Numerical Noise in Strong-Strong Beam-Beam Simulation

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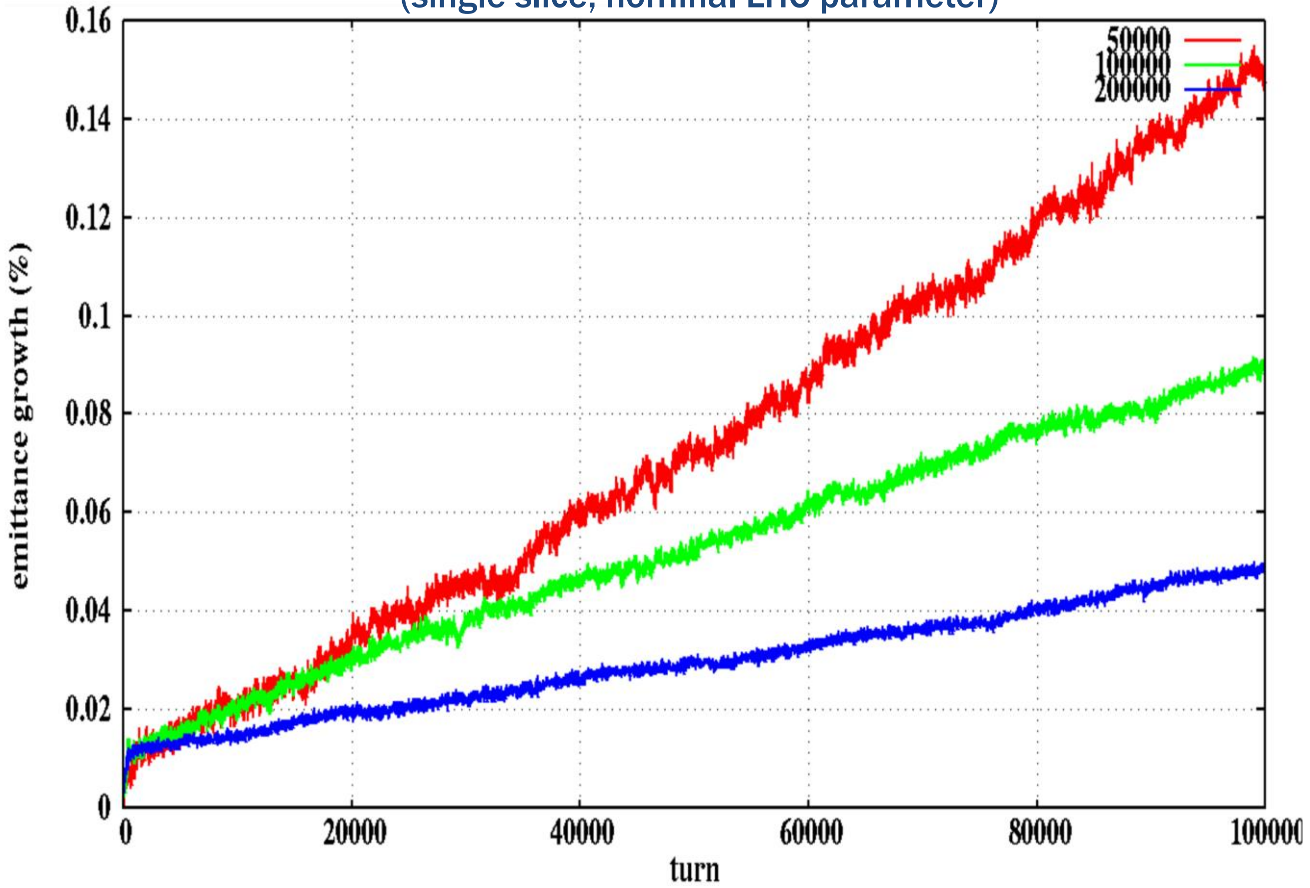
Outline

- Introduction
- Computational model of strong-strong beam-beam simulation
- Characterization of the numerical noise induced emittance growth
- A new method to mitigate the numerical noise
- Conclusions and future work

Artificial Numerical Emittance Growth Observed in Strong-Strong Beam-Beam Simulation

Emittance growth evolution with three numbers of macroparticles

(single slice, nominal LHC parameter)



Mathematical Model of Strong-Strong Beam-Beam Simulation

- Coupled Poisson-Vlasov Equations

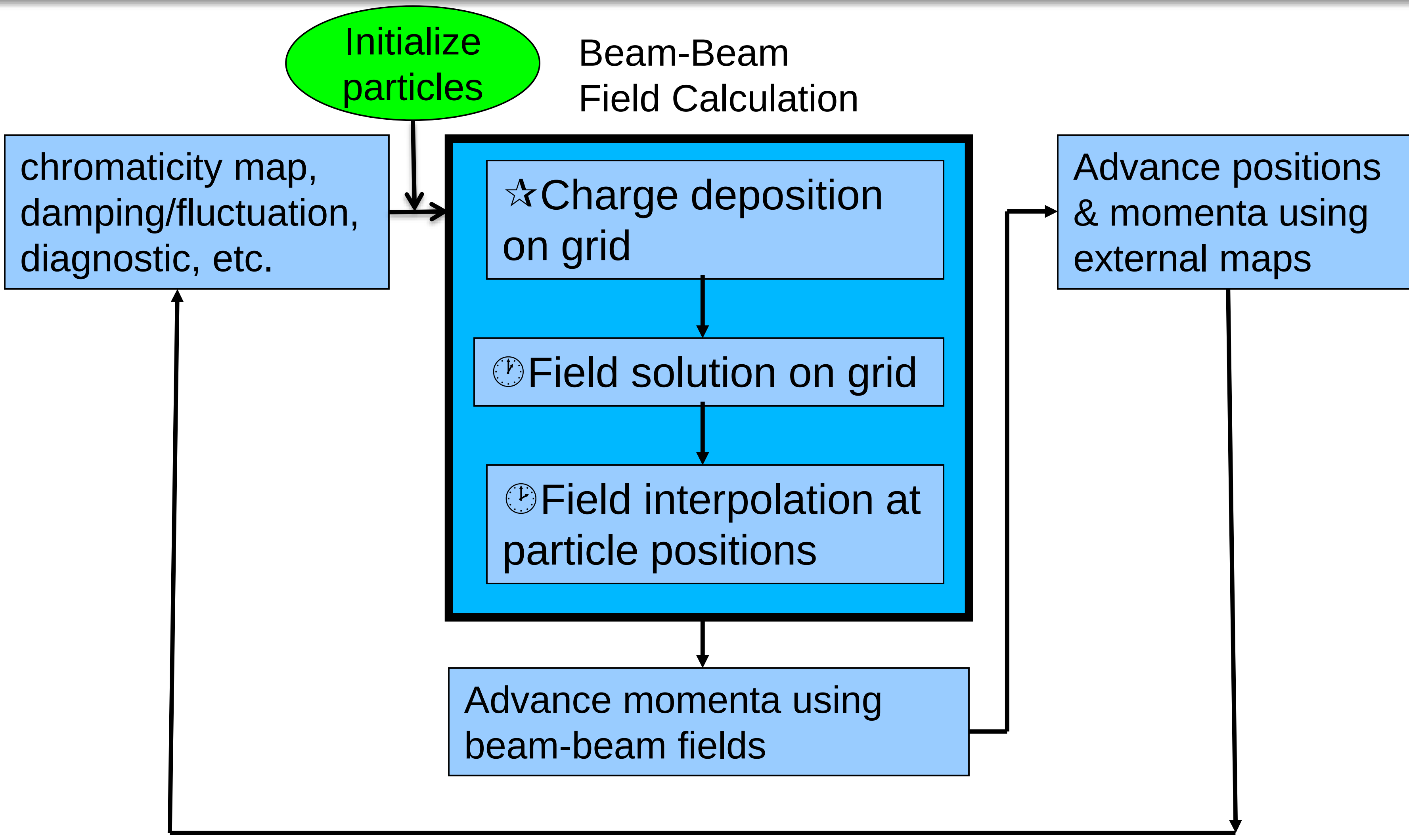
$$L{:}f_1(\mathbf{r},\mathbf{p}) = 0 \qquad L{:}f_2(\mathbf{r},\mathbf{p}) = 0$$

