Euclidean lattice gauge theories

the ugly, the bad, and the good

Berkeley
UNIVERSITY OF CALIFORNIA

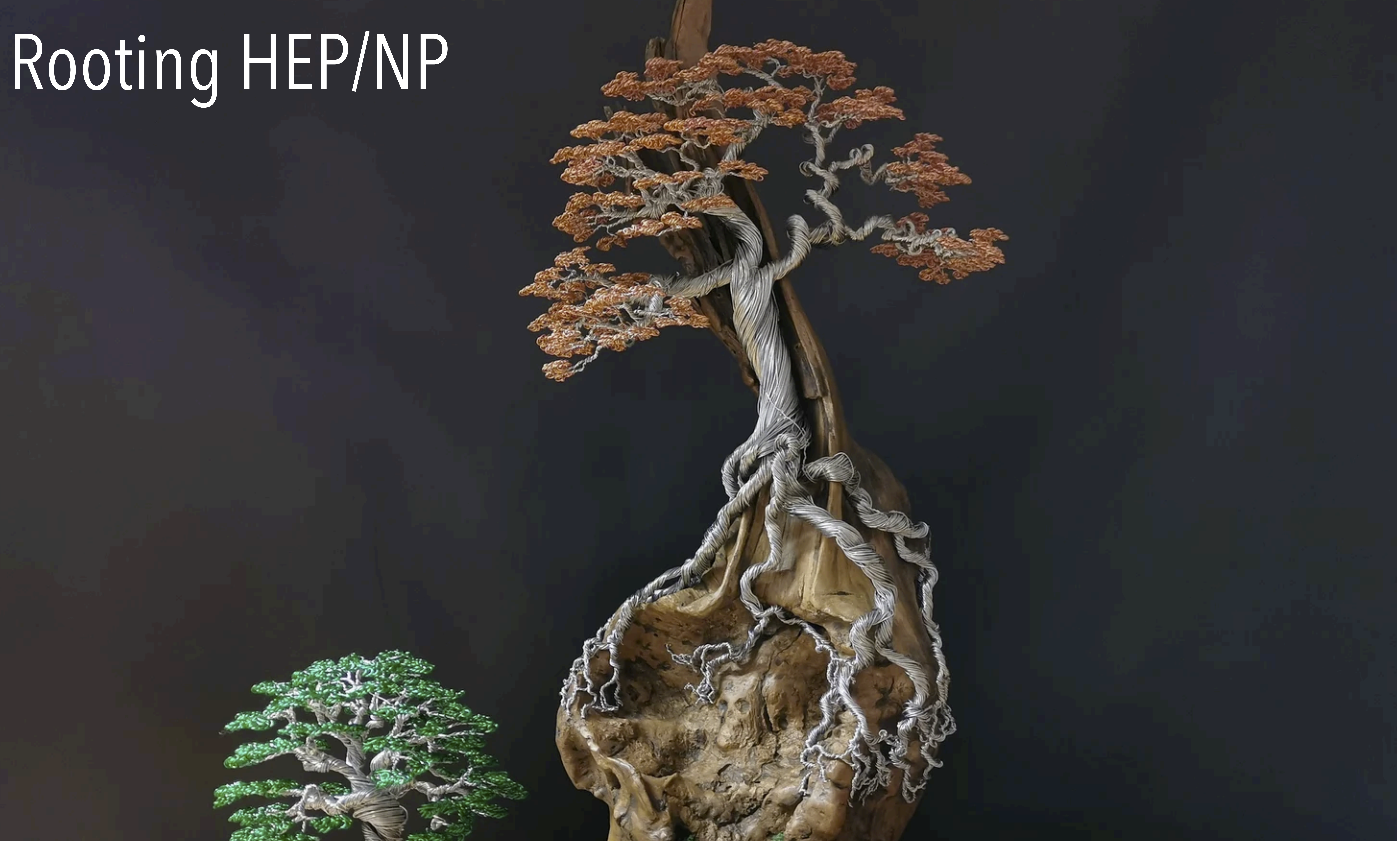
BERKELEY LAB

RAÚL BRICEÑO

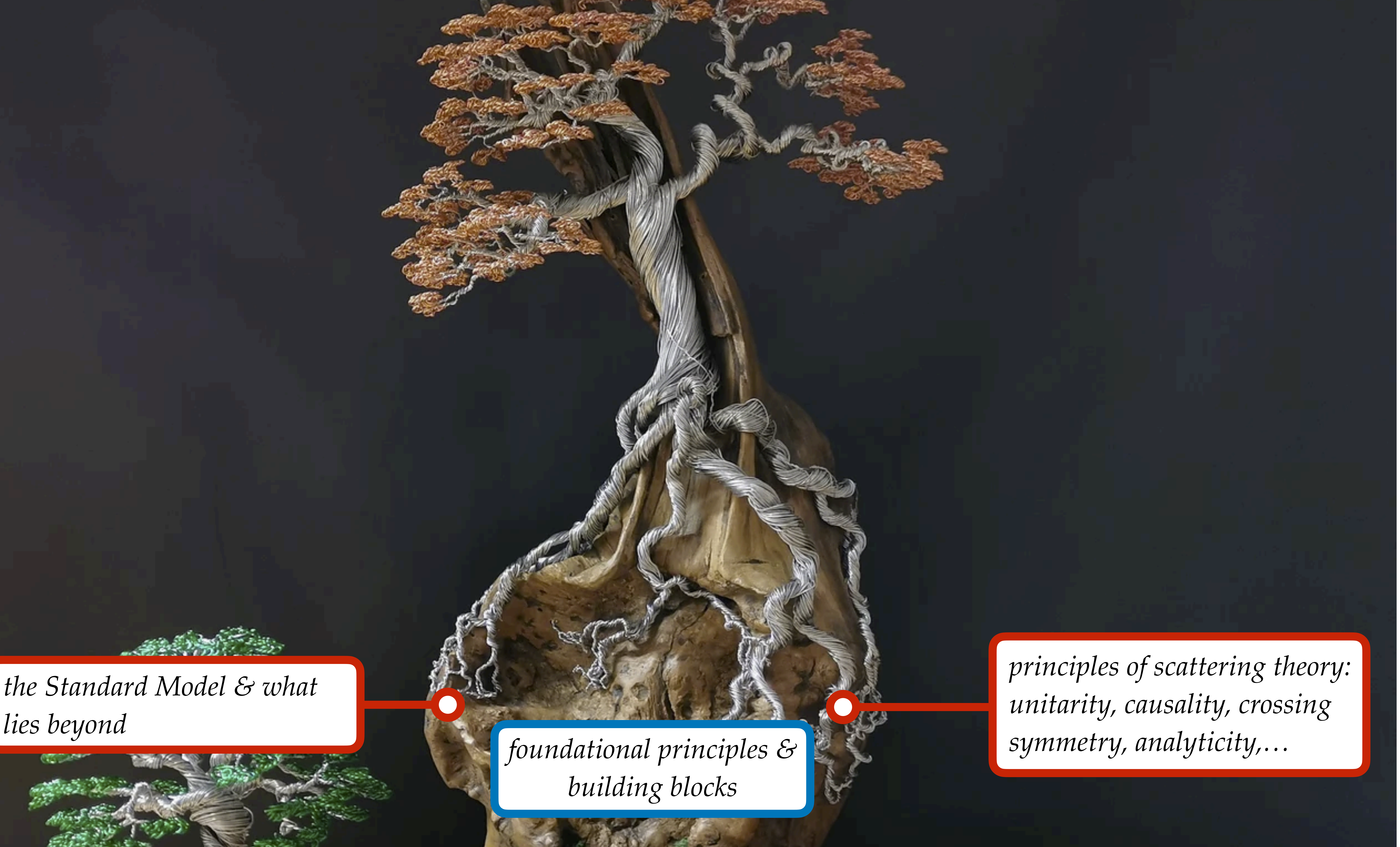
rbriceno@berkeley.edu

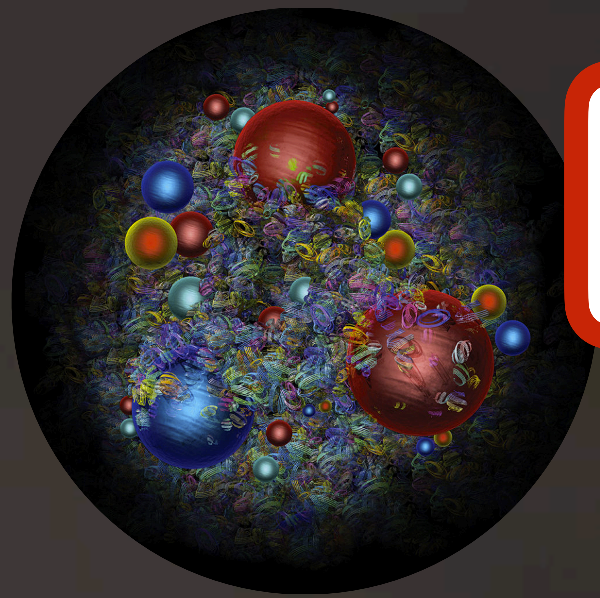
http://bit.ly/rbricenoPhD

Rooting HEP/NP



Rooting HEP/NP

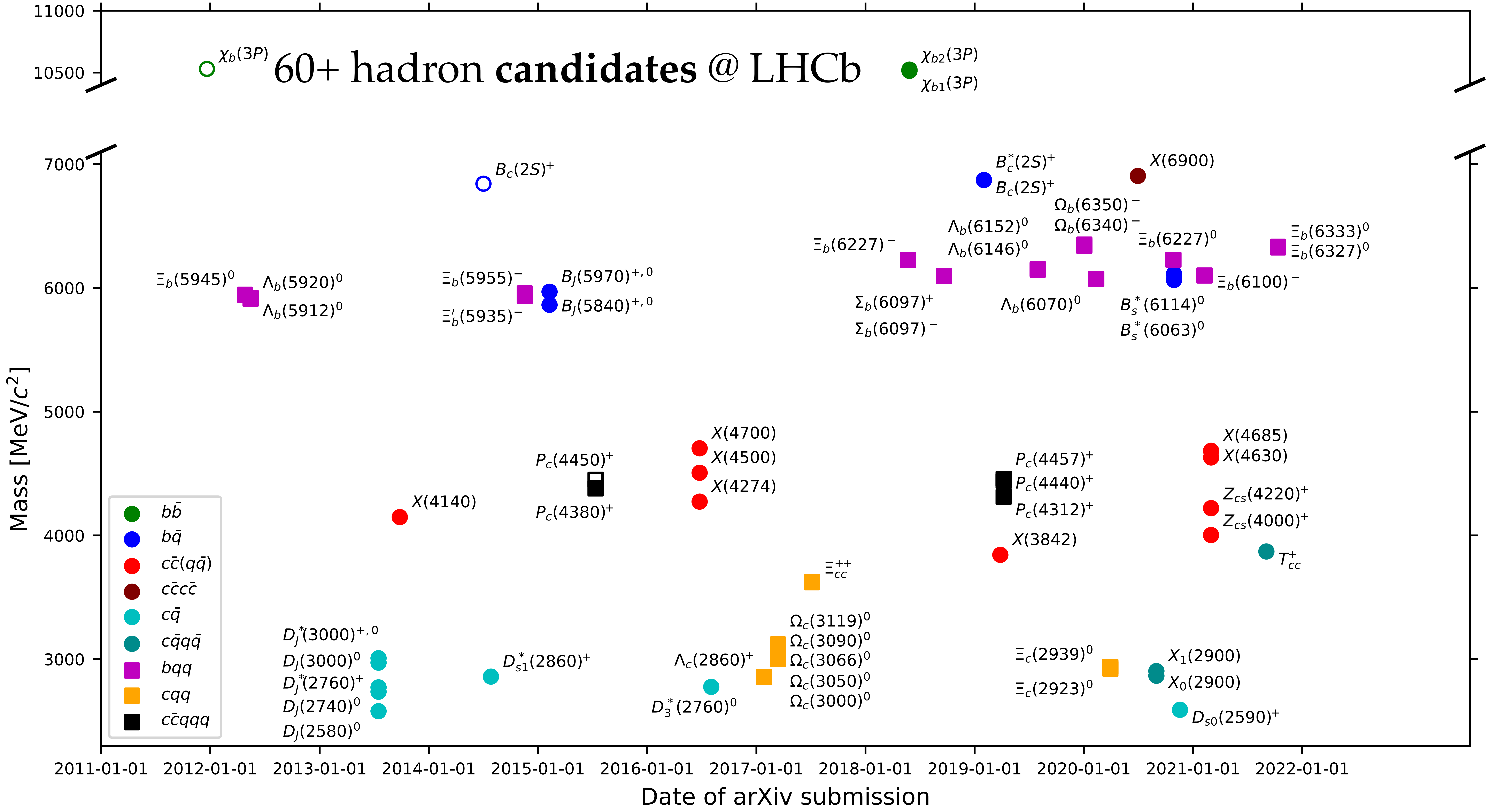





emergent phenomena
Nuclear astro physics:
equation of states…
Sanford Underground
Research Facility
800 miles
(1300 kilometers)
NEUTRINO
PRODUCTION
PARTICLE
DETECTOR
PROTON
ACCELERATOR
Fermilab
BSM searches
QCD spectrum
π_1
u
$\bar{s}$
s
d
d
$\bar{u}$
u
$\bar{d}$
the Standard Model & what
lies beyond
foundational principles &
building blocks
principles of scattering theory:
unitarity, causality, crossing
symmetry, analyticity,…

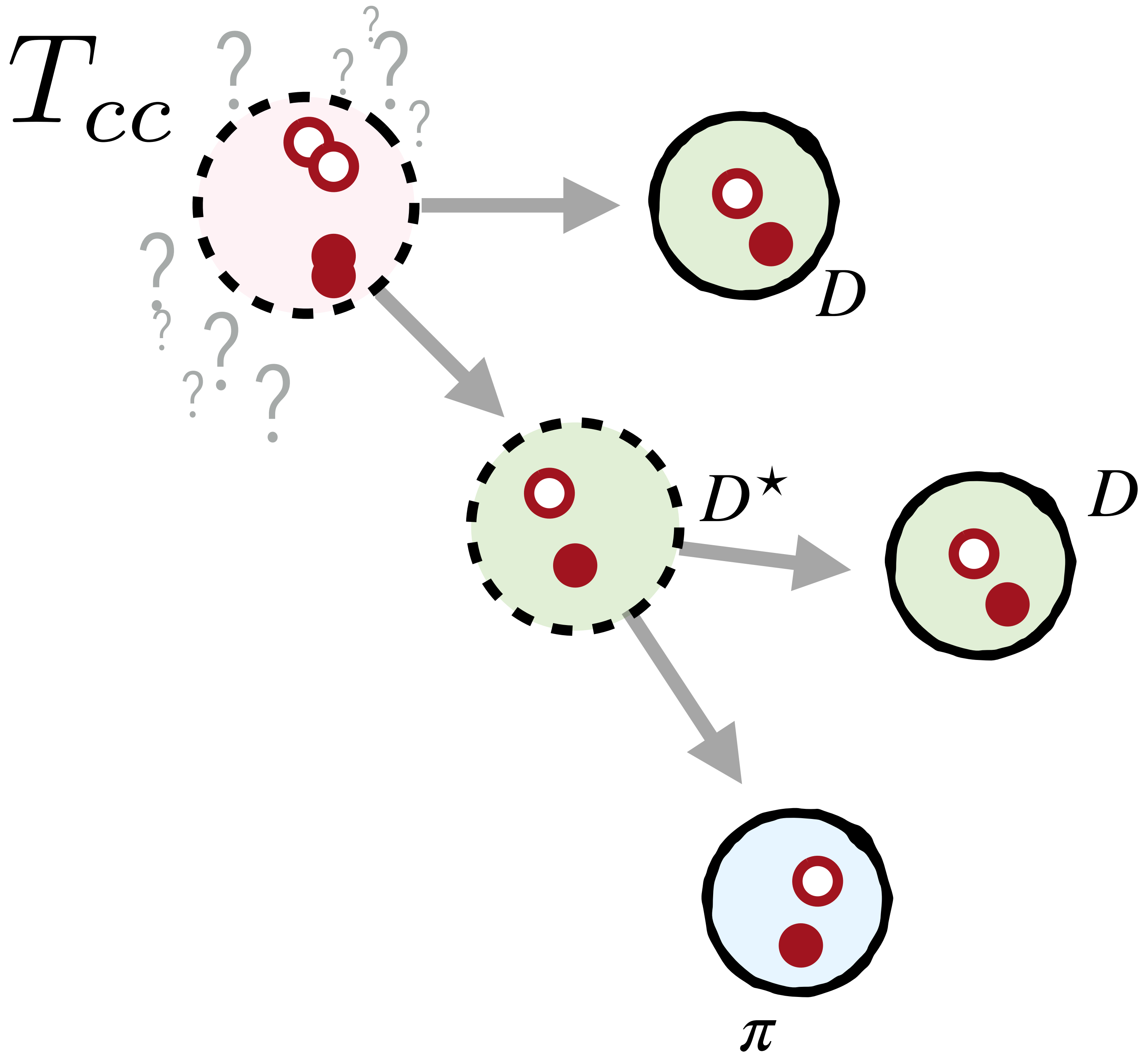

emergent phenomena
Nuclear astro physics: equation of states…
Sanford Underground Research Facility
800 miles (1300 kilometers)
Fermilab
NEUTRINO PRODUCTION
PARTICLE DETECTOR
PROTON ACCELERATOR
BSM searches
QCD spectrum
π_1
EFTs, pQCD,…
quantum computing
lattice QCD
scattering theory
the Standard Model & what lies beyond
foundational principles & building blocks
principles of scattering theory: unitarity, causality, crossing symmetry, analyticity,…

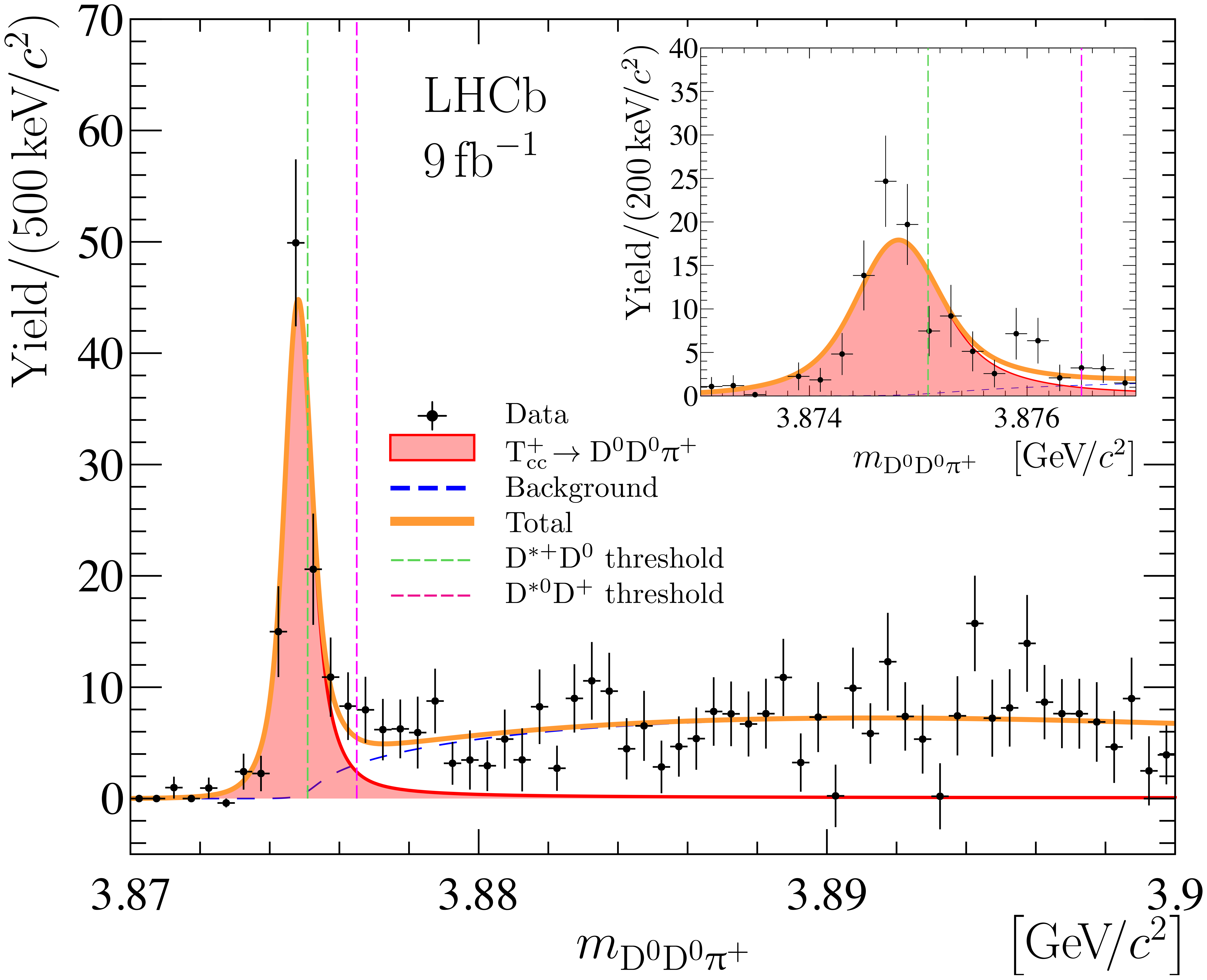
The particle zoo the remake



Numerous other experimental searches…

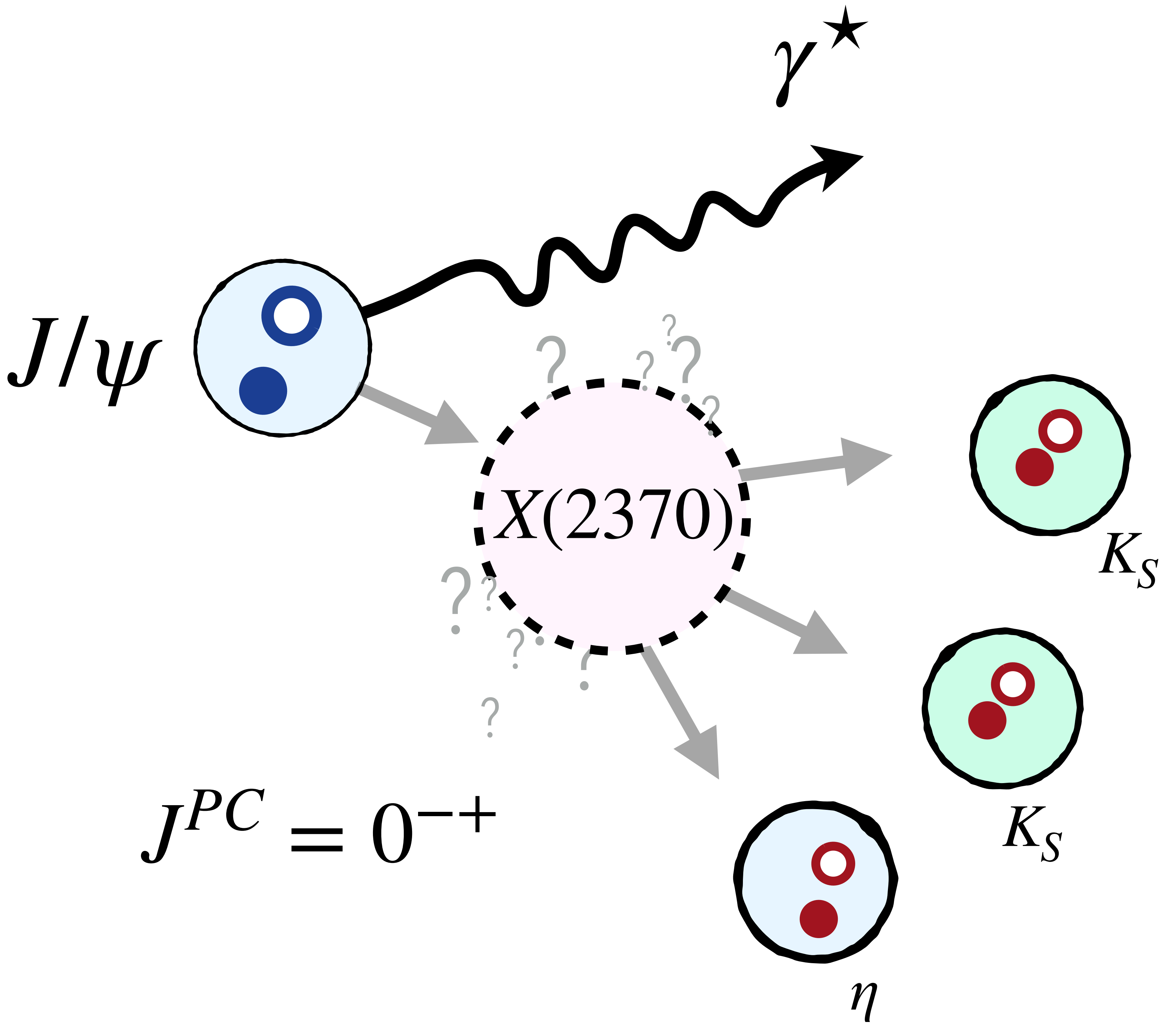
Tetraquarks?

