Workshop on Beam-Beam Effects in Circular Colliders
Berkeley 2018

Numerical and experimental studies of coherent beam-beam modes: stability and decoherence

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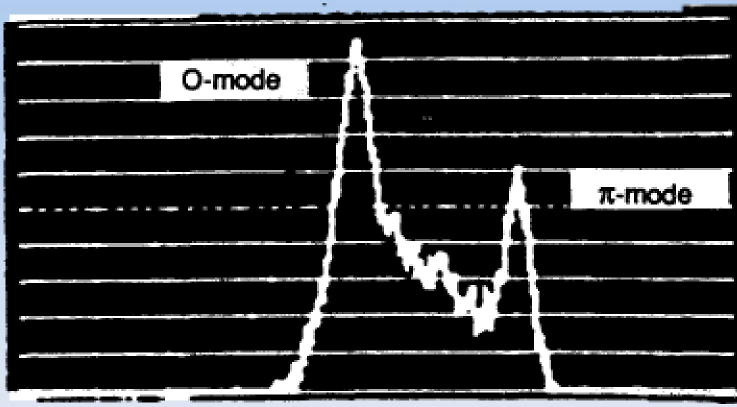
Content

- Coherent beam-beam modes
- Decoherence and emittance growth
- Beam-beam instabilities
- Conclusion

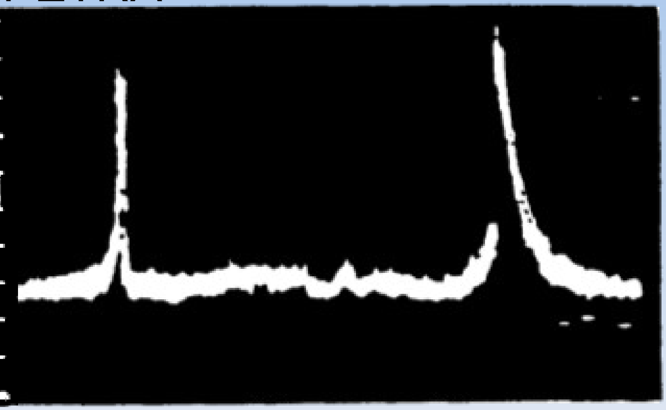
Observations

(1)

TRISTAN

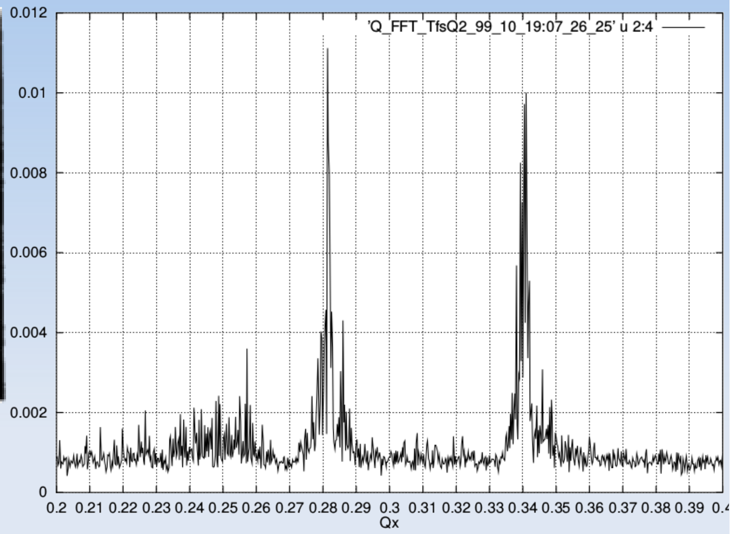


PETRA

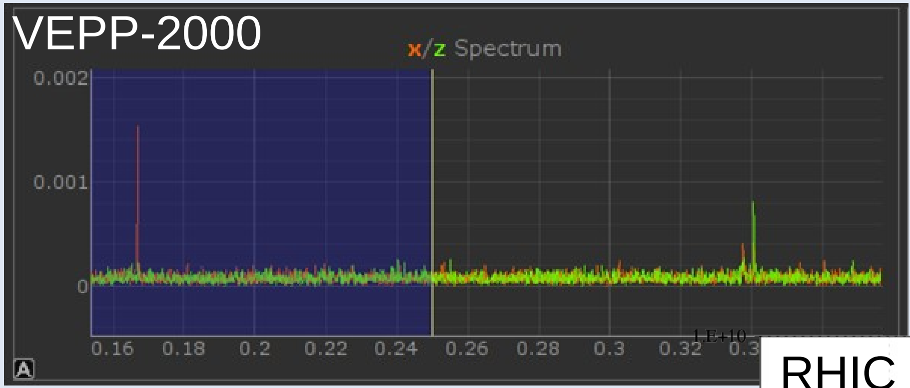


Vertical eigenfrequencies of two colliding bunches.

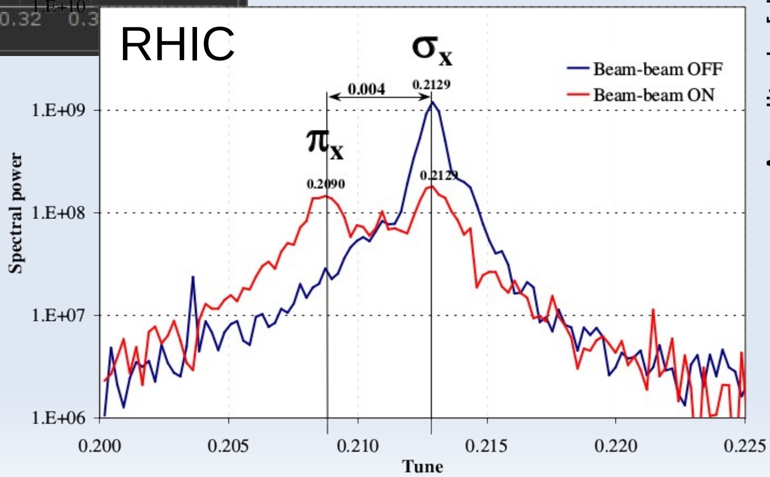
LEP



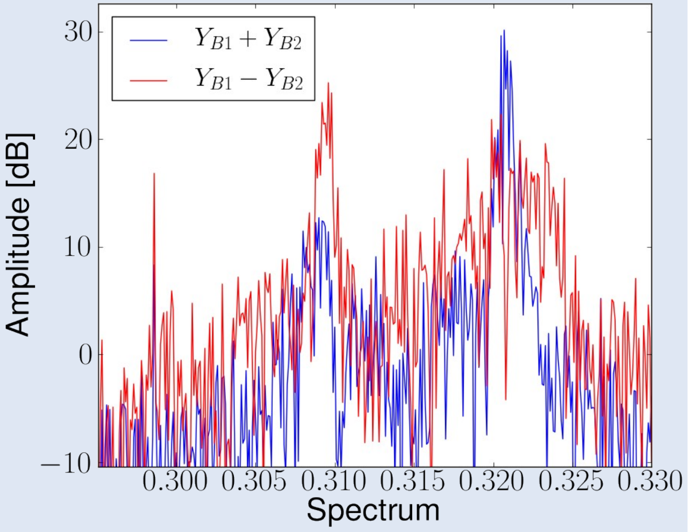
VEPP-2000



RHIC



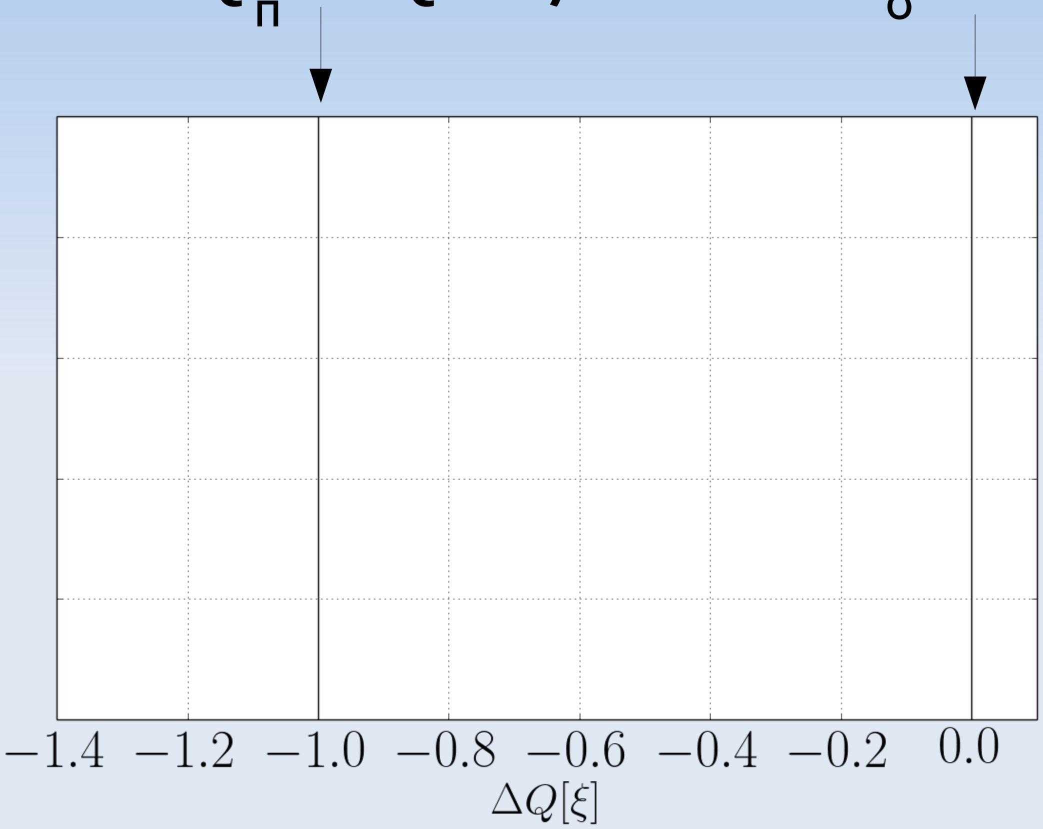
LHC



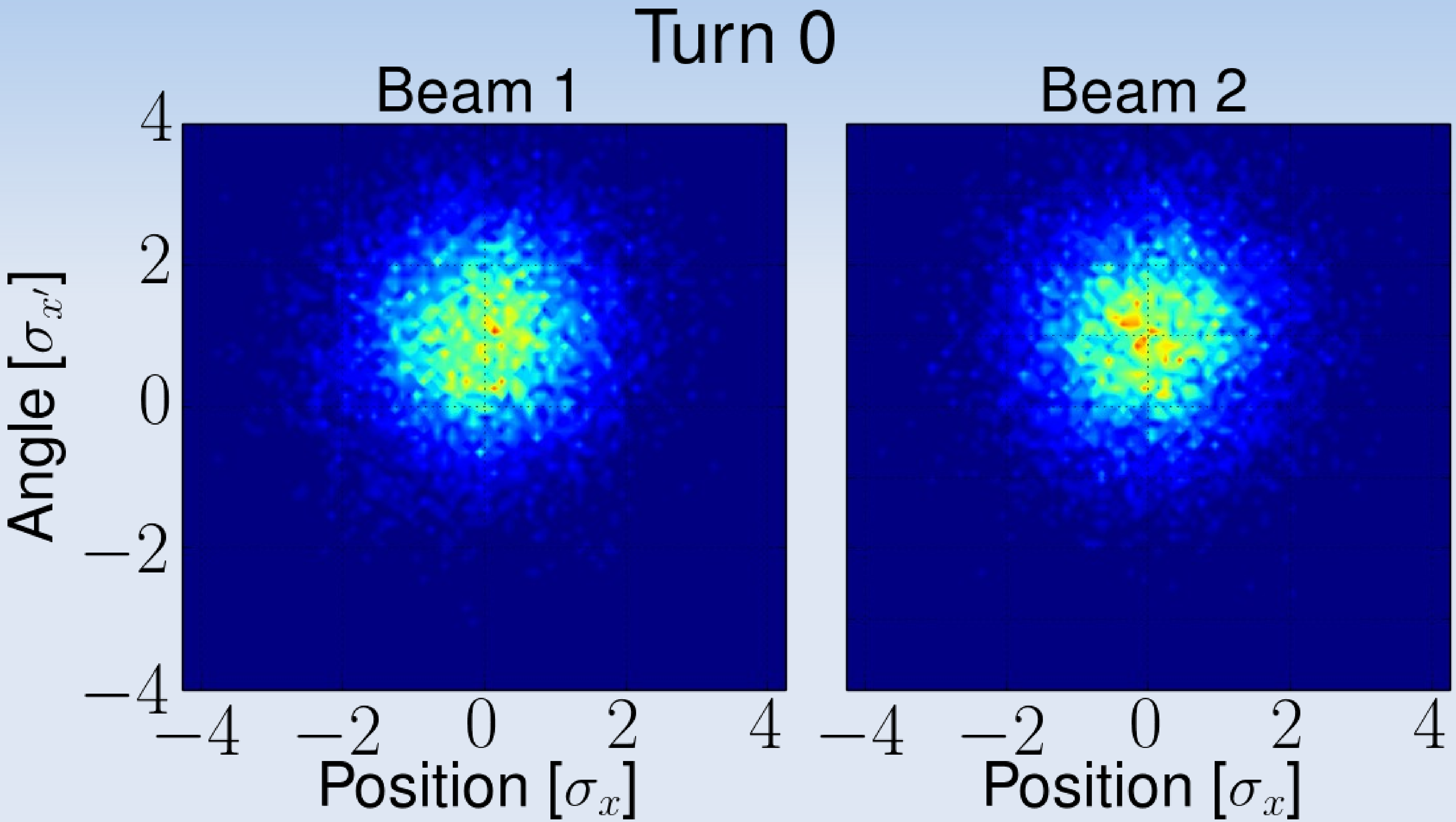
Coherent mode spectrum

Rigid bunch model :

$Q_\pi = Q - \xi$ $Q_\sigma = Q$

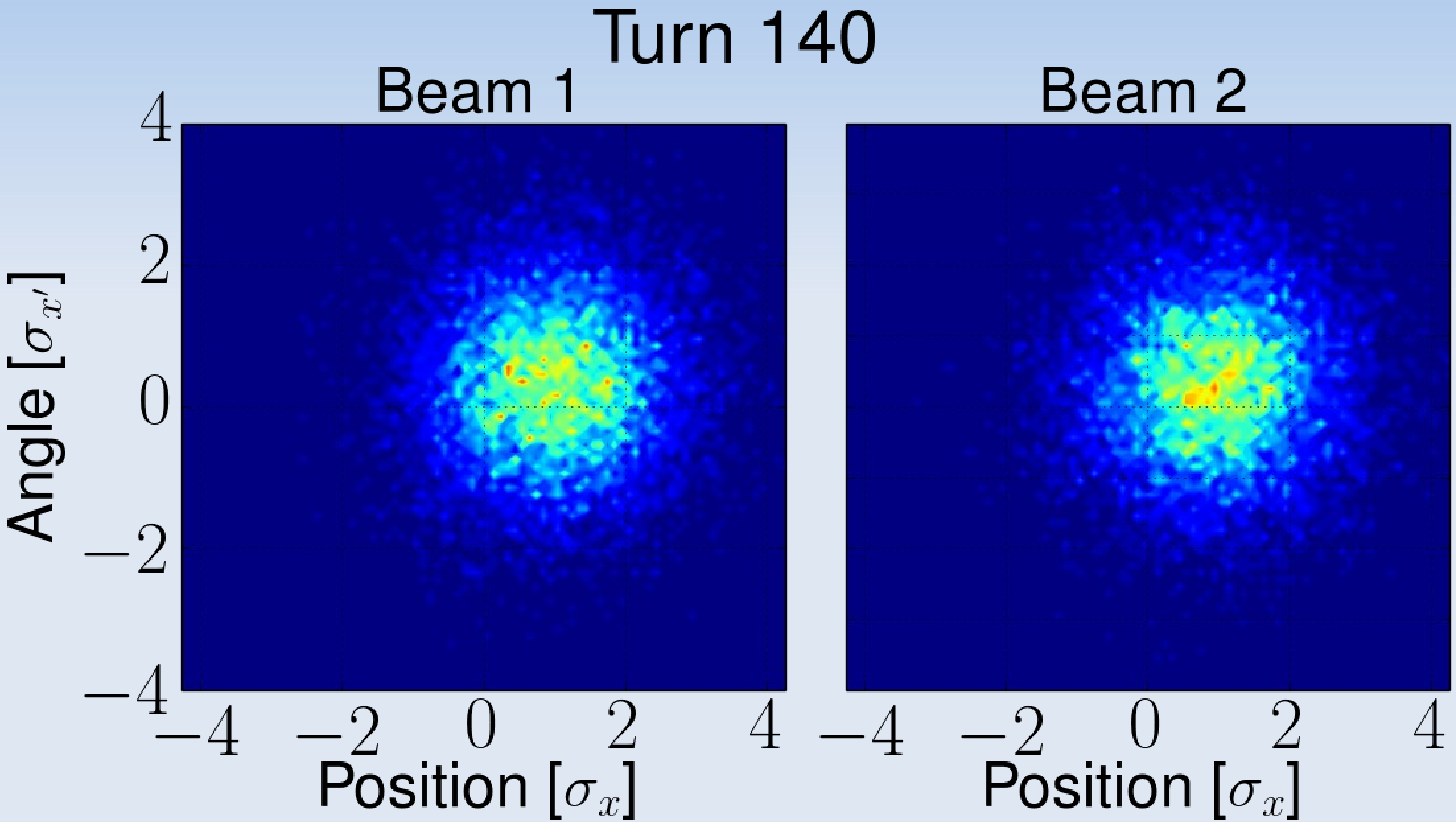


σ-mode

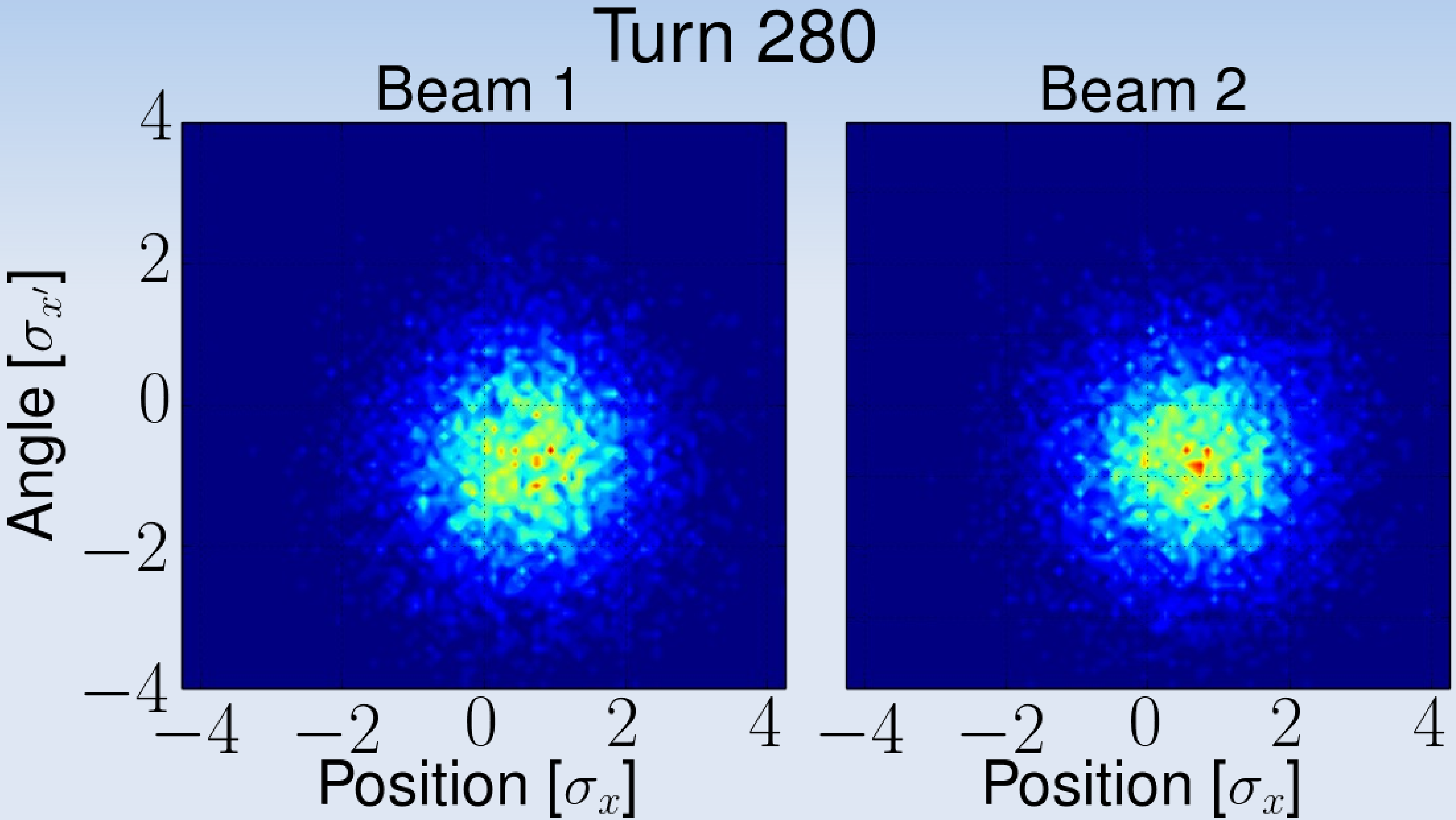


Turn 0
Beam 1
Beam 2
Angle $[\sigma_{x'}]$
Position $[\sigma_x]$
Position $[\sigma_x]$
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

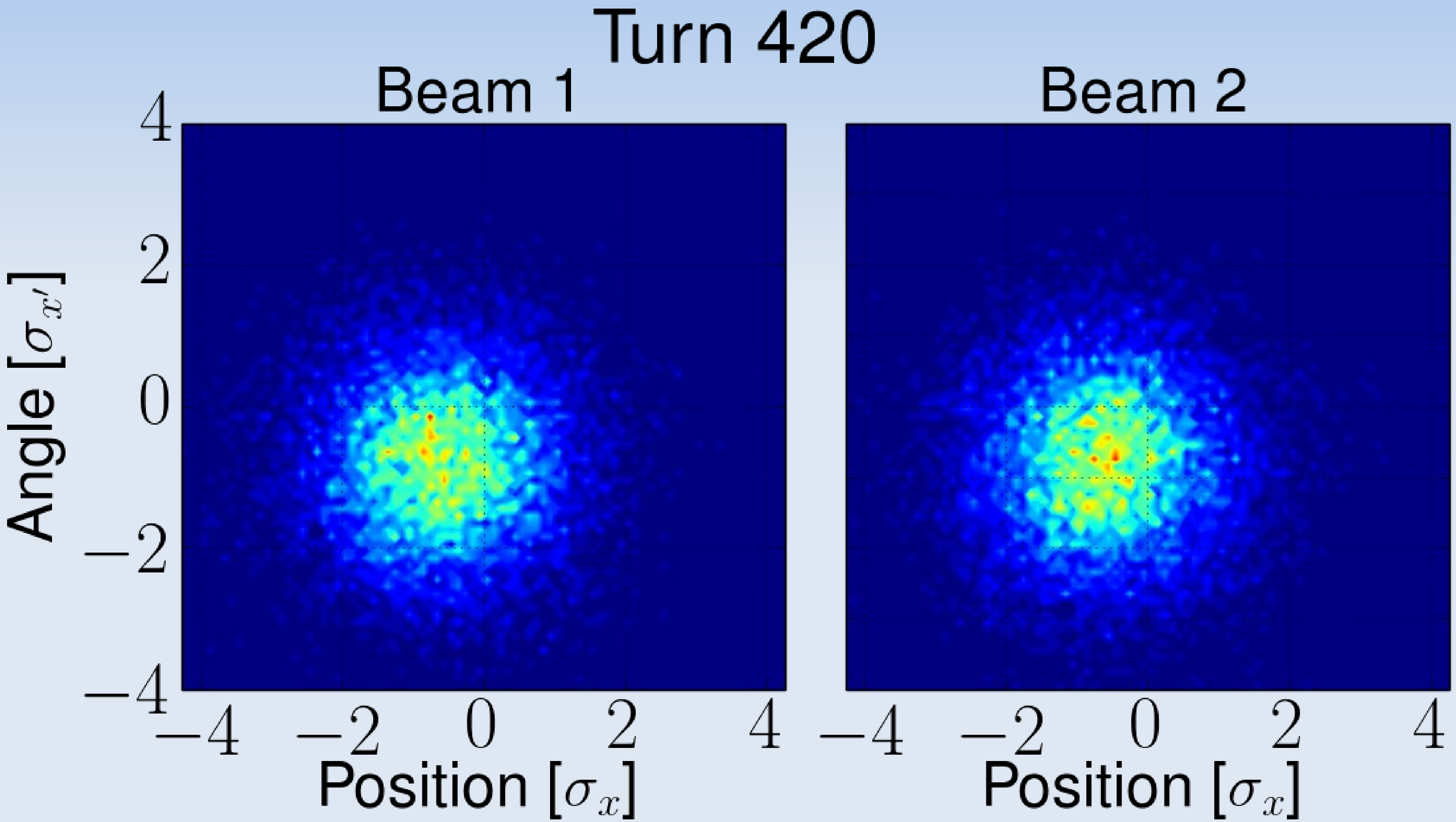
σ-mode


Turn 140
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4

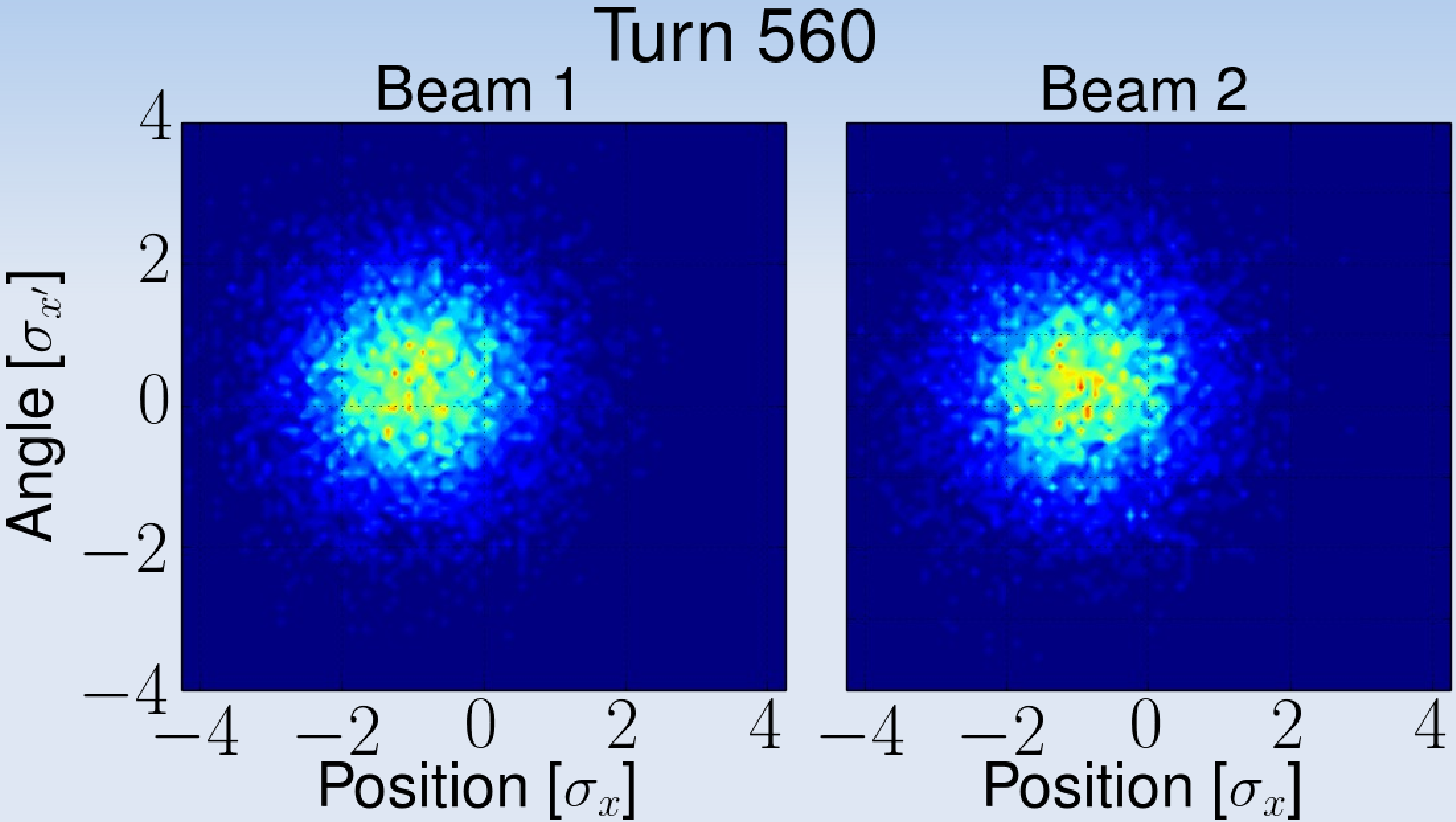
σ-mode


Turn 280
Beam 1
Beam 2
4
2
0
−2
−4
Angle [$\sigma_{x'}$]
−4
−2
0
2
4
Position [σ_x]
−4
−2
0
2
4
Position [σ_x]

σ-mode

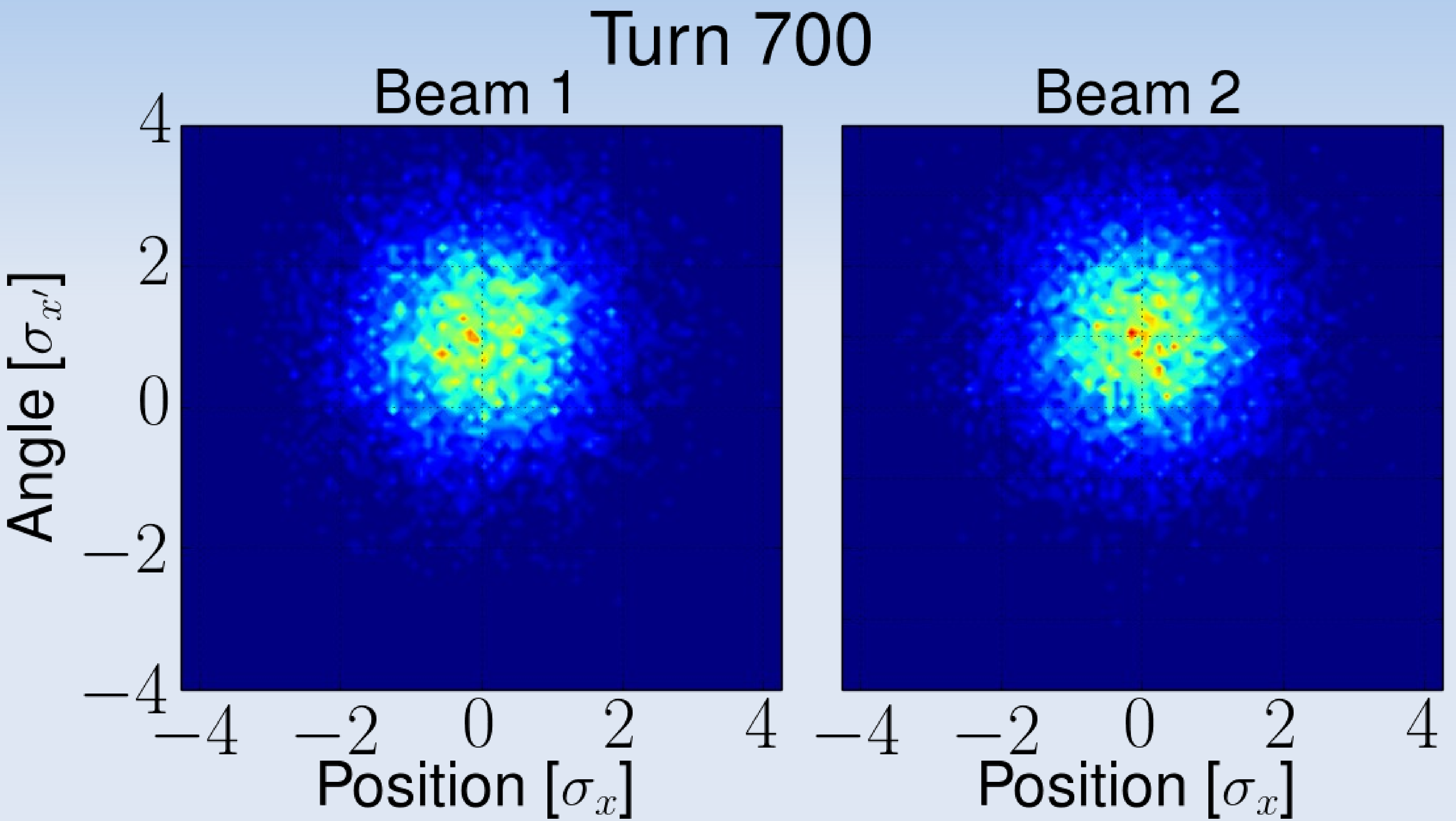

Turn 420
Beam 1
Beam 2
Angle $[\sigma_{x'}]$
Position $[\sigma_x]$
Position $[\sigma_x]$
4
2
0
−2
−4
−4 −2 0 2 4
−4 −2 0 2 4

σ-mode



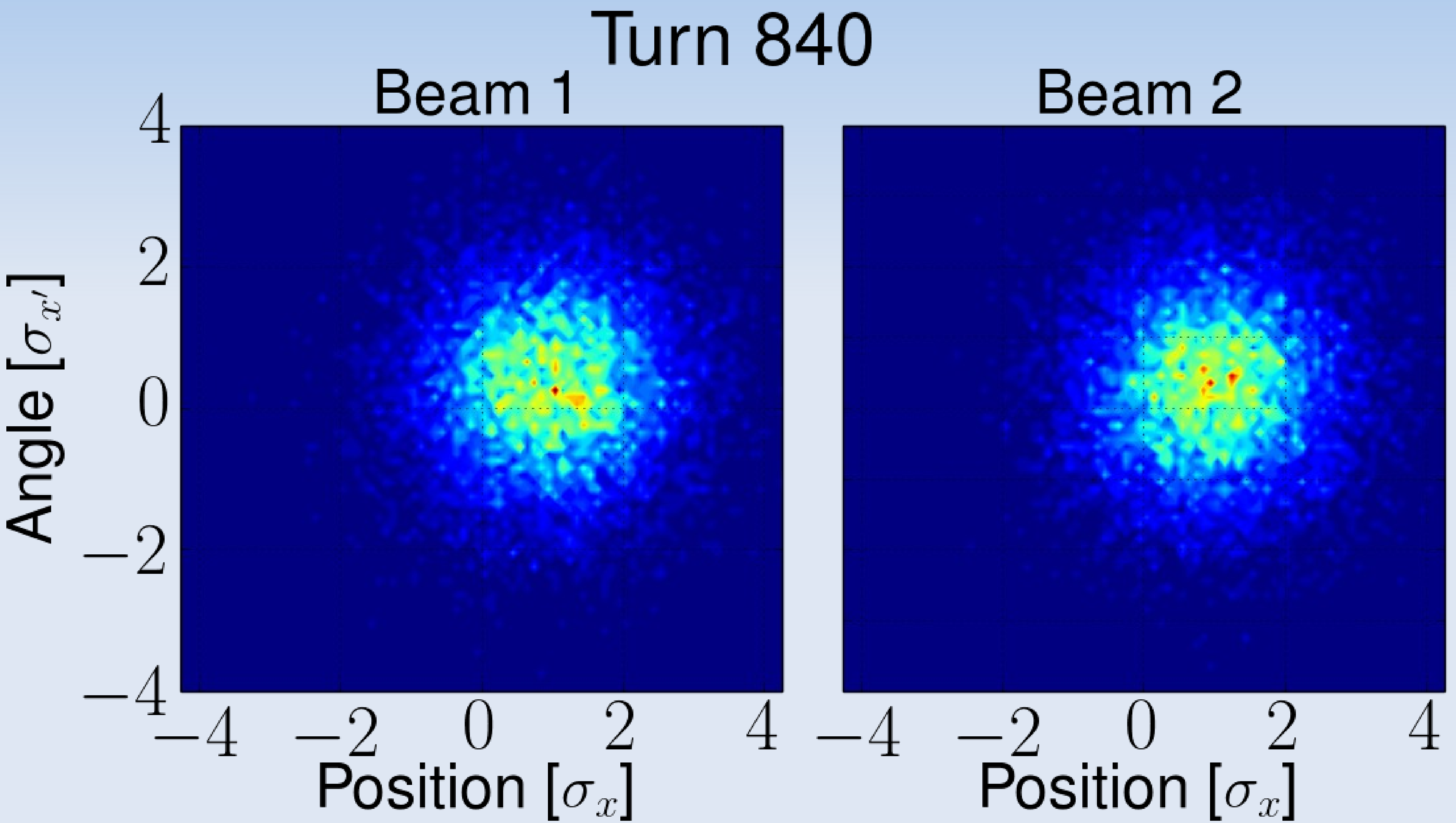
Turn 560
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
4
2
0
−2
−4
−4 −2 0 2 4
Position [σ_x]
−4 −2 0 2 4
Position [σ_x]