$$\nabla^2 \phi_1 = - \iiint f_2(r,p) d^3 p$$

$$\nabla^2 \phi_2 = - \iiint f_1(r,p) d^3 p$$

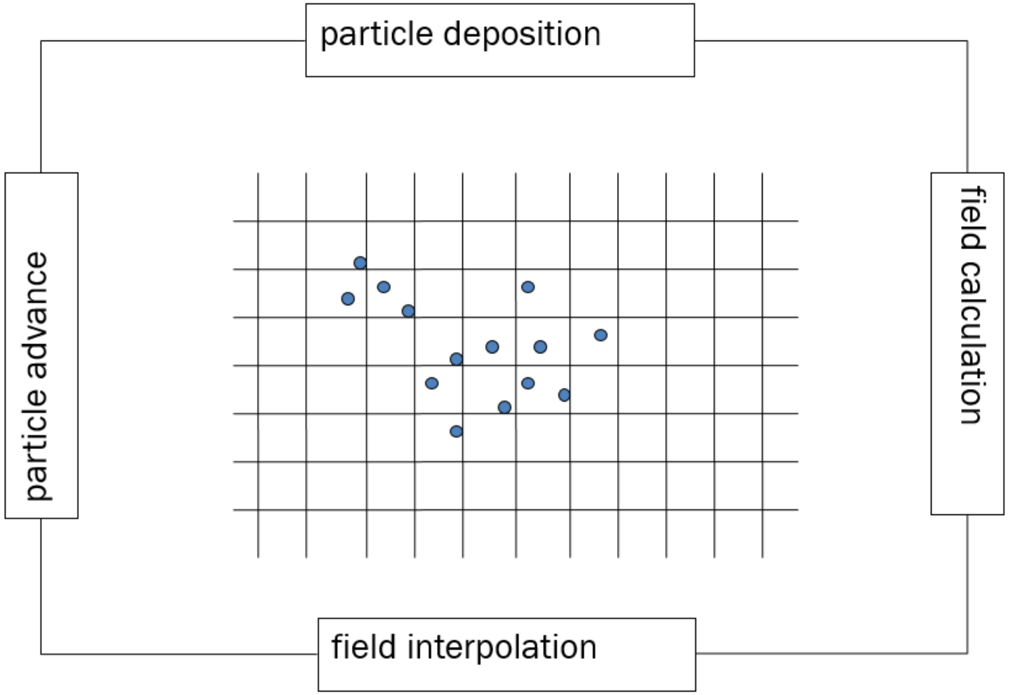
$$L = \frac{\partial}{\partial t} + \dot{r} \frac{\partial}{\partial r} + \dot{p} \frac{\partial}{\partial p}$$

Particle-In-Cell Based Numerical Beam-Beam Model



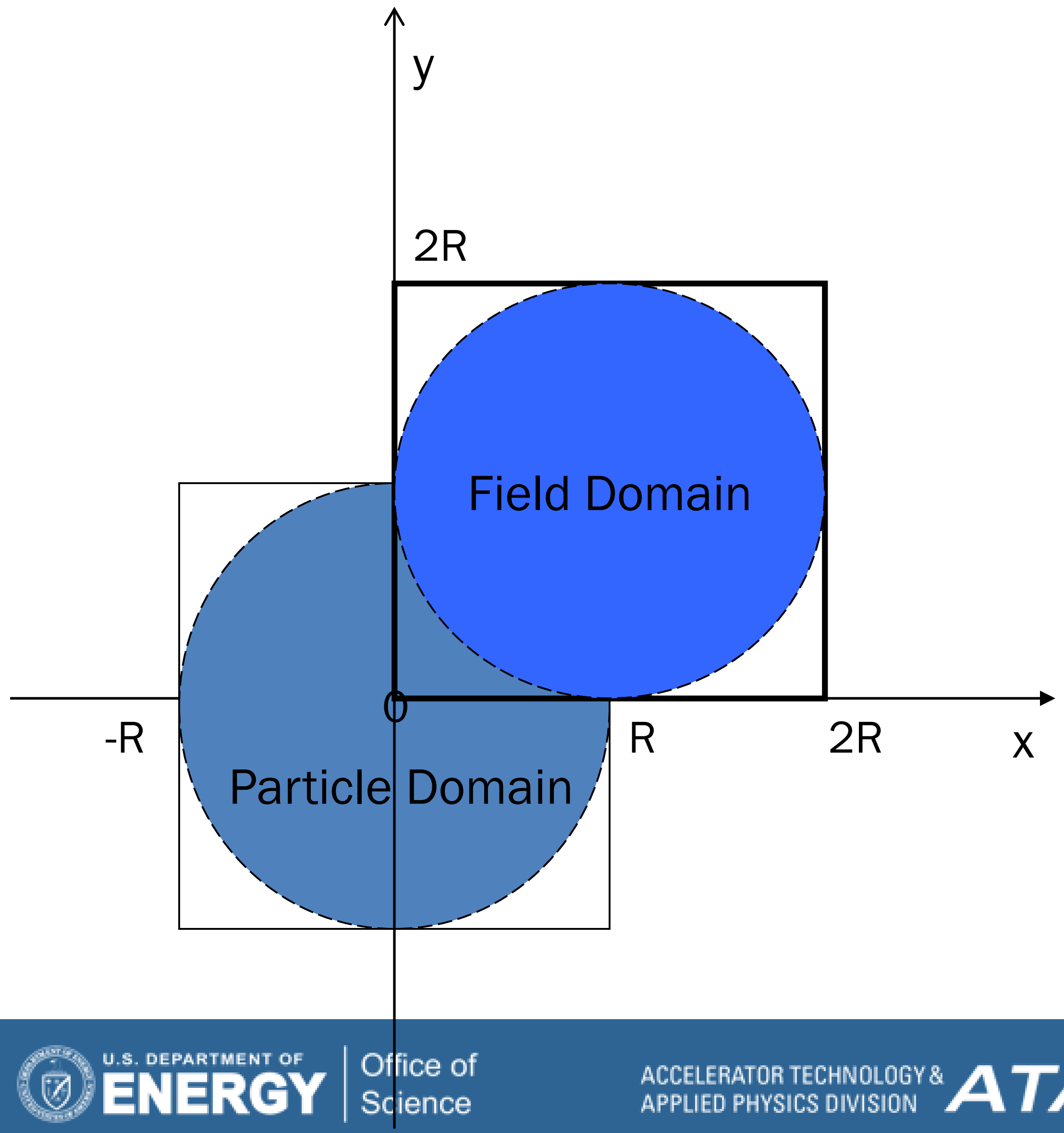
One Step PIC Model for Beam-Beam Force

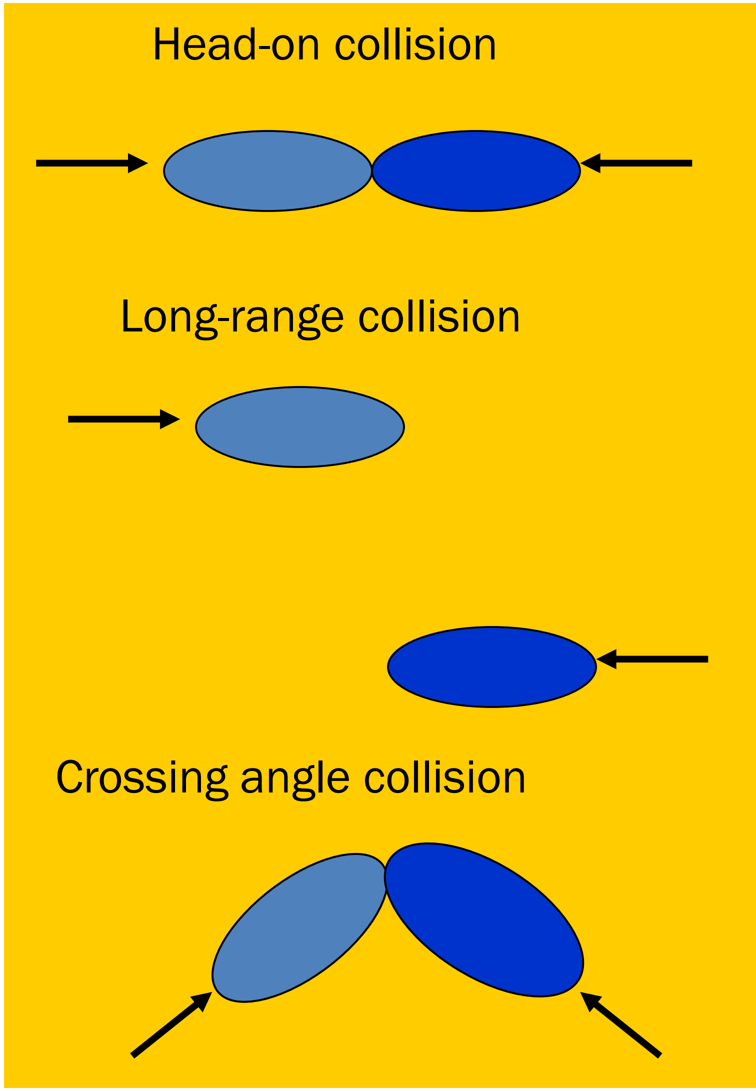
$$\rho_p = \sum_{i=1}^{n} q_i w(x_i - x_p)$$



