Sub-leading power N-jet operator anomalous dimensions

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Outline

- Introduction
- NLP N-jet operators, structure of the ADM
- Collinear anomalous dimension ($|F| = 1, 2, 3$)
- Soft mixing

MB, M. Garny, R. Szafron, J. Wang, 1712.04416, 1712.07462, 1808.04724, and in preparation

Motivations for NLP

- Precision (Loops, Legs, Powers) and new insights into amplitude structure
- Next-to-leading power (NLP, $\tau \to 0$) at NNLO

$$\hat{\sigma}^{\text{NNLO}}(\tau) \overset{\tau\to 0}{\sim} \delta(\tau) + \left[\frac{\ln^{3,2,1,0}}{\tau}\right]_+ + \ln^{3,2,1,0}\tau + \mathcal{O}(\tau)$$

  - Drell-Yan process near threshold
    [Del Duca, Laenen, Magnea, Vernazza, White, 1706.04018; Bonocore, Laenen, Magnea, Vernazza, White, 1706.04018, 1610.06842 and earlier papers]
  - Improving N-jettiness subtraction
    [Moult, Rothen, Stewart, Tackmann, Zhu, 1612.00450, 1710.03227, Ebert, Moult, Stewart, Tackmann, Vita, Zhu 1807.10764; Boughezal, Liu, Petriello, 1612.02911, 1802.00456]

- All-order resummation of NLP logs
  - Thrust distribution in $H \to gg$
    [Moult, Stewart, Vita, Zhu, 1804.04665]
  - Drell-Yan process near threshold
    [MB, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, Wang, 1809.10631]

Factorization at NLP

$$d\sigma = \sum_{a,b} C_a C_b^\star \otimes \prod_{i=1}^{N} J_a^{(i)} J_b^{(i)} \otimes S_{ab}$$

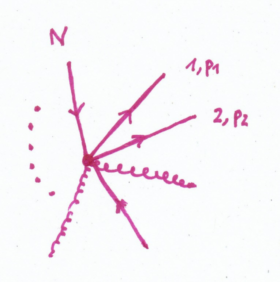
- SCET$_\mathrm{I}$ observables. Hard (Q), collinear ($Q\lambda$) and soft ($Q\lambda^2$) functions.
  (NLP rapidity factorization [Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 1812.08189])
- This talk: Anomalous dimensions of SCET$_\mathrm{I}$ operators $\rightarrow$ evolution of hard functions $C_{a,b}$.
  Mostly relevant from NLP-NLL.
- Factorization formula in dim reg at fixed order, resummation not generally understood.

N-jet amplitudes, leading power

Source of the hard process. N non-collinear directions defined by momenta

$$p_i^\mu = n_{+i} \cdot p_i \, \frac{n_{-i}^\mu}{2} + p_{\perp i} + n_{-i} \cdot p_i \, \frac{n_{+i}^\mu}{2}, \quad p_i^2 = 0, \quad \text{all } p_i \cdot p_j \sim Q^2$$

Log structure determined by IR singularities of the amplitude



N-jet operator in SCET

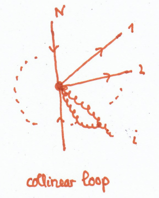
$$\mathcal{O}(x) = \int \prod_{i=1}^{N} dt_i \, C(\{t_i\}) \prod_{i=1}^{N} \psi_i(x + t_i n_{+i})$$

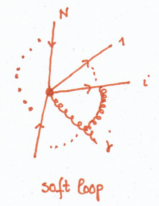
Log structure determined by the UV divergences of collinear and soft loops in SCET [Becher, Neubert, 2009]

$$Z_{\mathcal{O}} \prod_{i=1}^{N} \sqrt{Z_i} \langle 0 | \mathcal{O}(0) | \mathcal{M}(\{p_i\}) \rangle_{|\mathcal{L}^{(0)}_{\text{SCET}}} \stackrel{!}{=} \text{finite}$$

$$\langle 0|\mathcal{O}(0)|\mathcal{M}(\{p_i\})\rangle_{|\mathcal{L}^{(0)}_{\rm SCET}} = S(\{p_i\}) \prod_{i=1}^{N} J_i(p_i^2)$$

$$= 1 - \frac{\alpha_s}{4\pi}\left(\sum_{i,j,i\neq j} \frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\left[\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\frac{\mu^2}{-s_{ij}}\right] - \sum_i \mathbf{T}_i^2\,\frac{c_i}{\epsilon} + \mathcal{O}(\epsilon^0)\right)$$





SCET matrix element is scaleless without IR regulator, since all invariants are hard. Use small off-shellness p_i^2.
Colour conservation $\sum_i \mathbf{T}_i = 0$.

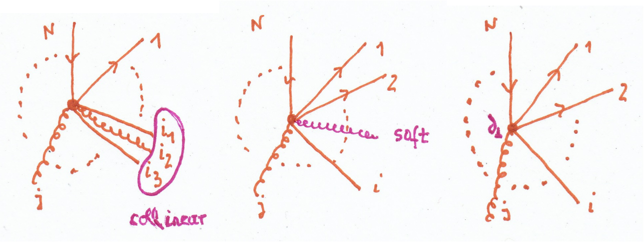
$$J_i(p_i^2) = 1 + \frac{\alpha_s}{4\pi}\,\mathbf{T}_i^2\left[\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\frac{\mu^2}{-p_i^2}+\frac{c_i}{\epsilon}\right]$$

$$S(\{p_i\}) = 1 + \frac{\alpha_s}{4\pi}\sum_{i,j,i\neq j}\frac{\mathbf{T}_i\cdot\mathbf{T}_j}{2}\left[\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\frac{-\mu^2 s_{ij}}{p_i^2 p_j^2}\right]$$

Note cancellation of IR regulator in pole parts. Required by consistency. UV anomalous dimension must not depend on IR reg. UV div non-local for J and S separately.

