SU(2) Yang–Mills Lattice Gauge Theory: from Classical to Quantum simulation

Pietro Silvi
Berkeley, September 2025

(2+1)D Ground State,
Tensor Networks
PRR **6**, 033057 (2024)

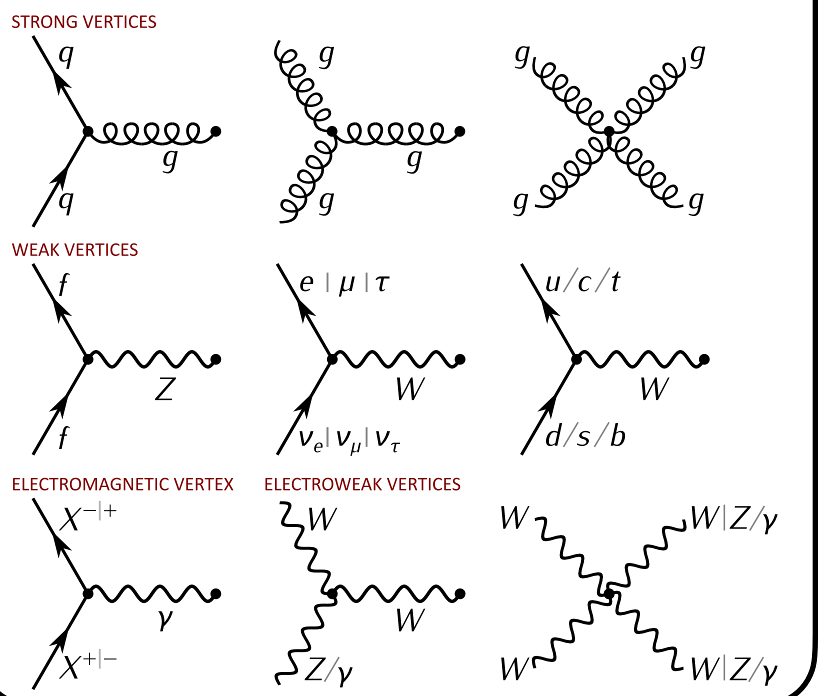
(1+1)D Trapped Ion
Quantum Simulator
PRX Quantum **5**, 040309 (2024)

WHY SIMULATE LGT?

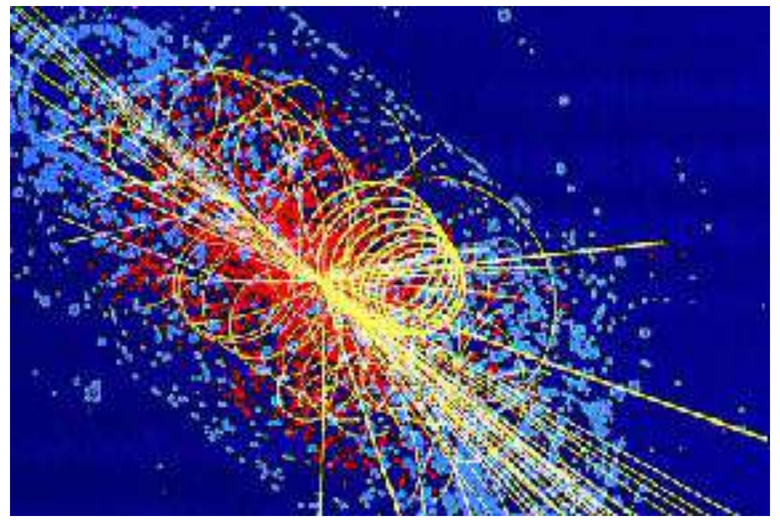
Gauge Theories are Local Symmetries
appearing in physics at different energy scales

$$\vec{\nabla} \cdot \vec{E} = \varrho$$

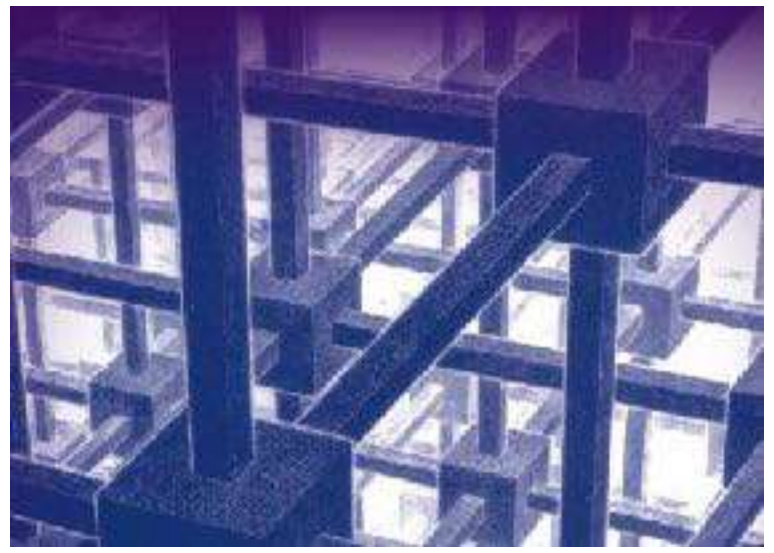
Theoretical particle physics



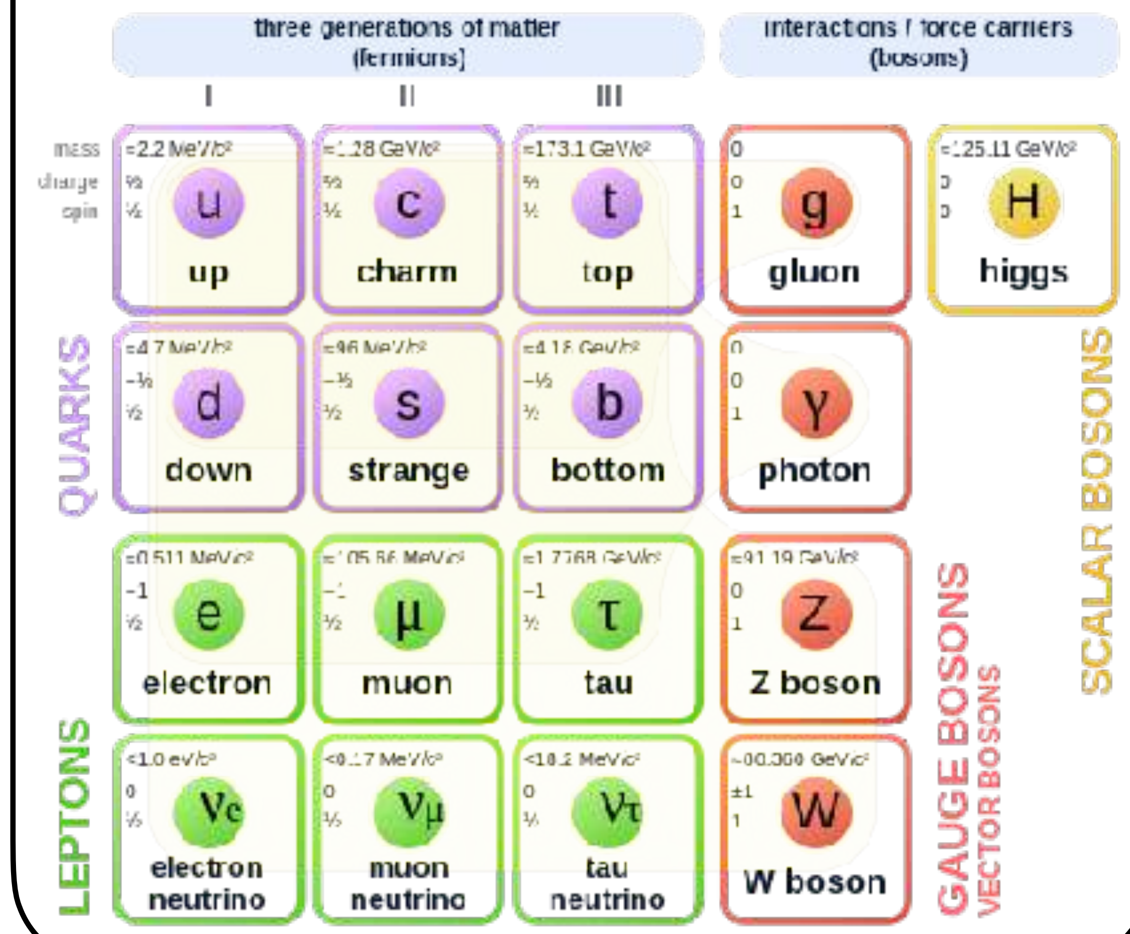
Experimental particle physics



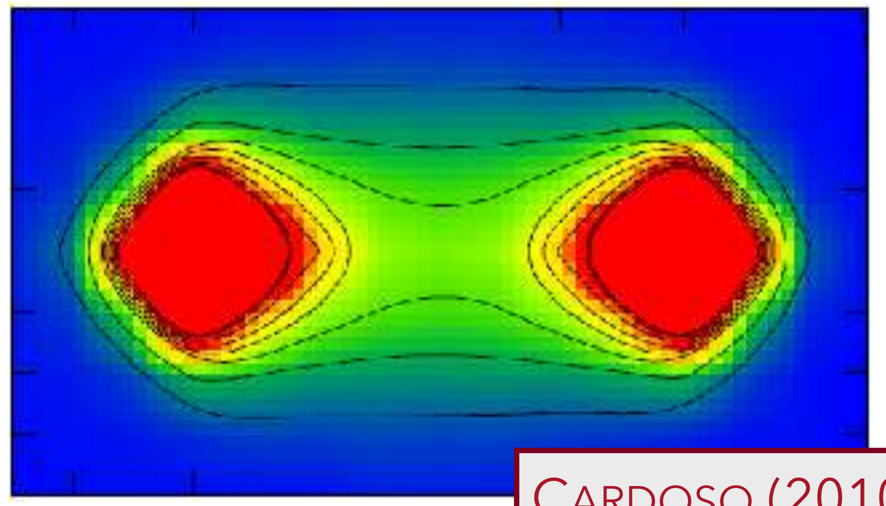
Beyond experimental and perturbative approaches: Lattice Gauge Theories



High-Energy Physics e.g. Standard Model

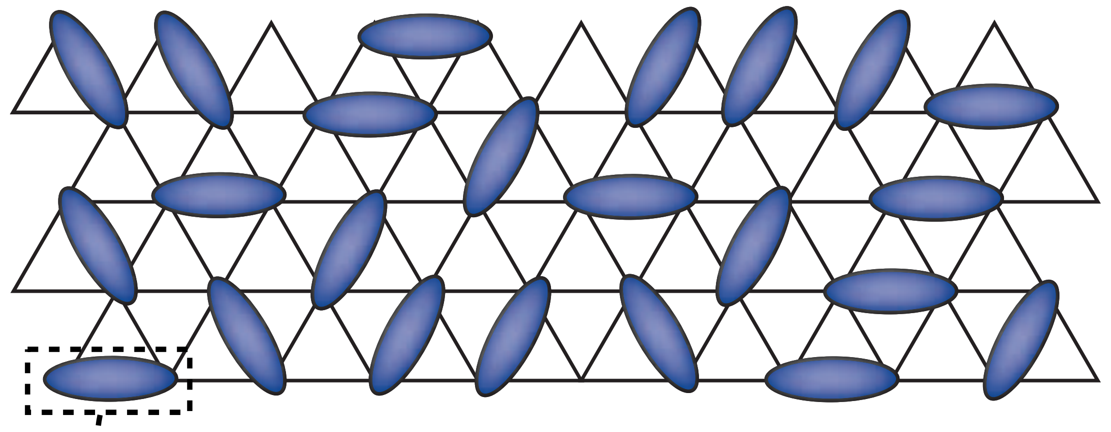


Monte Carlo Simulations

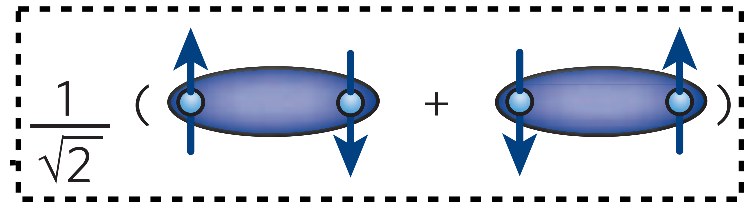


Cardoso (2010)

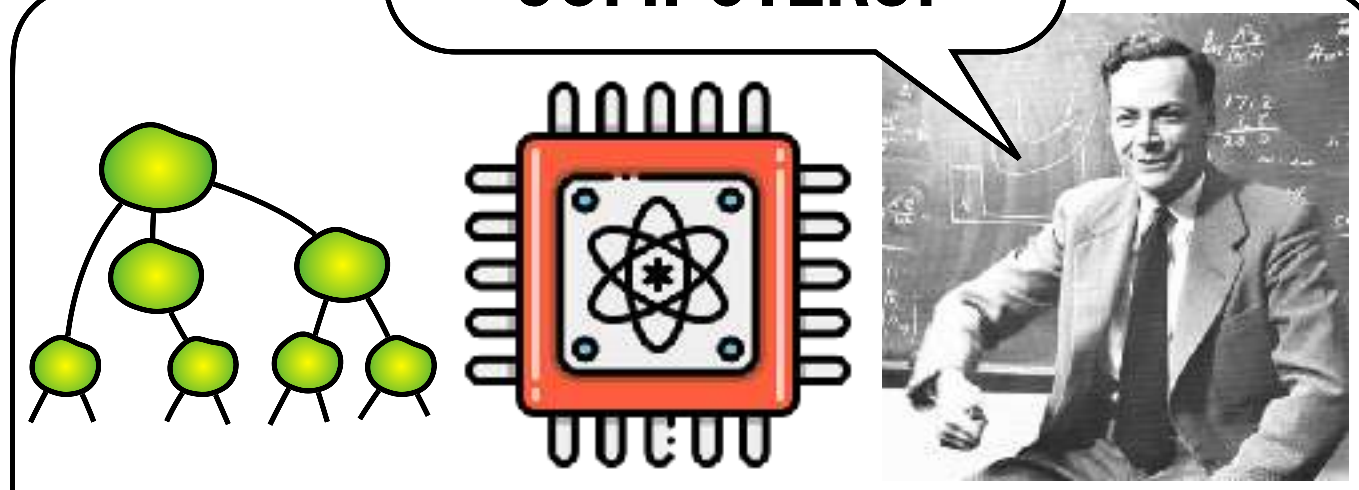
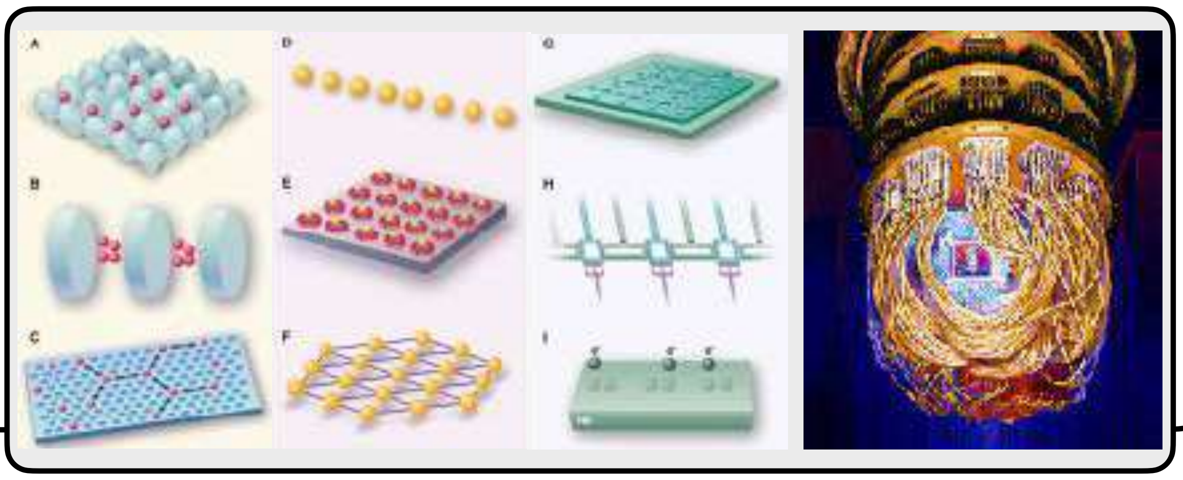
Condensed Matter Physics e.g. Quantum Spin Liquids



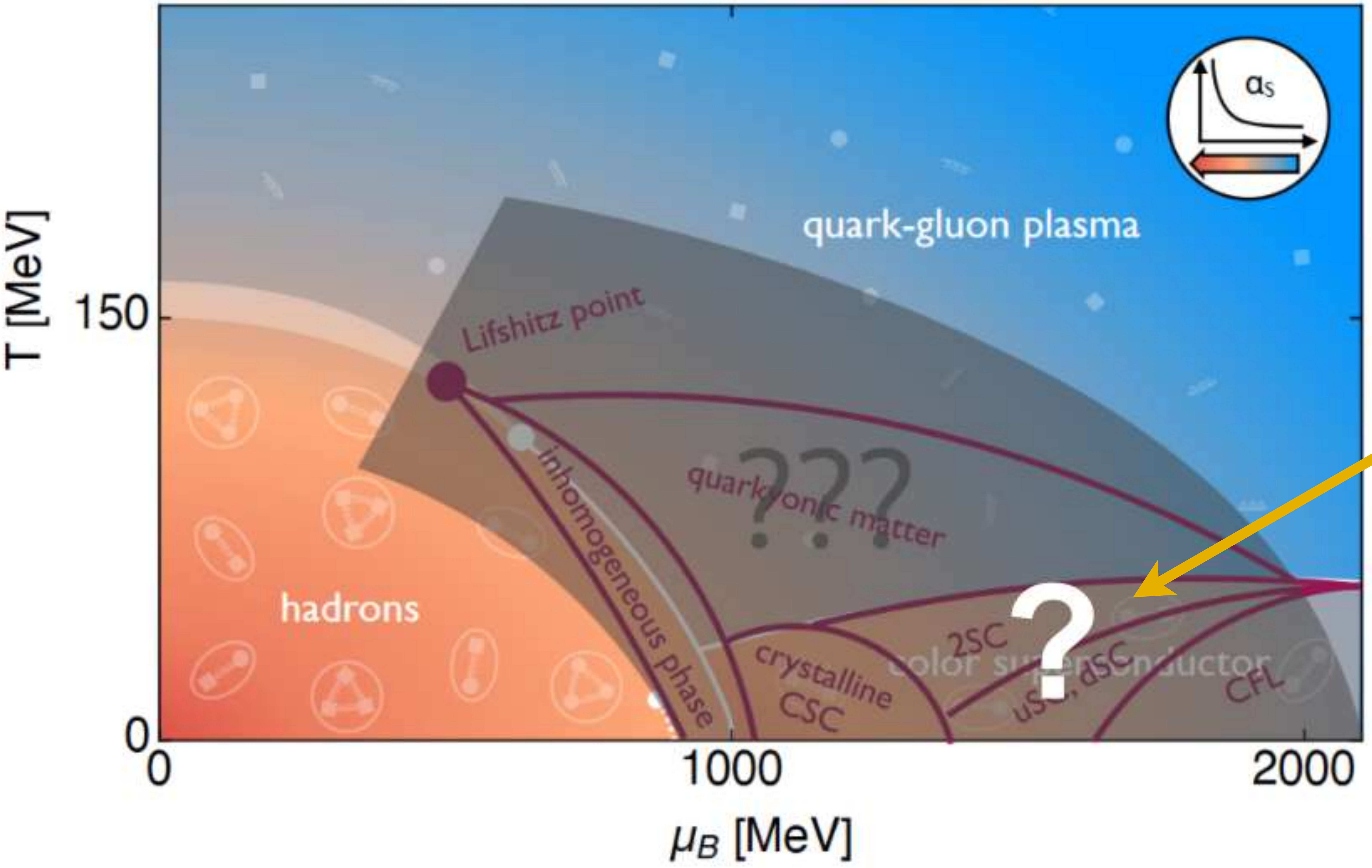
Balents (2010)



To simulate quantum physics, use "quantum" computers!