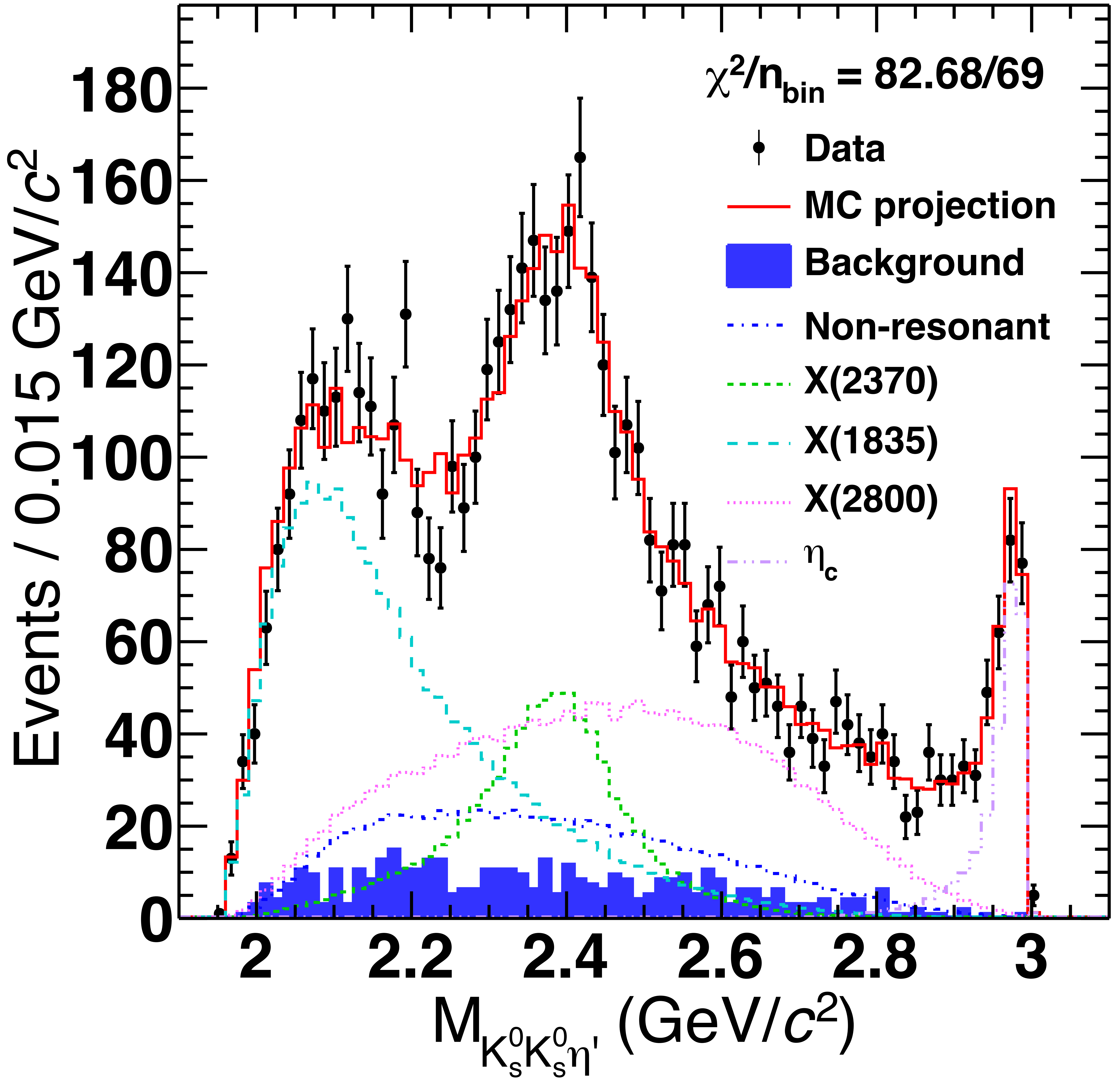




(2021)

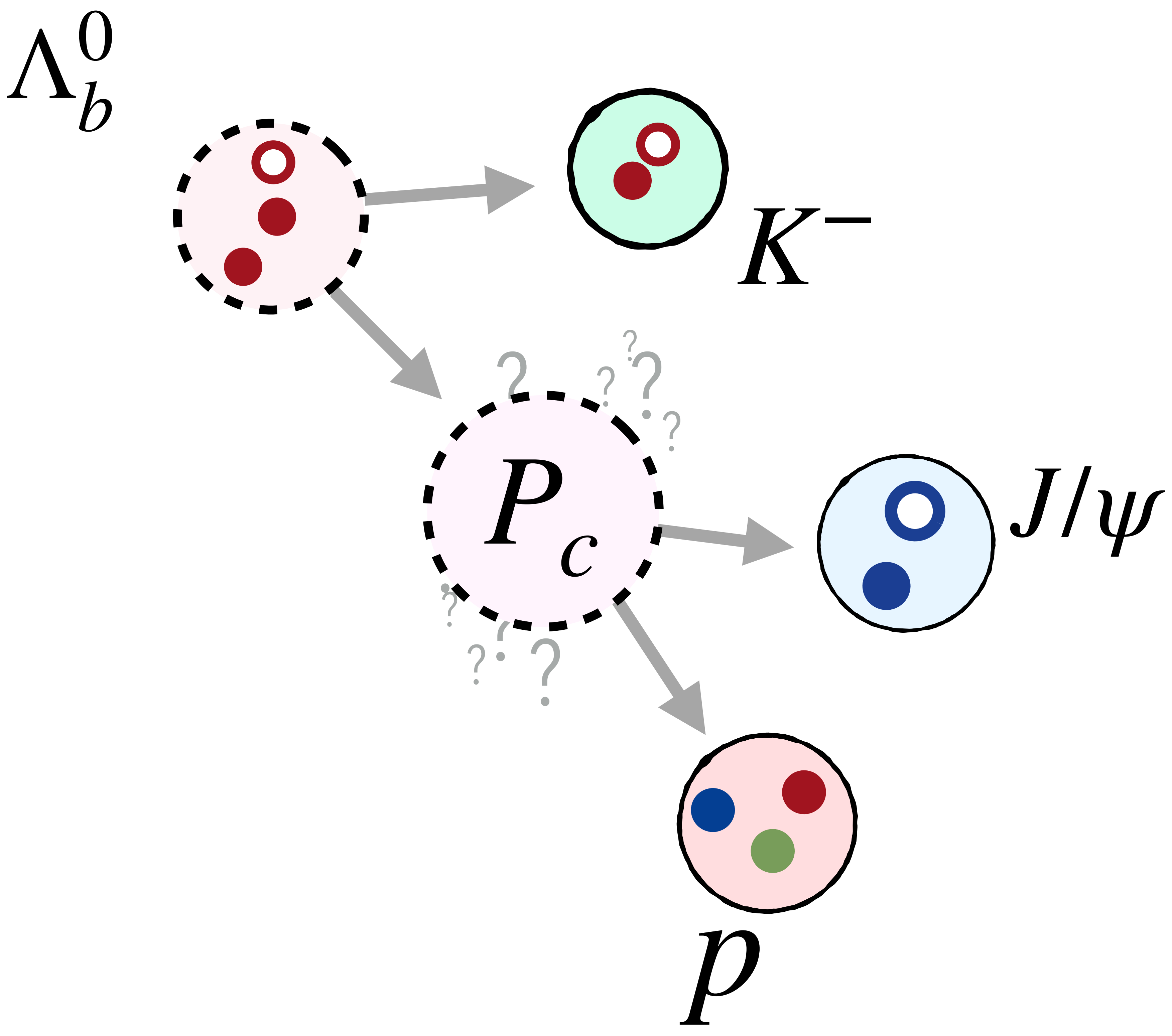
Glueballs?

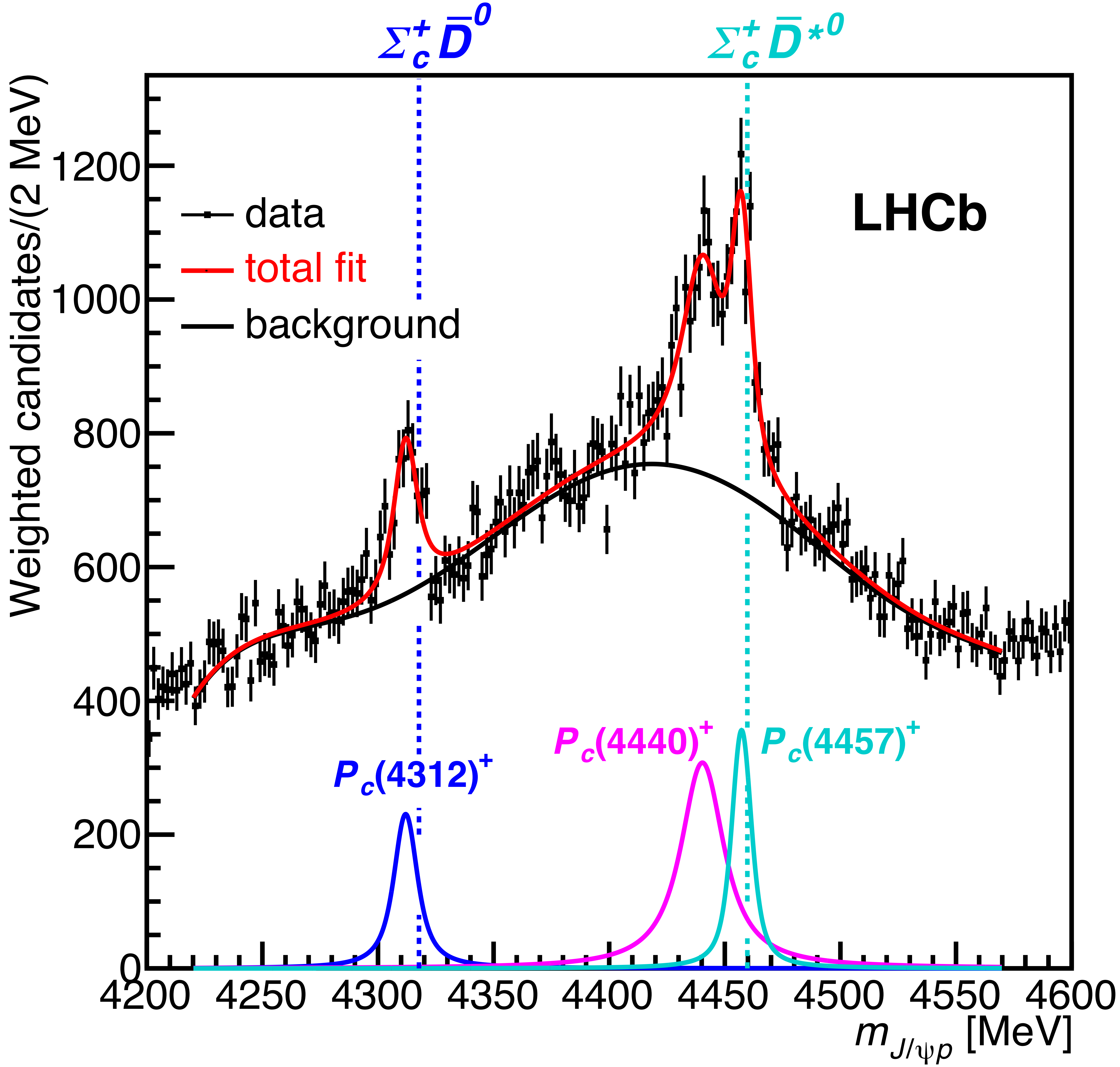




BESIII (2025)

Pentaquarks?

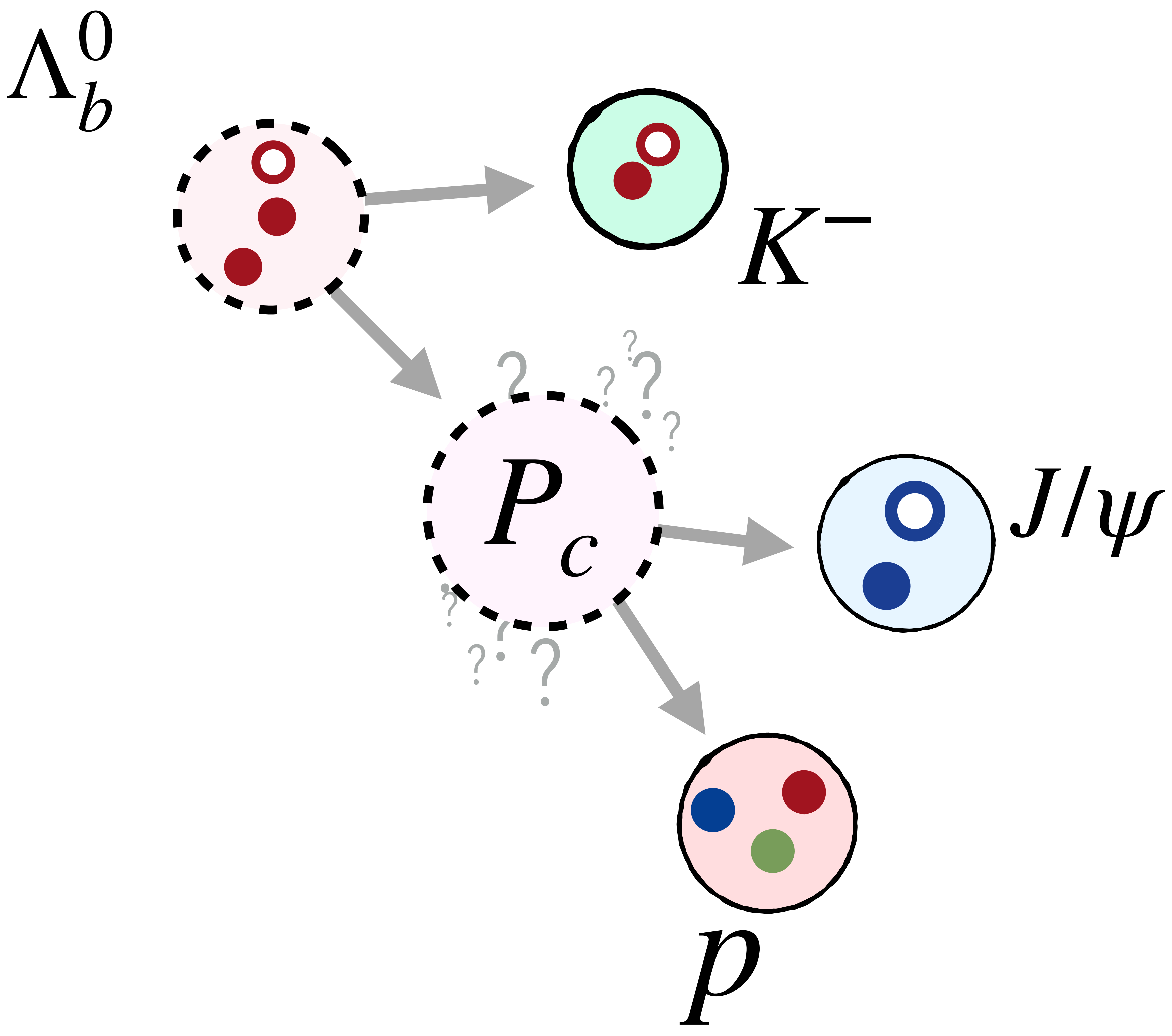




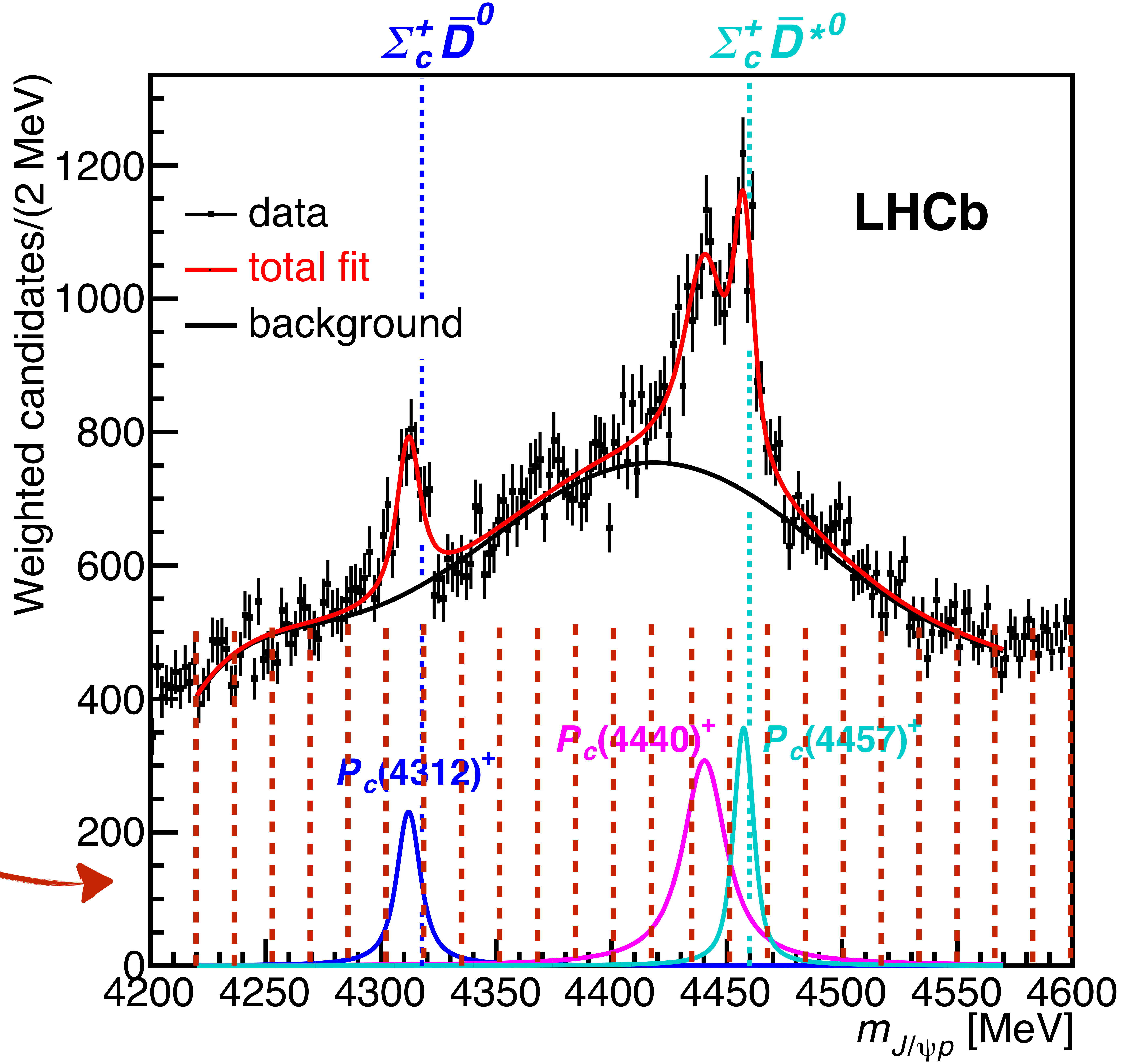
LHCb

(2019)

Pentaquarks?

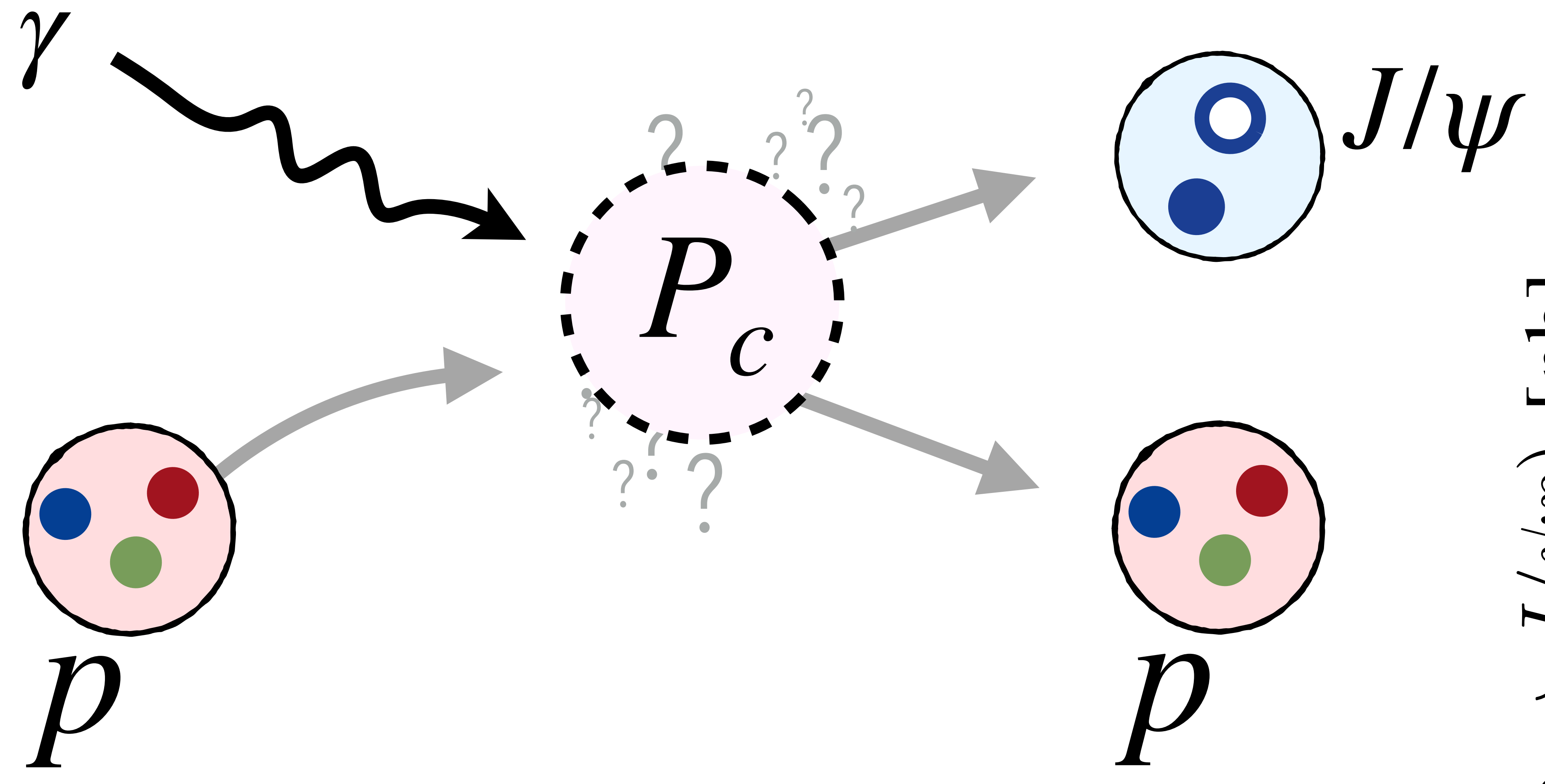


pions are cheap!

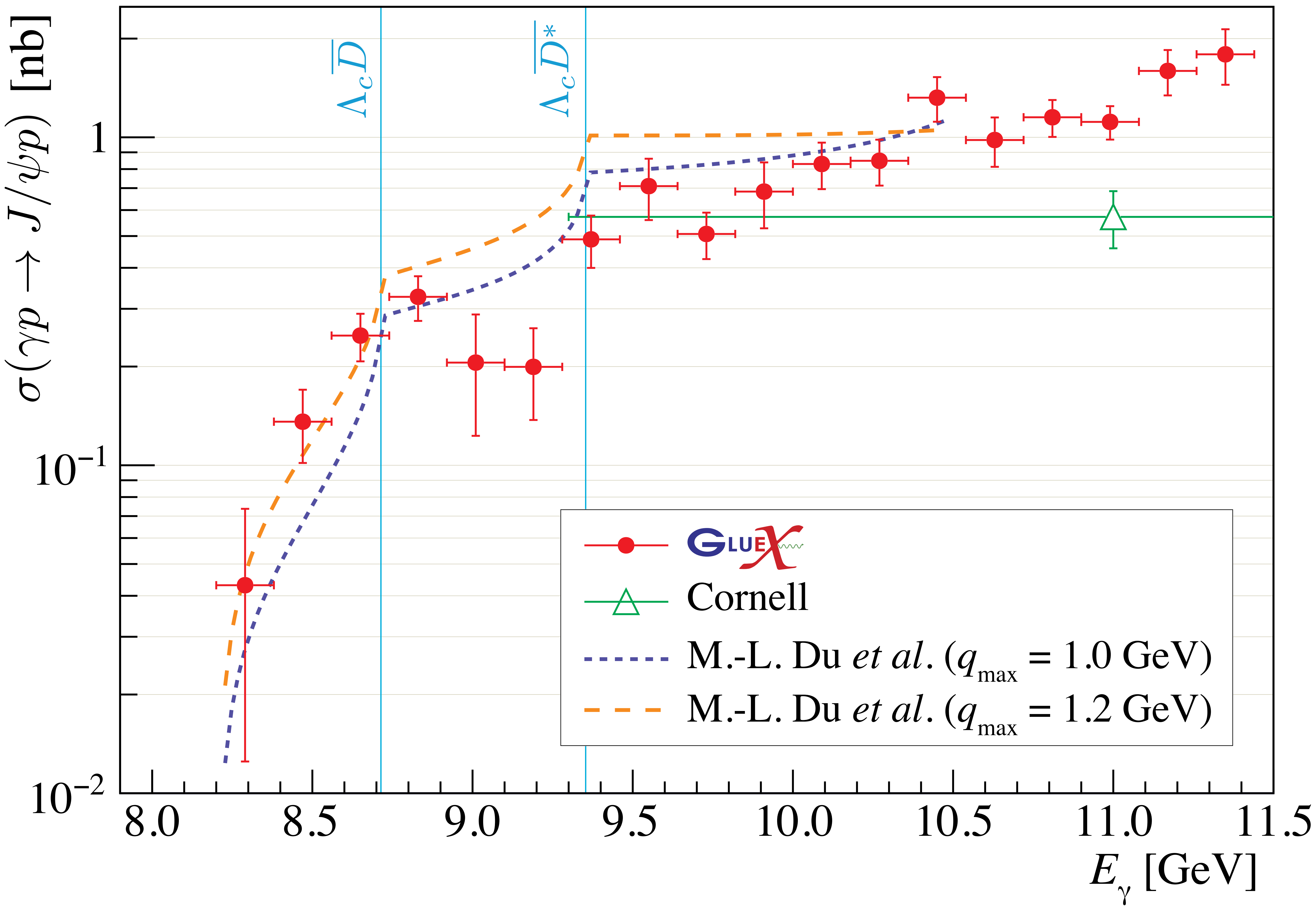


(2021)

Pentaquarks?



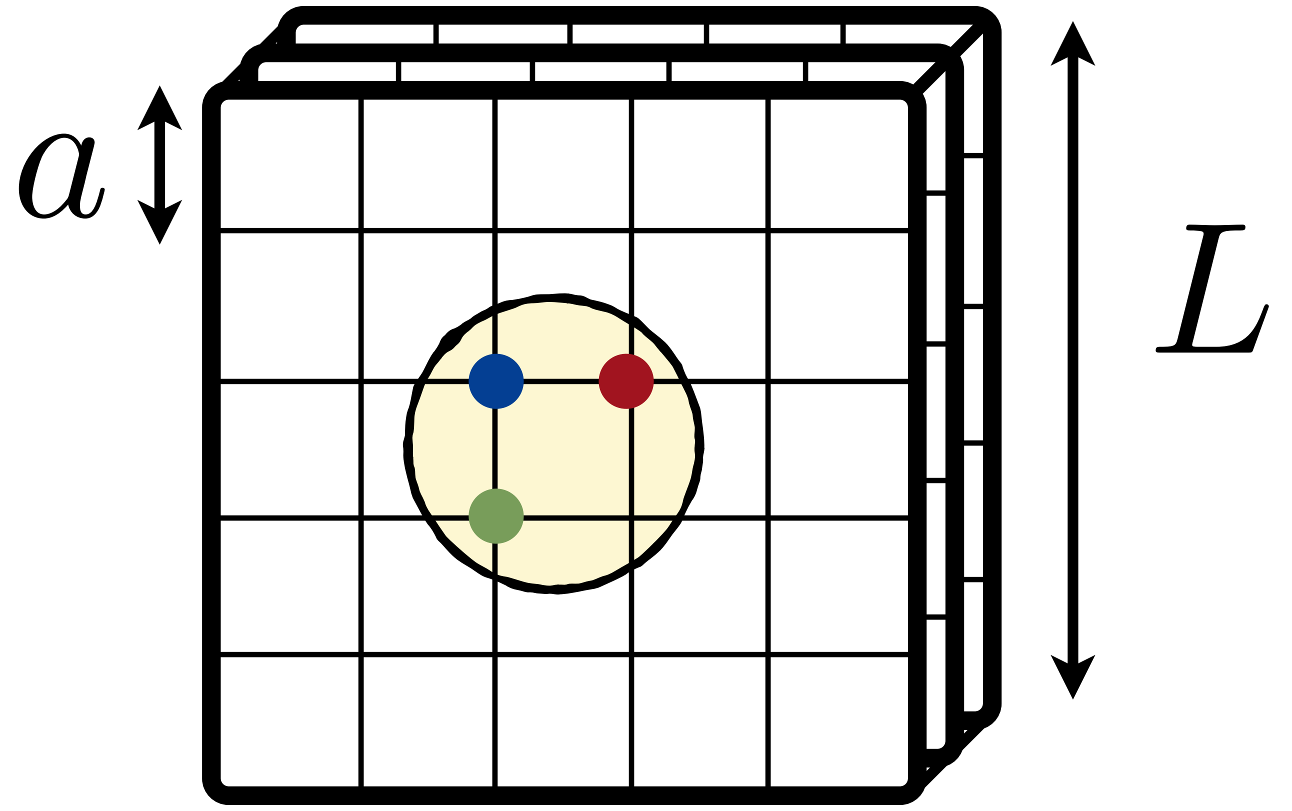
No smoking gun of pentaquarks at JLab



GlueX (2024)

Euclidean lattice gauge theories the ugly

- Euclidean spacetime: $t_M \to -it_E$
  - Monte Carlo sampling
- finite temporal extent: $T \sim 1 - 100$ fm
- finite spacial volume: $L \sim 1 - 10$ fm
- lattice spacing: $a \sim 0.03 - 0.1$ fm
- quark masses: $m_q \to m_q^{\text{phys}}$