NLP N-jet operators are the basic objects to match onto for NLP calculations.
If $p_\perp \sim \lambda Q$ and jet mass scale $p_J^2 \sim \lambda^2 Q^2$, need $\mathcal{O}(\lambda^2)$ in SCET expansion.
Consider a SCET_I situation $p_s^2 \ll p_J^2 \ll Q^2$.

- Matrix elements of LP N-jet operators with sub-leading soft and collinear interactions from $\mathcal{L}^{(1)}$, $\mathcal{L}^{(2)}$
- N-jet operators with 1) more than one collinear field of the same type in one direction, or 2) with additional soft fields, or 3) with derivatives.



Building blocks and basis of N-jet operators

Put $x = 0$, i.e. $\mathcal{O}(0)$.
Building blocks

	collinear quark	collinear gluon	soft fields
	$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i$	$\mathcal{A}^\mu_{\perp i}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i]$	$q_s(0), F^s_{\mu\nu}(0)$
	$\mathcal{O}(\lambda)$	$\mathcal{O}(\lambda)$	$\mathcal{O}(\lambda^3, \lambda^4)$

- Collinear gluon operator always transverse.
  $in_{+i} \cdot D_{c_i}$ can be eliminated by Wilson line identities and $in_{-i} \cdot D_{c_i+s}$ by equation of motion, e.g. for $n_{-i} \cdot \mathcal{A}_i$

- Soft covariant derivatives on collinear fields can be eliminated, e.g. $[in_- D_s, \mathcal{A}^\mu_\perp]$
  No soft fields in NLP operators.

- Sub-leading N-jet basis operators are constructed in the following way

  - every element collinear gauge *invariant* and soft gauge *covariant*
  - operate with $i\partial^\mu_{\perp i}$ on collinear building block
  - take products of several collinear building blocks in the same collinear sector, e.g.

$$\chi_i(t_{i1} n_{+i}) \chi_i(t_{i2} n_{+i}) \mathcal{A}^\mu_{c_i \perp}(t_{i3} n_{+i})$$

Notation

General form of the operator

$$\mathcal{O}(0) = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i}\, C(\{t_{ik_i}\}) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, \ldots t_{i_{n_i}})$$

- Notation: J^{An}, J^{Bn}, J^{Cn}, ...
  - – $A, B, C, \ldots$ refers to 1,2,3, ... fields in a given collinear direction
  - – n means $\mathcal{O}(\lambda^n)$ in a given collinear sector relative to A0
- At $\mathcal{O}(\lambda^2)$ up to two $\partial_\perp$ or up to three fields in one sector. Examples:

$$i\partial_{\perp i} i\partial_{\perp i}\chi_i \; (A2), \quad \chi(t_{i_1})\partial_{\perp i}\mathcal{A}_{\perp i}(t_{i_2}) \; (B2), \quad \chi(t_{i_1})\chi(t_{i_2})\chi(t_{i_3}) \; (C2)$$

- A 3-jet operator at $\mathcal{O}(\lambda^2)$ could then be, for example,

$$J^{(A0)}J^{(A0)}J^{(B2)}, \;\; J^{(B1)}J^{(A0)}J^{(A1)}, \ldots$$

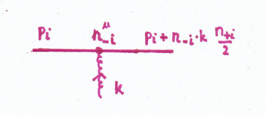
Employ the position-space SCET formalism [MB, Chapovsky, Diehl, Feldmann, 2002]

$$\mathcal{L}^{(0)}_{\text{SCET}} = \sum_{i=1}^{N} \mathcal{L}^{(0)}_{c_i} + \mathcal{L}_{\text{soft}}$$

$$\begin{aligned}\mathcal{L}_c(x) \;&=\; \bar\xi\left(in_-D_c + g_s n_- A_s(x_-) + i\not{D}_{\perp c}\frac{1}{in_+D_c}\,i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}^{(0)}_{c,\text{YM}} \\ &+\; \bar\xi\left(x^\mu_\perp n^\nu_-\, W_c\, g_s F^{s}_{\mu\nu} W^\dagger_c\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}^{(1)}_{\xi q+\text{YM}} + \mathcal{O}(\lambda^2)\end{aligned}$$

$$iD_c = i\partial + g_s A_c, \qquad x^\mu_- = \frac{1}{2} n_+\cdot x\, n^\mu_-$$

- Note multipole expansion of the soft field around x_- in collinear interactions. Guarantees eikonal propagator and soft-gluon decoupling via Wilson line field redefinition $\xi \to Y(x_-)\xi^{(0)}$ [Bauer, Pirjol, Stewart, 2001]
  Drops small momentum components at vertex.



- No purely collinear subleading interactions. At least one soft field in every vertex.