σ-mode



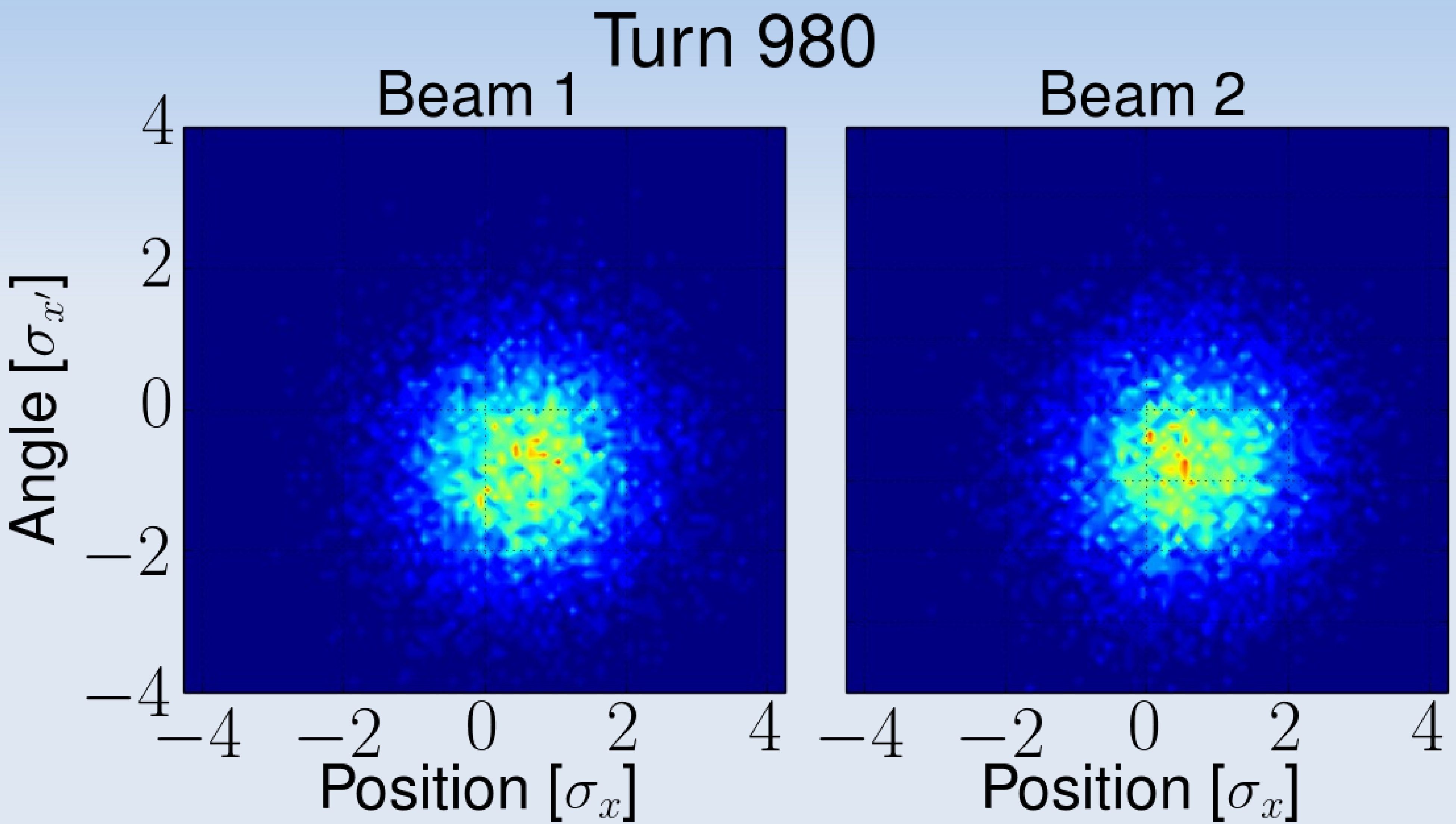
Turn 700
Beam 1
Beam 2
Angle $[\sigma_{x'}]$
Position $[\sigma_x]$
Position $[\sigma_x]$
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

σ-mode

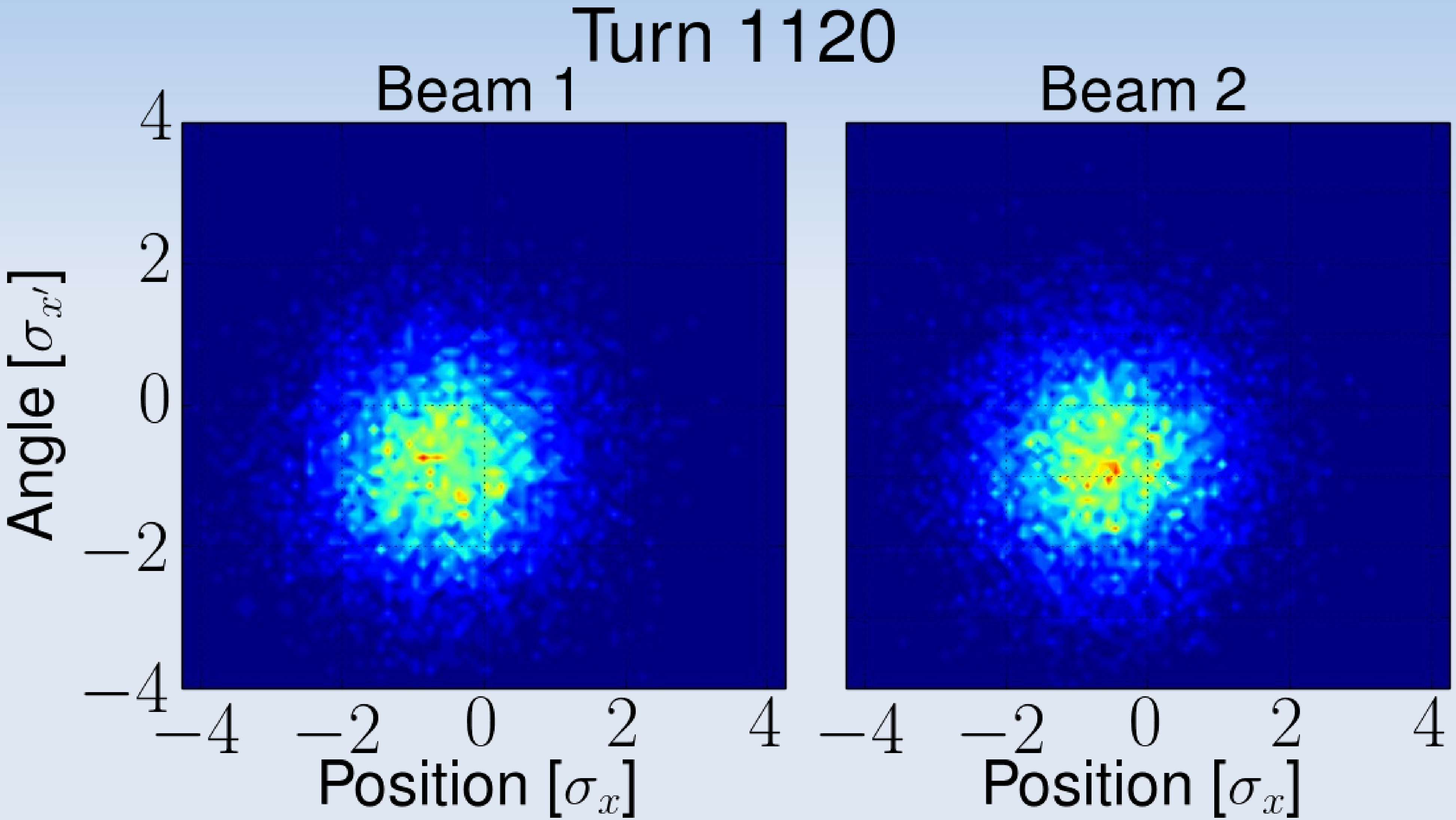


Turn 840
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4

σ-mode

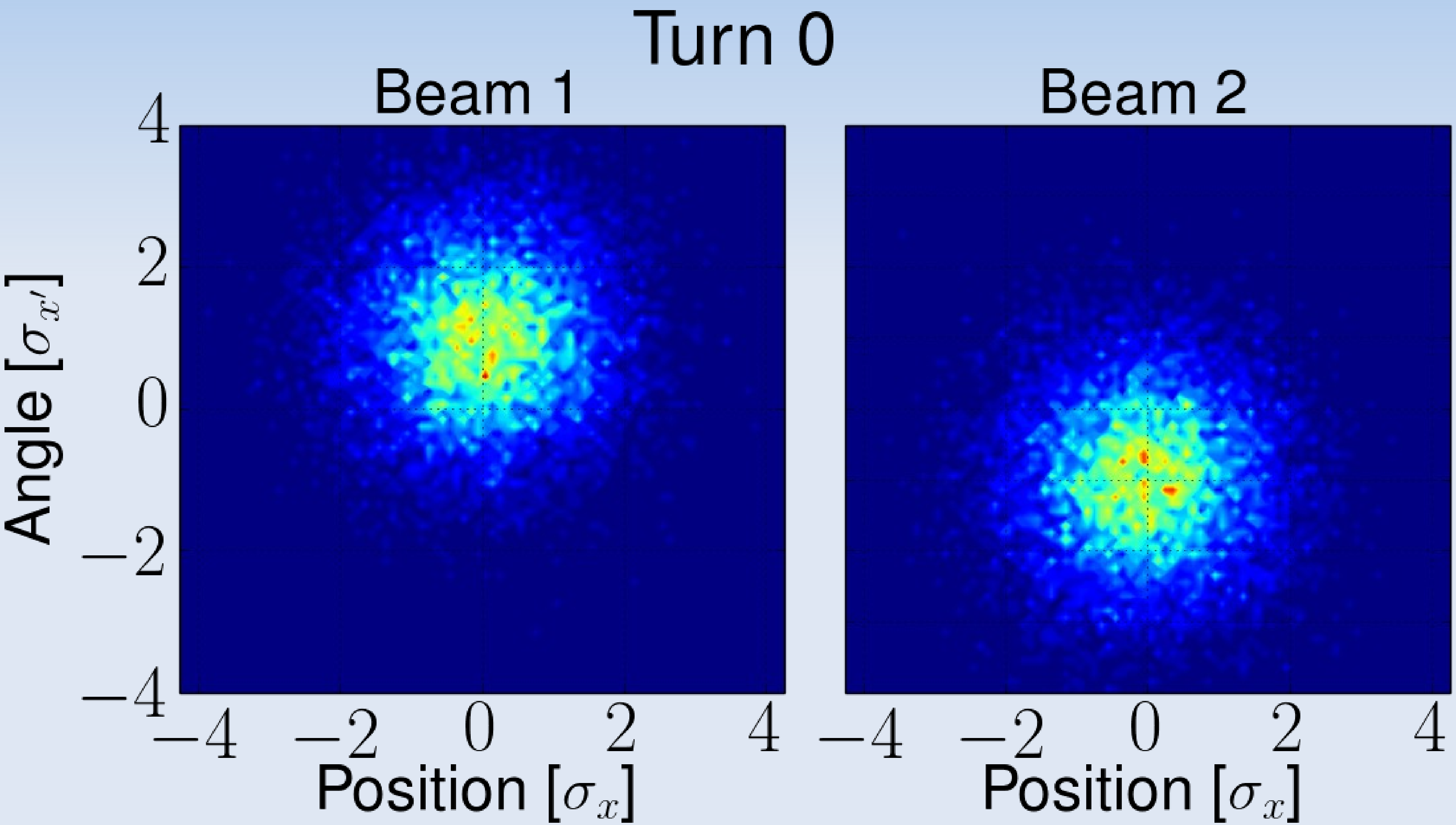

Turn 980
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

σ-mode

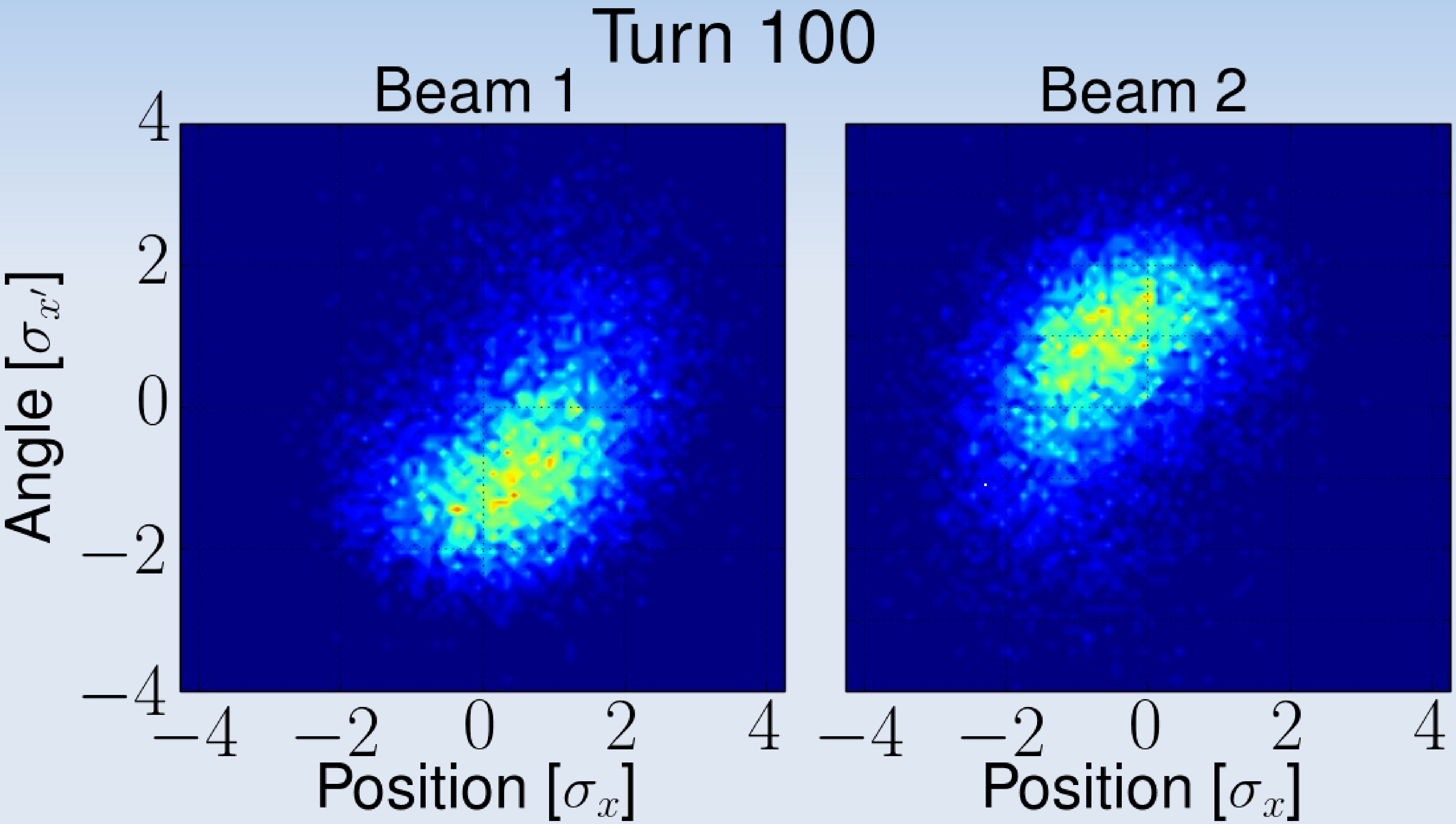


Turn 1120
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

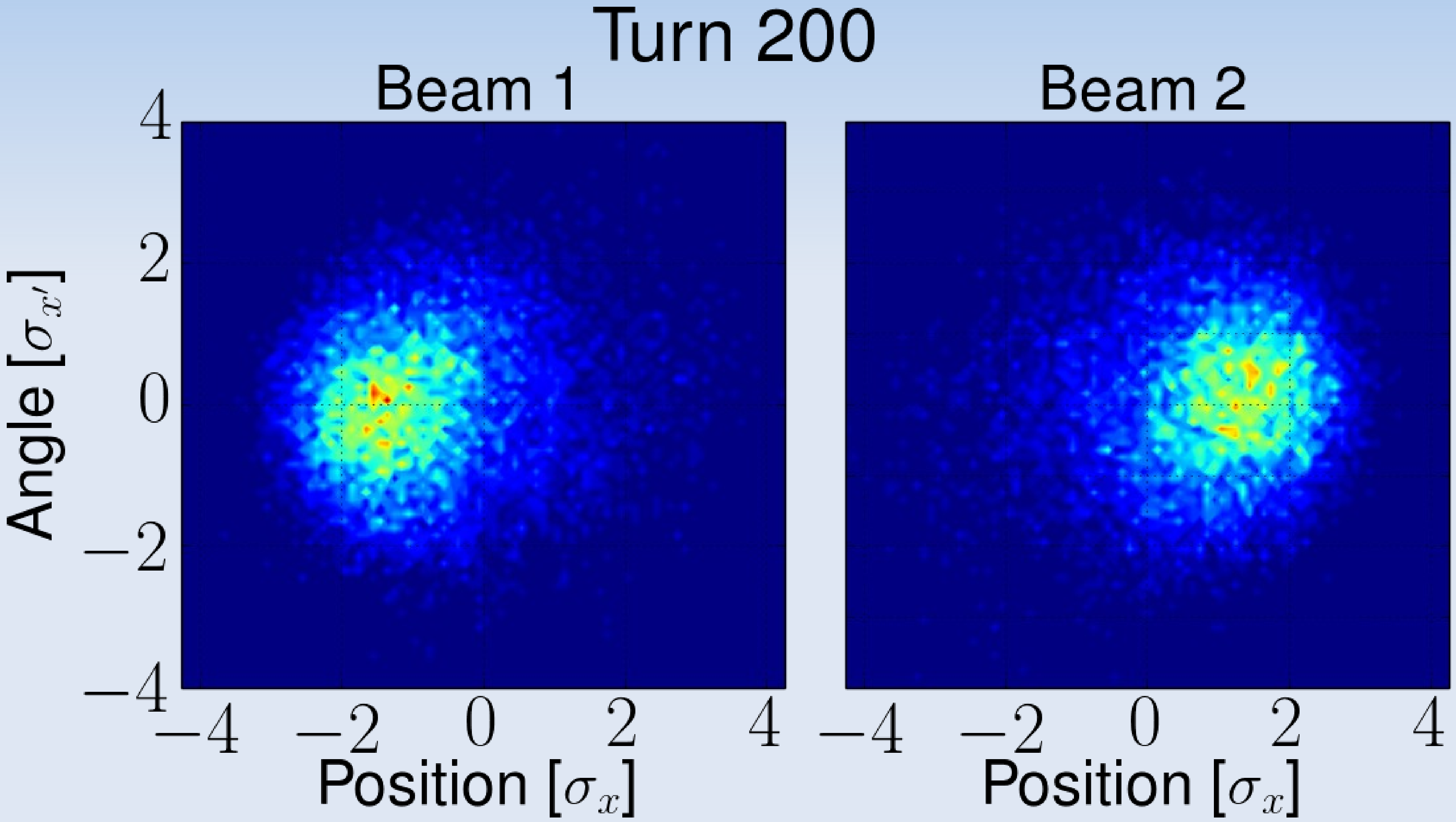
π mode


Turn 0
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

π mode

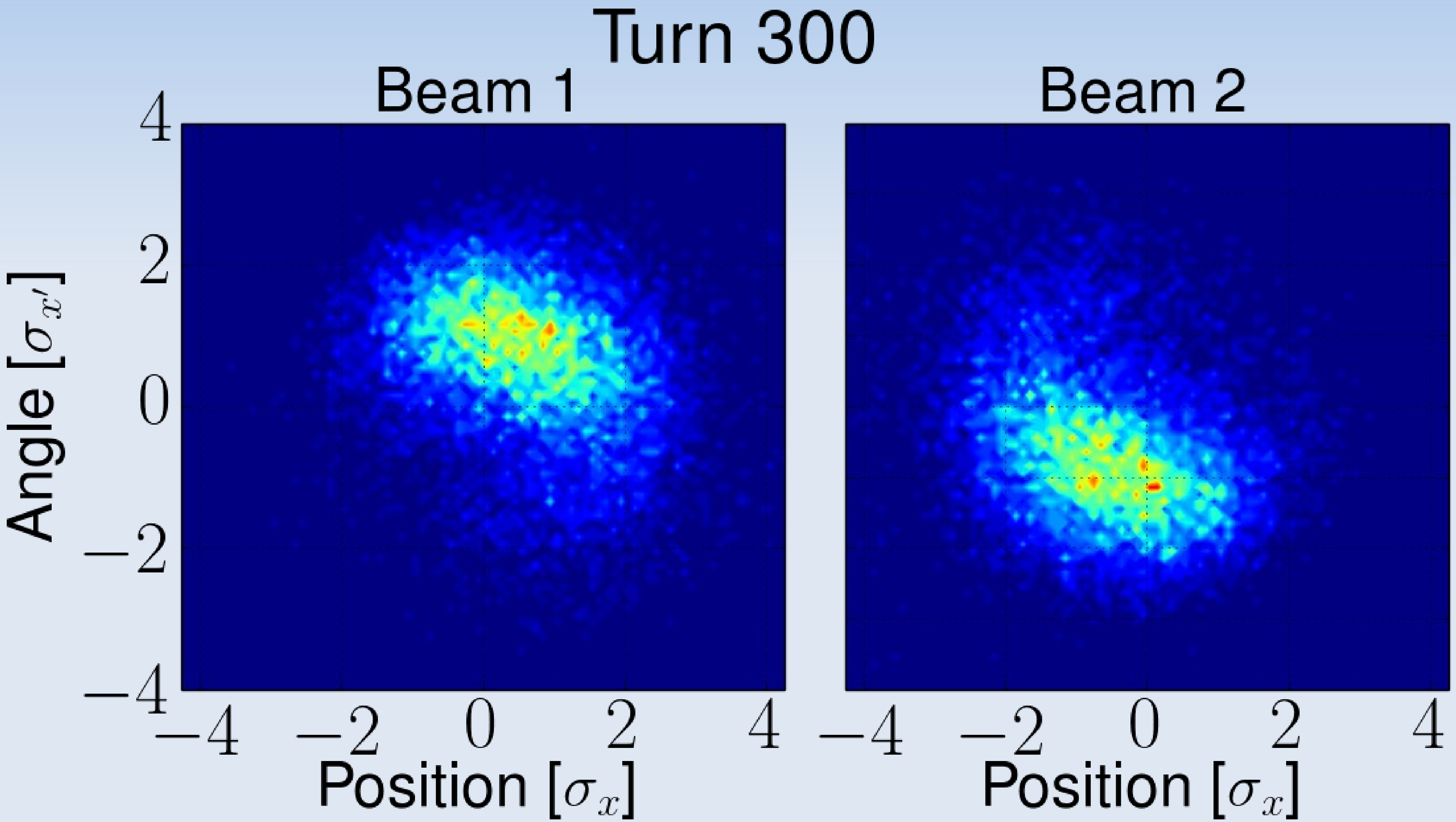

Turn 100
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
4
2
0
−2
−4
−4
−2
0
2
4
Position [σ_x]
−4
−2
0
2
4
Position [σ_x]

π mode

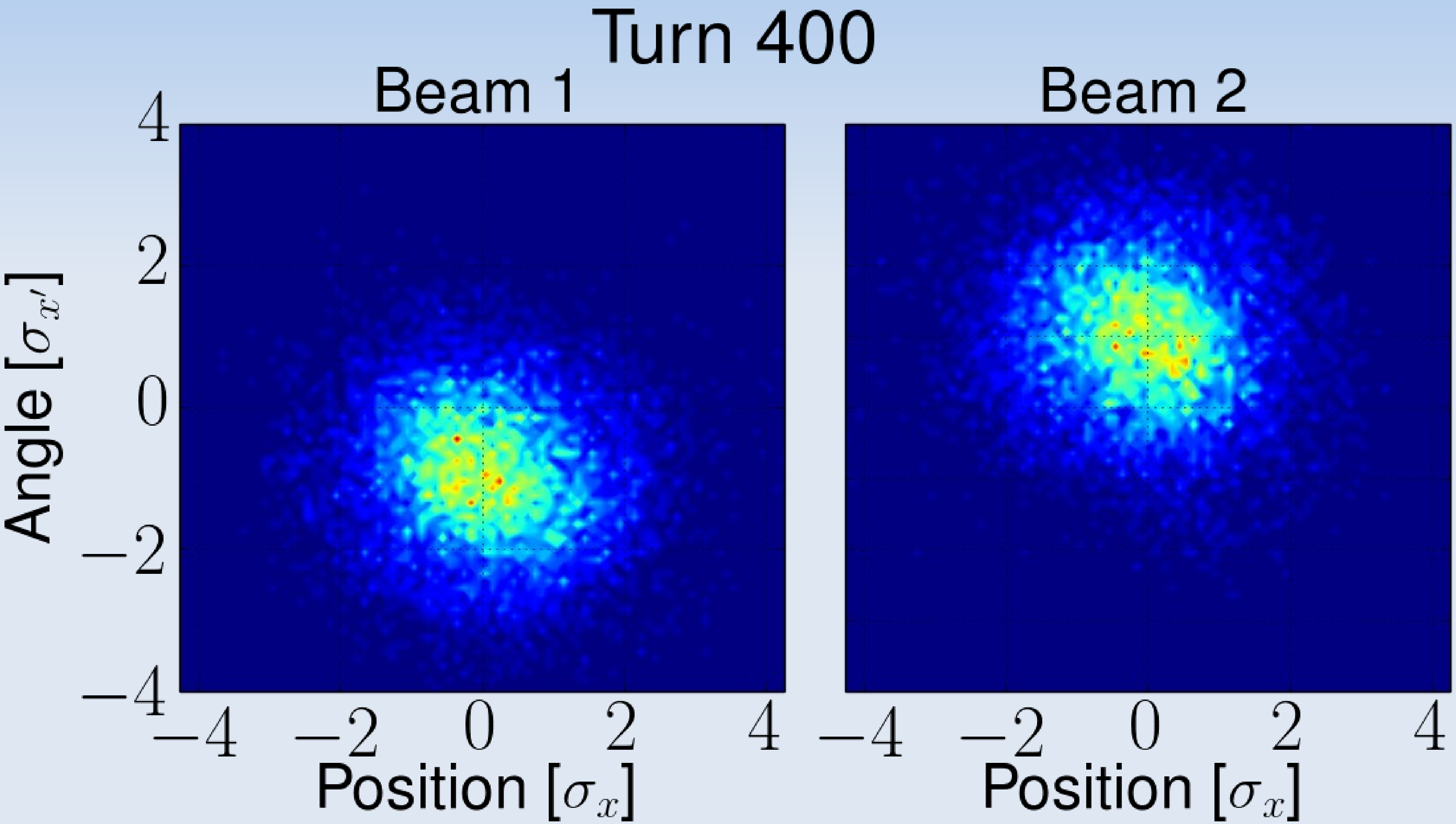


Turn 200
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

π mode

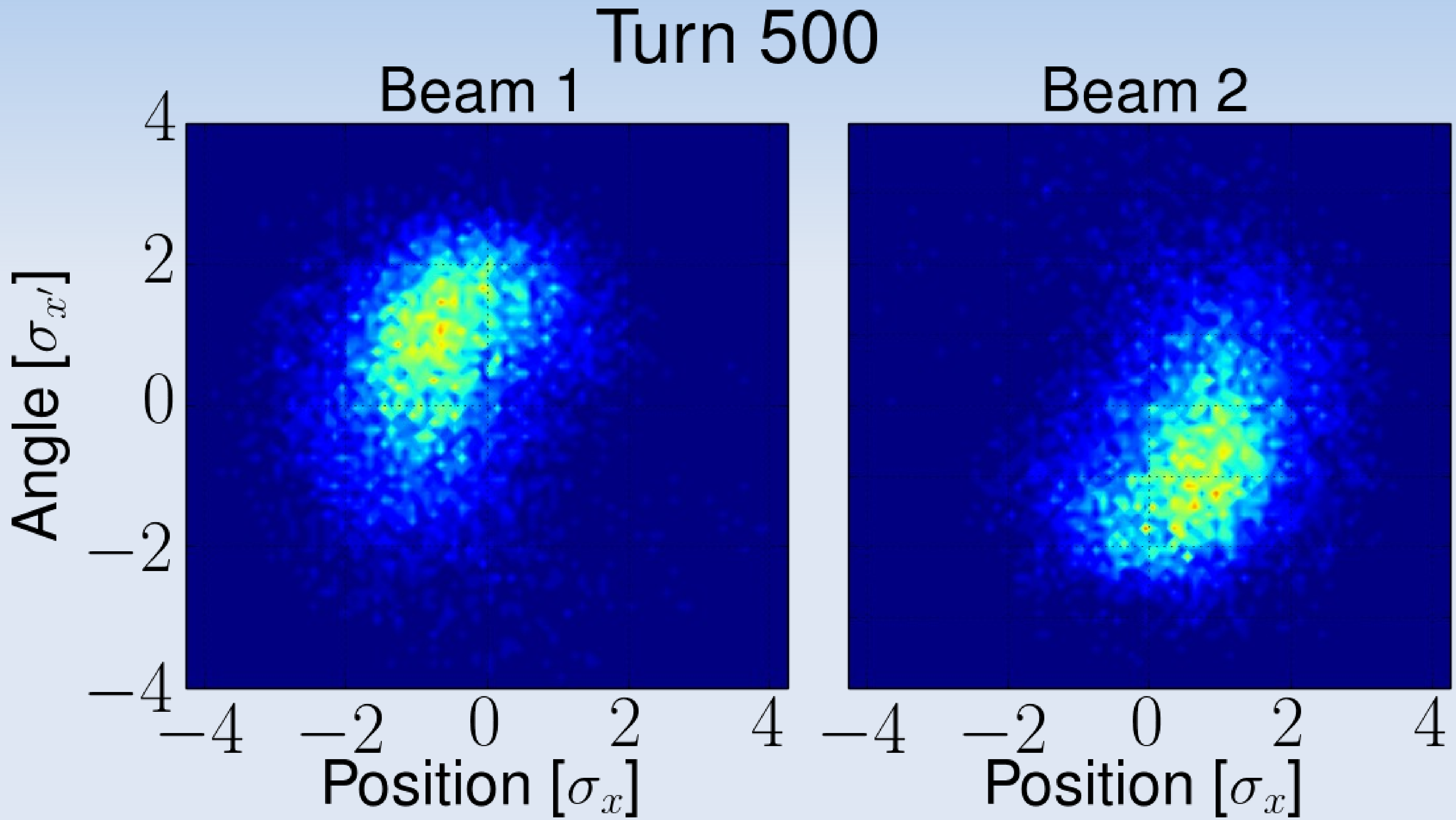

Turn 300
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

π mode

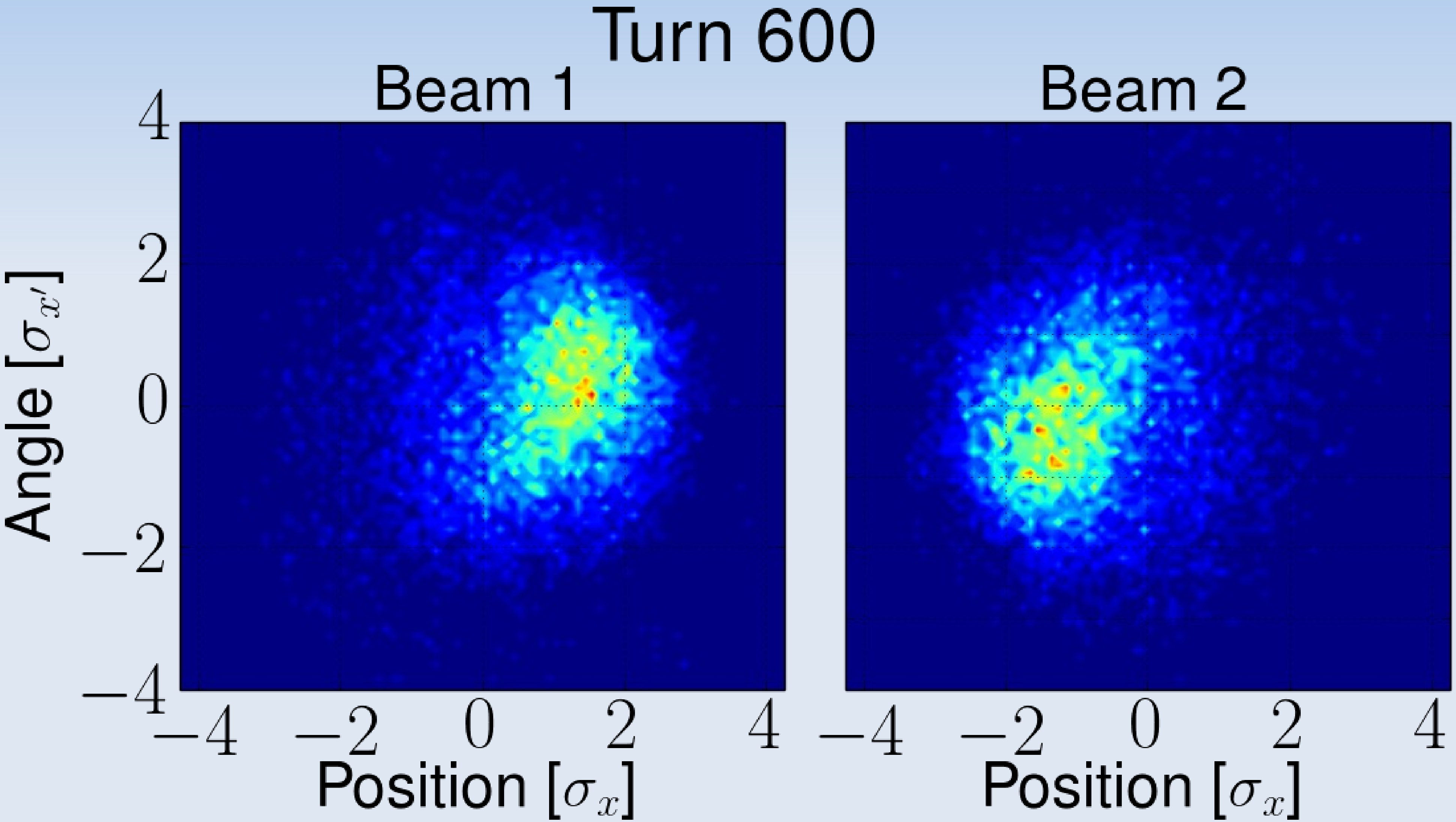


Turn 400
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
4
2
0
−2
−4
−4 −2 0 2 4
Position [σ_x]
−4 −2 0 2 4
Position [σ_x]