WHY SIMULATE LGT?



MAKE PREDICTIONS HERE?

NON-PERTURBATIVE

MONTECARLO SIGN-PROBLEM

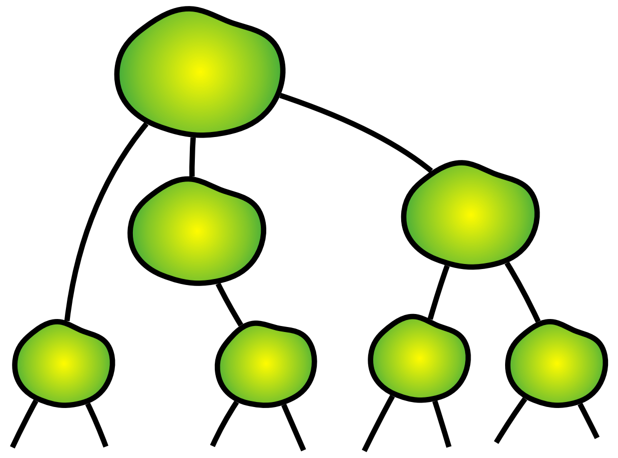
USE QUANTUM DEVICES

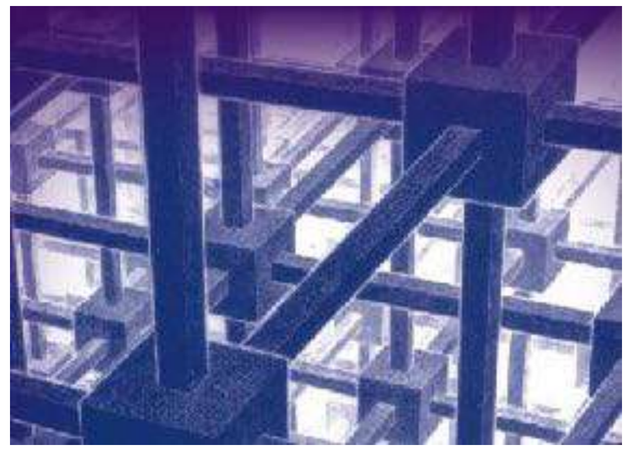
(OR QUANTUM-INSPIRED)

LGT @ PADUA QUANTUM GROUP

MAGNIFICO, RIGOBELLO, CATALDI, CALAJÒ, MONTANGERO

TENSOR NETWORKS FOR HIGH-DIMENSIONAL LATTICE GAUGE THEORIES IN HIGH-ENERGY PHYSICS





(2+1)D SU(2) Yang-Mills Lattice Gauge Theory at finite density via tensor networks

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(Dated: September 13, 2023)

We demonstrate the feasibility of Tensor Network simulations of non-Abelian lattice gauge theories in two spatial dimensions, by focusing on a (minimally truncated) SU(2) Yang-Mills model in Hamiltonian formulation, including dynamical matter. Thanks to our sign-problem-free approach, we characterize the phase diagram of the model at zero and finite baryon number, as a function of the bare mass and color charge of the quarks. Already at intermediate system sizes, we distinctly detect a liquid phase of quark-pair bound-state quasi-particles (baryons), whose mass is finite towards the continuum limit. Interesting phenomena arise at the transition boundary where color-electric and color-magnetic terms are maximally frustrated: for low quark masses, we see traces of potential deconfinement, while for high quark masses, we observe signatures of a possible topological order.

WHAT ABOUT NON-ABELIAN LATTICE GAUGE THEORIES?

• TOY MODEL FOR QCD

PHYSICAL REVIEW X 10, 041040 (2020)

Two-Dimensional Quantum-Link Lattice Quantum Electrodynamics at Finite Density

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We present an unconstrained tree-tensor-network approach to the study of lattice gauge theories in two spatial dimensions, showing how to perform numerical simulations of theories in the presence of fermionic matter and four-body magnetic terms, at zero and finite density, with periodic and open boundary conditions. We exploit the quantum-link representation of the gauge fields and demonstrate that a fermionic rishon representation of the quantum links allows us to efficiently handle the fermionic matter while finite densities are naturally enclosed in the tensor network description. We explicitly perform calculations for quantum electrodynamics in the spin-one quantum-link representation on lattice sizes of up to 16×16 sites, detecting and characterizing different quantum regimes. In particular, at finite density, we detect signatures of a phase separation as a function of the bare mass values at different filling densities. The presented approach can be extended straightforwardly to three spatial dimensions.

DOI: 10.1103/PhysRevX.10.041040

Subject Areas: Computational Physics, Particles and Fields, Quantum Physics

(2+1)D

(3+1)D

nature COMMUNICATIONS

ARTICLE

Check for updates

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Lattice quantum electrodynamics in (3+1)-dimensions at finite density with tensor networks

Giuseppe Magnifico[1,2], Timo Felser[1,2,3], Pietro Silvi[4,5] & Simone Montangero[1,2]

Gauge theories are of paramount importance in our understanding of fundamental constituents of matter and their interactions. However, the complete characterization of their phase diagrams and the full understanding of non-perturbative effects are still debated, especially at finite charge density, mostly due to the sign-problem affecting Monte Carlo numerical simulations. Here, we report the Tensor Network simulation of a three dimensional lattice gauge theory in the Hamiltonian formulation including dynamical matter: Using this sign-problem-free method, we simulate the ground states of a compact Quantum Electrodynamics at zero and finite charge densities, and address fundamental questions such as the characterization of collective phases of the model, the presence of a confining phase at large gauge coupling, and the study of charge-screening effects.

INTRODUCTION

HAMILTONIAN LATTICE GAUGE THEORIES

GENERAL DIRAC HAMILTONIAN:

MASS TERM PURE GAUGE TERM

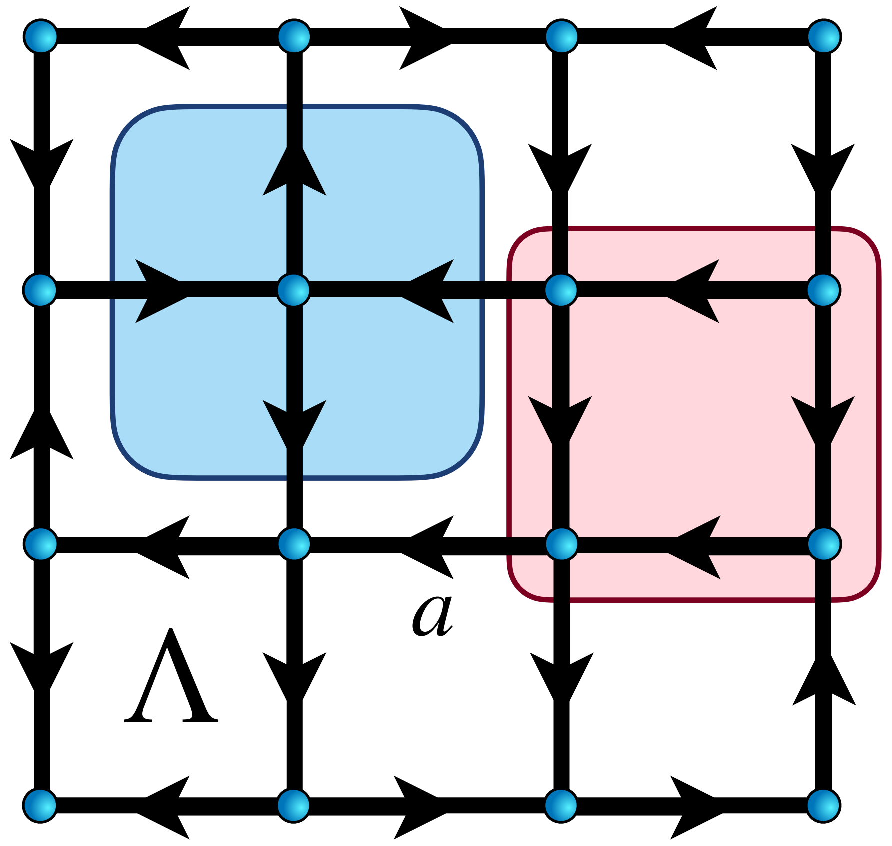
$$\tilde{H} \sim \int \psi^\dagger \gamma_0 \vec{\gamma}\left(-i\vec{\nabla} - g\vec{A}\right)\psi + \int m\psi^\dagger\gamma_0\psi + \int \left[E^2 + B^2\right]$$

$E_i = \dot{A}_i$

$B_i = \varepsilon_{ijk}\partial_j A_k - g[A_j, A_k]/2$

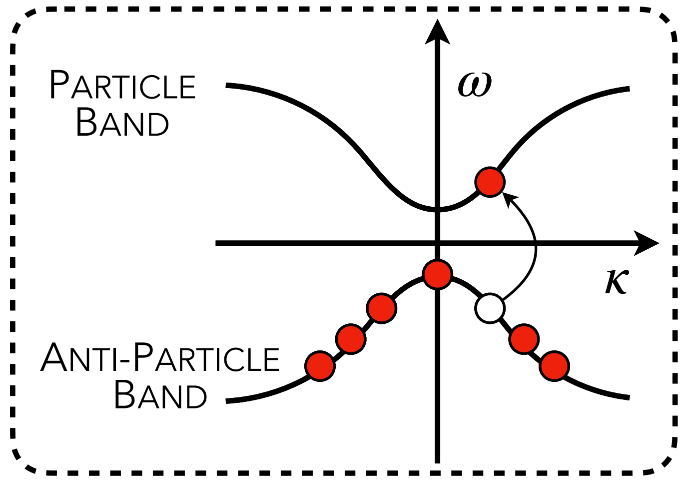
MATTER-GAUGE INT.

SPATIAL DISCRETIZATION: $x = ja$, $j \in \Lambda$. TIME IS CONTINUOUS



$$H = -\frac{c\hbar}{2a}\sum_{j,\mu}\left[e^{i\varphi(j,\mu)}\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \text{H.c.}\right] + m_0c^2\sum_j (-1)^j \psi_j^\dagger\psi_j + H_{\text{Pure}}$$

$$H_{\text{Pure}} = \frac{g^2 c\hbar}{2a}\sum_j\left(E^2_{j,\rightarrow} + E^2_{j,\uparrow}\right) - \frac{c\hbar}{2ag^2}\sum_{\square}\Re e\begin{pmatrix} & U^\dagger & \\ U^\dagger & & U \\ & U & \end{pmatrix}$$



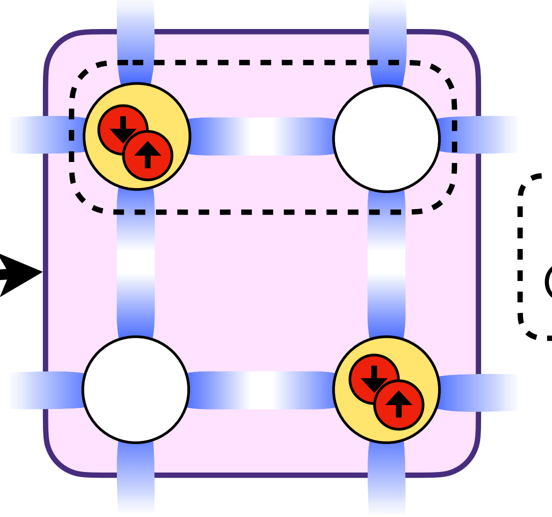
PARALLEL TRANSPORT $U_{j,j+\mu} = e^{igA_{j,j+\mu}}$

ELECTRIC CONTRIBUTION

MAGNETIC CONTRIBUTION

REGULARIZING FERMIONS VIA STAGGERED FERMIONS

SUSSKIND (1977)

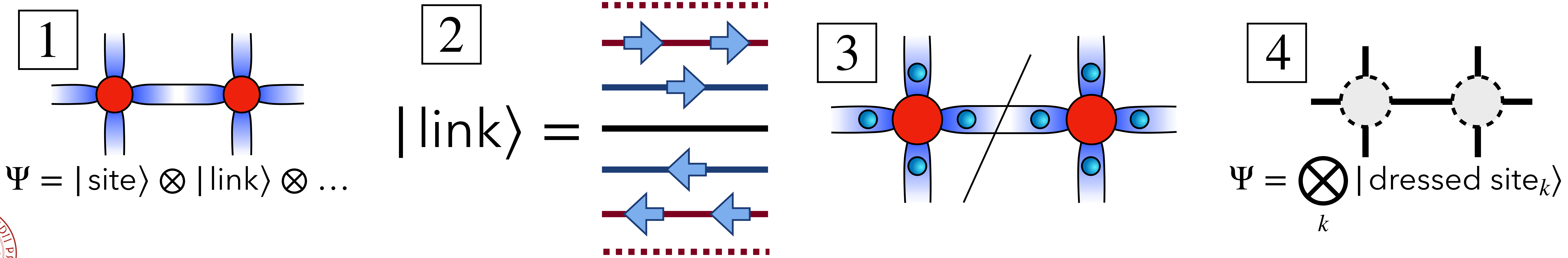


4 SPINOR COMPONENTS

QUANTUM SIMULATIONS OF LGT

CHANDRASEKHARAN, WIESE (1997)

1. HILBERT SPACE: $\mathscr{H}_m \otimes \mathscr{H}_G$ WITH GAUGE-INVARIANT STATES
2. GAUGE D.O.F TO **SPIN-LIKE OPERATORS**: E.G. QED $(E, U) \rightarrow (S^z, S^-)$
3. DECOMPOSE GAUGE LINKS INTO PAIRS OF **FERMIONIC RISHONS**
4. MERGE RISHONS & MATTER SITES INTO **BOSONIC GAUGE INVARIANT SITES**

