Double differential factorization in τ_0 and τ_1

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Work in progress with Goutam Das and Frank Tackmann

DESY

SCET
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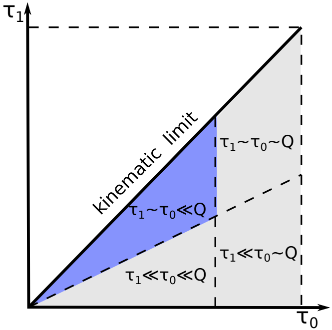
Introduction

- We want the double differential factorization for τ_0 and τ_1
  $\rightarrow$ measures 2 consecutive resolved emissions

- Some kinematic regions are already known
  - $\tau_1 \ll \tau_0 \ll Q$: known with SCET+ approach
    $\rightarrow$ strongly ordered emissions
    Pietrulewicz, Tackmann , Waalewijn 2016
  - $\tau_1 \ll \tau_0 \sim Q$: only τ_1 resummation
    $\rightarrow$ one hard emission
  - $\tau_1 \sim \tau_0 \sim Q$: purely fixed-order
    $\rightarrow$ two hard emissions



- $\tau_1 \sim \tau_0 \ll Q$ is the missing piece in the $\tau_0 \times \tau_1$ space
  $\rightarrow$ region with two unordered (but resolved) emissions

Motivation for $\tau_1 \sim \tau_0 \ll Q$

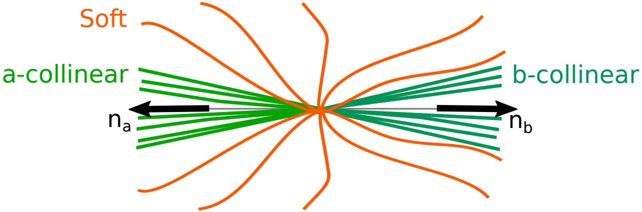
- Prototype for factorization beyond strongly ordered regime
  $\rightarrow$ using N-jettiness as a "simple" resolution variable
  $\rightarrow$ treatment of unordered emissions is currently unknown
  $\rightarrow$ relevant to increase accuracy of Parton Showers

- Immediate application: Geneva framework at NNLL' + NNLO
  $\rightarrow$ Needs full 0/1-jet and 1/2-jet separation

- Jet substructure : understanding cases with multiple observables with no hierarchy between them
  $\rightarrow$ Example : C- and D-parameters in the region $D \sim C^2 \ll 1$
  Larkoski, Procita 2019 – See Larkoski's talk later today
  $\rightarrow$Finite N-subjettiness ratios $\tau_{21} = \tau_2/\tau_1$ and $\tau_{32} = \tau_3/\tau_2$
  Napoletano, Soyez 2018

Notation and Setup

- Because $\tau_0 \ll Q \to$ power counting is defined by τ_0 only



- Definition of observables here

$$\tau_0 = \frac{1}{Q} \sum_{i \in \text{event}} \min\left(n_a \cdot p_i, n_b \cdot p_i\right), \quad \tau_1 = \frac{1}{Q} \sum_{i \in \text{event}} \min\left(n_1 \cdot p_i, n_a \cdot p_i, n_b \cdot p_i\right)$$

- Decompose N-jet as

$$\tau_N = \tau_{s,N} + \tau_{a,N} + \tau_{b,N}$$

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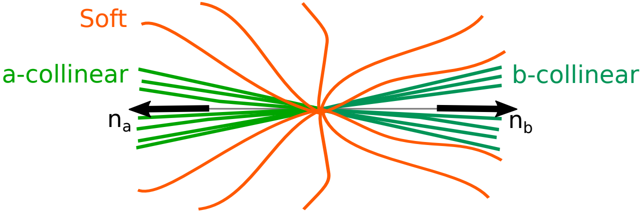

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- Decompose N-jet as

$$\tau_N = \tau_{s,N} + \tau_{a,N} + \tau_{b,N}$$

How to write τ_1 with these modes?