π mode

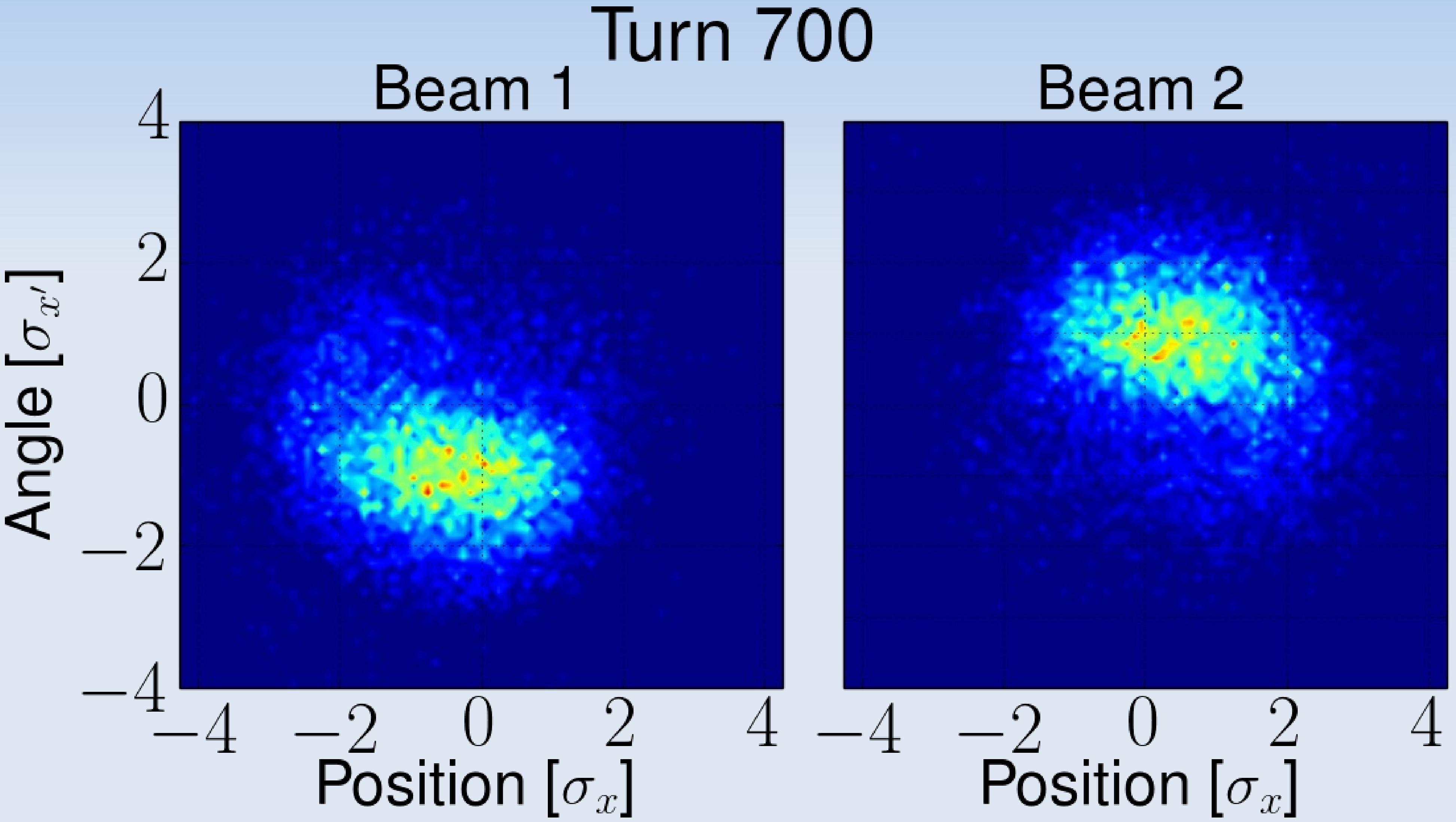

Turn 500
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

π mode



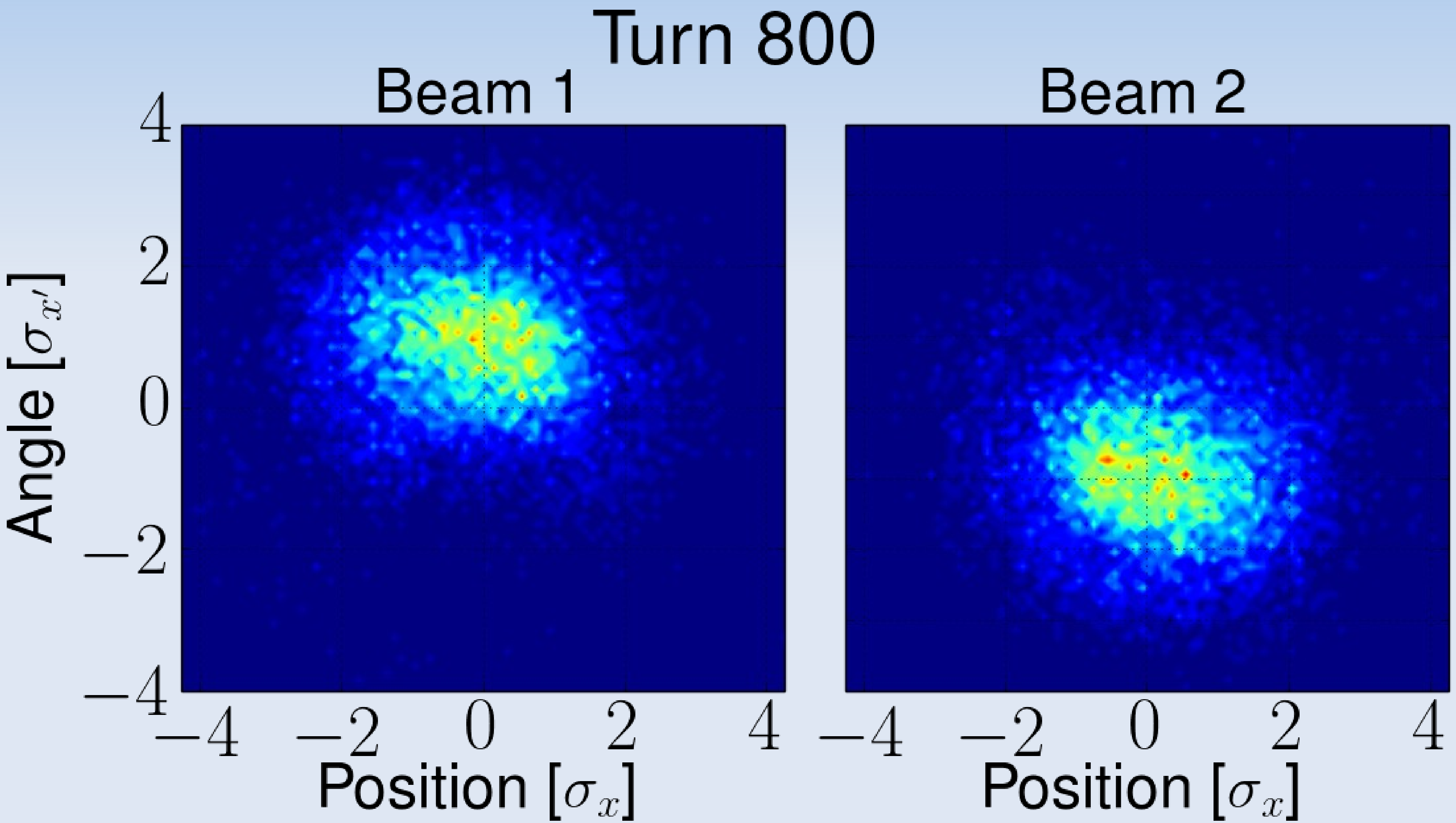
Turn 600
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
4
2
0
−2
−4
−4
−2
0
2
4
Position [σ_x]
−4
−2
0
2
4
Position [σ_x]

π mode

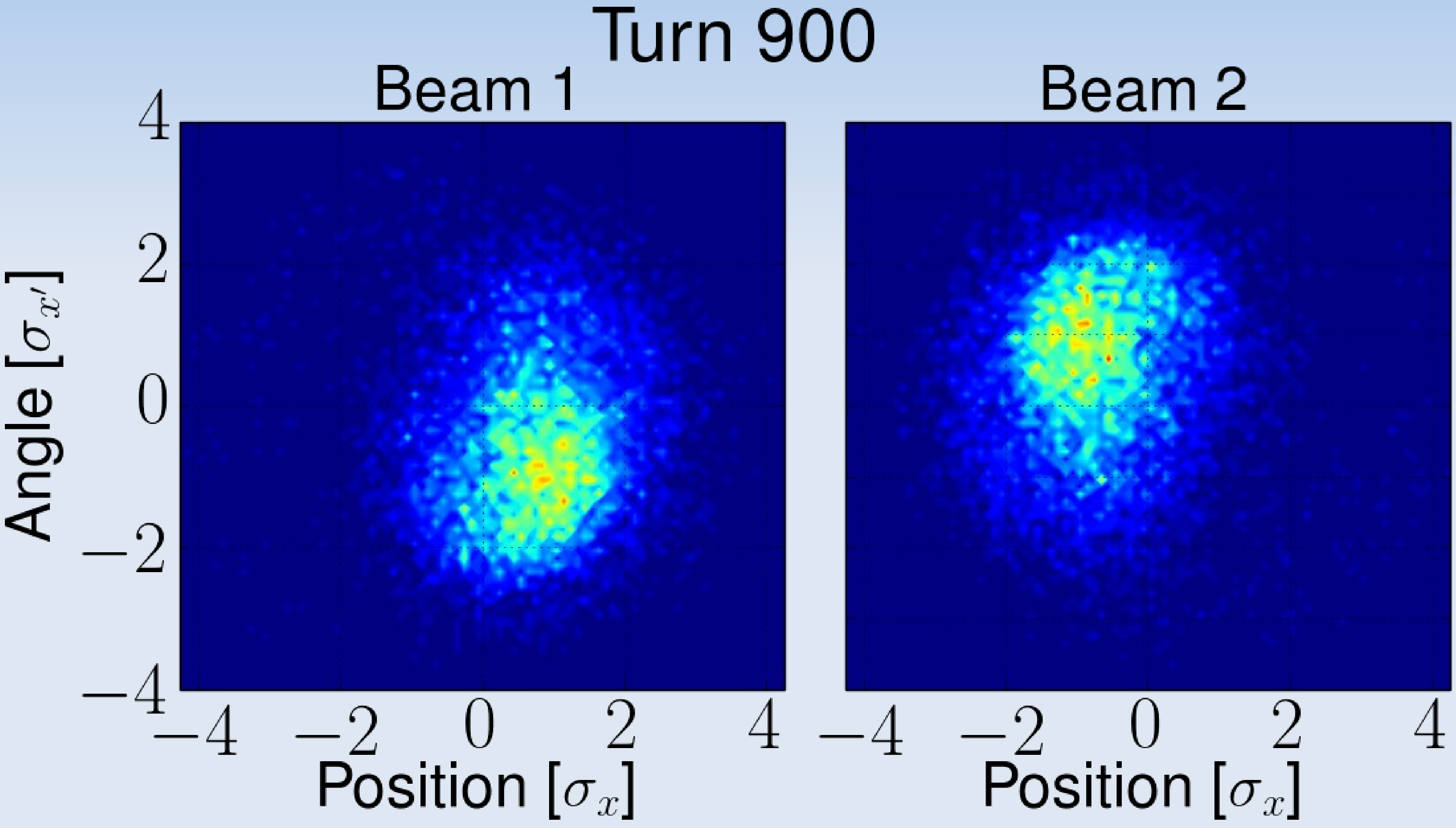


Turn 700
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
4
2
0
−2
−4
−4
−2
0
2
4
Position [σ_x]
−4
−2
0
2
4
Position [σ_x]

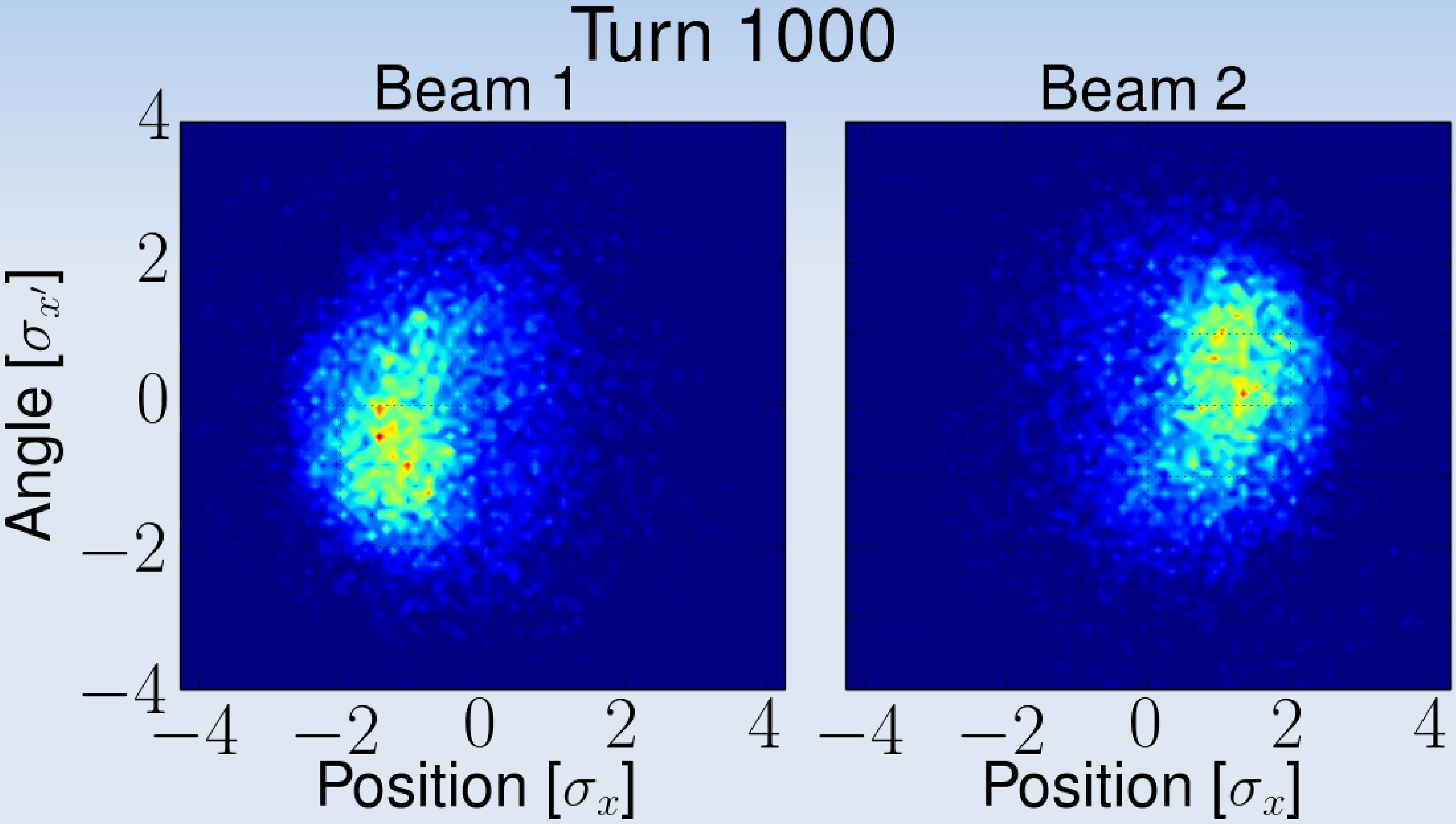
π mode


Turn 800
Beam 1
Beam 2
Angle $[\sigma_{x'}]$
Position $[\sigma_x]$
Position $[\sigma_x]$
4
2
0
−2
−4
−4
−2
0
2
4

π mode


Turn 900
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4

π mode

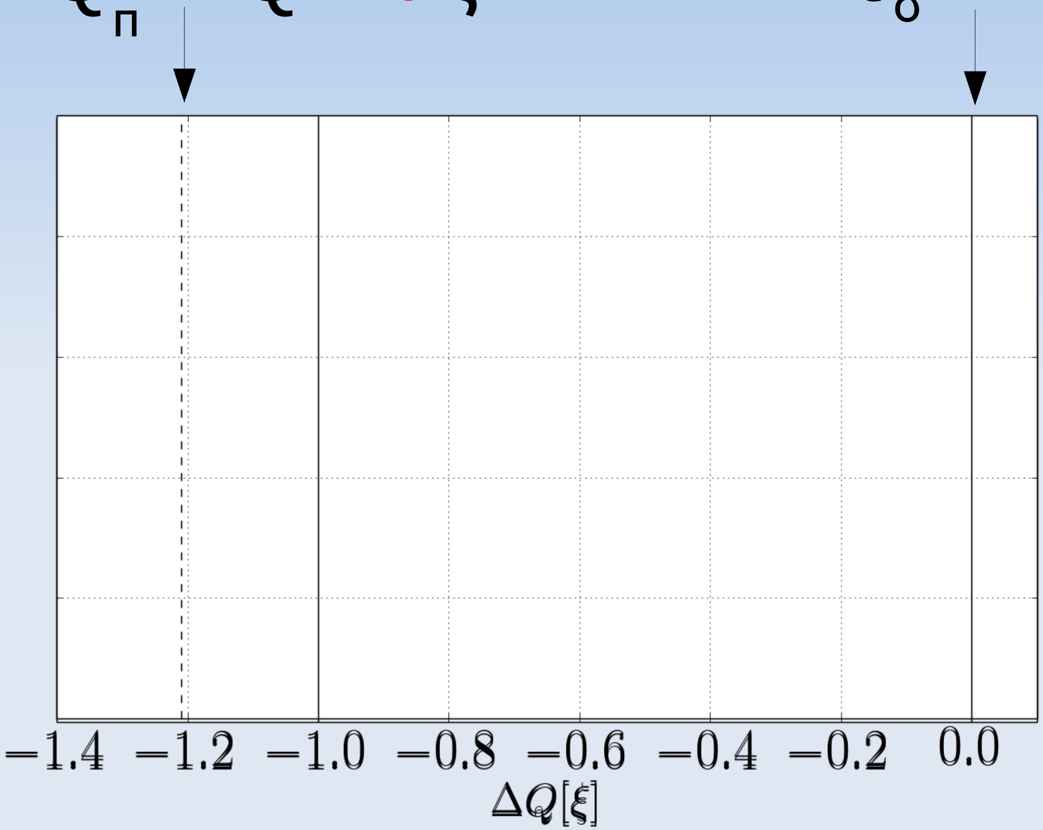


Turn 1000
Beam 1
Beam 2
Angle [$\sigma_{x'}$]
Position [σ_x]
Position [σ_x]
4
2
0
−2
−4
−4
−2
0
2
4
−4
−2
0
2
4

Coherent mode spectrum

Self-consistent computation :

$Q_{\pi} = Q - Y\xi$ $\qquad$ $Q_{\sigma} = Q$



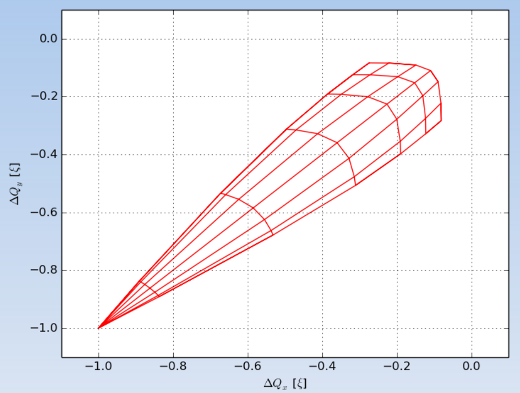
- The Yokoya factor Y is usually between 1.0 and 1.3 depending on the type of interaction (Flat, round, asymmetric, long-range, …) (1)

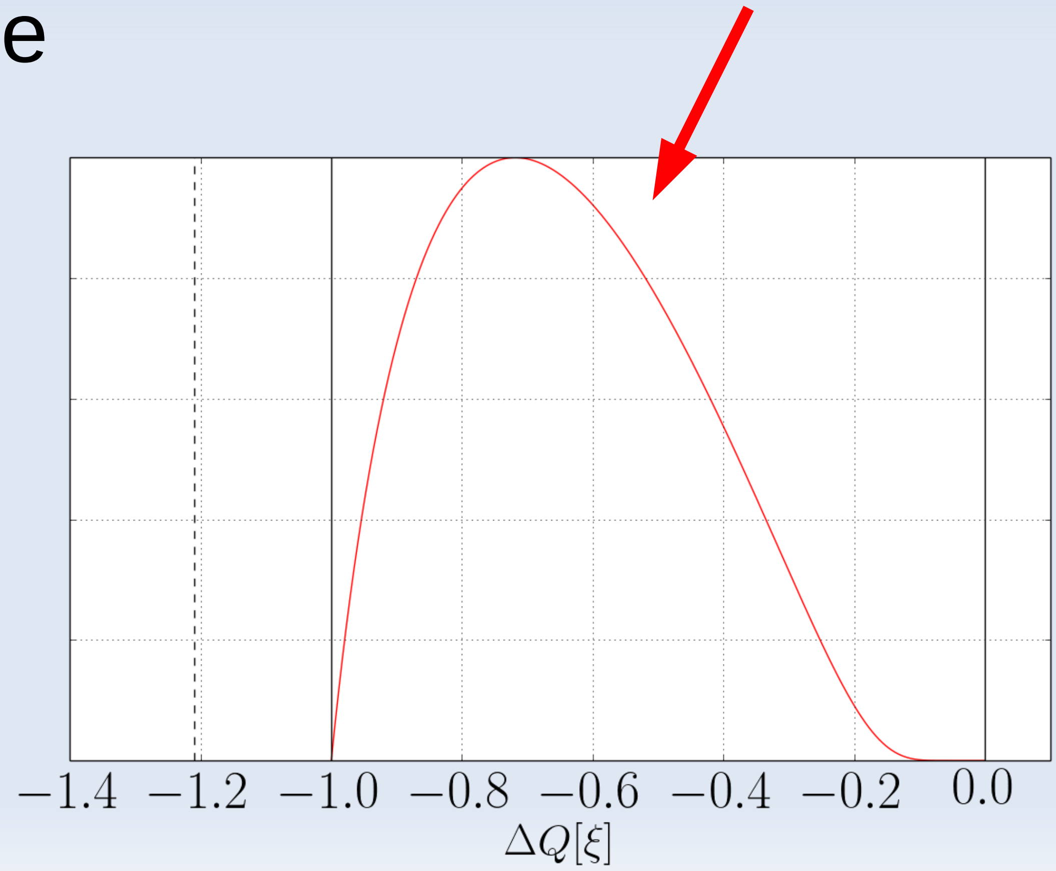
(in)coherent spectrum

- The non-linearity of beam-beam interactions result in a strong amplitude detuning
- The single particles generate a continuum of modes, the *incoherent spectrum*
- Both the σ and π mode are outside the incoherent spectrum

→ Absence of Landau damping

→ Improved feedback efficiency to prevent decoherence





Efficiency of the feedback to suppress emittance growth

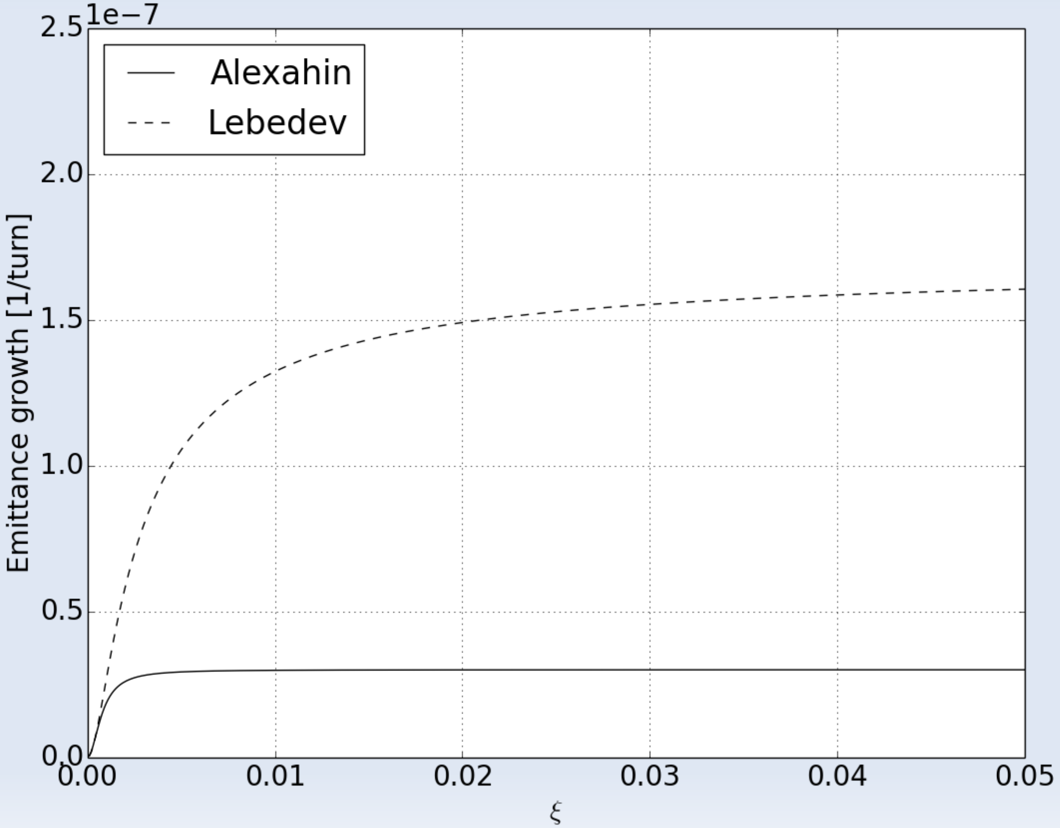
Lebedev's weak-strong model (3) :

$$\frac{1}{\epsilon_0}\frac{d\epsilon}{dt}=\left\langle \frac{\Delta^2}{2}\frac{4\pi^2\left(1-\frac{g}{2}\right)^2\Delta Q^2}{4\pi^2\left(1-\frac{g}{2}\right)^2\Delta Q^2+\left(\frac{g}{2}\right)^2}\right\rangle$$

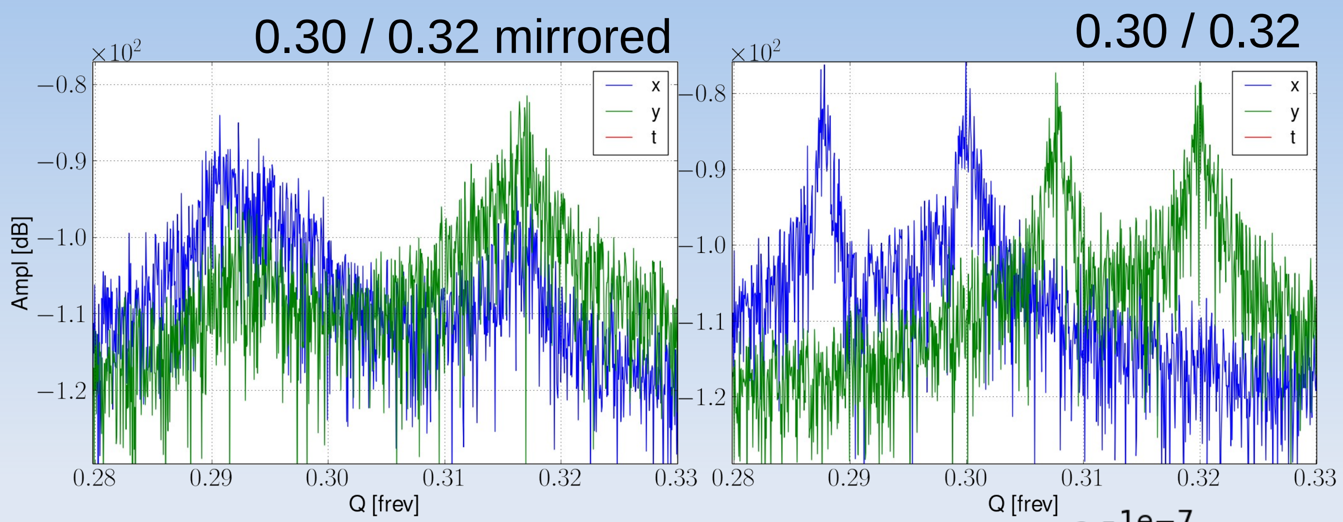
Alexahin's strong-strong model (4) :

$$\frac{1}{\epsilon_0}\frac{d\epsilon}{dt}=\frac{\Delta^2(1-s_0)}{4\left(1+\frac{g}{2\pi\xi}\right)^2}$$

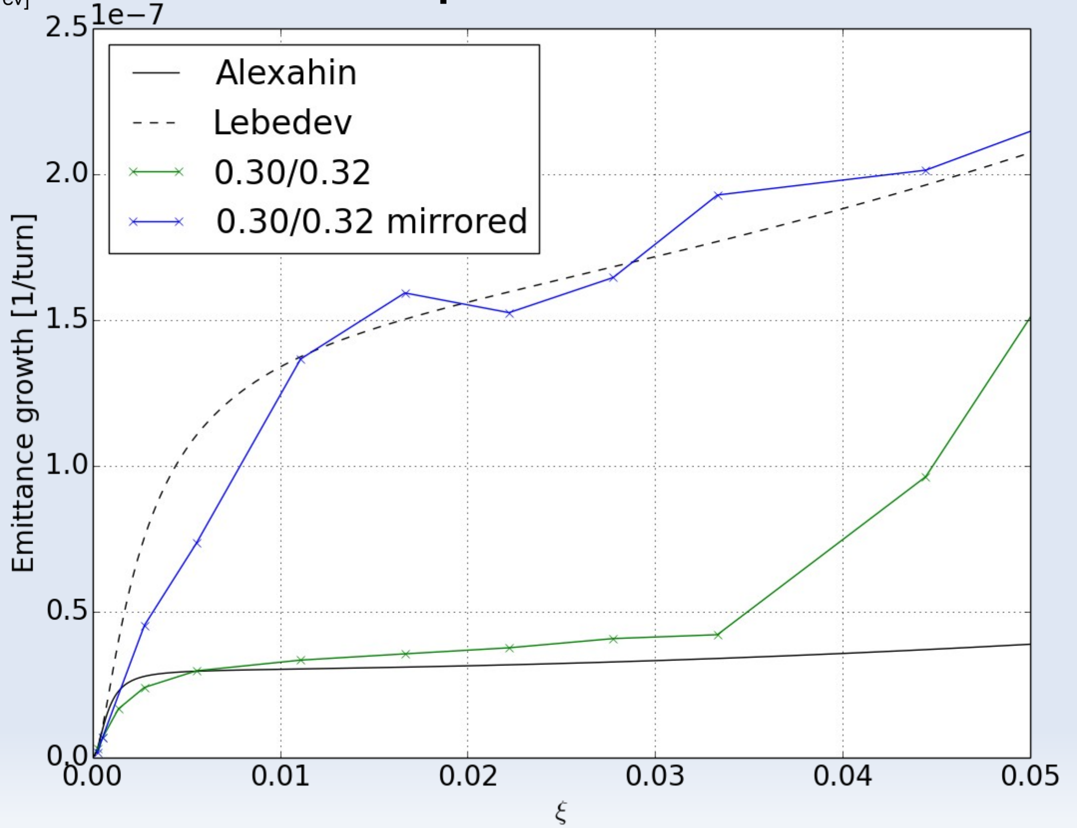
- When ξ >> g, the Alexahin's model predicts a significant reduction of the emittance growth due to decoherence with respect to the Lebedev's model
- Alexahin's formula is predicted to break down when coherent modes enter the incoherent spectrum (4)



Mirrored tune



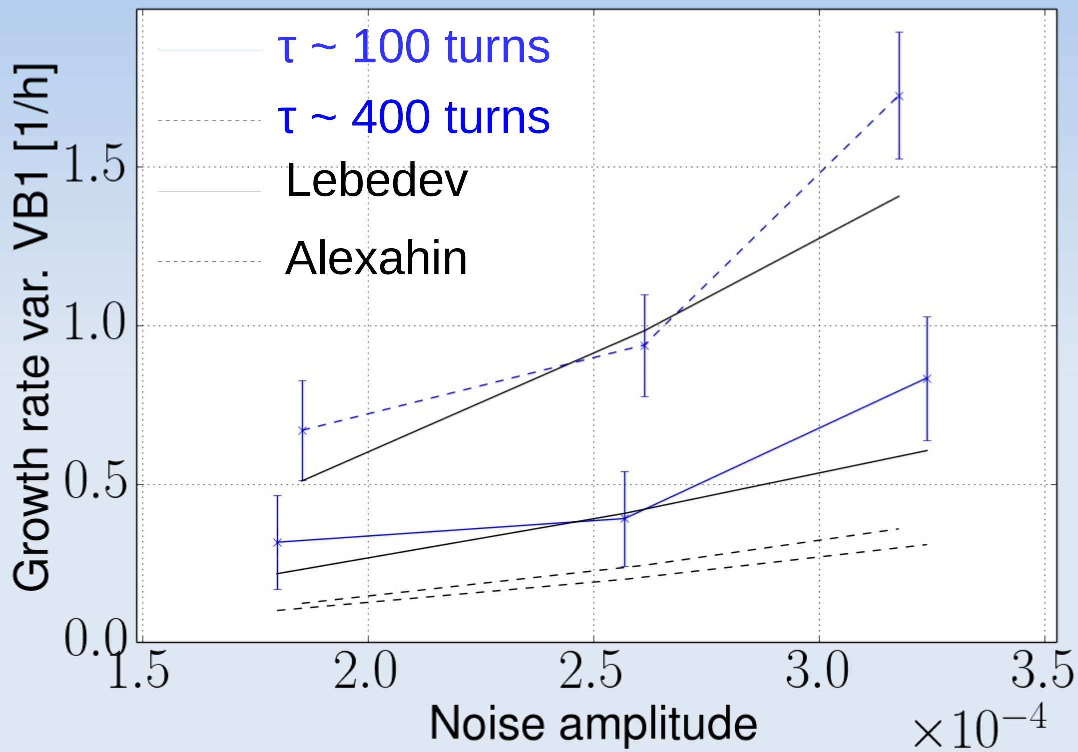
- Mirroring the tune of the two beams moves all coherent modes inside the incoherent spectrum



- Despite the strong-strong nature of the configuration :
  - Alexahin forumla does not apply due to the interaction of coherent and incoherent spectrum
  - Lebedev's weak-strong forumla is accurate
- In a realistic configurations, the two models provide upper/lower bounds for the emittance growth

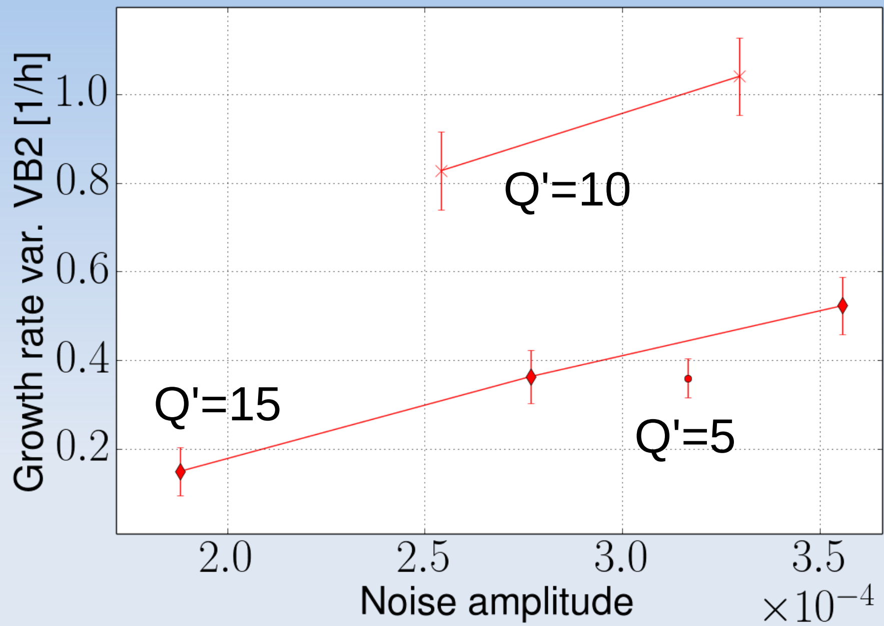
Measurement at the LHC

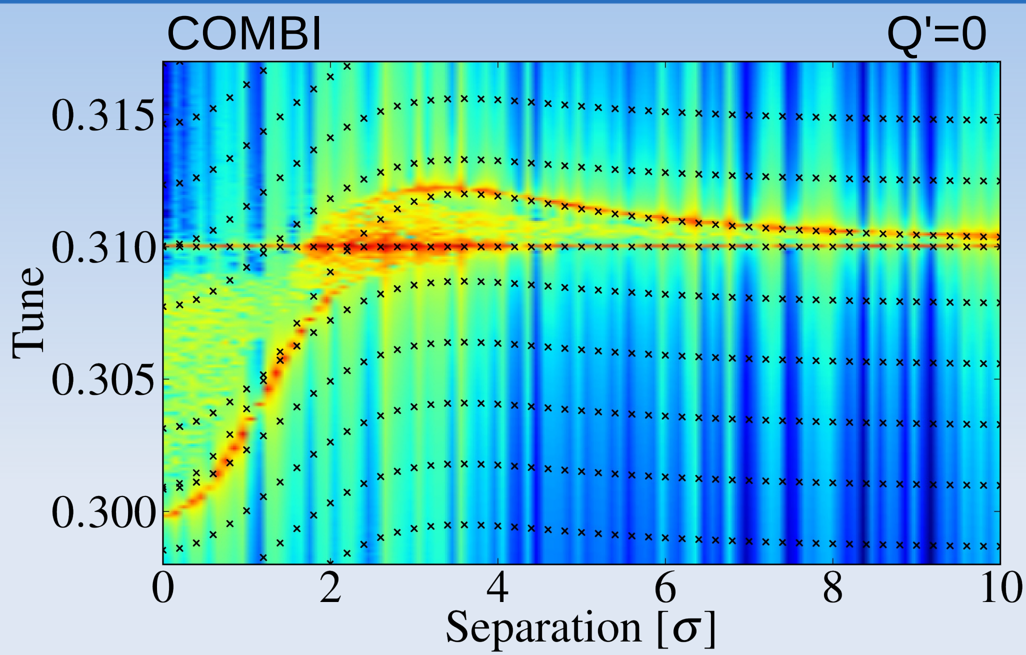
ΔQ~0.01



- The emittance growth measured when introducing controlled noise on colliding bunches experiencing different gains is compatible with Lebedev's formula but not Alexahin's, despite the strong-strong configuration
  - Several effects may bring the coherent modes inside the incoherent spectrum, even in simple configurations (11)

