}^+_{\perp\nu}(z_-)\right]|0\rangle$$



For details on renormalization of soft functions and resummation see Robert's talk.

Result for the power suppressed amplitude: C_F

$$i\,C_F\,\gamma_{\perp\rho}\,\frac{1}{(n_+p)(n_-k)}\Bigg((n_+k)n_{-\nu}\left(\frac{3}{\epsilon}+2-\frac{3}{2}\zeta(2)\epsilon+\left(-\zeta(2)-\frac{21\zeta(3)}{3}-4\right)\epsilon^2\right)$$
$$+(n_-k)n_{+\nu}\left(-\frac{1}{\epsilon}-3+(-6+\frac{1}{2}\zeta(2))\epsilon+\left(\frac{3}{2}\zeta(2)-12+\frac{7\zeta(3)}{3}\right)\epsilon^2\right)$$
$$+k_{\perp\nu}\left(+\frac{2}{\epsilon}-1+(-6-\zeta(2))\epsilon+\left(+\frac{\zeta(2)}{2}-\frac{14\zeta(3)}{3}-16\right)\epsilon^2\right)$$
$$+[\not{k}_\perp,\gamma_{\perp\nu}]\left(+\frac{1}{2}+\epsilon+\left(-\frac{1}{4}\zeta(2)+2\right)\epsilon^2\right)\Bigg)$$

$$i\,C_F\,n_{-\rho}\,\frac{1}{n_-l}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(+1+4\epsilon-\frac{1}{2}(\zeta(2)-20)\epsilon^2\right)$$

$$i\,C_F\,n_{+\rho}\,\frac{1}{n_+p}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(-1-4\epsilon+\frac{1}{2}(\zeta(2)-20)\epsilon^2\right)$$

Result for the power suppressed amplitude: C_A

$$
\begin{aligned}
&i\,C_A\,\gamma_{\perp\rho}\,\frac{1}{(n_+p)(n_-k)}\Bigg((n_+k)n_{-\nu}\Bigg(-\frac{1}{2\,\epsilon^2}-\frac{3}{2\,\epsilon}+\frac{1}{4}(\zeta(2)-18)\\
&+\frac{1}{12}(9\zeta(2)+14\zeta(3)-48)\epsilon+\frac{1}{32}(72\zeta(2)+112\zeta(3)+47\zeta(4)-288)\epsilon^2\Bigg)\\
&+(n_-k)n_{+\nu}\Bigg(-\frac{1}{2\epsilon^2}-\frac{3}{2\epsilon}+\frac{1}{4}(\zeta(2)+2)+\frac{1}{12}(9\zeta(2)+14\zeta(3)-24)\epsilon\\
&-\frac{1}{32}(8\zeta(2)-112\zeta(3)-47\zeta(4)+32)\epsilon^2\Bigg)\\
&+k_{\perp\nu}\Bigg(-\frac{1}{\epsilon^2}-\frac{3}{\epsilon}+\frac{1}{2}(\zeta(2)-8)\\
&+\left(\frac{3\zeta(2)}{2}+\frac{7\zeta(3)}{3}-6\right)\epsilon+\left(2\zeta(2)+7\zeta(3)+\frac{47\zeta(4)}{16}-10\right)\epsilon^2\Bigg)\\
&+[\not{k}_\perp,\gamma_{\perp\nu}]\left(\frac{1}{4}\left(-2-4\epsilon+(\zeta(2)-8)\epsilon^2\right)\right)\Bigg)
\end{aligned}
$$

$$
i\,C_A\,n_{-\rho}\,\frac{1}{n_-l}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(+\frac{1}{\epsilon}+2-\frac{1}{2}(\zeta(2)-6)\epsilon+\left(-\zeta(2)-\frac{7\zeta(3)}{3}+5\right)\epsilon^2\right)
$$

$$
i\,C_A\,n_{+\rho}\,\frac{1}{n_+p}\left(\gamma_{\perp\nu}-\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}\right)\left(-\frac{1}{\epsilon}-2+\frac{1}{2}(\zeta(2)-6)\epsilon+\left(\zeta(2)+\frac{7\zeta(3)}{3}-5\right)\epsilon^2\right)
$$