$$E_i = \sum_{i=1}^{M} E_p w(x_i - x_p)$$

Two Beams Might Collide with Different Geometries





Efficient Green's Function Method to Solve the Poisson Equation for Beam-Beam Force Calculation (1)

$$\phi(r) = \int G(r, r')\rho(r')dr'$$

$$\phi(r_i) = h\sum_{i'=1}^{N} G(r_i - r_{i'})\rho(r_{i'})$$

$$G(x, y) = -\frac{1}{2}\log(x^2 + y^2)$$

Direct summation of the convolution scales as N^4 !!!!

N – grid number in each dimension

Efficient Green's Function Method to Solve the Poisson Equation for Beam-Beam Force Calculation (2)

Hockney's Algorithm:- scales as $(2N)^2\log(2N)$

- Ref: Hockney and Easwood, ***Computer Simulation using Particles***, McGraw-Hill Book Company, New York, 1985.

$$\phi_c(r_i) = h\sum_{i'=1}^{2N} G_c(r_i - r_{i'})\rho_c(r_{i'})$$

$$\phi(r_i) = \phi_c(r_i) \quad \text{for i} = 1, N$$

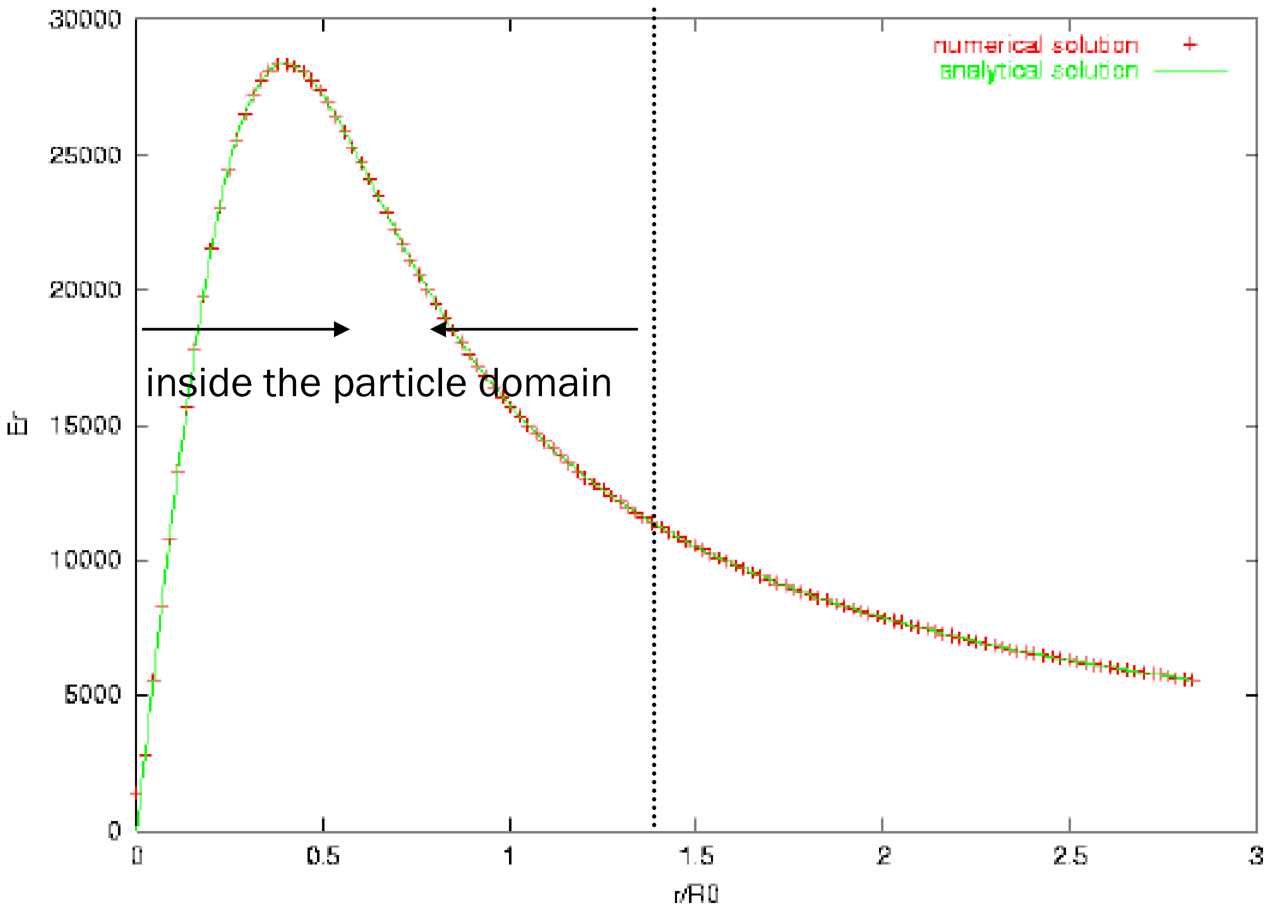
Shifted Green function Algorithm:

$$\phi_F(r) = \int G_s(r,r')\rho(r')dr'$$

$$G_s(r,r') = G(r + r_s, r')$$

Good Agreement between the Numerical Solution from the Shifted Green Function and the Analytical Solution

E_x



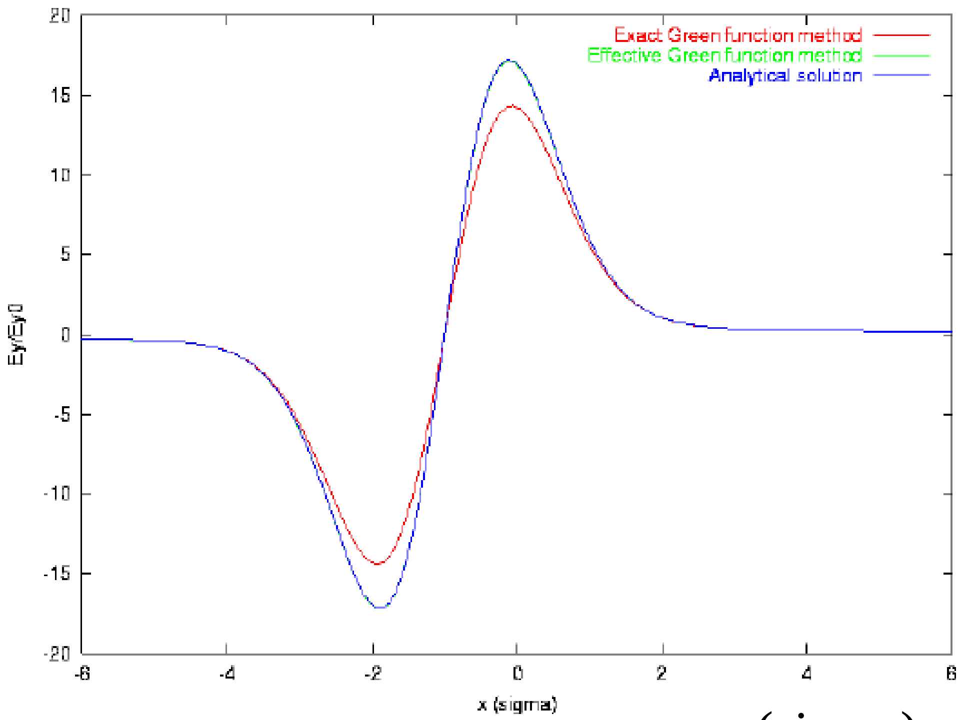
radius

Efficient Green's Function Method to Solve the Poisson Equation for Large Aspect Ratio Beam (3)