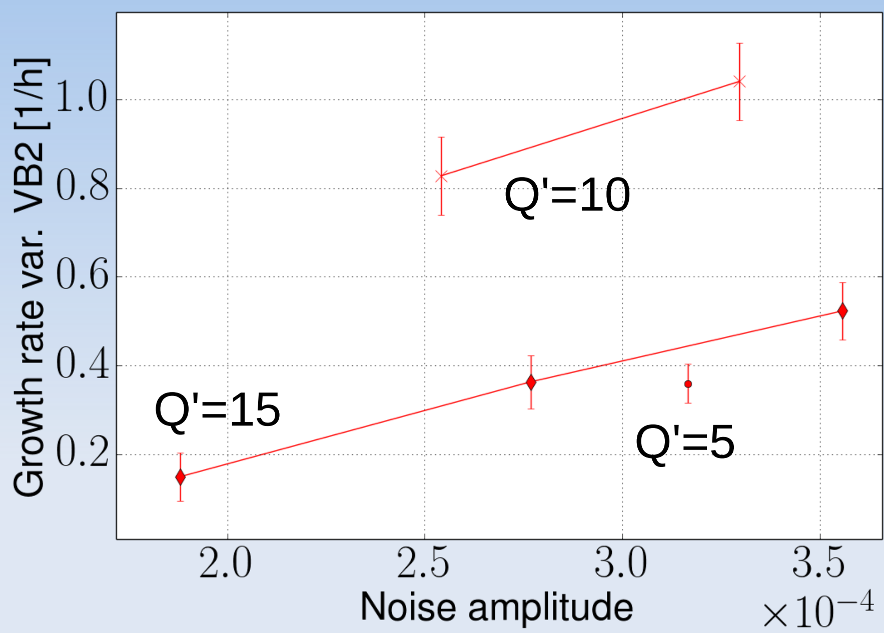
Effect of chromaticity

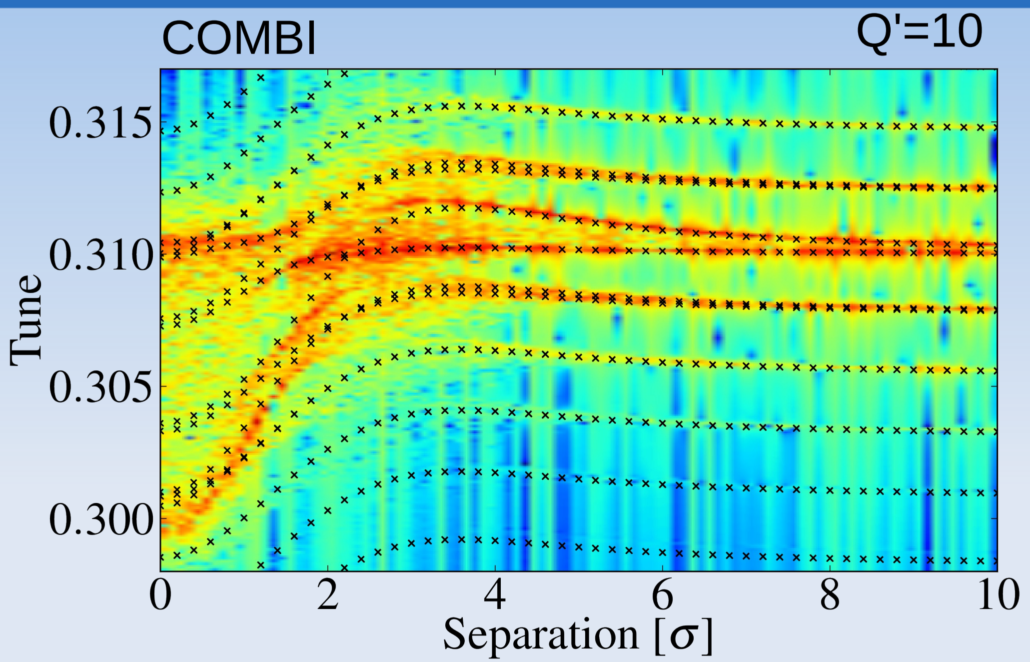




- The measured impact of chromaticity is non-trivial

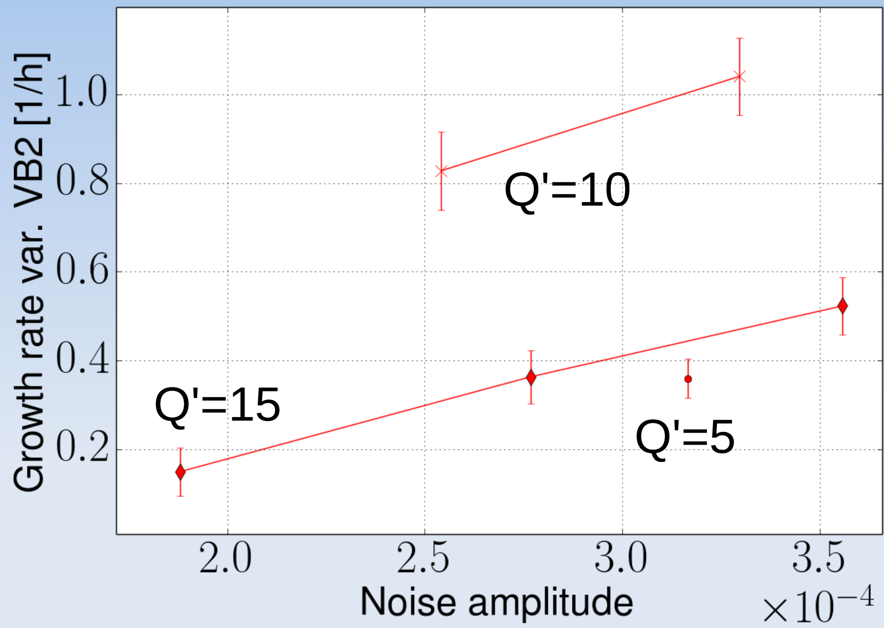
Effect of chromaticity

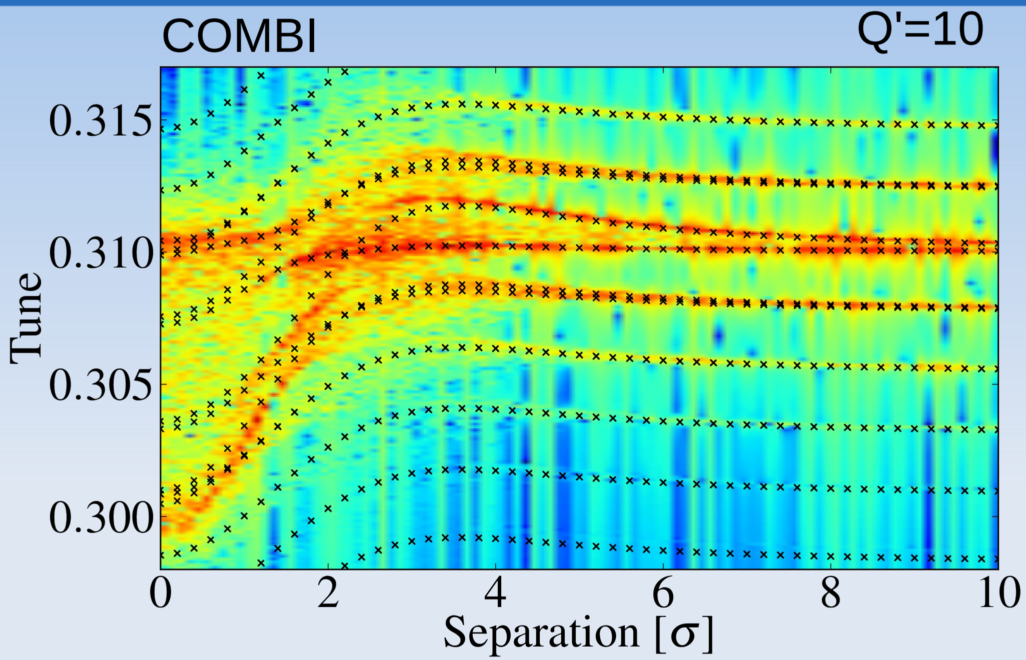




- The measured impact of chromaticity is non-trivial

Effect of chromaticity

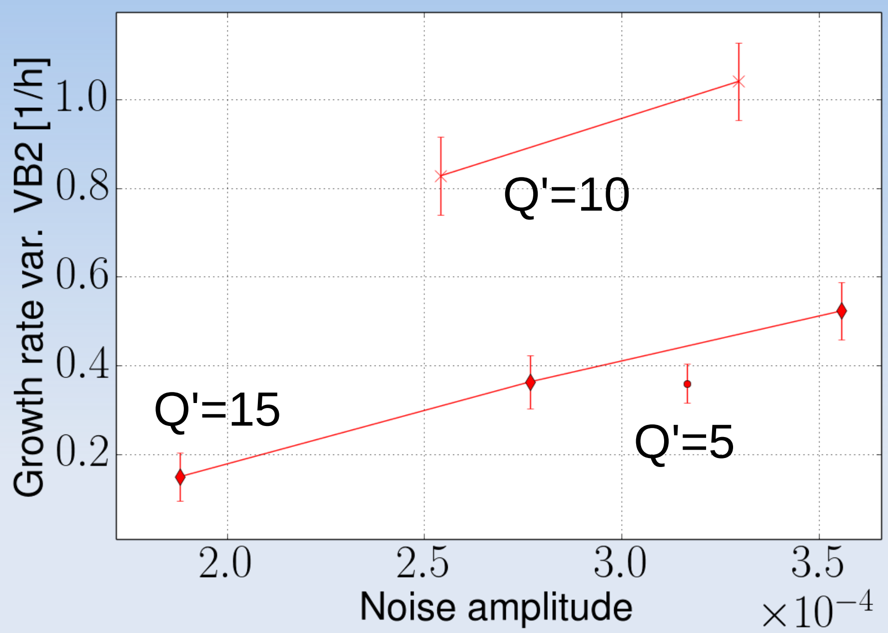


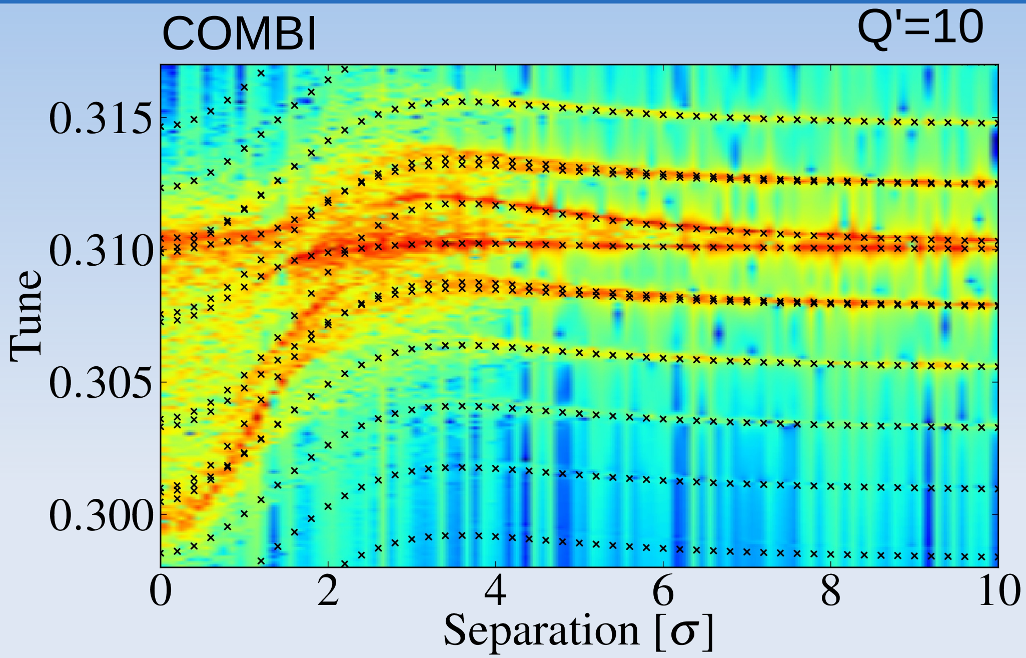


- The measured impact of chromaticity is non-trivial

  → This experiment confirms the difficulty to achieve the S-S mechanism for the reduction of the emittance growth, even in the S-S regime

Effect of chromaticity





- The measured impact of chromaticity is non-trivial

  → This experiment confirms the difficulty to achieve the S-S mechanism for the reduction of the emittance growth, even in the S-S regime

  → HL-LHC design is conservatively based on the W-S model

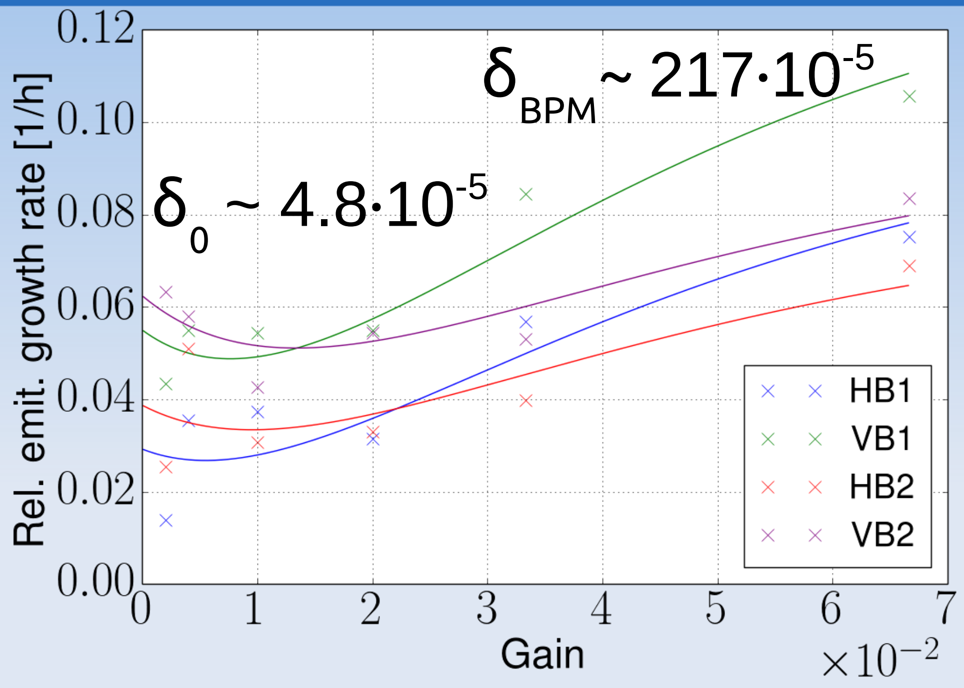
Identifying the noise source

“...Operation of the feedback in presence of strong non-linearities, such as octupoles, must be avoided...”, LHC design report

Identifying the noise source

"...Operation of the feedback in presence of strong non-linearities, such as octupoles, must be avoided...", LHC design report

$$\frac{1}{\epsilon}\frac{d\epsilon}{dt} = \frac{1}{2}\left(\delta_0^2 + G^2\delta_{BPM}^2\right)\left\langle \frac{4\pi^2\left(1-\frac{G}{2}\right)^2\Delta Q^2}{4\pi^2\left(1-\frac{G}{2}\right)\Delta Q^2 + \left(\frac{G}{2}\right)^2} \right\rangle$$



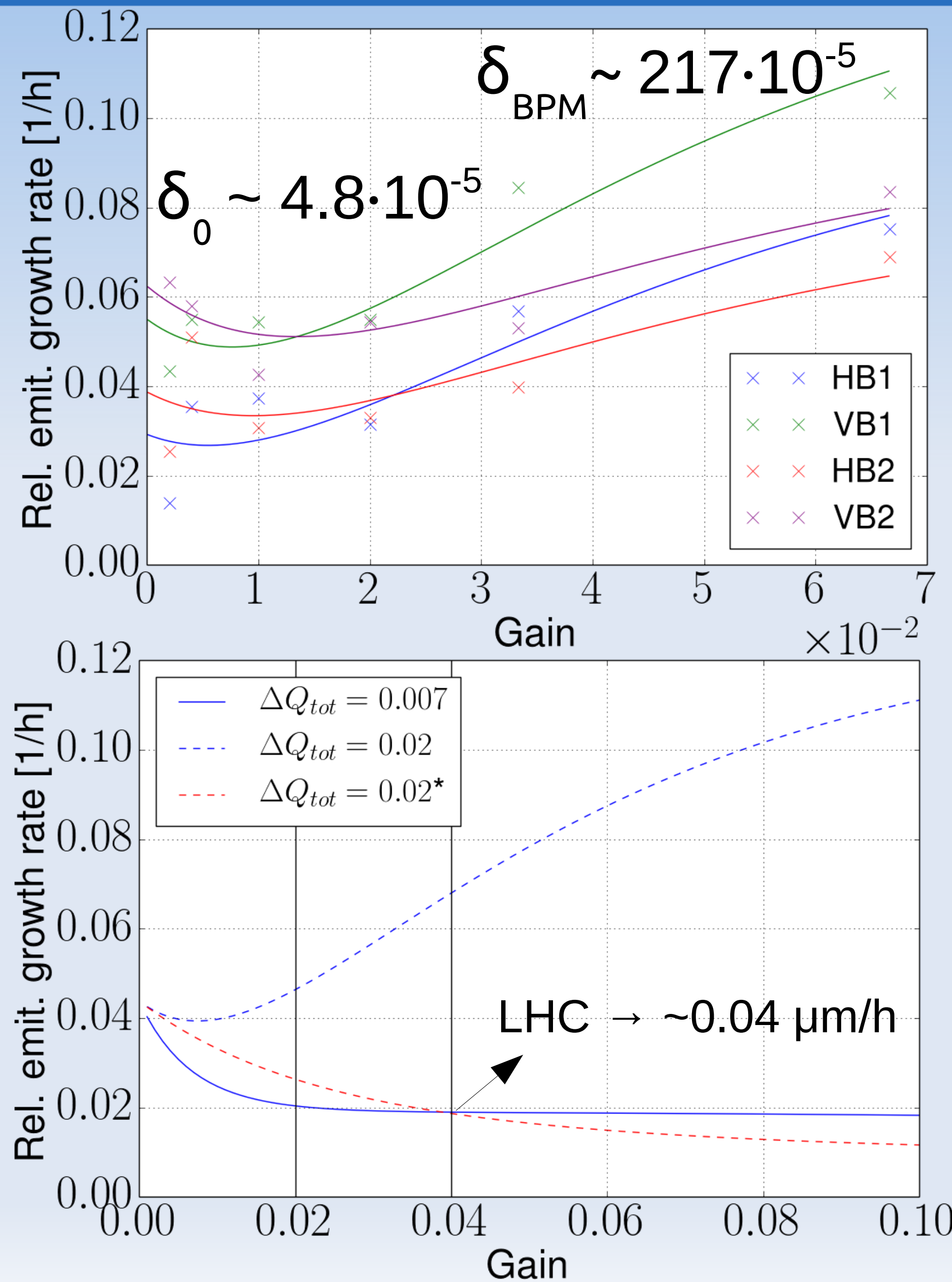
Identifying the noise source

"...Operation of the feedback in presence of strong non-linearities, such as octupoles, must be avoided...", LHC design report

$$\frac{1}{\epsilon}\frac{d\epsilon}{dt} = \frac{1}{2}\left(\delta_0^2 + G^2\delta_{BPM}^2\right)\left\langle \frac{4\pi^2\left(1-\frac{G}{2}\right)^2 \Delta Q^2}{4\pi^2\left(1-\frac{G}{2}\right)\Delta Q^2 + \left(\frac{G}{2}\right)^2} \right\rangle$$

- In the LHC the feedback is efficient at suppressing the emittance growth from other sources (PC ripple)

  → High gain favourable



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"...Operation of the feedback in presence of strong non-linearities, such as octupoles, must be avoided...", LHC design report

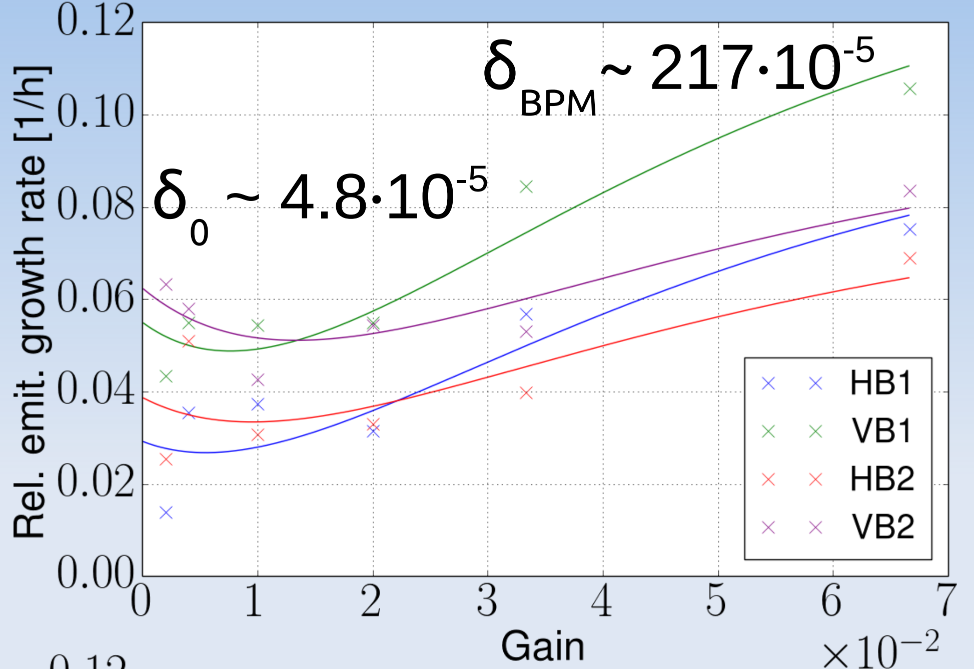
$$\frac{1}{\epsilon}\frac{d\epsilon}{dt} = \frac{1}{2}\left(\delta_0^2 + G^2\delta_{BPM}^2\right)\left\langle \frac{4\pi^2\left(1-\frac{G}{2}\right)^2\Delta Q^2}{4\pi^2\left(1-\frac{G}{2}\right)\Delta Q^2 + \left(\frac{G}{2}\right)^2} \right\rangle$$

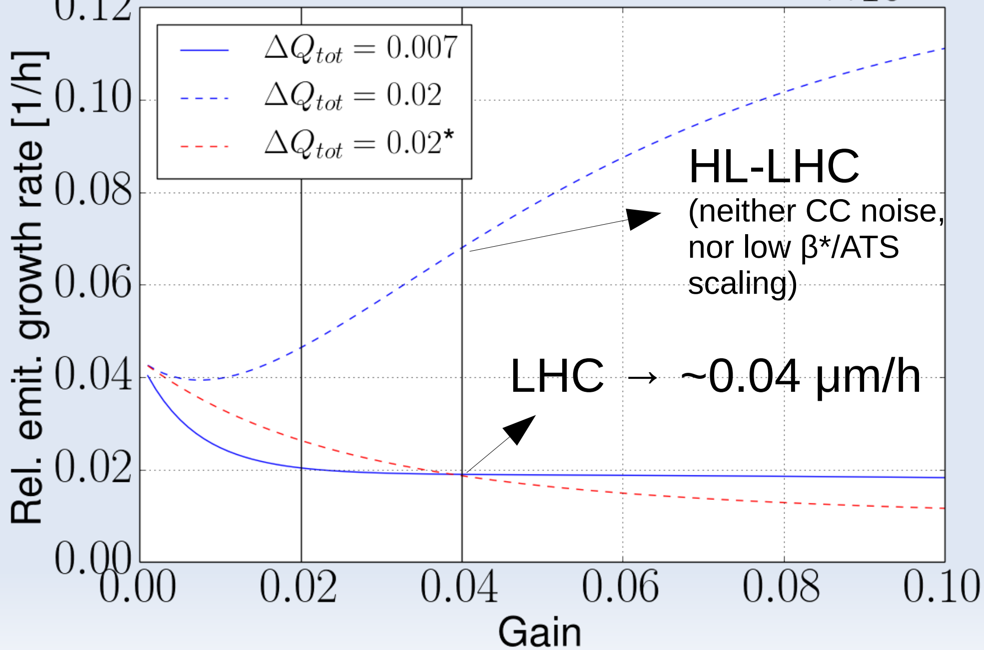
- In the LHC the feedback is efficient at suppressing the emittance growth from other sources (PC ripple)

  → High gain favourable

- In the HL-LHC, it becomes less efficient due to the large tune spread, yet introduces the same noise

  → Low gain favourable





Identifying the noise source

"...Operation of the feedback in presence of strong non-linearities, such as octupoles, must be avoided...", LHC design report

$$\frac{1}{\epsilon}\frac{d\epsilon}{dt} = \frac{1}{2}\left(\delta_0^2 + G^2\delta_{BPM}^2\right)\left\langle \frac{4\pi^2\left(1-\frac{G}{2}\right)^2\Delta Q^2}{4\pi^2\left(1-\frac{G}{2}\right)\Delta Q^2 + \left(\frac{G}{2}\right)^2} \right\rangle$$

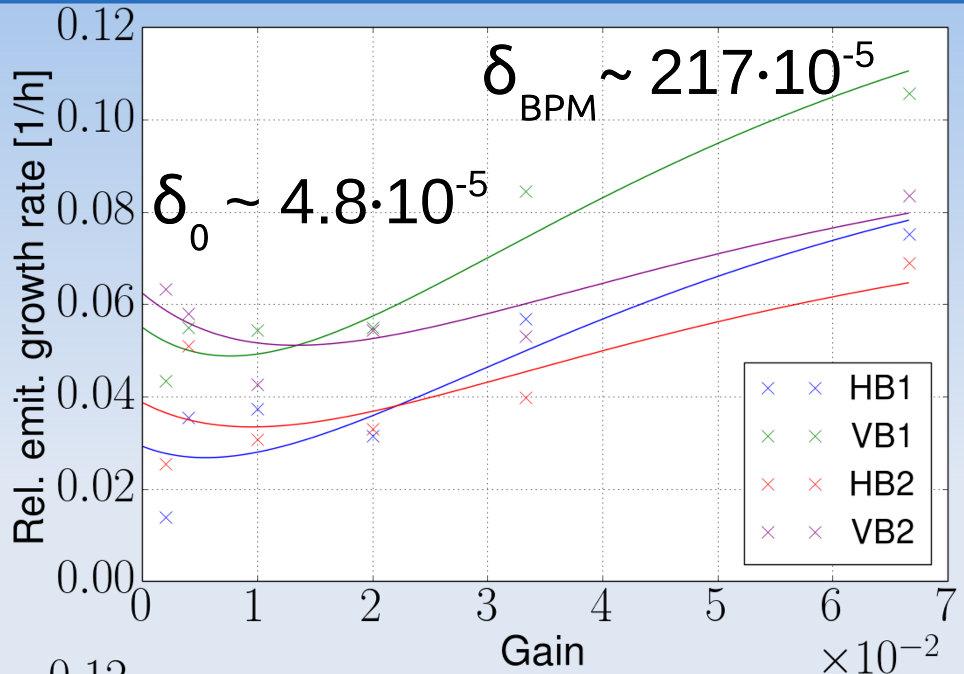
- In the LHC the feedback is efficient at suppressing the emittance growth from other sources (PC ripple)

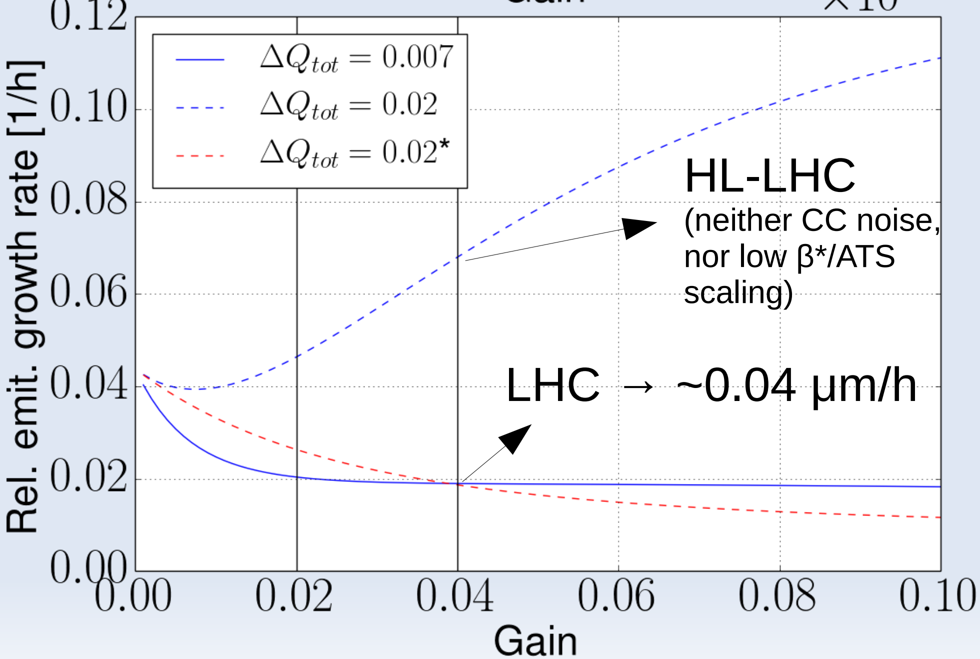
  → High gain favourable

- In the HL-LHC, it becomes less efficient due to the large tune spread, yet introduces the same noise

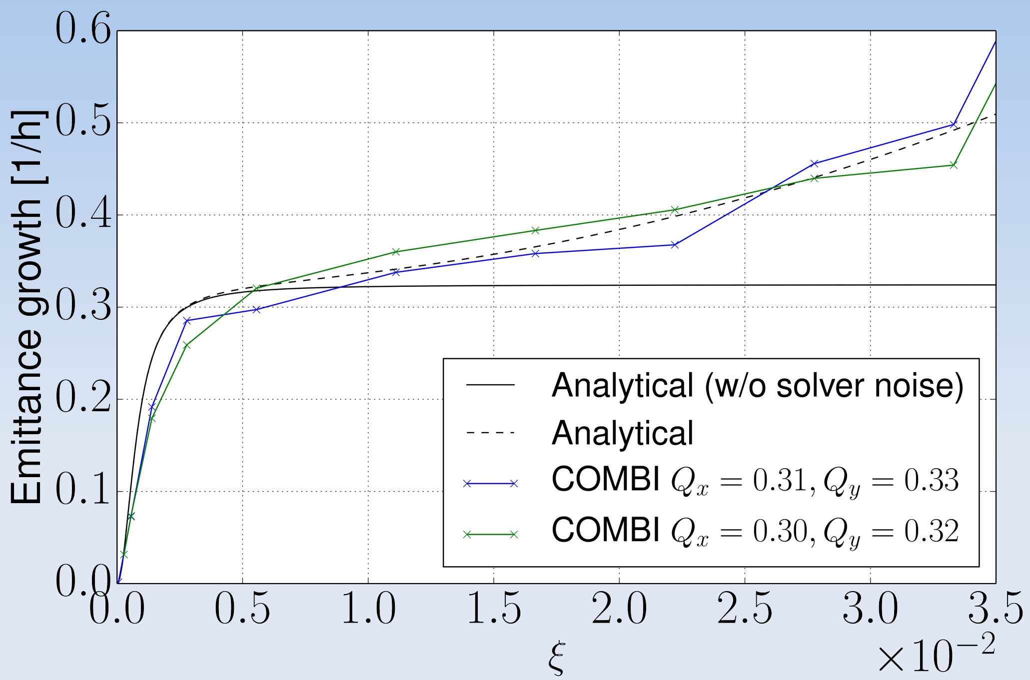
  → Low gain favourable

  → Recution of δ_{BPM} needed to recover the good behaviour



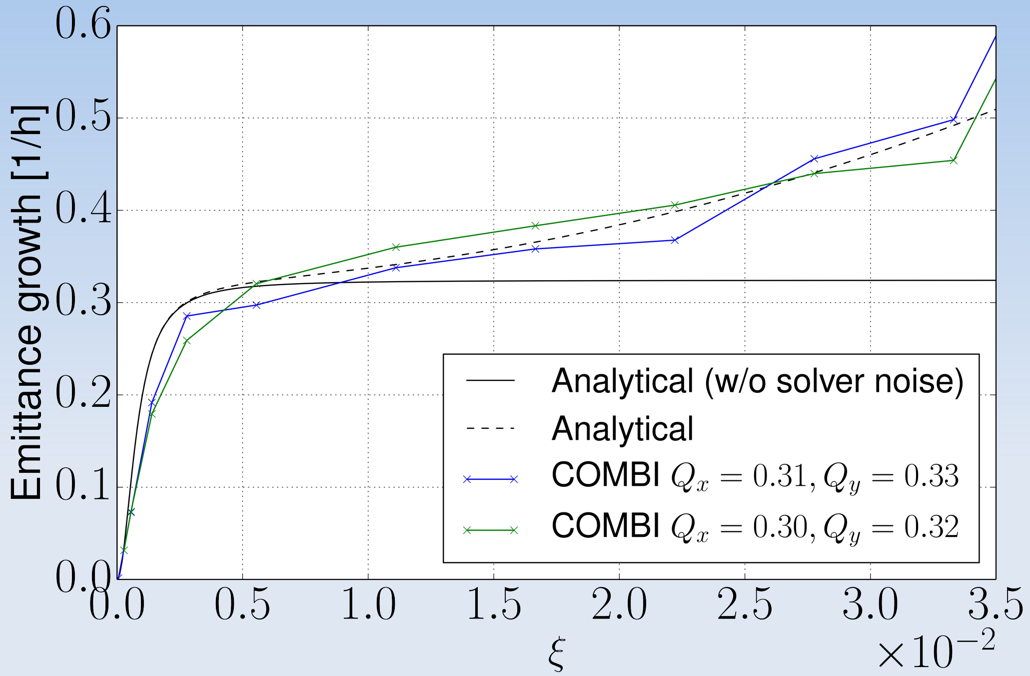


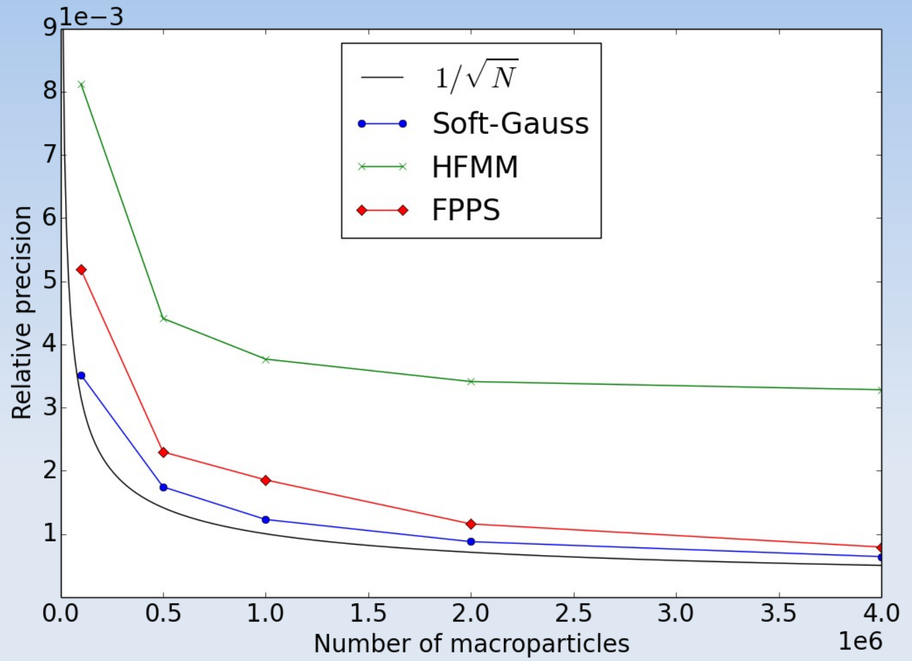
Solver noise



- The noise induced by the field solver is large for configurations with large beam-beam parameters