“Inspiration” : Fully Recursive N-jettiness

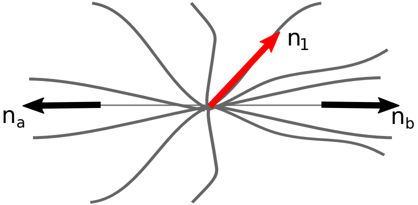
- Fully recursive τ_0^{FR} is given by

  Step 1 Compute τ_1, with the resp. axis n_1

  Step 2 Emissions divided in subgroups – depends on which axis the have “chosen”

  Step 3 Add contribution

$$\Delta^{\mathrm{FR}} = \left|\sum \vec{p}\right| - \left|\sum p^z\right|$$



- Corollary: we can always write $\tau_0^{FR} = \tau_1 + \Delta^{FR}$

  Definition of τ_0 used for *pp* collision in GENEVA (Alioli and al. 2015)

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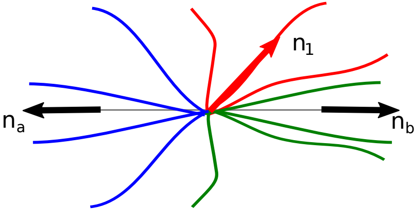

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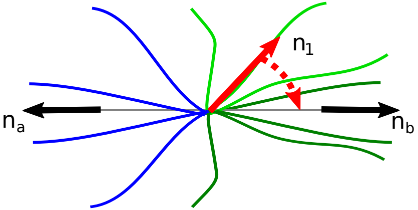

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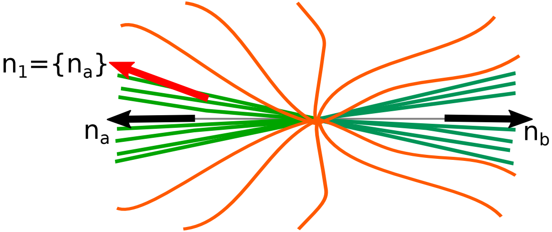
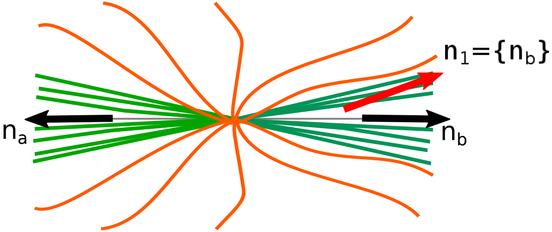
1-jettiness axis choice

- We select the axis n_1 that minimizes the 1-jettiness
- Possibilities for the axis choice are $\rightarrow$ in the a-collinear sector



1-jettiness axis choice

- We select the axis n_1 that minimizes the 1-jettiness
- Possibilities for the axis choice are $\rightarrow$ in the b-collinear sector



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- We select the axis n_1 that minimizes the 1-jettiness
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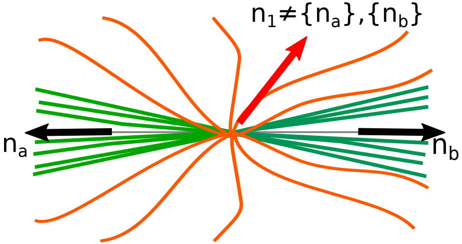

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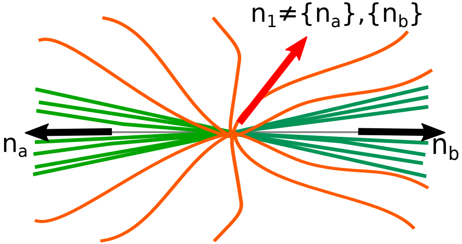


- From the power-counting point of view

$$\{n_a\} \equiv \left(1, \frac{\vec{p}_a}{|\vec{p}_a|}\right), \qquad p_a \sim (p_a^+, p_a^-, p_{a\perp}) \sim Q^2(\lambda^2, 1, \lambda)$$

Factorization

- Decompose axis minimization as

$$\tau_1 = \min\Bigg[\min_{n_1=\{n_a\}}\left(\sum_{i\in event}\min\left(n_a\cdot p_i, n_b\cdot p_i, n_1\cdot p_i\right)\right),\\ \min_{n_1=\{n_b\}}\left(\sum_{i\in event}\min\left(n_a\cdot p_i, n_b\cdot p_i, n_1\cdot p_i\right)\right),\\ \min_{n_1\neq\{n_a\},\{n_b\}}\left(\sum_{i\in event}\min\left(n_a\cdot p_i, n_b\cdot p_i, n_1\cdot p_i\right)\right)\Bigg]$$