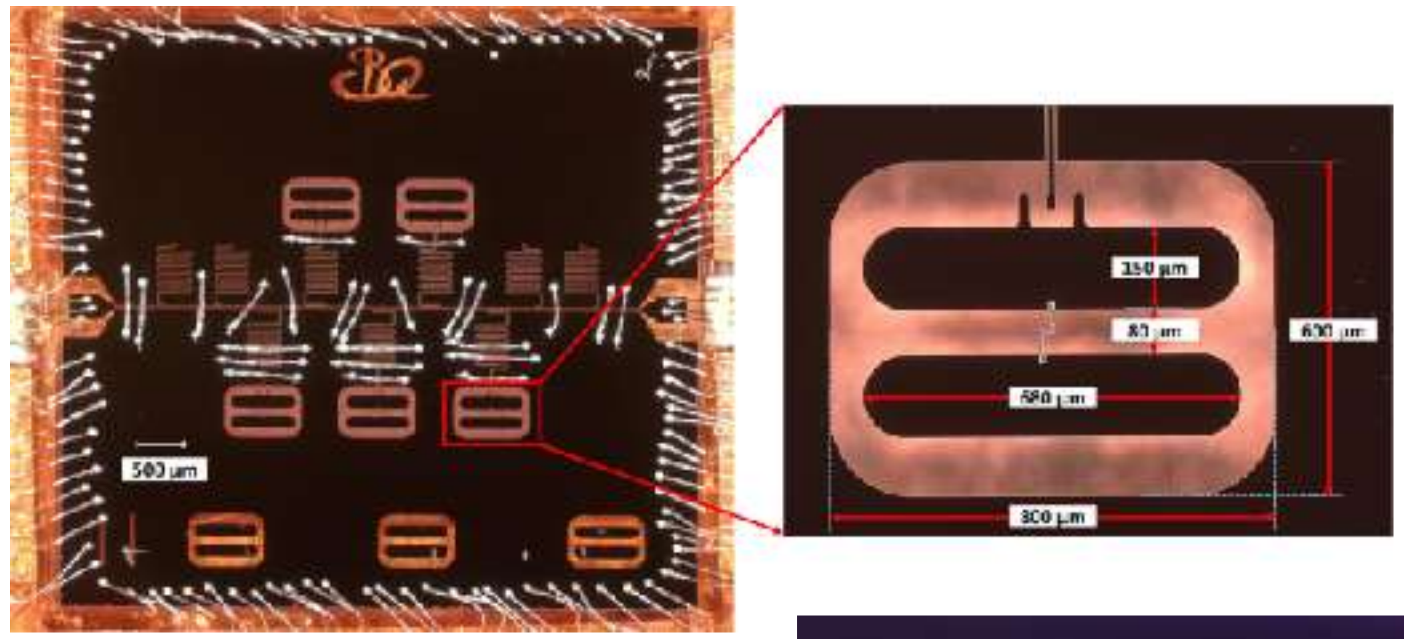


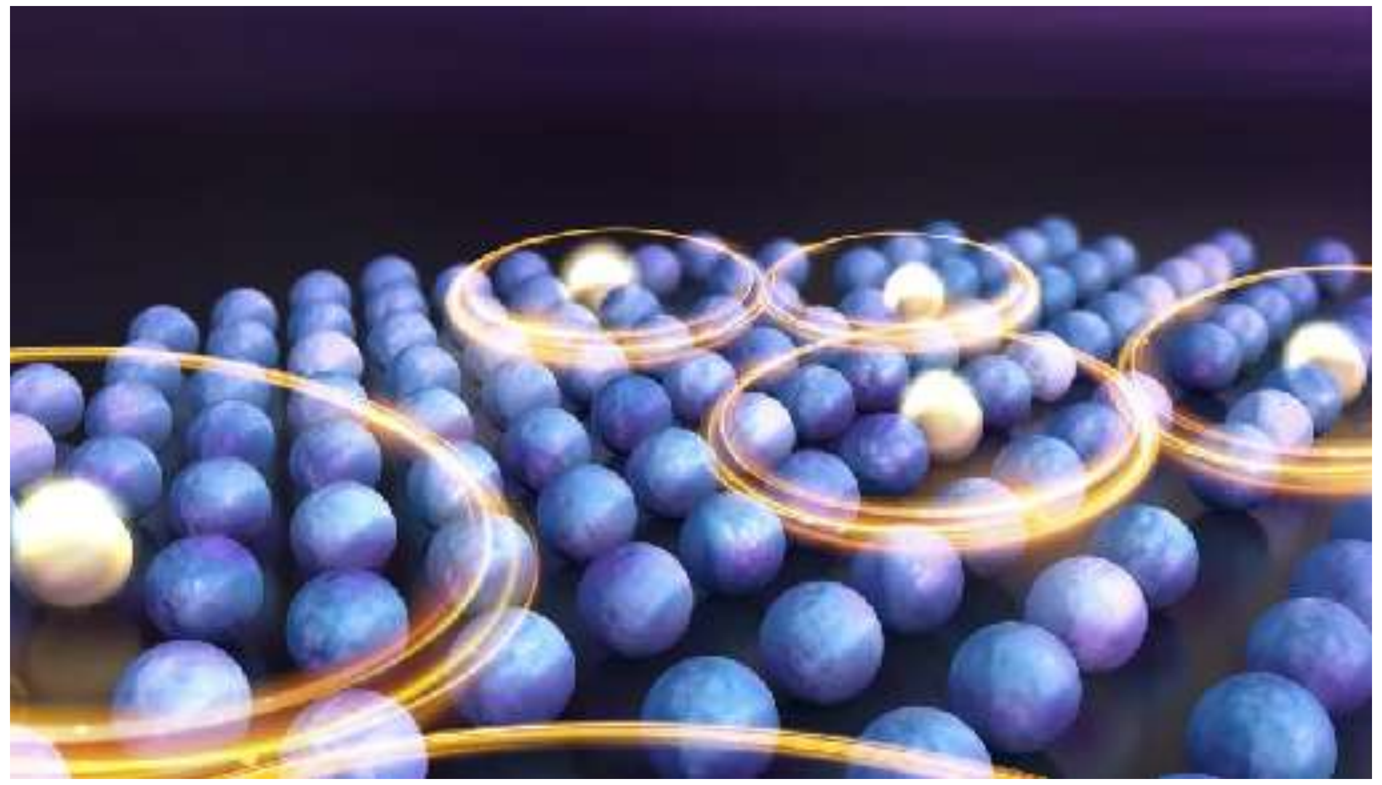
PROGRAMMABLE QUANTUM DEVICES

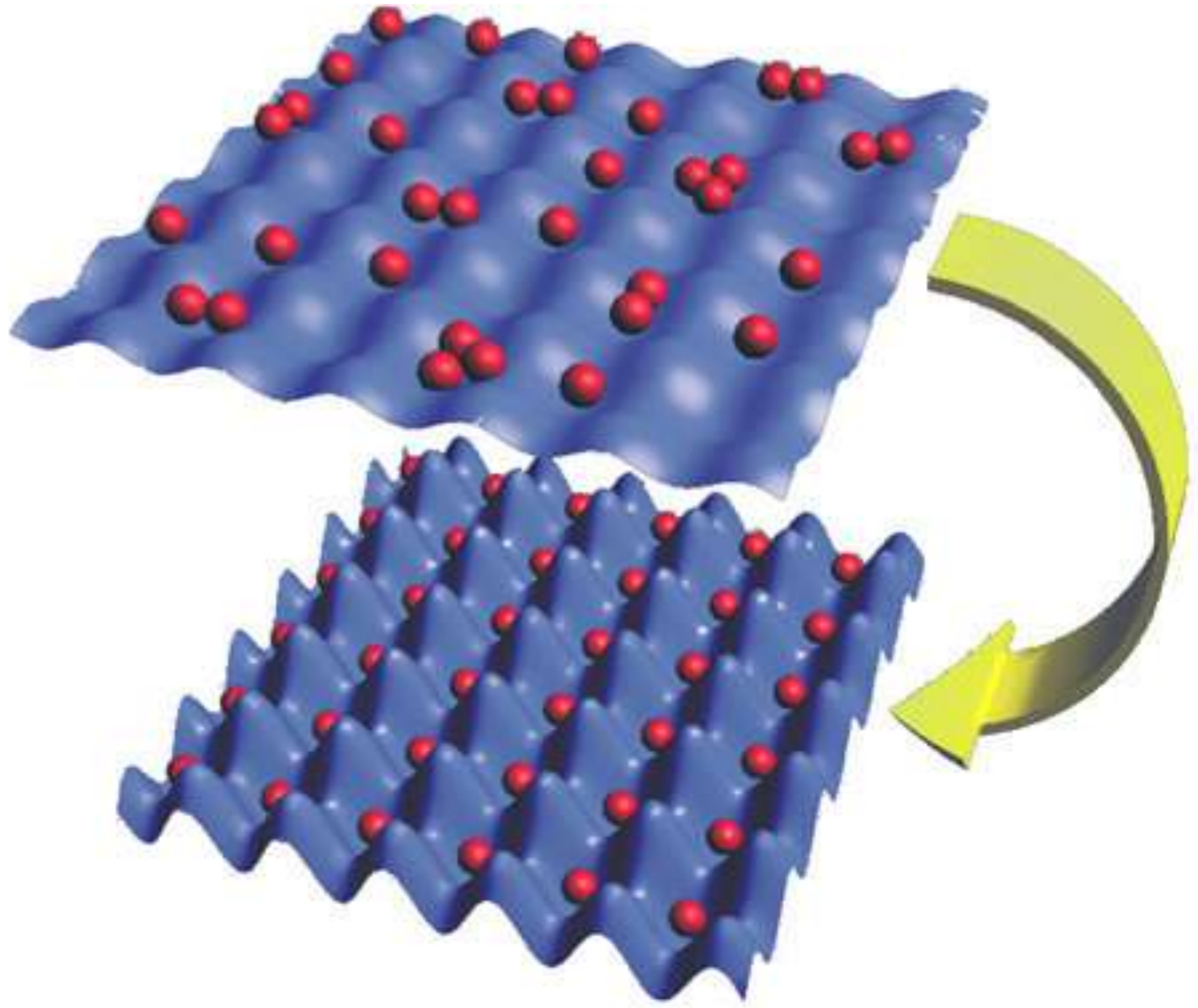
THEY HARNESS THE POWER OF THE QUANTUM WORLD:

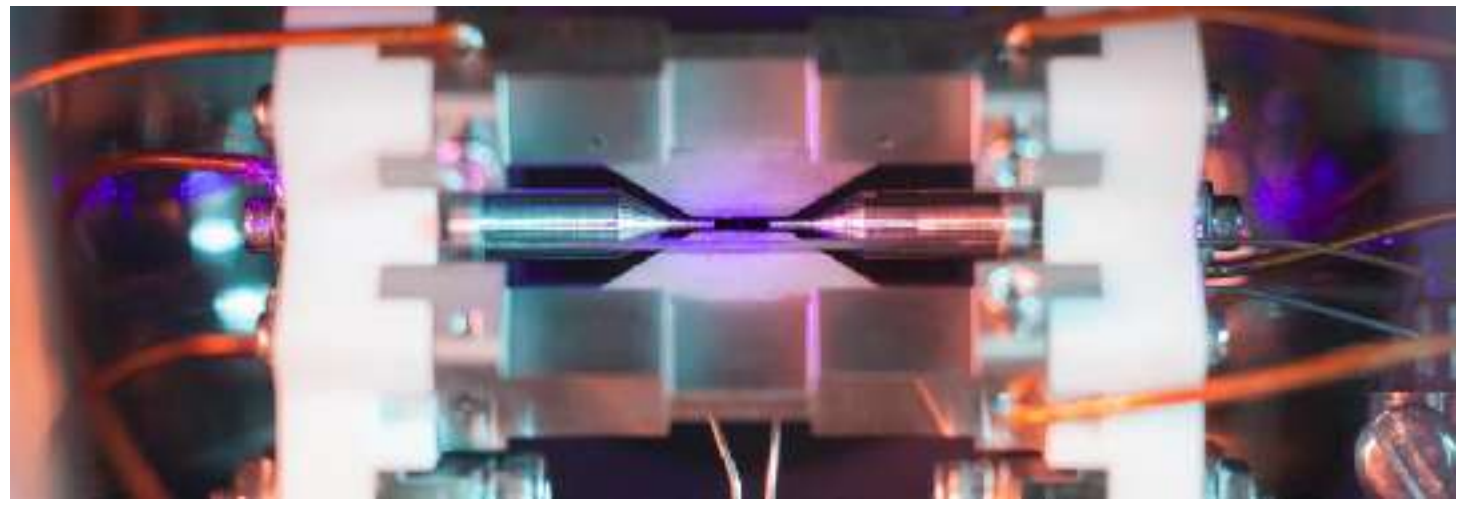
SUPERPOSITION, INTERFERENCE, ENTANGLEMENT

- Superconducting Qubits
- Rydberg Atoms in Optical Tweezers
- Trapped Ions
- Optical Lattices
- Photonic Circuits
- Nitrogen-Vacancy Centers in Diamond
- … and more!







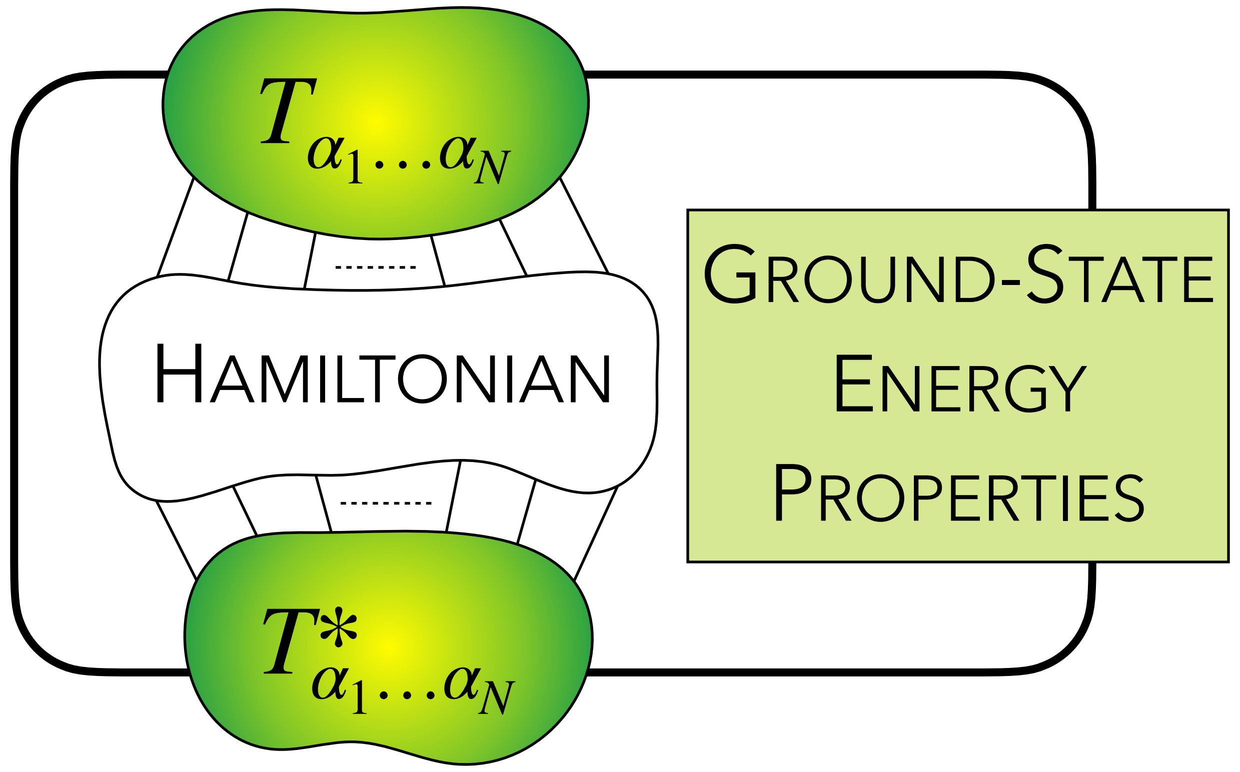


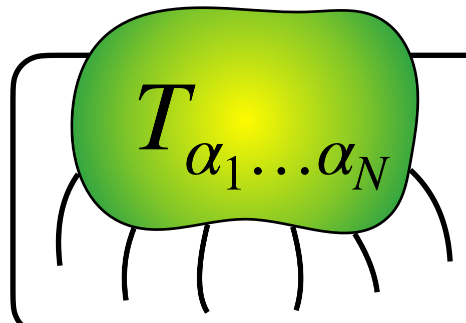
TENSOR NETWORK METHODS

The Hilbert space of N-Body Systems *grows exponentially*.

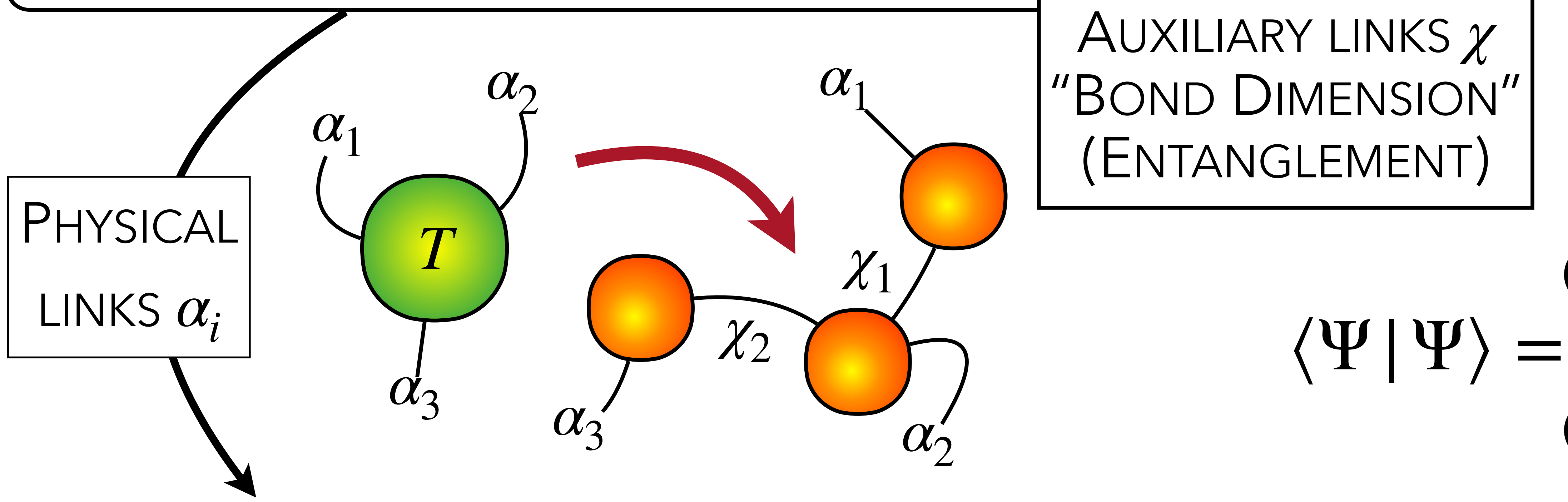
Exact Diagonalization (ED) is not sustainable for large N