General structure of the ADM (at one-loop)

$$\Gamma_{PQ}(x,y) = \delta_{PQ}\delta(x-y)\left[-\gamma_{\rm cusp}(\alpha_s)\sum_{i<j}\sum_{l,k}\mathbf{T}_{i_k}\cdot\mathbf{T}_{j_l}\,\ln\left(\frac{-s_{ij}x_{i_k}x_{j_l}}{\mu^2}\right)+\sum_i\sum_k\gamma_{i_k}(\alpha_s)\right]$$
$$+2\sum_i\delta^{[i]}(x-y)\gamma^i_{PQ}(x,y)+2\sum_{i<j}\delta(x-y)\gamma^{ij}_{PQ}$$

- Operators $[\mathcal{O}(\lambda^2)]$

$$P = J^{(A0,A2)},\, J^{(A1,A1)},\, J^{(A1,B1)},\, J^{(A0,B2)},\, J^{(A0,C2)},\, J^{(B1,B1)},$$
$$T(J^{(A0,A0)},\mathcal{L}^{(1)},\mathcal{L}^{(1)}),\; T(J^{(A0,A0)},\mathcal{L}^{(2)}),\; T(J^{(A0,A1)},\mathcal{L}^{(1)}),\; T(J^{(A0,B1)},\mathcal{L}^{(1)})$$

- Off-shell IR regulator $p^2_{i_k}$ cancels upon summing soft+collinear
- Operator mixing, collinear anomalous dimension $\gamma^i_{PQ}(x,y)$ a matrix in the spin, colour and momentum labels within a collinear sector.

$$\delta^{(i)}(x_i-y_i)=\prod_{k=1}^{n_i}\delta(x_{ik_i}-y_{ik_i}) \qquad \delta(x-y)=\prod_{i=1}^{N}\delta^{(i)}(x_i-y_i) \qquad \delta^{[i]}(x-y)=\prod_{j=1,j\neq i}^{N}\delta^{(j)}(x_j-y_j)$$

General structure of the ADM (at one-loop)

$$\Gamma_{PQ}(x,y) = \delta_{PQ}\delta(x-y)\left[-\gamma_{\rm cusp}(\alpha_s)\sum_{i<j}\sum_{l,k}\mathbf{T}_{i_k}\cdot\mathbf{T}_{j_l}\,\ln\left(\frac{-s_{ij}x_{i_k}x_{j_l}}{\mu^2}\right)+\sum_i\sum_k\gamma_{i_k}(\alpha_s)\right]$$
$$+2\sum_i\delta^{[i]}(x-y)\gamma^i_{PQ}(x,y)+2\sum_{i<j}\delta(x-y)\gamma^{ij}_{PQ}$$

- Note similarity of $1/\epsilon^2$ and $1/\epsilon\times\ln\dfrac{-s_{ij}x_{i_k}x_{j_l}}{\mu^2}$ to LP. The $\ln\mu^2$ is $\sum_{i<j}(\sum_k\mathbf{T}_{i_k})\cdot(\sum_l\mathbf{T}_{j_l})$ and involves only total colour charge in every collinear sector.
- Collinear contribution depends only on single sectors, but within each sector on x_{i_k}. complicated expressions hidden in collinear term $\gamma^i_{PQ}(x,y)$.
- Soft contributions connect two sectors i,j and have dipole form. Loops with LP soft interaction contribute (only) to the first line. The NLP soft contribution γ^{ij}_{PQ} arises only from mixing of time-ordered products into currents.

$$\gamma^i=\begin{pmatrix}\gamma^i_{PQ} & 0\\ 0 & \gamma^i_{P'Q'}\end{pmatrix},\qquad \gamma^{ij}=\begin{pmatrix}0 & 0\\ \gamma^{ij}_{T(P')Q} & 0\end{pmatrix}.$$

Collinear anomalous dimension

(1) Status: Fermion number $|F| = 1, 2, 3$ of J_i completed. This includes quark (anti-quark) jets ($F = \pm 1$), gluons jets $F = 0$ in progress.

(2) Recall: no subleading-power purely collinear Lagrangian interactions

(3) No mixing between An and Bn operators. "An" anomalous dimension can be expressed in terms of LP A0 anomalous dimension.

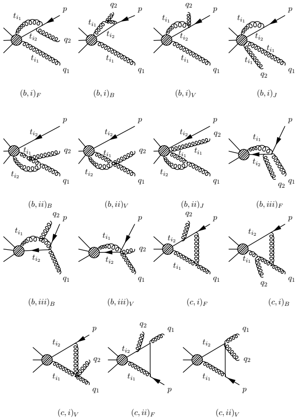
(4) Example: $\mathcal{O}(\lambda^2)$, $F_i = 1$

$\gamma^i_{PQ} =$

	$J^{A2}_{\partial\partial\chi}$	$J^{B2}_{\mathcal{A}\partial\chi}$	$J^{B2}_{\partial(\mathcal{A}\chi)}$	$J^{C2}_{\mathcal{A}\mathcal{A}\chi}$	$J^{C2}_{\chi\bar{\chi}\chi}$
$J^{A2}_{\partial\partial\chi}$	0	0	0	0	0
$J^{B2}_{\mathcal{A}\partial\chi}$	0	(4.7)	(4.8)	(4.22)	(4.30)
$J^{B2}_{\partial(\mathcal{A}\chi)}$	0	0	(4.9)	0	0
$J^{C2}_{\mathcal{A}\mathcal{A}\chi}$	0	0	0	(4.32)	(4.33)
$J^{C2}_{\chi\bar{\chi}\chi}$	0	0	0	(4.35)	(4.34)

