Soft gluon evolution beyond leading order

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Motivation

- Higher log resummation only been performed for the narrow class of global observables (e.g. no hard phase-space cuts)
- In non-global observables soft emissions can resolve the direction and color information of energetic particles
- In this talk I will show our recent computation of higher-logarithmic terms for non-global observables

Interjet energy flow

Single log observable: collinear logs cancel out inside the jets

$$\Sigma_\Omega(Q_\Omega, Q) = \frac{1}{\sigma}\int_0^{Q_\Omega} dE_t \, \frac{d\sigma}{dE_t} \qquad (1)$$

$E_t =$ Energy outside jets

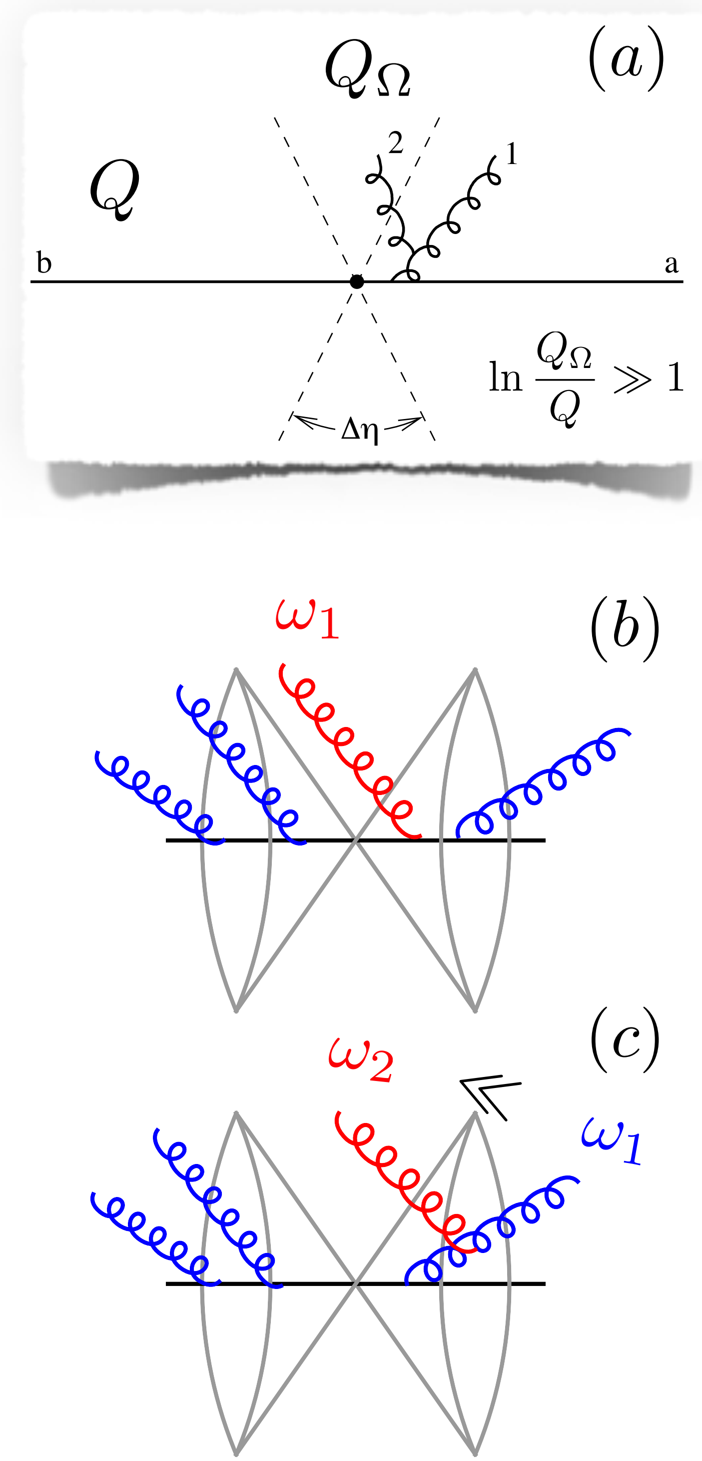
Two sources of single logarithms:

1. From primary emissions

$$\alpha_s \int_{Q_\Omega}^{Q} \frac{d\omega_1}{\omega_1} \sim \alpha_s \ln \frac{Q}{Q_\Omega} \qquad (2)$$

2. From secondary emissions

$$\alpha_s^2 \int_{Q_\Omega}^{Q} \frac{d\omega_2}{\omega_2} \int_{\omega_2}^{Q} \frac{d\omega_1}{\omega_1} \sim \alpha_s^2 \ln^2 \frac{Q}{Q_\Omega} \qquad (3)$$



Soft gluon evolution at LO

- The leading logarithms arise from configuration in which the emitted gluons are strongly ordered

$$E_1 \gg E_2 \gg \cdots \gg E_m$$

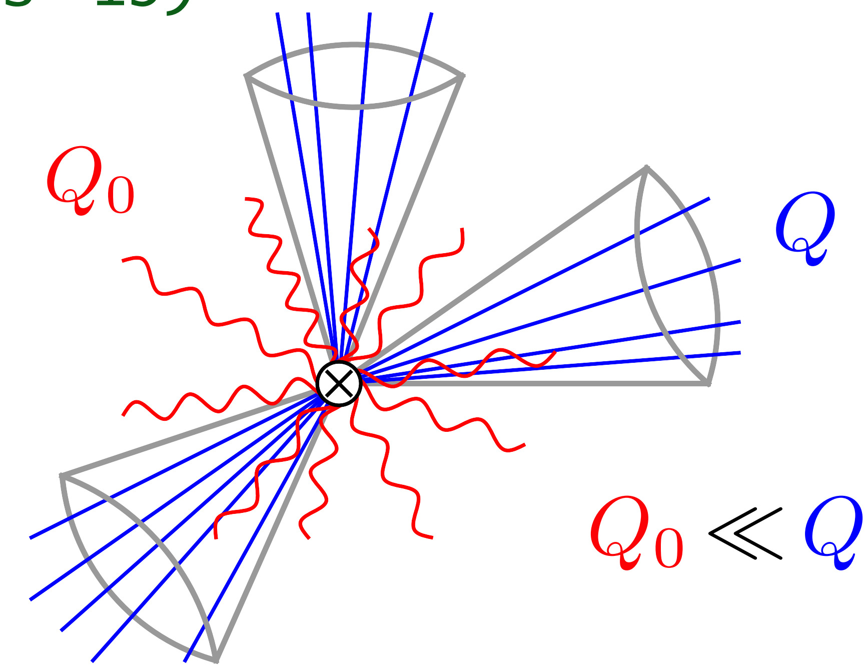
- In the large-Nc limit, multi-gluon emission amplitudes become simple
- Dasgupta-Salam shower (Dasgupta & Salam 2001)
- Banfi-Marchesini-Smye eqation (Banfi, Marchesini & Smye 2002)

$$\partial_{\hat{L}} G_{kl}(\hat{L}) = \int \frac{d\,\Omega(n_j)}{4\pi} W_{kl}^{j} \left[\Theta_{\rm in}^{n\bar{n}}(j)\, G_{kj}(\hat{L})\, G_{jl}(\hat{L}) - G_{kl}(\hat{L})\right]$$

- Dress gluon expansion (Larkoski et.al.'15), finite Nc (Hatta et.al.'15, Martinez et.al.'18), rapidity logs (Becher et.al.'17), double NGLs resummation(Hatta et.al.'18), reduced density matrix (Neill et.al. '18), automation (Balsiger et.al.'18), clustering effects(Neill'18) . . .