Results for power suppressed amplitude: Soft

$$i\,C_A\gamma_{\perp\rho}\,\frac{1}{(n_+p)(n_-k)}\Bigg((n_+k)\,n_{-\nu}\left(+\frac{1}{\epsilon^2}+\frac{\zeta(2)}{2}-\frac{7}{3}\zeta(3)\epsilon-\frac{1}{16}39\zeta(4)\epsilon^2\right)$$

$$+(n_-k)\,n_{+\nu}\,\Big(0\Big)$$

$$+k_{\perp\nu}\left(\frac{1}{\epsilon^2}+\frac{1}{2}\zeta(2)-\frac{7}{3}\zeta(3)\epsilon-\frac{39}{16}\zeta(4)\epsilon^2\right)$$

$$+[\not{k}_\perp,\gamma_{\perp\nu}]\left(+\frac{1}{2\,\epsilon^2}+\frac{1}{4}\zeta(2)-\frac{7}{6}\zeta(3)\epsilon-\frac{1}{32}39\zeta(4)\epsilon^2\right)\Bigg)$$

$$g\,\frac{C_A}{2}\,t^b\,\frac{1}{(n_+p)}\,n_+^\rho\,\frac{i\alpha\,e^{\epsilon\gamma_E}}{(4\pi)}\,\frac{1}{\epsilon^2}\,\frac{\Gamma[1-\epsilon]^3}{\Gamma[2-2\epsilon]}\,\Gamma[1+\epsilon]^2$$

$$\times\left[-(1-2\epsilon)\frac{\not{k}_\perp n_{-\nu}}{(n_-k)}+2\gamma_{\perp\nu}+\frac{2\not{k}_\perp k_{\perp\nu}}{(n_-k)(n_+k)}+\frac{\not{k}_\perp n_{+\nu}}{(n_+k)}\,(1-2\epsilon)\right]$$

Results for power suppressed amplitude: soft × hard

$$
\begin{aligned}
iC_F\,\gamma_\perp^\rho\,\frac{1}{(n_+p)(n_-k)}\Bigg(&(n_+k)n_{-\nu}\left(\frac{2}{\epsilon^2}+\frac{1}{\epsilon}+5-\frac{1}{6}\pi^2+\mathcal{O}(\epsilon)\right)\\
&+(n_-k)n_{+\nu}\left(+\frac{2}{\epsilon}+3+\mathcal{O}(\epsilon)\right)\\
&+k_{\perp\nu}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\frac{\pi^2}{6}+8+\mathcal{O}(\epsilon)\right)\\
&+[\not{k}_\perp,\gamma_{\perp\nu}]\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}-\frac{\pi^2}{12}+4+\mathcal{O}(\epsilon)\right)\Bigg)
\end{aligned}
$$

$$
i\,gt^b\;n_+^\rho C_F\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8+\frac{\pi^2}{6}+\mathcal{O}(\epsilon)\right)
\frac{1}{(n_+p)(n_-k)}\left(\not{k}_\perp n_{-\nu}-(n_-k)\gamma_{\perp\nu}\right)
$$

NLP factorization formula

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$\begin{aligned} \hat{\sigma}(z) \quad = \quad & \sum_{\rm terms} \int d\omega_i d\bar{\omega}_i d\omega_i' d\bar{\omega}_i'\, D(-\hat{s}; \omega_i, \bar{\omega}_i) D^*(-\hat{s}; \omega_i', \bar{\omega}_i') \\ & \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3\, 2\sqrt{Q^2 + \vec{q}^{\,2}}}\, \frac{1}{2\pi} \int d^4x\, e^{i(x_a p_A + x_b p_B - q)\cdot x} \\ & \times \widetilde{S}(x; \omega_i, \bar{\omega}_i, \omega_i', \bar{\omega}_i') \end{aligned}$$

and

$$\begin{aligned} D(-\hat{s}; \omega_i, \bar{\omega}_i) \quad = \quad & \int d(n_+ p_i) d(n_- \bar{p}_i)\, C(n_+ p_i, n_- \bar{p}_i) \\ & \times J(n_+ p_i, x_a n_+ p_A; \omega_i)\, \bar{J}(n_- \bar{p}_i, -x_b n_- p_B; \bar{\omega}_i) \end{aligned}$$