(5) Spin-dependent, momentum-fraction-dependent, colour structures, ugly.

$$\gamma^{i}_{\mathcal{A}^{\mu a}\partial^{\nu}\xi,\mathcal{A}^{\sigma d}\mathcal{A}^{\lambda e}\xi}(x,y_1,y_2) = -\frac{\alpha_s}{8\pi}\, I^{\mu\nu\sigma\lambda}_{ade}(x,y_1,y_2)$$



where the kernel $I^{\mu\nu\sigma\lambda}_{ade}(x,y_1,y_2)$ is a sum of terms of the form

$$I^{\mu\nu\sigma\lambda}_{ade}(x,y_1,y_2)|_{(b,iii)_B}$$

$$= \frac{if^{abe}}{2}\bar{x}\left(\theta(x-y_2)\frac{\bar{x}}{\bar{y}_2} + \theta(y_2-x)\frac{x}{y_2}\right)$$

$$\left\{\left(2g_\perp^{\nu\lambda}\gamma_\perp^{\mu}\gamma_\perp^{\sigma} - \frac{2y_2}{x}g_\perp^{\mu\nu}\gamma_\perp^{\lambda}\gamma_\perp^{\sigma} - \frac{1+y_2}{\bar{x}}g_\perp^{\mu\lambda}\gamma_\perp^{\nu}\gamma_\perp^{\sigma}\right)\frac{t^b t^d}{y_1+y_3}\right.$$

$$\left.+\left(2g_\perp^{\nu\lambda}\gamma_\perp^{\sigma}\gamma_\perp^{\mu} - \frac{2y_2}{x}g_\perp^{\mu\nu}\gamma_\perp^{\sigma}\gamma_\perp^{\lambda} - g_\perp^{\mu\lambda}\gamma_\perp^{\sigma}\gamma_\perp^{\nu}\right)\frac{t^d t^b}{y_2+y_3-x}\right\}$$

$$+\,(y_1 d\sigma \leftrightarrow y_2 e\lambda)$$

Three-particle collinear C2 anomalous dimension

$$
\begin{aligned}
\gamma^{i}_{\mathcal{A}^\mu\mathcal{A}^\nu\chi_\alpha,\mathcal{A}^\rho\mathcal{A}^\sigma\chi_\beta}(x_1,x_2,y_1,y_2) \;=\; & \frac{1}{1-y_2}\delta(x_2-y_2)g_\perp^{\nu\sigma}\gamma^{i}_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\rho\chi_\beta}\left(\frac{x_1}{1-x_2},\frac{y_1}{1-y_2}\right) \\
& + \frac{1}{1-y_1}\delta(x_1-y_1)g_\perp^{\mu\rho}\gamma^{i}_{\mathcal{A}^\nu\chi_\alpha,\mathcal{A}^\sigma\chi_\beta}\left(\frac{x_2}{1-x_1},\frac{y_2}{1-y_1}\right) \\
& + \frac{1}{1-y_3}\delta(x_3-y_3)\delta_{\alpha\beta}\gamma^{i}_{\mathcal{A}^\mu\mathcal{A}^\nu,\mathcal{A}^\rho\mathcal{A}^\sigma}\left(\frac{x_1}{1-x_3},\frac{y_1}{1-y_3}\right) \\
& + (y_1,\rho,b_1)\leftrightarrow(y_2,\sigma,b_2)
\end{aligned}
$$

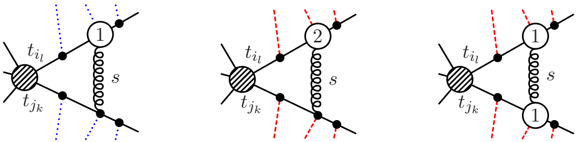
- At one-loop only two of the three lines can be connected.
- Anomalous dimension is a sum of $\mathcal{O}(\lambda)$ B1 anomalous dimensions for all pairs of lines with rescaled momentum fractions, since now $x_1+x_2+x_3=1$, $y_1+y_2+y_3=1$.

Soft time-ordered product mixing

(1) Recall: At one-loop, sub-leading-power Lagrangian interactions necessarily give soft loops, since there are no soft fields in the operators.

(2) The non-cusp contribution to the soft anomalous dimension, γ^{ij}_{PQ} is generated <u>only</u> by sub-leading power Lagrangian insertions.

(3) Soft loops vanish, if gluon is attached to two collinear lines in the same direction.



(4) Find that the single insertions of $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(2)}$ vanish (more on this below), in particular no $\mathcal{O}(\lambda)$ mixing.
Double $\mathcal{L}^{(1)}$ insertion is non-zero.