Solver noise

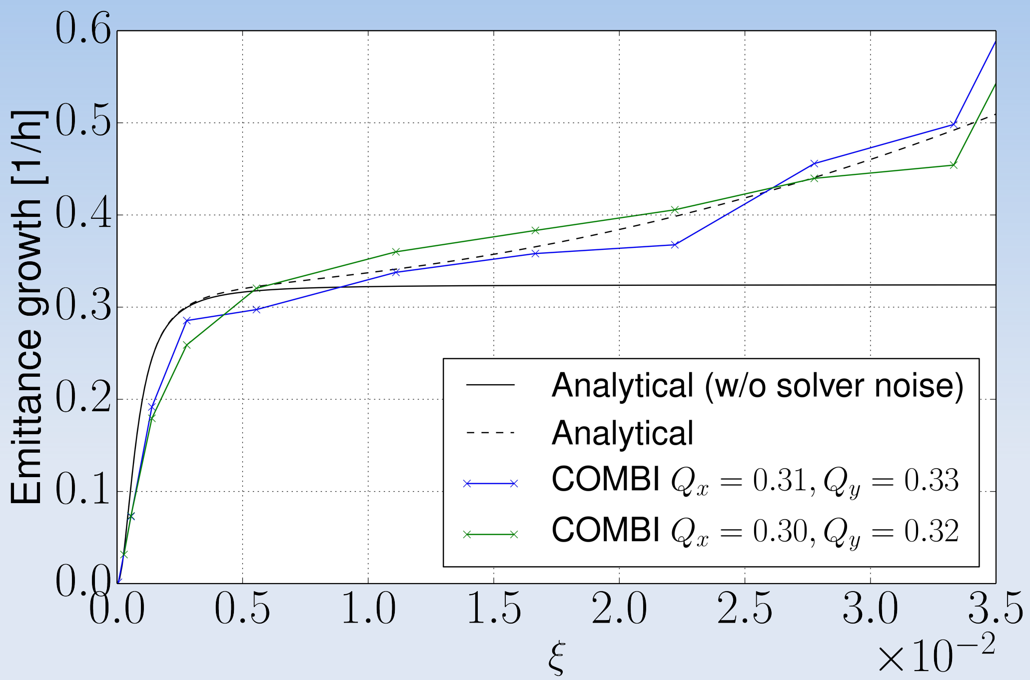


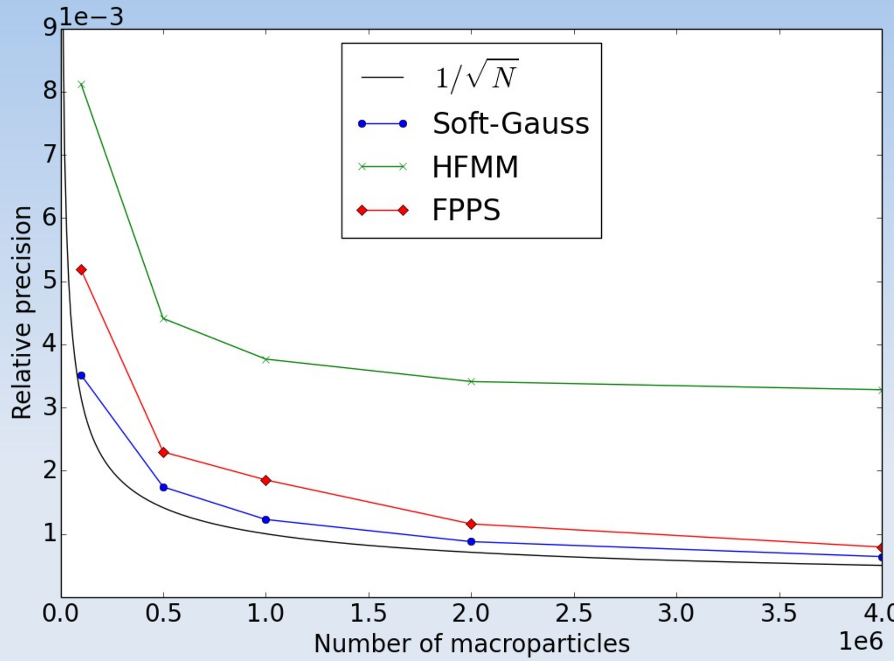


- The noise induced by the field solver is large for configurations with large beam-beam parameters

  → Need large number of macroparticles at the expense of comoutational power

Solver noise





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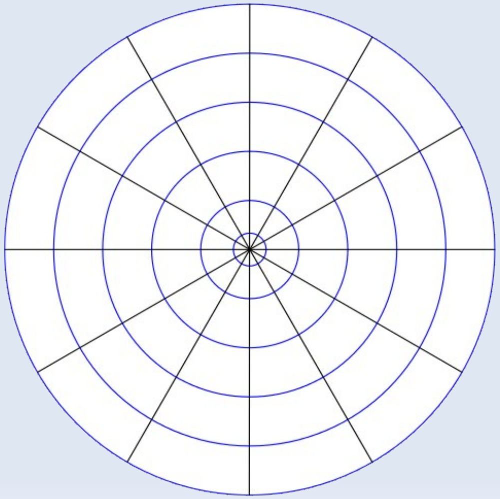
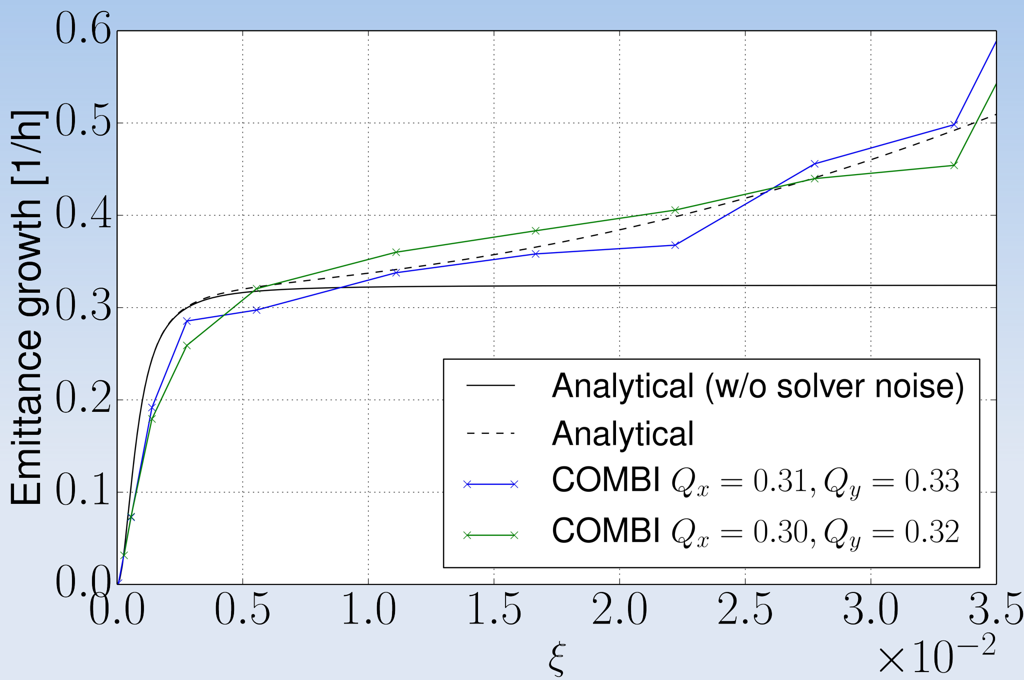
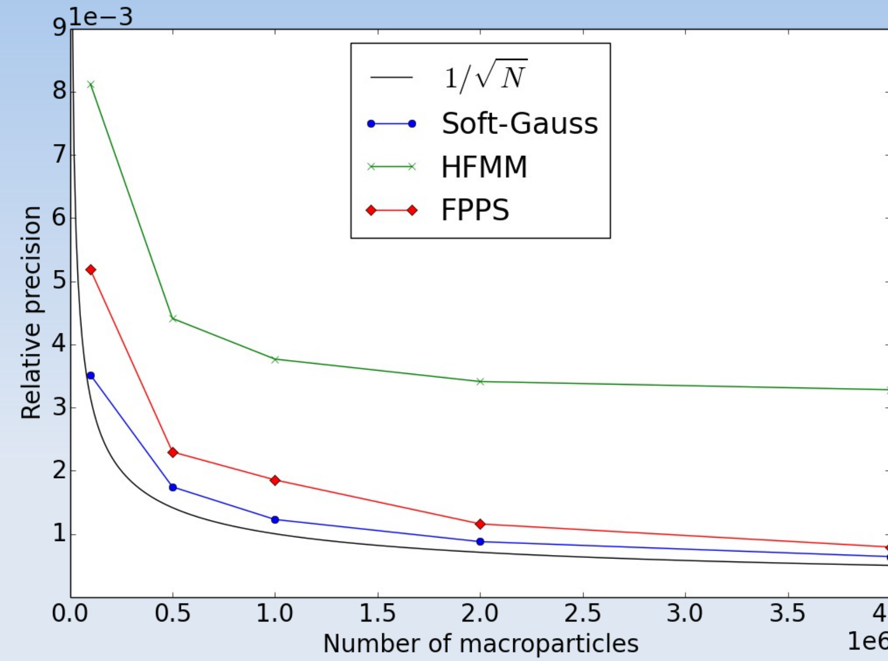


Solver noise





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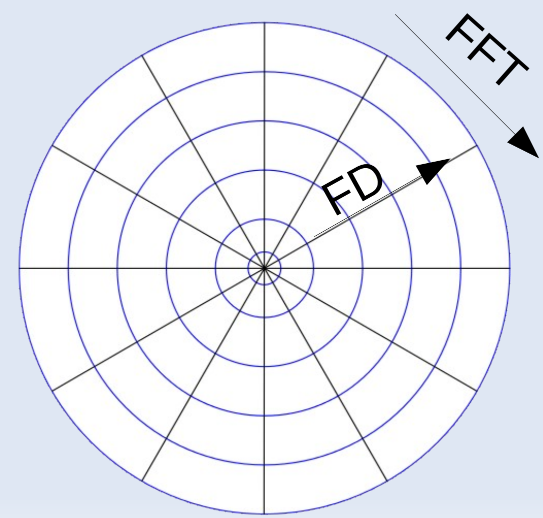

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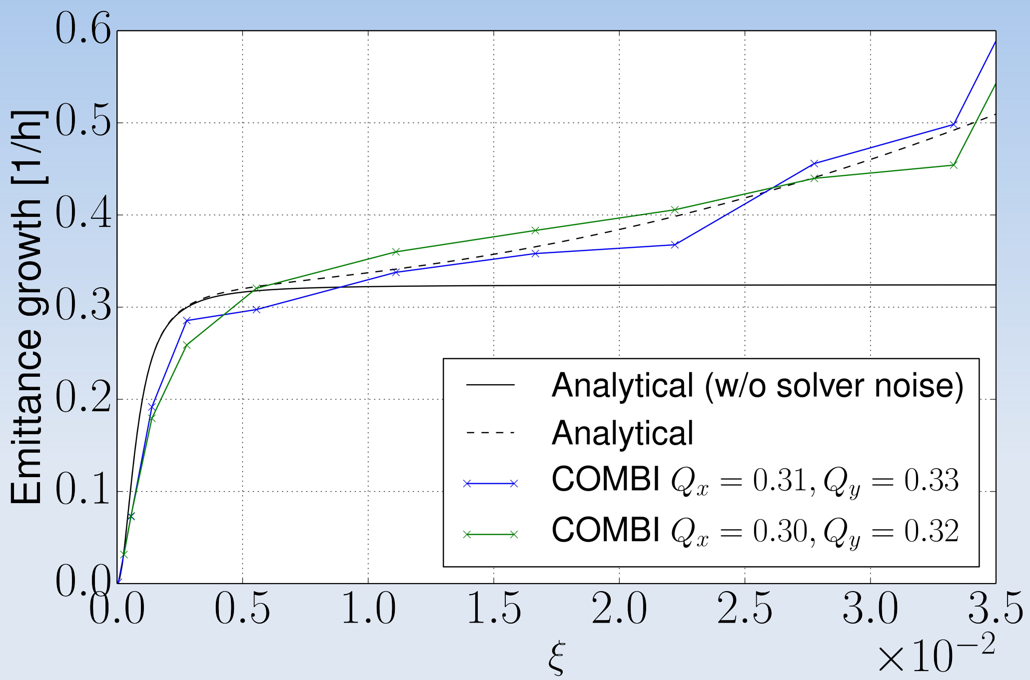

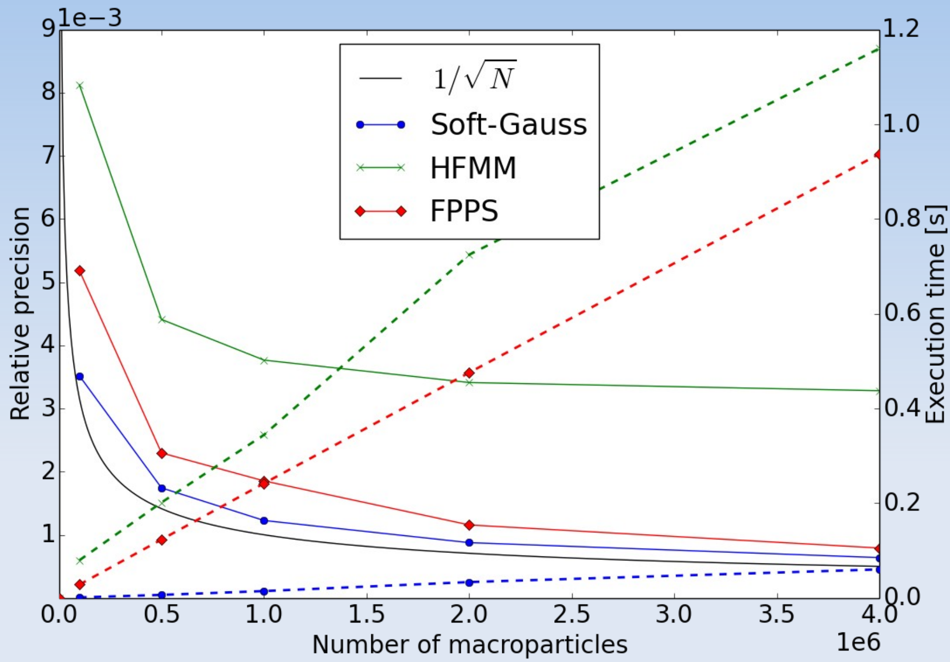

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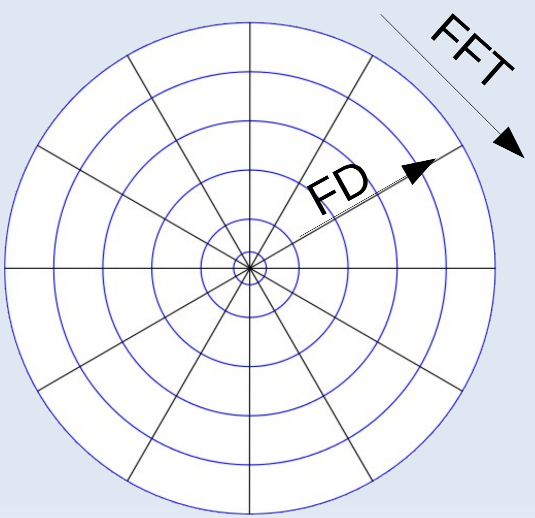
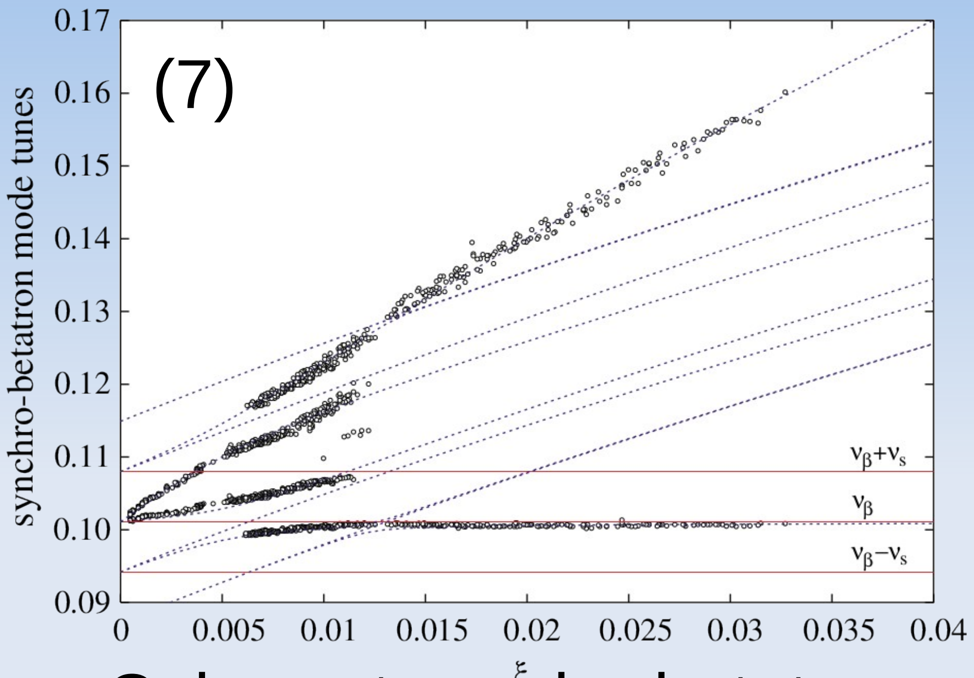
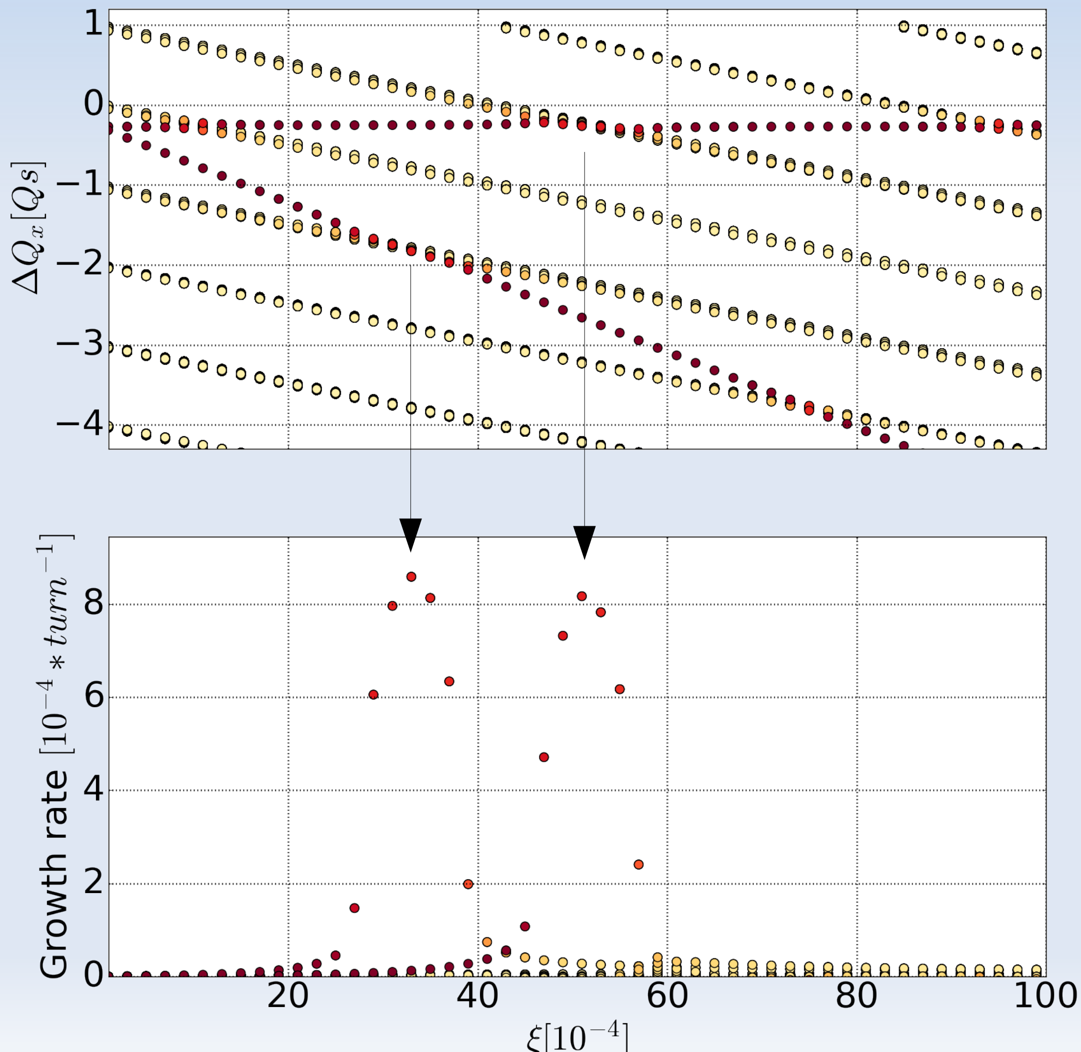


Beam-beam instabilities



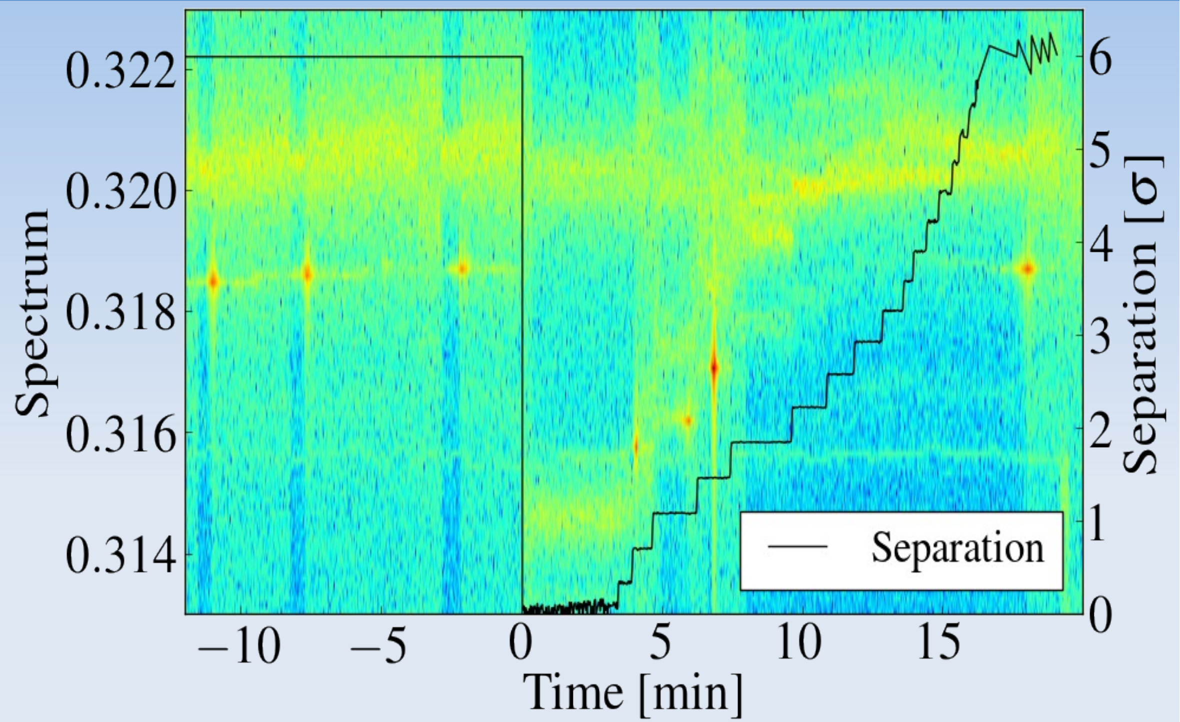
- Coherent synchrobetatron beam-beam modes were predicted based on the circulant matrix model and demonstrated experimentally at VEPP-2000 (7)

- The same model including the impedance showed a TMCI-like instability due to beam-beam interaction (BBMCI) (6)



Measurement at LHC

- Instability observed for intermediate separations
- Stability ensured by the transverse feedback
- In agreement with the models



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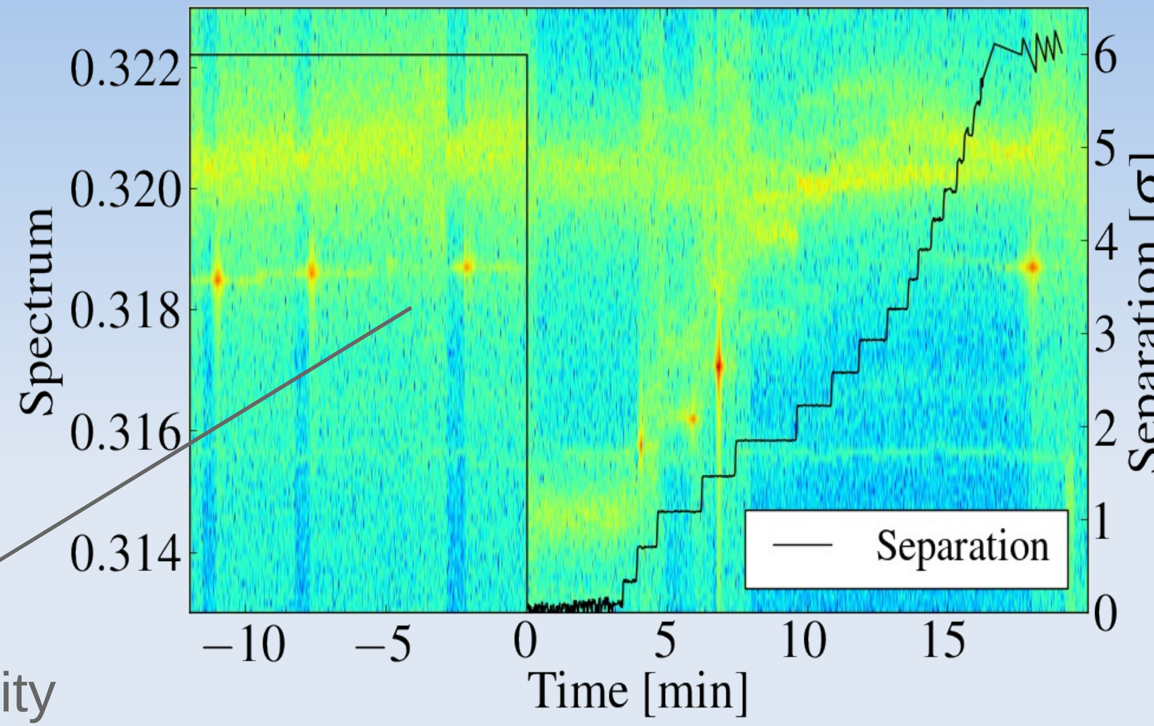
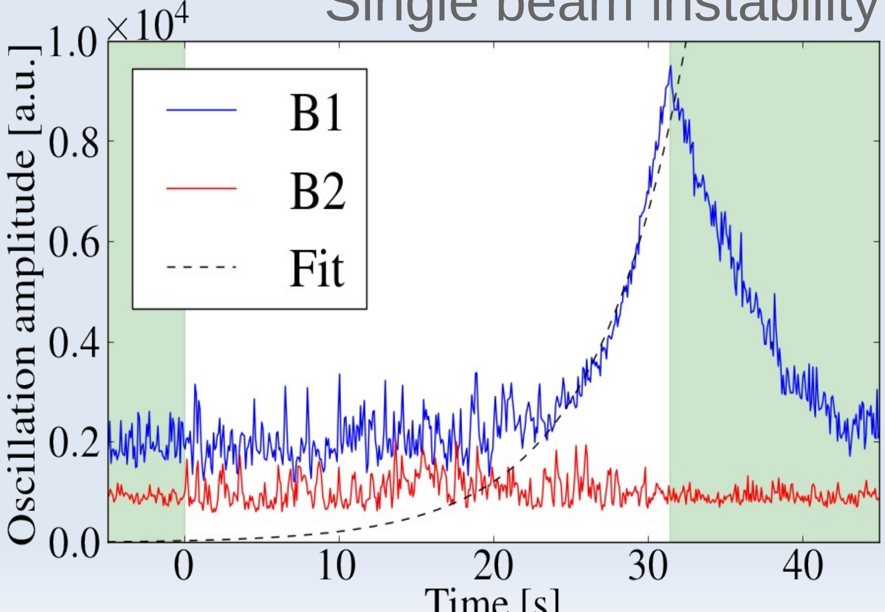


Single beam instability



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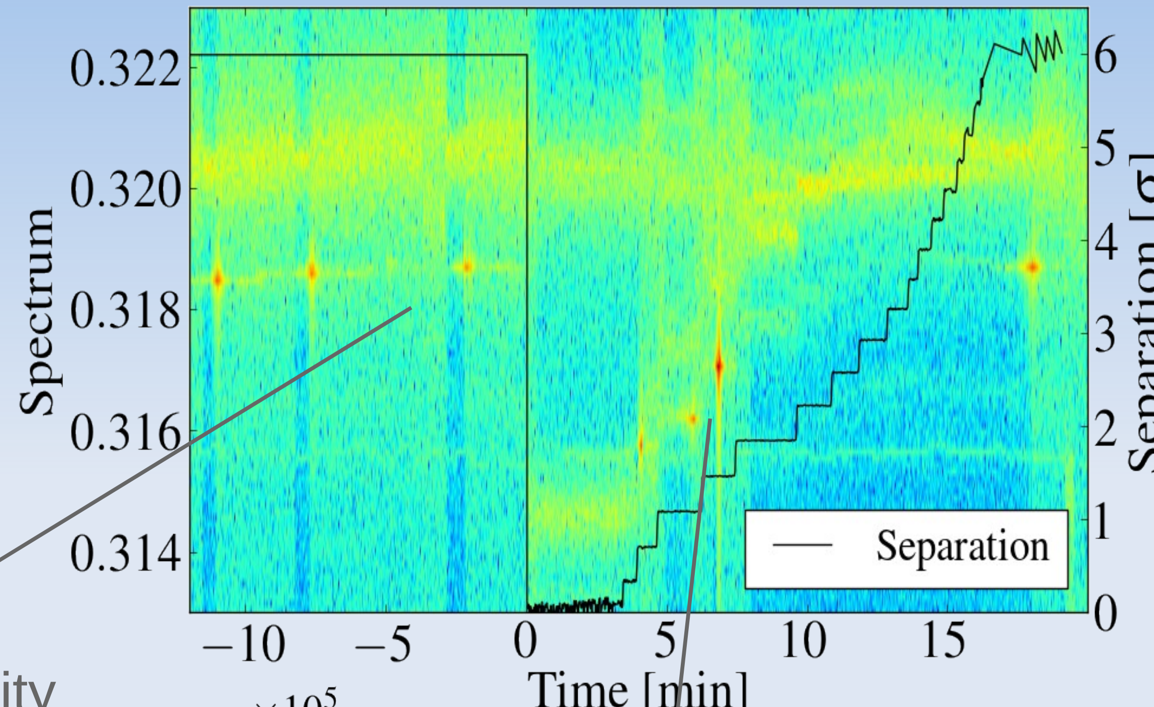
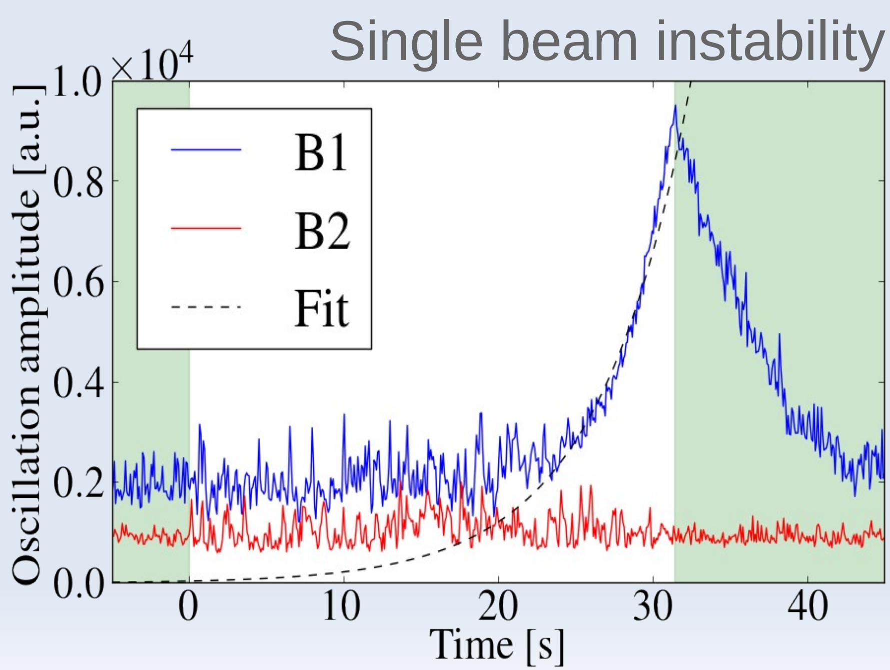
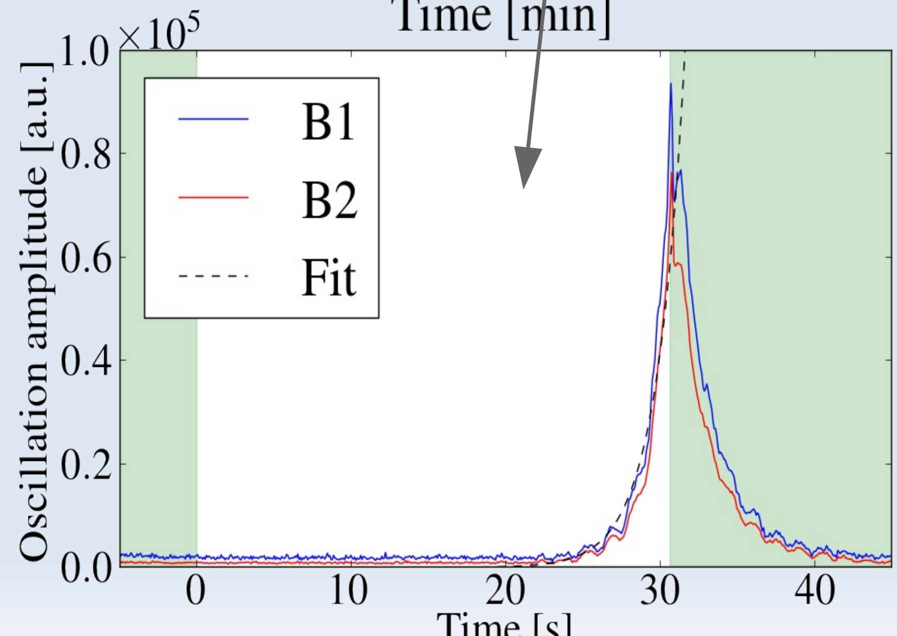
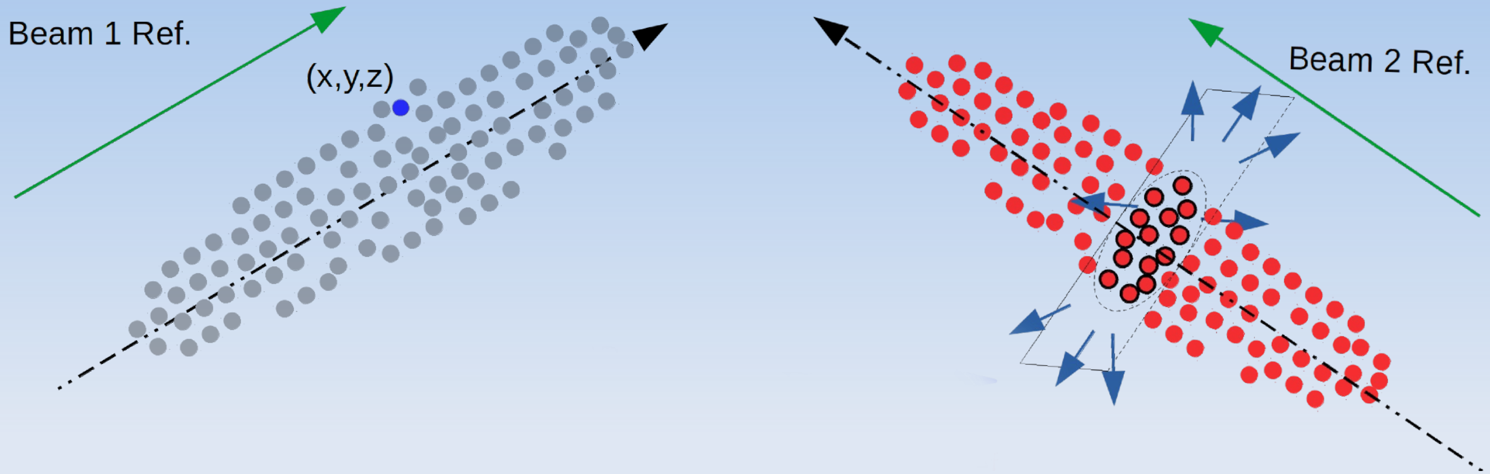