- Divide and conquer $\rightarrow$ prove factorization for each piece

Factorization

- Decompose axis minimization as

$$\tau_1 = \min \Bigg[\min_{n_1=\{n_a\}} \left(\sum_{i\in event} \min\left(n_a \cdot p_i, n_b \cdot p_i, n_1 \cdot p_i\right) \right), \min_{n_1=\{n_b\}} \left(\sum_{i\in event} \min\left(n_a \cdot p_i, n_b \cdot p_i, n_1 \cdot p_i\right) \right), \min_{n_1\neq\{n_a\},\{n_b\}} \left(\sum_{i\in event} \min\left(n_a \cdot p_i, n_b \cdot p_i, n_1 \cdot p_i\right) \right) \Bigg]$$

- Divide and conquer $\rightarrow$ prove factorization for each piece

Start with $n_1 = \{n_a\}$ case

Factorization for $n_1 = \{n_a\}$

- Reminder : we can write $\tau_N = \tau_{s,N} + \tau_{a,N} + \tau_{b,N}$
  1. For **a-collinear** piece
  $$\tau_{1,a} = \min\left(n_a \cdot b_a, n_b \cdot b_a, n_1 \cdot b_a\right) = b_a^+ - \delta\tau_a$$
  2. For **b-collinear** piece
  $$\tau_{1,b} = \min\left(n_a \cdot b_b, n_b \cdot b_b, n_1 \cdot b_b\right) = b_b^+$$
  3. For **soft** piece:
  $\{n_a\} = n_a + \lambda^2 \vec{n} \quad \rightarrow \quad n_1 \cdot k_s = n_a \cdot k_s(1 + \mathcal{O}(\lambda^2))$
  $$\tau_{1,s} = \min\left(n_a \cdot k_s, n_b \cdot k_s, n_1 \cdot k_s\right) = k_s$$

If $n_1 \in \{n_a\}$, then $\tau_1 = \tau_0 - \delta\tau_a$
Only emissions in the a-collinear sector matter

Factorization for $n_1 \neq \{n_a\}, \{n_b\}$

- Suppose $n_1 \neq \{n_a\}, \{n_b\}$
  1. **Collinear** pieces → same reasoning as before

$$\tau_{1,a} = b_a^+, \qquad \tau_{1,b} = b_b^+$$

  2. **Soft** emissions

$$\tau_{1,s} = \min\left(n_a \cdot k_s, n_b \cdot k_s, n_1 \cdot k_s\right) = k_s - \delta\tau_s$$

Factorization for $n_1 \neq \{n_a\}, \{n_b\}$

- Suppose $n_1 \neq \{n_a\}, \{n_b\}$
  1. **Collinear** pieces $\rightarrow$ same reasoning as before

$$\tau_{1,a} = b_a^+, \qquad \tau_{1,b} = b_b^+$$

  2. **Soft** emissions

$$\tau_{1,s} = \min\left(n_a \cdot k_s, n_b \cdot k_s, n_1 \cdot k_s\right) = k_s - \delta\tau_s$$

$$\text{If } n_1 = \{n_a\}, \qquad \text{then } \tau_1 = \tau_0 - \delta\tau_a$$

$$\text{If } n_1 = \{n_b\}, \qquad \text{then } \tau_1 = \tau_0 - \delta\tau_b$$

$$\text{If } n_1 \neq \{n_a\}, \{n_b\}, \qquad \text{then } \tau_1 = \tau_0 - \delta\tau_s$$

Axis in a sector implies $\delta\tau_i$ in the same sector

Factorization

- Write the 1-jettiness like

$$\tau_1 = \min \Big[b_b^+ + k_s + \min_{n_1=\{n_a\}} \left(b_a^+ - \delta\tau_a \right), \\ b_a^+ + k_s + \min_{n_1=\{n_b\}} \left(b_b^+ - \delta\tau_b \right), \\ b_a^+ + b_b^+ + \min_{n_1\neq\{n_a\},\{n_b\}} \left(k_s - \delta\tau_s \right) \Big]$$