$$\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'})\rho_c(r_{i'})$$

$$\boxed{G_i(r,r') = \oint G_s(r,r')\,dr'}$$

E_y

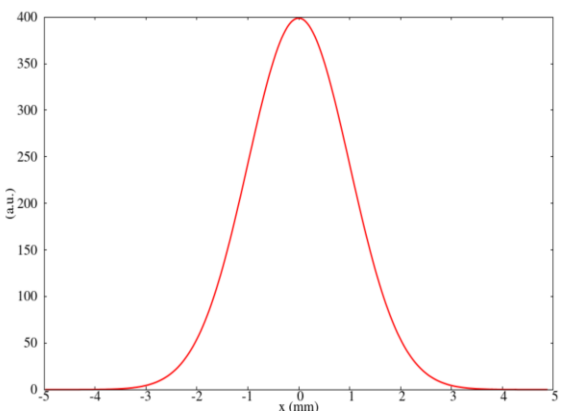


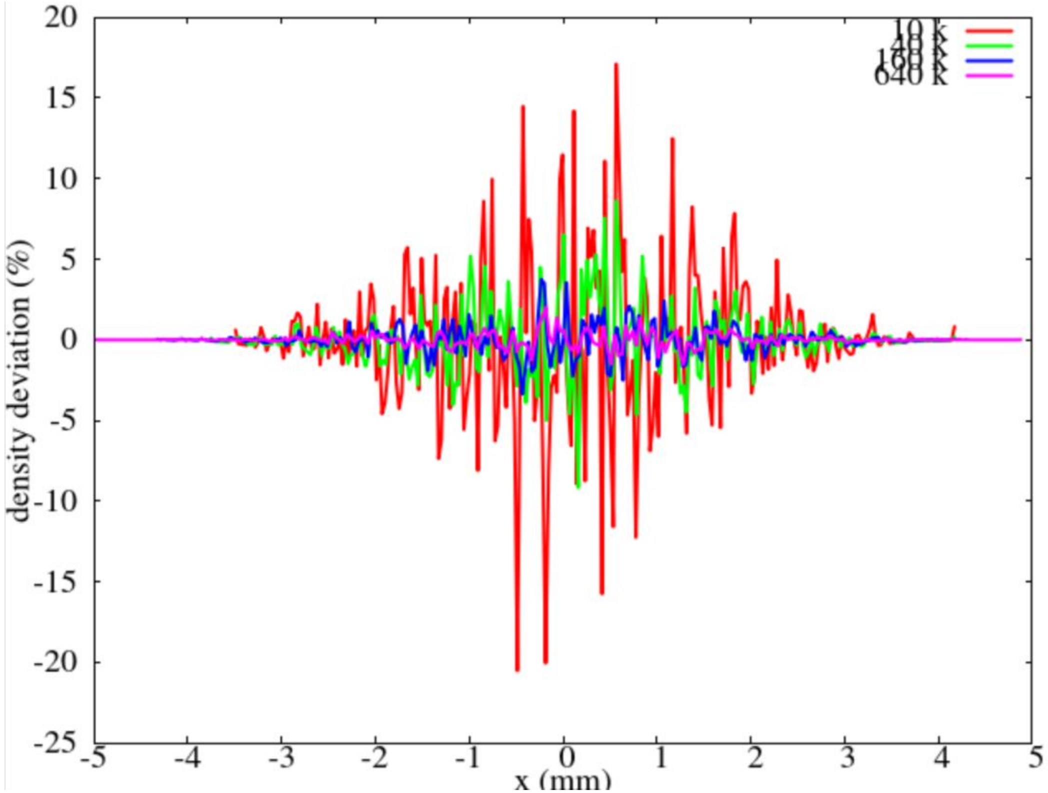
x (sigma)

Finite Number of Macroparticle Sampling Results in Density Fluctuation on Grid

Relative deviation of the density from the macroparticle sampling, linear deposition on 256 grid points and the analytical function

$$<\delta\rho^2>\sim\frac{1}{N}$$





A Random Diffusion Model to Estimate the Numerical Noise Induced Emittance Growth

$$\left(\frac{d\varepsilon}{dt}\right)_{sim} \propto \frac{D_{sim}}{\varepsilon}$$

$$D_{sim} = \frac{N}{N_{sim}} D_{real}$$

$$D_{real} \propto \frac{N}{\sigma^2}$$

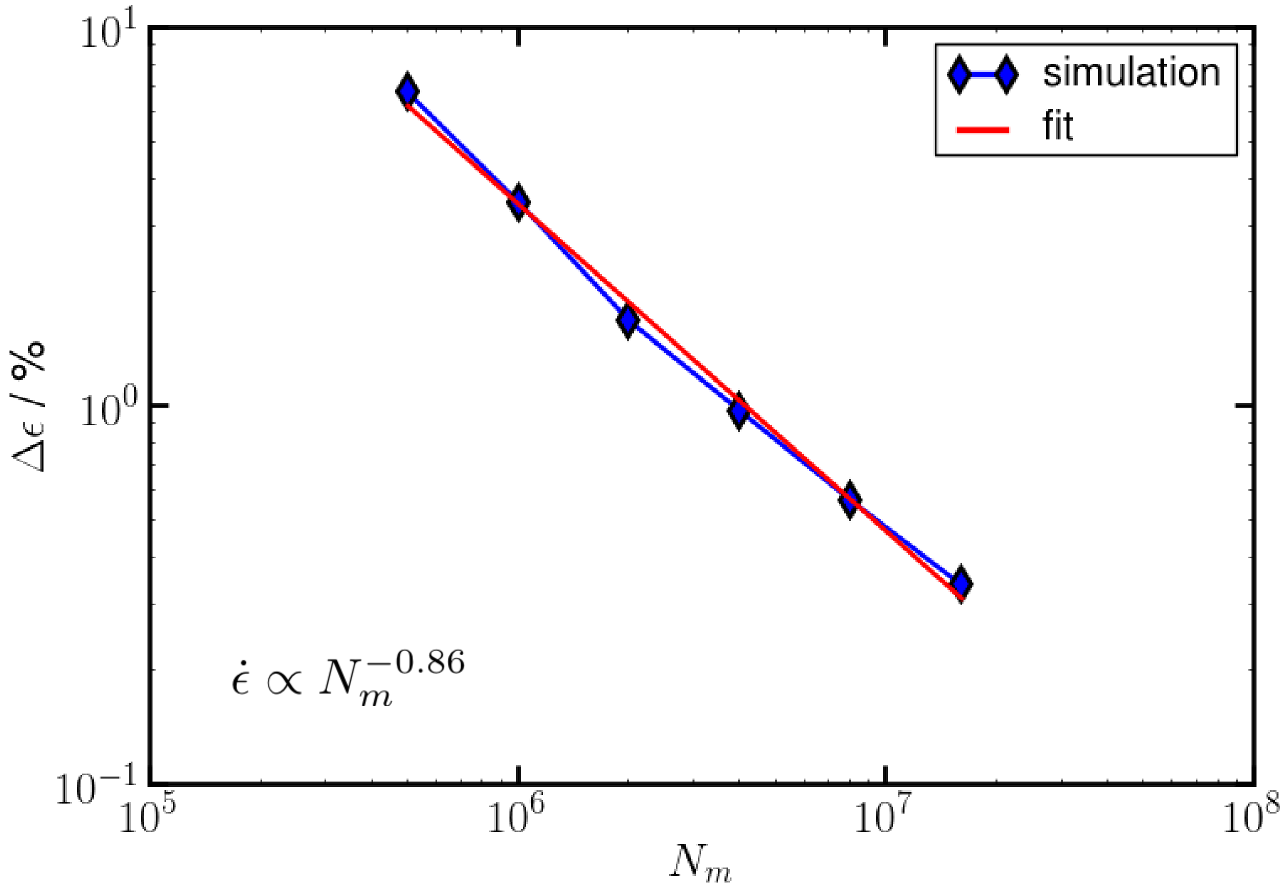
$$\left(\frac{d\varepsilon}{dt}\right)_{sim} \propto \frac{N^2}{\varepsilon^2 N_{sim}}$$