Soft time-ordered product mixing

The following mixings exist through the double insertion

- (A0,A0) $\to$ (A1,A1)

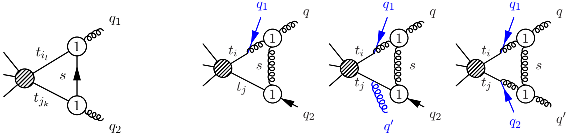
$$\gamma^{ij}_{\left(J^{T1}_{\chi,\xi}\right)_i \left(J^{T1}_{\chi,\xi}\right)_j, \left(J^{A1}_{\partial^\mu \chi}\right)_i \left(J^{A1}_{\partial^\nu \chi}\right)_j} = \frac{2\alpha_s}{\pi}\,\mathbf{T}_i\cdot\mathbf{T}_j \underbrace{\left(g^{\mu\nu} - \frac{n_{i-}^\nu n_{j-}^\mu}{n_{i-}n_{j-}}\right)\frac{1}{(n_{i-}n_{j-})P_iP_j}}_{G^{\mu\nu}_{ij}}$$

- (A0,A0) $\to$ (B1,A1), (A1,B1), (B1,B1). Example:

$$\gamma^{ij\;(y_{i_1},\,y_{j_1})}_{\left(J^{T1}_{\chi_\alpha,\mathrm{YM}}\right)_i \left(J^{T1}_{\chi_\beta,\mathrm{YM}}\right)_j, \left(J^{B1}_{\mathcal{A}_b^\mu \chi_\gamma}\right)_i \left(J^{B1}_{\mathcal{A}_c^\nu \chi_\delta}\right)_j}$$

$$= -\frac{\alpha_s}{2\pi} f^{bda} f^{cea}\,\mathbf{T}_i^d\mathbf{T}_j^e\,G^{ij}_{\lambda\kappa}\left(\frac{2g^{\mu\lambda}_{\perp i}}{y_{i_1}} - \gamma^\lambda_{\perp i}\gamma^\mu_{\perp i}\right)_{\alpha\gamma}\left(\frac{2g^{\nu\kappa}_{\perp j}}{y_{j_1}} - \gamma^\kappa_{\perp j}\gamma^\nu_{\perp j}\right)_{\beta\delta}$$

- Soft quark exchange vanishes for massless quarks.



- Take i, j directions back-to-back, $n^\mu_{j\pm} = n^\mu_{i\mp} \equiv n^\mu_\pm$, allow $p_{\perp i}, p_{\perp j} \neq 0$. One particle in each direction (A0)

$$s_{ij} = 2p_i \cdot p_j = \underbrace{n_+ \cdot p_i n_- \cdot p_j}_{s^{(0)}_{ij} = 2P_i P_j} + 2p_{\perp i} \cdot p_{\perp j} + \mathcal{O}(\lambda^4)$$

$$\Gamma = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\ln \frac{s^{(0)}_{ij}}{\mu^2} + \frac{2p_{\perp i} \cdot p_{\perp j}}{s^{(0)}_{ij}} + \ldots \right) \quad \Rightarrow \quad \gamma^{ij}_{PQ} = -\frac{2\alpha_s}{\pi} \mathbf{T}_i \cdot \mathbf{T}_j \frac{g^{\mu\nu}_\perp}{(n_- n_+) P_i P_j}$$

Same structure as the the time-ordered product mixing into (A1,A1).

- Take i, j directions non-back-to-back, $p_{\perp i} \neq 0$ but $p_{\perp j} = 0$ for simplicity. One particle in each direction (A0)

$$s_{ij} = s^{(0)}_{ij} + n_{j+} \cdot p_j n_{j-} \cdot p_{\perp i} + \mathcal{O}(\lambda^2)$$

$$\Gamma = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \left(\ln \frac{s^{(0)}_{ij}}{\mu^2} + \frac{n_{j+} \cdot p_j n_{j-} \cdot p_{\perp i}}{s^{(0)}_{ij}} + \ldots \right) \quad \Rightarrow \quad \gamma^{ij}_{PQ} = -\frac{\alpha_s}{\pi} \mathbf{T}_i \cdot \mathbf{T}_j \frac{n^\mu_{j-}}{(n_{i-} n_{j-}) P_i}$$

There must be $\mathcal{O}(\lambda)$ mixing into (A1,A0).
In conflict with previous results.

RPI in SCET

- RPI invariant operator is

$$\int dsdt\, \bar{\chi}_j(sn_{j-}) \left[1 + \frac{2t}{n_{i-}n_{j-}}\, n_{j-} \cdot \partial_{\perp i}\right] \chi_i(tn_{i-})$$

  Momentum-space coefficient relation

$$C^{(A1,A0)} = \frac{2}{n_{i-}n_{j-}} \frac{\partial}{\partial n_{i+}p_i} C^{(A0,A0)}$$

- Implies the RGE equation

$$\frac{d}{d\ln\mu} C^{(A1,A0)} = -\left[\gamma_{\rm cusp} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{s_{ij}^{(0)}}{\mu^2} + \text{non-cusp}\right] C^{(A1,A0)} - \gamma_{\rm cusp} \mathbf{T}_i \cdot \mathbf{T}_j \frac{2}{(n_{i-}n_{j-})n_{i+}p_i} C^{(A0,A0)}$$

  The inhomogeneous term arises from $\frac{\partial}{\partial n_{i+}p_i}$ acting on the cusp logarithm.
  It implies $\mathcal{O}(\lambda)$ mixing into (A1,A0).

- The counterterm related to this AD is also required to reproduce the IR poles of the on-shell QCD amplitude.