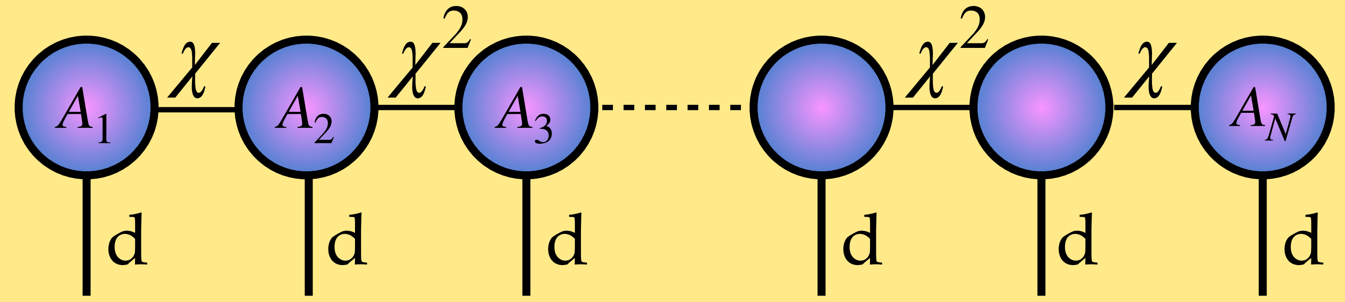
$$\dim\mathcal{H} = d^N$$





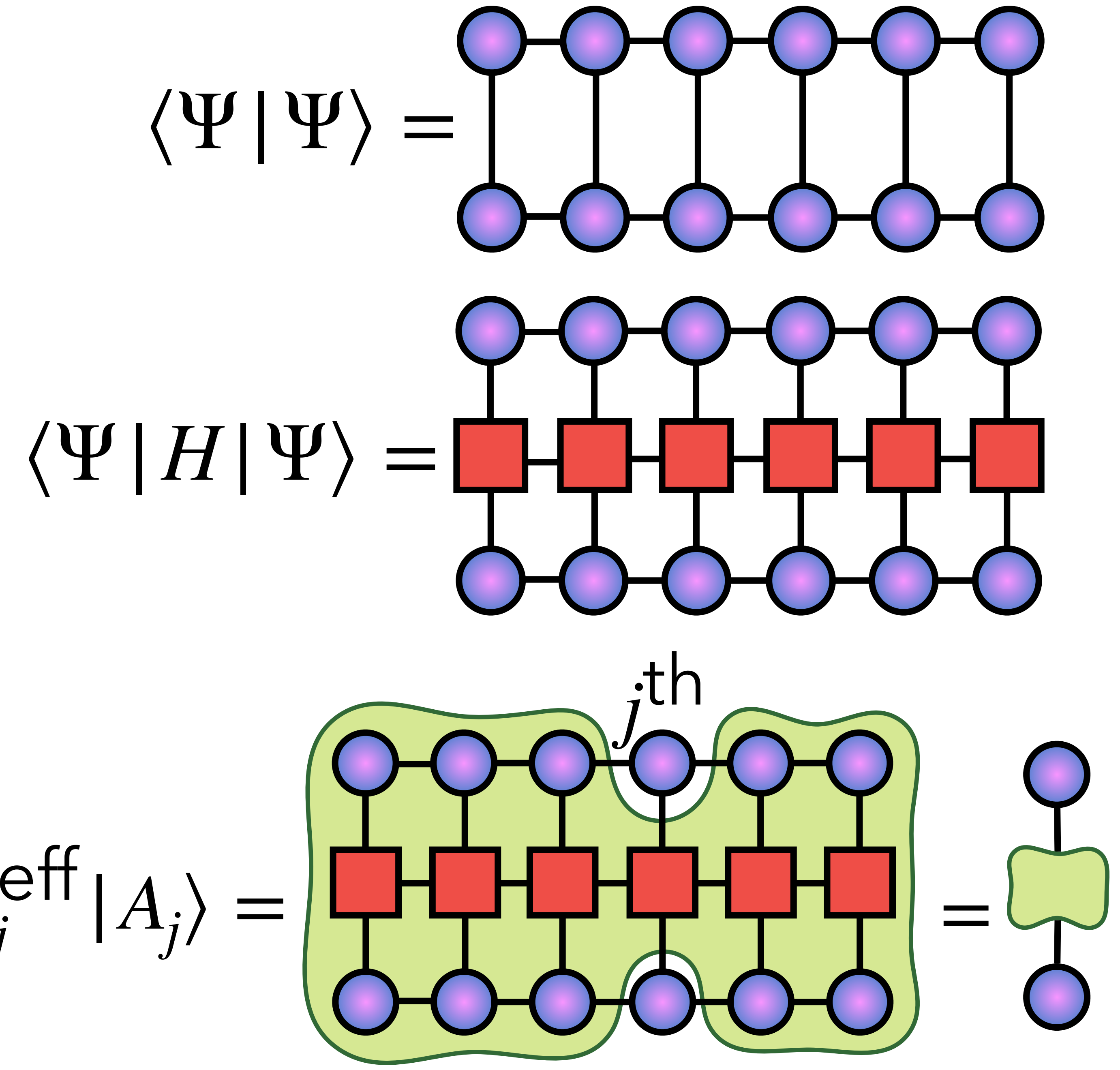
$$\left|\Psi_{\mathrm{QMB}}\right\rangle = \sum_{\alpha_1\ldots\alpha_N} T_{\alpha_1\ldots\alpha_N}\left|\alpha_1\ldots\alpha_N\right\rangle$$





$$\left|\Psi_{\mathrm{MPS}}\right\rangle = \sum_{\alpha_1\ldots\alpha_N} \mathrm{Tr}\left[A_1^{\alpha_1}\ldots A_N^{\alpha_N}\right]\left|\alpha_1\ldots\alpha_N\right\rangle$$

MATRIX PRODUCT STATE

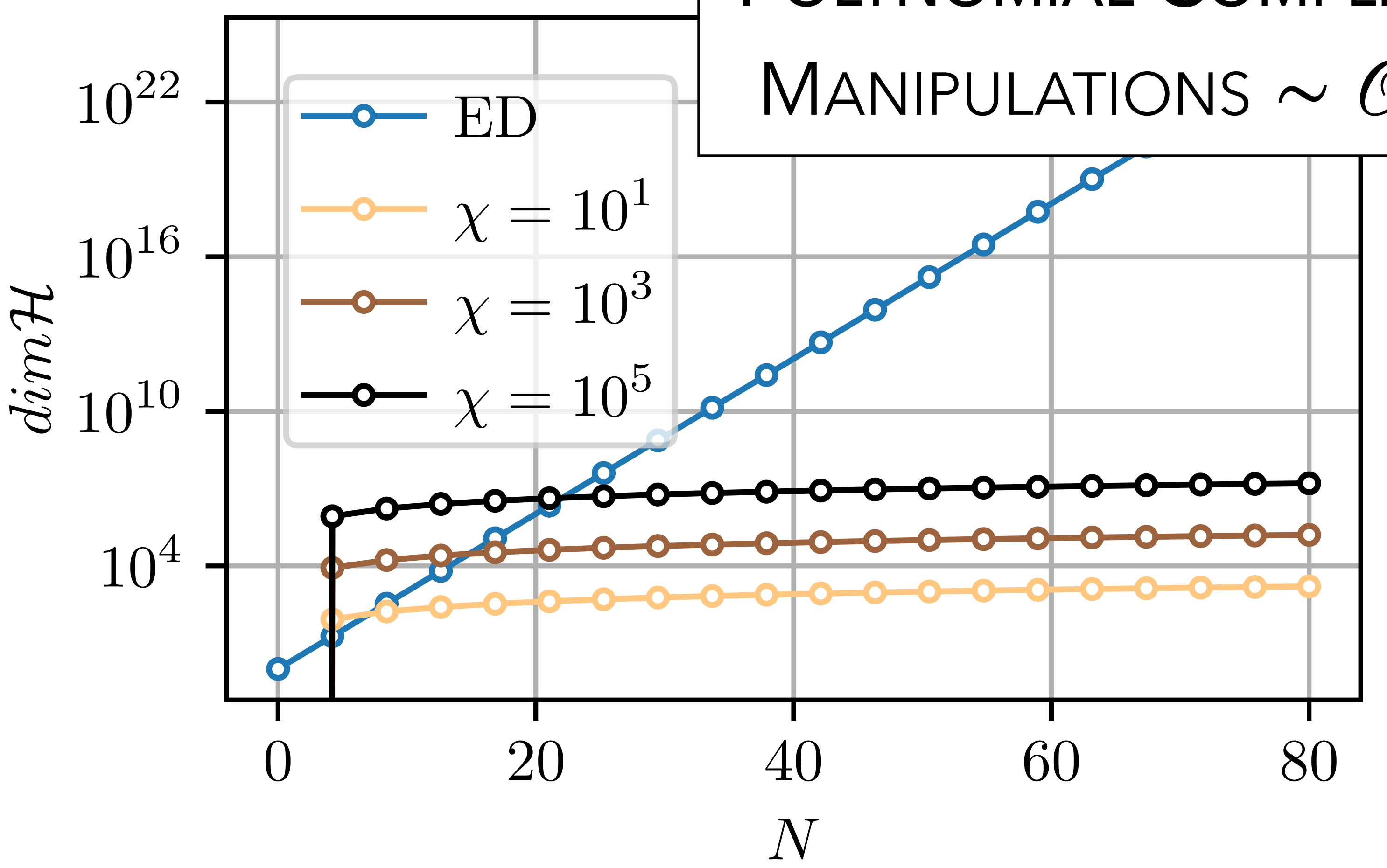


GROUND-STATE SEARCH VARIATIONAL ALGORITHM

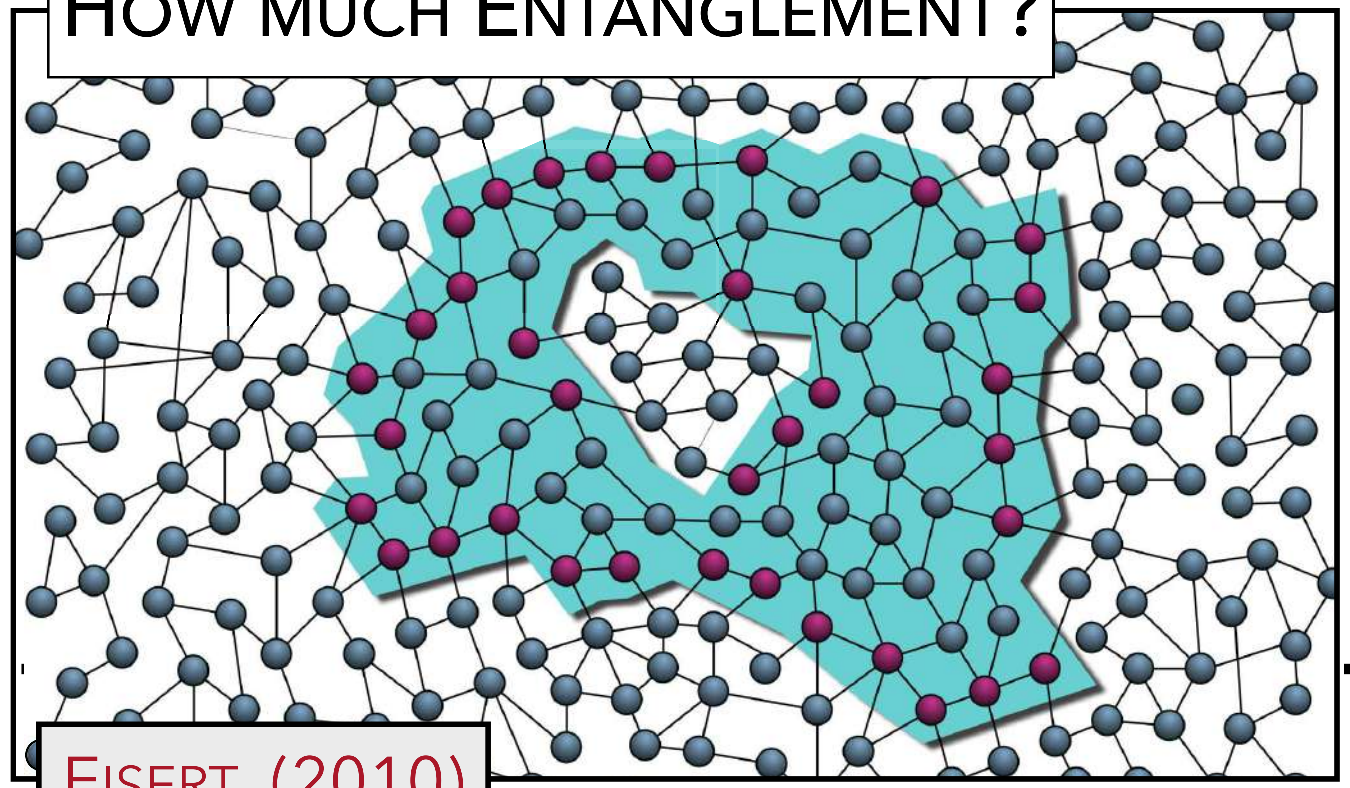
1. Initialize (randomly) $|\Psi\rangle = \{A_1\ldots, A_N\}$
2. Measure $E(0) = \langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle$
3. n^{th}Sweep Sequence: $\forall j \in \{1,\ldots,N\}$, optimize the tensor A_j and update $|\Psi\rangle$
4. Measure $E(n) = \langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle$
5. Compute $\delta E = E(n) - E(n-1)$
6. If $\delta E < \varepsilon$, convergence is reached, otherwise, go back to step 2.

TENSOR NETWORK ANSÄTZE

Polynomial Complexity: Manipulations $\sim \mathcal{O}(\chi^k)$

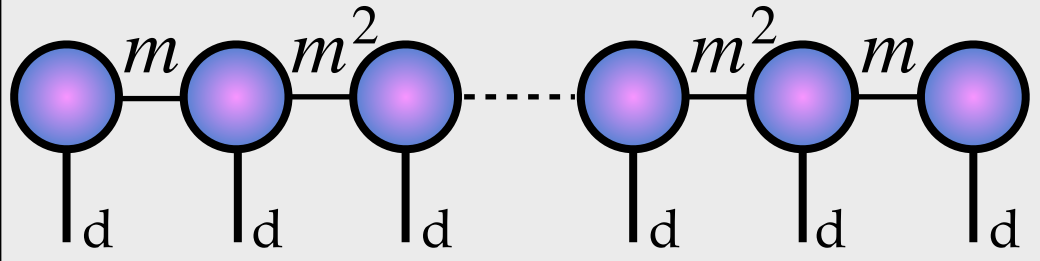


How much Entanglement?



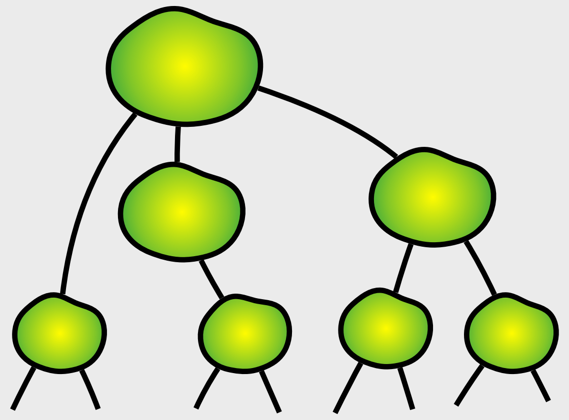
Eisert, (2010)

1D — **Matrix Product States (MPS)**



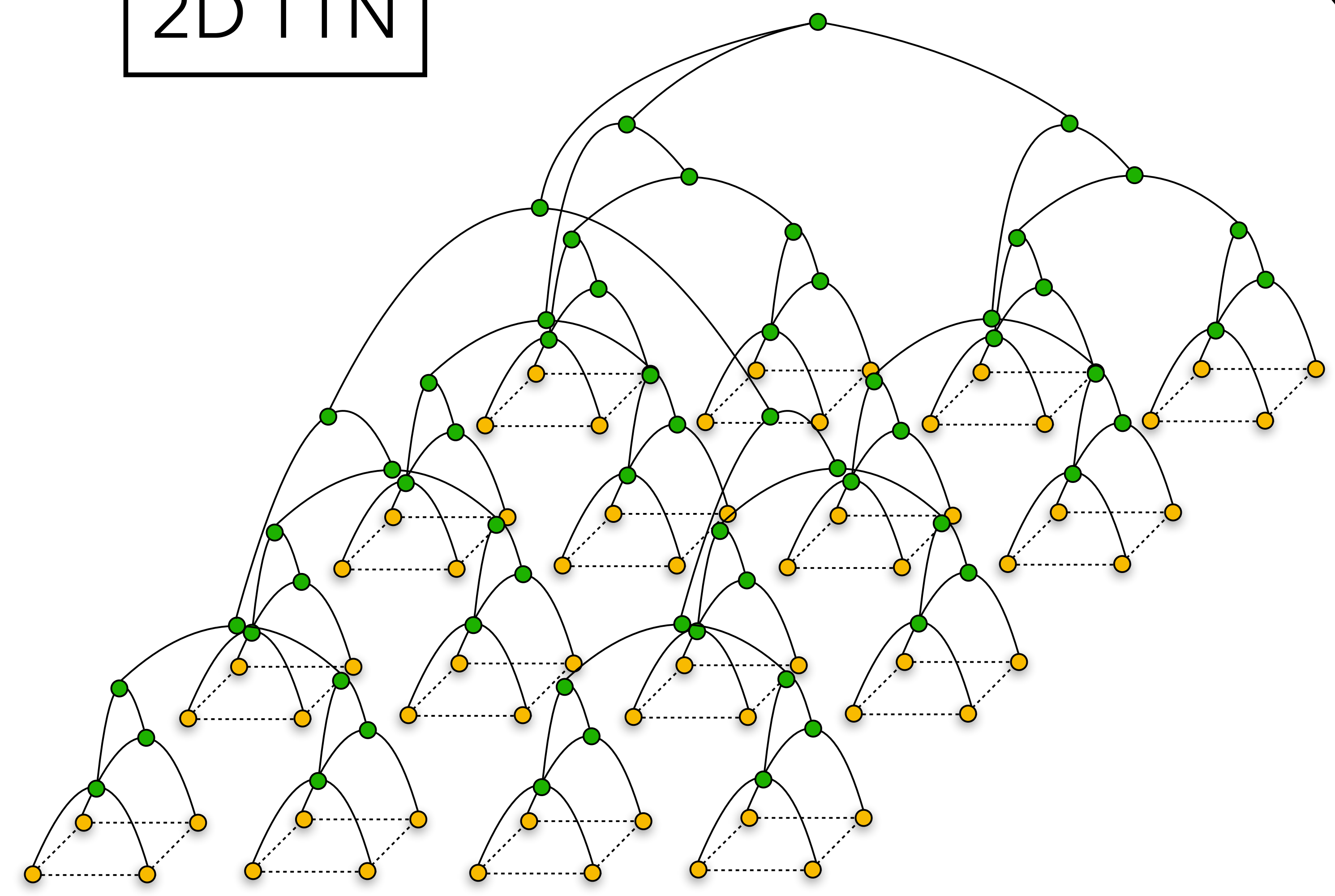
Implementing Area Law
Low Complexity $\sim \mathcal{O}(\chi^3)$