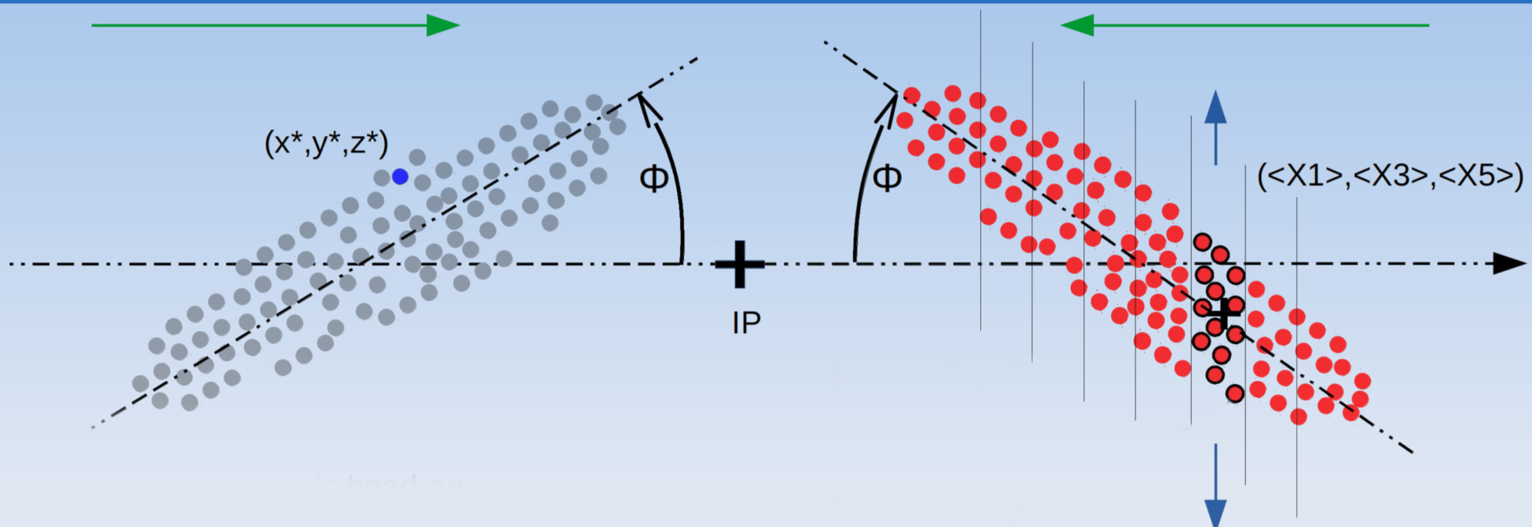


Efficient 6D models



- The 6D coherent beam-beam model is based on Hirata's weak-strong 6D kick (10)

Efficient 6D models



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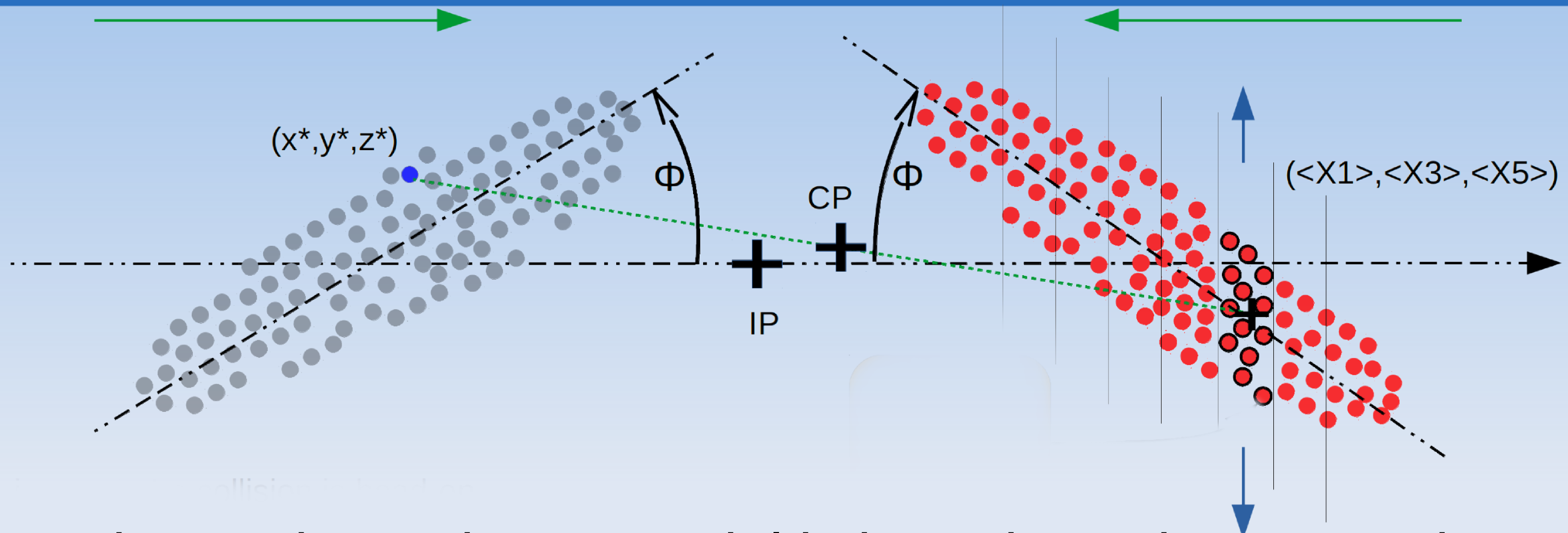

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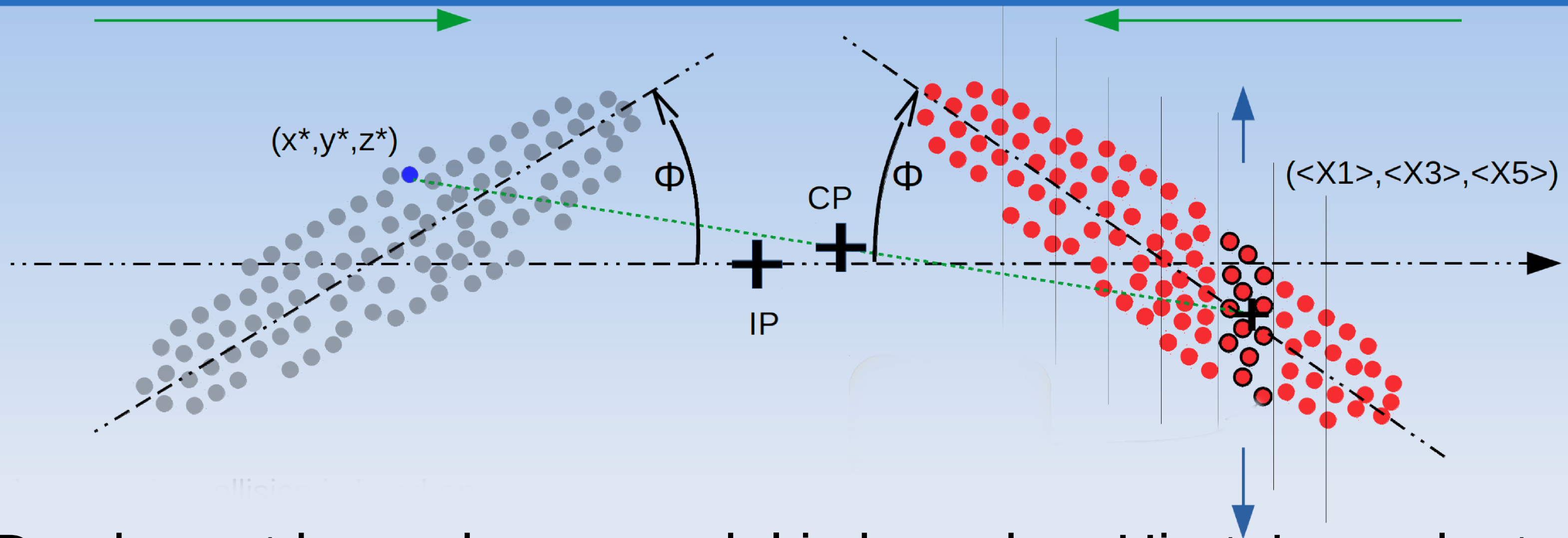

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- The macroparticles of both beams are boosted and the moments of the slices are computed in the boosted frame
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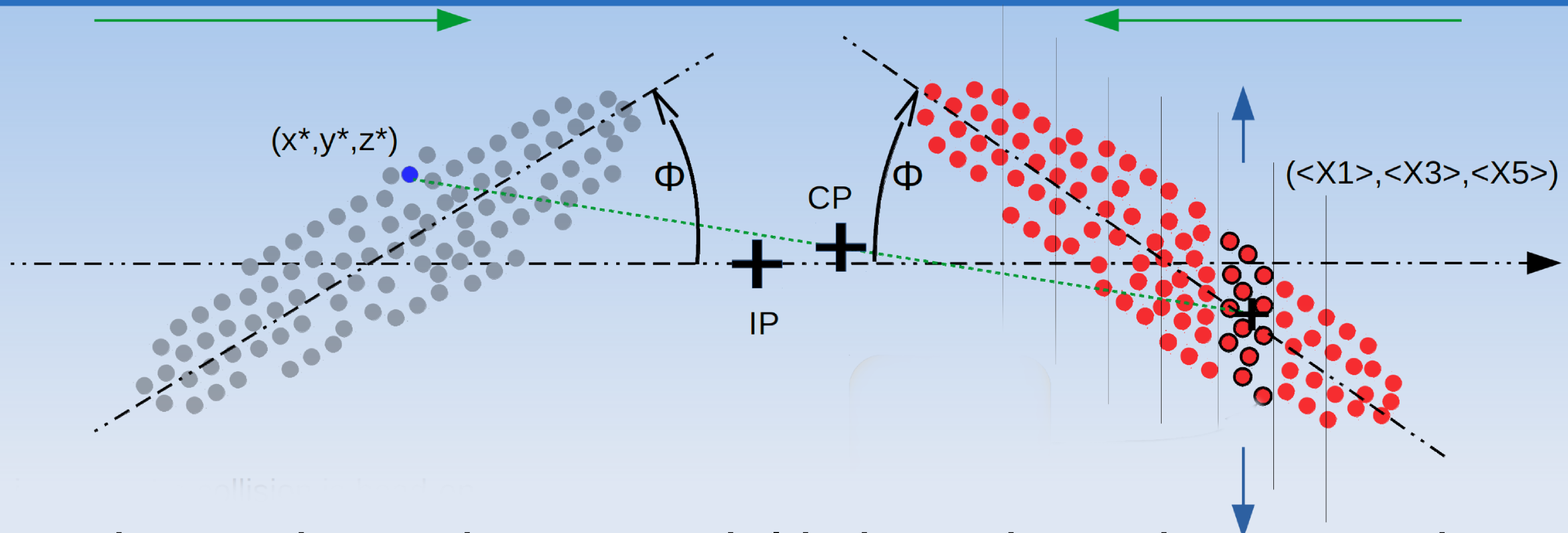
  - Compute de moments of the slices once and compute their forces on all particles of the other beam
  - Fast (~2s), but inaccurate for strong beam-beam forces

Efficient 6D models



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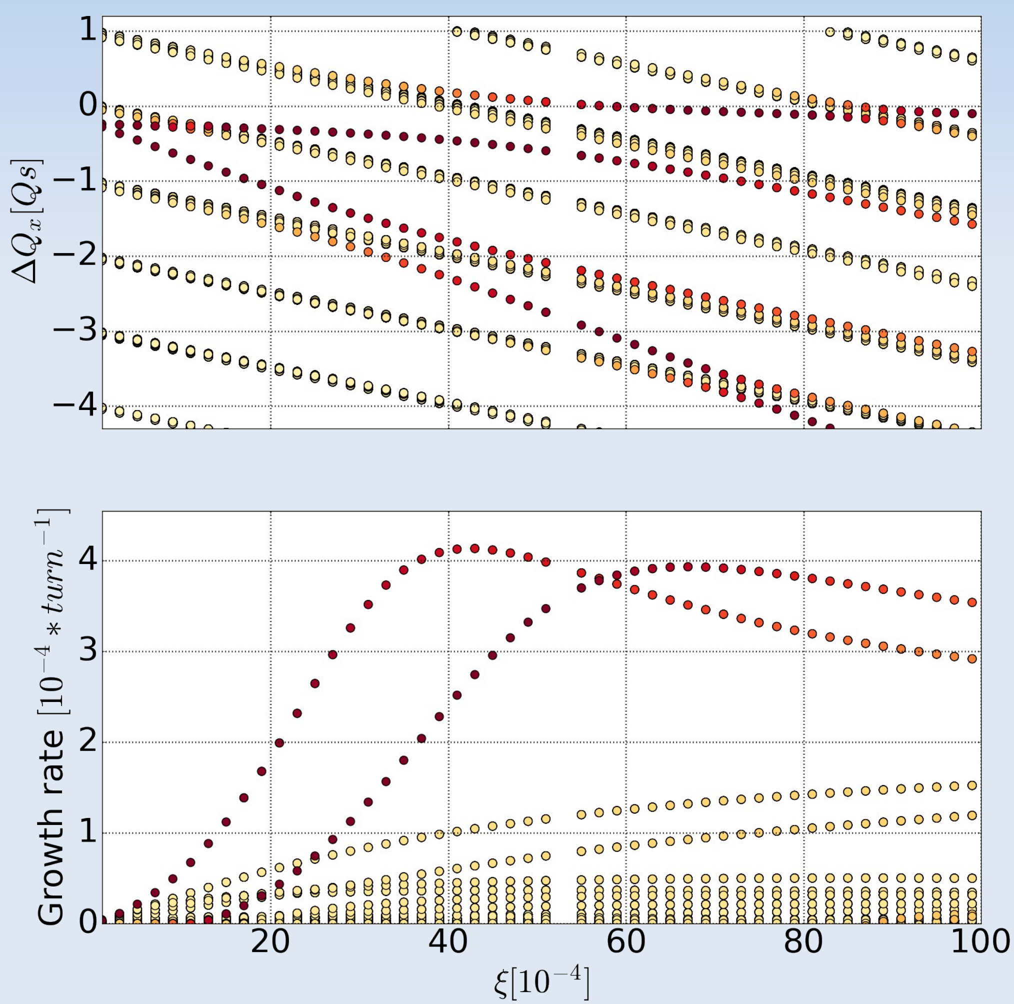
- Frozen model :
  - Compute de moments of the slices once and compute their forces on all particles of the other beam
  - Fast (~2s), but inaccurate for strong beam-beam forces
- Full model :
  - Iteratively compute the interaction of pair of slices and update the moments accordingly
  - Slow (~13s), but fully accurate

Landau damping by synchrotron sidebands

- In configurations with strong synchrobetatron coupling (e.g. HL-LHC), the instability occur at any beam-beam tune shift

(8)



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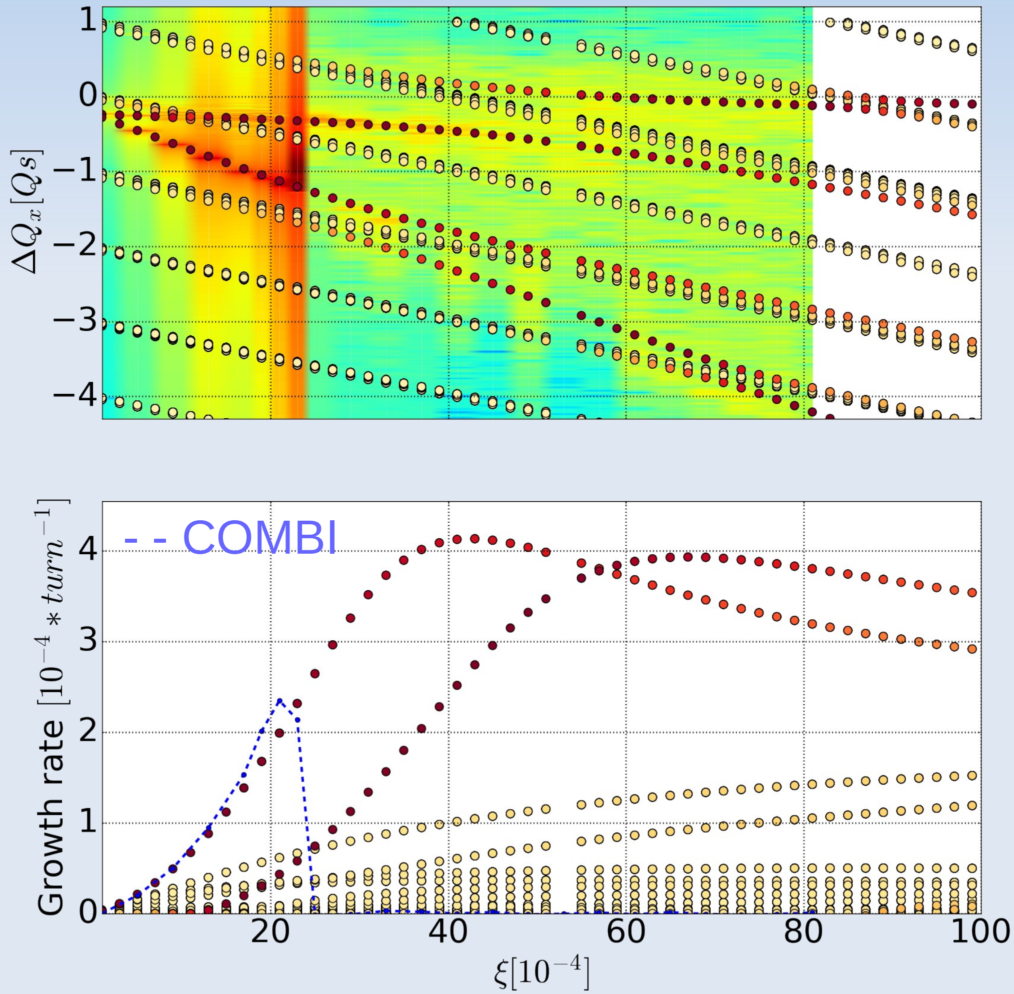

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- In configurations with strong synchrobetatron coupling (e.g. HL-LHC), the instability occur at any beam-beam tune shift
- Landau damping from the beam-beam induced tune spread is usually effective if the coherent modes frequency is within the incoherent tune spread and its side bands (4,9)

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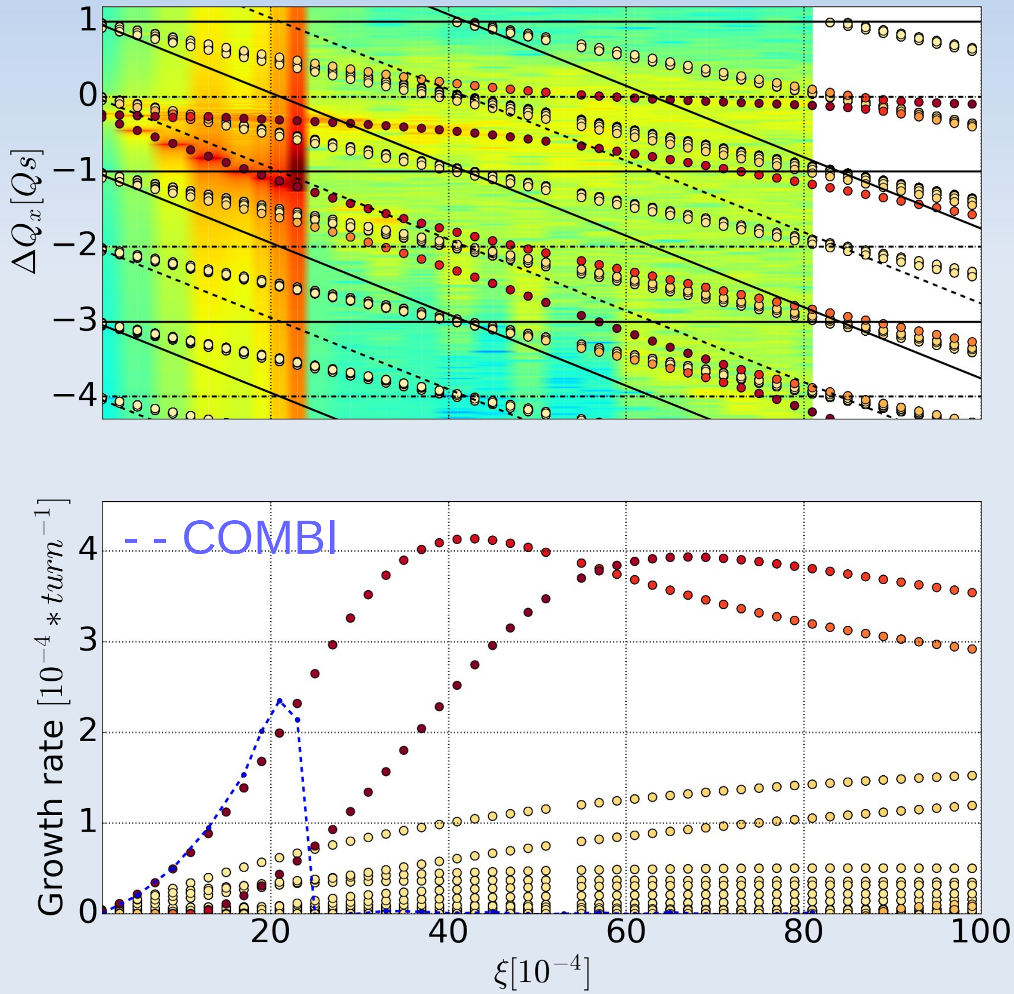

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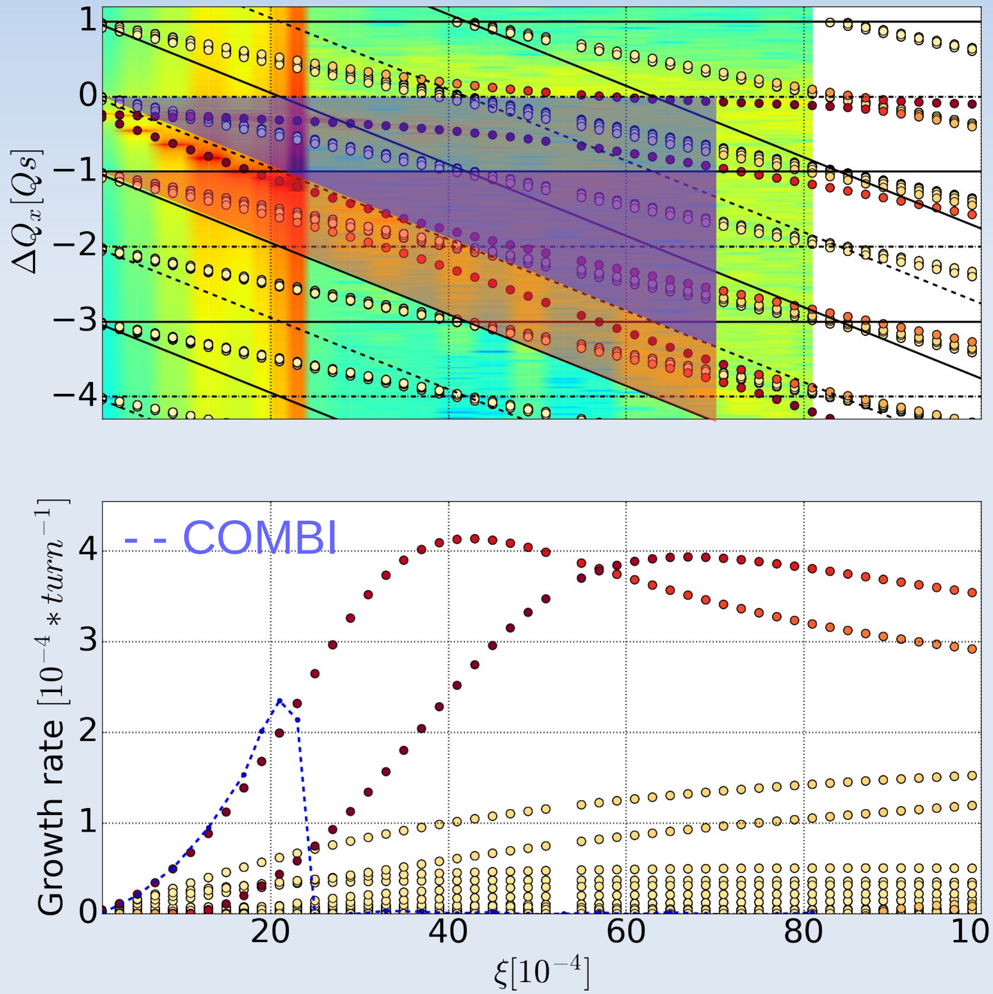

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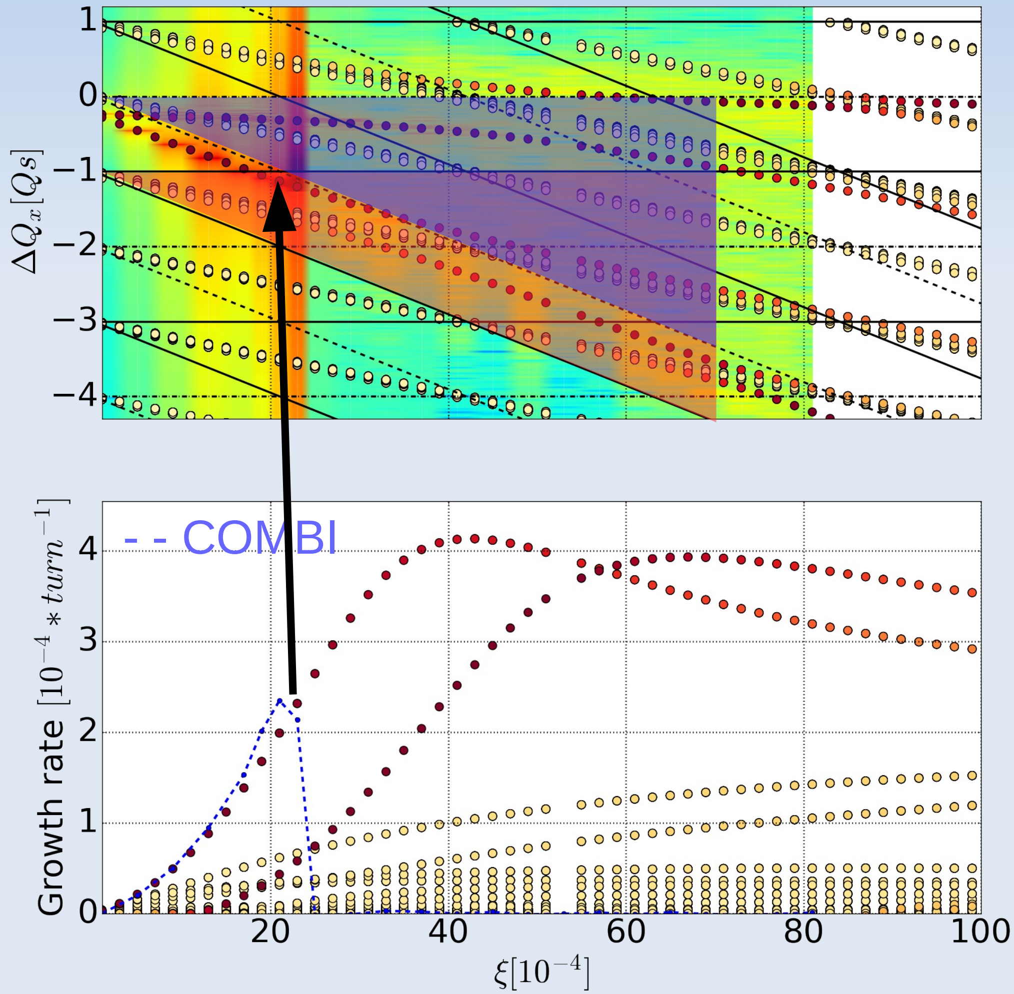

Landau damping by synchrotron sidebands

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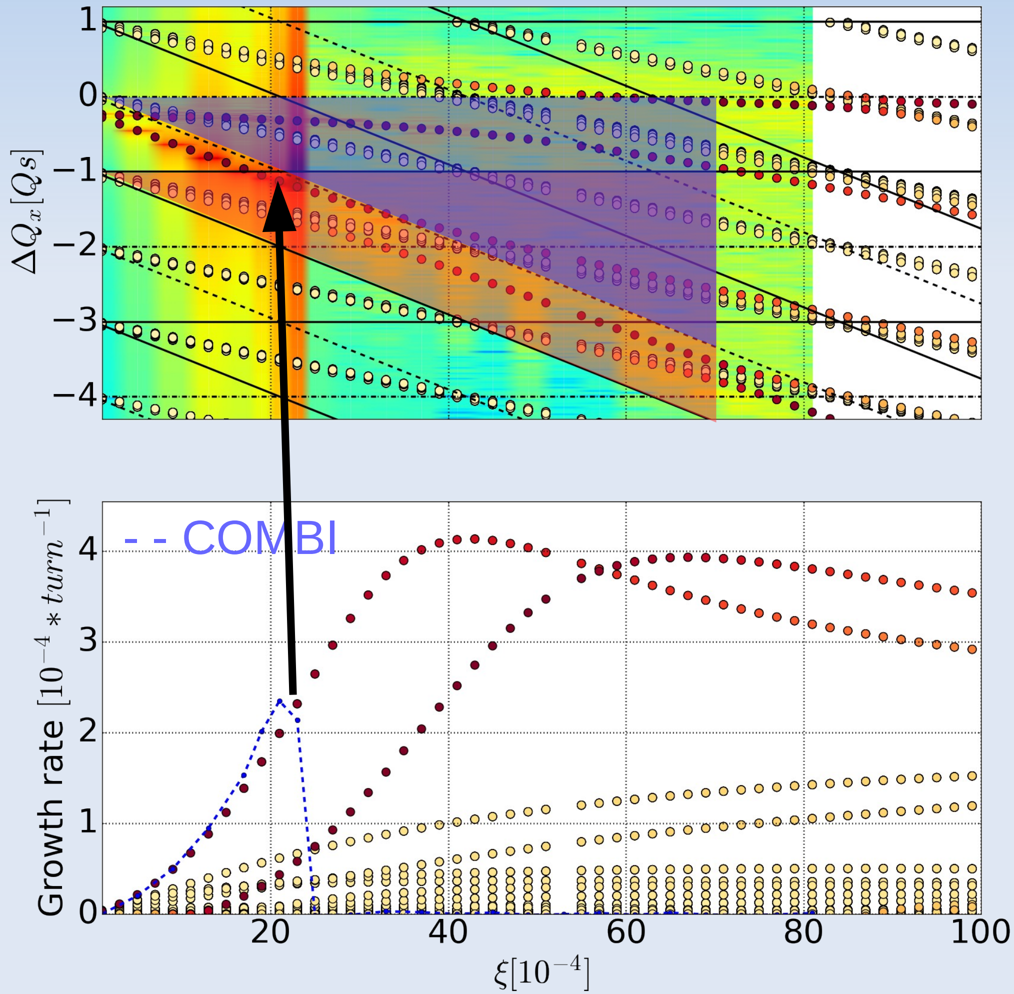
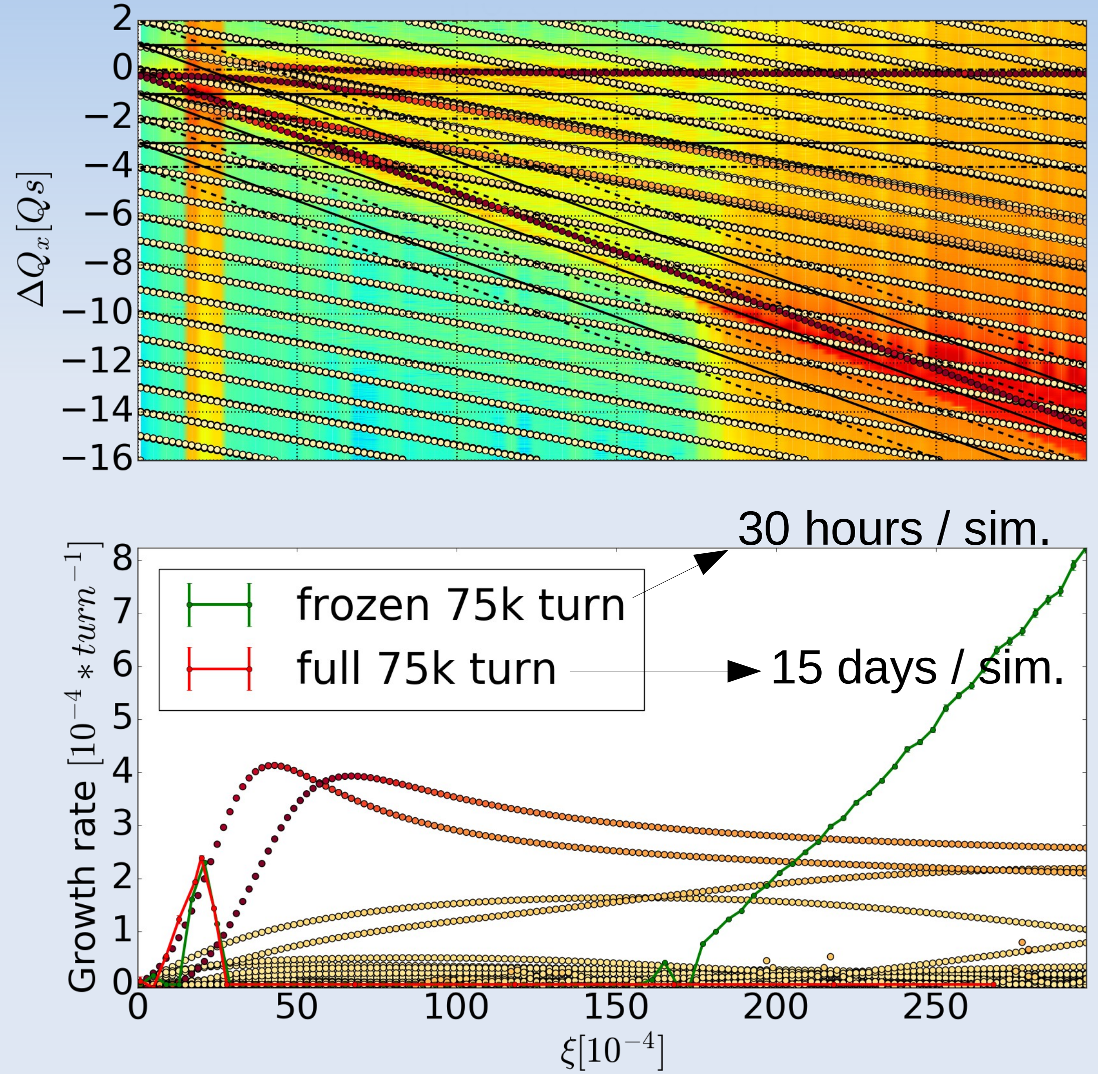
- Landau damping from the beam-beam induced tune spread is usually effective if the coherent modes frequency is within the incoherent tune spread and its side bands (4,9)
  - Can become an issue in the case of head-on tune spread compensation with an e⁻ lens



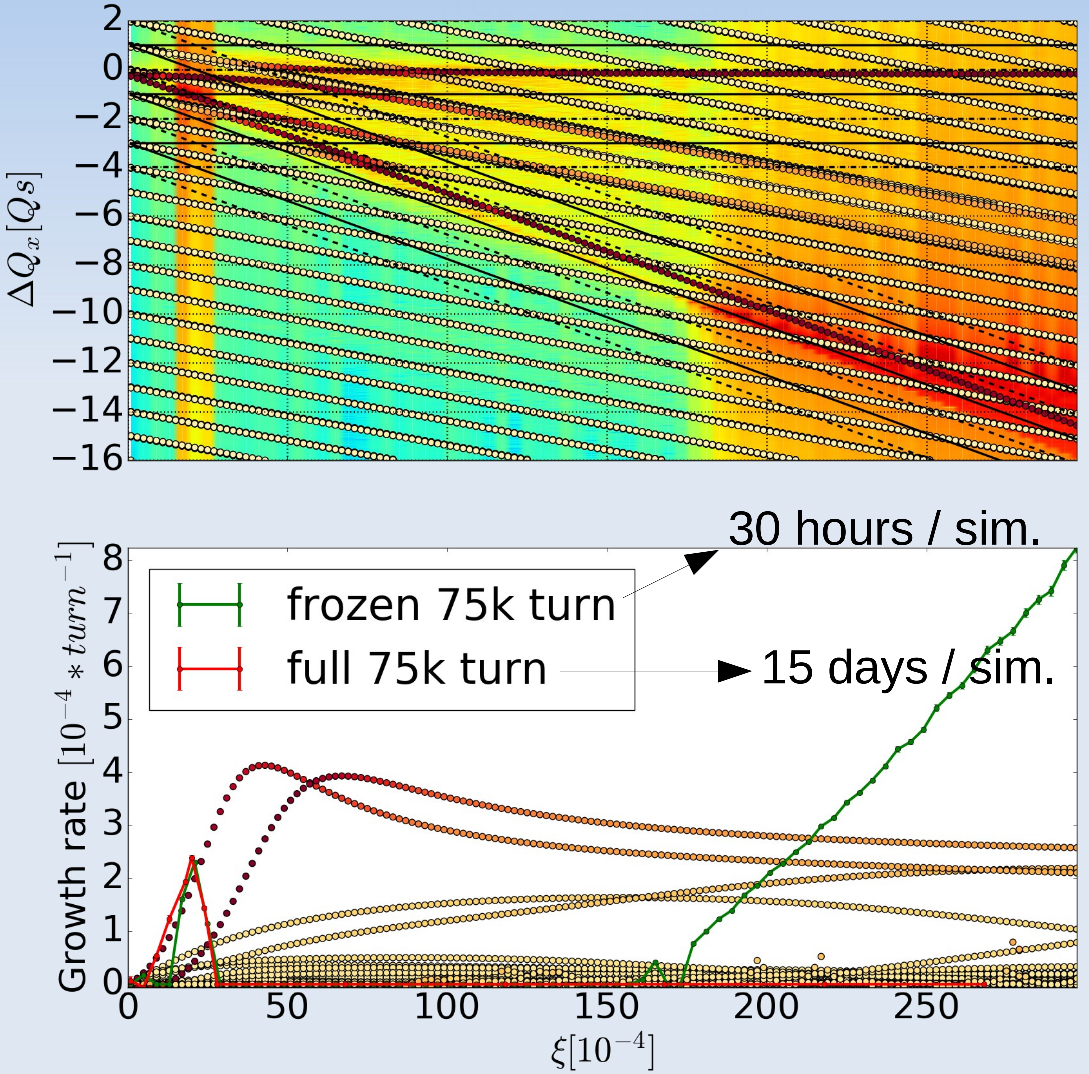
Limit of the frozen model

- An instability incompatible with the circulant matrix model arise for large beam-beam parameters