Factorization in SCET

(Becher, Neubert, Rothen & DYS '15)



- For k jets process at lepton collider

$$d\sigma(Q, Q_0) = \sum_{m=k}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

$$\{\underline{n}\} = \{n_1, n_2, \cdots, n_m\}$$

- Soft function:

$$\boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) = \sum\!\!\!\!\!\!\!\int_{X_s} \langle 0 | \, \boldsymbol{S}_1^\dagger(n_1) \, \ldots \, \boldsymbol{S}_m^\dagger(n_m) \, | X_s \rangle \langle X_s | \, \boldsymbol{S}_1(n_1) \, \ldots \, \boldsymbol{S}_m(n_m) \, | 0 \rangle \, \theta(Q_0 - E_{\rm out})$$

- Hard function: integrating over the energies of the hard particles, while keeping their direction fixed

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) = \frac{1}{2Q^2} \sum_{\rm spins} \prod_{i=1}^{m} \int \frac{dE_i \, E_i^{d-3}}{(2\pi)^{d-2}} \, |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})| (2\pi)^d \, \delta\Big(Q - \sum_{i=1}^{m} E_i\Big) \, \delta^{(d-1)}(\vec{p}_{\rm tot}) \, \Theta_{\rm in}\big(\{\underline{p}\}\big)$$

- $\otimes$ indicates integration over the direction of the energetic partons

- $\langle \cdots \rangle$ taking the color trace

Resummation in SCET

Evolving hard function from $\mu_h \sim Q$ **to** $\mu_s \sim Q_0$

$$\sigma(Q, Q_0) = \sum_{l=2}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}'\}, Q, \mu_h) \otimes \sum_{m \geq l}^{\infty} \boldsymbol{U}_{lm}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu_s) \big\rangle$$

Resummation in SCET

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$$\boldsymbol{U}(\{\underline{n}\}, \mu_s, \mu_h) = \mathbf{P} \exp \left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^H(\{\underline{n}\}, \mu) \right]$$

Resummation in SCET

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$$\{\underline{n}'\} = \{n_1,\ldots,n_l\}$$

$$\{\underline{n}\} = \{n_1,\ldots,n_l,n_{l+1},\ldots,n_m\}$$

$$\boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h) = \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^H(\{\underline{n}\},\mu)\right]$$

Resummation in SCET

Evolving hard function from $\mu_h \sim Q$ to $\mu_s \sim Q_0$

$$\otimes = \int d\Omega_1 \cdots d\Omega_l$$

$$\hat{\otimes} = \int d\Omega_{l+1} \cdots d\Omega_m$$

$$\sigma(Q, Q_0) = \sum_{l=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}'\}, Q, \mu_h) \otimes \sum_{m \geq l}^{\infty} \boldsymbol{U}_{lm}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu_s) \rangle$$

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Leading Log Resummation

$$\sigma^{\mathrm{LL}}(Q,Q_0) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_2(\{n_1,n_2\},Q,\mu_h) \otimes \boldsymbol{U}_{2m}(\{\underline{n}\},\mu_s,\mu_h) \,\hat{\otimes}\, \mathbf{1} \rangle$$

One-loop anomalous dim. :
$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\boldsymbol{V}_m = 2 \sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_k)}{4\pi} W_{ij}^k,$$

$$\boldsymbol{R}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} W_{ij}^{m+1} \, \Theta_{\mathrm{in}}(n_{m+1}),$$

dipole: $$W_{ij}^k = \frac{n_i \cdot n_j}{(n_i \cdot n_k)(n_j \cdot n_k)}$$

virtual: $$\boldsymbol{V}_m \mathcal{H}_m \sim \sum_a \boldsymbol{T}_i^a \cdot \boldsymbol{T}_j^a |\mathcal{M}_m\rangle \langle \mathcal{M}_m| + |\mathcal{M}_m\rangle \langle \mathcal{M}_m| \sum_a \boldsymbol{T}_i^a \cdot \boldsymbol{T}_j^a$$

real: $$\boldsymbol{R}_m \mathcal{H}_m \sim \boldsymbol{T}_i^a |\mathcal{M}_m\rangle \langle \mathcal{M}_m| \, \boldsymbol{T}_j^a$$

RG equation: $$\frac{d}{dt} \boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t) \boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t) \boldsymbol{R}_{m-1}$$

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)} \frac{\alpha}{4\pi}$$

RG evolution = Parton Shower

$$\mathcal{H}_m(t) = \mathcal{H}_m(t_0)\, e^{(t-t_0)\boldsymbol{V}_m} + \int_{t_0}^{t} dt'\, \mathcal{H}_{m-1}(t')\, \boldsymbol{R}_{m-1}\, e^{(t-t')\boldsymbol{V}_m}$$

What is a shower?

A parton shower consists of three main features:

1. An ordering variable which defines the sequence according to which emissions are generated (such as k_t, angle, virtuality).
2. A branching probability $P(S_n.v)$ of finding a state S_n with n partons at scale v, which evolves as

$$\frac{dP(\mathcal{S}_n, v)}{d\ln 1/v} = -f(\mathcal{S}_n, v)P(\mathcal{S}_n, v)\,.$$

3. A kinematic mapping $\mathcal{M}$ from state $\mathcal{S}_n$ to $\mathcal{S}_{n+1}$

$$\mathcal{S}_{n+1} = \mathcal{M}(\mathcal{S}_n, v; i, j, \underbrace{z, \phi}_{\text{emission}})\,.$$

with an associated "splitting" weight function $d\mathcal{P}(\mathcal{S}_n, v; i, j, z, \phi)$, governing relative probabilities of new states.

"Renormalization Scale"

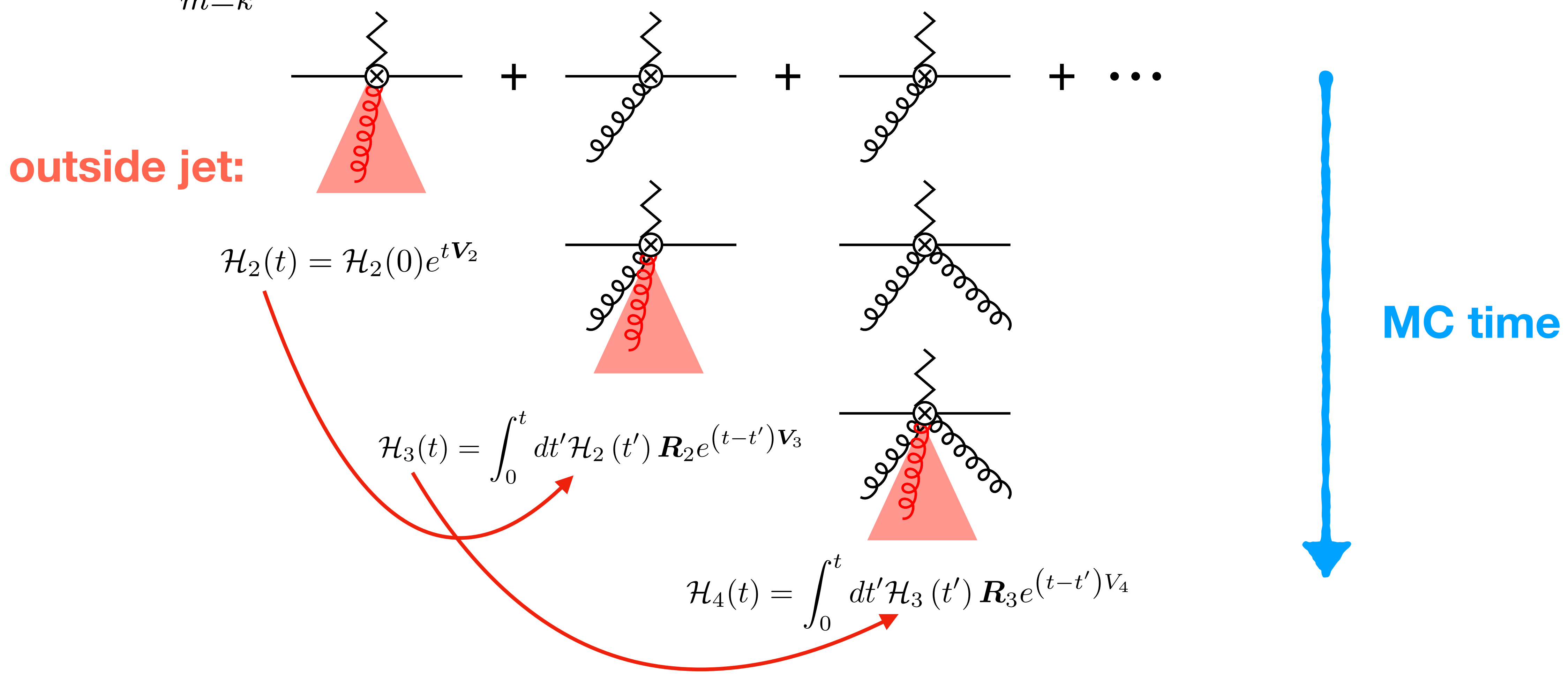
$\boldsymbol{V}_m$

$\boldsymbol{R}_m$

Frédéric Dreyer **Dreyer' talk in QCD@LHC2018** 3/12

Leading Log Shower

$$\sigma_{\rm LL}(Q,Q_0) = \sum_{m=k}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\},Q,\mu_s)\,\hat{\otimes}\,\mathbf{1}\rangle = \langle \mathcal{H}_2(t) + \int \frac{d\Omega_1}{4\pi}\mathcal{H}_3(t) + \int \frac{d\Omega_1}{4\pi}\int \frac{d\Omega_2}{4\pi}\mathcal{H}_4(t) + \ldots\rangle$$



1. start at $t=0$ with initial event $E=\{n_1,n_2\}$ and weight $w=1$
2. increase Δt according to $V_E\exp(-V_E t)$
3. choose a dipole $\{n_i,n_j\}$ with probability V_{ij}/V_E
4. generate a new vector, if it's inside the jet, add it to the event, and return step 2. Otherwise, add the weight factor at time t, go to step 1