The bad #1

the vast majority of observables are directly inaccessible

e.g. low-lying spectrum

$$C_{2\text{pt.}}(T,t) = \text{Tr}\left[e^{-\hat{H}T}\,\mathcal{O}(t)\,\mathcal{O}^\dagger(0)\right]$$

knowing what's the right operator requires diagonalizing the Hamiltonian

The bad #1

the vast majority of observables are directly inaccessible

e.g. low-lying spectrum

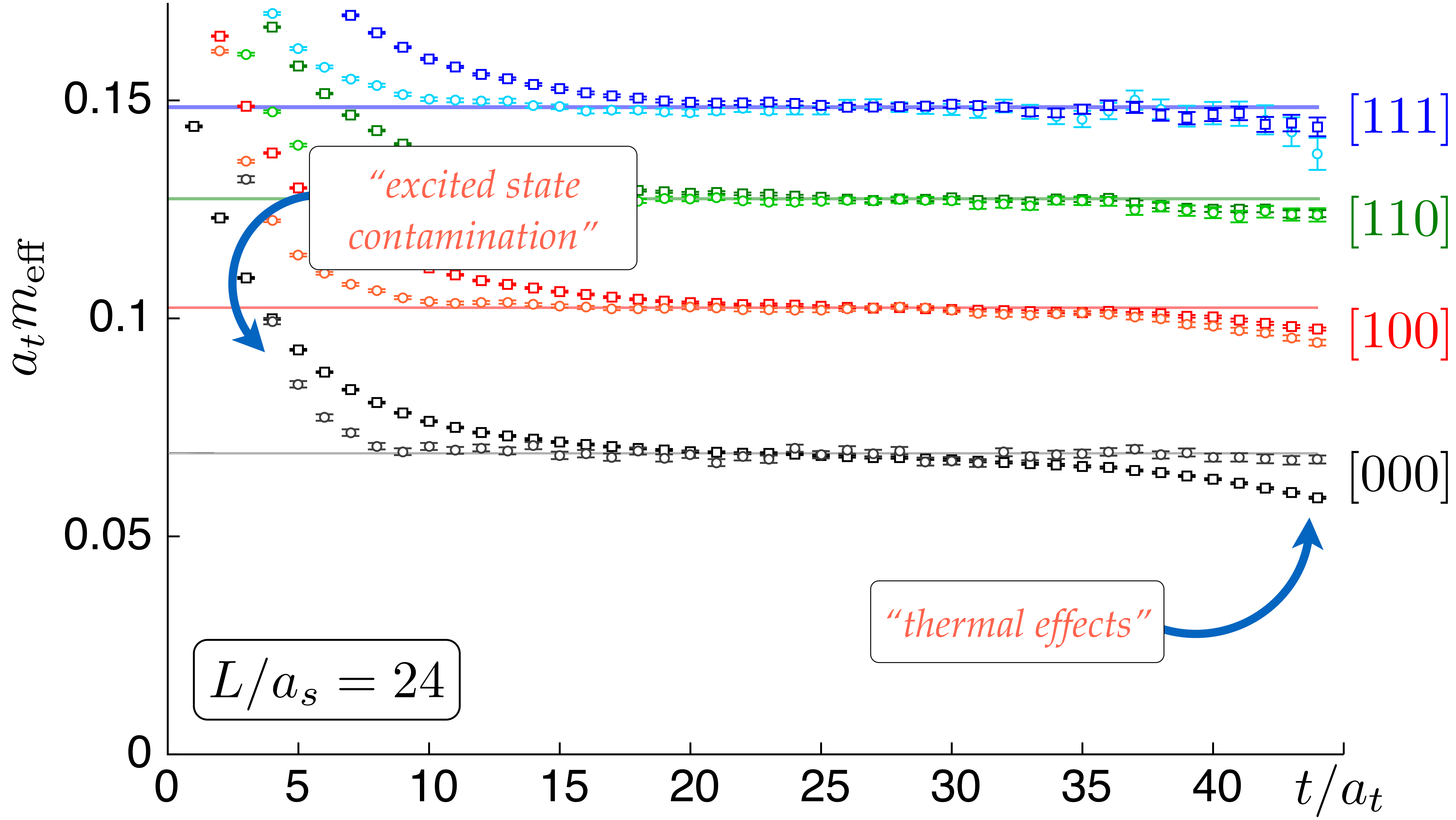
$$C_{2\text{pt.}}(T,t) = \text{Tr}\left[e^{-\hat{H}T}\,\mathcal{O}(t)\,\mathcal{O}^\dagger(0)\right]$$

$$= \sum_{m,n} e^{-E_m(T-t)} e^{-E_n t} \langle E_m|\,\mathcal{O}(0)|E_n\rangle\langle E_n\,\mathcal{O}^\dagger(0)|E_n\rangle$$

$$\longrightarrow e^{-E_0 t}\langle 0|\,\mathcal{O}(0)|E_0\rangle\langle E_0\,\mathcal{O}^\dagger(0)|0\rangle\left(1+\mathcal{O}\left(e^{-(E_1-E_0)t}\right)\right)$$

large T and t limit

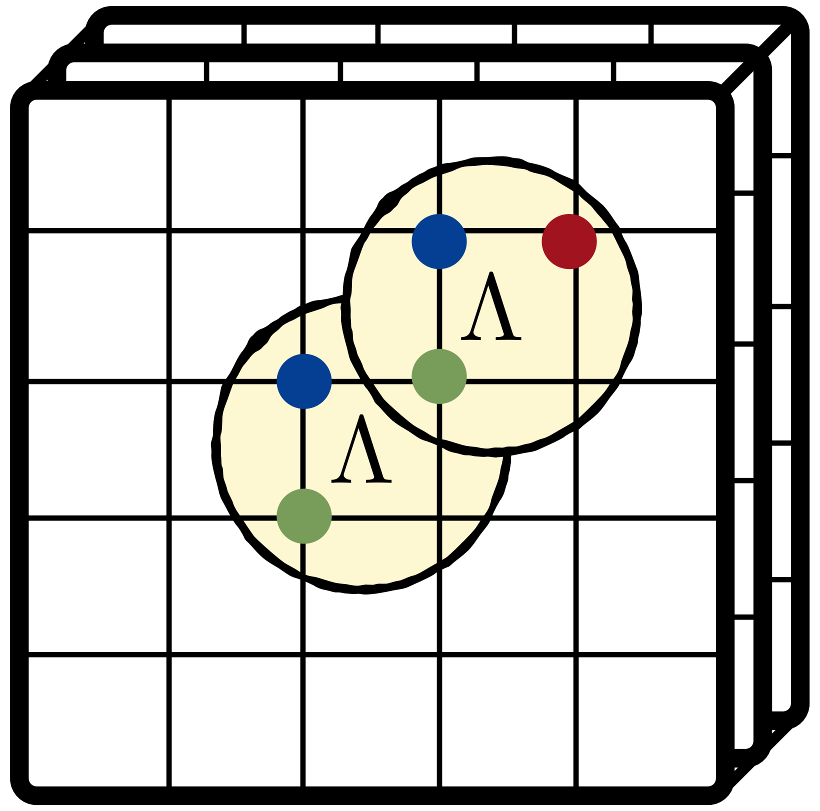
effective mass: $a\,m_{\text{eff}}(t) = \log\left(\frac{C_{2\text{pt.}}(t)}{C_{2\text{pt.}}(t+1)}\right)$

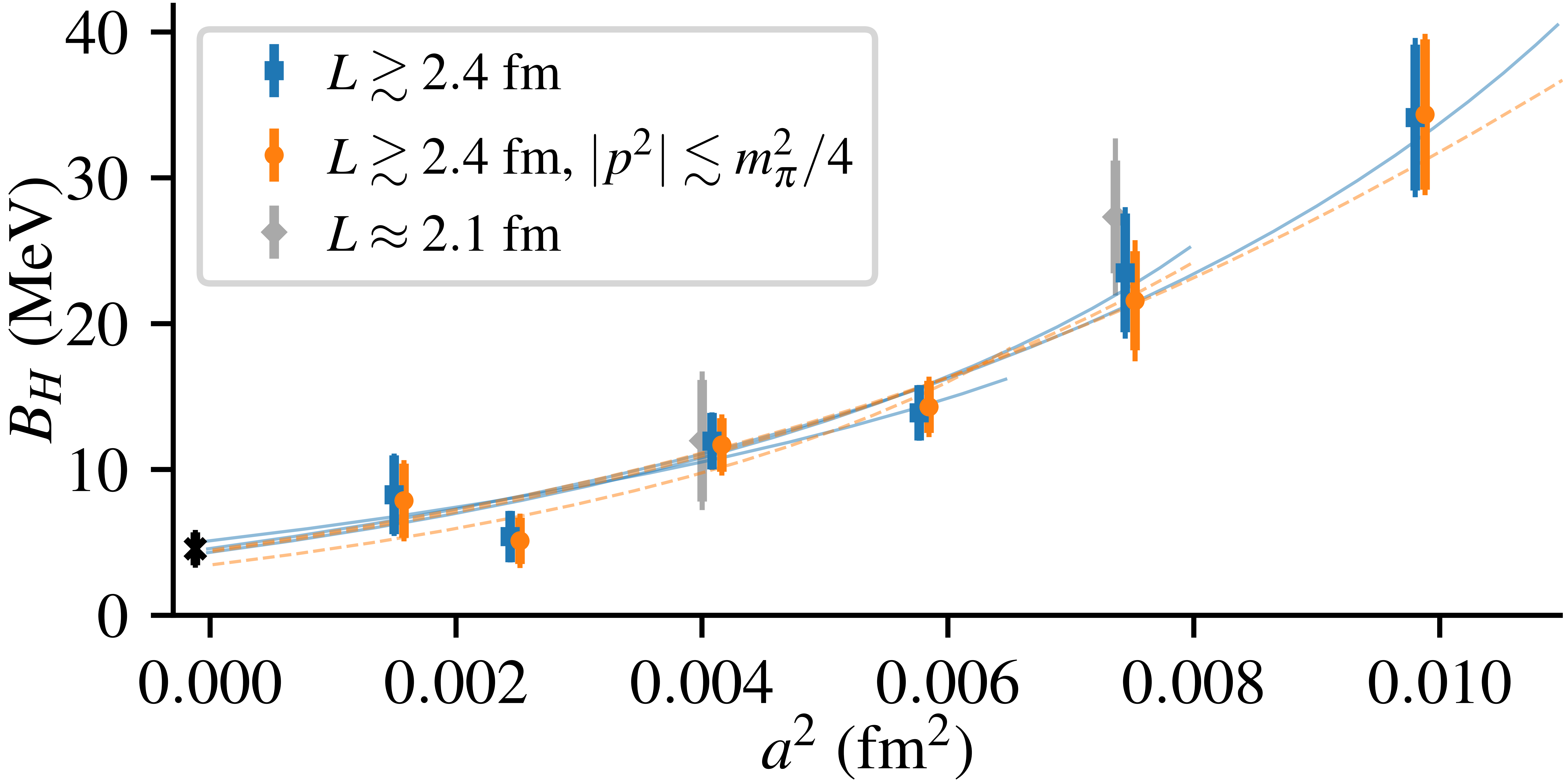


Dudek, Edwards, Thomas (2012)

More bad

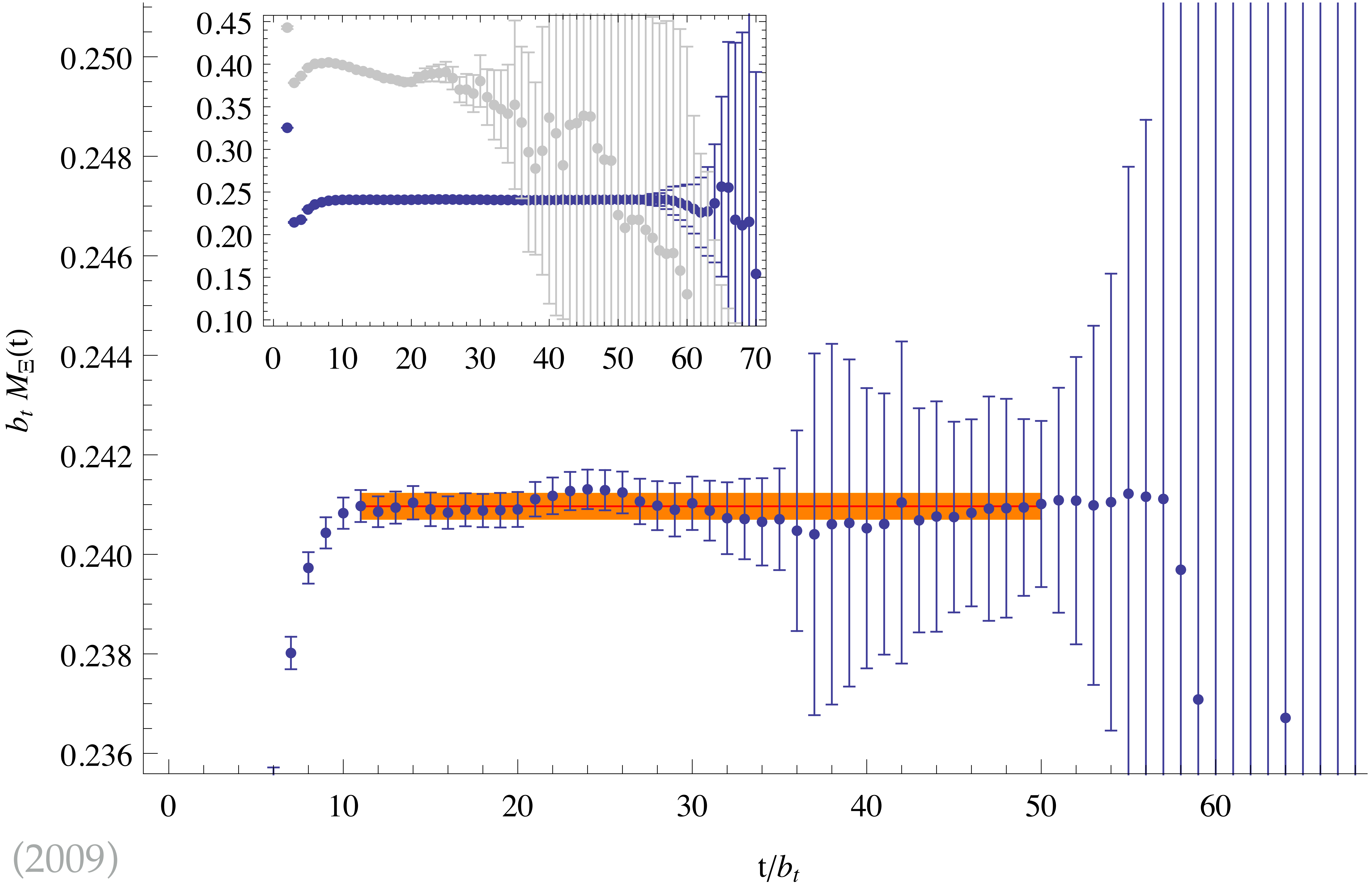
- Nielsen–Ninomiya theorem [doublers/chiral symmetry] and other discretization effects,





More bad

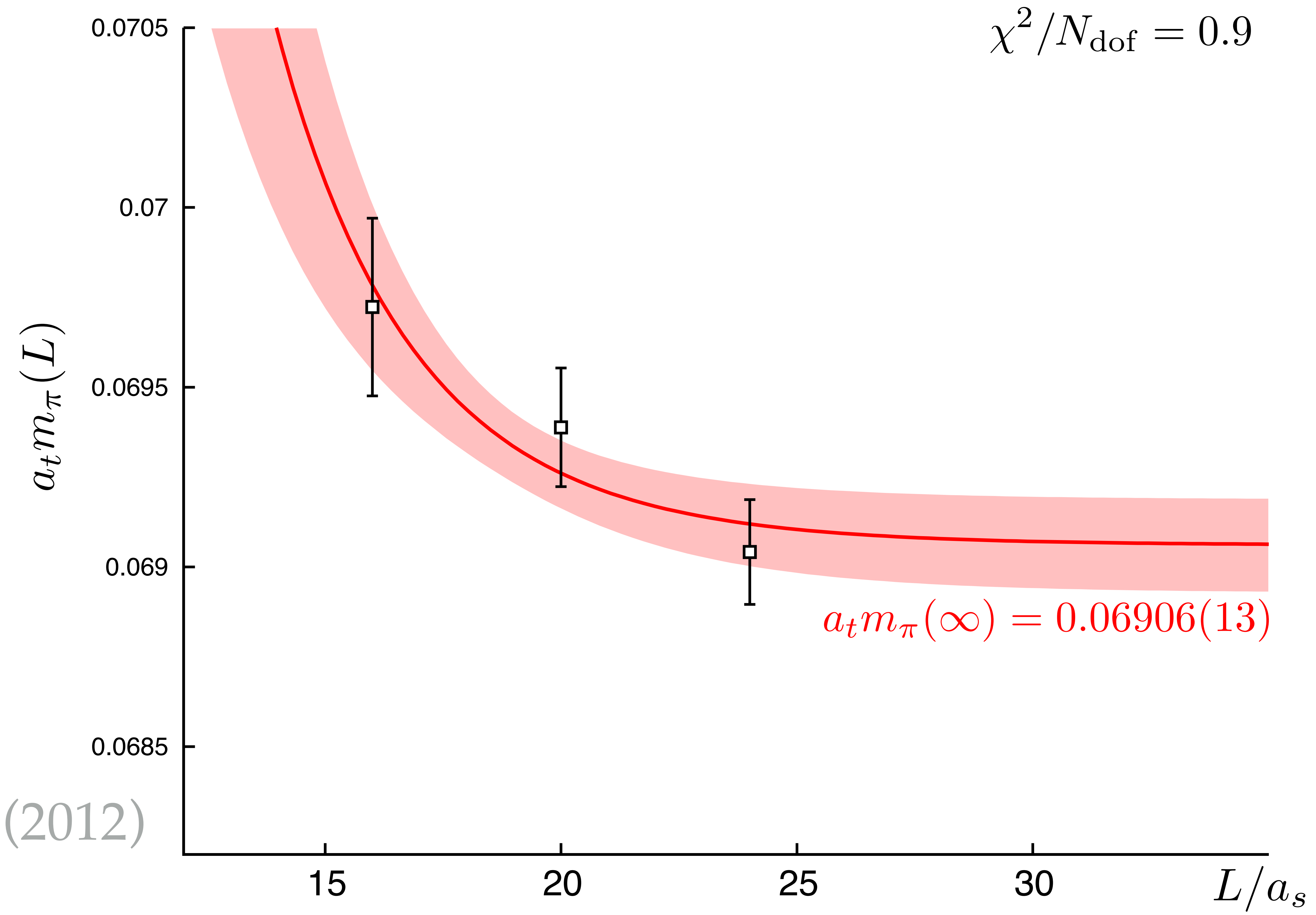
- Nielsen–Ninomiya theorem [doublers/chiral symmetry] and other discretization effects,
- signal to noise,



NPLQCD (2009)

More bad

- Nielsen–Ninomiya theorem [doublers/chiral symmetry] and other discretization effects,
- signal to noise,
- IR and UV symmetry breaking effects,
- contraction costs,
- exponentially suppressed volume effects,
- no asymptotic states,
- critical slowing down,
- sign problems,
- …

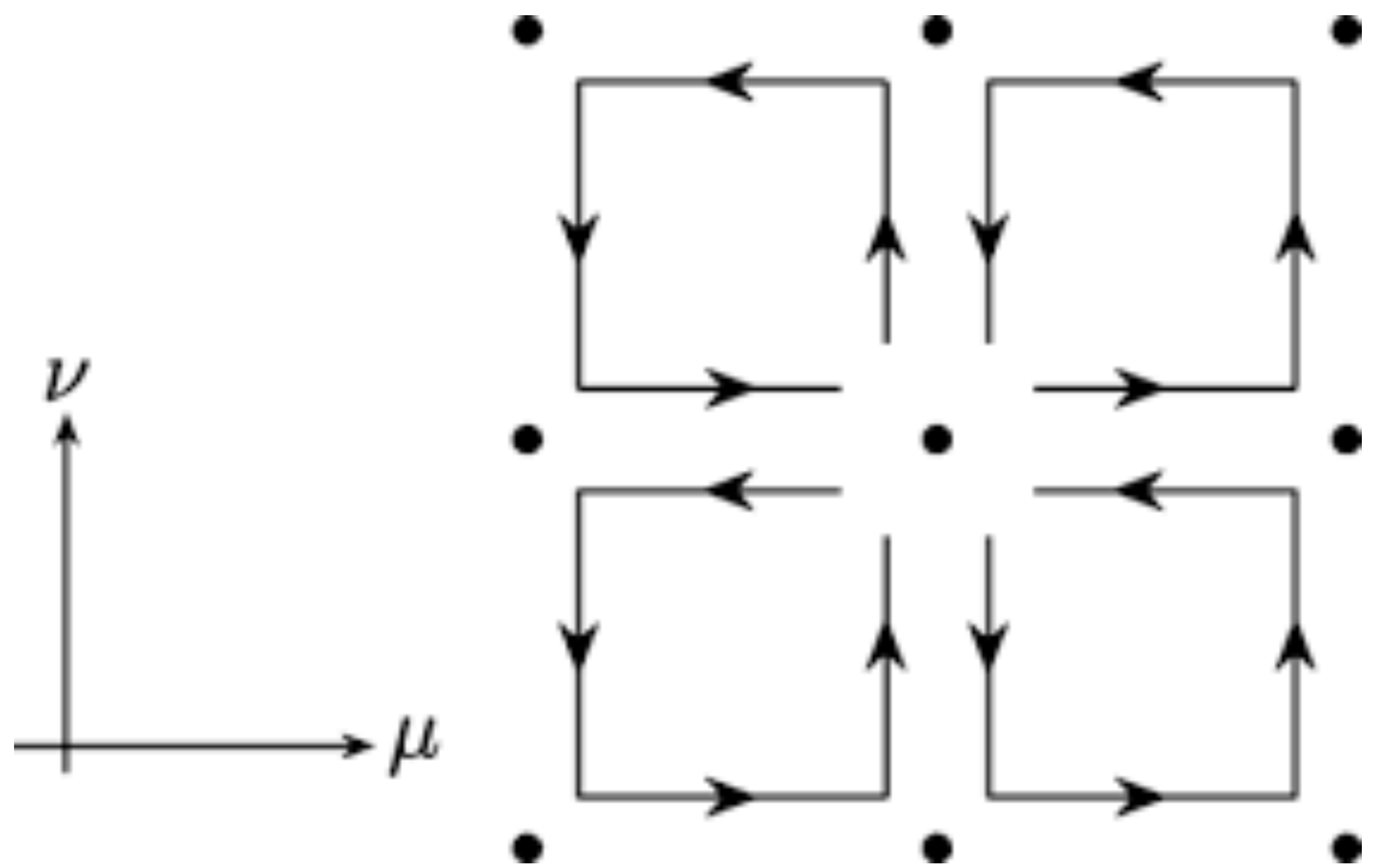


Dudek, Edwards, Thomas (2012)

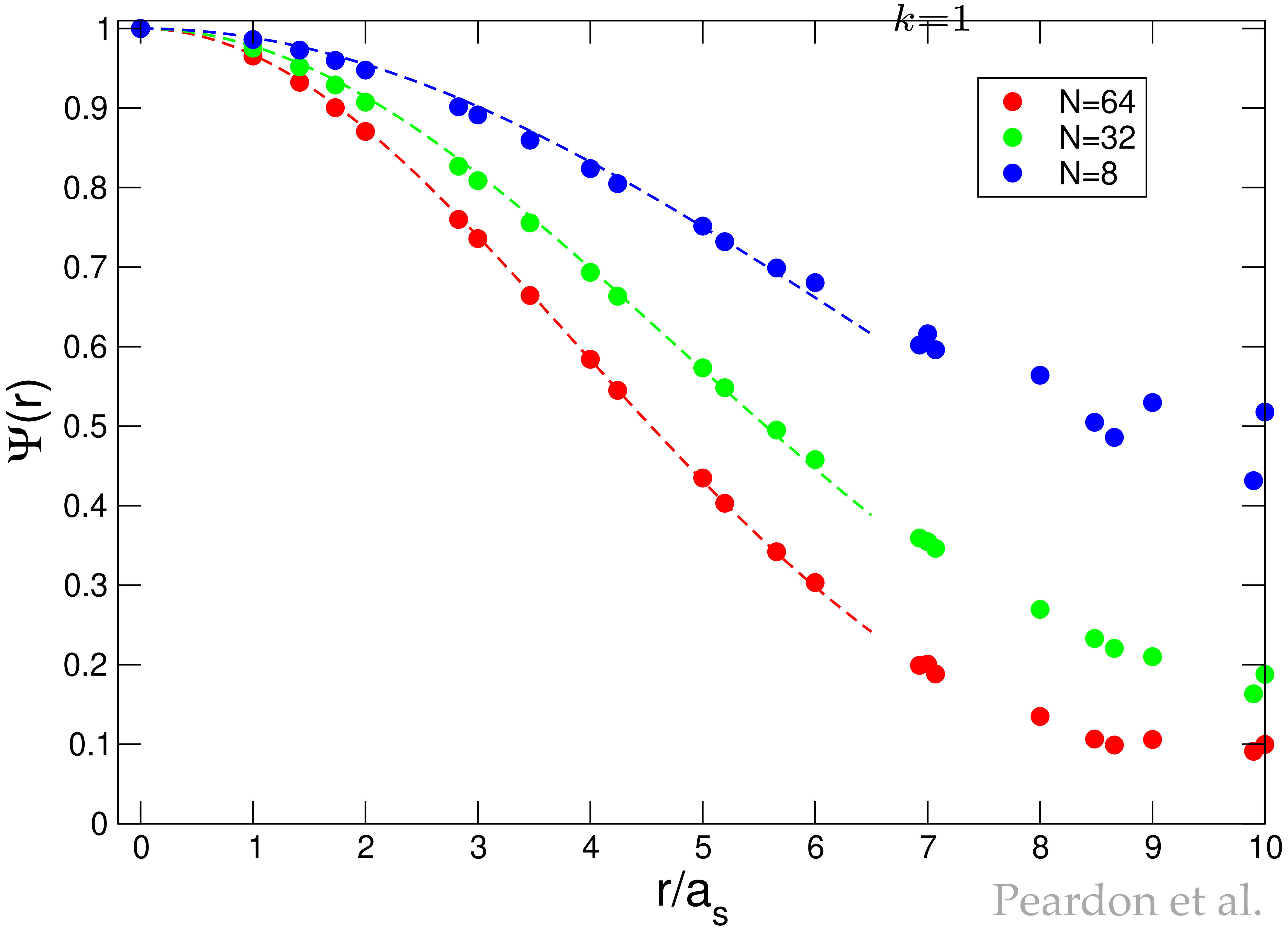
The good - algorithmic

- ☑ Improvement in actions (clover, HISQ, domain wall, overlap, twisted mass, …),
- ☑ Improvements in invertors (multigrid, deflation , all mode averaging, mixed-precision…),
- ☑ Improvements in operators (non-local, multi-hadron, …),
- ☑ Smart smearing (distillation, gradient flow, …)
- ☑ ….

$$\Box(t) = V(t)V^\dagger(t) \implies \Box_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t) v_y^{(k)\dagger}(t)$$



clover term



Peardon et al.