Tree Tensor Networks (TTN)

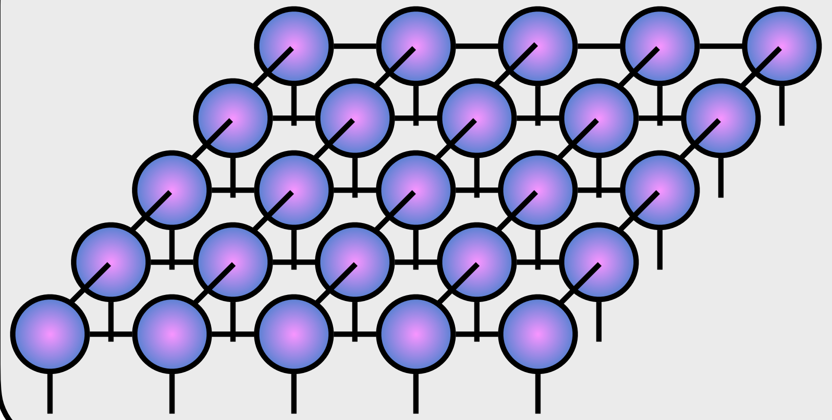


Not implementing
Area Law
Low Algorithmic
Complexity $\sim \mathcal{O}(\chi^4)$

2D TTN



2D — **Projected Entangled Pair States (PEPS)**



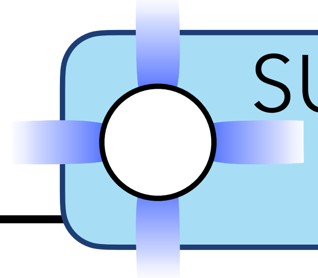
Naturally implementing
Entanglement Area Law
Too High Algorithmic
Complexity $\sim \mathcal{O}(\chi^8)$

SU(2) YM LATTICE HAMILTONIAN

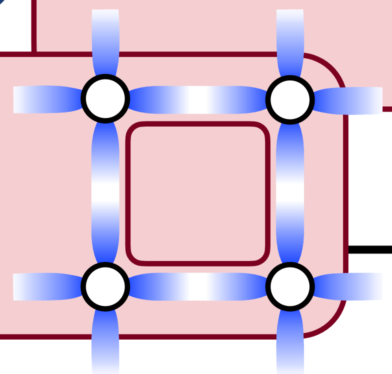
(2+1)D SU(2) YANG MILLS LGT

$$H^{SU(2)}_{LYM} = \frac{ch}{2a}\sum_{\alpha,\beta}\sum_{j,\mu}\left[e^{ik(j,\mu)}\,\psi^{\dagger}_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m_0c^2\sum_j(-1)^{j_x+j_y}\sum_{\alpha}\psi^{\dagger}_{j\alpha}\psi_{j\alpha}$$

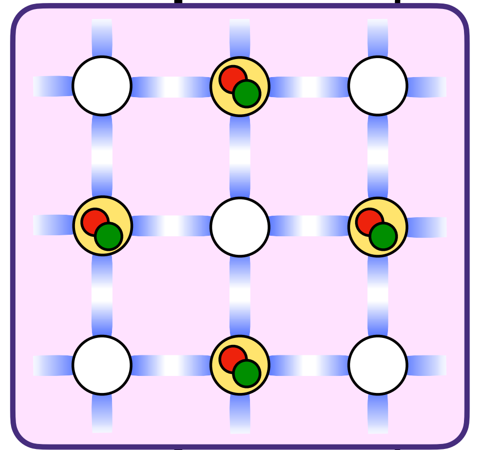
$$+\frac{g^2c\hbar}{2a}\sum_j\left(E^2_{j,j+\mu_x}+E^2_{j,j+\mu_y}\right) - \frac{8c\hbar}{ag^2}\sum_j\sum_{\alpha,\beta,\gamma,\delta}\mathfrak{Re}\begin{pmatrix} & U^{\dagger}_{\gamma\delta} & \\ U^{\dagger}_{\gamma\alpha} & & U_{\beta\gamma} \\ & U_{\alpha\beta} & \end{pmatrix}$$



SU(2) ELECTRIC ENERGY



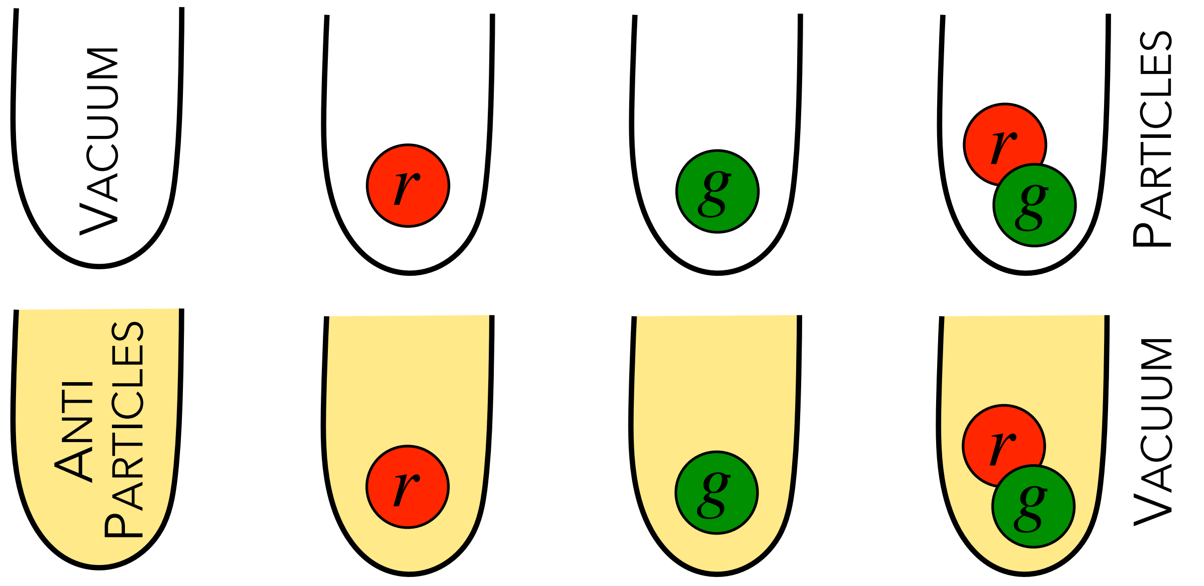
SU(2) MAGNETIC INTERACTION



4d-MATTER HILBERT SPACE $\mathscr{H}_M$
SU(2)-COLOR 1/2 FLAVORLESS DIRAC FERMIONS

QUARKS (ANTIQUARKS-HOLES) r g

EVEN SITES + ODD SITES −



BARE VACUUM

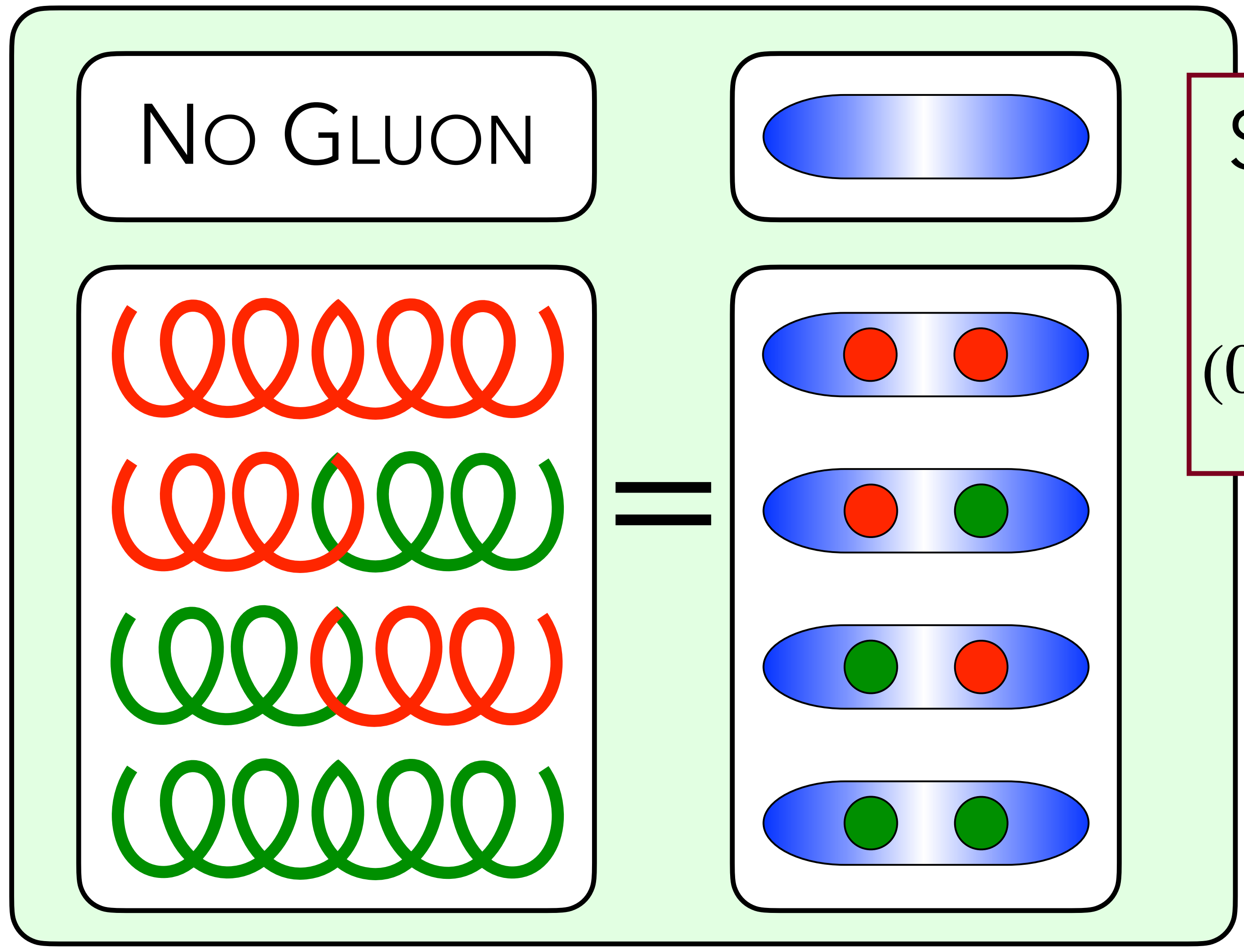
"BARYONS"

"ANTI BARYONS"

QUARK BARE MASS m_0

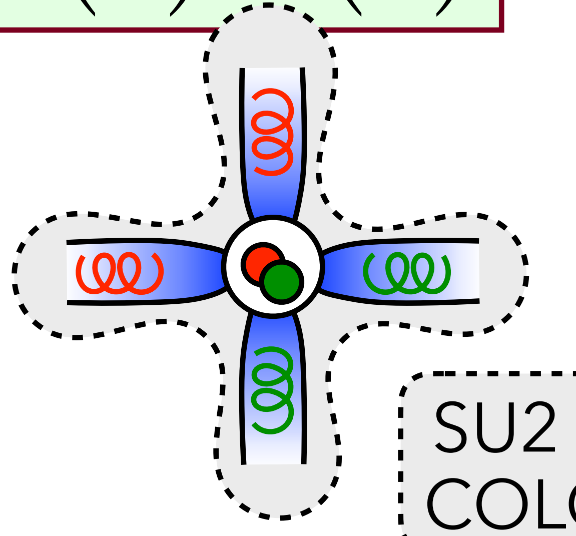
GAUGE COUPLING $g \propto q_c\sqrt{a}$

QUARK CHARGE q_c



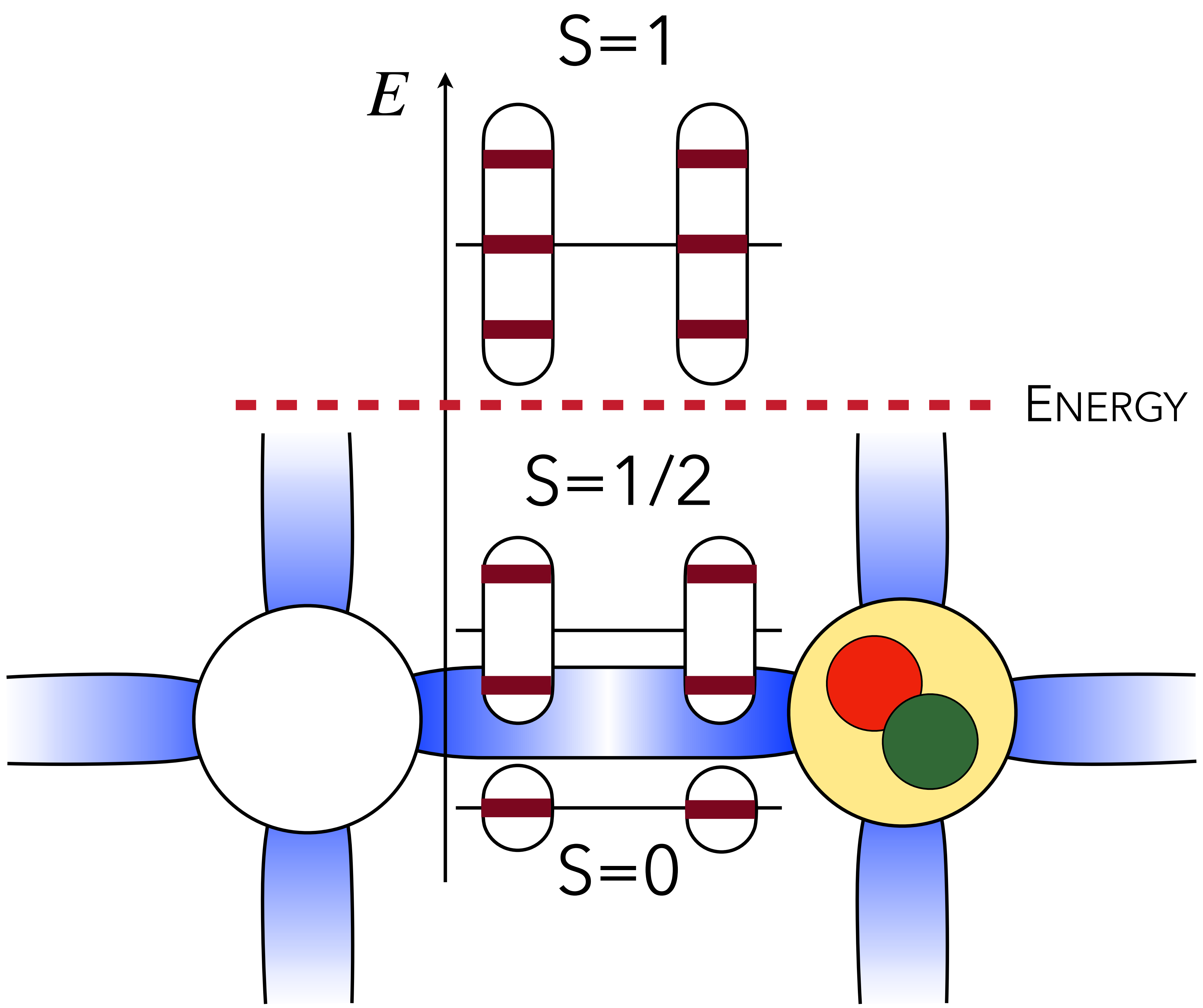
SU(2) Gauge Links (color+anticolor)