From LL to NLL: Sub-leading NGLs

- In order to resum sub-leading NGLs, one needs
  - One-loop soft function $\mathcal{S}_m^{(1)}$
  - One-loop hard function $\mathcal{H}_2^{(1)}$ and tree level hard function $\mathcal{H}_3^{(1)}$
  - Two-loop anomalous dimensions: $\Gamma^{(2)} = \begin{pmatrix} \boldsymbol{v}_2 & \boldsymbol{r}_2 & \boldsymbol{d}_2 & 0 & \dots \\ 0 & \boldsymbol{v}_3 & \boldsymbol{r}_3 & \boldsymbol{d}_2 & \dots \\ 0 & 0 & \boldsymbol{v}_4 & \boldsymbol{r}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{v}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

    See Caron-Huot '15 + Herranen '16
  - Monte-Carlo implementation of all ingredients

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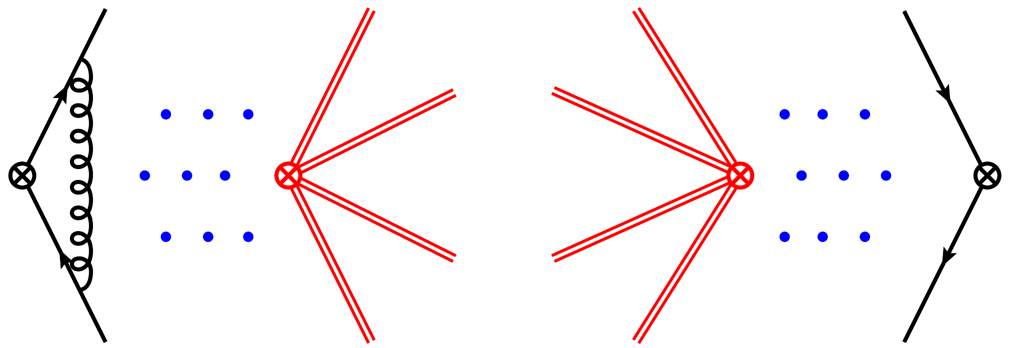
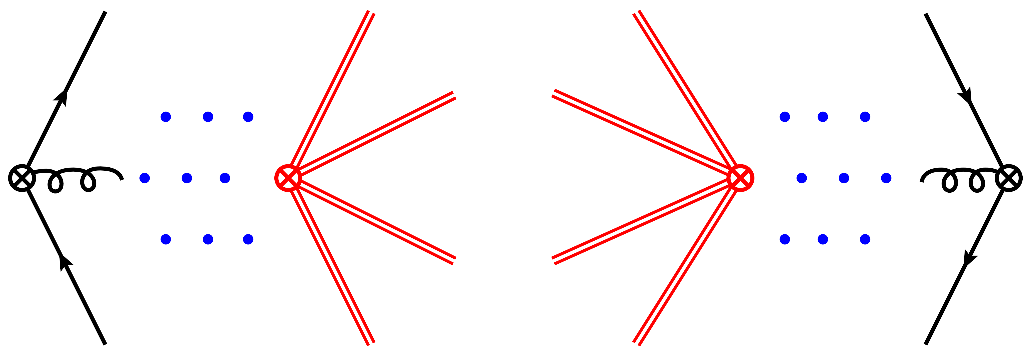
LL′ resummation

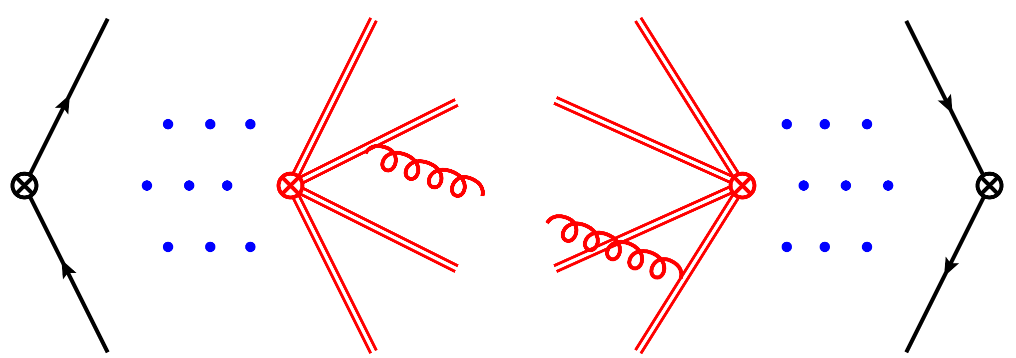
$\mu_h \sim Q \longrightarrow \mu_s \sim Q_0$



$\sim \quad \boldsymbol{\mathcal{H}}_2^{(1)} \otimes \boldsymbol{U}_{2m} \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m^{(0)}$



$\sim \quad \boldsymbol{\mathcal{H}}_3^{(1)} \otimes \boldsymbol{U}_{3m} \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m^{(0)}$



$\sim \quad \boldsymbol{\mathcal{H}}_2^{(0)} \otimes \boldsymbol{U}_{2m} \,\hat{\otimes}\, \boldsymbol{\mathcal{S}}_m^{(1)}$

Soft corrections

One-loop soft function corrections:

$$\sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(t) \hat{\otimes} \boldsymbol{\mathcal{S}}_m^{(1)} \rangle = \langle \boldsymbol{\mathcal{H}}_2(t) \boldsymbol{\mathcal{S}}_2^{(1)} + \int \frac{d\Omega_1}{4\pi} \boldsymbol{\mathcal{H}}_3(t) \boldsymbol{\mathcal{S}}_3^{(1)} + \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \boldsymbol{\mathcal{H}}_4(t) \boldsymbol{\mathcal{S}}_4^{(1)} + \dots \rangle$$

Definition:

$$\frac{\alpha_s}{4\pi} \boldsymbol{\mathcal{S}}_m^{(1)}(\{\underline{n}\}, Q_0, \epsilon) = -g_s^2 \, \tilde{\mu}^{2\epsilon} \sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} \int \frac{d^d k}{(2\pi)^{d-1}} \delta(k^2) \theta(k^0) \frac{n_i \cdot n_j}{n_i \cdot k \, n_j \cdot k} \Theta_{\text{out}}(n_k) \theta(Q_0 - E_k)$$

$$\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} \to -\frac{N_c}{2} \, \delta_{i,j\pm 1}$$

c.m. frame of parent dipole (n_i, n_j)

$$\mathcal{S}_m^{(1)}(\{\underline{n}\}, Q_0, \mu) = \frac{N_c}{2} \sum_{i,j=1}^{m} \delta_{i,j\pm 1} \int d\hat{y} \int_0^{2\pi} \frac{d\hat{\phi}}{2\pi} \left[-4 \ln \frac{\mu}{Q_0} + 4 \ln \frac{2\,|\sin \hat{\phi}|}{f_{ij}(\hat{\phi}, \hat{y})} \right] \Theta_{\text{out}}^{\text{lab}}(\hat{y}, \hat{\phi})$$

weight factor

Soft corrections

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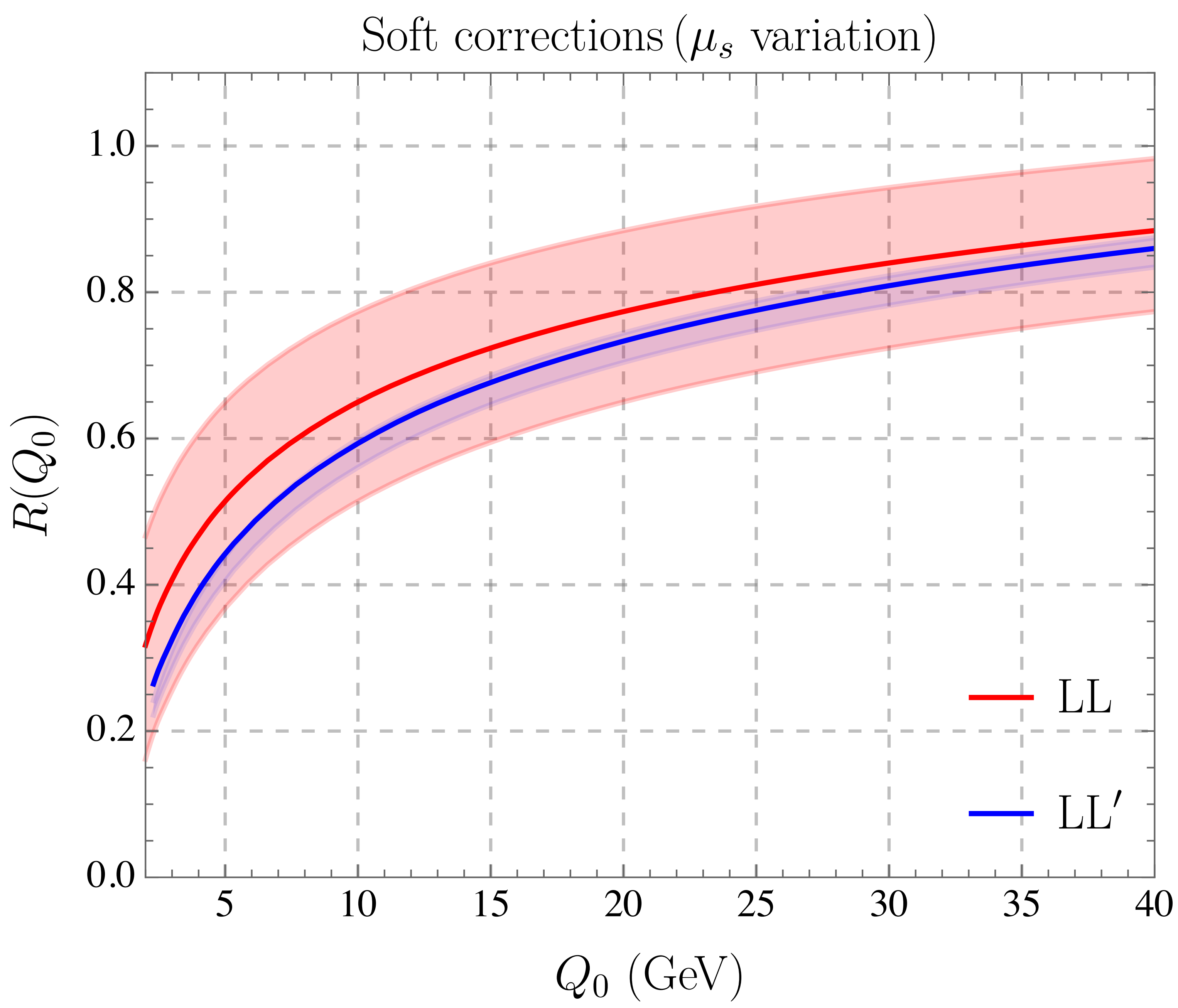
weight factor