More good

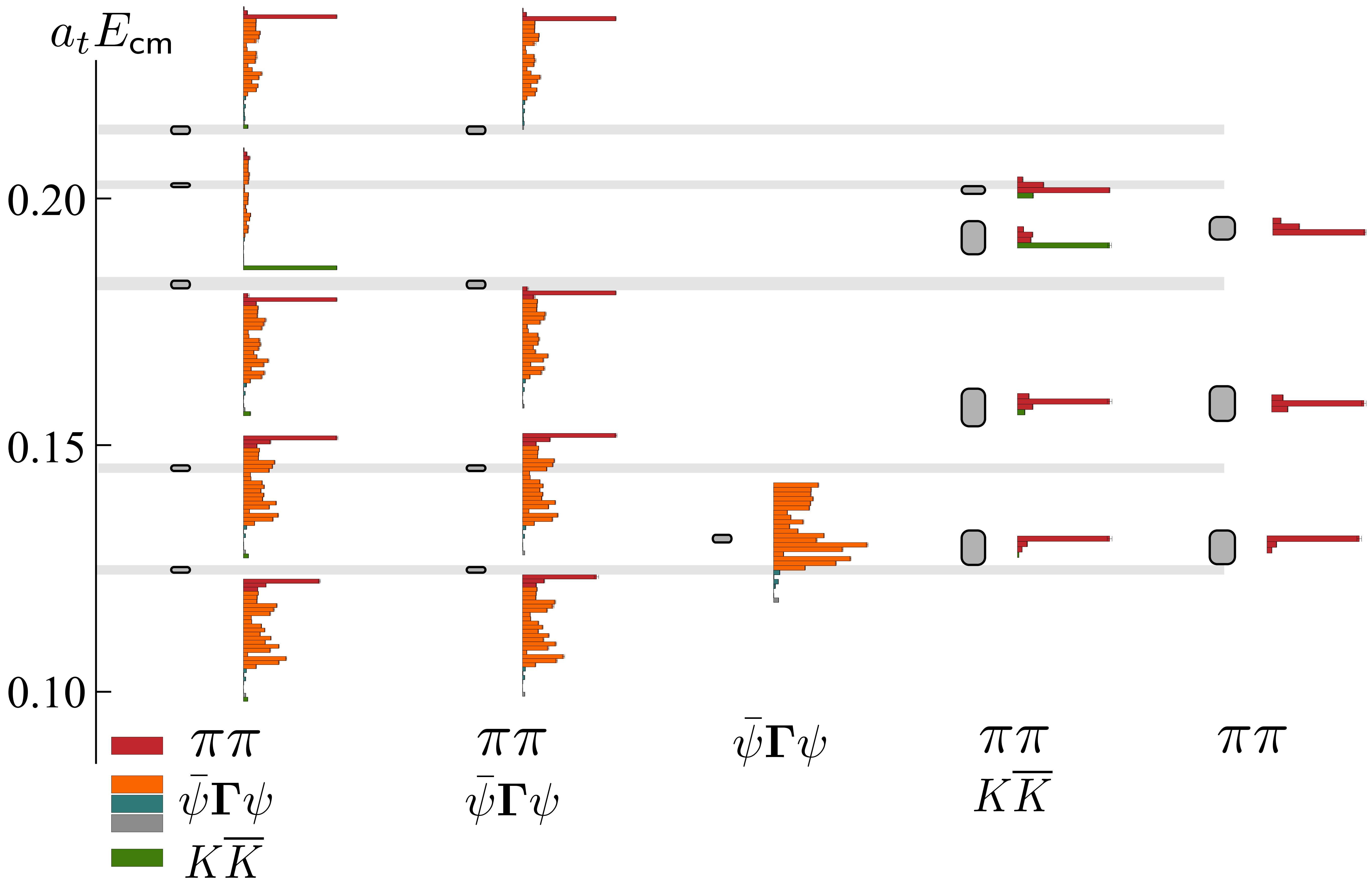
- ☑ Generalized eigenvalue problem (GEVP),
  - ☑ large basis of ops,

$$\mathcal{O}_b \sim \bar{q}\,\Gamma_b\,q, \pi\pi, K\overline{K}, \ldots,$$

  - ☑ “diagonalization”,

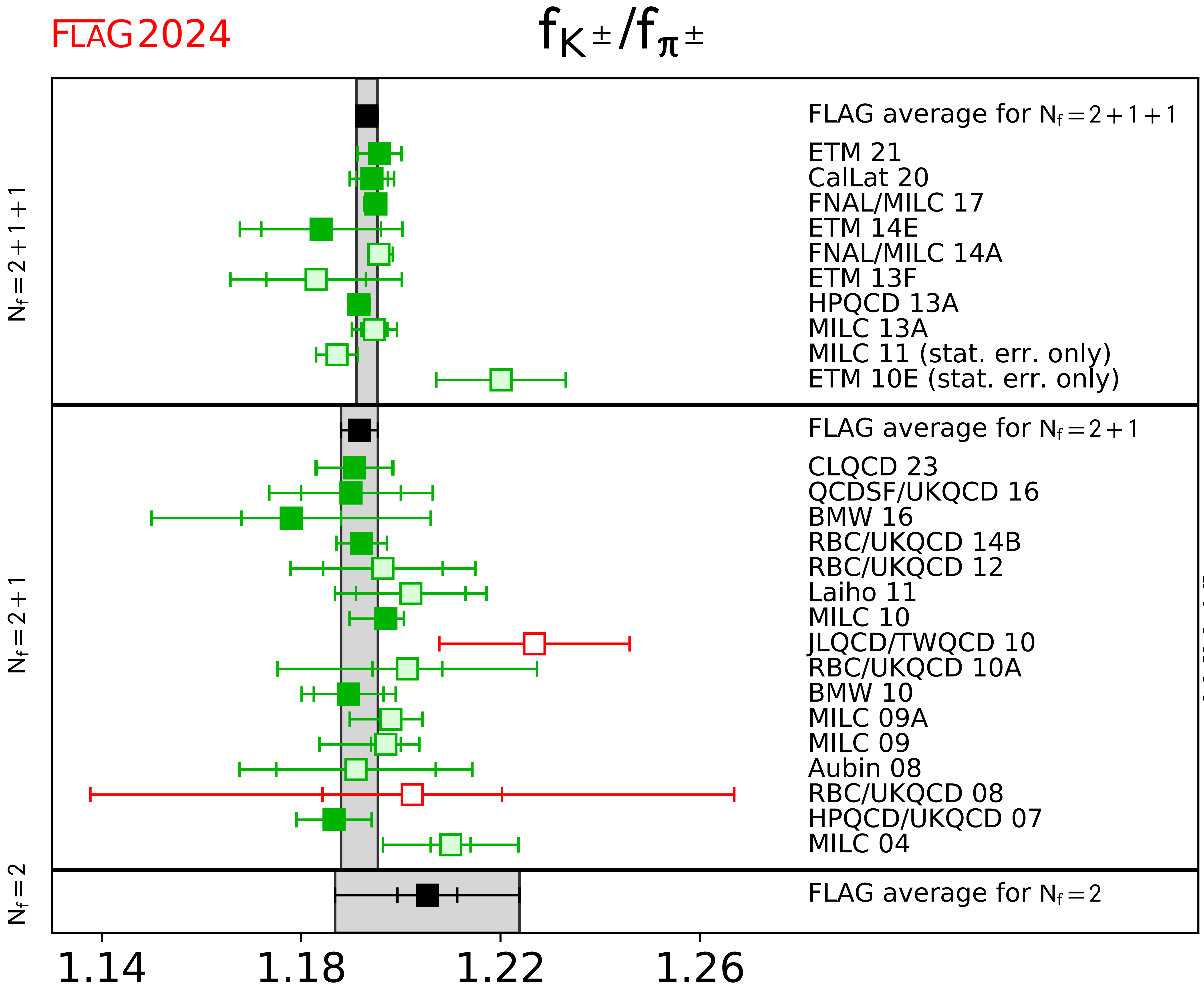
$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z^*_{a,n} e^{-E_n t}$$

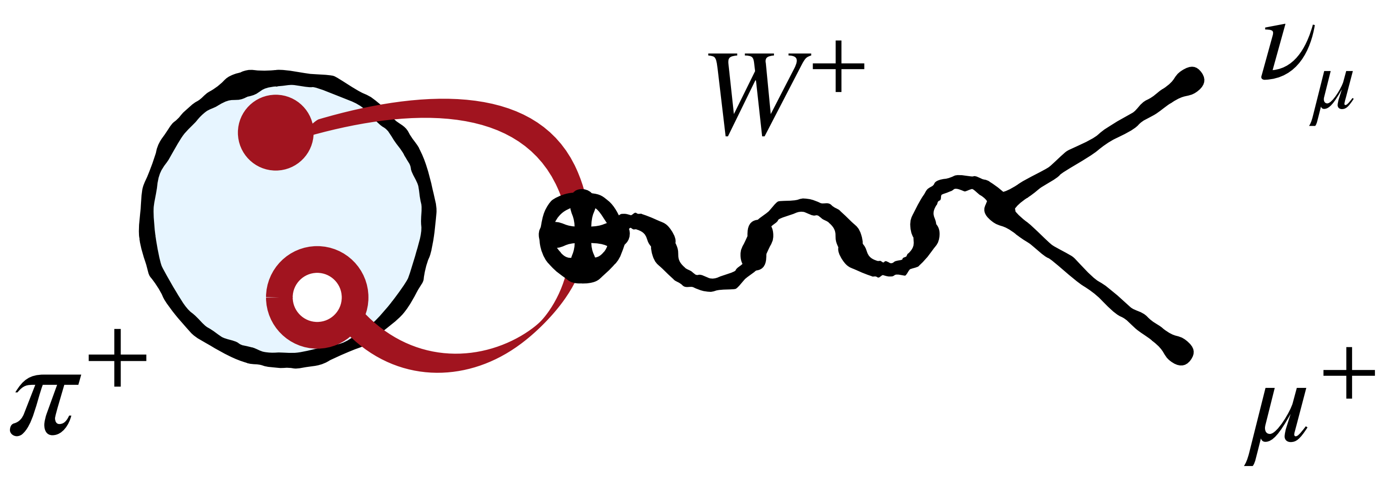
$$C(t)\vec{v}^{(n)}(t,t_0) = \lambda_n(t,t_0)C(t_0)\vec{v}^{(n)}(t,t_0)$$

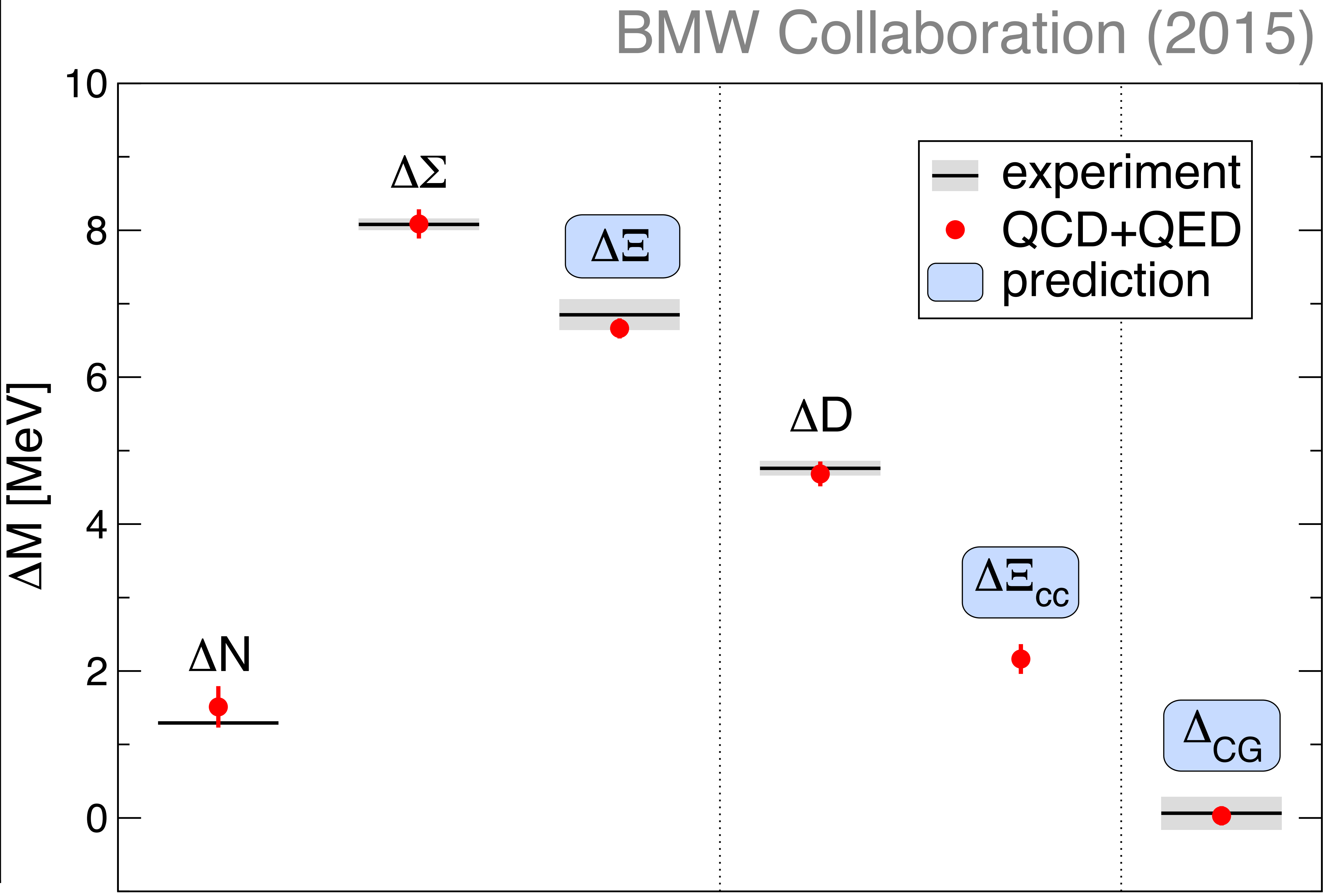


Wilson, RB, Dudek, Edwards, & Thomas (2015)

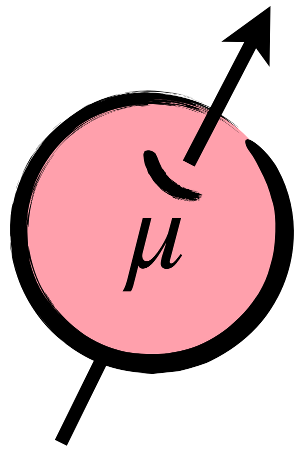
Precision era



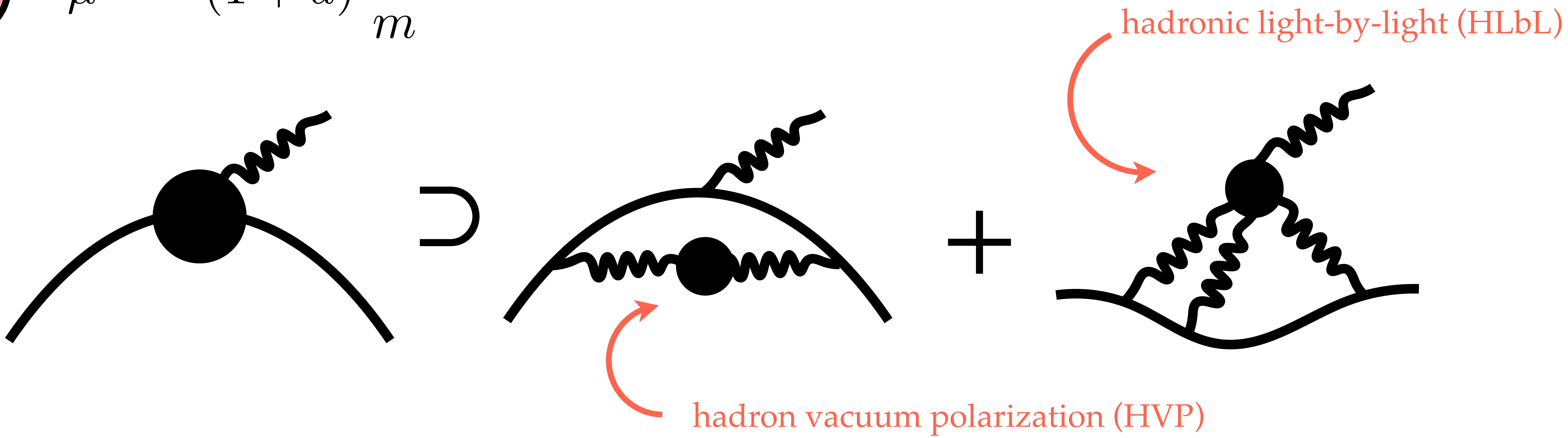




g-2

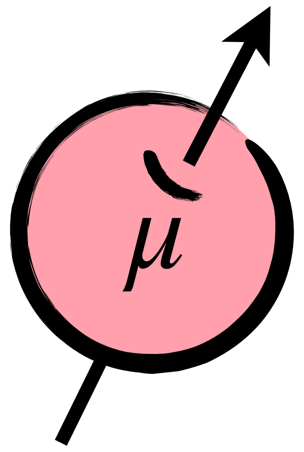


$$\mu = -(1+a)\frac{e\,S}{m}$$

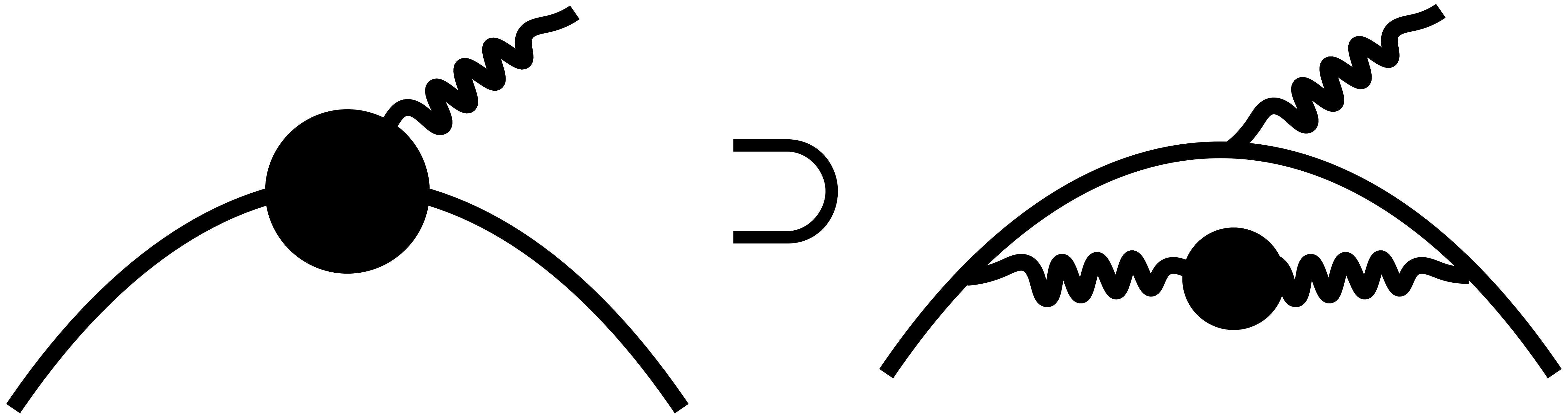


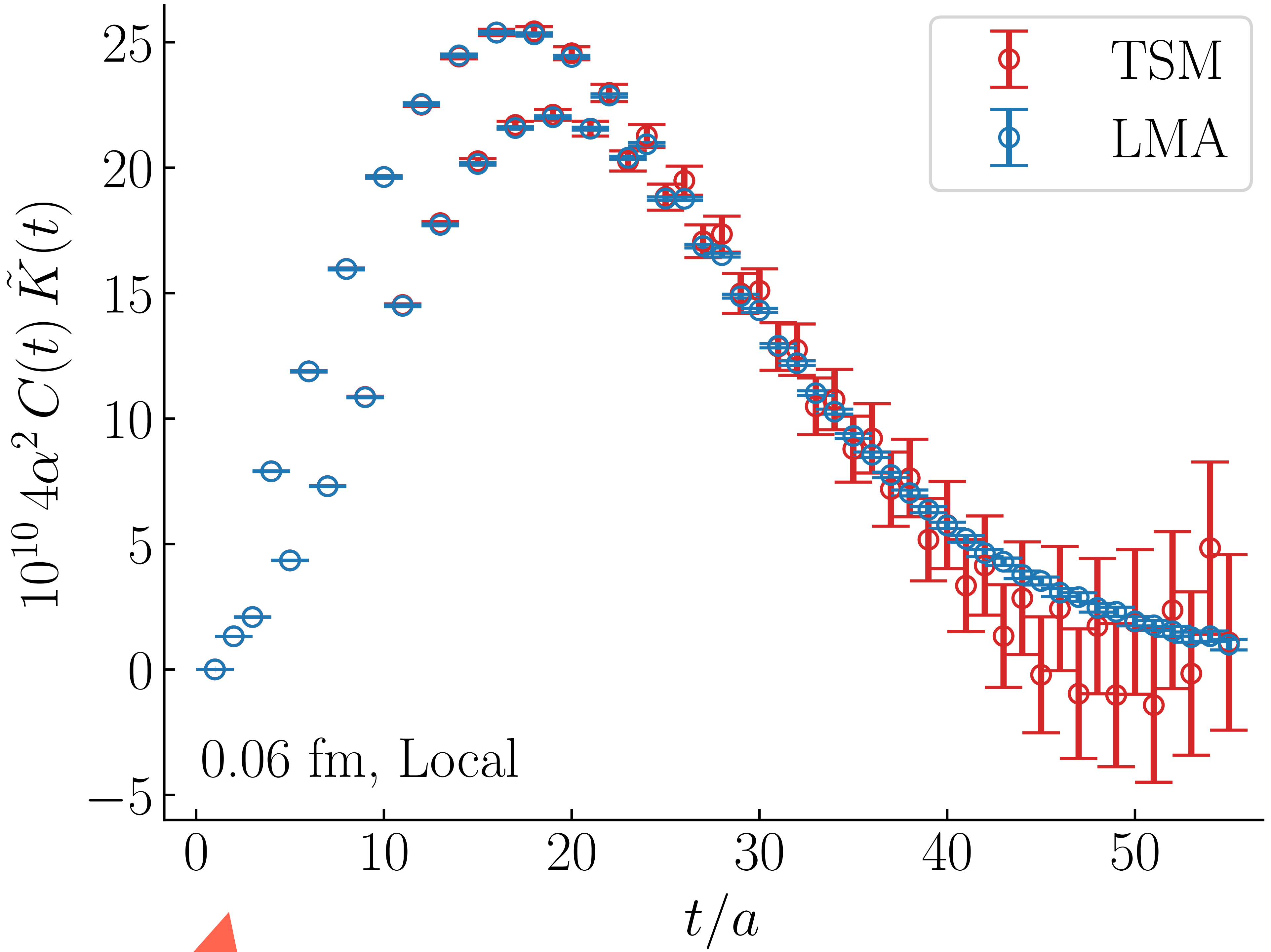
g-2

$$4\alpha^2 \int dt\, C(t)\, \tilde{K}(t)$$ [Blum (2003)]



$$\mu = -(1+a)\frac{e\,S}{m}$$

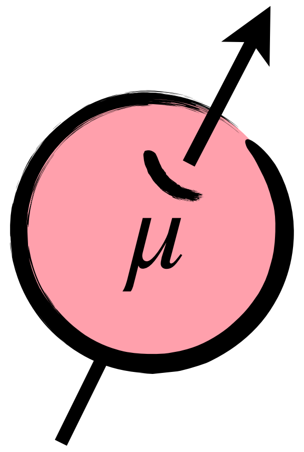




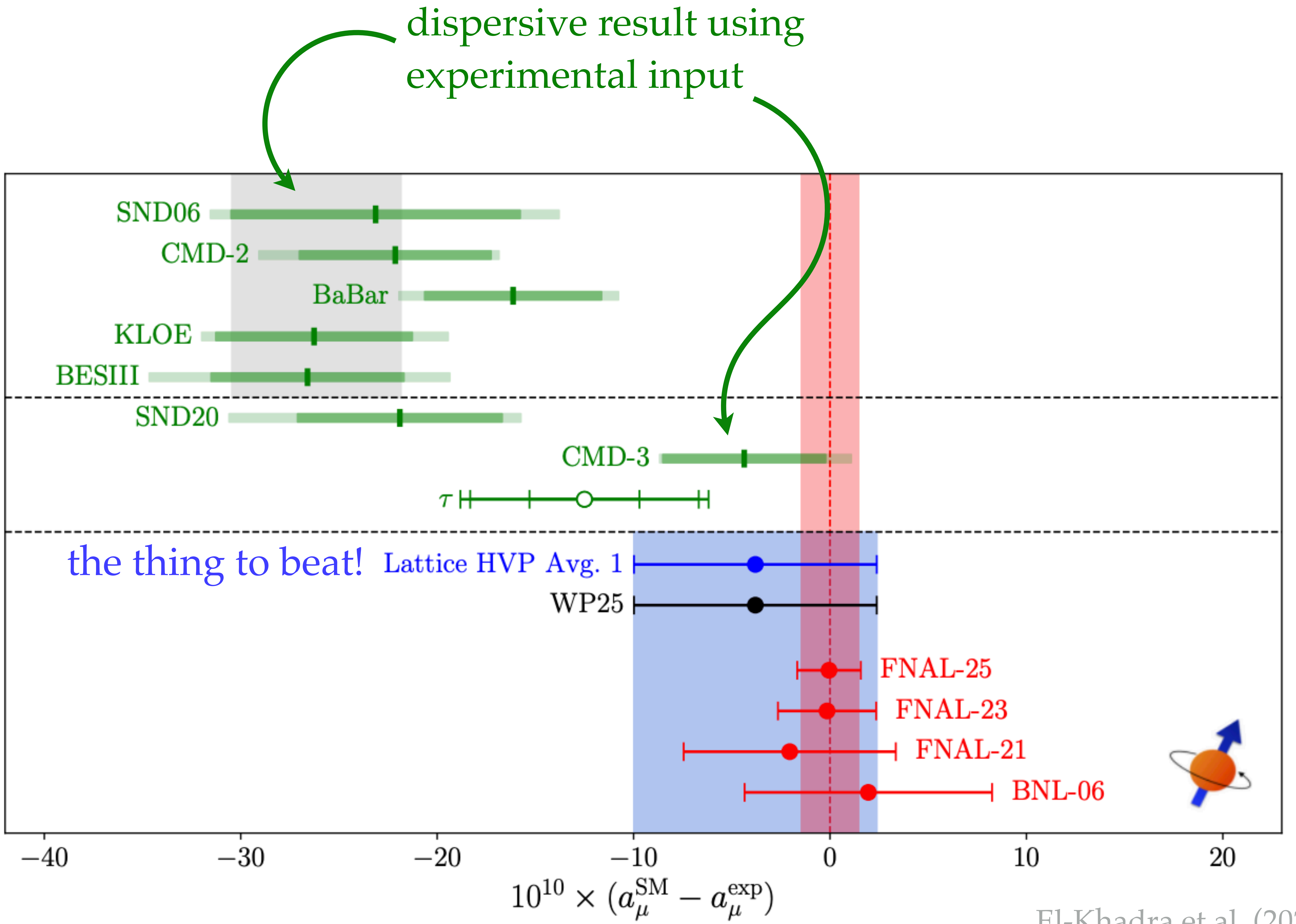
$$C_{\text{2pt.}}(t) = \langle \mathcal{J}^{\mu}(t)\mathcal{J}^{\nu}(0)\rangle$$

Fermilab Lattice,
HPQCD, and MILC (2024)

g-2

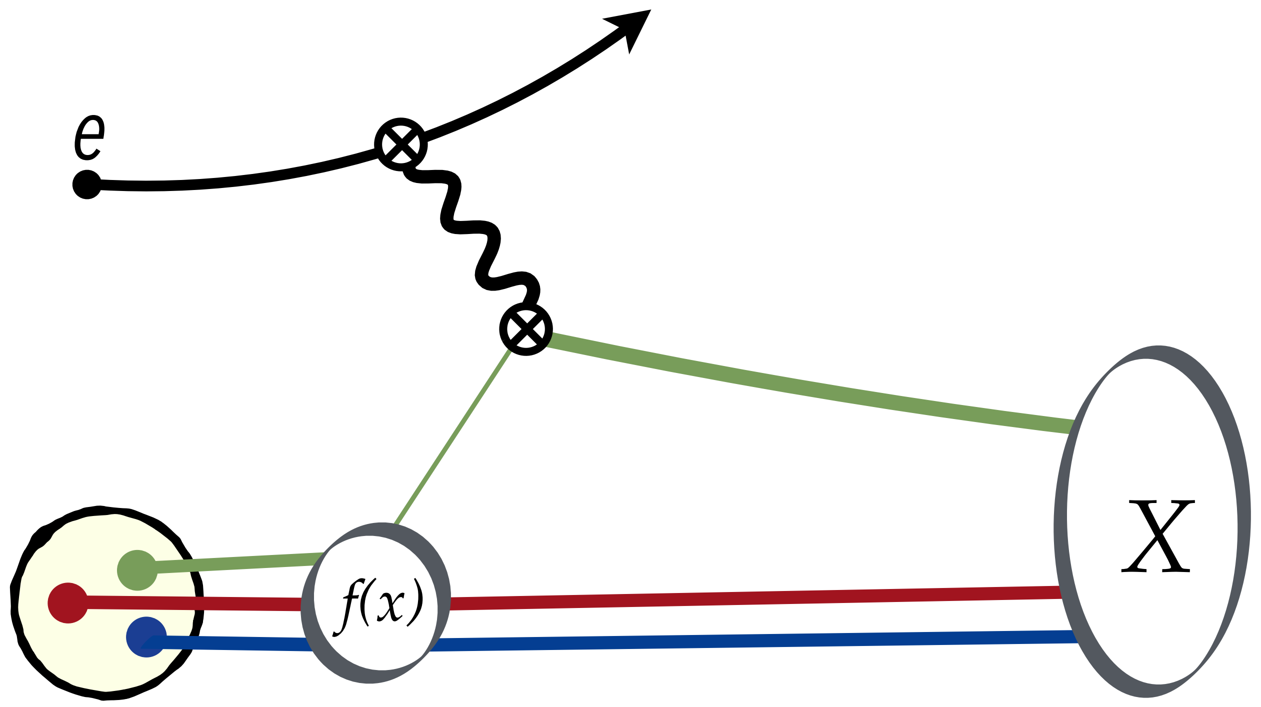


$$\mu = -(1+a)\frac{e\,S}{m}$$



El-Khadra et al. (2025)