Factorization

- Write the 1-jettiness like

$$\tau_1 = \min \Big[\tau_0 + \min_{n_1=\{n_a\}} (-\delta\tau_a)\,, \\ \tau_0 + \min_{n_1=\{n_b\}} (-\delta\tau_b)\,, \\ \tau_0 + \min_{n_1\neq\{n_a\},\{n_b\}} (-\delta\tau_s) \Big]$$

- Write the 1-jettiness like

$$\tau_1 = \tau_0 - \max\Big[\max_{n_1=\{n_a\}} (\delta\tau_a), \max_{n_1=\{n_b\}} (\delta\tau_b), \max_{n_1\neq\{n_a\},\{n_b\}} (\delta\tau_s) \Big]$$

Factorization

- Write the 1-jettiness like

$$\tau_1 = \tau_0 - \max\left[\Delta_a, \Delta_b, \Delta_s\right]$$

Complete separation of contributions from different modes

Factorization

- Write the 1-jettiness like

$$\tau_1 = \tau_0 - \max\left[\Delta_a, \Delta_b, \Delta_s\right]$$

Complete separation of contributions from different modes

- The rest of factorization is similar to pure τ_0 case
- Need new Beam and Soft functions that depend on Δ_i measurements

$$\frac{d^2\sigma}{d\tau_0 d\tau_1} = \sigma_0 \int db_a^+ db_b^+ d\Delta_a d\Delta_b d\Delta_s B_i^{(a)}(b_a^+, \Delta_a, x_a) B_j^{(b)}(b_b^+, \Delta_b, x_b)$$
$$\times S(\tau_0 - b_a^+ - b_b^+, \Delta_s)\, \delta(\tau_1 - \tau_0 + \max(\Delta_i))$$

Features

$$\frac{d^2\sigma}{d\tau_0 d\tau_1} = \sigma_0 \int db_a^+ db_b^+ d\Delta_a d\Delta_b d\Delta_s B_i^{(a)}(b_a^+, \Delta_a, x_a) B_j^{(b)}(b_b^+, \Delta_b, x_b) \times S(\tau_0 - b_a^+ - b_b^+, \Delta_s)\, \delta(\tau_1 - \tau_0 + \max(\Delta_i))$$

- We must recover τ_0 SCET factorization

$$\int d\tau_1 \frac{d^2\sigma}{d\tau_0 d\tau_1} = \frac{d\sigma}{d\tau_0}$$

- Integration over $\delta(\tau_1 - \dots) \to$ integration over Δ_i measurements
- Implies condition on Beam and Soft functions

$$\int d\Delta_a B_i^{(a)}(b_a^+, \Delta_a, x_a) = B_i^{(a)}(b_a^+, x_a)$$

$$\int d\Delta_s S(k_s, \Delta_s) = S(k_s)$$

Features

- At NLO $\mathcal{O}(\alpha_s)$, we know that there is no 1-jettiness contribution

$$\frac{d^2\sigma^{NLO}}{d\tau_0 d\tau_1} = \frac{d\sigma}{d\tau_0}\delta(\tau_1)$$

- The 1-loop Beam and Soft functions are given by

$$\begin{aligned} B_i^{\text{1-loop}}(b_a^+, x_a, \Delta_a) &= B_i^{\text{1-loop}}(b_a^+, x_a)\delta(\Delta_a - b_a) \\ S^{\text{1-loop}}(k_s, \Delta_s) &= S^{\text{1-loop}}(k_s)\delta(\Delta_s - k_s) \end{aligned}$$

- Auxiliary measurement $\delta(\tau_1)$ does not change structure of RGEs $\rightarrow$ resummation up to NLL' is unchanged

Features

- First unkown order is at $\mathcal{O}(\alpha_s^2) \to$ needs 2-loop functions
  Work in progress
- Cross-terms are known

$$\frac{d\sigma}{d\tau_0 d\tau_1} = \sigma_0 \int db_a^+ \, db_b^+ \, dk_s \; B_i^{1-loop}(b_a^+, x_a) B_i^{tree}(b_b^+, x_b) \times S^{1-loop}(k_s)\delta(\tau_0 - b_a^+ - b_b^+ - k_s)\delta(\tau_1 - \min(b_a^+, k_s))$$

$$= \sigma_0 \frac{f_j(x_b)}{x_b E_{CM}^2} \Theta\left(\frac{\tau_0}{2} - \tau_1\right) \times \left[B_i^{1-\mathrm{loop}}(\tau_1, x_a) S^{1-\mathrm{loop}}(\tau_0 - \tau_1) + B_i^{1-\mathrm{loop}}(\tau_0 - \tau_1, x_a) S^{1-\mathrm{loop}}(\tau_1)\right].$$