Test of the Numerical Noise Induced Emittance Growth Using Nominal LHC Parameters

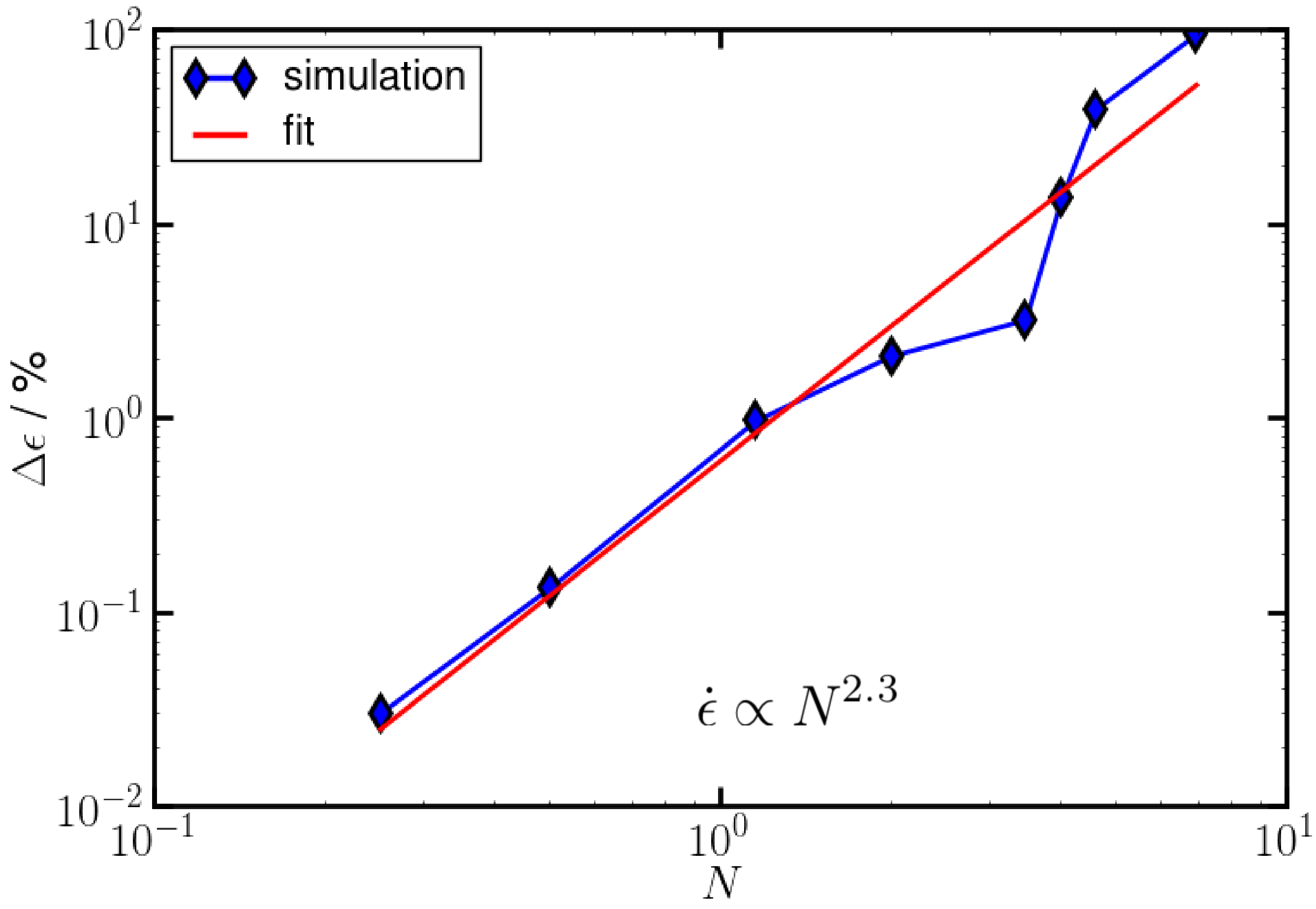
Numerical parameters	
N_m	4×10^{6}
# turns	1×10^{5}
# mesh cells	128×128
Long. slices	1

Physical parameters	
IPs	1
E	7 TeV
ε	0.5 nm
σ_z	7.5 cm
$\delta p/p$	1.1×10^{-4}
β^*	55 cm
ξ	0.015

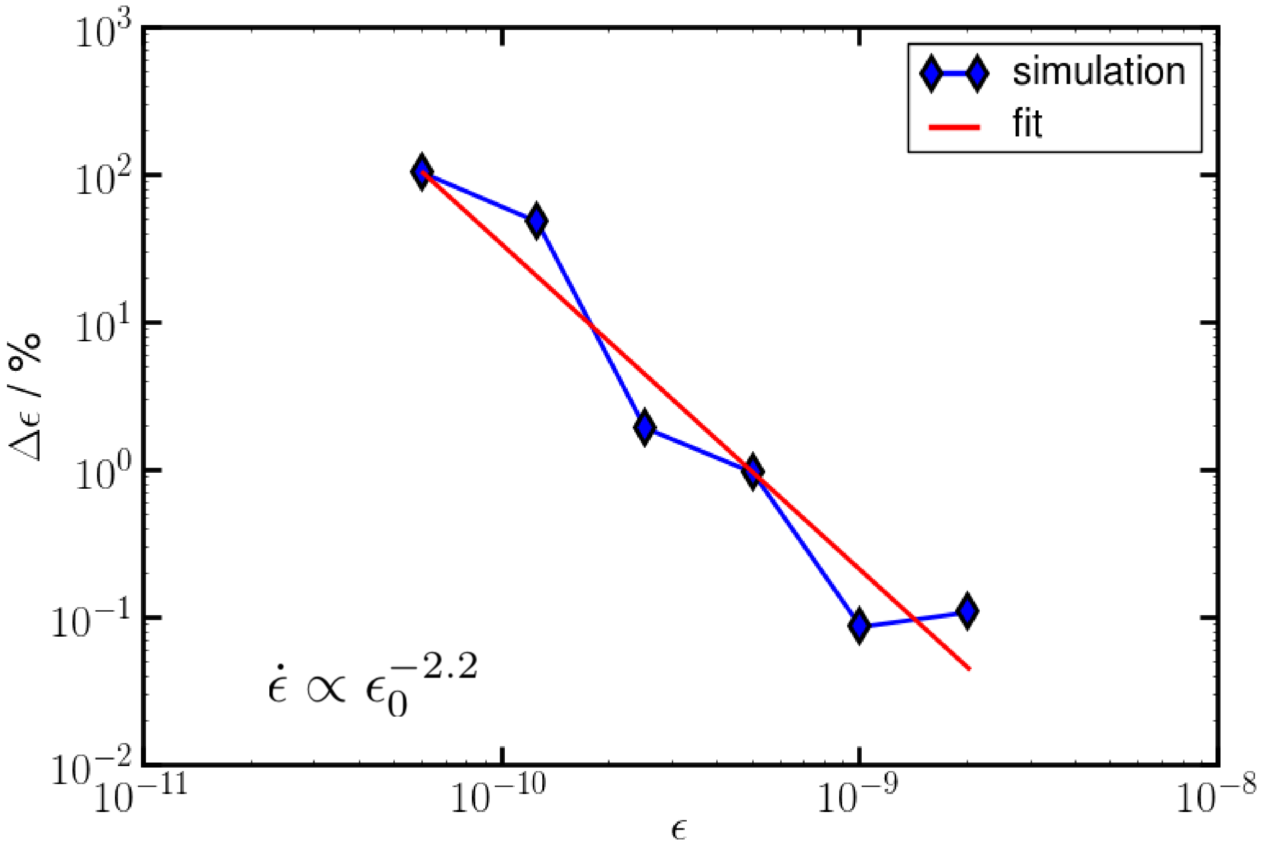
NI-Emittance Growth vs. # of Macroparticles Agrees Well with the Analytical Estimate