Soft time-ordered product mixing at $\mathcal{O}(\lambda)$ revisited

- Only possibility is the soft loop with one insertion of $\mathcal{L}^{(1)}$.

$$\mathcal{L}_\xi^{(1)} = \bar{\xi}\left(x_\perp^\mu n_-^\nu\, g_s F^{\rm s}_{\mu\nu}\right)\frac{\not{n}_+}{2}\,\xi$$

$$\tilde{\mathcal{L}}_\xi^{(1)} = \bar{\xi}\left(i\not{D}_{\perp c}\frac{1}{in_+D_c}\,g\not{A}_{\perp \rm us} + g\not{A}_{\perp \rm s}\frac{1}{in_+D_c}\,i\not{D}_{\perp c} + \left[(x_\perp\partial)\,(gn_-A_{\rm s})\right]\right)\frac{\not{n}_+}{2}\,\xi$$

The 2nd Lagrangian arises in the direct expansion of the quark Lagrangian. The 1st form was obtained in [MB, Chapovsky, Diehl, Feldmann, hep-ph/0206152] by the field redefinition

$$\xi' = (1 + g_s x_\perp \cdot A_s)\xi$$

- Alternatively,

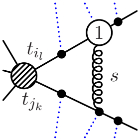
$$\tilde{\mathcal{L}}_\xi^{(1)} = \mathcal{L}_\xi^{(1)} + \Delta\mathcal{L}^{(1)}_{\rm eom}$$

$$\Delta\mathcal{L}^{(1)}_{\rm eom} = \bar{\xi}\,\left[ig_s x_\perp A_{\rm s},\; in_-D + i\not{D}_{\perp c}\frac{1}{in_+D_c}i\not{D}_{\perp c}\right]\frac{\not{n}_+}{2}\,\xi.$$

[To avoid dealing with the YM part of the Lagrangian, we assume abelian gauge fields for simplicity.]

Soft time-ordered product mixing at $\mathcal{O}(\lambda)$ revisited

- Calculate the soft mixing graph with $\tilde{\mathcal{L}}_\xi^{(1)}$



- Relevant integral is (off-shell IR regularization)

$$-i\tilde{\mu}^{2\epsilon}\int\frac{d^dl}{(2\pi)^d}\,\frac{n_{j+}p_j}{l^2(p_i^2-n_{i+}p_in_{i-}l)^2(p_j^2-n_{j+}p_jn_{j-}l)}$$
$$\times\Big(\underbrace{-\left[n_{j-}p_{\perp i}\,n_{+i}p_i\,n_{-i}l-n_{+i}p_i\,n_{i-}n_{j-}\,p_{\perp i}l\right]}_{\text{from }\mathcal{L}_\xi^{(1)}}+\underbrace{n_{j-}p_{\perp i}\,p_i^2}_{\text{from }\Delta\mathcal{L}_{\rm eom}^{(1)}}\Big)$$

$$=\frac{1}{4\pi^2}\,\frac{n_{j-}\,p_{\perp i}}{n_{i-}n_{j-}\,n_{i+}p_i}\left(\frac{\mu^2 s_{ij}^{(0)}}{p_i^2p_j^2}\right)^{\epsilon}\frac{1}{\epsilon}\neq 0$$

- Off-shell term contributes due to p_i^2/p_i^2. UV divergence is non-local. Introduce a counterterm for mixing of eom operator into a "physical operator"

$$T(J^{(A0)},\Delta\mathcal{L}_{\rm eom}^{(1)})\to J^{(A1)}$$

Does off-shell Lagrangian mixing into currents make sense?

(1) Does $\Delta\mathcal{L}^{(1)}_{\text{eom}}$ contribute to on-shell amplitudes?

(2) Violates the [Kluberg-Stern, Zuber, 1975] theorem that eom operators do not mix into "physical operators" (i.e. that don't vanish by eom), i.e. the block-triangular structure of ADM matrix.

(3) Uniqueness. We could use $\mathcal{L}^{(1)}_{\xi} + \text{const.} \times \Delta\mathcal{L}^{(1)}_{\text{eom}}$ and get an arbitrary coefficient for the mixing counterterm

Does off-shell Lagrangian mixing into currents make sense?

(1) Does $\Delta\mathcal{L}^{(1)}_{\text{eom}}$ contribute to on-shell amplitudes? – No.

On-shell soft integrals are scaleless and vanish.
Using LSZ:

$$\lim_{p^2\to 0}(p^2)^{-\epsilon} \times \frac{i}{p^2} \times p^2 = 0$$

Must take $p^2 \to 0$ before $\epsilon \to 0$.

(2) Violates the [Kluberg-Stern, Zuber, 1975] theorem that eom operators do not mix into "physical operators" (i.e. that don't vanish by eom), i.e. the block-triangular structure of ADM matrix.

(3) Uniqueness. We could use $\mathcal{L}^{(1)}_{\xi} + \text{const.} \times \Delta\mathcal{L}^{(1)}_{\text{eom}}$ and get an arbitrary coefficient for the mixing counterterm

Does off-shell Lagrangian mixing into currents make sense?