Partonic physics



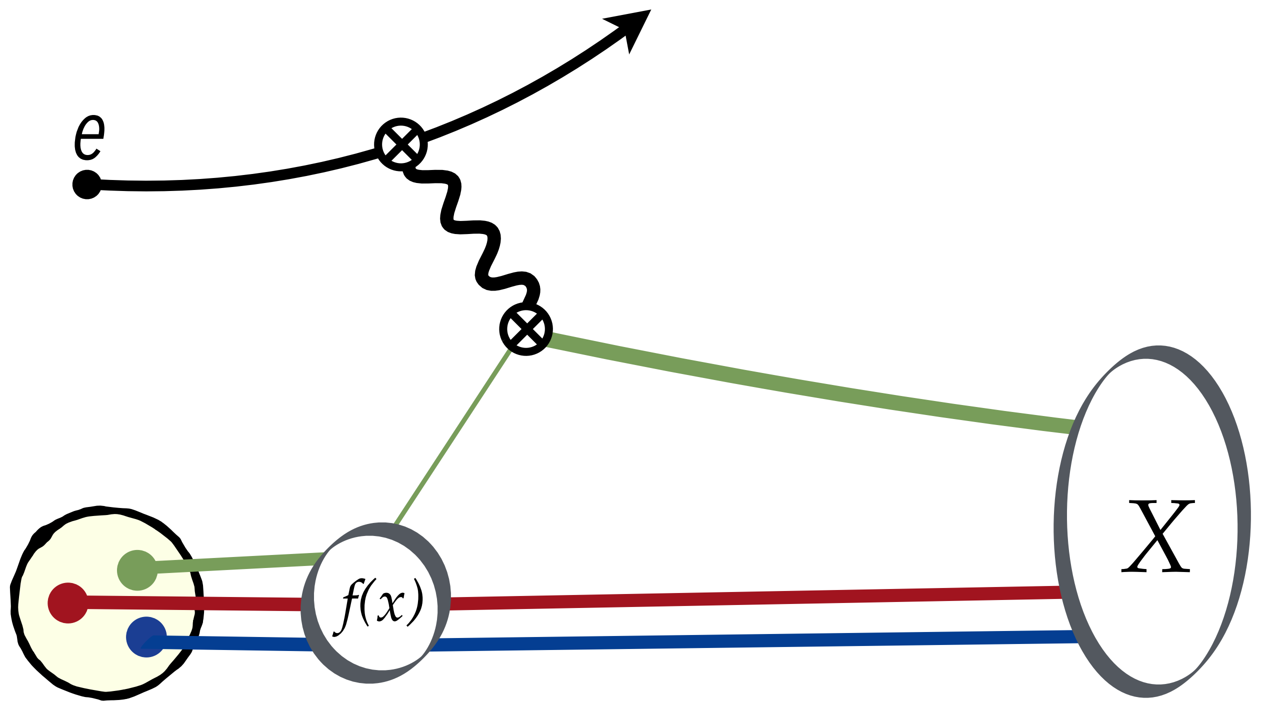
$$\sim \int_0^1 dx\, f_{q/P}(x)\, \sigma_{eq \to eq}$$

Light cone parton distribution functions (PDFs)

$$f_{q/p}(x) = \frac{1}{4\pi} \int dz^- \, e^{ixP^+ z^-} \, \langle P| \, \bar{\psi}_q(0) \, \gamma^+ \, \mathcal{W}(0, z^-) \, \psi_q(z^-) \, |P\rangle$$

inaccessible on Euclidean lattices

Partonic physics



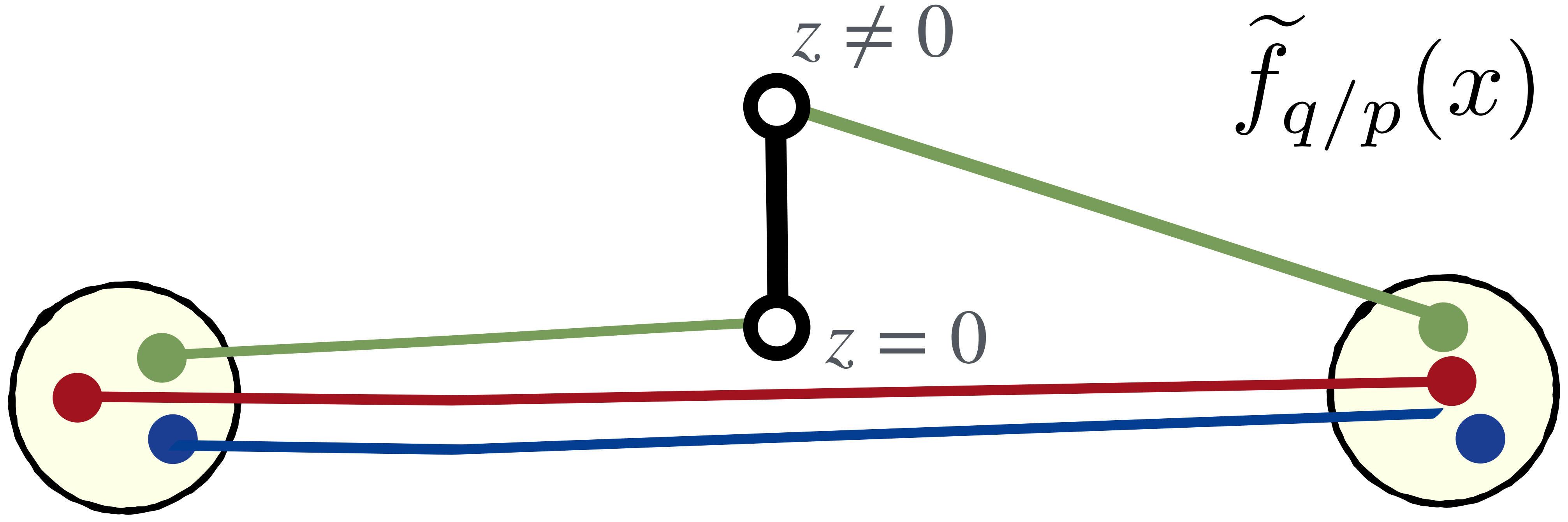
$$\sim \int_0^1 dx\, f_{q/P}(x)\, \sigma_{eq\to eq}$$

Light cone parton distribution functions (PDFs)

$$f_{q/p}(x) = \frac{1}{4\pi}\int dz^-\, e^{ixP^+z^-}\, \langle P|\, \bar{\psi}_q(0)\, \gamma^+\, \mathcal{W}(0,z^-)\, \psi_q(z^-)\, |P\rangle$$

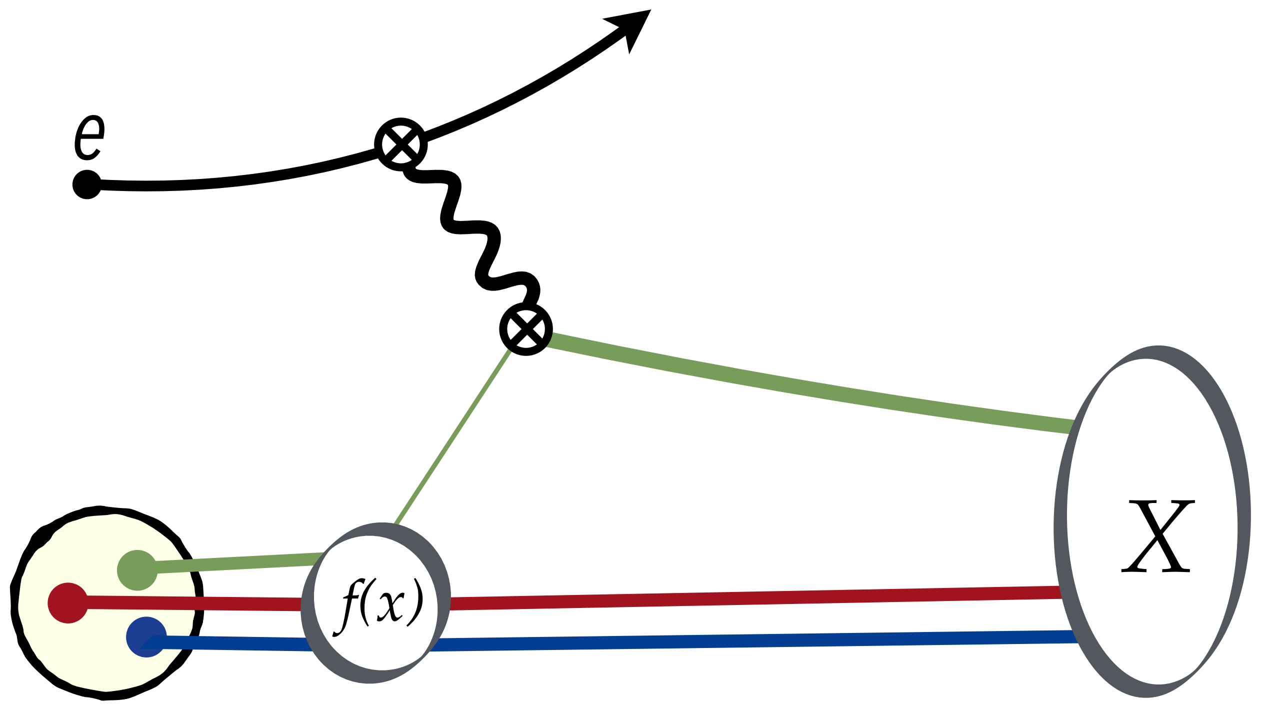
match perturbatively

Quasi/pseudo PDFs [Ji (2013), Radyushkin (2017)]:

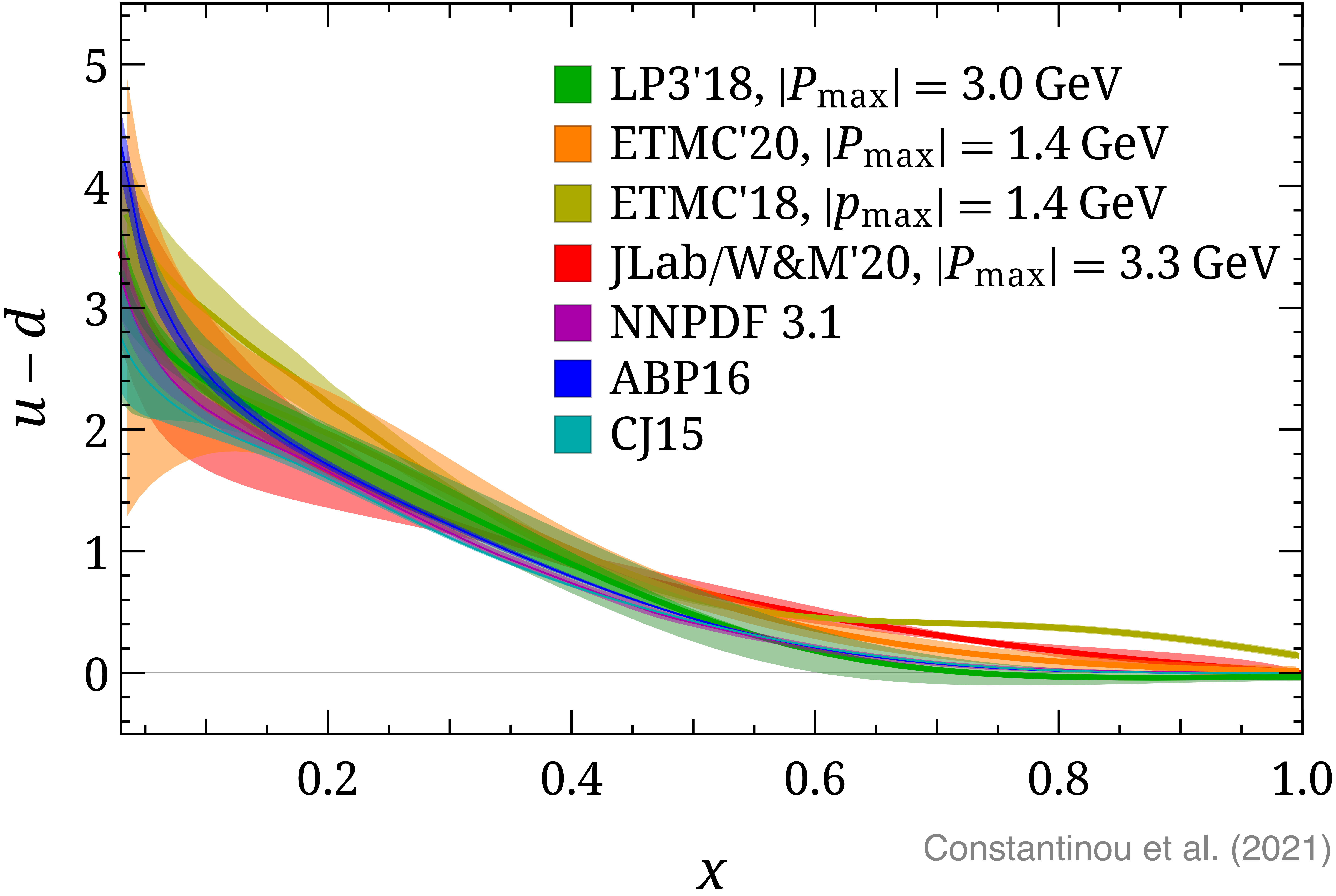


$$\widetilde{f}_{q/p}(x) = \frac{1}{4\pi}\int dz\, e^{ixP_z z}\, \langle P|\, \bar{\psi}_q(0)\, \gamma^z\, \mathcal{W}(0,z)\, \psi_q(z)\, |P\rangle$$

Partonic physics

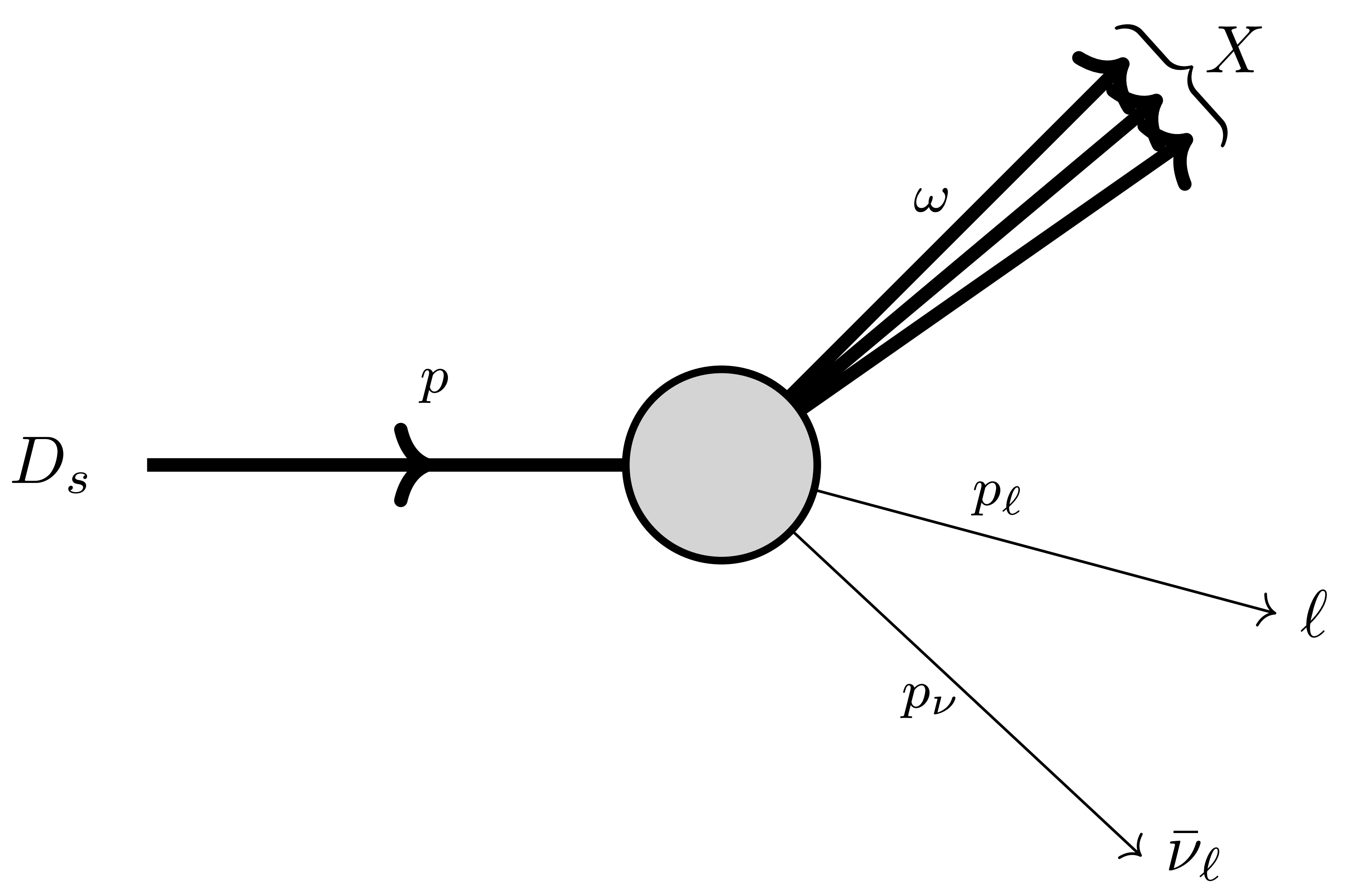


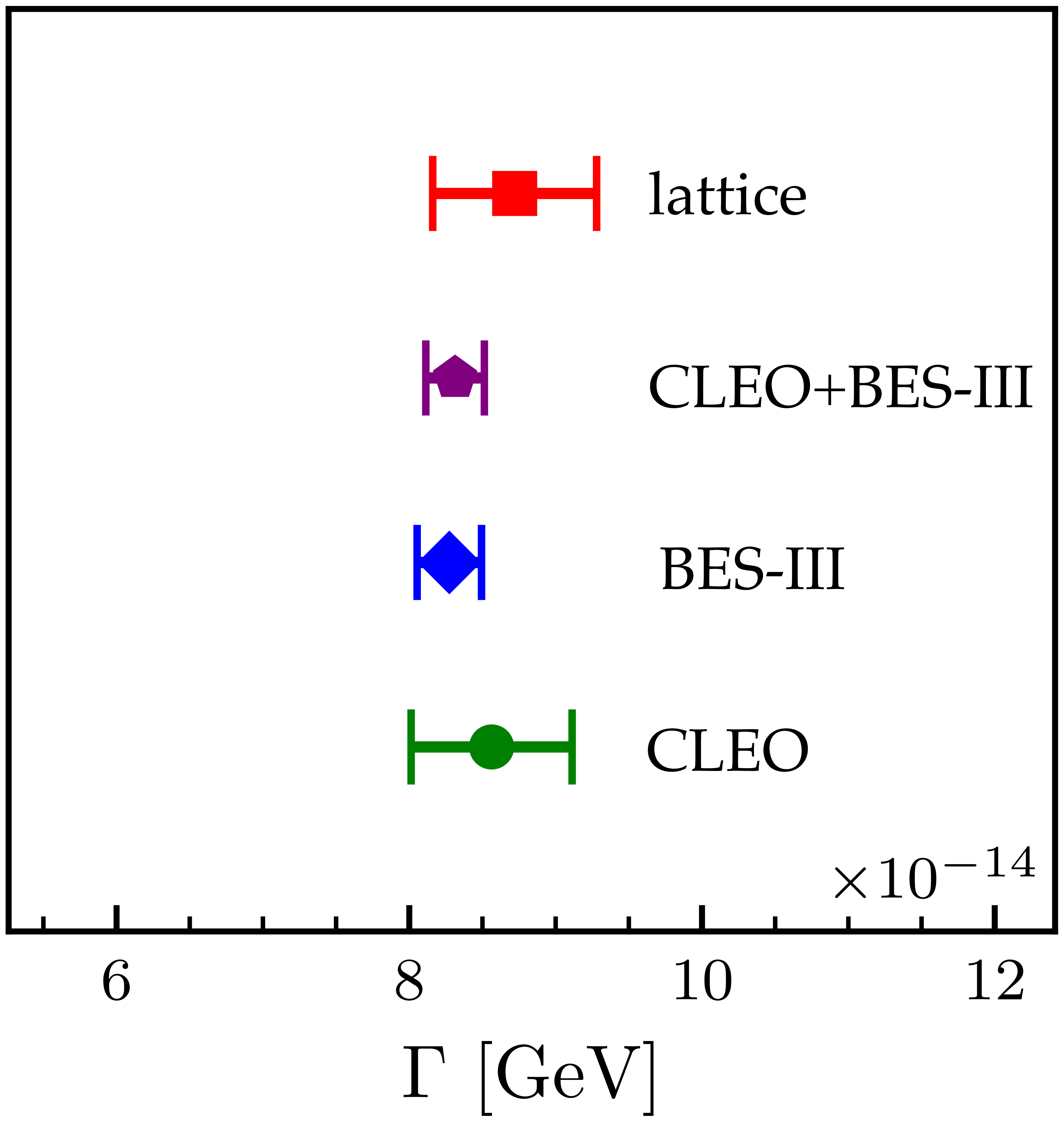
now an industry standard!



Constantinou et al. (2021)

Inclusive reactions





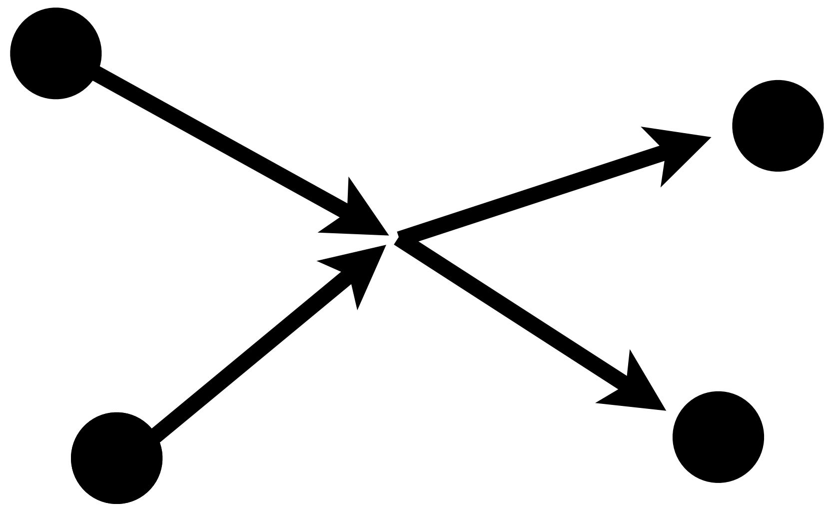
Euclidean correlator

$$\sum_x \langle D_s(p)|T\{J_\mu^\dagger(t,\vec{x})\, J_\nu(0)\}|D_s(p)\rangle = \int_0^\infty d\omega_0\, e^{-m_{D_s}\omega_0 t}\, \rho_L(\omega_0)$$

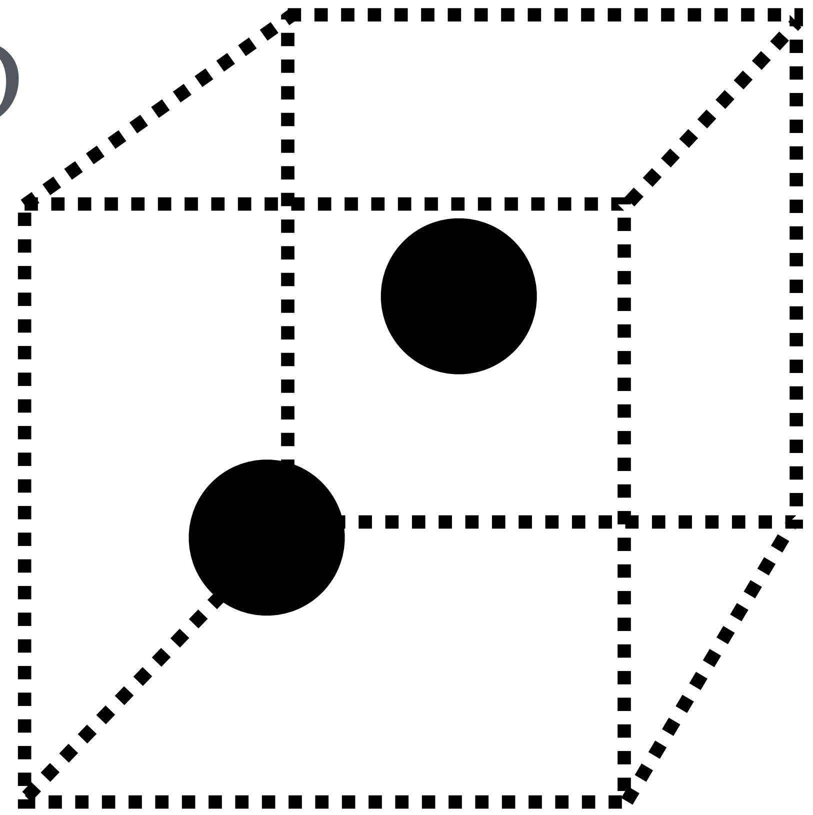
De Santis et al. (2025)