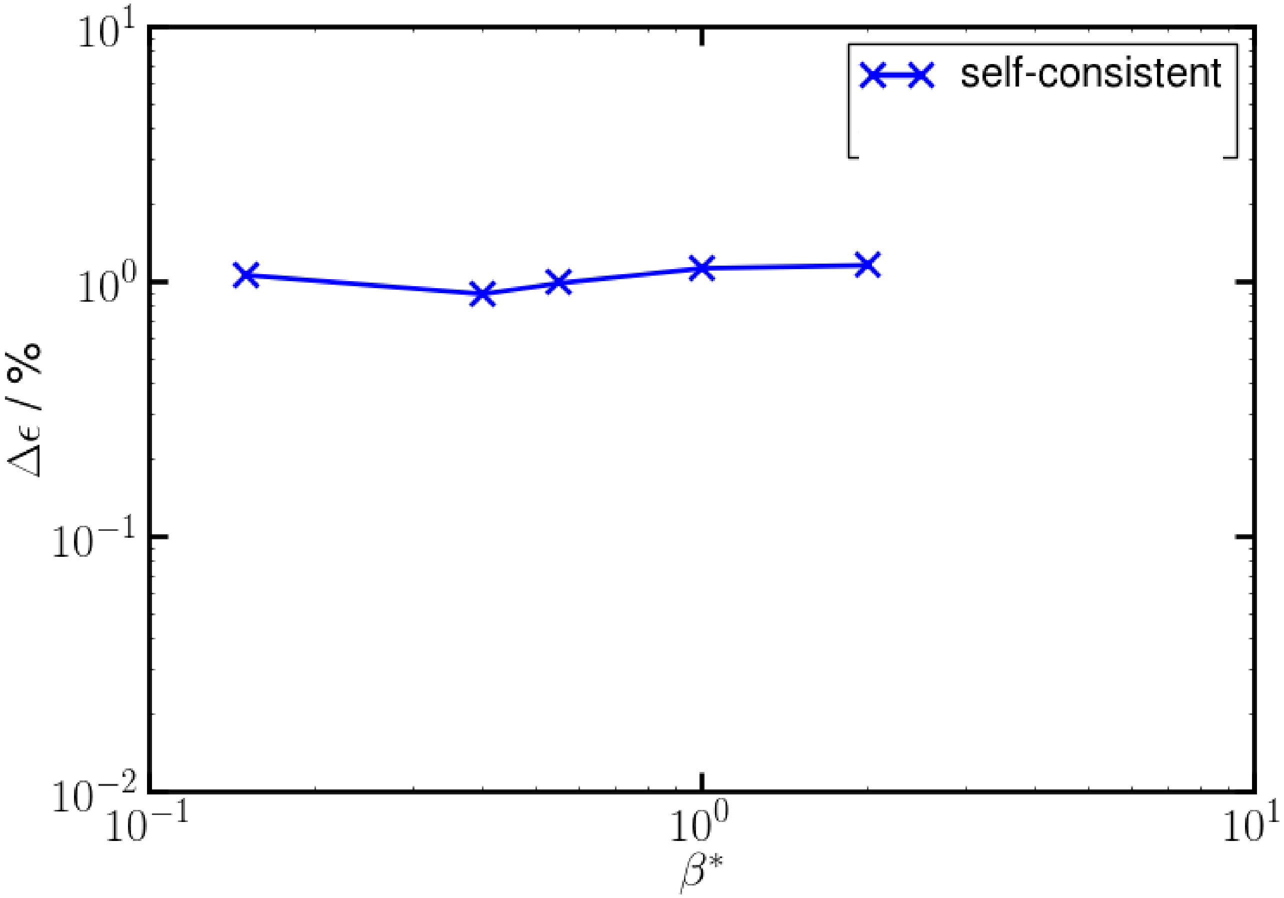
NI-Emittance Growth vs. Bunch Intensity Agrees Well with the Analytical Estimate



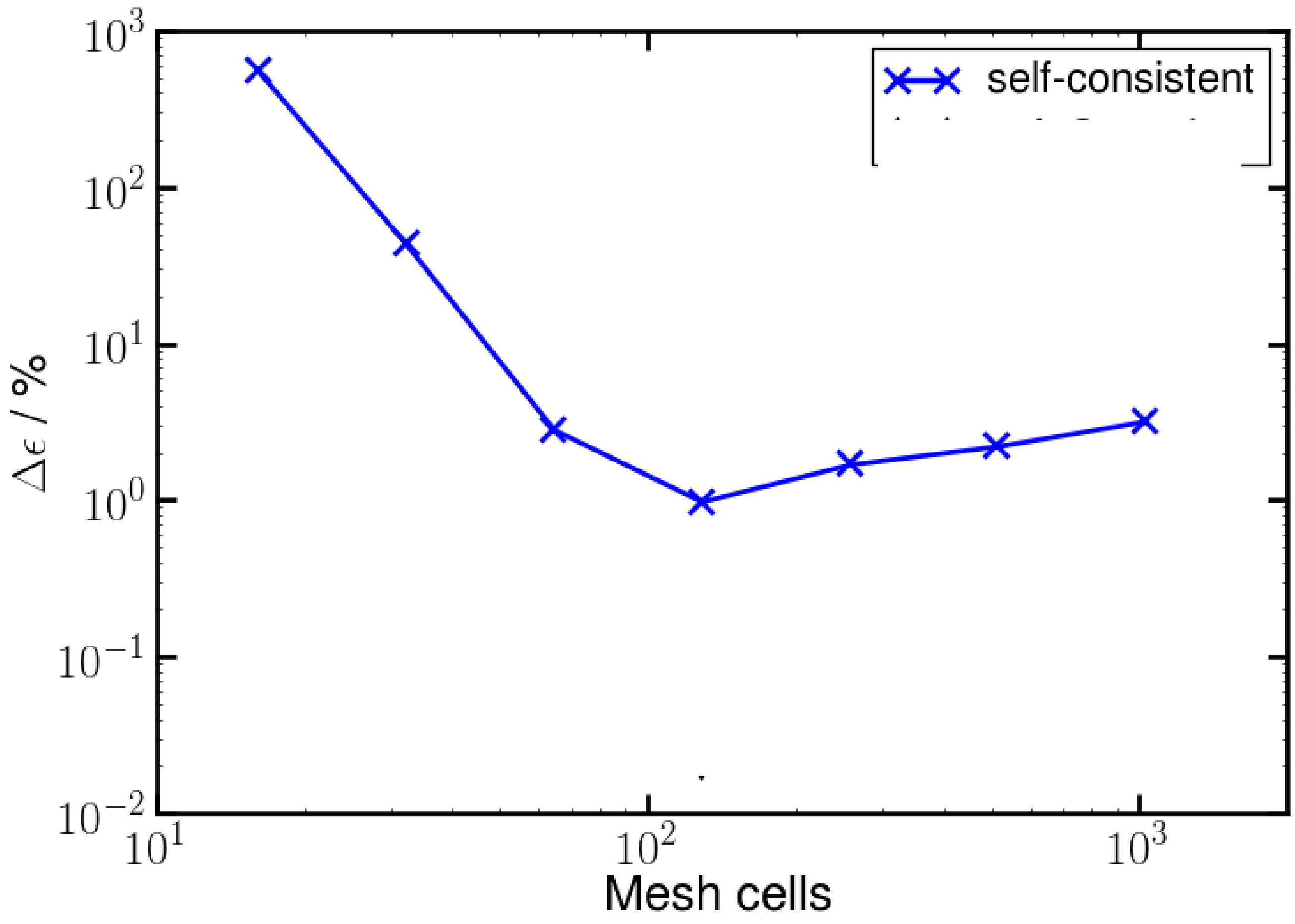
NI-Emittance Growth vs. Emittance Agrees Well with the Analytical Estimate