Soft corrections

$$R(Q_0) \equiv \int_0^{Q_0} dE_s \, \frac{1}{\sigma_{\rm tot}} \frac{d\sigma}{dE_s}$$

Bands from variation of soft scales by factor two around $\mu_s = Q_0$



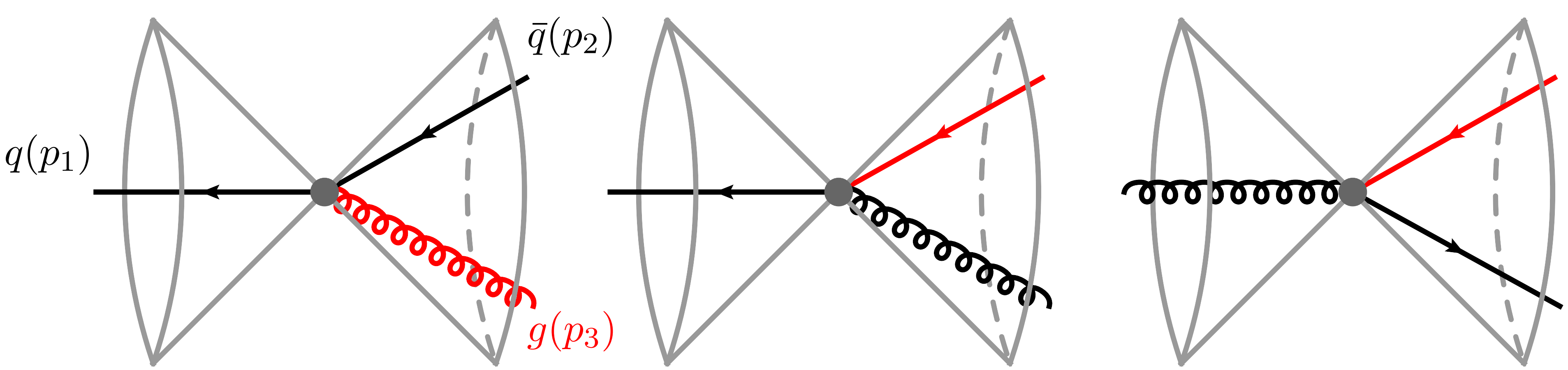
Hard corrections

Virtual corrections to $\mathcal{H}_2$ give trivial refactors

$$\sum_{m=2}^{\infty} \left\langle \boldsymbol{\mathcal{H}}_2(\{n_1,n_2\},Q,\mu_h) \otimes \boldsymbol{U}_{2m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1}\right\rangle = \sigma_0 H_2(Q,\mu_h)\left\langle \boldsymbol{U}_{2m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1}\right\rangle$$

$$H_2(Q^2,\mu) = 1 + \frac{\alpha_s}{4\pi}C_F\left[-8\ln^2\frac{\mu}{Q} - 12\ln\frac{\mu}{Q} - 16 + \frac{7}{3}\pi^2\right]$$

$\mathcal{H}_3$ is a function of two angles



I: $E_1 > E_2 > E_3$ II : $E_1 > E_3 > E_2$ III: $E_3 > E_1 > E_2$

Hard corrections

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$$\sum_{m=2}^{\infty} \left\langle \boldsymbol{\mathcal{H}}_2(\{n_1, n_2\}, Q, \mu_h) \otimes \boldsymbol{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \mathbf{1} \right\rangle = \sigma_0 H_2(Q, \mu_h) \left\langle \boldsymbol{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \mathbf{1} \right\rangle$$

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LL shower

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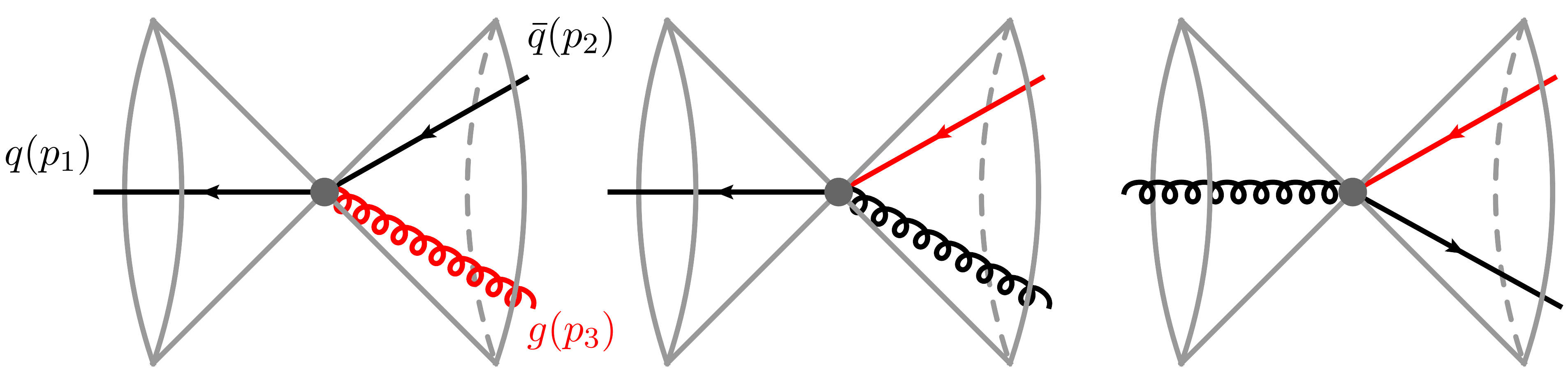

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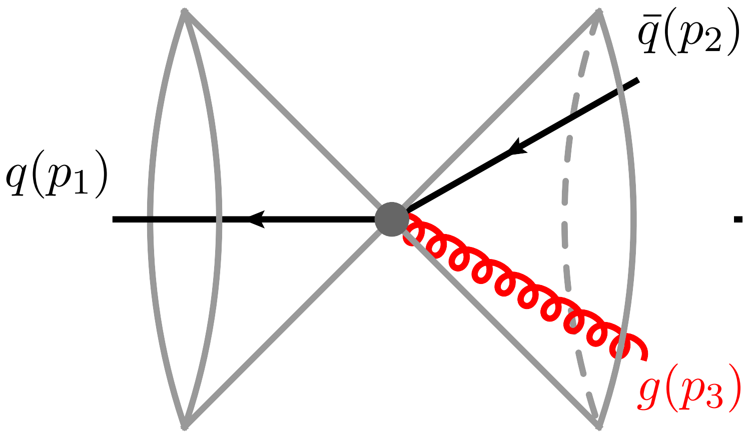
Hard corrections

In Region I, we parameterize



$$\hat{\theta}_2 \equiv \tan\frac{\theta_2}{2} = u\,v, \qquad \hat{\theta}_3 \equiv \tan\frac{\theta_3}{2} = v$$

the angular convolution

$$\left\langle \boldsymbol{\mathcal{H}}_3^{(1)}(\{\underline{n}\}, Q, \mu_h) \otimes \widehat{\boldsymbol{\mathcal{S}}}_3(\{\underline{n}\}, \mu_h) \right\rangle = \int_0^1 du \int_0^1 dv \left\langle \boldsymbol{\mathcal{H}}_3^{(1)}(u, v, Q, \mu_h) \widehat{\boldsymbol{\mathcal{S}}}_3(u, v, \mu_h) \right\rangle$$