$(0)-(0)\oplus\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)$



SU2 GAUSS LAW: COLOR SINGLETS

MINIMALLY TRUNCATED SU(2) FIELDS

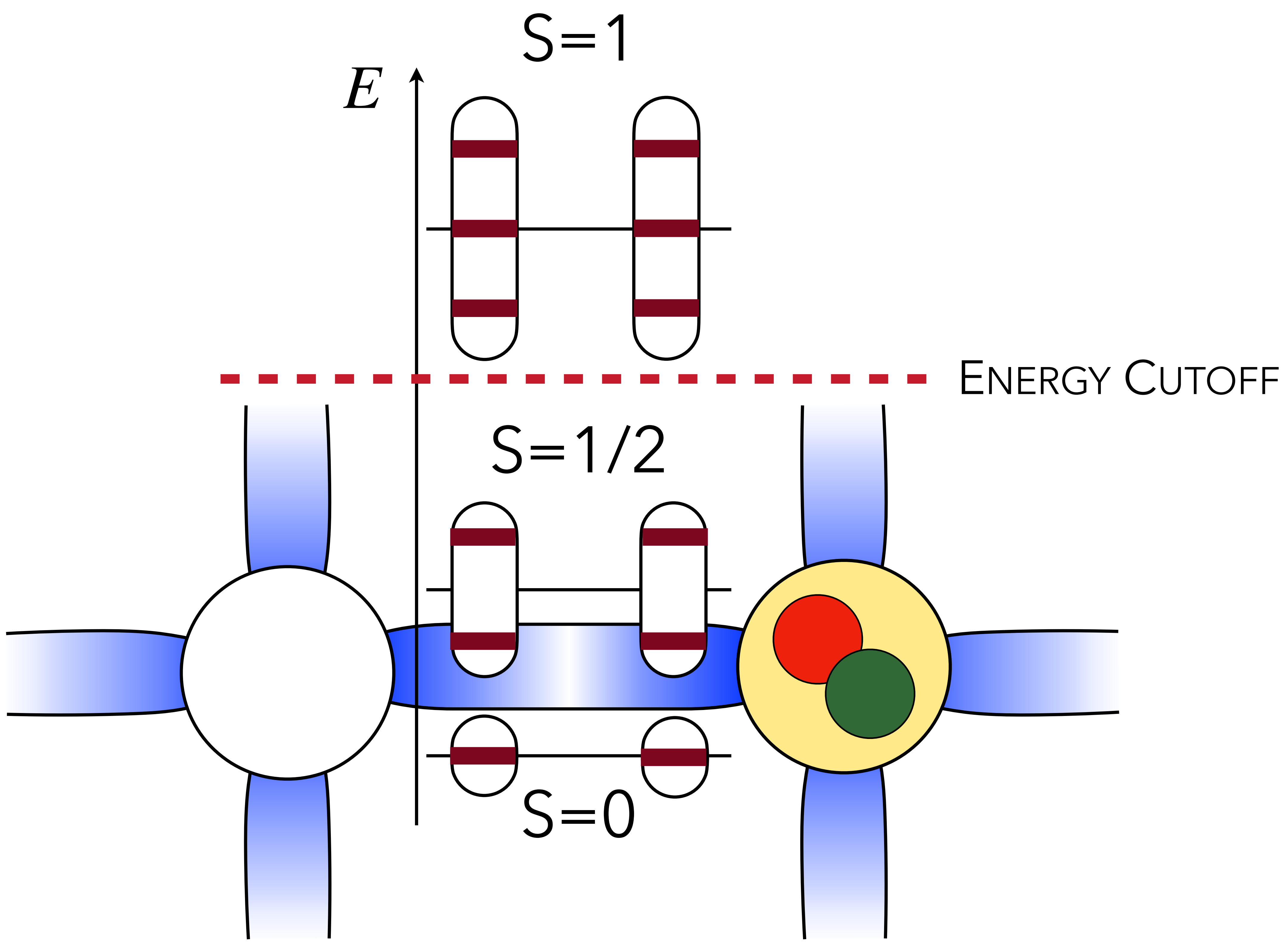


EXPRESS GAUGE FIELD IN "ELECTRICAL BASIS"

$$|j; m_L, m_R\rangle$$

- $j \in \mathbb{N}/2$ COLOR ("SPIN") SHELL
- $m_L, m_R \in \{-j \dots j\}$

MINIMALLY TRUNCATED SU(2) FIELDS



EXPRESS GAUGE FIELD IN "ELECTRICAL BASIS"

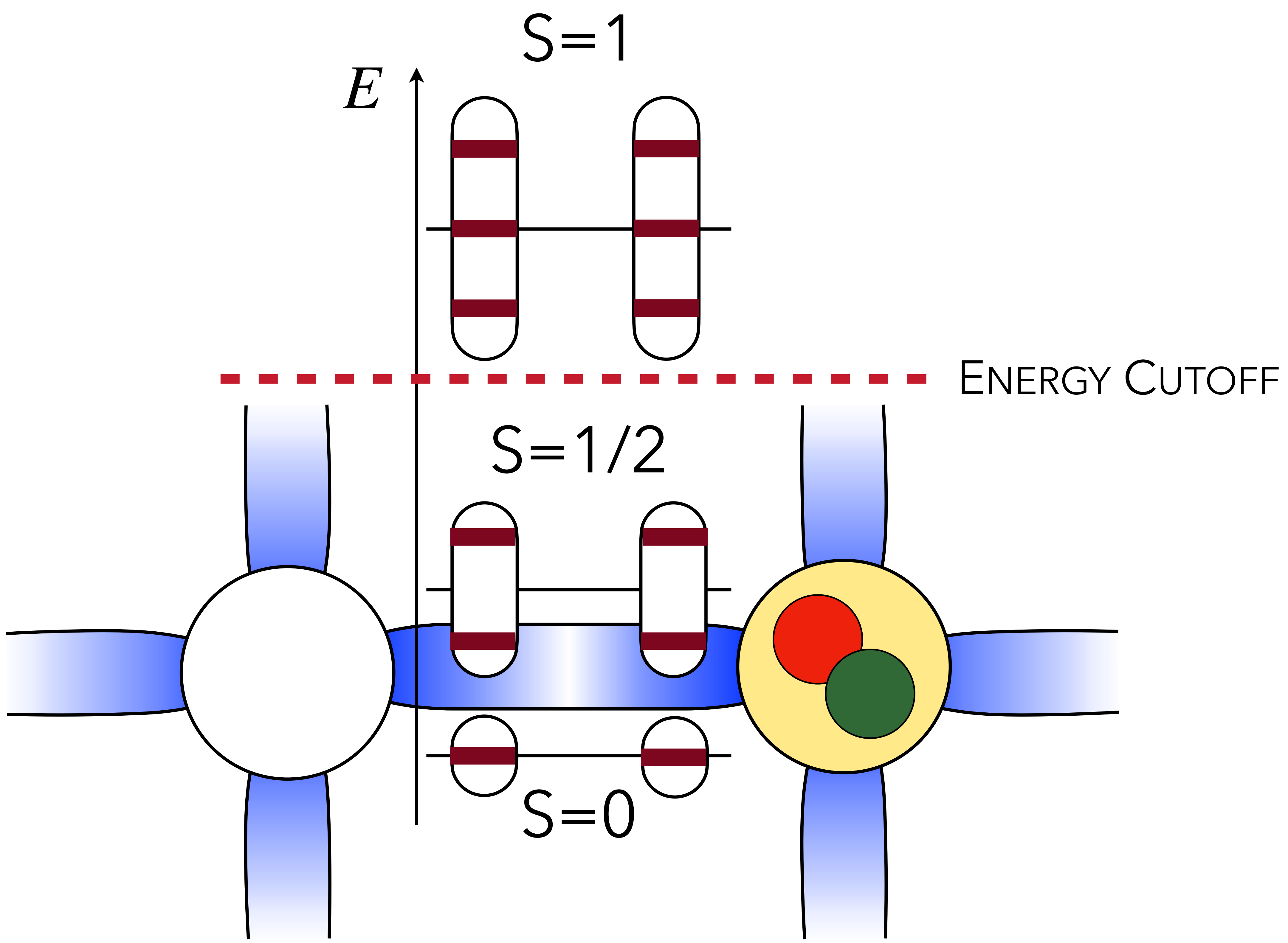
$$|j; m_L, m_R\rangle$$

IN THIS BASIS:

$$E^2|j; m_L, m_R\rangle = j(j+1)|j; m_L, m_R\rangle$$

THE ELECTRIC ENERGY DENSITY IS DIAGONAL
(QUADRATIC CASIMIR)

MINIMALLY TRUNCATED SU(2) FIELDS



EXPRESS GAUGE FIELD IN "ELECTRICAL BASIS"

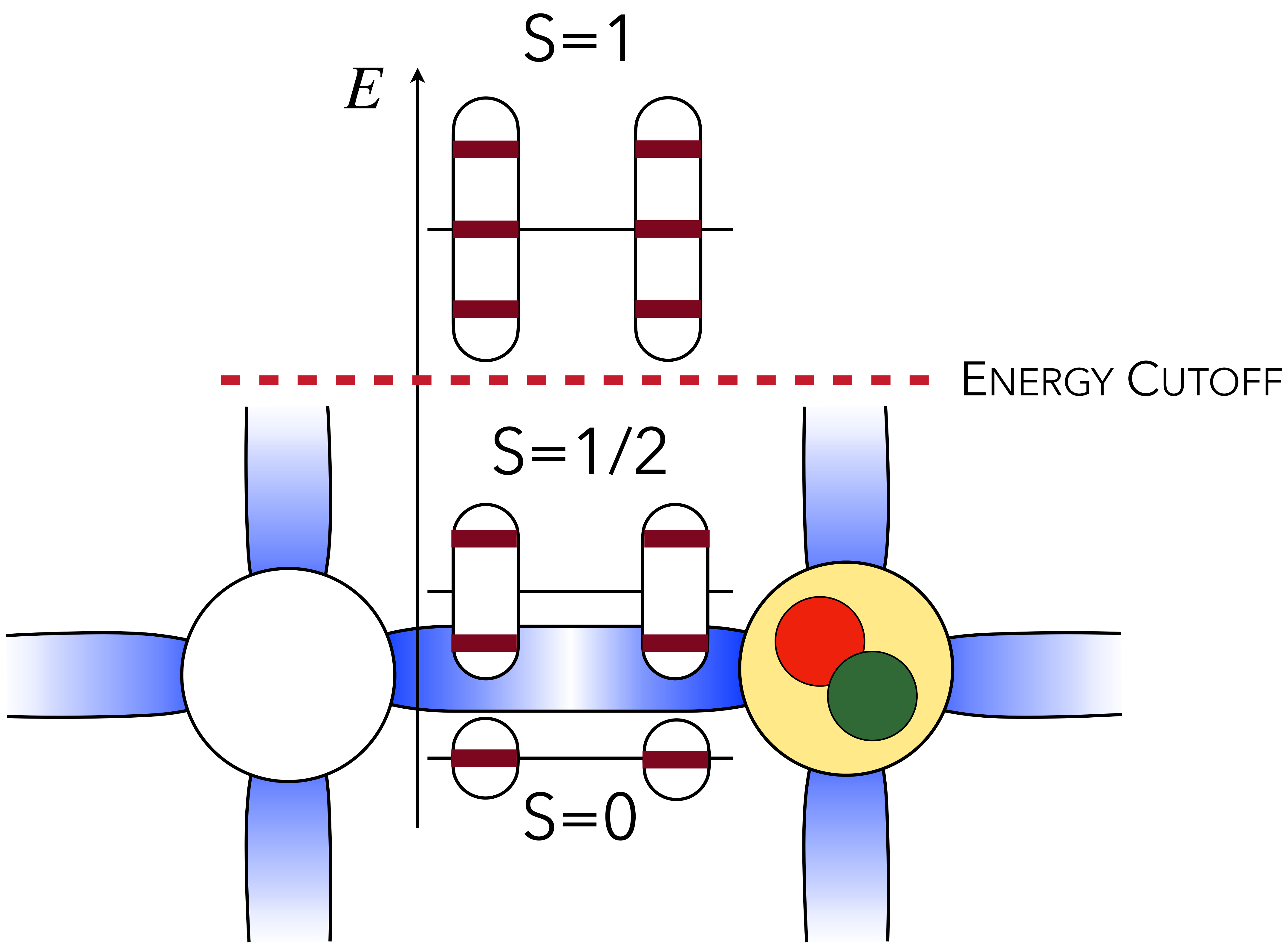
$$|j; m_L, m_R\rangle$$

IN THIS BASIS:

$$U_{\alpha\beta}|j; m_L m_R\rangle = \begin{cases} |j+\frac{1}{2}; \ldots\rangle \\ |j-\frac{1}{2}; \ldots\rangle \end{cases}$$

U RAISES AND LOWERS THE COLOR SHELL

MINIMALLY TRUNCATED SU(2) FIELDS



EXPRESS GAUGE FIELD IN "ELECTRICAL BASIS"

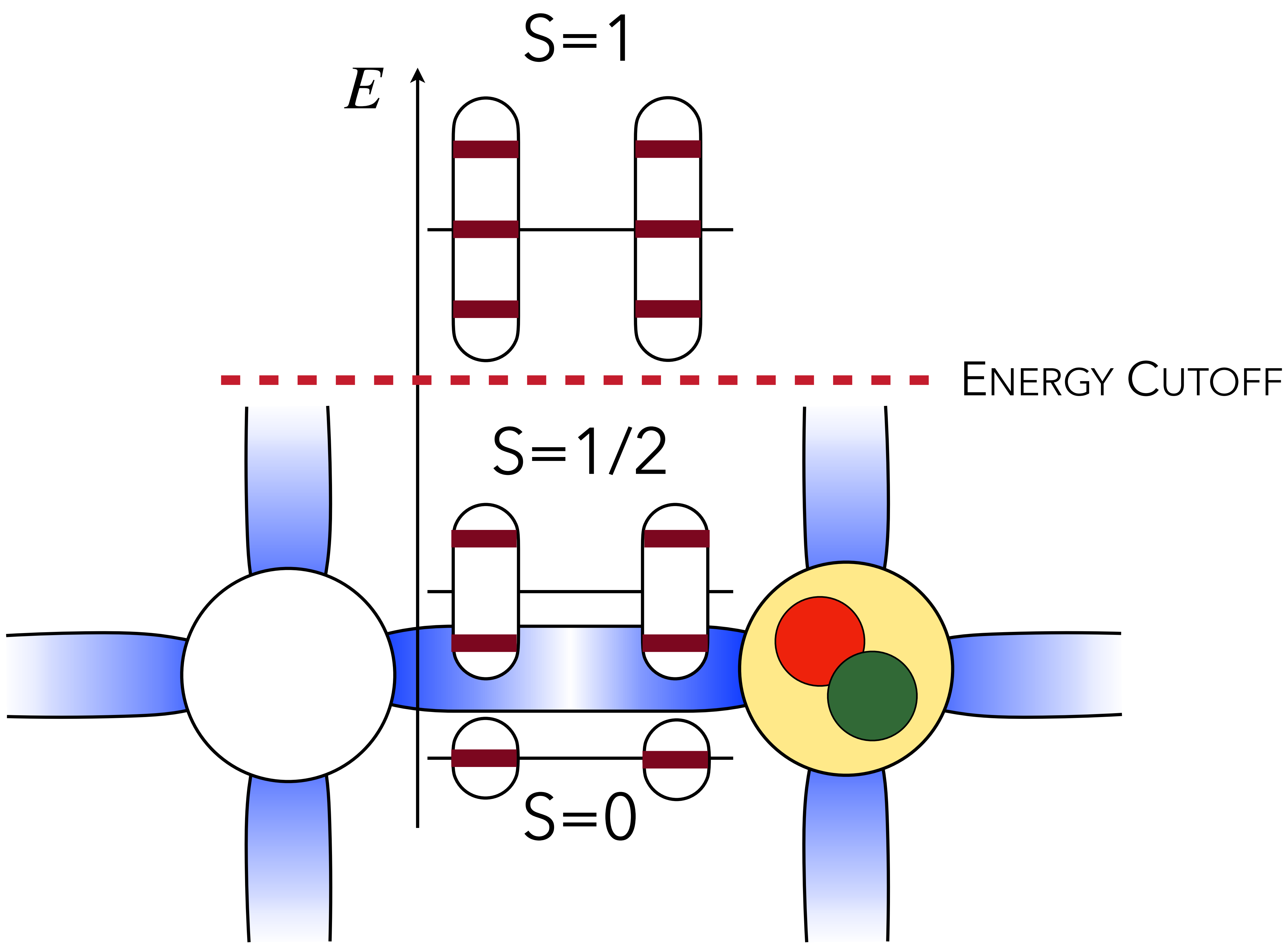
$$|j; m_L, m_R\rangle$$

IN THIS BASIS:

$$\langle j'; m'_L m'_R | U_{\alpha\beta} | j; m_L m_R \rangle =$$

$$\bar{C}^{j,m_L}_{\frac{1}{2},\alpha;j',m'_L} \, C^{j',m'_R}_{j,m_R;\frac{1}{2},\beta}$$