- Let $F(x)$ be a composite operator, $K(y,x)$ a c-number kernel, and $\partial_S F$ the eom operator

$$\partial_S F = \int d^d y \, \frac{\delta S}{\delta \chi(y)} K(y,z) F(x)$$

[Kluberg-Stern, Zuber, 1975] show for the generating functional of 1PI functions with one insertion of $\partial_S F$ that

$$\Gamma^{(L),\mathrm{div}}_{\partial_S F} = \int d^d y \, \frac{\delta S}{\delta \chi(y)} K(y,z) \Gamma_F(x) \Big|_{(L),\mathrm{div}}$$

- If $\Gamma^{(L),\mathrm{div}}_F$ and $K(x,y)$ are polynomial in momentum space, this implies that that $\Gamma^{(L),\mathrm{div}}_{\partial_S F}$ is of the form $\sum_{F'} Z_{FF'} \partial_S F'$, which proves the theorem.

- But in our case $F = T(J^{(A0)}, g_s A_s(x_{i-}) \chi_i(x))$ and $K(x,y) = x_\perp \delta(x-y)$. Then

$$\Gamma^{(1-loop),\mathrm{div}}_F \propto (p^2)^{-\epsilon}/\epsilon^2 \quad \text{and} \quad K(p) \propto \partial/\partial p_\perp$$

which cancels the p^2 from $\delta S/\delta \chi$. The theorem is violated because the assumption that the divergence is local is violated, which in turn happens due to the $1/\epsilon^2$ pole.

Does off-shell Lagrangian mixing into currents make sense?

(1) Does $\Delta\mathcal{L}_{\rm eom}^{(1)}$ contribute to on-shell amplitudes? – No.

On-shell soft integrals are scaleless and vanish.
Using LSZ:

$$\lim_{p^2\to 0} (p^2)^{-\epsilon} \times \frac{i}{p^2} \times p^2 = 0$$

Must take $p^2 \to 0$ before $\epsilon \to 0$.

(2) Violates the [Kluberg-Stern, Zuber, 1975] theorem that eom operators do not mix into "physical operators" (i.e. that don't vanish by eom), i.e. the block-triangular structure of ADM matrix.

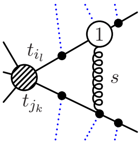
Yes, but the assumptions of the theorem do not hold.

(3) Uniqueness. We could use $\mathcal{L}_\xi^{(1)} + \text{const.} \times \Delta\mathcal{L}_{\rm eom}^{(1)}$ and get an arbitrary coefficient for the mixing counterterm

The SCET Lagrangian is not renormalized (Lorentz invariance! [MB, Chapovsky, Diehl, Feldmann, 2002]).
Coefficient is uniquely fixed by matching to QCD <u>off-shell</u>

Check: extra collinear emission

- Case-I on-shell amplitude
  - $\Delta\mathcal{L}^{(1)}_{\text{eom}}$ does not contribute
  - The counterterm from $T(J^{(A0)}, \Delta\mathcal{L}^{(1)}_{\text{eom}}) \to J^{(A1)}$ mixing is needed to renormalize the amplitude.



  - IR divergences of the QCD amplitude are correctly reproduced and include a purely collinear contribution from the matrix element of a B1 operator. This includes the pole of a divergent convolution.
- Case-II off-shell Green function
  - In the sum of all soft contributions a non-local pole term $\frac{1}{\epsilon} \times \frac{1}{p^2}$ is left-over.
  - This cancels with the collinear contribution from a B1 operators.
  - Similar cancellations in the presence of an extra collinear emission already occur at leading power, but the dependence on p^2 is logarithmic

Summary

I Cusp part of the one-loop ADM for NLP N-jet operators has a simple universal form, which is a straightforward generalization of the LP form

II Collinear anomalous dimensions depend only on single directions, are spin-dependent and algebraically complicated.

Existing results for $|F| = 1, 2, 3$, gluon jet $F = 0$ some time (hopefully) soon

III Soft mixing is subtle and interesting due to a violation of the Kluberg-Stern-Zuber theorem on non-mixing of eom operators into physical operators when the UV divergence is extracted using an off-shell IR regulator.

Soft anomalous dimension in [MB, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04724] receives additional contributions still to be determined from off-shell operators. This is necessary for consistent matching to QCD.

IV Any other non-dim reg IR regulator (requires to determine the UV anomalous dimension) will lead to a complication of some sort for algebraic reasons.

Extra slides

$\mathcal{O}(\lambda^{1,2})$ NLP N-jet operator do not contain soft fields

- Collinear gluon operator always transverse. $in_{+i} \cdot D_{c_i}$ can be eliminated by Wilson line identities and $in_{-i} \cdot D_{c_i+s}$ by equation of motion, e.g.

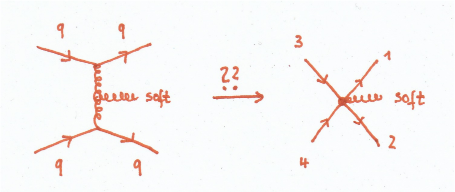
$$
\begin{aligned}
(n_- \mathcal{A})_{ij} \;&=\; -\frac{2}{in_+\partial}(i\partial_{\perp\nu}\mathcal{A}^\nu_\perp)_{ij} - \frac{2}{(in_+\partial)^2}[\mathcal{A}^\nu_\perp, (in_+\partial\mathcal{A}_{\perp\nu})]_{ij} \\
&\quad - \frac{2g_s^2}{(in_+\partial)^2}\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)\overline{\chi}_k \frac{\not{n}_+}{2}\chi_l\,,
\end{aligned}
$$

- Soft covariant derivatives on collinear fields can be eliminated, e.g.