LL shower from 3-parton configuration: $\widehat{\boldsymbol{\mathcal{S}}}_3(u, v, \mu_h) = \sum_{m=3}^{\infty} \boldsymbol{U}_{3m}(\{\underline{n}\}, \mu_s, \mu_h) \,\hat{\otimes}\, \mathbf{1}$

Complication: $\mathcal{H}_3^{(1)}(u, v, Q, \mu_h)$ is a distribution

$$\mathcal{H}^{(1)}_{3,\mathrm{I}}(u,v,Q,\mu) = C_F\Bigg\{ \left[4\ln^2\frac{\mu}{Q} - \frac{\pi^2}{6}\right]\delta(u)\delta(v) - 8\ln\frac{\mu}{Q}\delta(u)\left(\frac{1}{v}\right)_+ + 8\,\delta(u)\left(\frac{\ln v}{v}\right)_+ + \left[-\ln\frac{\mu}{Q}F(u,0) + \frac{2u^2}{(1+u)^3} - F(u,0)\ln(1+u)\right]\delta(v)\left(\frac{1}{u}\right)_+ + F(u,0)\delta(v)\left(\frac{\ln u}{u}\right)_+ + F(u,v)\left(\frac{1}{u}\right)_+\left(\frac{1}{v}\right)_+ \Bigg\}\Theta_{\mathrm{in}}(v)$$

Use slicing method:

$$\int_0^1 dv\left[\frac{1}{v}\right]_+ \widehat{\mathcal{S}}_3(v) = \int_0^1 \frac{dv}{v}\left[\widehat{\mathcal{S}}_3(v) - \widehat{\mathcal{S}}_2\right] = \int_{v_0}^1 \frac{dv}{v}\widehat{\mathcal{S}}_3(v) + \ln v_0\,\widehat{\mathcal{S}}_2 + \mathcal{O}(v_0)$$

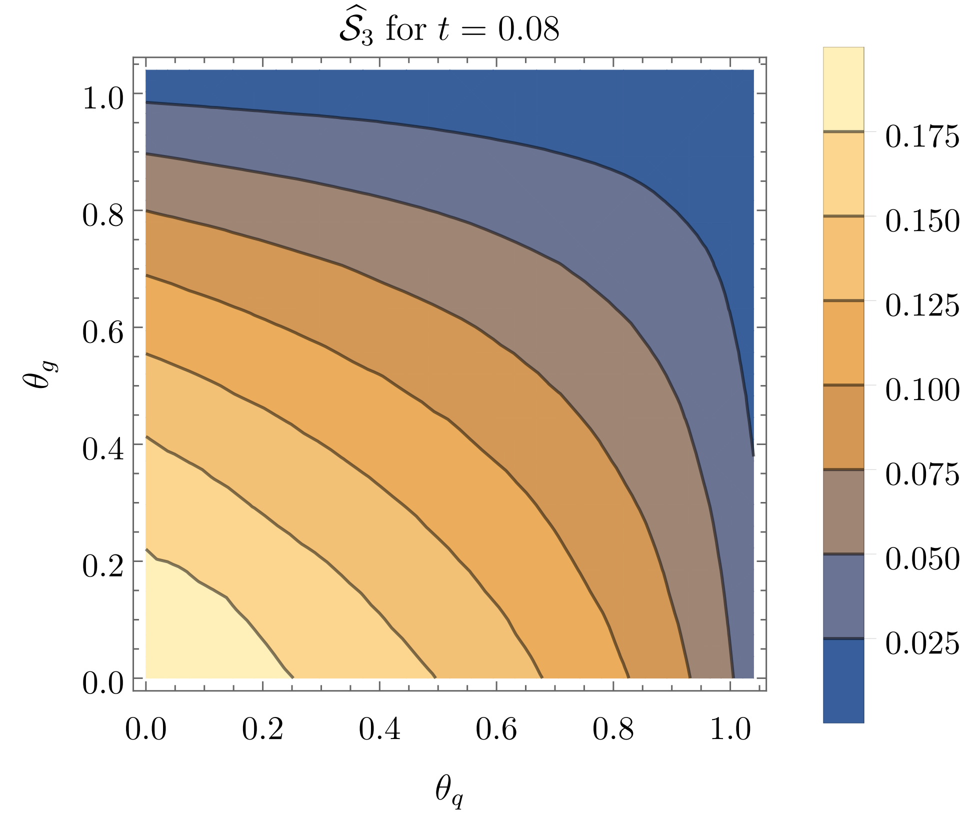
$$\widehat{\mathcal{S}}_3(v=0) = \widehat{\mathcal{S}}_2$$

Works well for the 3-parton configuration.

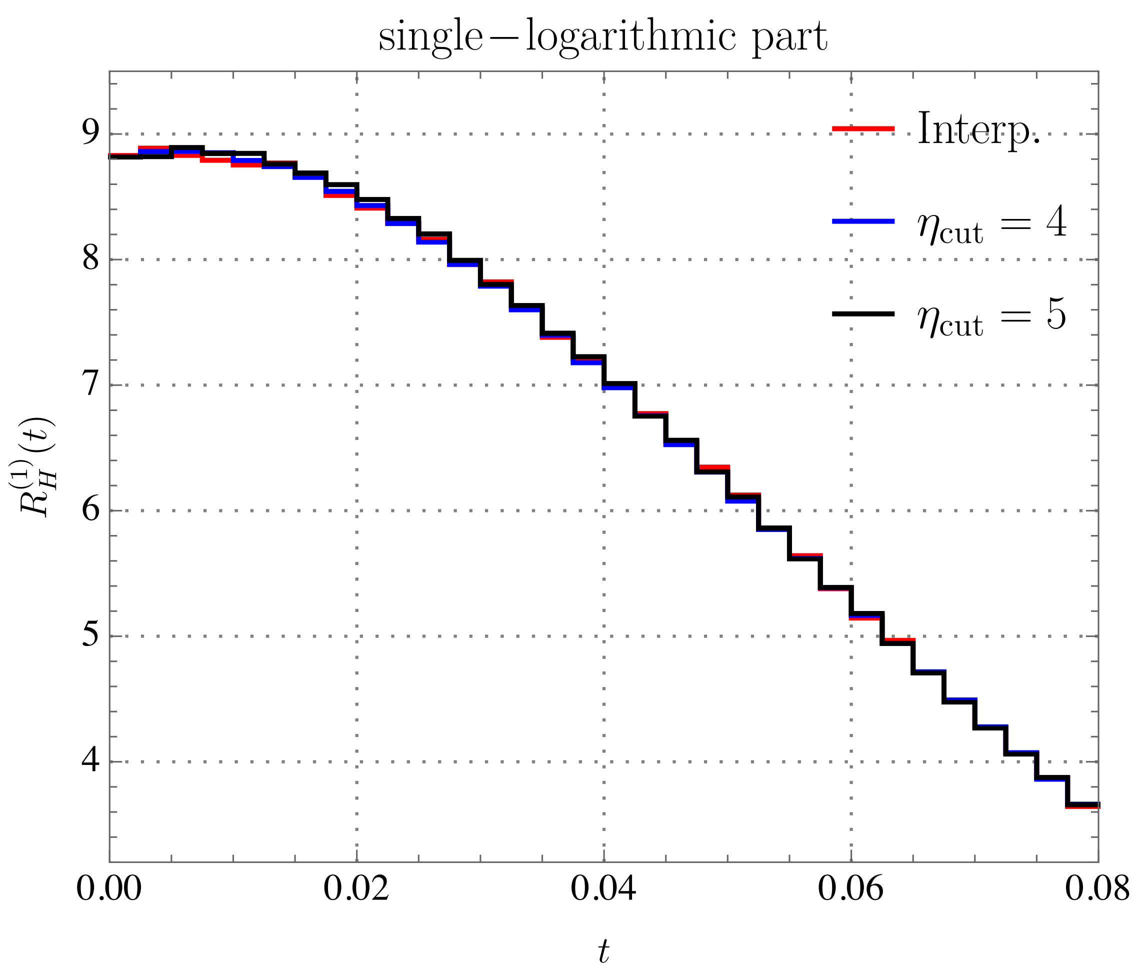
Alternate scheme:

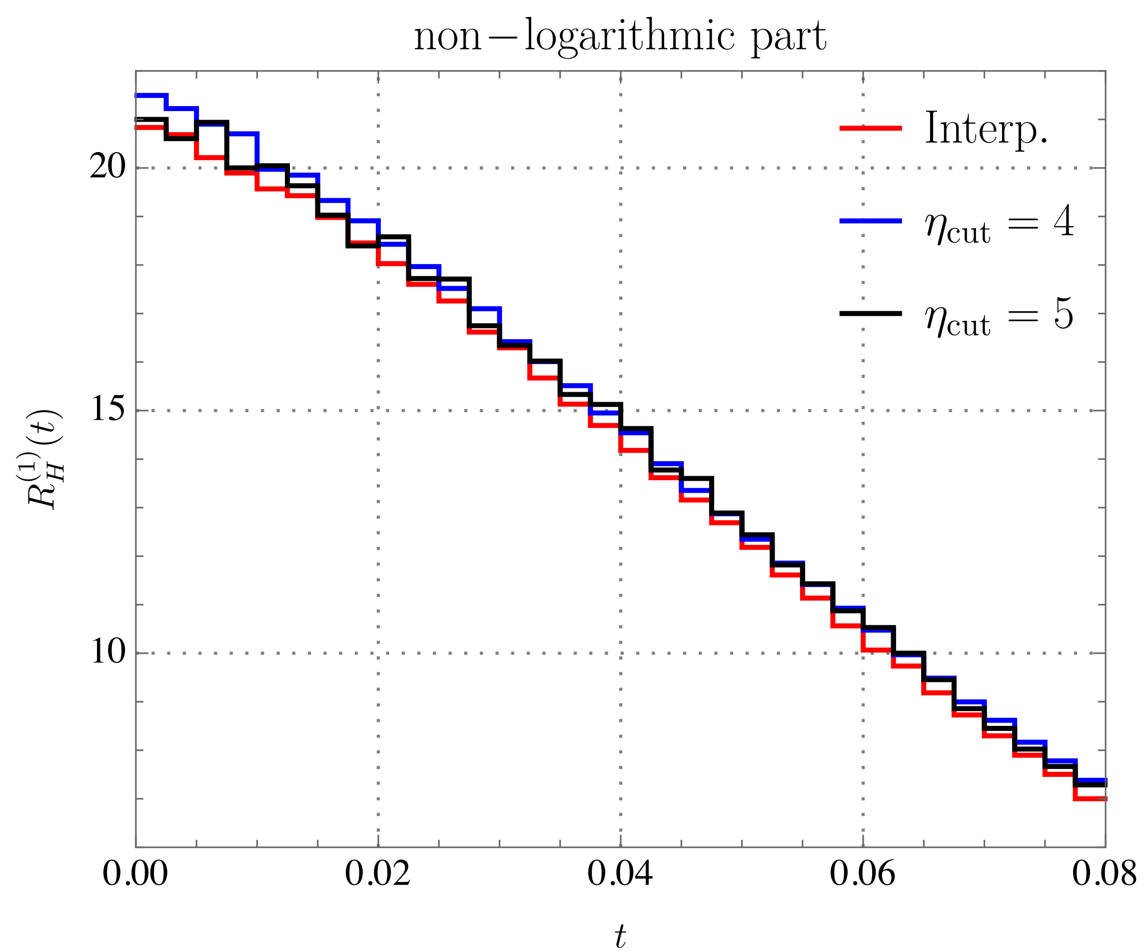
$$\hat{\boldsymbol{\mathcal{S}}}_3(u,v,\mu_h) = \sum_{m=3}^{\infty} \boldsymbol{U}_{3m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1}$$