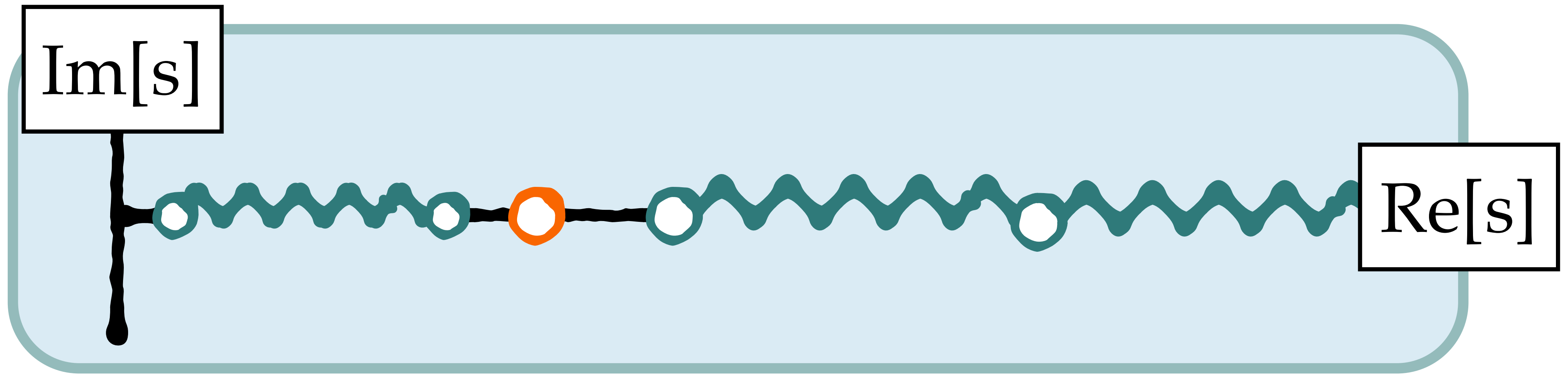
scattering via Euclidean lattices

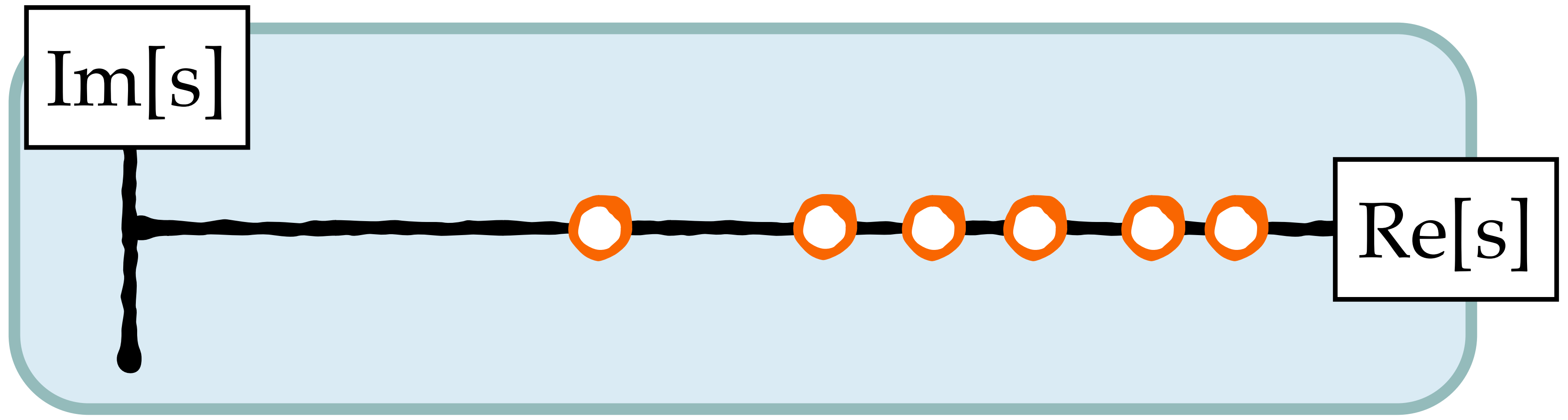
Scattering theory



Lattice QCD

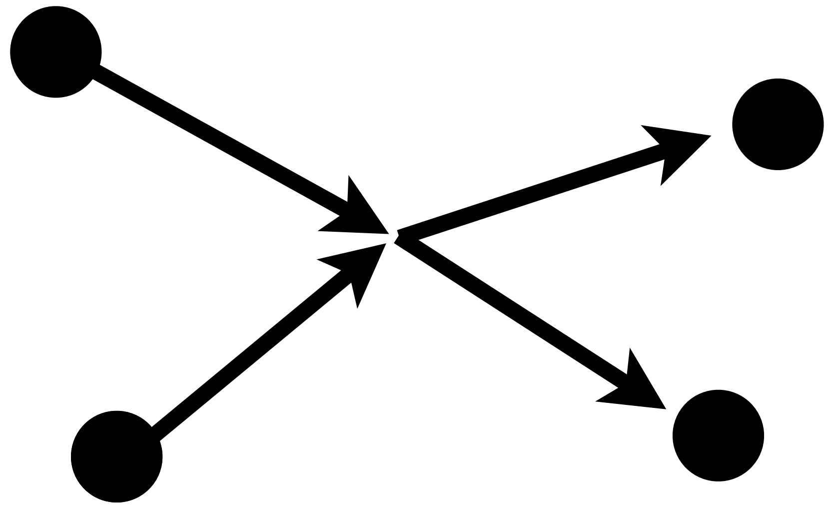


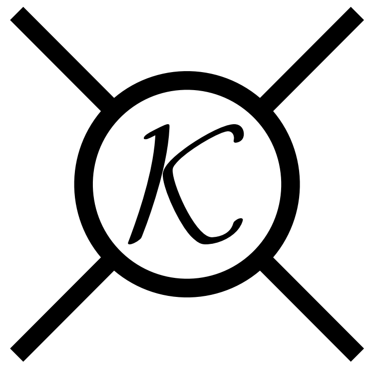




scattering via Euclidean lattices

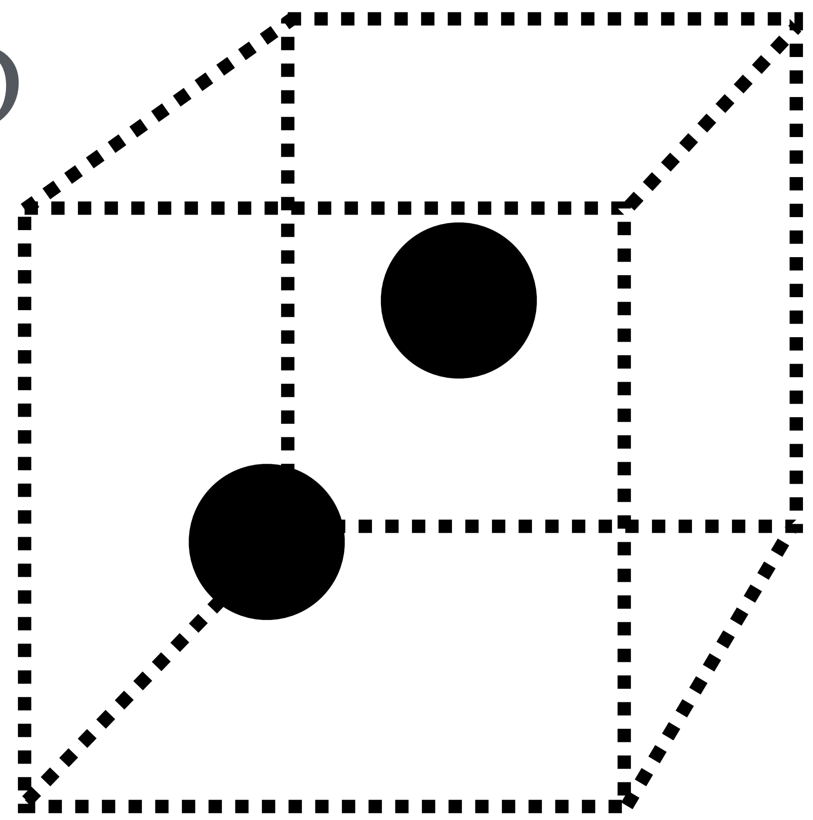
Scattering theory





short-distance dynamics

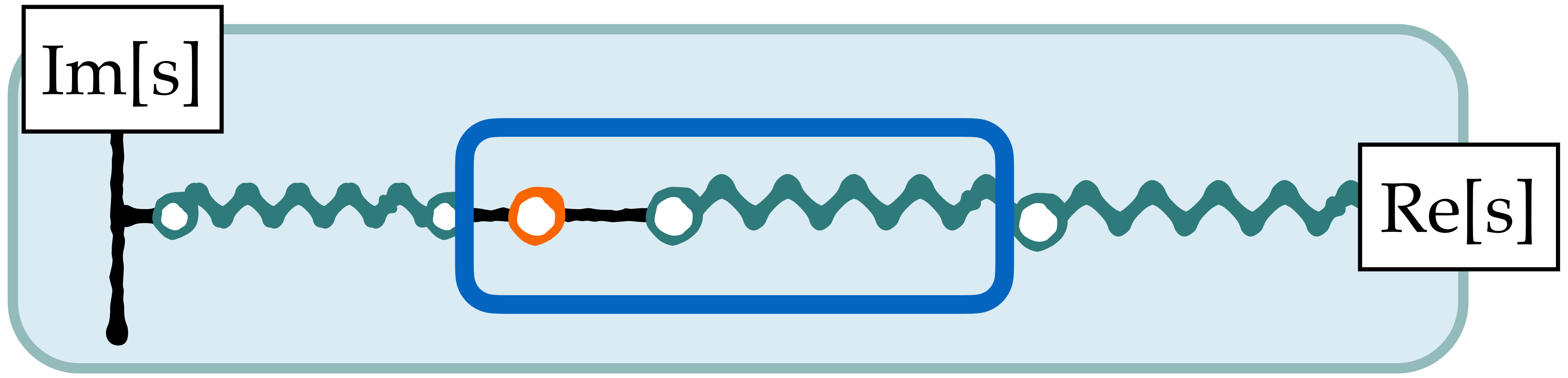
Lattice QCD

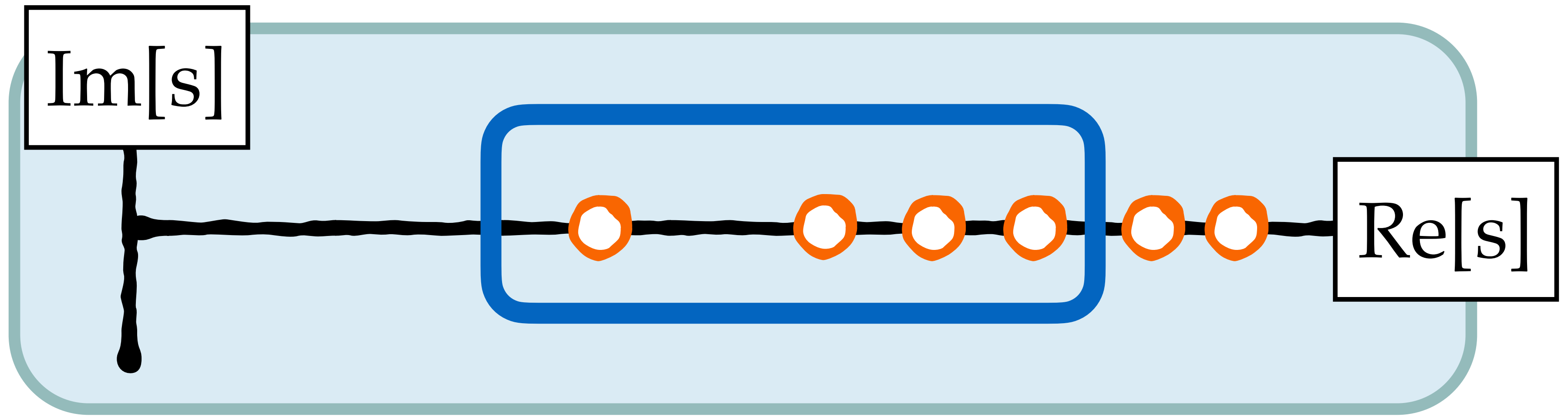


Two-step non-perturbative correspondence:

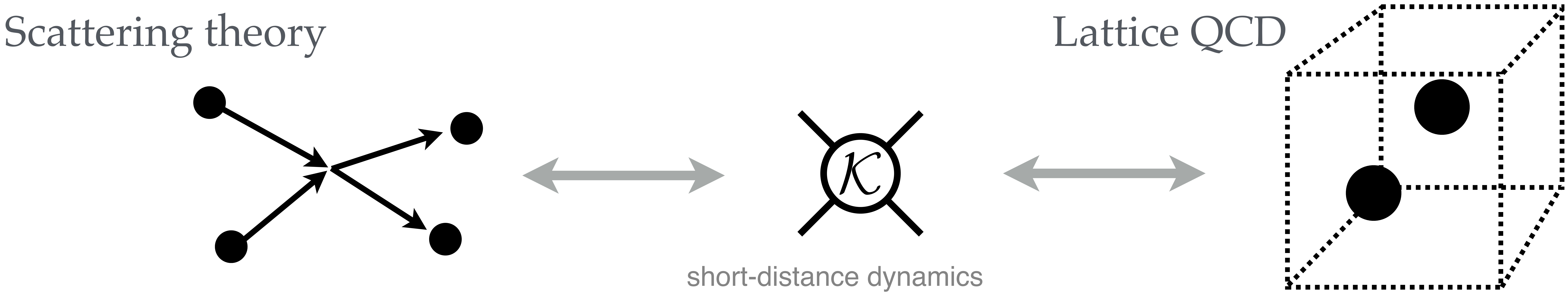
Spectrum to K matrices: $$\det\left[F^{-1}(P,L)+\mathcal{K}(P^2)\right]=0$$

applicable only in a restricted kinematic region





scattering via Euclidean lattices

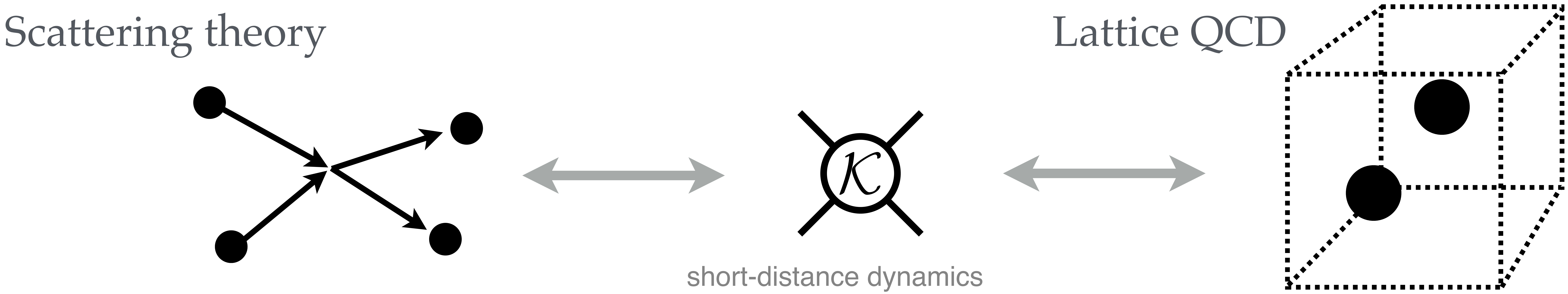


Two-step non-perturbative correspondence:

Spectrum to K matrices: $$\det\left[F^{-1}(P,L)+\mathcal{K}(P^2)\right]=0$$

absence of asymptotic states

scattering via Euclidean lattices



Two-step non-perturbative correspondence:

Spectrum to K matrices: $$\det\left[F^{-1}(P,L)+\mathcal{K}(P^2)\right]=0$$

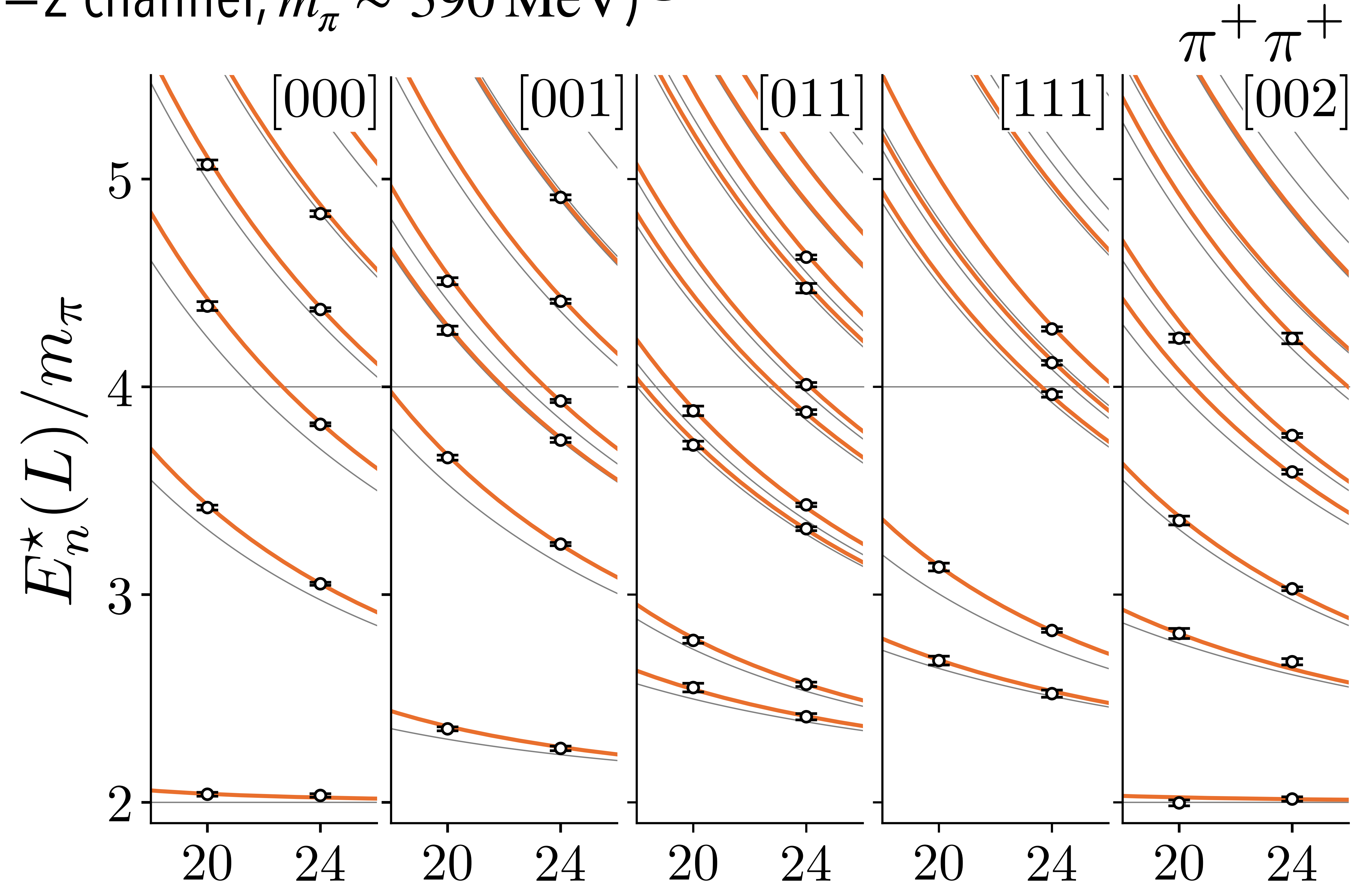
K matrices to amplitudes: $$\mathcal{M}=\mathcal{M}\left[\mathcal{K}\right]$$

e.g. two-body scattering scattering $(2m)^2 \leq P^2 \leq (3m)^2$ $$\mathcal{M}=\frac{1}{\mathcal{K}^{-1}-i\frac{q}{16\pi\sqrt{s}}}$$

…in general, this is an integral equation relation…

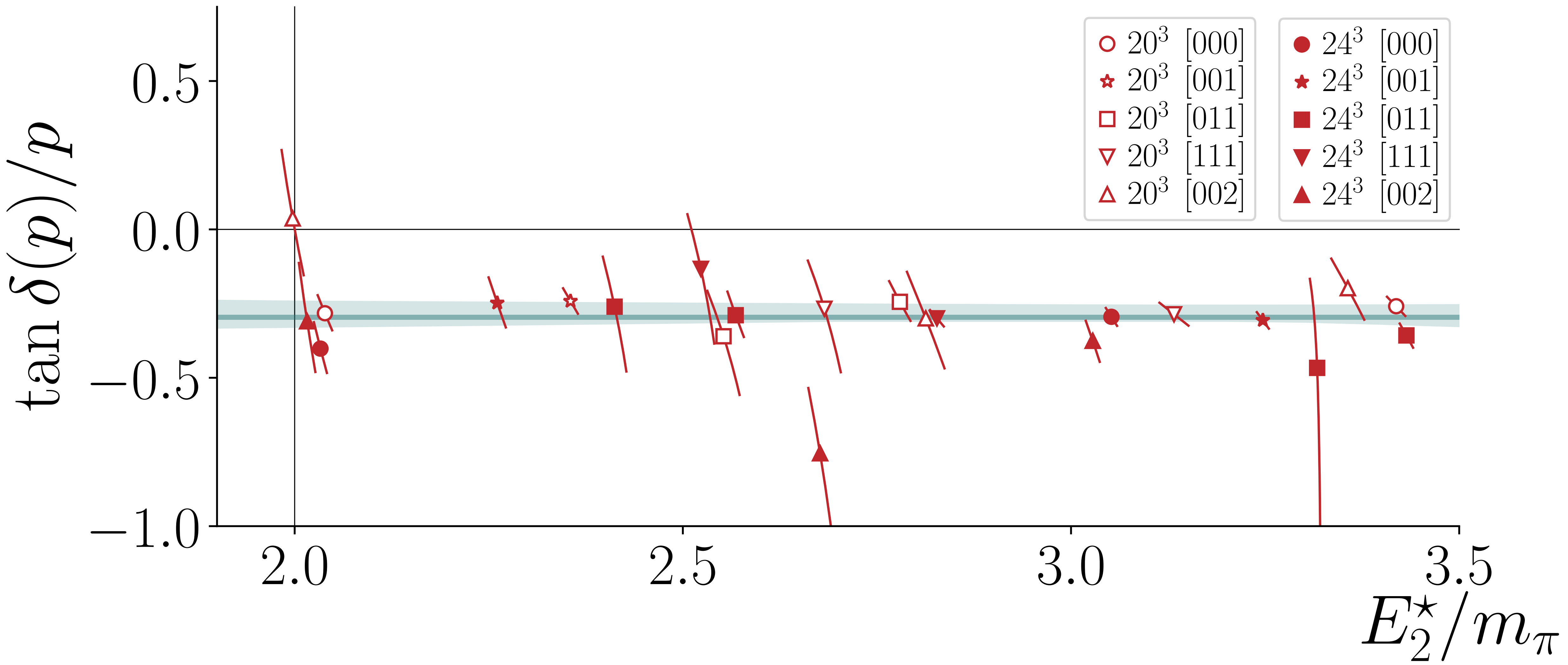
$\pi\pi$ scattering

(I=2 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



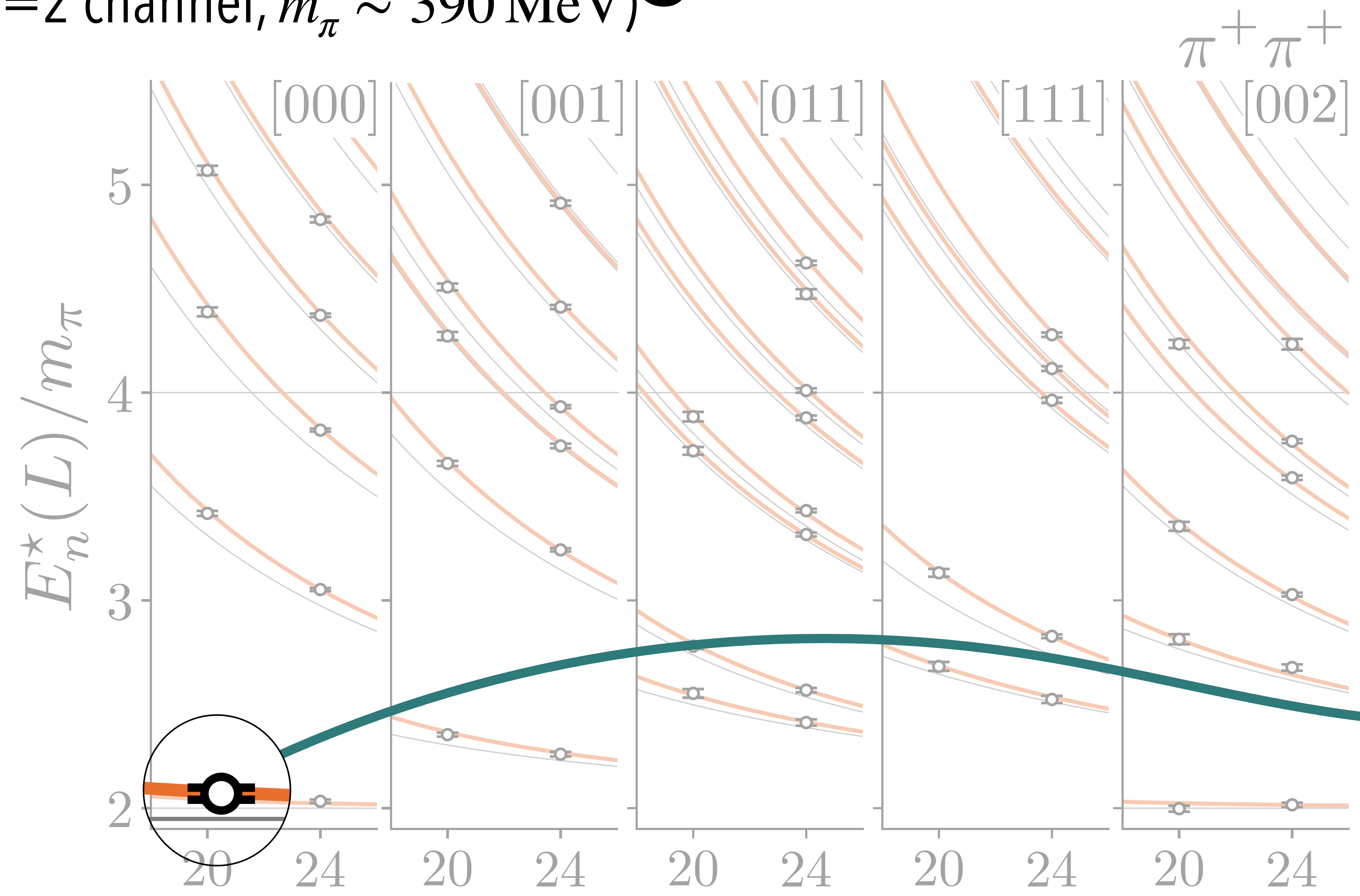
$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$



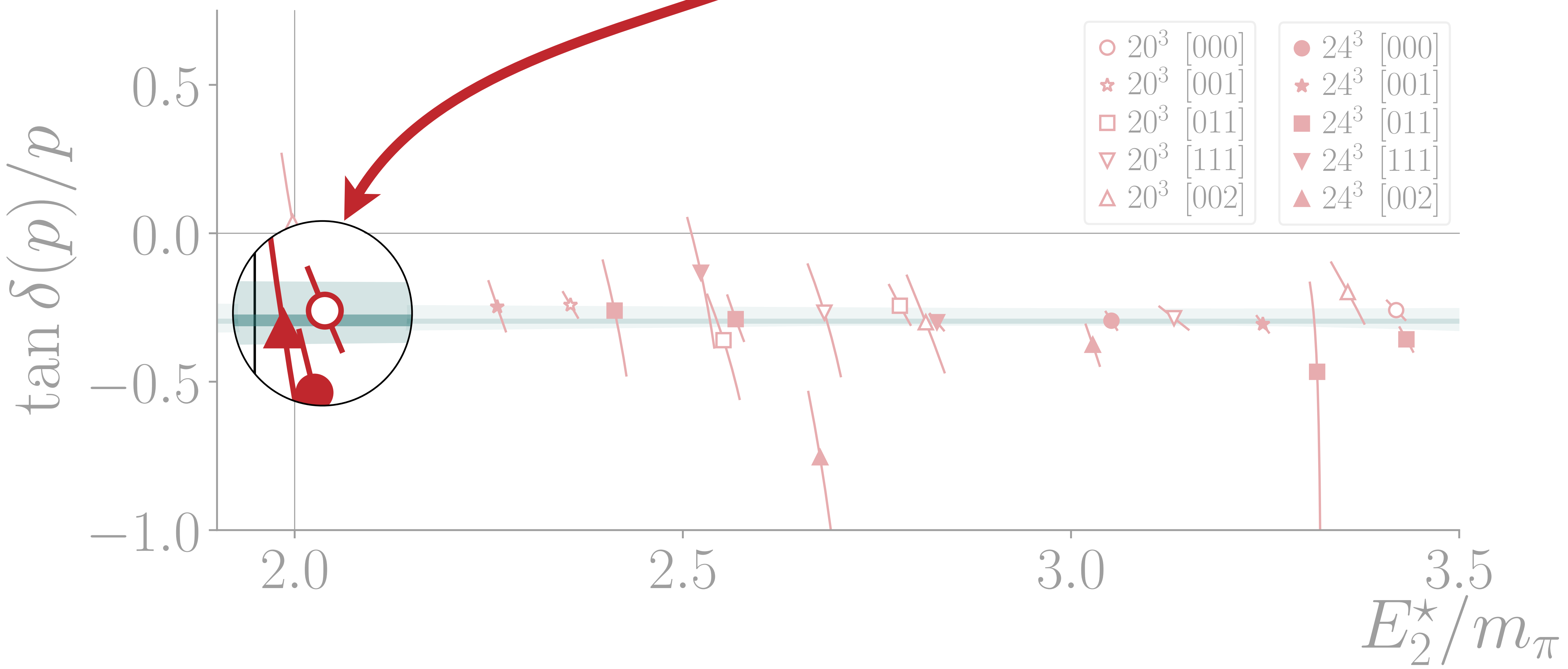
Hansen, RB, Edwards, Thomas, & Wilson (2020)

$\pi\pi$ scattering
(I=2 channel, $m_\pi \sim 390\,\mathrm{MeV}$)



$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip}$$

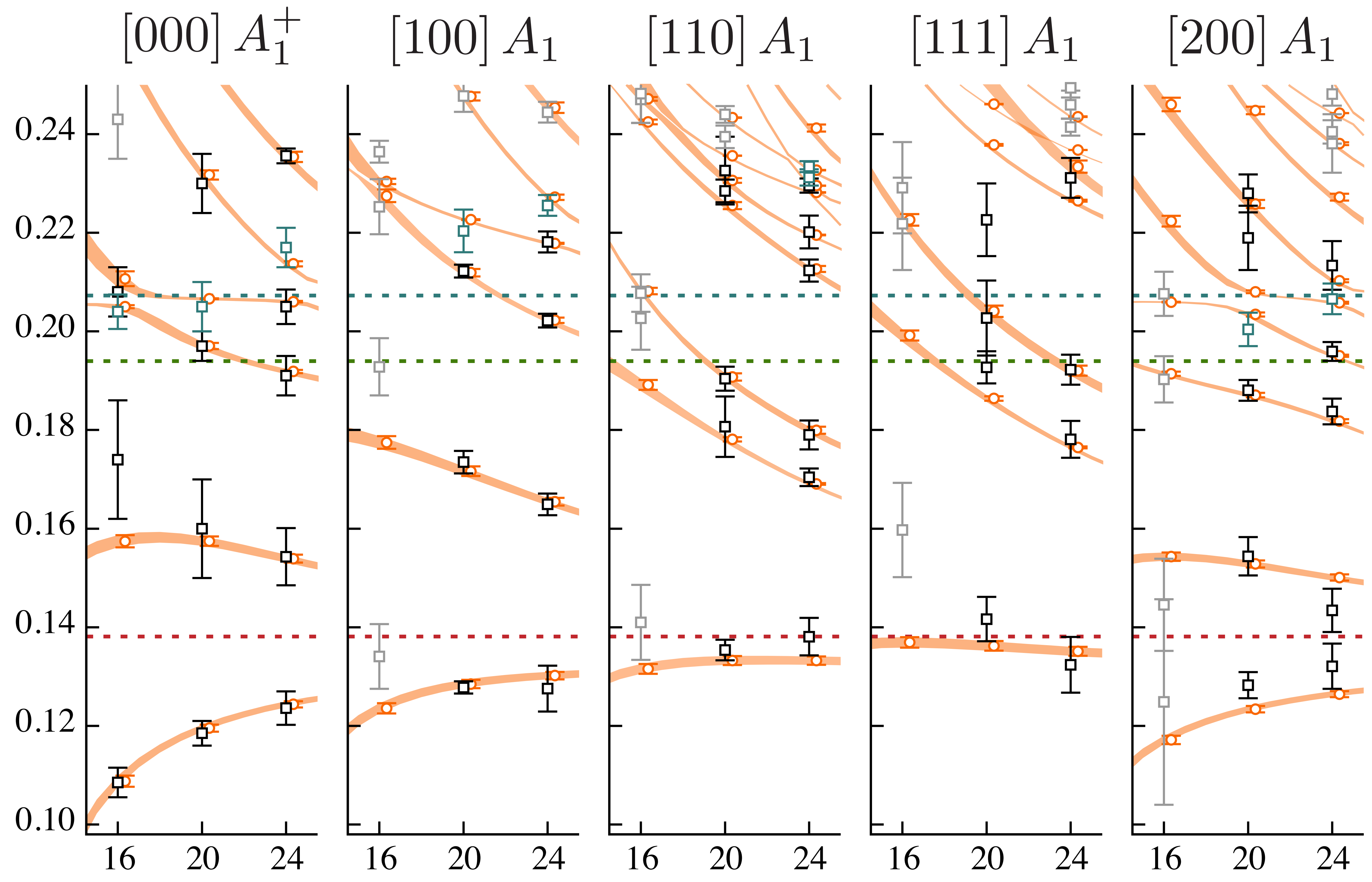


Hansen, RB, Edwards, Thomas, & Wilson (2020)