$$
\begin{aligned}
\left([in_- D_s, \mathcal{A}^\mu_\perp]\right)_{ij} \;&=\; \frac{1}{2}i\partial^\mu_\perp(n_-\mathcal{A})_{ij} + \frac{1}{2}\left([\mathcal{A}^\mu_\perp, n_-\mathcal{A}]\right)_{ij} + \frac{1}{2in_+\partial}\left([(in_+\partial\mathcal{A}^\mu_\perp), n_-\mathcal{A}]\right)_{ij} \\
&\quad + \frac{1}{in_+\partial}\left([i\partial^\nu_\perp + \mathcal{A}^\nu_\perp, [i\partial^\mu_\perp + \mathcal{A}^\mu_\perp, i\partial_{\perp\nu} + \mathcal{A}_{\perp\nu}]]\right)_{ij} \\
&\quad + \frac{g_s^2}{2in_+\partial}\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)\Bigg(\bar{\chi}_k\gamma^\mu_\perp \frac{1}{in_+\partial}(\not{\mathcal{A}}_\perp)_{ll'}\frac{\not{n}_+}{2}\chi_{l'} \\
&\quad + \bar{\chi}_{k'}(\not{\mathcal{A}}_\perp)_{k'k}\frac{1}{in_+\partial}\gamma^\mu_\perp\frac{\not{n}_+}{2}\chi_l + 2\bar{\chi}_k\frac{i\partial^\mu_\perp}{in_+\partial}\frac{\not{n}_+}{2}\chi_l\Bigg)
\end{aligned}
$$

$\Rightarrow$ Soft fields can appear only in the form of products of soft building blocks at $x = 0$ and soft derivative on them. Starts at $\mathcal{O}(\lambda^3)$

$\mathcal{O}(\lambda^{1,2})$ NLP N-jet operator do not contain soft fields (2)



Does not generate a soft operator. Graph is reproduced in SCET by time-ordered products $T(J^{(A0)}, \mathcal{L}_\xi^{(2)}), T(J^{(A1)}, \mathcal{L}_\xi^{(1)})$

- Checked that LBK amplitude is reproduced. No need to invoke gauge invariance/Ward identity to fix the local term. Automatic in SCET.
- Previous work on NLP operator bases
    - [MB, Campanario, Mannel, Pecjak, hep-ph/0411395]

      Operator with soft heavy quark fields as source of large energy. Different, there are $\mathcal{O}(\lambda^2)$ operators with soft gluon fields.
    - [Kolodrubetz, Moult, Stewart, 1601.02607; Feige, Kolodrubetz, Moult, Stewart, 1703.03411]

      Helicity basis in label SCET – rather different: subleading purely collinear interactions, two-point insertions, soft building blocks.
      Position space SCET looks simpler in this respect.

SCET Lagrangian, sub-leading power

$$\mathcal{L}^{(1)} = \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, gF^{\rm us}_{\mu\nu} W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi + \bar{q}\,W_c^\dagger\, i\not{D}_{\perp c}\,\xi - \bar{\xi}\, i\overleftarrow{\not{D}}_{\perp c} W_c\, q + \mathcal{L}^{(1)}_{\rm YM}$$

$$\begin{aligned}\mathcal{L}^{(2)} = &\ \frac{1}{2}\,\bar{\xi}\left((n_- x)\, n_+^\mu n_-^\nu\, W_c\, gF^{\rm us}_{\mu\nu} W_c^\dagger + x_\perp^\mu x_{\perp\rho} n_-^\nu\, W_c\, [D^\rho_{\rm us}, gF^{\rm us}_{\mu\nu}]\, W_c^\dagger\right)\frac{\not{n}_+}{2}\,\xi \\ &+ \frac{1}{2}\,\bar{\xi}\left(i\not{D}_{\perp c}\,\frac{1}{in_+D_c}\, x_\perp^\mu \gamma_\perp^\nu\, W_c\, gF^{\rm us}_{\mu\nu} W_c^\dagger + x_\perp^\mu \gamma_\perp^\nu\, W_c\, gF^{\rm us}_{\mu\nu} W_c^\dagger\,\frac{1}{in_+D_c}\, i\not{D}_{\perp c}\right)\frac{\not{n}_+}{2}\,\xi + \mathcal{L}^{(2)}_{\xi q} + \mathcal{L}^{(2)}_{\rm YM}\end{aligned}$$

- Algorithm for construction to higher orders. [MB, Feldmann, 2002]
- Invariance soft and (separate for every direction) collinear gauge transformation

$$\begin{array}{llll}\text{Collinear:} & A_c \to U_c\, A_c\, U_c^\dagger + \frac{i}{g}\, U_c\left[D_{\rm us}(x_-), U_c^\dagger\right], & \xi \to U_c\,\xi, & \\ & A_{\rm us} \to A_{\rm us}, & q \to q, & \\ & & & (1) \\ \text{Soft:} & A_c \to U_{\rm us}(x_-)\, A_c\, U^\dagger_{\rm us}(x_-), & \xi \to U_{\rm us}(x_-)\,\xi, & \\ & A_{\rm us} \to U_{\rm us}\, A_{\rm us}\, U^\dagger_{\rm us} + \frac{i}{g}\, U_{\rm us}\left[\partial, U^\dagger_{\rm us}\right], & q \to U_{\rm us}\, q. & \end{array}$$