Smooth function of two angles

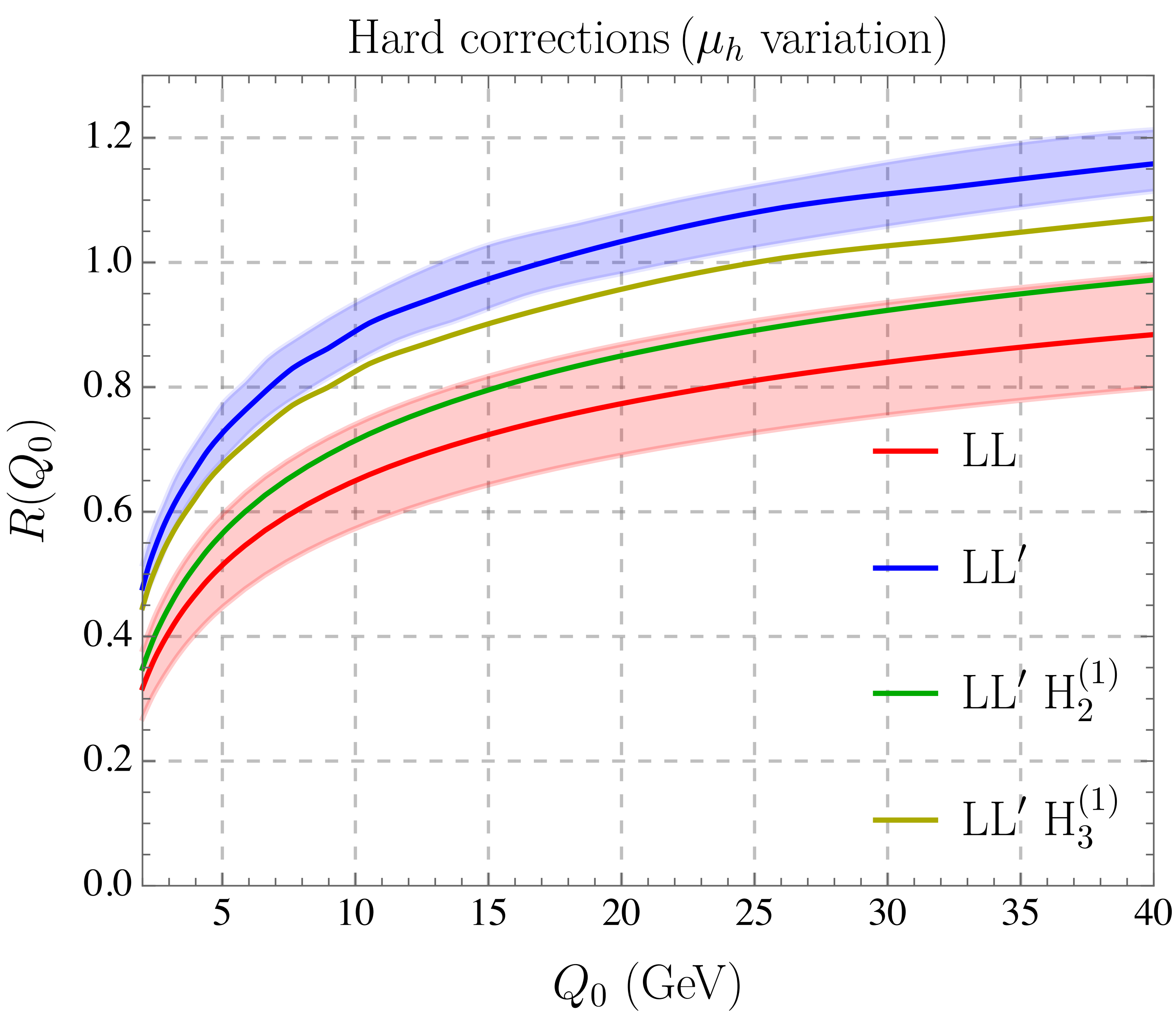


Interpolate $\hat{\mathcal{S}}_3(u,v,\mu_h)$, then perform angular convolution





Hard corrections

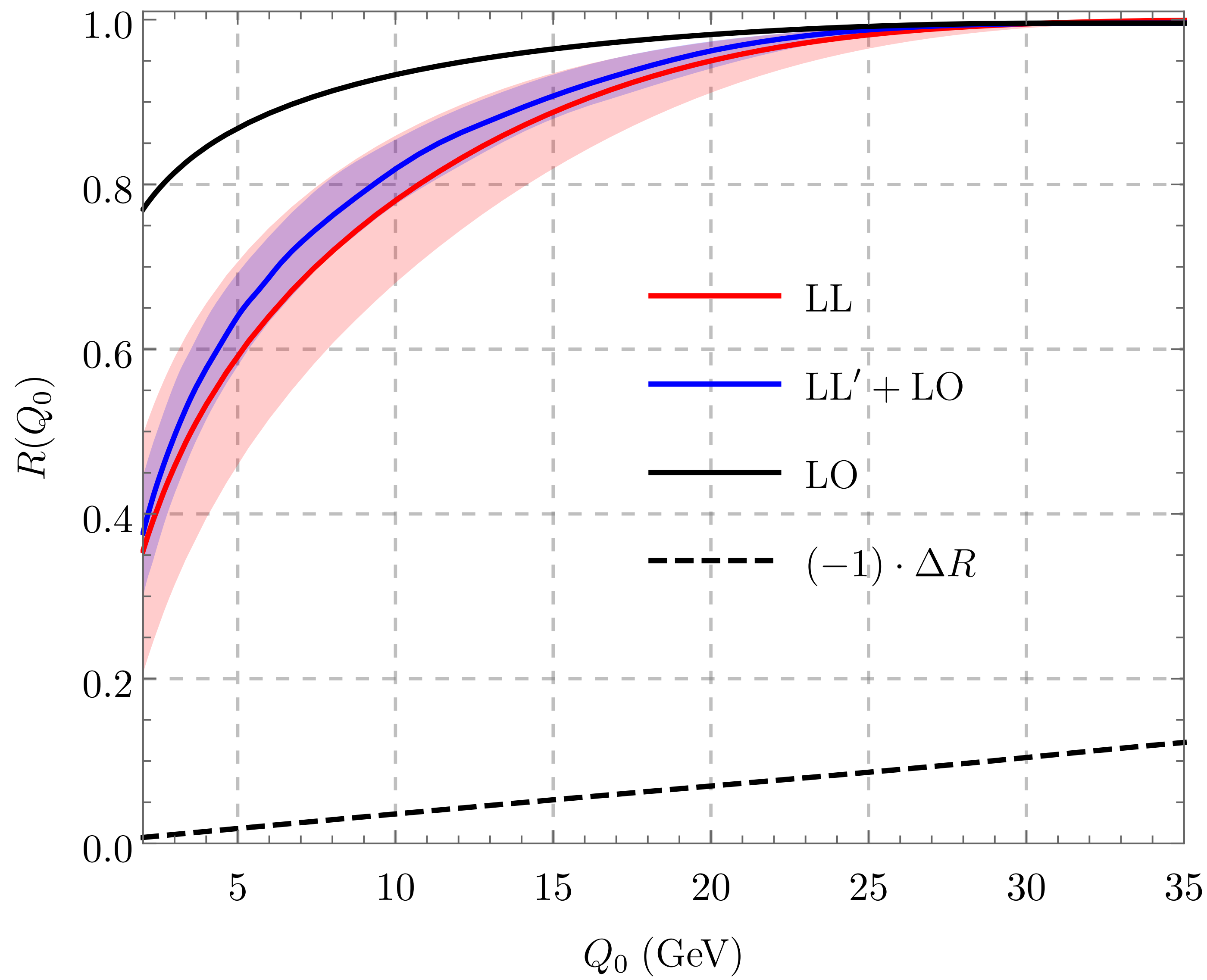


Bands from variation of hard scales by factor two around $\mu_h = Q$

Largest corrections from $\mathcal{H}_3$

Large corrections at large Q_0 must be compensated by power corrections

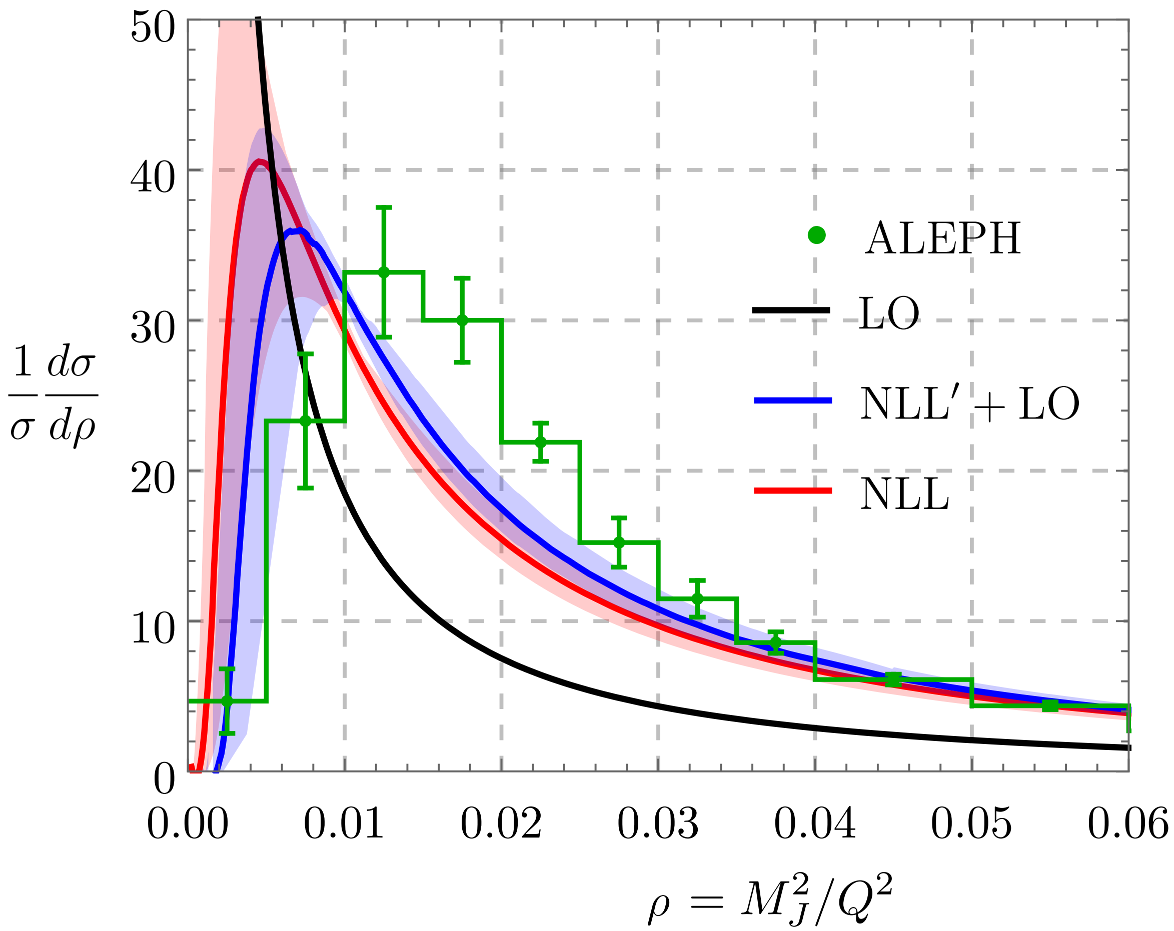
LL′ + LO resummation

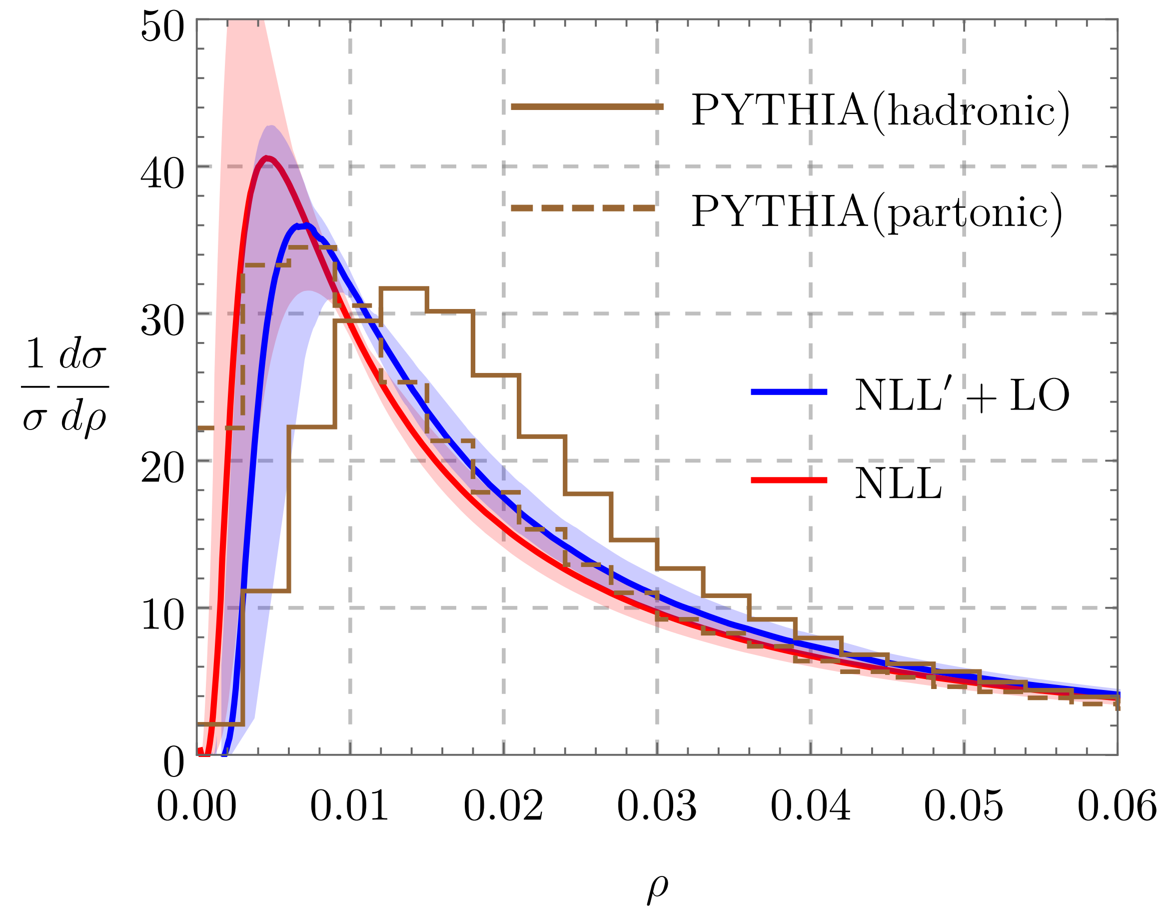


- We match LL′ to LO and use a profile function to switch off resummation
- Hard corrections at large Q_0 get cancelled by matching to LO.
- No Exp. data

NLL′ resummation for jet mass

(Balsiger, Becher, DYS,'19)





- Peak at $\rho \sim 0.006$ corresponds to $\mu_s \sim 0.5\text{GeV}$. Non-perturbative effects are important and shift the peak
- Partonic PYTHIA is close to NLL′+LO

Conclusions and outlook

- First results for non-global observables which go beyond leading logarithms
  - full one-loop corrections to matching coefficients
  - implemented in MC framework
  - high order corrections improve results significantly
- Next steps: NNLL resummation for double log observables
  - Two-loop anomalous dimension & Monte-Carlo implementation
  - More complicated processes at the LHC
  - Automation framework. Interface with NLO generators?