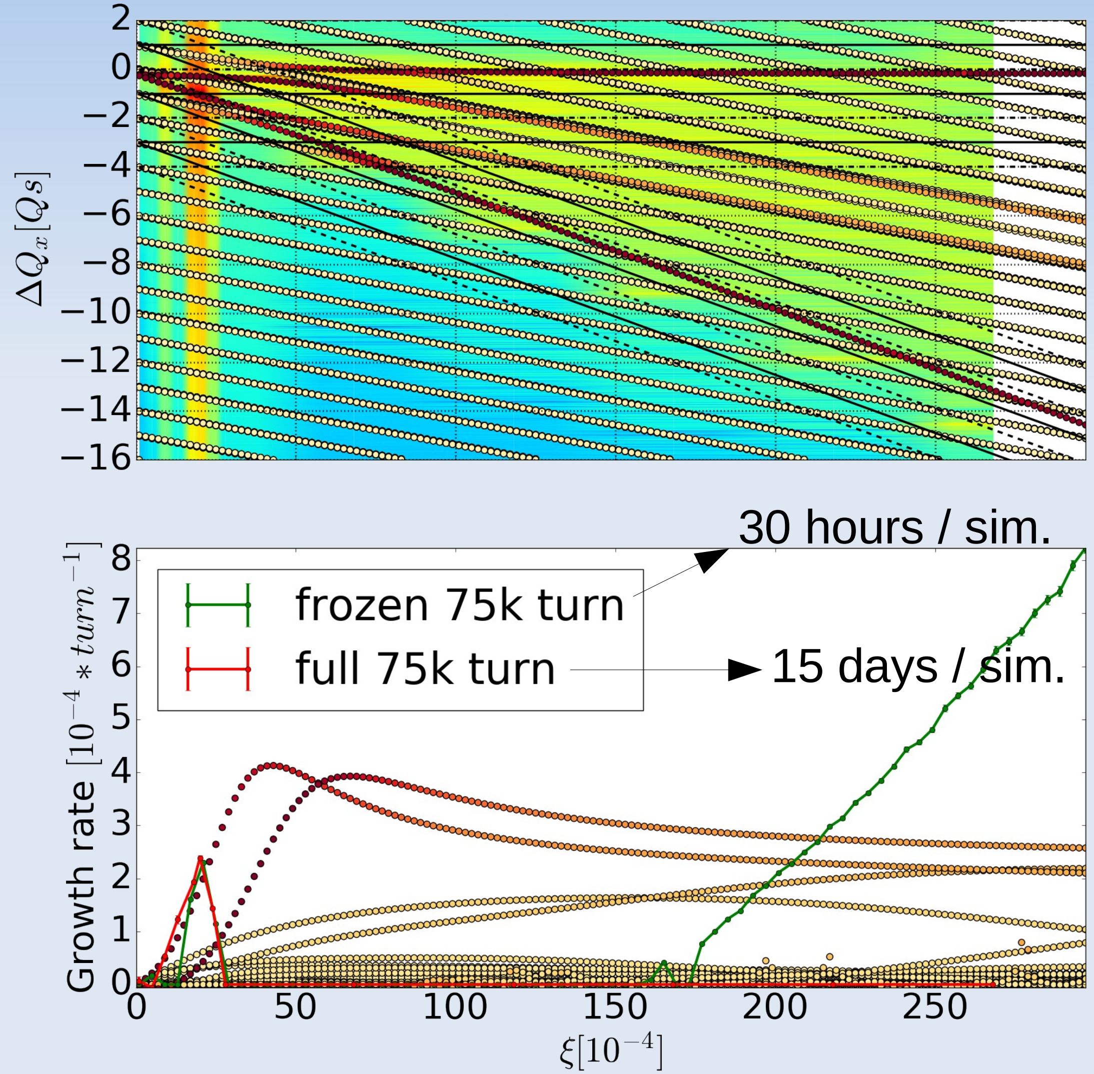
Limit of the frozen model

- An instability incompatible with the circulant matrix model arise for large beam-beam parameters
  - The frozen model does not accurately model the interaction when the kick modifies significantly the slice behaviour within a single interaction



Limit of the frozen model

- An instability incompatible with the circulant matrix model arise for large beam-beam parameters
  - The frozen model does not accurately model the interaction when the kick modifies significantly the slice behaviour within a single interaction
- The proper behaviour is recovered with the full 6D model



Conclusion

- In most realistic conditions (LHC) the emittance growth of colliding beams can be described with Lebedev's weak-strong formula, despite the strong-strong nature of the configuration
  - For the first time the model was verified experimentally in configurations relevant for HL-LHC and future hadron colliders (ΔQ~0.02)
  - The detailed modeling of these effects requires efficient low noise Poisson solver → The Fast Polar Poisson Solver shows good performance
- The existence of mode coupling instabilities of colliding beams was demonstrated experimentally in the LHC
  - The efficiency of the transverse feedback to suppress it was also verified
- The presence of import synchrobetatron coupling in the LHC generalises the instability to all beam-beam tune shifts
  - Landau damping by synchrotron side bands, predicted qualitatively, can be quantified using macroparticle simulations

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Decoherence of the σ mode

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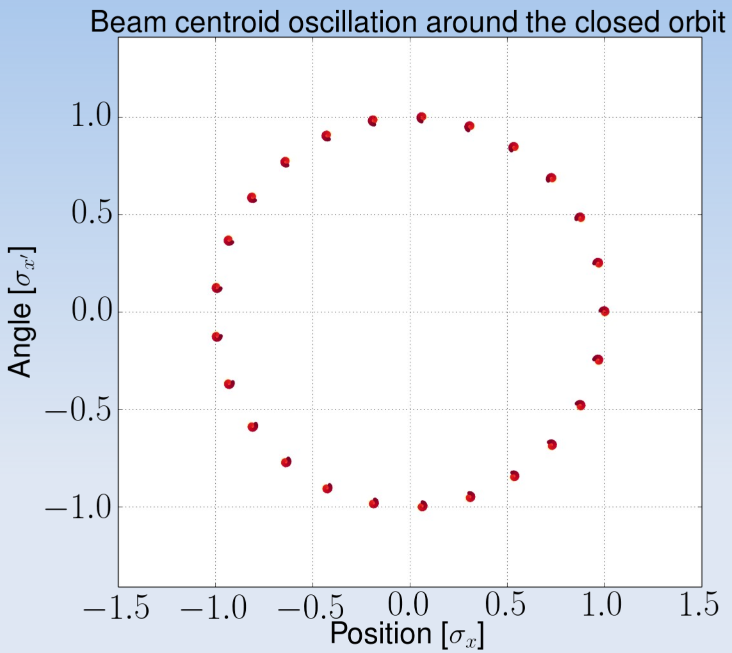

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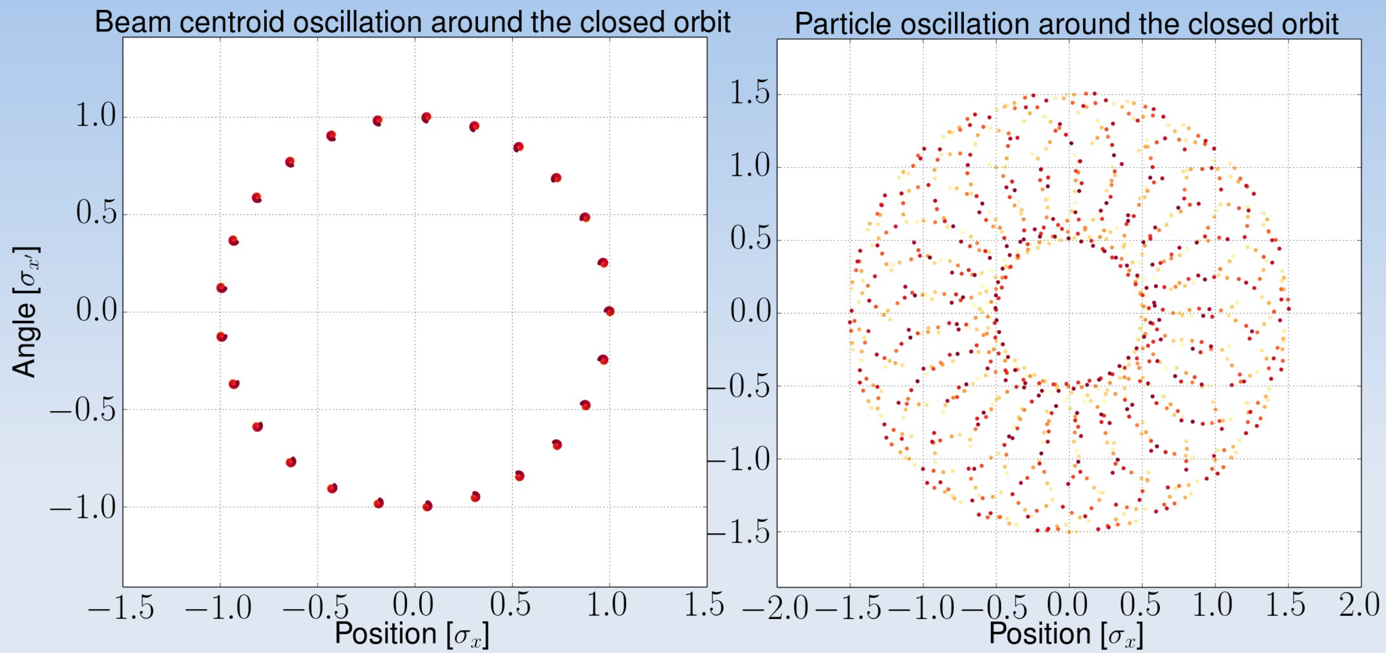

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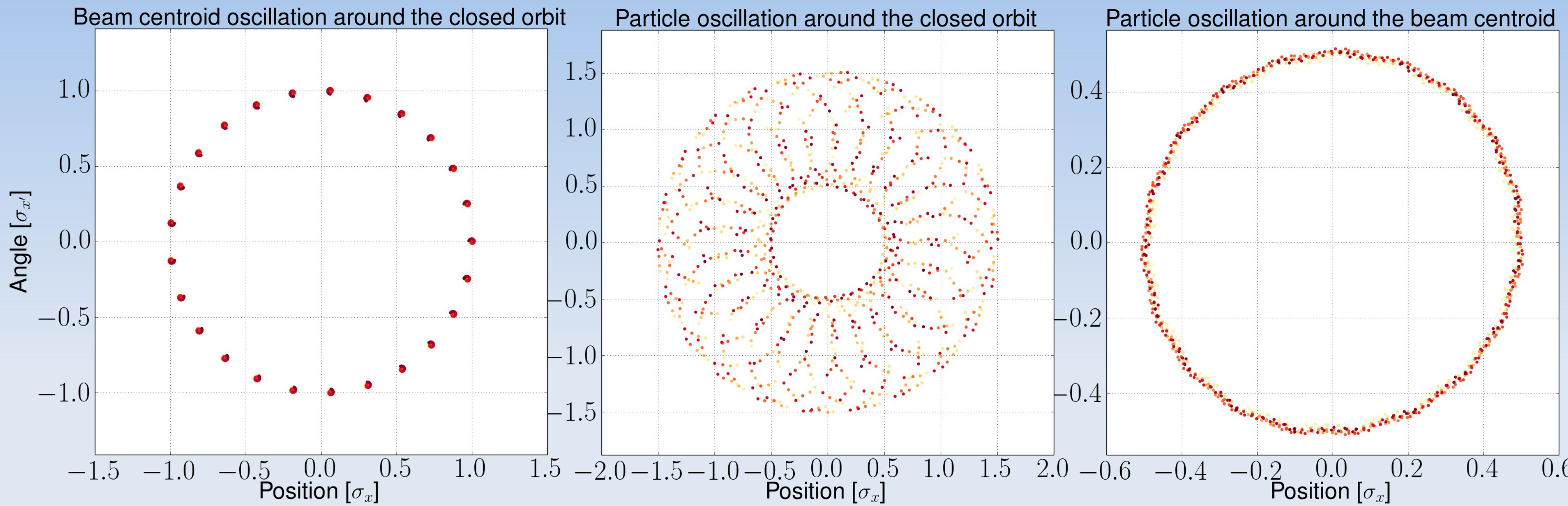

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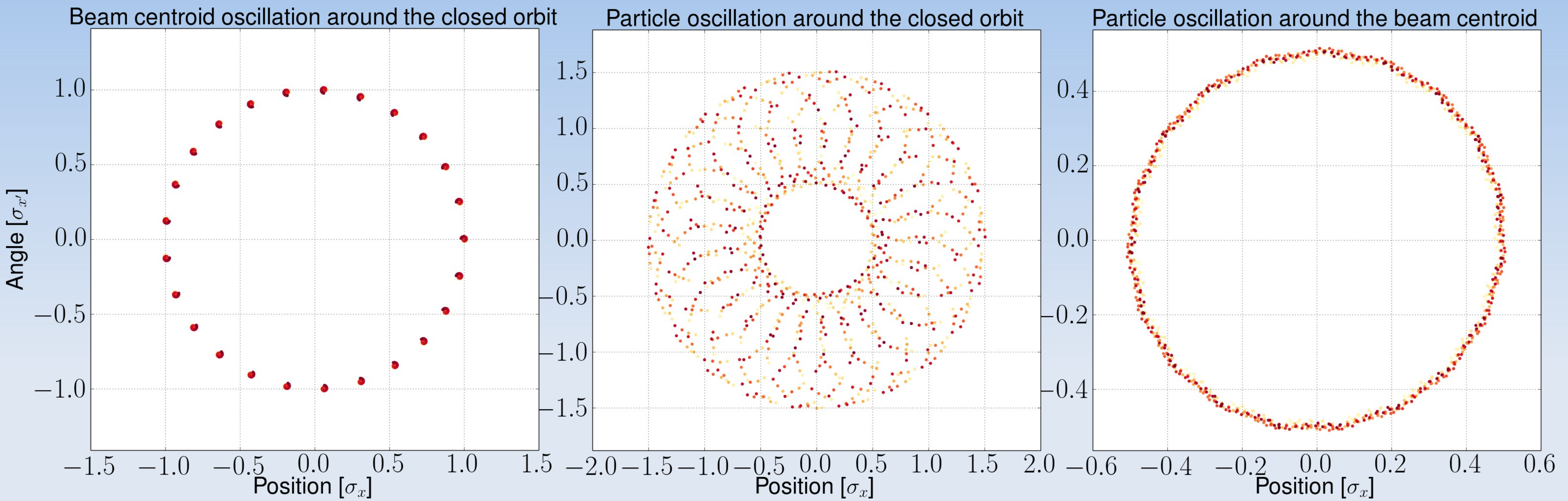
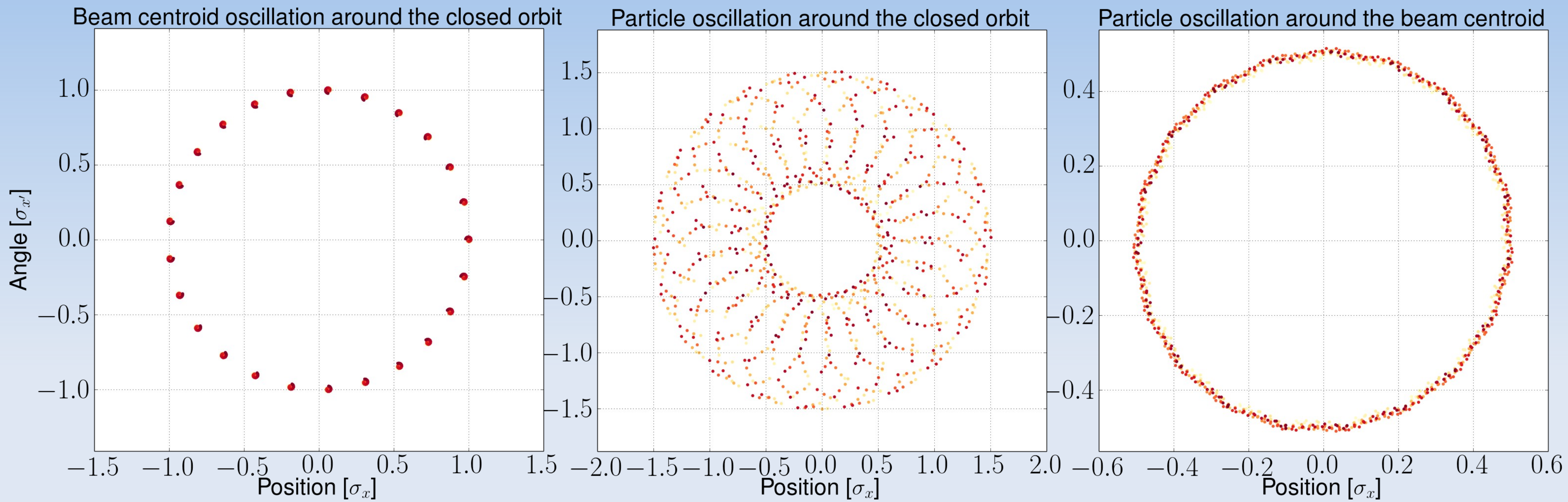


- The single particle motion is the linear composition of the centroid position and the position with respect to the centroid position

→ The single particle motion does not change the coherent force

Decoherence of the σ mode



- The single particle motion is the linear composition of the centroid position and the position with respect to the centroid position

  → The single particle motion does not change the coherent force

- The incoherent and coherent motion are decoupled → Absence of decoherence

Decoherence of the π mode

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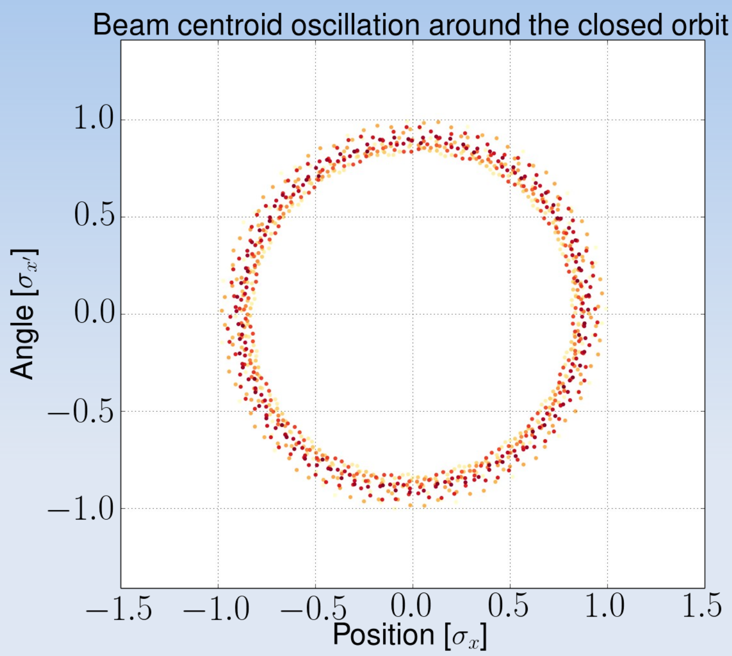

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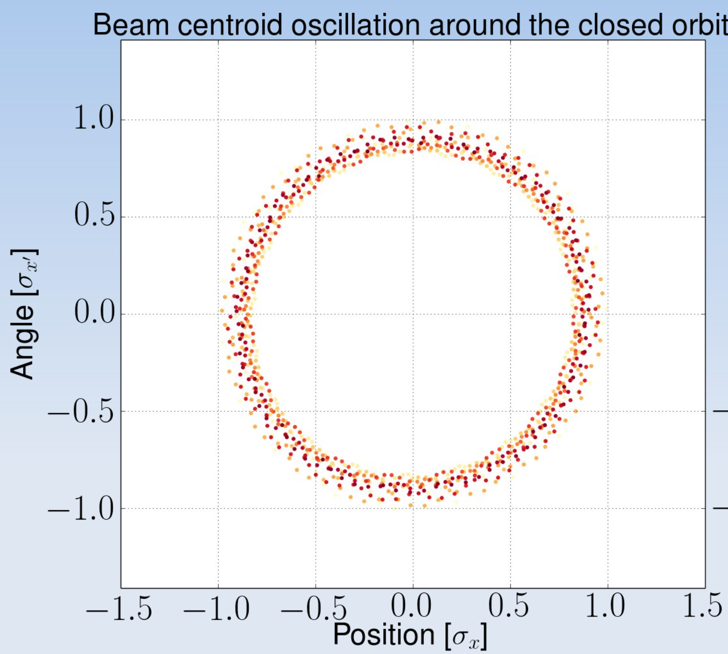

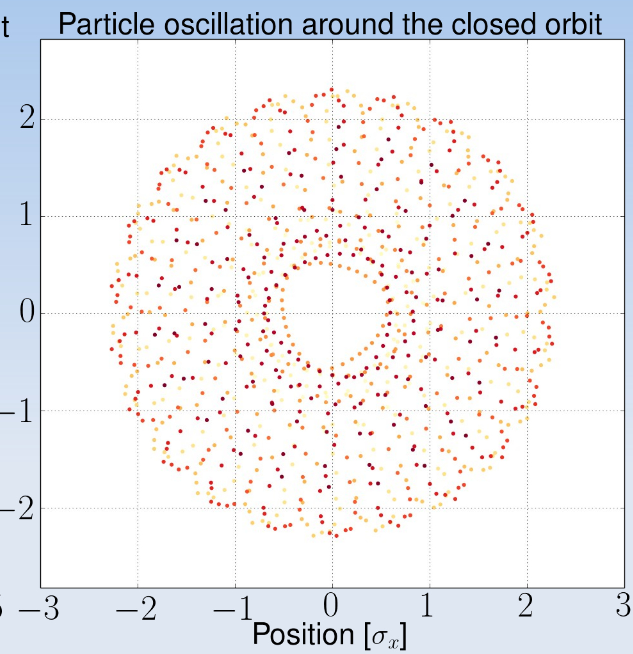

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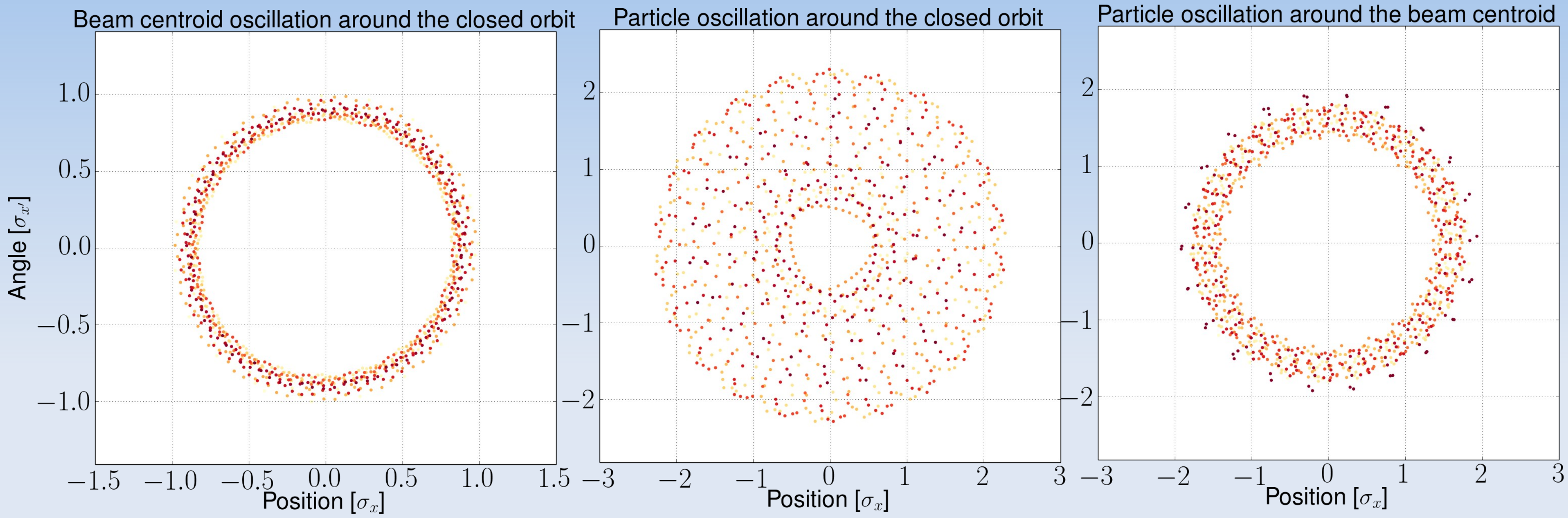

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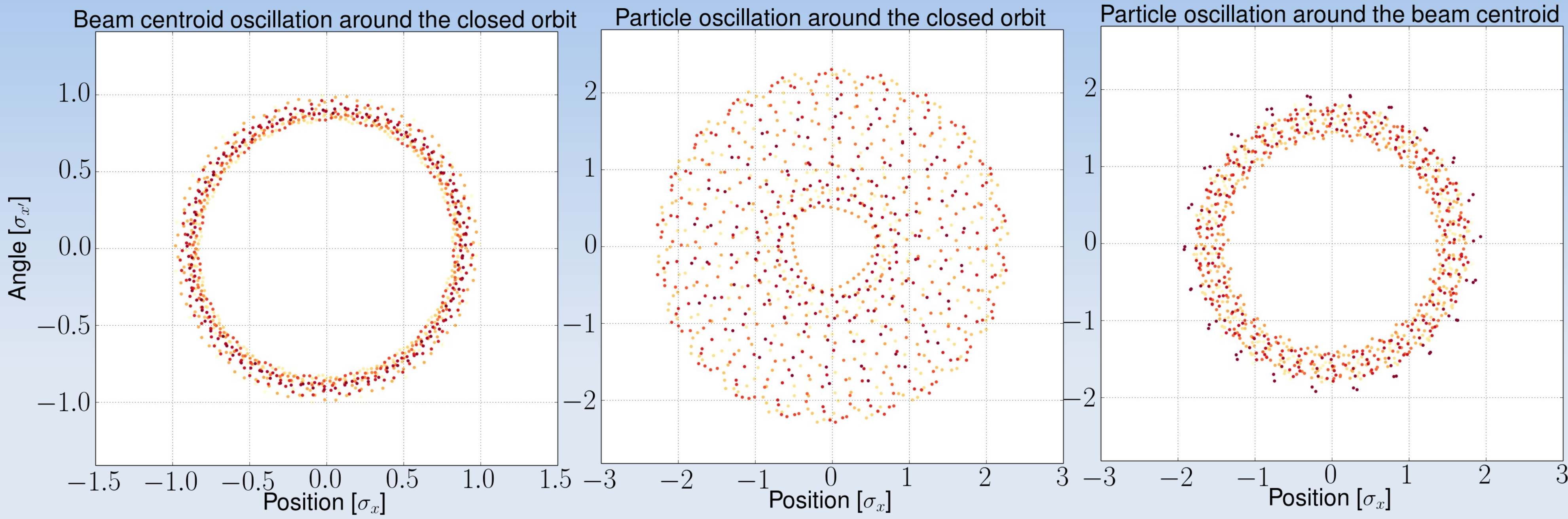
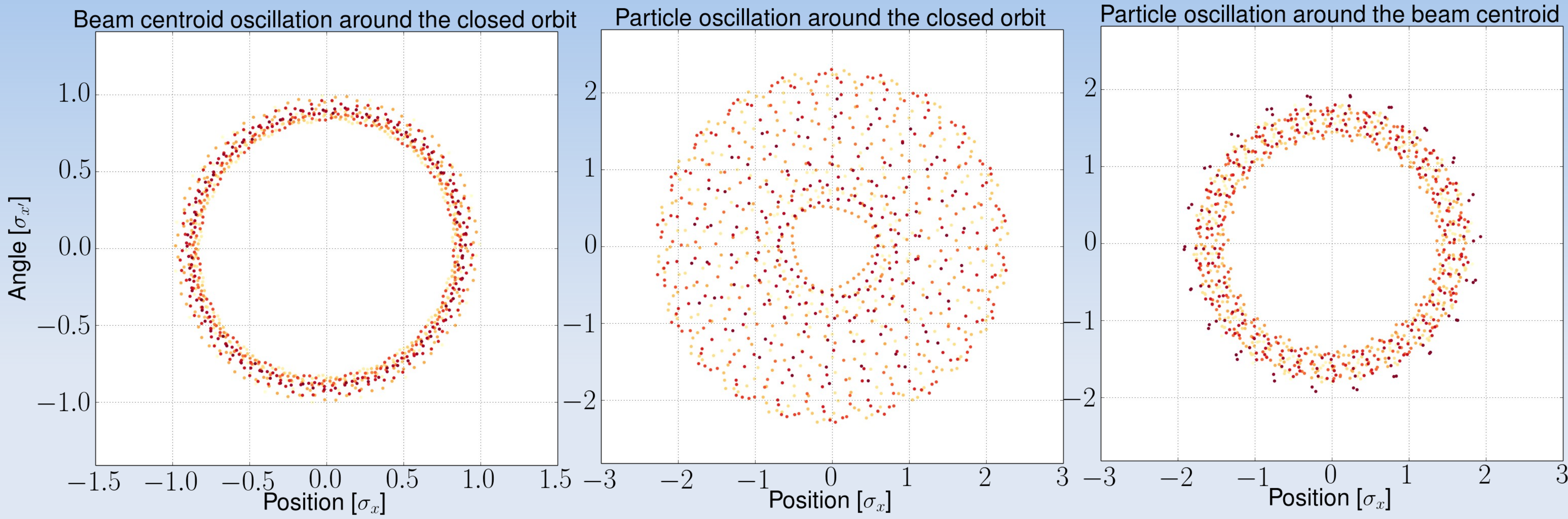


- Again, the single particle motion is 'regular' with respect to the bunch centroid

Decoherence of the π mode



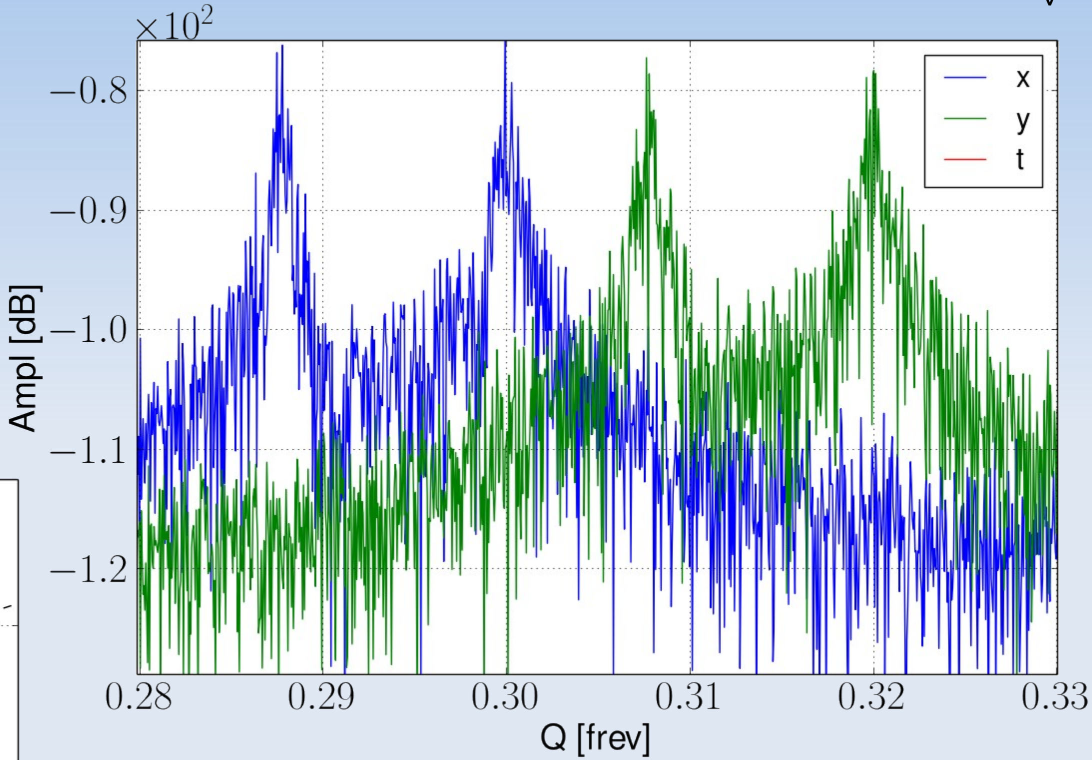
- Again, the single particle motion is 'regular' with respect to the bunch centroid

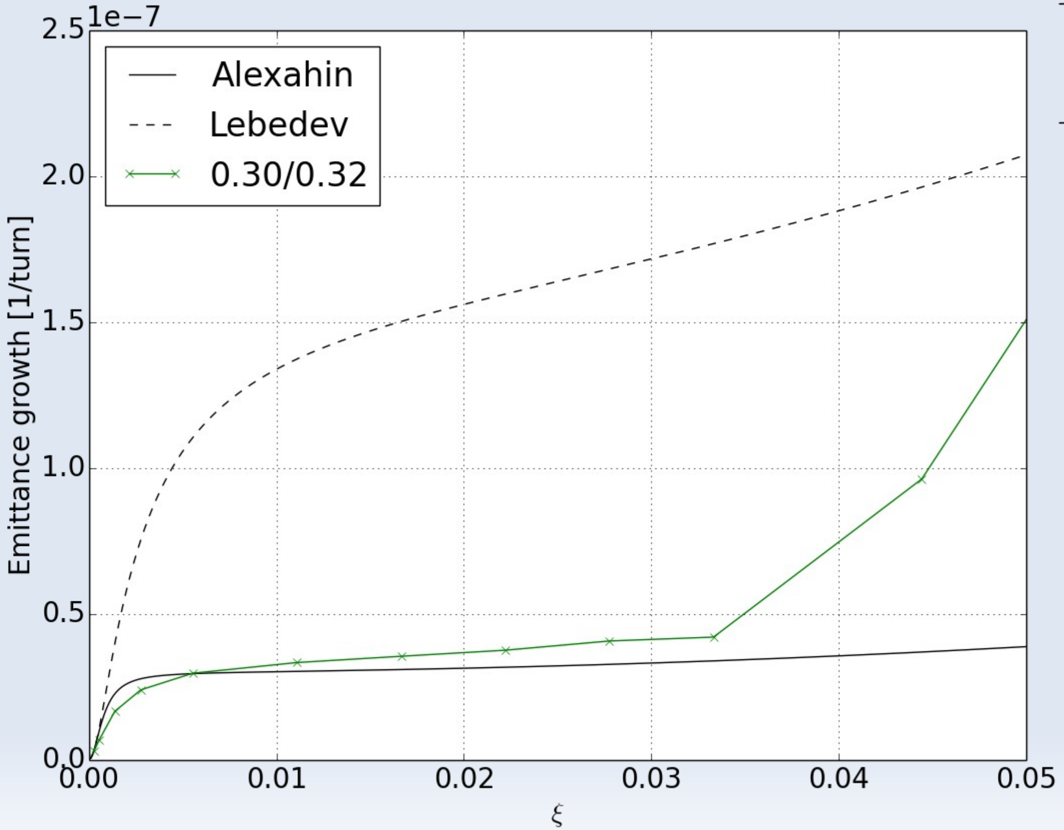
  → Absence of decoherence

  - A slight emittance growth still exists due to the mismatch of the distribution

2^{nd} order effets

0.32 (σ_v)