NI-Emittance Growth Shows Independence of Beta*

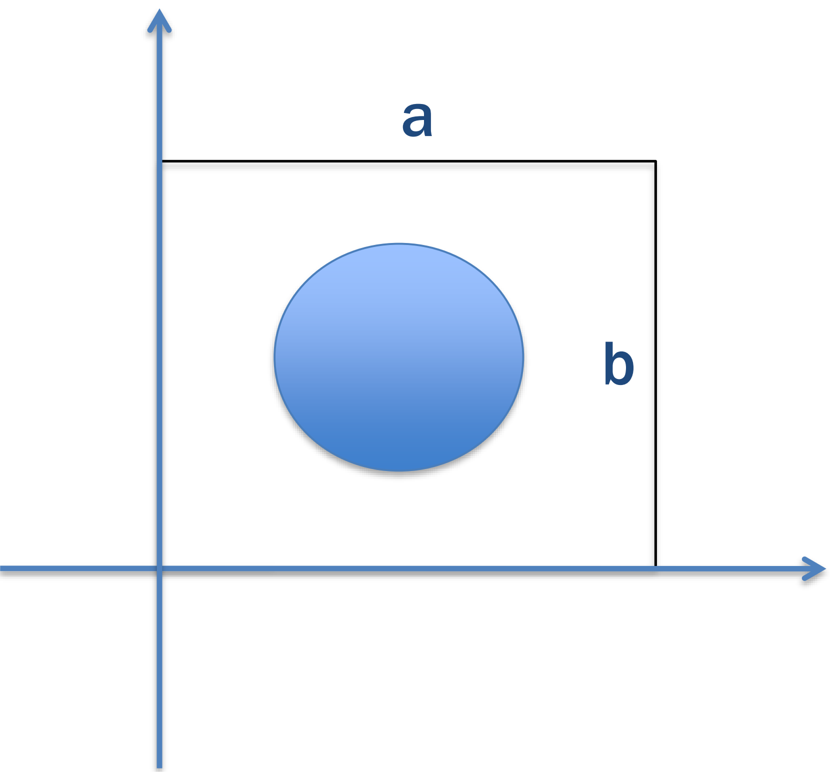


Numerical Grid Cells also Affects the NI-Emittance



Solving the Poisson's Eq. Using a Spectral Method

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -4\pi\rho,$$



$$\rho(x,y) = \sum_{l=1}^{N_l}\sum_{m=1}^{N_m} \rho^{lm} \sin(\alpha_l x)\sin(\beta_m y)$$

$$\phi(x,y) = \sum_{l=1}^{N_l}\sum_{m=1}^{N_m} \phi^{lm} \sin(\alpha_l x)\sin(\beta_m y),$$

$$\rho^{lm} = \frac{4}{ab}\int_0^a \int_0^b \rho(x,y)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$

$$\phi^{lm} = \frac{4}{ab}\int_0^a \int_0^b \phi(x,y)\sin(\alpha_l x)\sin(\beta_m y)dxdy,$$

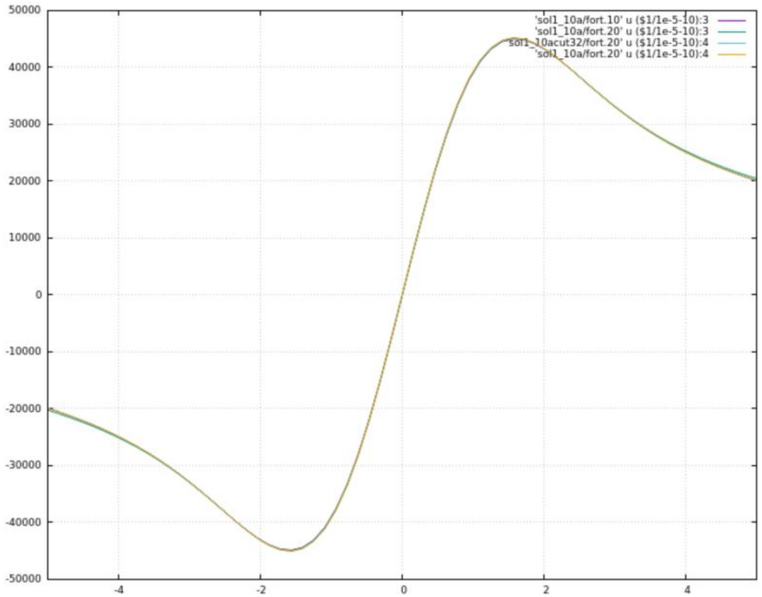
where $\alpha_l = l\pi/a$ and $\beta_m = m\pi/b$.

$$\phi^{lm} = \frac{4\pi\rho^{lm}}{\gamma_{lm}^2}$$

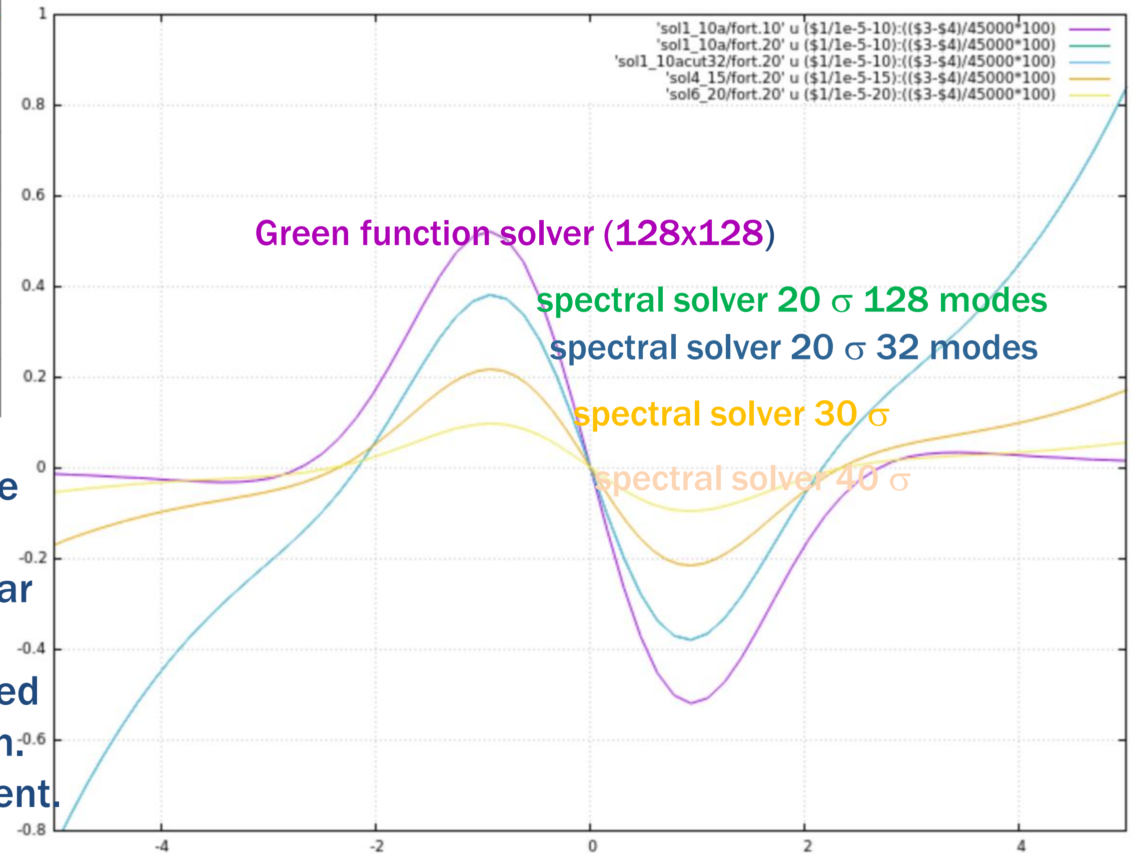
where $\gamma_{lm}^2 = \alpha_l^2 + \beta_m^2$.