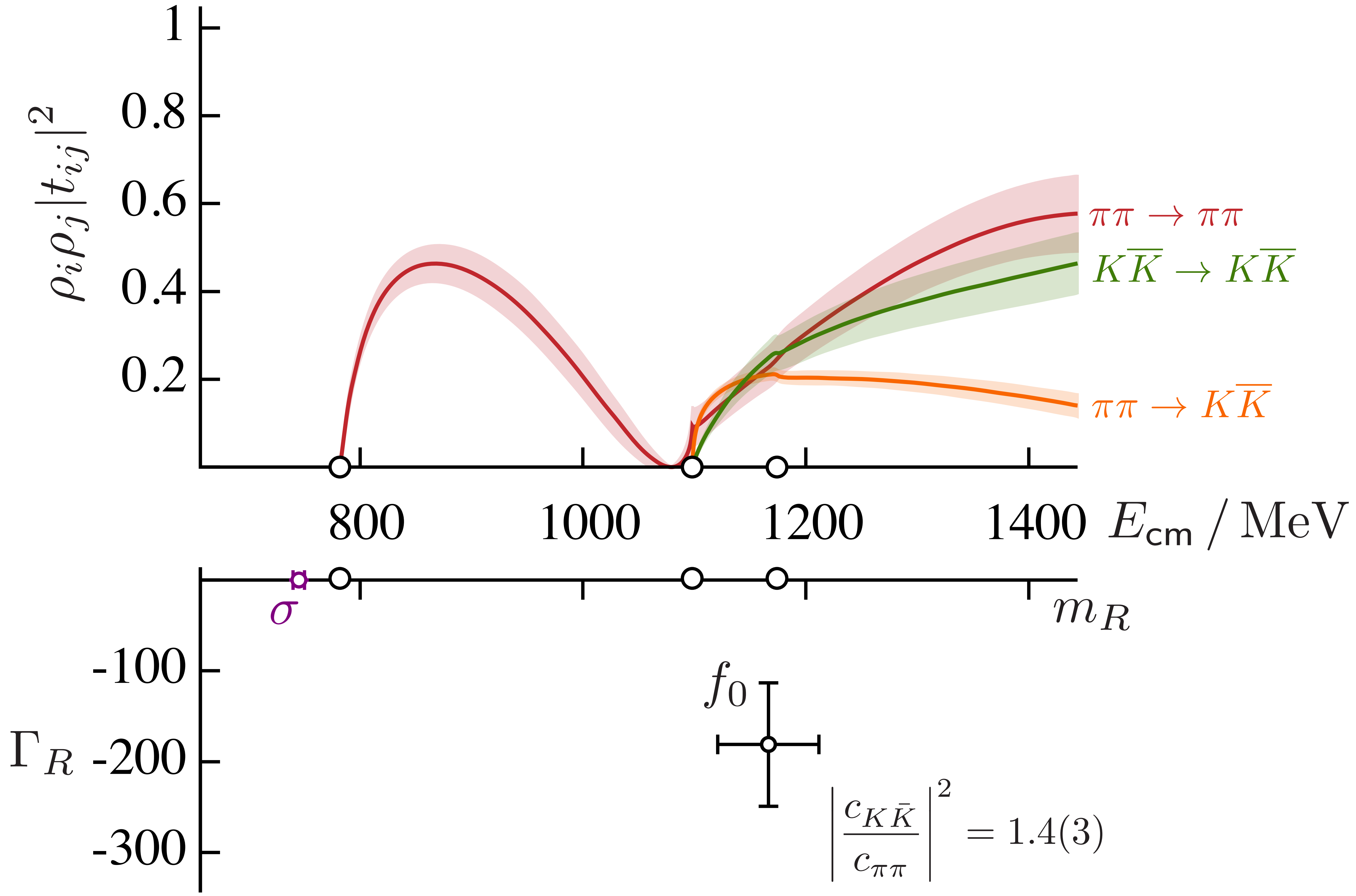
Coupled $\pi\pi$, $K\overline{K}$ scattering

(I=0 channel, $m_\pi \sim 390\,\text{MeV}$)

- ☑ Above $K\overline{K}$-threshold, spectrum satisfies: $$\det\begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$
- ☑ No one-to-one correspondence,
- ☑ Parameterize amplitude and perform global fit.

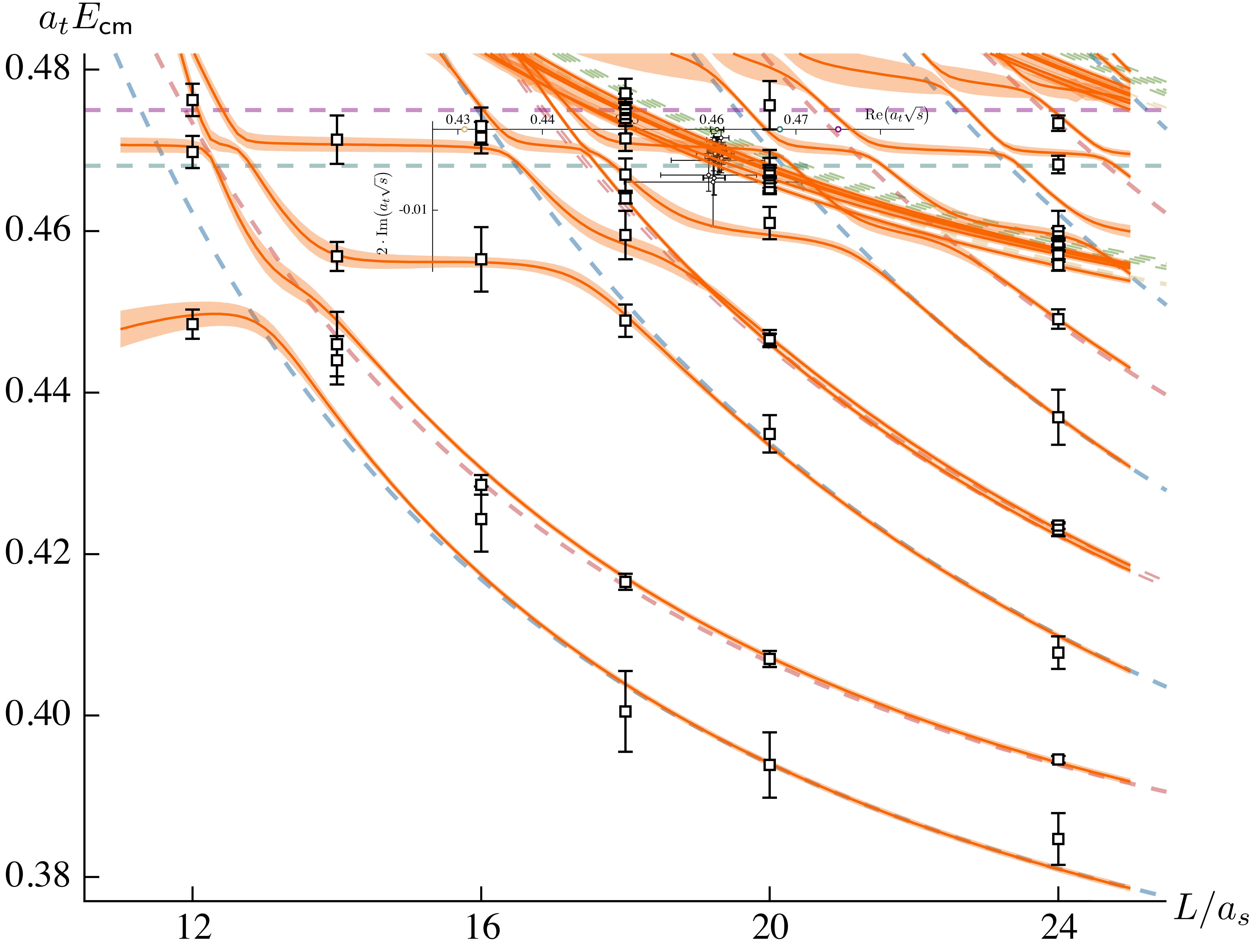


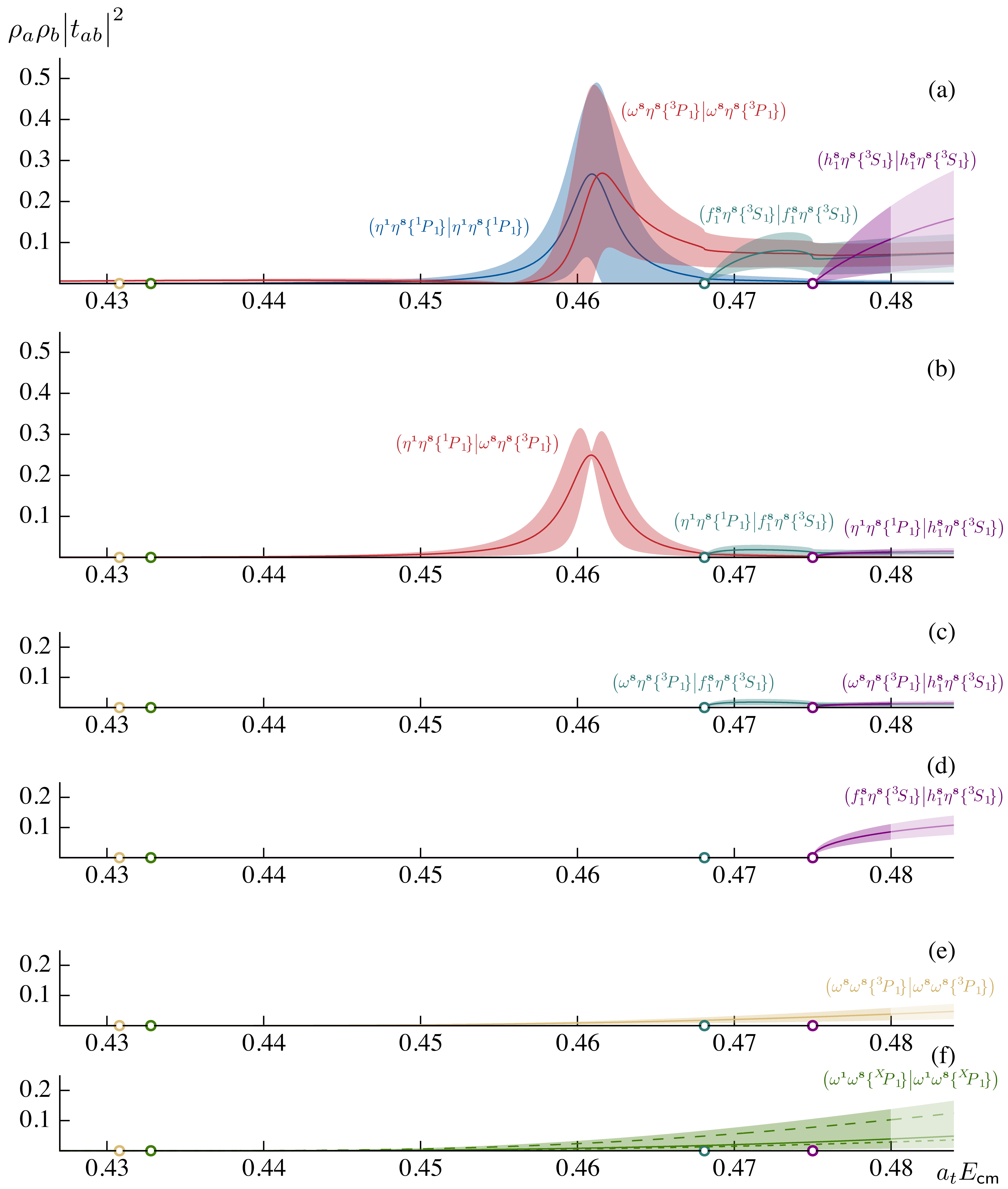
RB, Dudek, Edwards, & Wilson (2017)



π_1 resonance

($m_\pi \sim 700\,\mathrm{MeV}$)

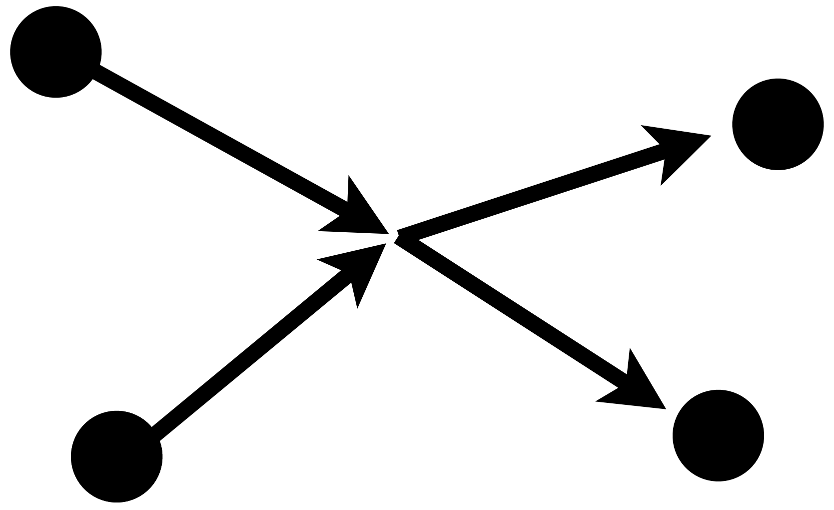


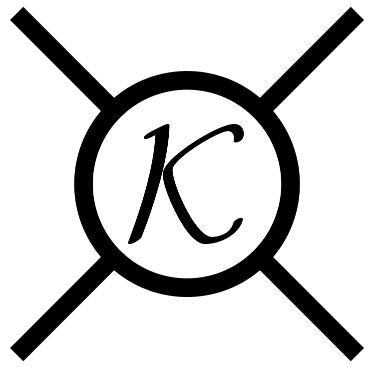


Woss, Dudek, Edwards, Thomas, Wilson (2025)

scattering via Euclidean lattices

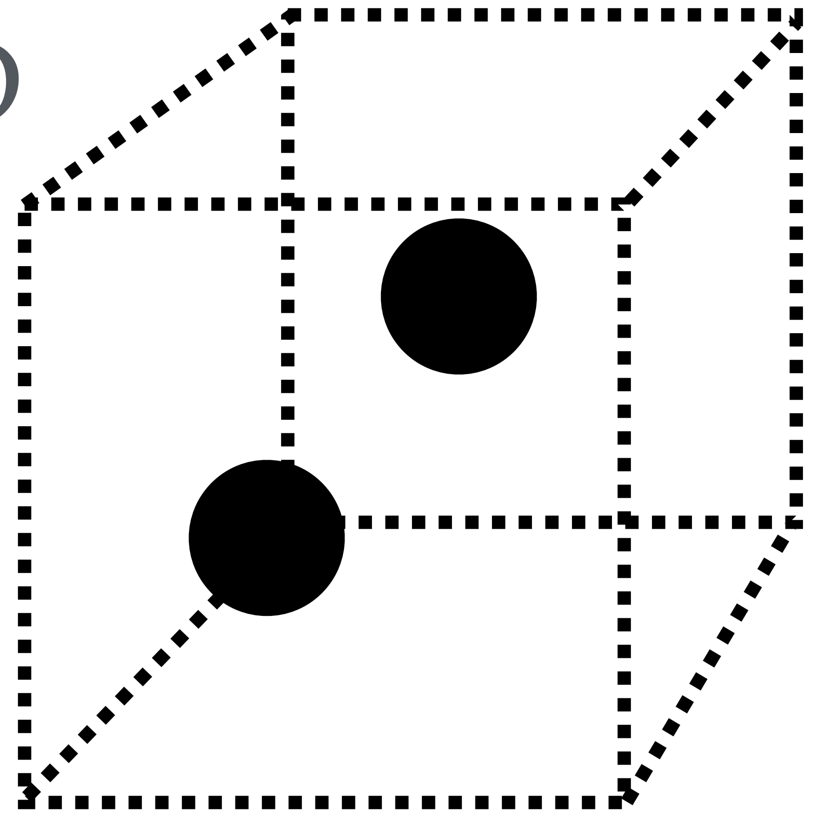
Scattering theory





short-distance dynamics

Lattice QCD



Two-step non-perturbative correspondence:

Spectrum to K matrices: $$\det\left[F^{-1}(P,L)+\mathcal{K}(P^2)\right]=0$$

K matrices to amplitudes: $$\mathcal{M}=\mathcal{M}\left[\mathcal{K}\right]$$

Bottlenecks in this program:

- ☐ ~~accessing the finite-volume spectrum,~~
- ☐ deriving and implementing these equations.

scattering via QC

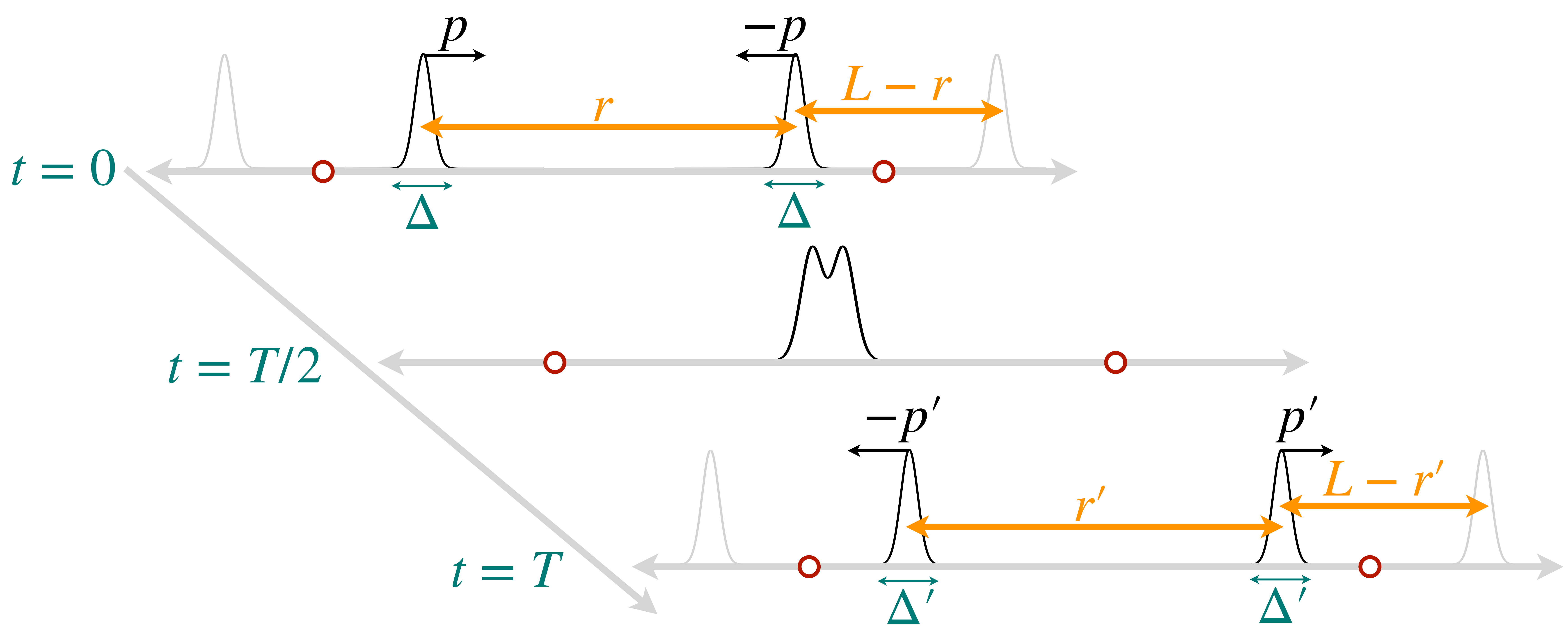
Proposal for studying reactions with real-time evolution:

- [ ] ~~finite-volume formalism~~ no real quantum advantage!

scattering via QC

Proposal for studying reactions with real-time evolution:

- ☐ ~~finite-volume formalism~~ *no real quantum advantage!*
- ☐ wave packets [Jordan Lee, & Preskill (2014)]



scattering via QC

Proposal for studying reactions with real-time evolution:

- ☐ ~~finite-volume formalism~~ *no real quantum advantage!*
- ☐ wave packets [Jordan Lee, & Preskill (2014)]
- ☐ real-time estimators for scattering observables (RESOs)

[RB, Guerrero, Hansen, & Sturzu (2021), Burbano, Carrillo, Urek, Ciavarella, RB (2025)]

claim: any scattering amplitude can systematically be obtained from these matrix elements

$$\mathcal{T} = \int_{\mathcal{V}} \prod_n d\mathcal{V}_n \, e^{iq_n \cdot x_n} \langle P_f | \mathrm{T} \left[\prod_{n'} \mathcal{J}_n(x_{n'}) \right] | P_i \rangle$$

$+ i\epsilon$
+ binning
+ boost averaging

Recap of RESOs

Four-point functions in a finite, Minkowski spacetime

$$\mathcal{T} \sim \int_0^T d^4x\, e^{it(\omega+i\epsilon)} \langle n_f | \mathcal{J}(t)\, \mathcal{J}(0) | n_i \rangle_\infty$$

[only considering one time ordering, introduced ϵ as a regulator]

$$= \sum_n \int_0^T d^4x\, e^{it(E_f+\omega-E_n+i\epsilon)} \langle n_f | \mathcal{J}(0) | n \rangle \langle n | \mathcal{J}(0) | n_i \rangle_\infty$$

[inserting a complete set of discrete finite-volume states]

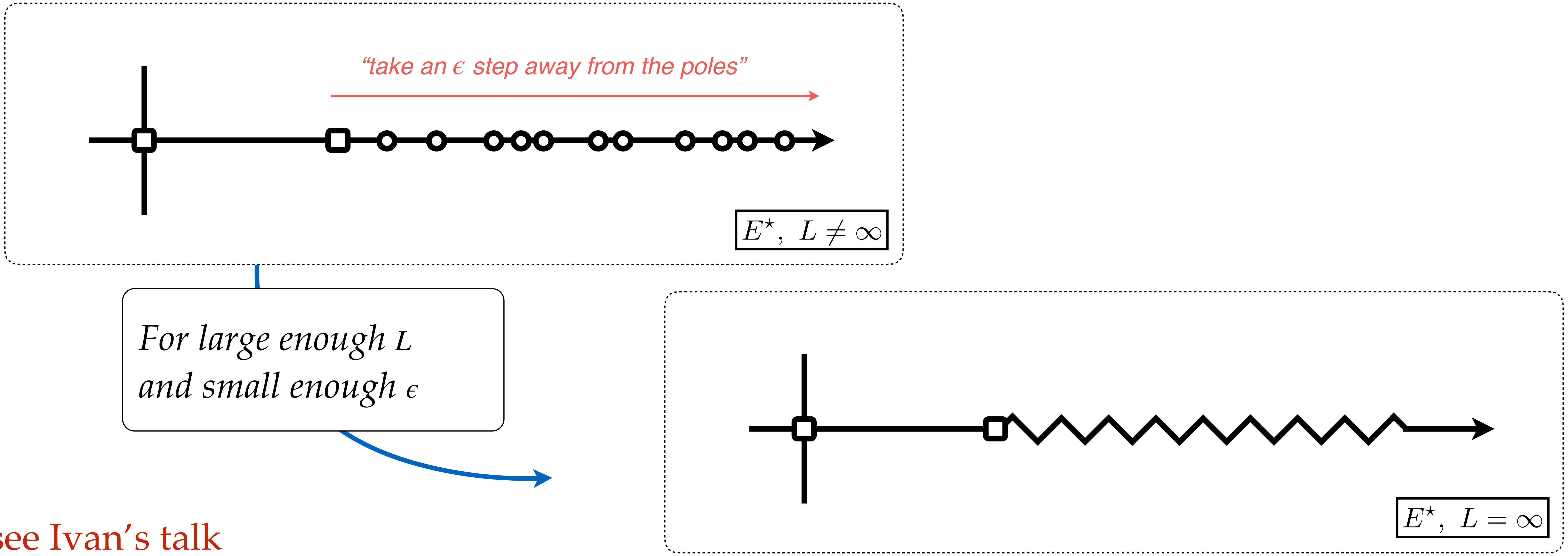
$$\approx \sum_n i \frac{\langle n_f | \mathcal{J}(0) | n \rangle \langle n | \mathcal{J}(0) | n_i \rangle_\infty}{(E_f + \omega - E_n + i\epsilon)} \left(1 - e^{T(iE_f + i\omega - iE_n - \epsilon)}\right)$$

for general proof, see Ivan's talk

Constructing RESOs

- ☑ Determine time-dependent matrix elements [easier said than done 😅]
- ☑ Introduce an $i\epsilon$ by hand [makes sense 🙂]

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} d\tau \, e^{iq_0 t - \epsilon |t|} \langle n_f | \, T[\mathcal{J}_2(t) \, \mathcal{J}_1(0)] \, | n_i \rangle_L$$



for general proof, see Ivan's talk

Constructing RESOs

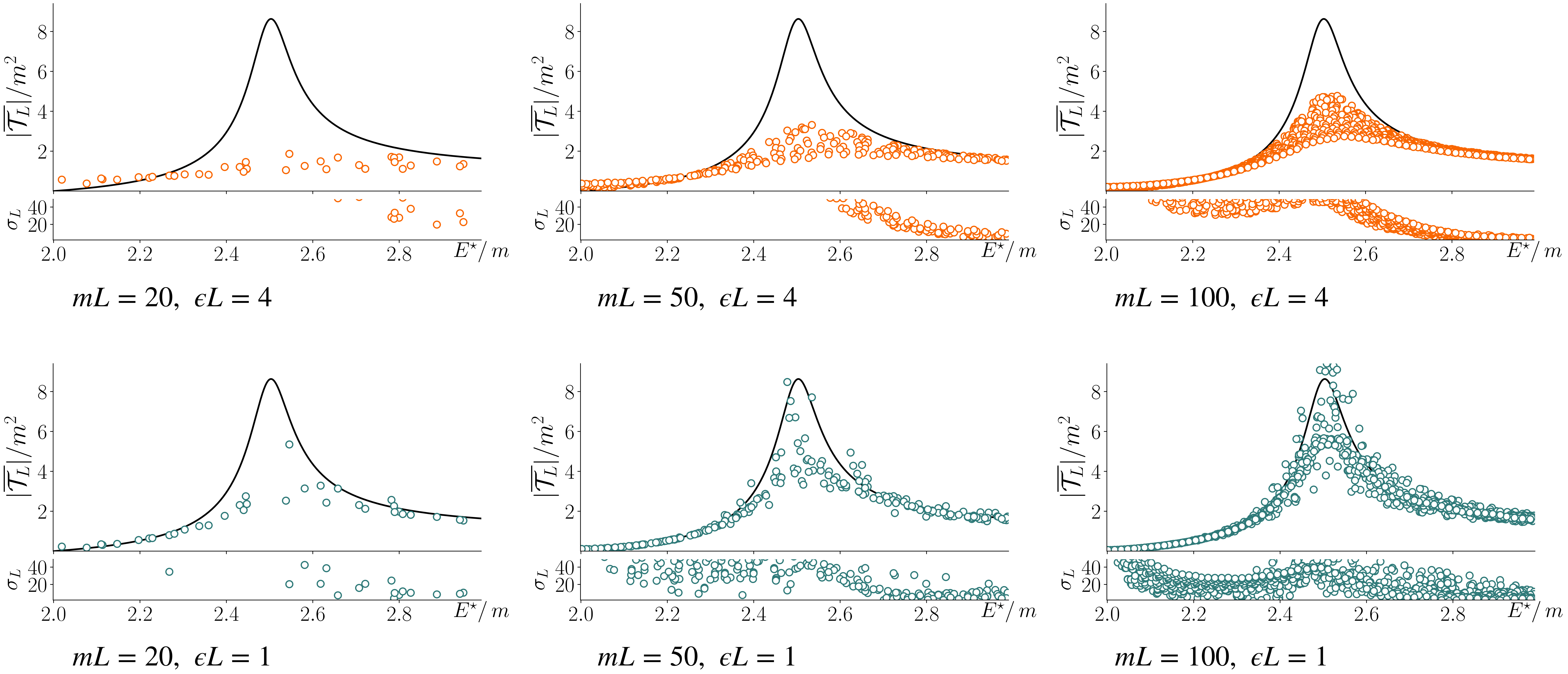
- ☑ Determine time-dependent matrix elements [easier said than done 😅]
- ☑ Introduce an $i\epsilon$ by hand [makes sense 🙂]

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} d\tau\, e^{iq_0 t - \epsilon|t|}\, \langle n_f |\, T[\mathcal{J}_2(t)\, \mathcal{J}_1(0)]\, | n_i \rangle_L$$

- ☑ Binning [“*wave packets like*”] [makes sense 🙂]
- ☑ Impose / enhance symmetry: [uh? 🤔]
  - ☑ Physical amplitudes only depend on Lorentz scalars.
  - ☑ Boost average $\Longleftrightarrow$ reduce finite-volume effects.