- We recover the $\tau_1 < \tau_0/2$ limit
  $\to$ intrinsic property of the $\tau_1 \sim \tau_0$ regime

Matching : reminder for large-τ_0

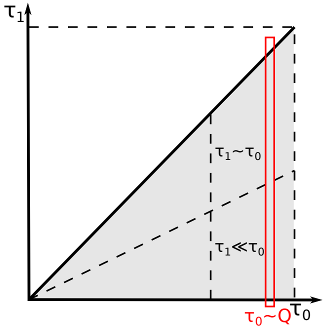
$$\tau_1 \ll \tau_0 \sim Q \rightarrow \tau_1 \sim \tau_0 \sim Q$$

Match τ_1 factorization with fixed-order calculation

- For a given τ_0:

$$d\sigma^{match} = d\sigma^{(1)} + \left(d\sigma^{FO} - \left[d\sigma^{(1)}\right]_{FO}\right)$$

- With $\left[d\sigma^{(1)}\right]_{FO}$ such that
  - $\left[d\sigma^{(1)}\right]_{FO} \rightarrow d\sigma^{(1)}$ when $\tau_1 \sim \tau_0$
  - $\left[d\sigma^{(1)}\right]_{FO} \rightarrow d\sigma^{FO}$ when $\tau_1 \ll \tau_0$
- Set scales to turn off resummation $\mu_{(1)} \rightarrow \mu_{FO}$



Matching with SCET+

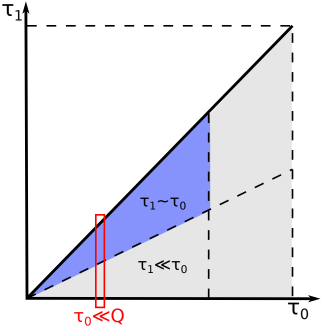
$\tau_1 \ll \tau_0 \ll Q \to \tau_1 \sim \tau_0 \ll Q$

Match our factorization with existing SCET+ factorization

- For a given τ_0:

$$d\sigma^{match} = d\sigma^{+} + \left(d\sigma^{(0,1)} - \left[d\sigma^{+} \right]_{\text{“}FO\text{”}} \right)$$

- “FO” has to include resummation in τ_0
- With $[d\sigma^{+}]_{\text{“}FO\text{”}}$ such that
    - $[d\sigma^{+}]_{\text{“}FO\text{”}} \to d\sigma^{+}$ when $\tau_1 \sim \tau_0$
    - $[d\sigma^{+}]_{\text{“}FO\text{”}} \to d\sigma^{(0,1)}$ when $\tau_1 \ll \tau_0$
- Set scales to “turn off” $\log(\tau_1/\tau_0)$ resummation $\mu_{+} \to \mu_{(0,1)}$

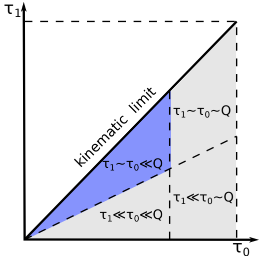


Work in progress

Conclusion

Recap

- Presented the factorization of the double differential $\frac{d\sigma}{d\tau_0 d\tau_1}$ in the region $\tau_0 \sim \tau_1$
- Completes the description of the two resolved emissions phase space
  $\rightarrow$ including **unordered emissions**



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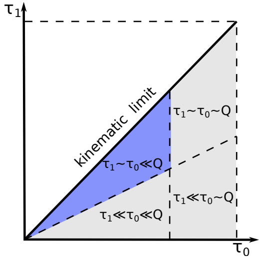

Future

- Fully understand details of matching to SCET+
- Calculate soft $S(k_s, \Delta_s)$ and beam $B(x_i, b_i, \Delta_i)$ functions at $\mathcal{O}(\alpha_s^2)$