Spectral Method Shows Good Solution of Electric Fields of a Gaussian Distribution

Beam-Beam Field

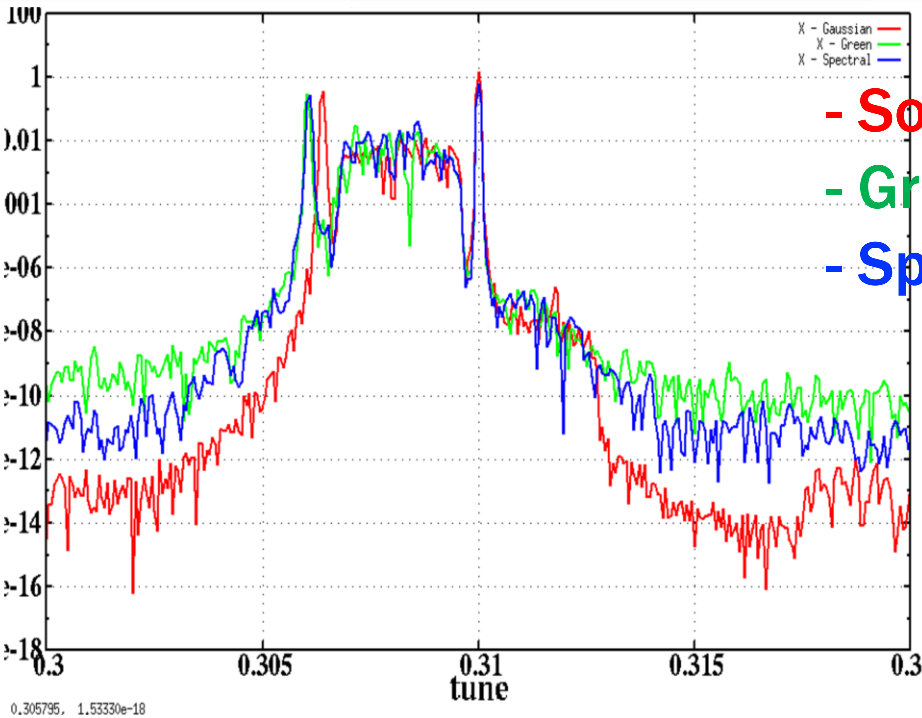


Relative Errors (%)

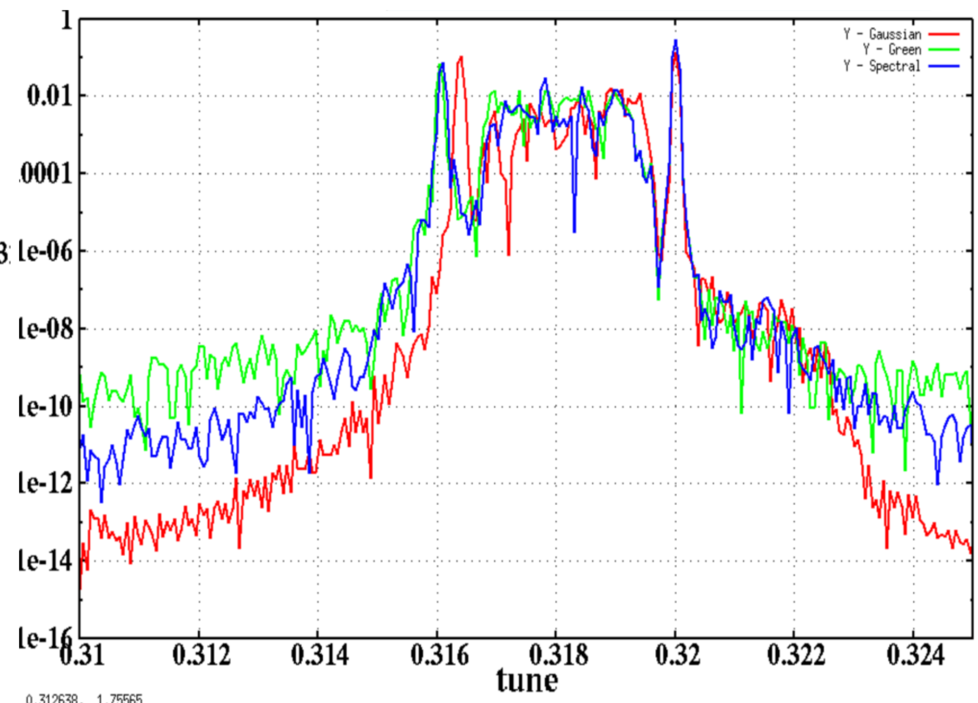


- Spectral solver yields less errors in the core
- Spectral solver yields larger errors near the edge
- The error near the edge can be reduced by using larger computational domain.
- A smaller number of modes is sufficient.

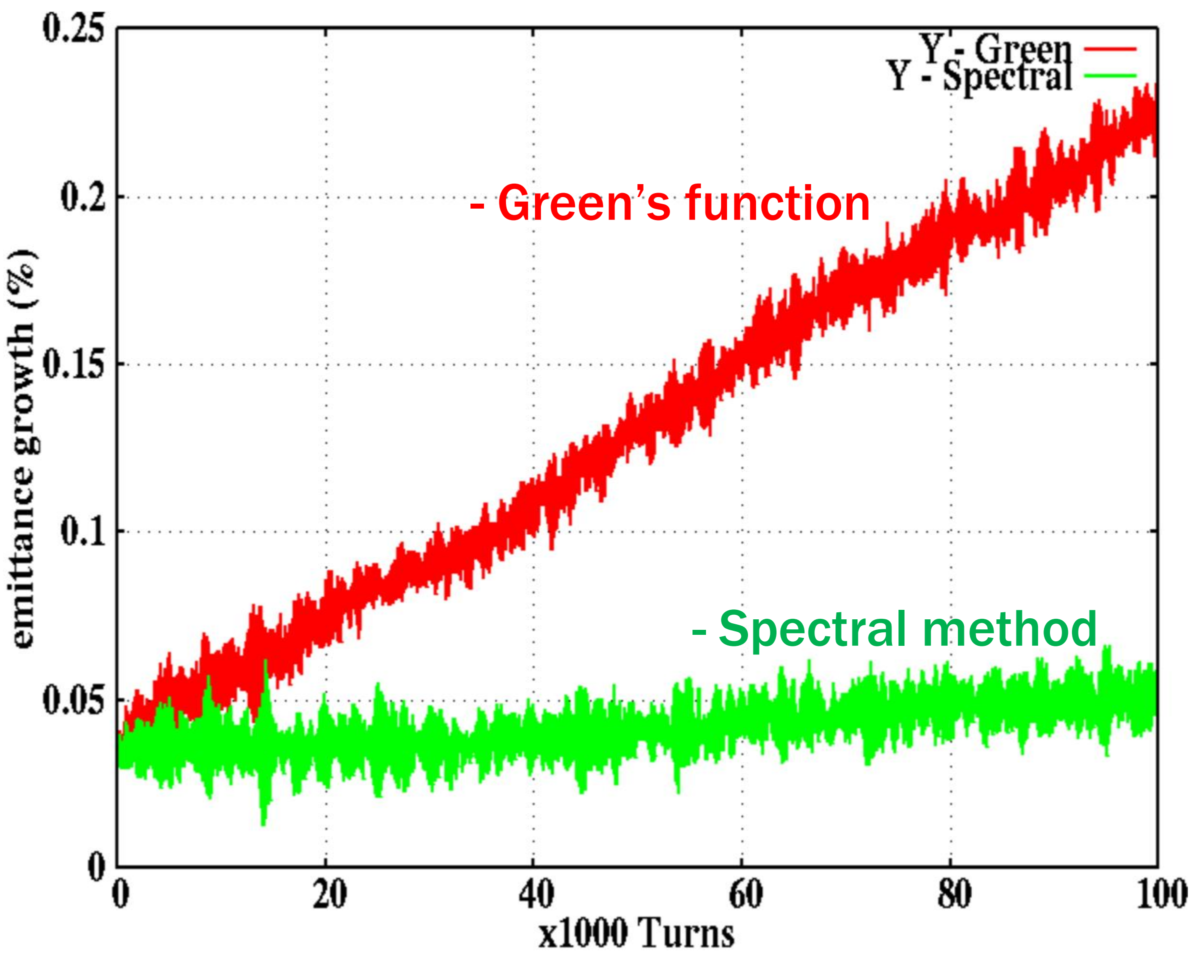
Spectral Method Produces Correct Power Spectral of Coherent Modes



- Soft Gaussian model
- Green's function method (128x128)
- Spectral method (32x32)



Spectral Method Shows Much Less Numerical Emittance Growth than the Green's Function Method



Conclusions and Future Work

- Numerical noise from finite number of marcoparticles causes significant artificial emittance growth.
- The NI-emittance growth scales as expected with # of macropartilces, bunch intensity and emittance.
- Using a spectral method, the NI-emittance can be significantly mitigated.

➢ **Parallelization**
➢ **Extension to 3D**
➢ **Extension to fully symplectic model**