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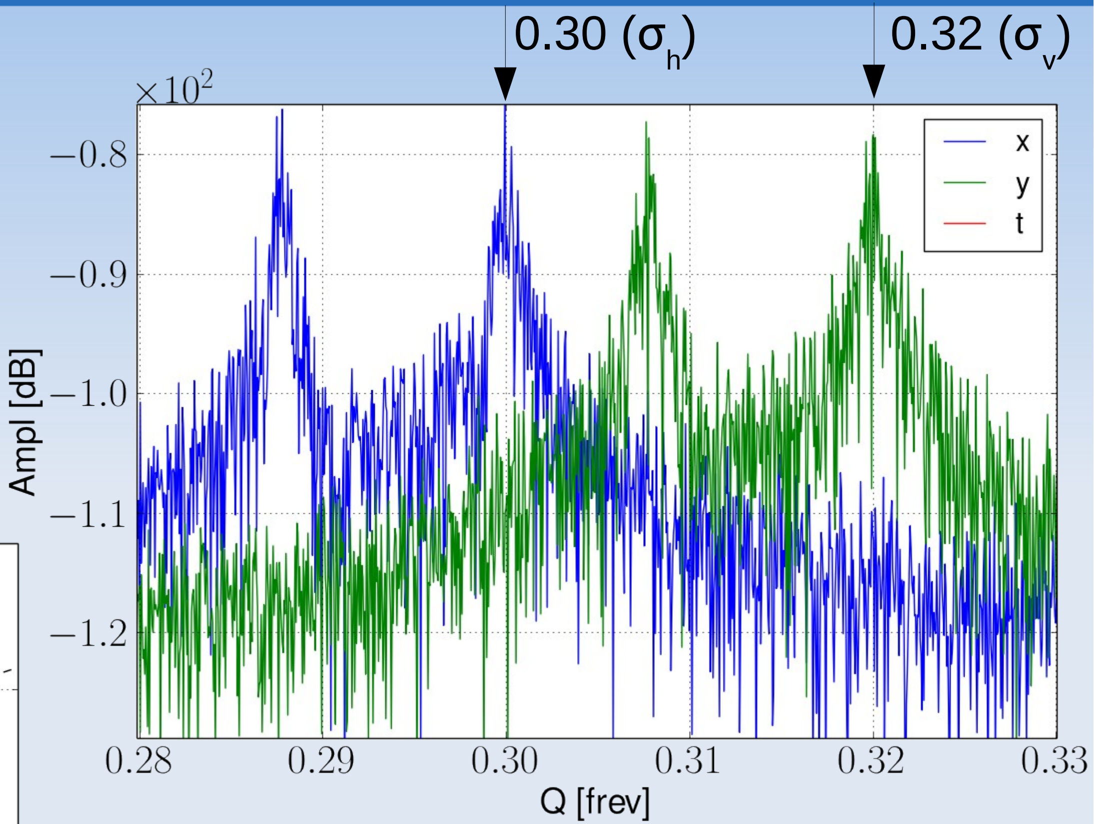
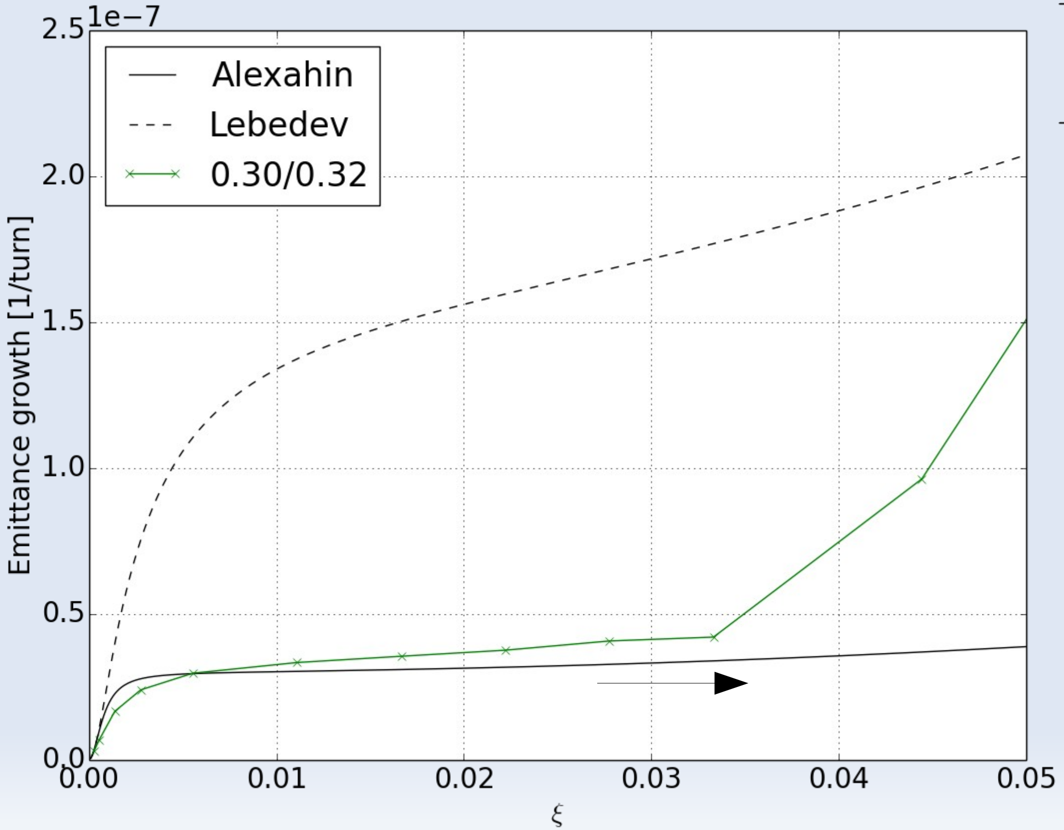
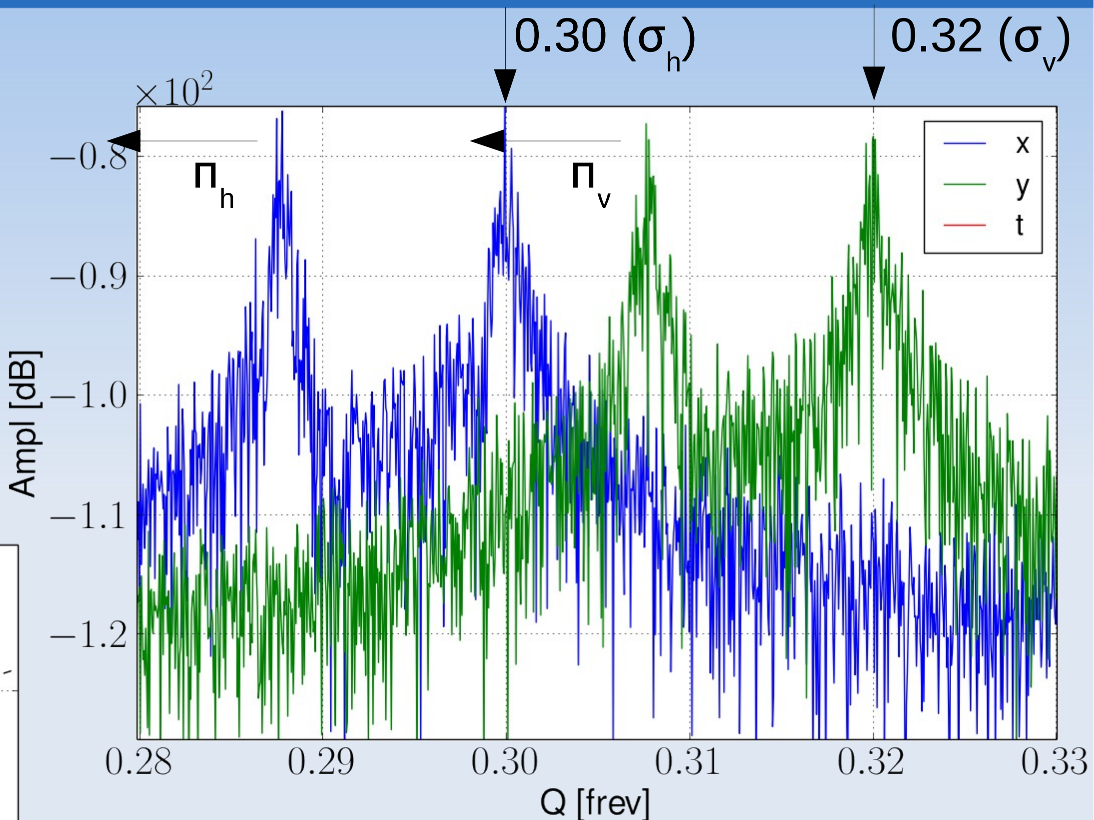
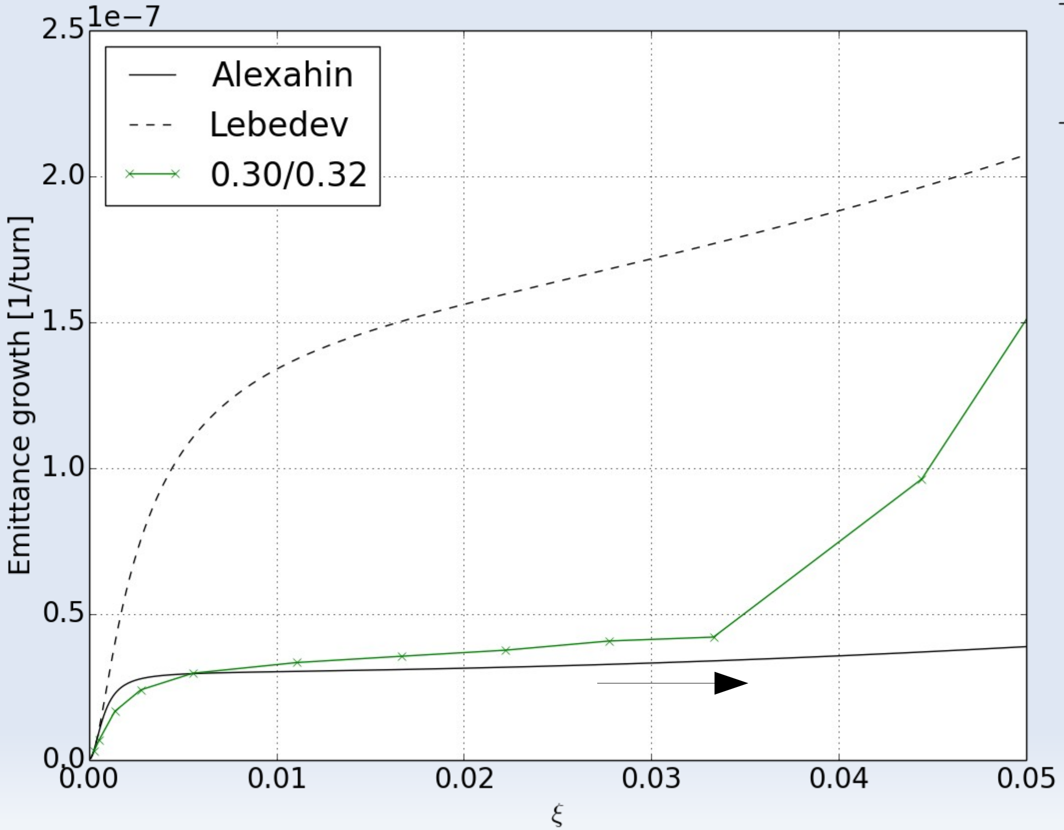




2nd order effets

- When the shift of the π-mode exceeds the tune separation between the plane, the coupling due to the beam-beam force is sufficient to break Alexahin's formula





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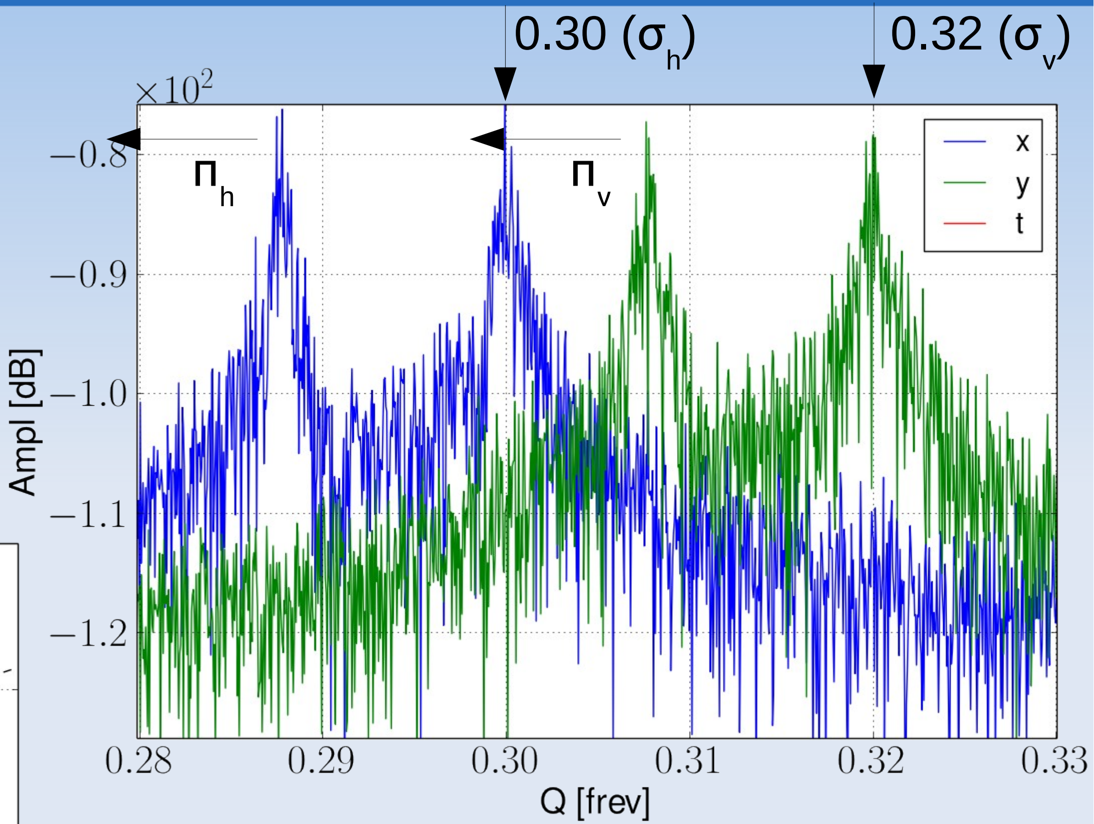
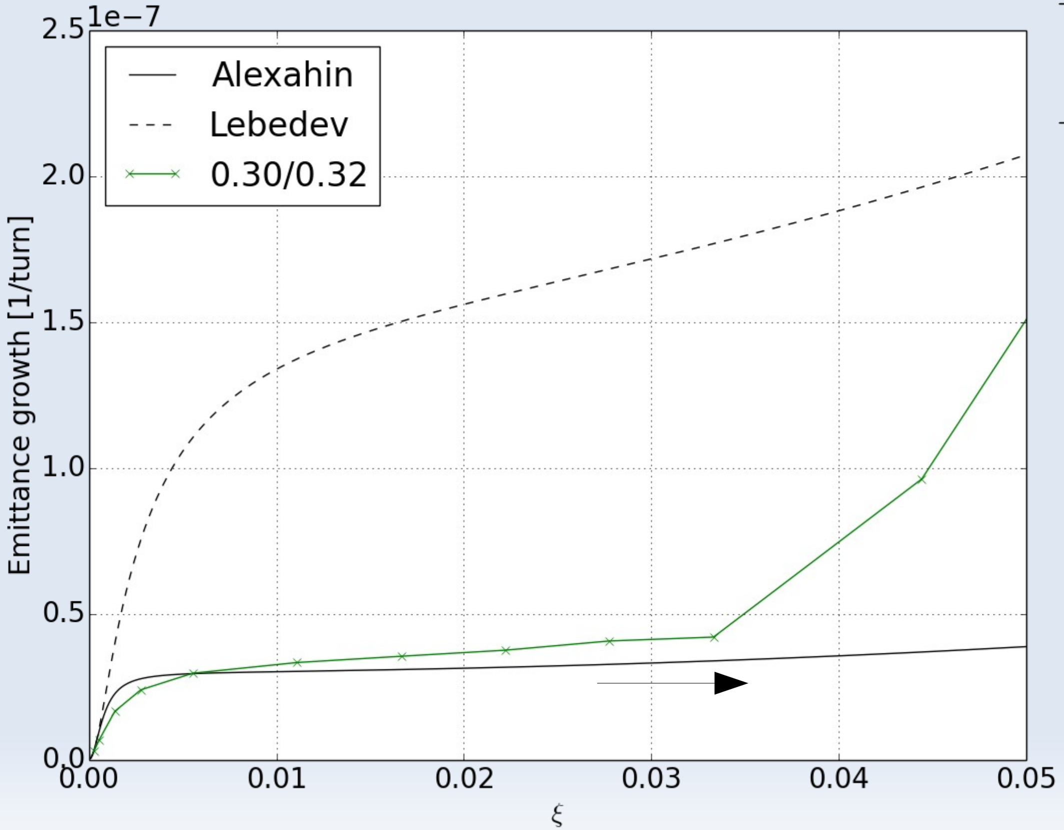
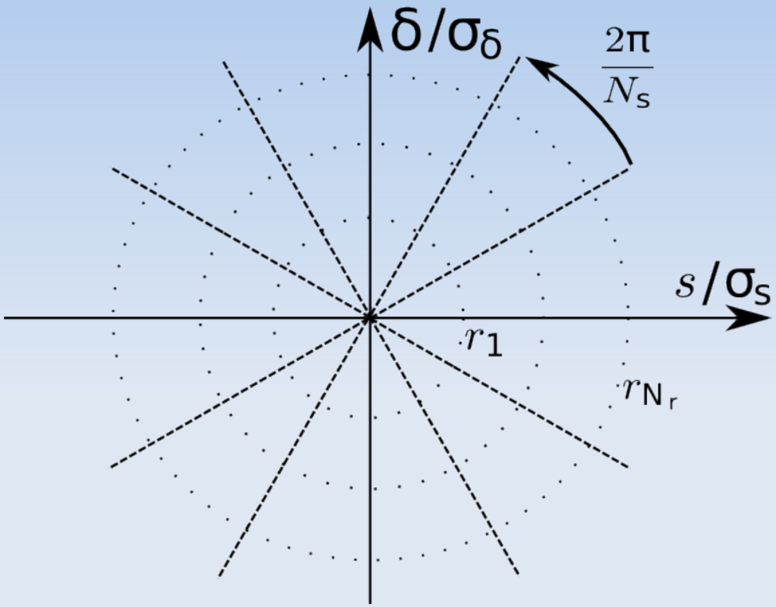




- Head-on beam-beam interactions do not generate coupling at first order → limitation of the theoretical model

The circulant matrix model basis

- Polar discretisation of the longitudinal phase space in cells (slices and rings)
  - The dynamical variables are the transverse positions and momentum (1 or 2 planes) of the cells
  - The synchrotron motion corresponds to a rotation of the slices → circulant matrix
  - The basis can be easily extended to describe several bunches per beam
- Initially developed to study the stabilisation of the TMCI with a feedback [V.V. Danilov] and for coherent synchrobetatron beam-beam modes in VEPP-2M [E.A. Perevedentsev]



$$\underline{x}(t) = M^{t}_{One\,turn}\,\underline{x}(0) = \sum_j e^{-2\pi i Q_j t}\,\underline{v}_j$$

The unperturbed circulant matrix

$$M_{1\mathrm{b}} = \frac{1}{N_r N_s} \mathbb{I}_{N_r} \otimes P_{N_s}^{N_s Q_s} \otimes B_0(2\pi Q_{y,0})$$

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Unperturbed betatron motion (w/o chromaticity)

$$B_0 = \begin{pmatrix} cos(2\pi Q) & \beta sin(2\pi Q) \\ \frac{-1}{\beta} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix}$$

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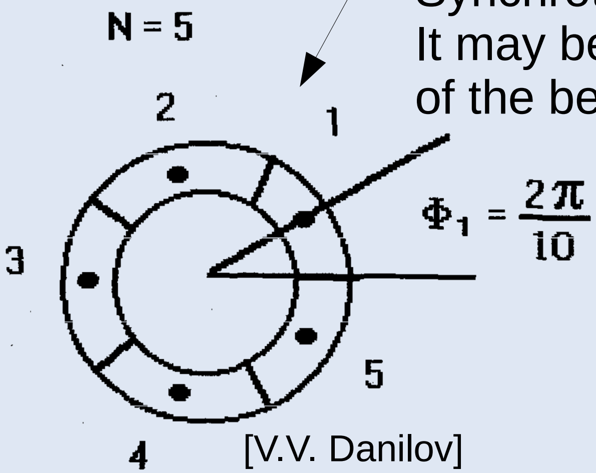
Synchrotron motion within each ring
It may be extended to include the chromatic shift of the betatron phase (see backup)



[V.V. Danilov]

$$P_{N_s} = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ 1 & & & 0 & 1 \end{pmatrix}$$

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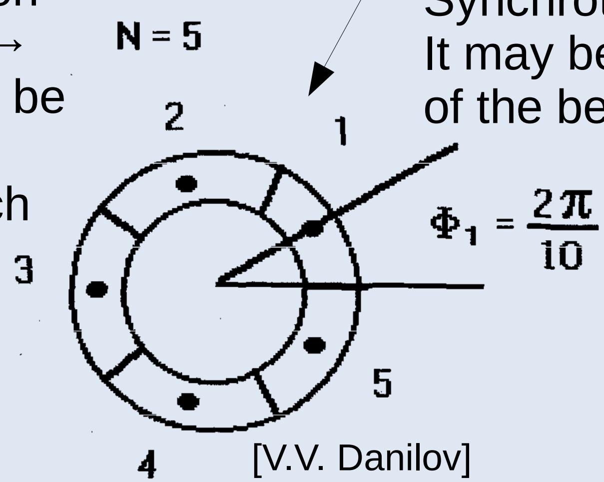
Identical synchrotron tune for each ring → the matrix can also be constructed with a different Qs for each ring



[V.V. Danilov]

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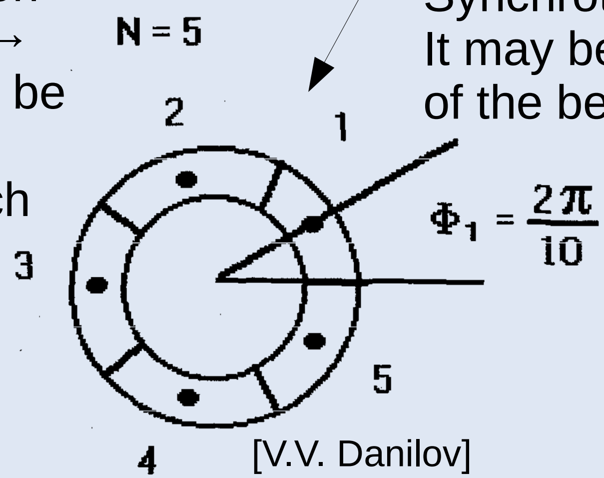

[V.V. Danilov]

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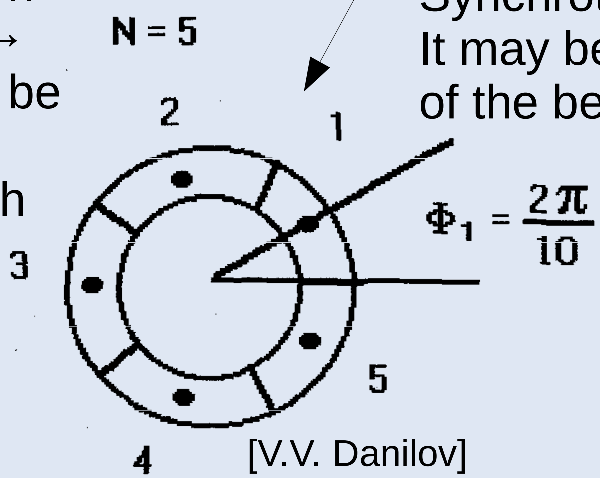

[V.V. Danilov]

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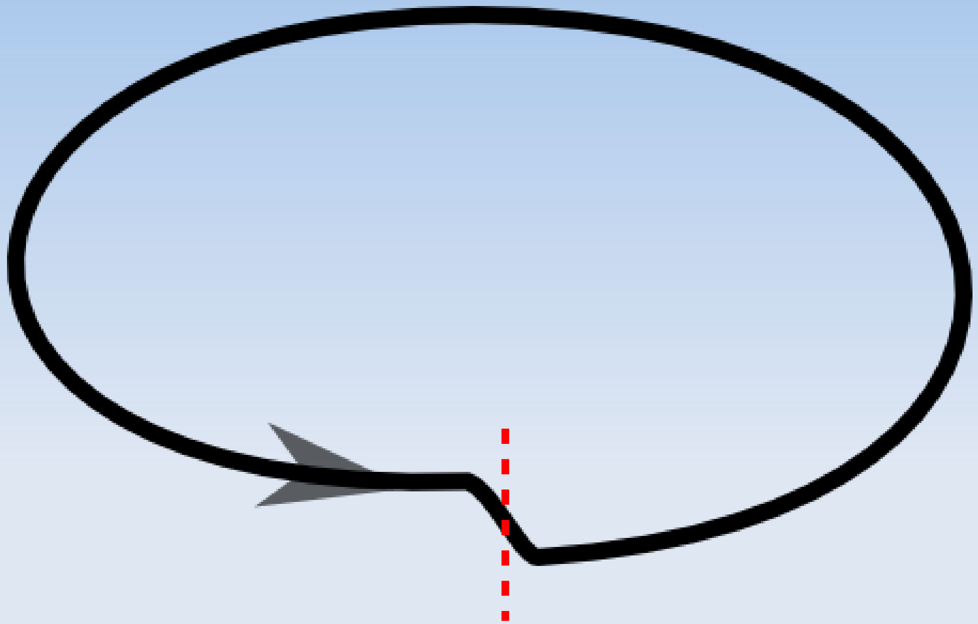
$$P_{N_s} = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ 1 & & & 0 & 1 \end{pmatrix}$$

For multiple identical beam / bunches : $M = \mathbb{I}_{N_{\mathrm{beam}}} \otimes \mathbb{I}_{N_{\mathrm{bunch}}} \otimes M_{1\mathrm{b}}$

In practice, the matrix of each beam/bunch can be build based on different parameters

Example : Beam-beam interaction



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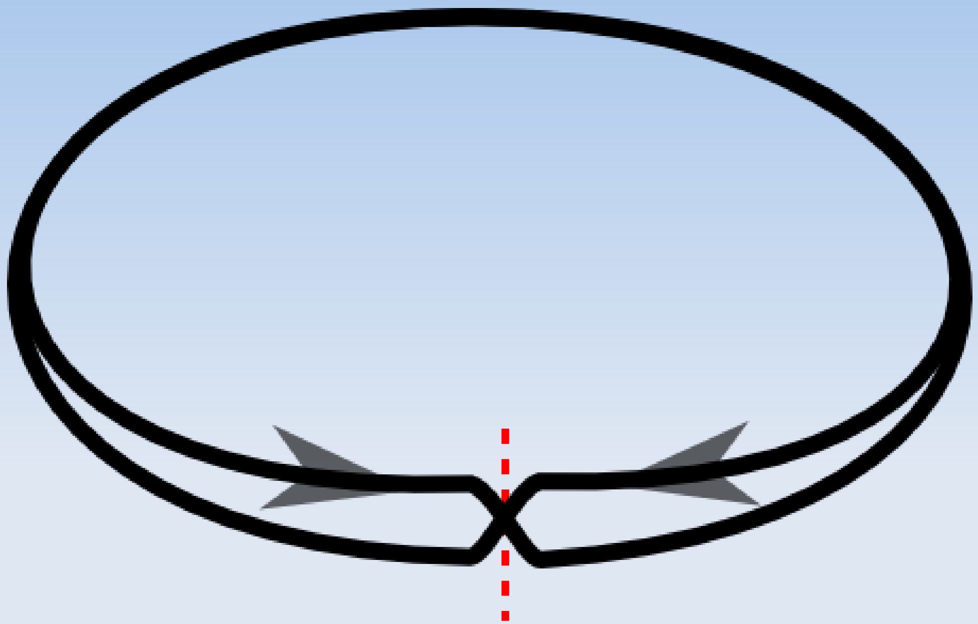

$$\begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) & 0 & 0 \\ -\sin(2\pi Q) & \cos(2\pi Q) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q) & \sin(2\pi Q) \\ 0 & 0 & -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t}$$

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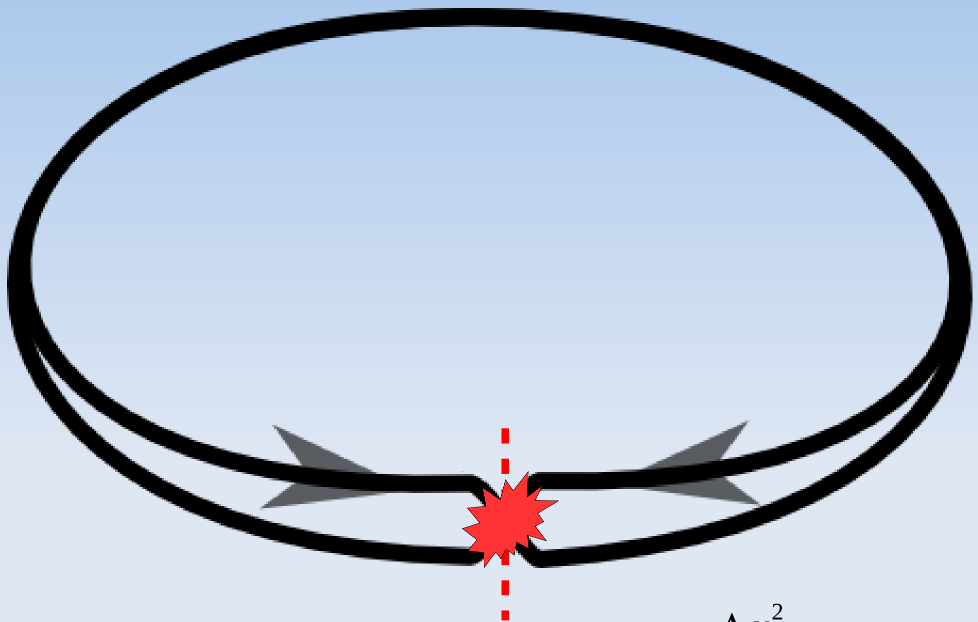

$$\Delta x'_{B1} = \frac{-2 r_0 N}{\gamma_r} \frac{1}{\Delta x}\left(1 - e^{\frac{-\Delta x^2}{4\sigma^2}}\right) \approx k\left(x_{B1} - x_{B2}\right)$$

(linearised coherent force)

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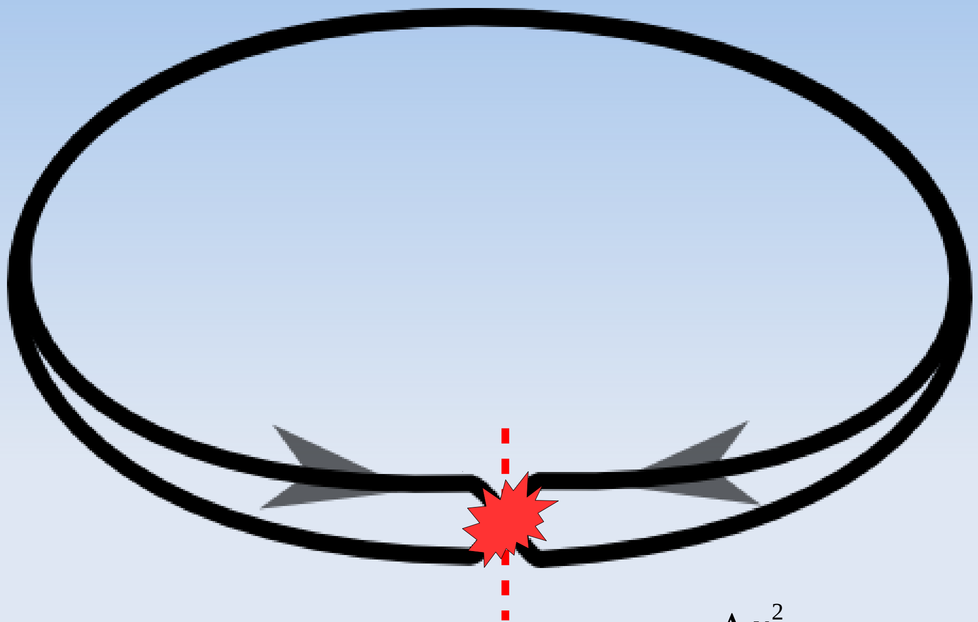

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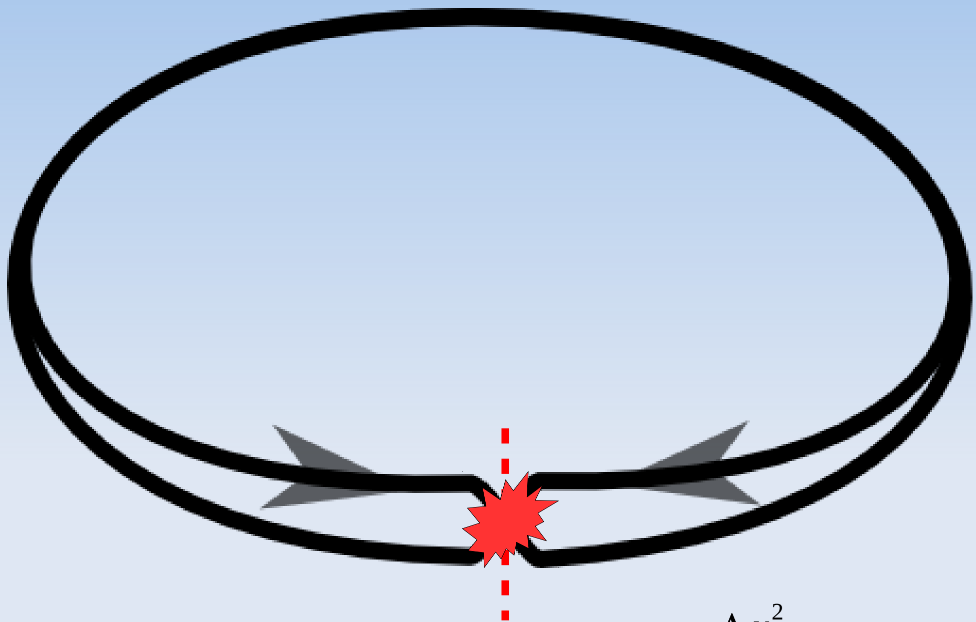

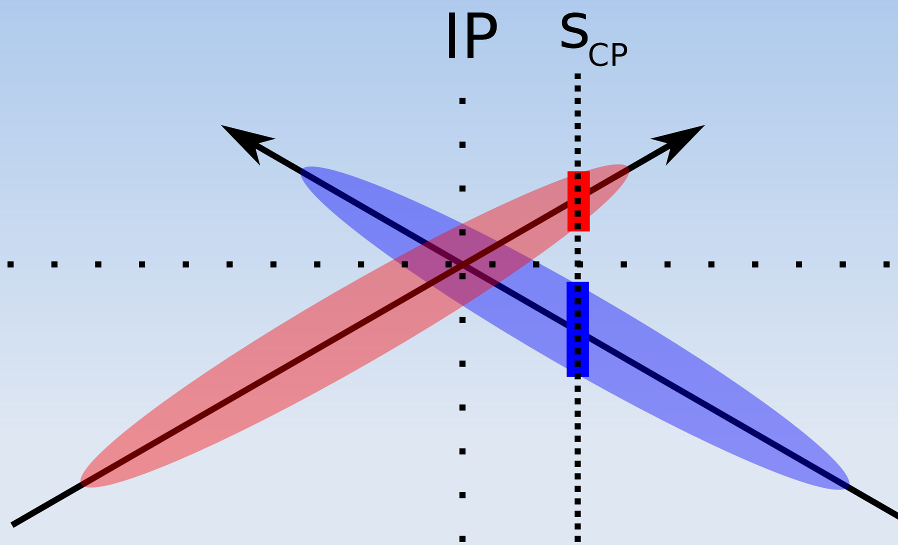

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→ This procedure is extended to binary collision of all the cells (possibly including the crossing angle and the hourglass effects)

Effect of an electron lens

- In the presence of an electron lens that compensates fully the tune spread due to the beam-beam interactions, Landau damping is suppressed for the BBMCI