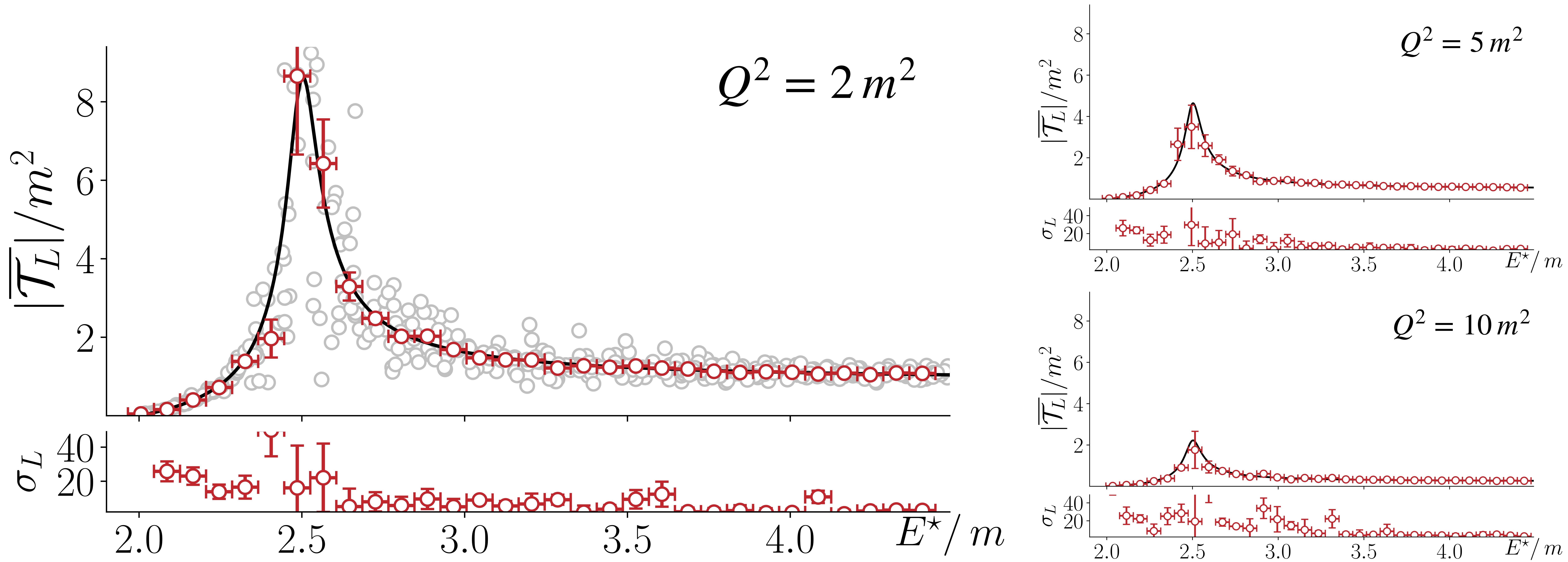
for general proof, see Ivan’s talk

Toy model investigation for $\mathscr{T}$



Following the recipe

By averaging over $mL = [20,25,30]$ boost with $d \leq mL$, and binning in energy and virtualities.




misc.
g-2
scattering
electroweak
spectro
structure

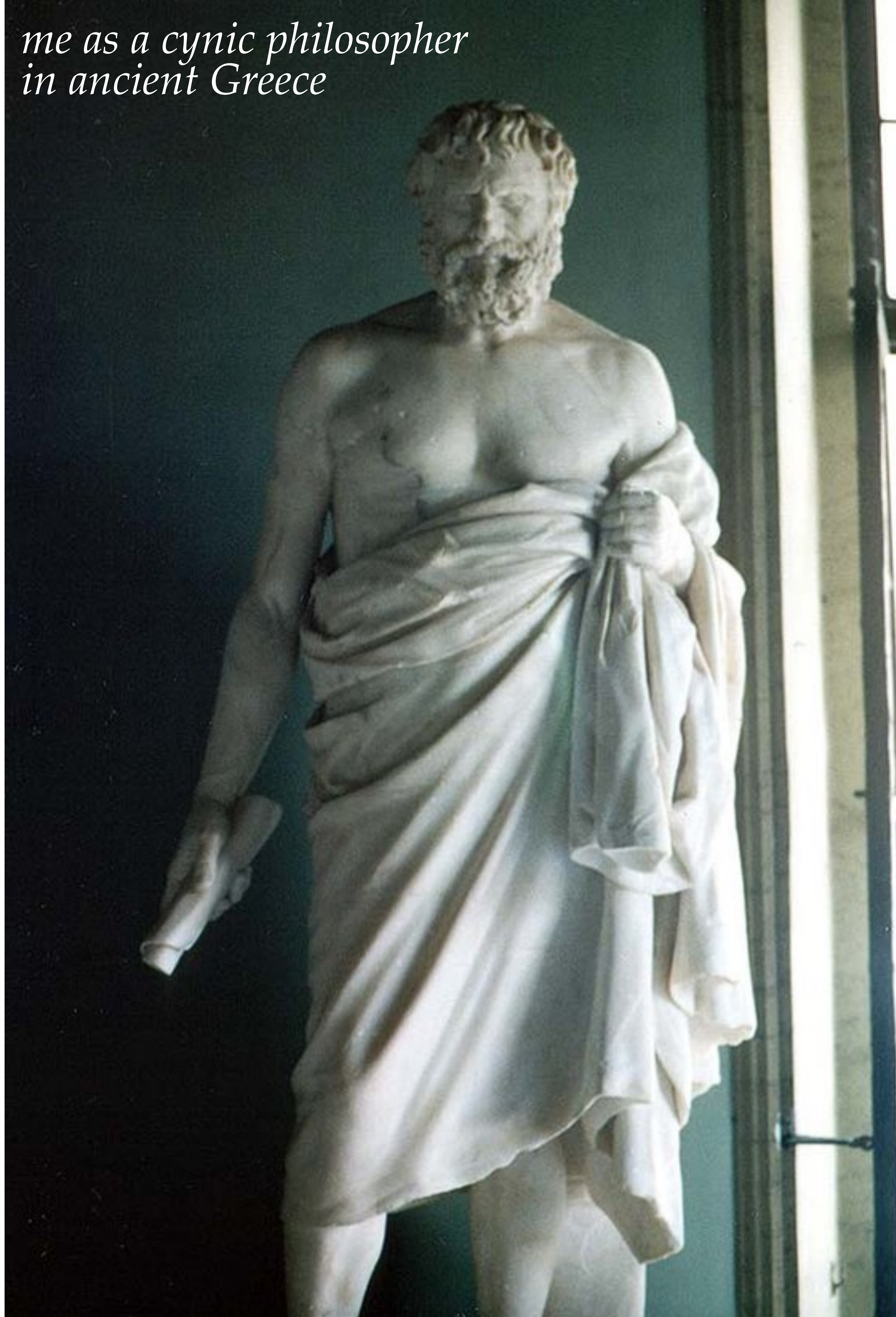
finite temp.
misc.
elastic scattering
g-2
couplings
leptonic
decays
moments
form factors
semi-
leptonic
upper bounds
of energies
multi-particle
ops
scattering
Euclidean
15yrs ago
electroweak
spectro
structure

finite temp.
physical point
gradient flow
QED+QCD
ML for critical slowing down
finite-chemical potential
LO. Hadron vacuum polarization HVP
NLO. Hadron vacuum polarization HVP
hadronic light-by-light
elastic scattering
inelastic scattering
3body scattering
coupled 2- and 3-body scattering
structure of scattering states
Euclidean today
couplings
moments
form factors
Parton distribution functions
Transverse momentum distributions
Generalized parton distributions
leptonic decays
semi-leptonic
multi-hadron weak decays
long-range matrix elements
inclusive decays
upper bounds of energies
multi-particle ops
tetraquarks
pentaquarks
lower bounds of energies

opportunity for QC

I came into this field as a cynic

…but the progress is making me increasingly hopeful



*"The Cynics rejected all conventional desires for wealth, power, **glory**, social recognition, conformity, and worldly possessions and even flouted such conventions openly and derisively in public."*

opportunity for QC

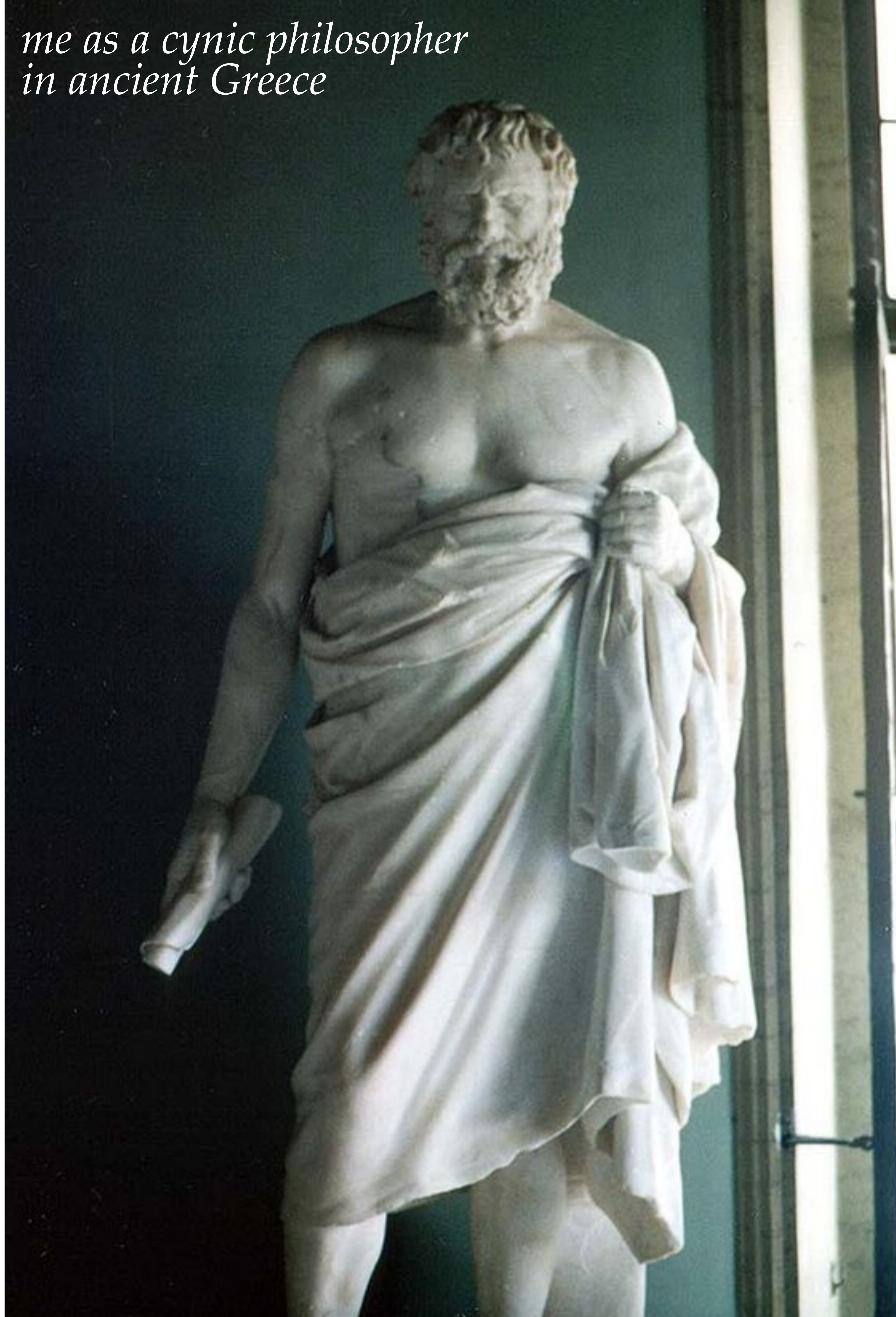
I came into this field as a cynic

Other things I have been cynical about

- ☑ the Higgs discovery,
- ☑ gravitational wave discovery,
- ☑ lattice QCD having an impact in g-2,
- ☐ dark matter discovery.

Meaning you have $\geq 75\ \%$ of achieving glory!

Godspeed!



take home message / call to arms

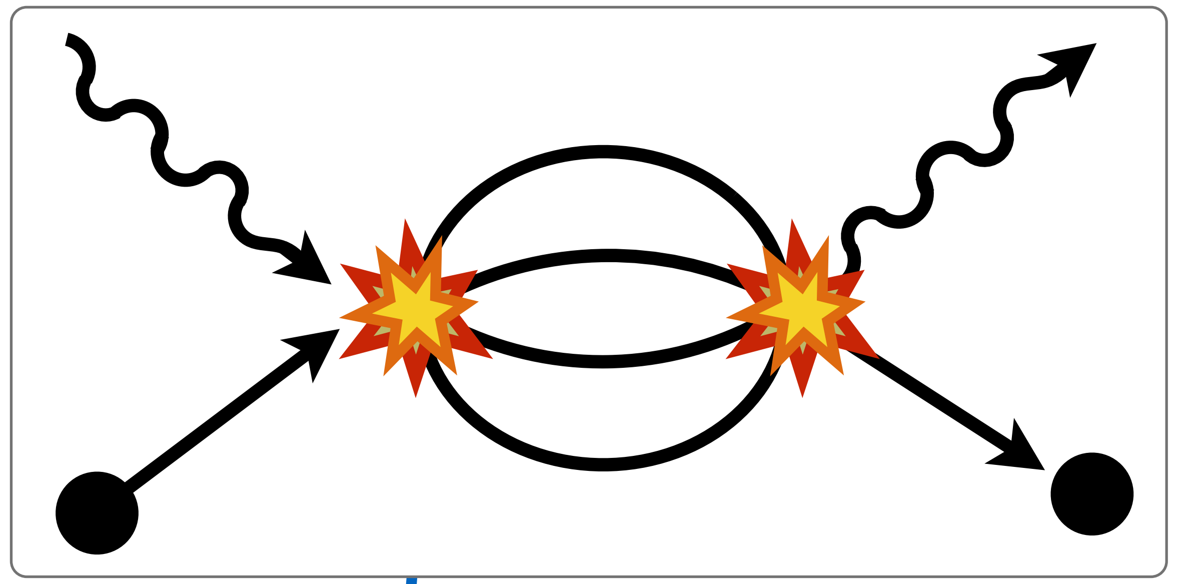
despite all the reasons as to why LQCD should not tell give us insight into observables that aren't directly accessible, LQCD has given us a whole lot more than was previously imaginable.

Do not undersestimate the speed at which LQCD community could progress to obtain quantities that are today unimaginable.

So move fast!

Exclusive vs. inclusive reactions

- If exclusive and *interesting*
  - After developing increasingly complex formalism…
  - Lattice QCD **will always win**
- Inclusive reactions, QC methods *may* be needed and worth investigating.



Infinite-volume reactions

- complex functions,
- kinematic singularities,
- due to intermediate on-shell states.

Hamiltonian frameworks

Four-point functions in a finite, Minkowski spacetime

$$\mathcal{T} \sim \int_0^T d^4x\, e^{it(\omega+i\epsilon)} \langle n_f | \mathcal{J}(t)\, \mathcal{J}(0) \, | n_i \rangle_\infty$$

[only considering one time ordering, introduced ϵ as a regulator]

$$= \sum_n \int_0^T d^4x\, e^{it(E_f+\omega-E_n+i\epsilon)} \langle n_f | \mathcal{J}(0) \, | n \rangle \langle n \, | \mathcal{J}(0) \, | n_i \rangle_\infty$$

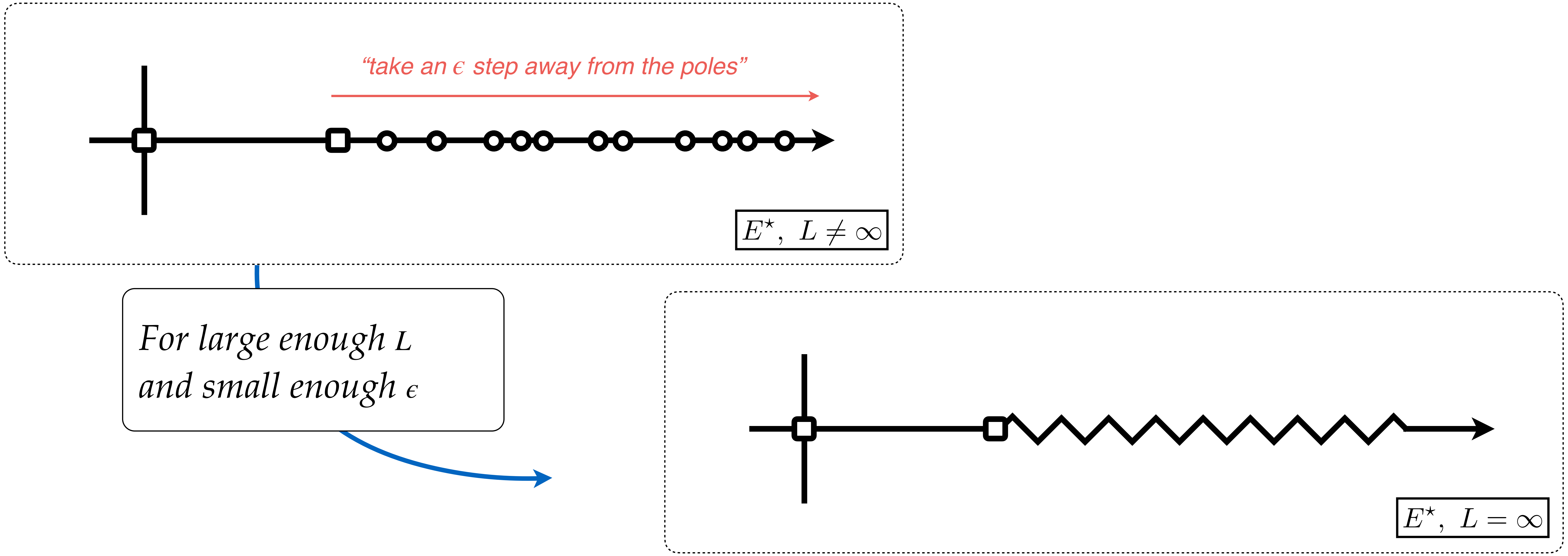
[inserting a complete set of discrete finite-volume states]

$$\approx \sum_n i \frac{\langle n_f | \mathcal{J}(0) \, | n \rangle \langle n \, | \mathcal{J}(0) \, | n_i \rangle_\infty}{(E_f + \omega - E_n + i\epsilon)} \left(1 - e^{T(iE_f + i\omega - iE_n - \epsilon)} \right)$$

Constructing reliably estimators

- ☑ Determine time-dependent matrix elements [easier said than done 😅]
- ☑ Introduce an $i\epsilon$ by hand

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} d\tau \, e^{iq_0 t - \epsilon|t|} \langle n_f | \, T[\mathcal{J}_2(t) \, \mathcal{J}_1(0)] \, | n_i \rangle_L$$



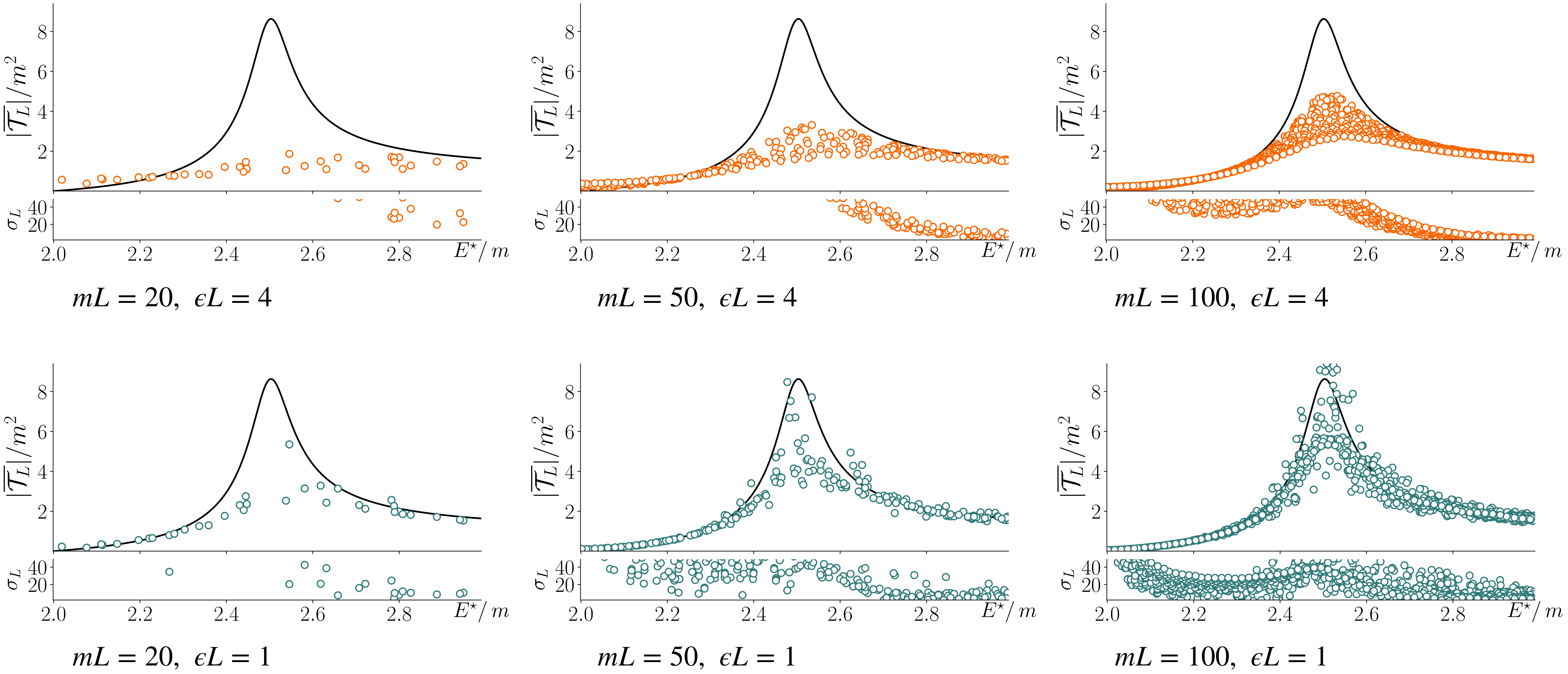
Constructing reliably estimators

- ☑ Determine time-dependent matrix elements [easier said than done 😅]
- ☑ Introduce an $i\epsilon$ by hand [makes sense 🙂]

$$\mathcal{T}_L(\epsilon) \sim \int_{-\infty}^{\infty} d\tau\, e^{iq_0 t - \epsilon|t|} \langle n_f | \, T[\mathcal{J}_2(t)\, \mathcal{J}_1(0)] \, | n_i \rangle_L$$

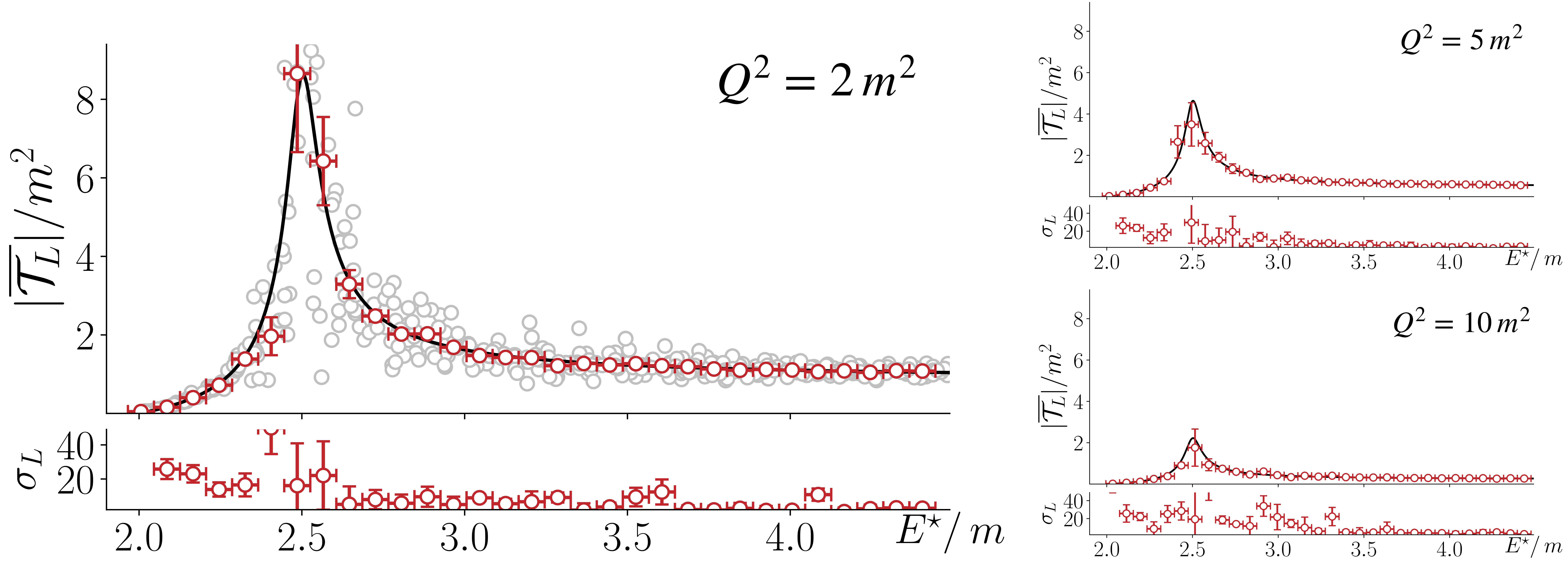
- ☑ Binning [wave packets like] [makes sense 🙂]
- ☑ Exploit symmetry: [uh? 🤔]
  - ☑ Physical amplitudes only depend on Lorentz scalars.
  - ☑ Boost average

Toy model investigation for $\mathscr{T}$



Following the recipe

By averaging over $mL = [20,25,30]$ boost with $d \leq mL$, and binning in energy and virtualities.



A call to arms

- ☑ Euclidean LGTs has been remarkably successful despite it's ugliness,
- ☑ Thanks for the fields ingenuity, the space of observables that cannot be accessed directly or indirectly is increasingly shrinking,
- ☑ It's really hard for me to predict what won't be accessed in the future,
- ☑ For Minkowski LGTs to have phenomenological impact, it needs to move quick!

Carrillo, Guerrero, Sturzu, RB (to appear)
RB, Guerrero, Hansen, Sturzu (2020)