Thank you

Backup

Glauber Phase in V_m

$$\mathrm{Im}\left[\boldsymbol{V}_m\right] = -\,2\pi \sum_{(ij)} \left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)\Pi_{ij}$$

$\Pi_{ij}=1$ if (ij) both incoming or outgoing
$\Pi_{ij}=0$ otherwise

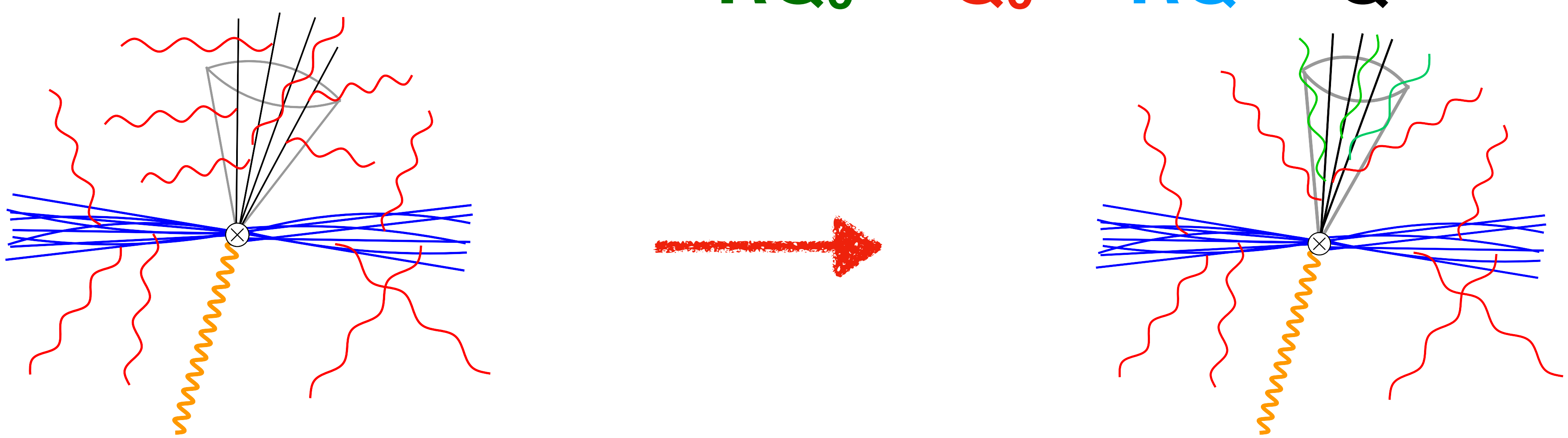
Amplitudes conserve color charge $\sum_i \boldsymbol{T}_i = 0$

- If all particles outgoing $\Pi_{ij} = 1$ and the sum vanishes. No Glauber phases in e^+e^- ep!
- But sum is non-zero for pp $T_1 + T_2 \to T_3 + \ldots + T_m$

Jet radius resummation

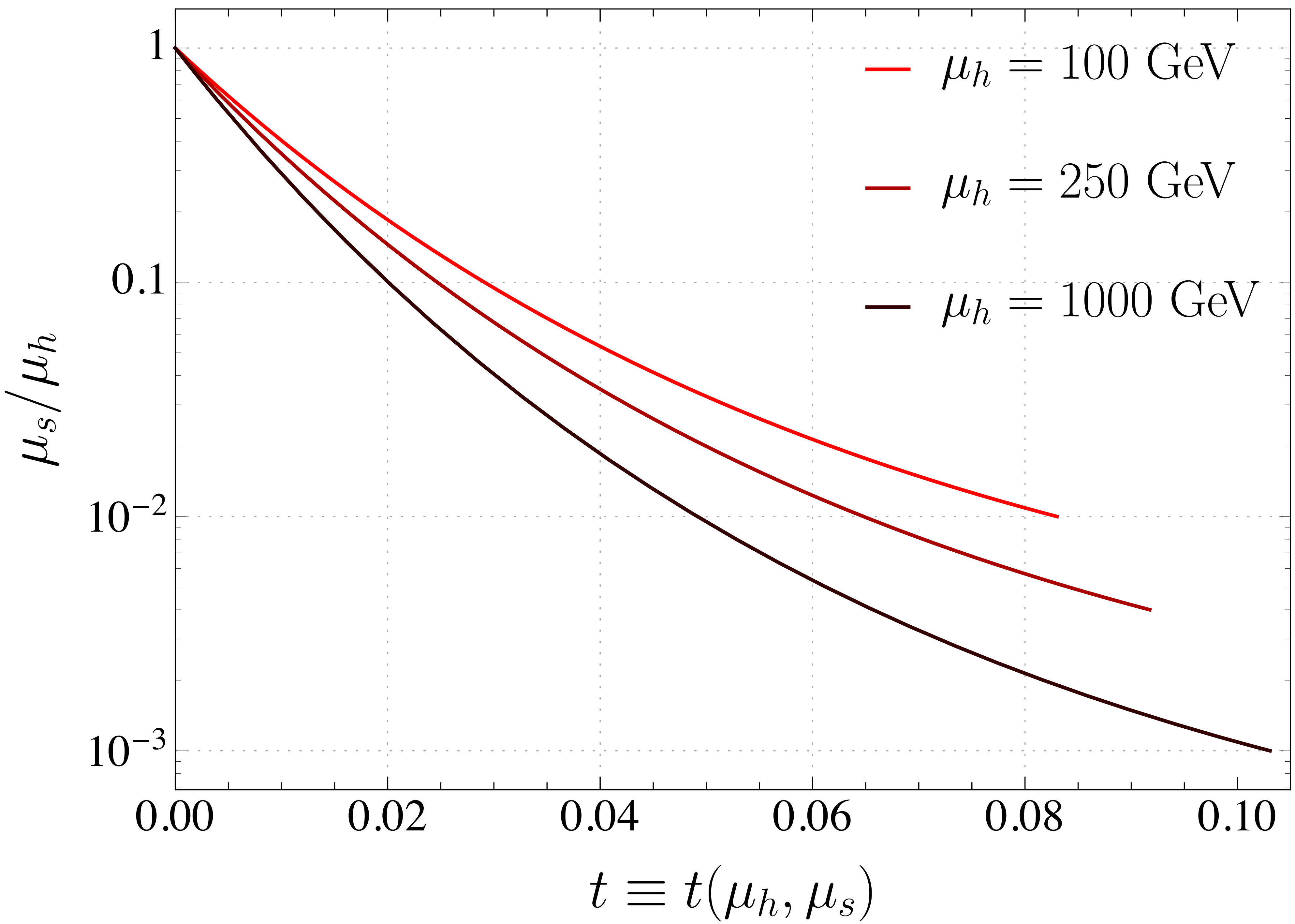
- Exclusive jet cross section:
  - e+e-: (Becher, Neubert, Rothen & DYS `15, Chien, Hornig & Lee `15)
  - pp: threshold resummation (Liu, Moch & Ringer `15); $q_T$ resummation (Buffing, Kang, Lee & Liu `17)

$Q_0 \ll Q$ $\quad\quad$ $RQ_0 \ll Q_0 \ll RQ \ll Q$



$$\mathcal{H}_{m+2}(\{n,\bar{n},\underline{n}\},Q) \to H(Q)\mathcal{J}_m(\{\underline{n}\},RQ)$$

After taking small R limit, small-angle soft emissions can not resolve the color structure inside initial states. Super-leading logs are power suppressed by jet radius R



1
0.1
10^{-2}
10^{-3}
μ_s/μ_h
0.00
0.02
0.04
0.06
0.08
0.10
$t \equiv t(\mu_h, \mu_s)$
$\mu_h = 100$ GeV
$\mu_h = 250$ GeV
$\mu_h = 1000$ GeV