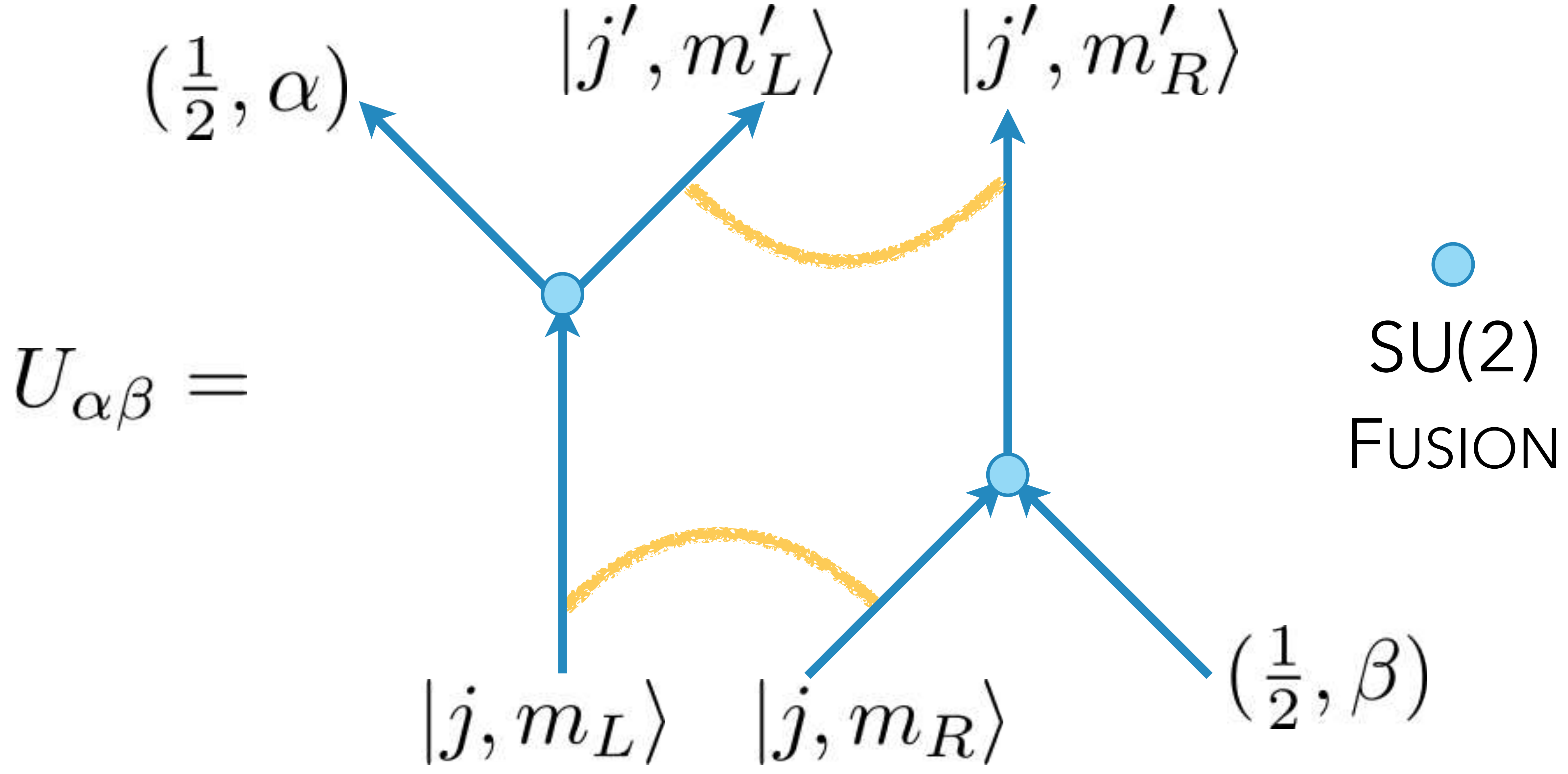
MINIMALLY TRUNCATED SU(2) FIELDS



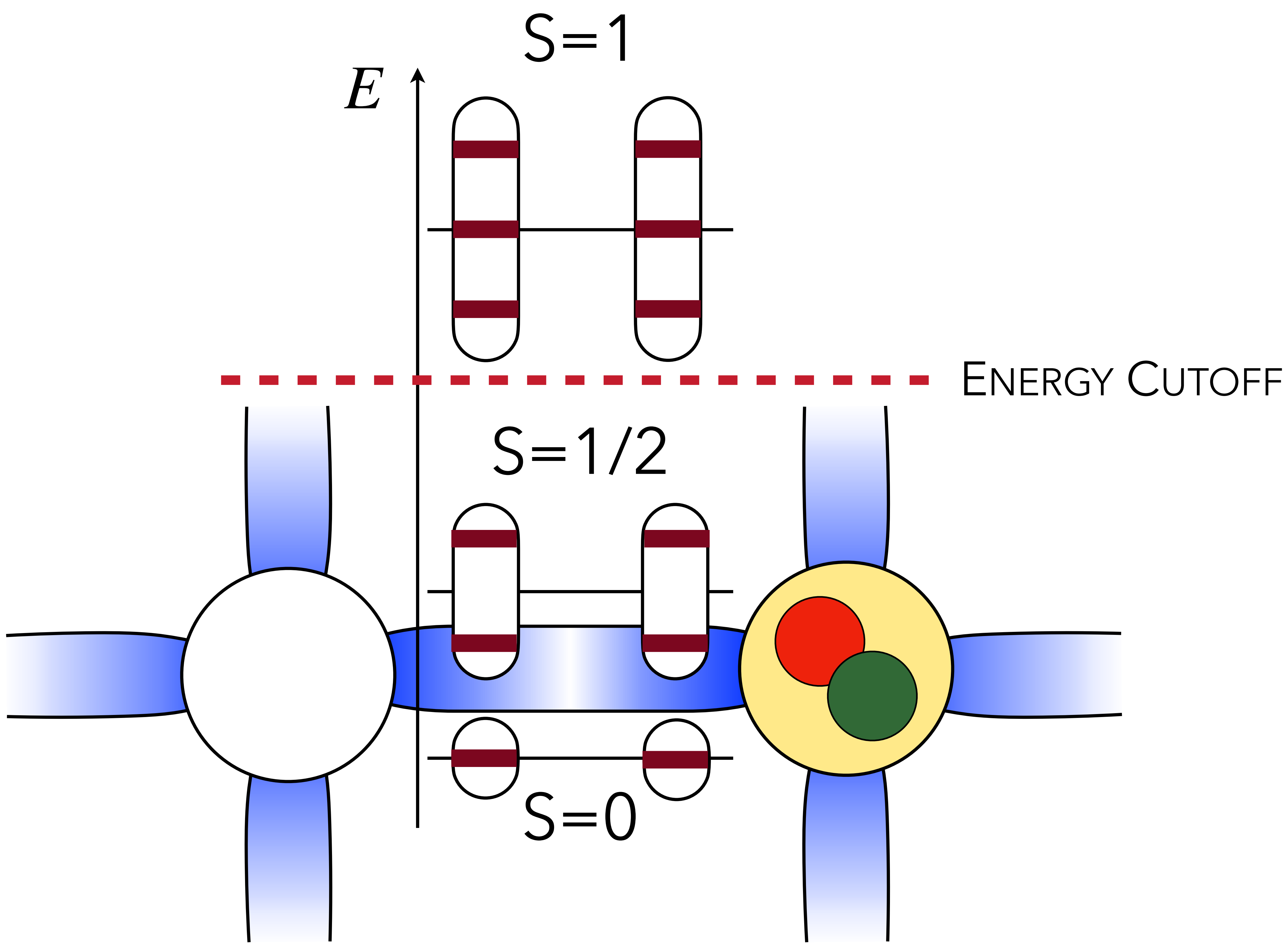
Express Gauge Field in "Electrical Basis"

$$|j; m_L, m_R\rangle$$

In this basis:



MINIMALLY TRUNCATED SU(2) FIELDS



CUTOFF HERE

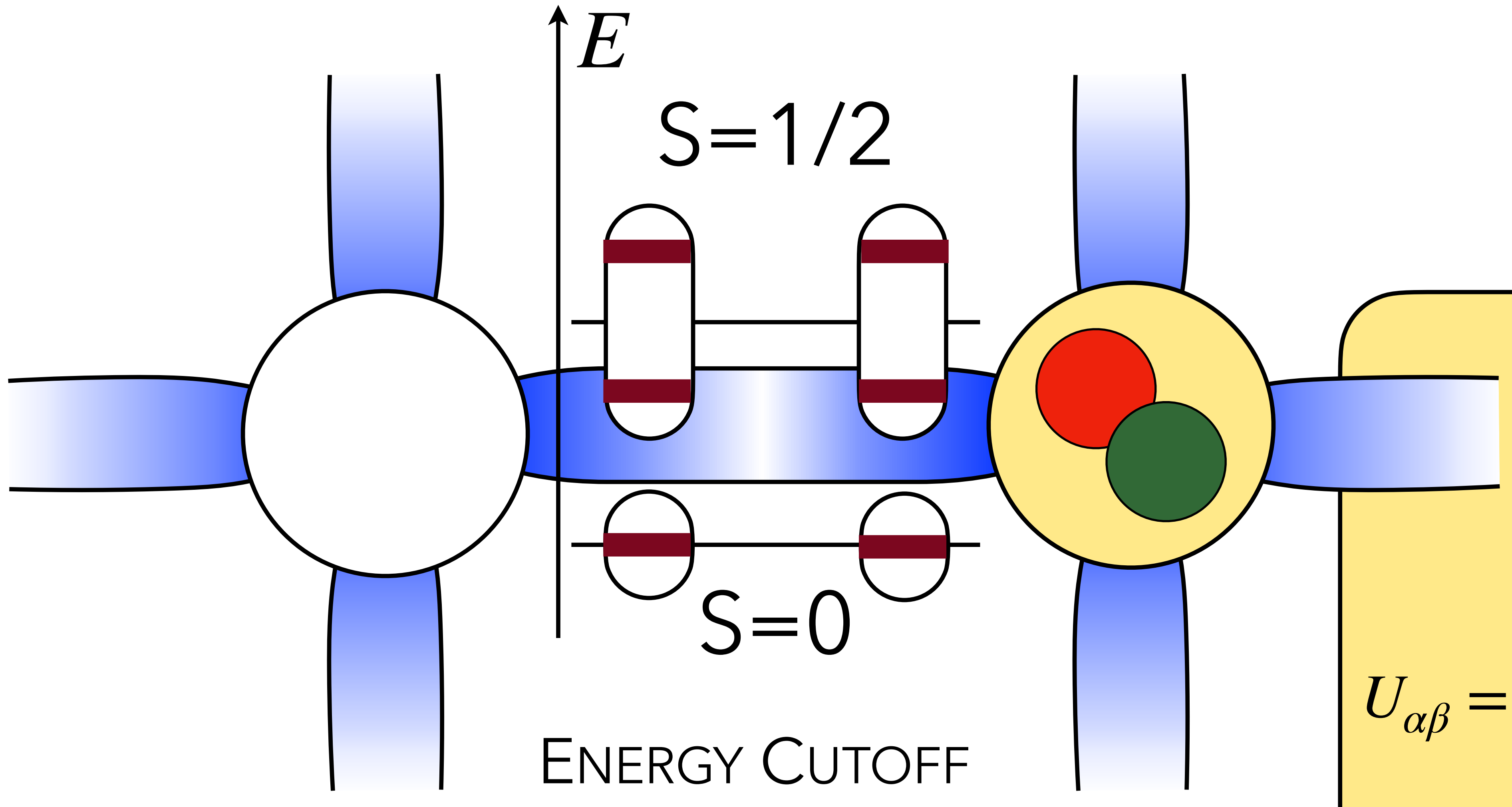
GOOD APPROXIMATION ONLY IN THE STRONG (LATTICE) COUPLING REGIME

$$g \gg 1$$

MINIMALLY TRUNCATED SU(2) LINKS

5dim SU(2) Gauge Link Hilbert space $\mathscr{H}_G$

$\mathscr{H}_{\text{link}} =$ [(0,0)-IRREP: $|0,0\rangle$] [$\left(\frac{1}{2},\frac{1}{2}\right)$-IRREP: $|r,r\rangle$ $|r,g\rangle$ $|g,r\rangle$ $|g,g\rangle$]



SU(2) Gauge Invariance

$$\left[\vec{L}, U_{\alpha\beta}\right] = -\frac{1}{2}\sum_{\gamma=r,g}\vec{\sigma}_{\alpha\gamma}U_{\gamma\beta} \qquad \left[\vec{R}, U_{\alpha\beta}\right] = +\frac{1}{2}\sum_{\gamma=r,g}U_{\alpha\gamma}\vec{\sigma}_{\gamma\beta}$$

LEFT- and RIGHT- generators of SU(2) transformations

$$\vec{L} = \frac{1}{2}\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \vec{\sigma}\otimes\mathbb{1} \end{array}\right) \qquad \vec{R} = \frac{1}{2}\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \mathbb{1}\otimes\vec{\sigma} \end{array}\right)$$

SU(2) Parallel Transport

$$U_{\alpha\beta} = \frac{1}{2}\begin{pmatrix} 0 & -\delta_{g,\alpha}\delta_{r,\beta} & +\delta_{g,\alpha}\delta_{g,\beta} & -\delta_{r,\alpha}\delta_{r,\beta} & +\delta_{r,\alpha}\delta_{g,\beta} \\ +\delta_{r,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ +\delta_{r,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \\ -\delta_{g,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ -\delta_{g,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

SU(2) Casimir Operator

$$E^2_{j,j+\mu} = \frac{3}{4}\left(\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

ZOHAR, BURRELLO (2015)

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F$$

$$\zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

EXOTIC FERMIONS

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

$$P = \left(\begin{array}{c|cc} 1 & & \\ \hline & -1 & \\ & & -1 \end{array}\right)$$

$$\zeta_r = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F$$

$$\zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

LOCALLY: THEY INVERT A PARITY

$$P\zeta_a P = -\zeta_a$$

EXOTIC FERMIONS

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LOCALLY: THEY INVERT A PARITY

$$P\zeta_a P = -\zeta_a$$

GLOBALLY: THEY GET JORDAN-WIGNER STRINGS ATTACHED

$$\zeta_a = \ldots \otimes 1_{j-2} \otimes 1_{j-1} \otimes (\square)_j \otimes P_{j+1} \otimes P_{j+2} \otimes \ldots$$

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

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$$\zeta_{g} = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

THIS MAKES THEM MUTUALLY ANTICOMMUTE

$$\{\zeta_{a,j}, \zeta_{b,j'\neq j}\} = 0$$

GLOBALLY: THEY GET JORDAN-WIGNER STRINGS ATTACHED

$$\zeta_a = \ldots \otimes 1_{j-2} \otimes 1_{j-1} \otimes (\Box)_j \otimes P_{j+1} \otimes P_{j+2} \otimes \ldots$$

SU(2) GAUGE LINKS VIA RISHONS

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$$\zeta_g = \frac{1}{\sqrt{2}}\left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

THREE LOCAL RISHON STATES

- EMPTY (EVEN)
- RED (ODD)
- GREEN (ODD)

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

THE PARALLEL TRANSPORT IN TERMS OF FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta}$$

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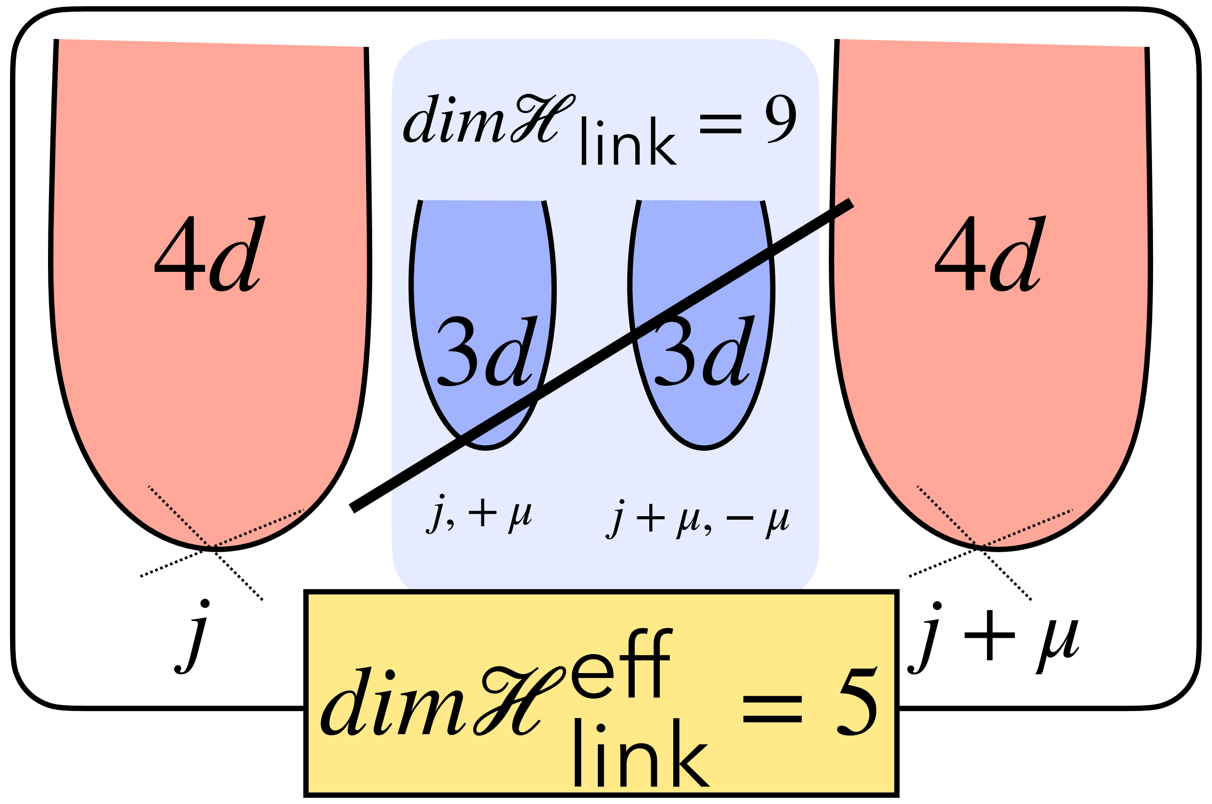
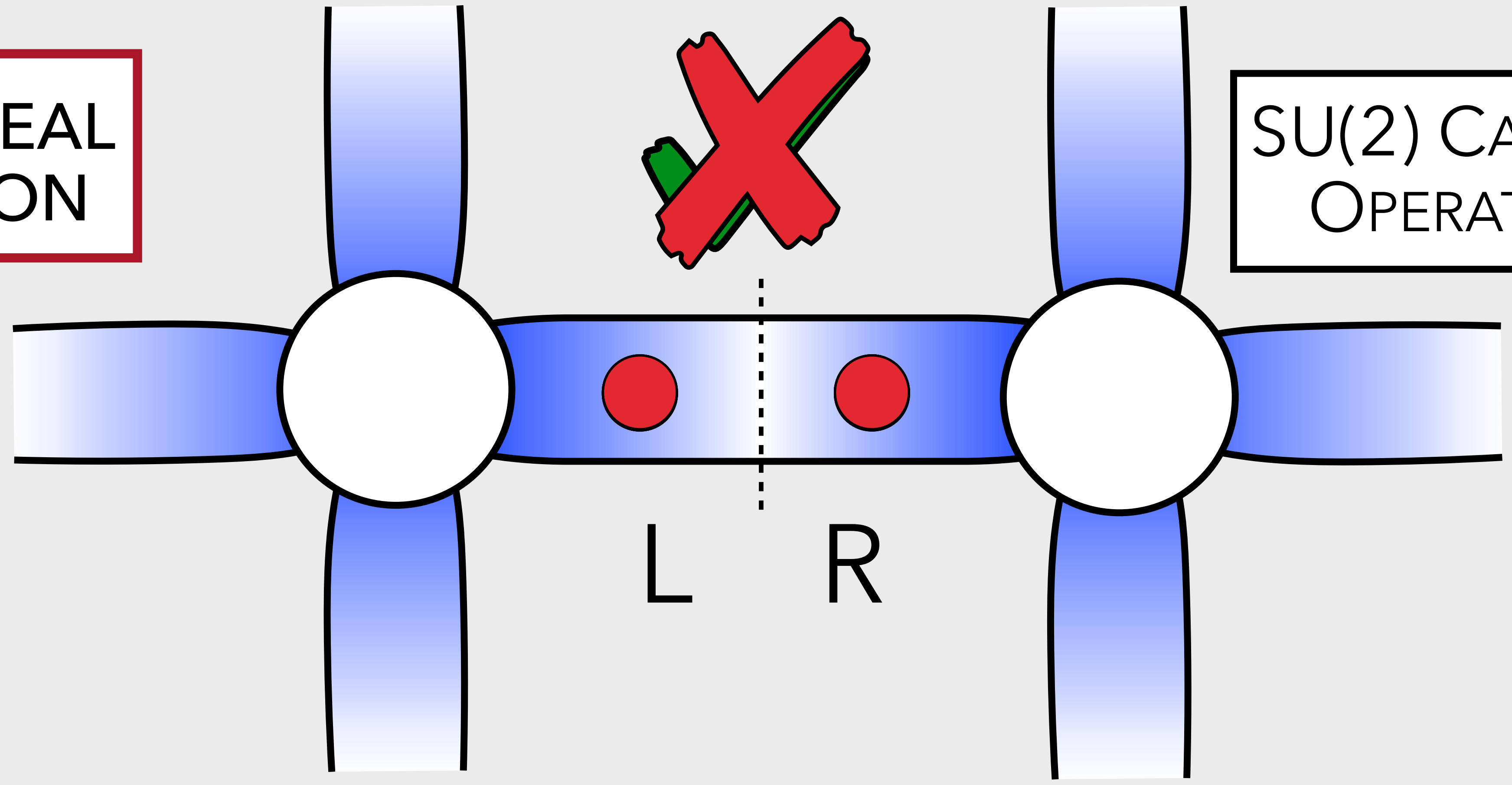


ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$

SU(2) QUASI-REAL REPRESENTATION

SU(2) CASIMIR OPERATOR



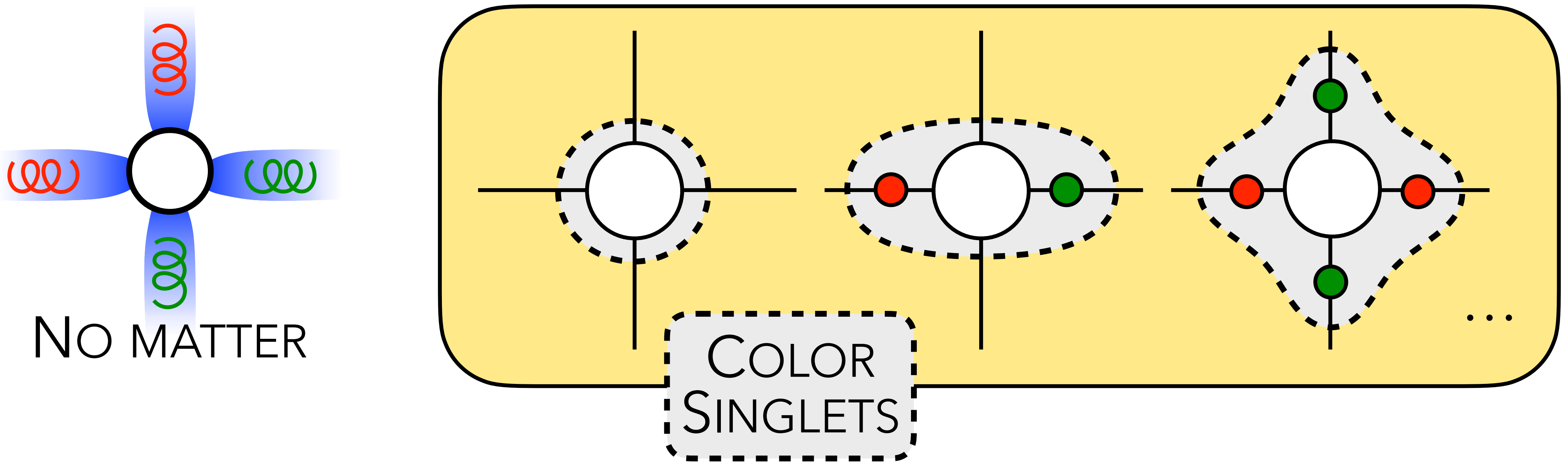
ζ Rishon-based Parallel Transport ✓

$$\zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \frac{1}{2}\begin{pmatrix} 0 & -\delta_{g,\alpha}\delta_{r,\beta} & +\delta_{g,\alpha}\delta_{\downarrow\beta} & -\delta_{r,\alpha}\delta_{r,\beta} & +\delta_{r,\alpha}\delta_{g,\beta} \\ +\delta_{r,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ +\delta_{r,\alpha}\delta_{r,\beta} & & & & \\ -\delta_{g,\alpha}\delta_{g,\beta} & & & & \\ -\delta_{g,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

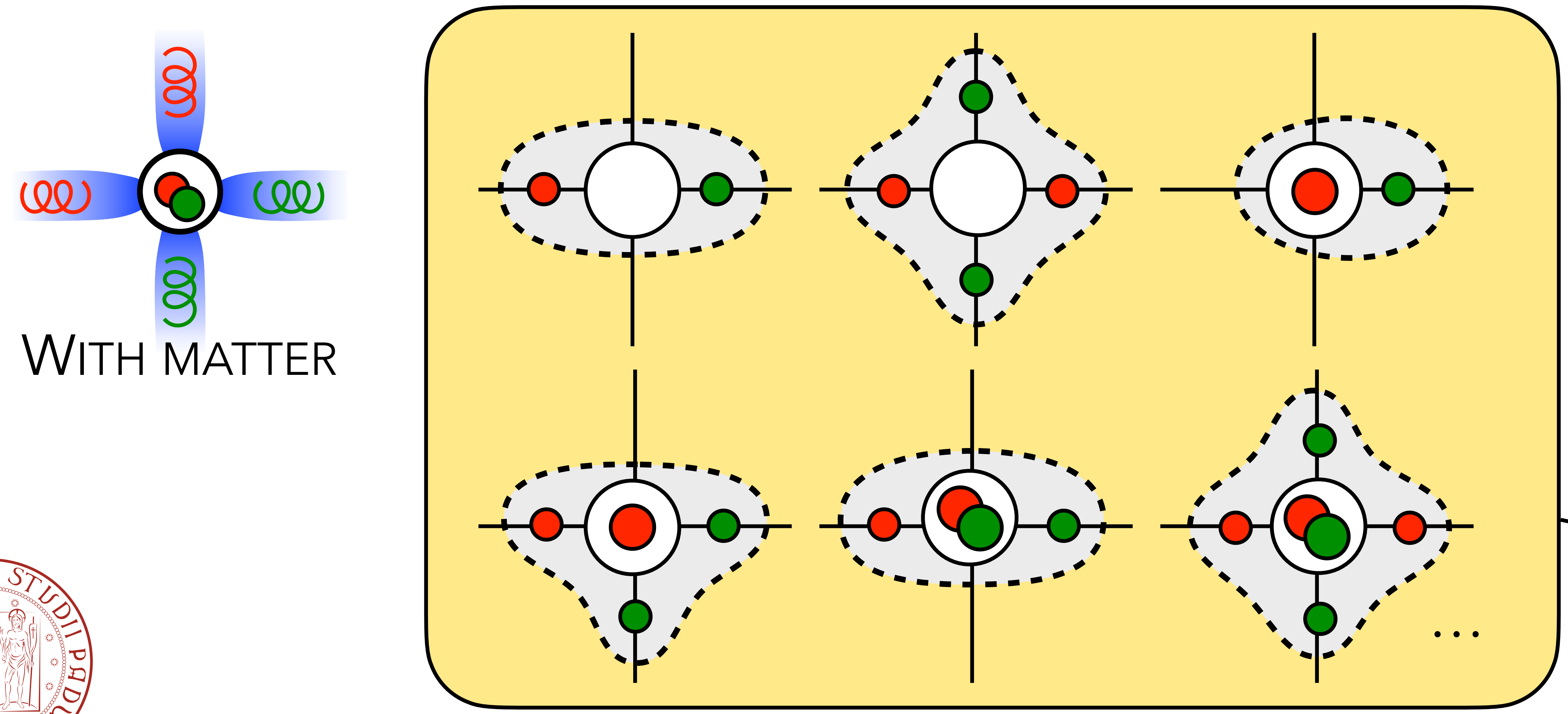
WE CAN GENERALIZE IT TO ANY SPIN-REPRESENTATION!

GAUSS LAW AND DEFERMIONIZATION

Pure Theory: 9 SU(2) Gauge Invariant States

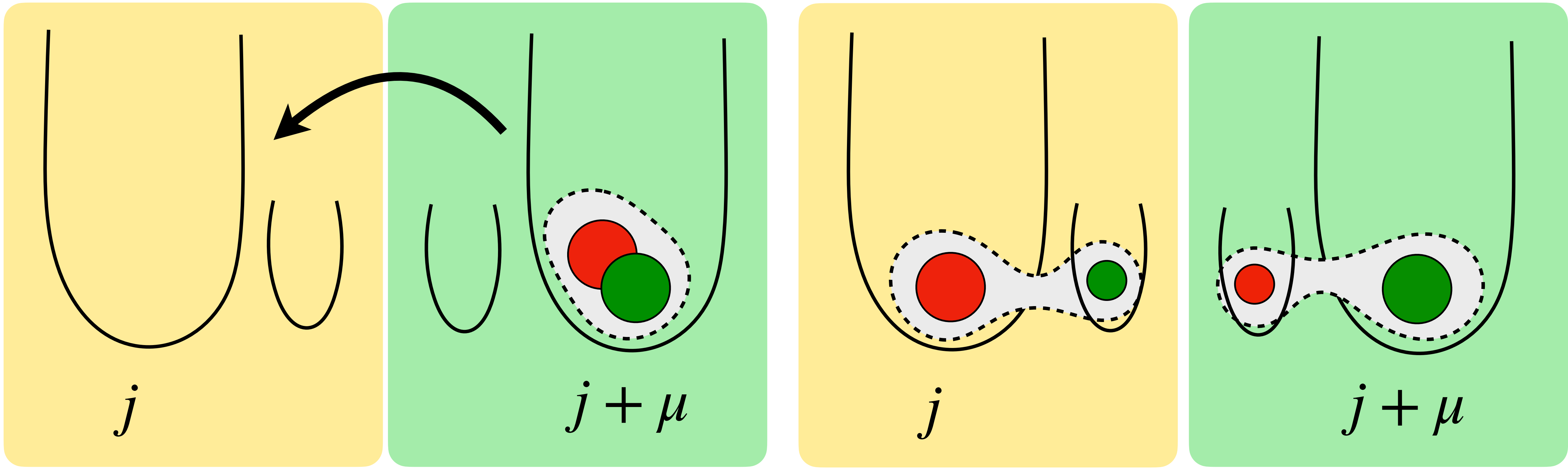


Full Theory: 30 SU(2) Gauge Invariant States

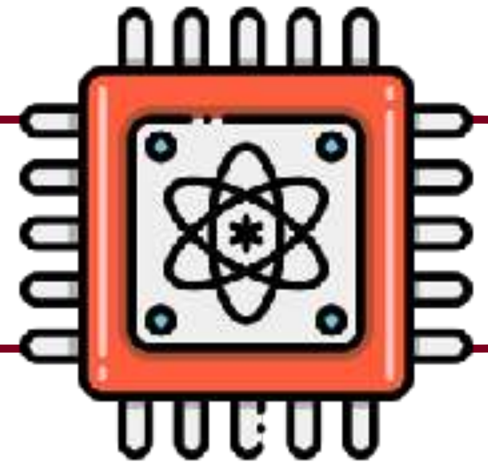


Every Hamiltonian term is made of an **even number of Fermion Operators**: We got a **Bosonic** theory!

$$\psi^{\dagger}_{j,r}\, U^{r,r}_{j,j+\mu}\, \psi_{j+\mu,r} \longrightarrow \psi^{\dagger}_{j,r}\, \zeta^{r}_{j,\mu}\; \zeta^{r\dagger}_{j+\mu,-\mu}\, \psi_{j+\mu,r}$$

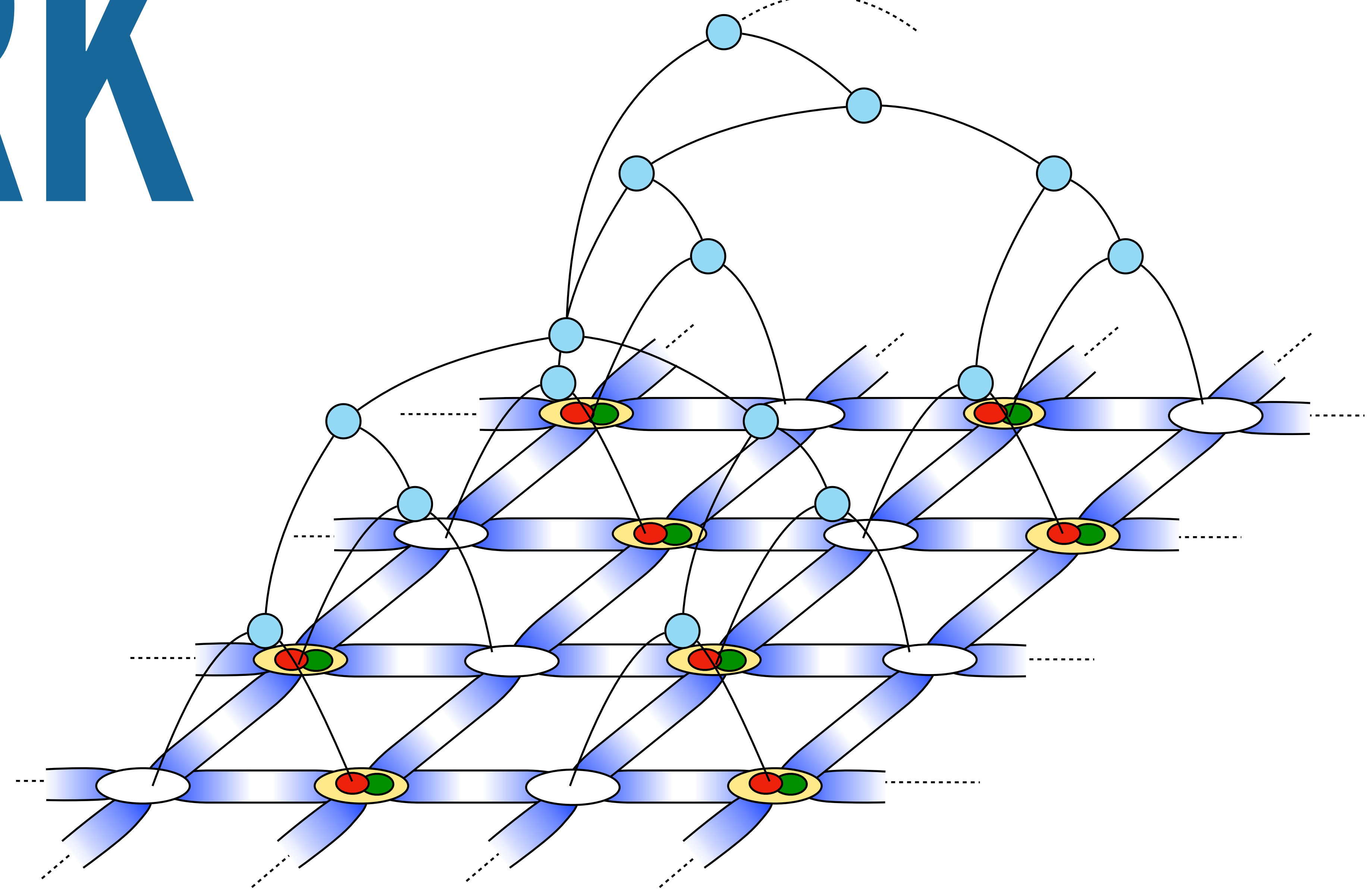


5QuBit Site on a Quantum Computer!



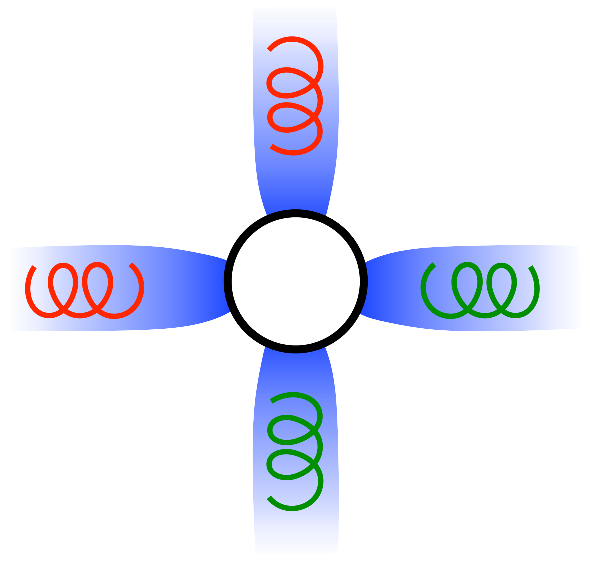
TENSOR NETWORK SIMULATION (2+1)D

$$c = \hbar = 1 \qquad H \longrightarrow H \cdot a$$

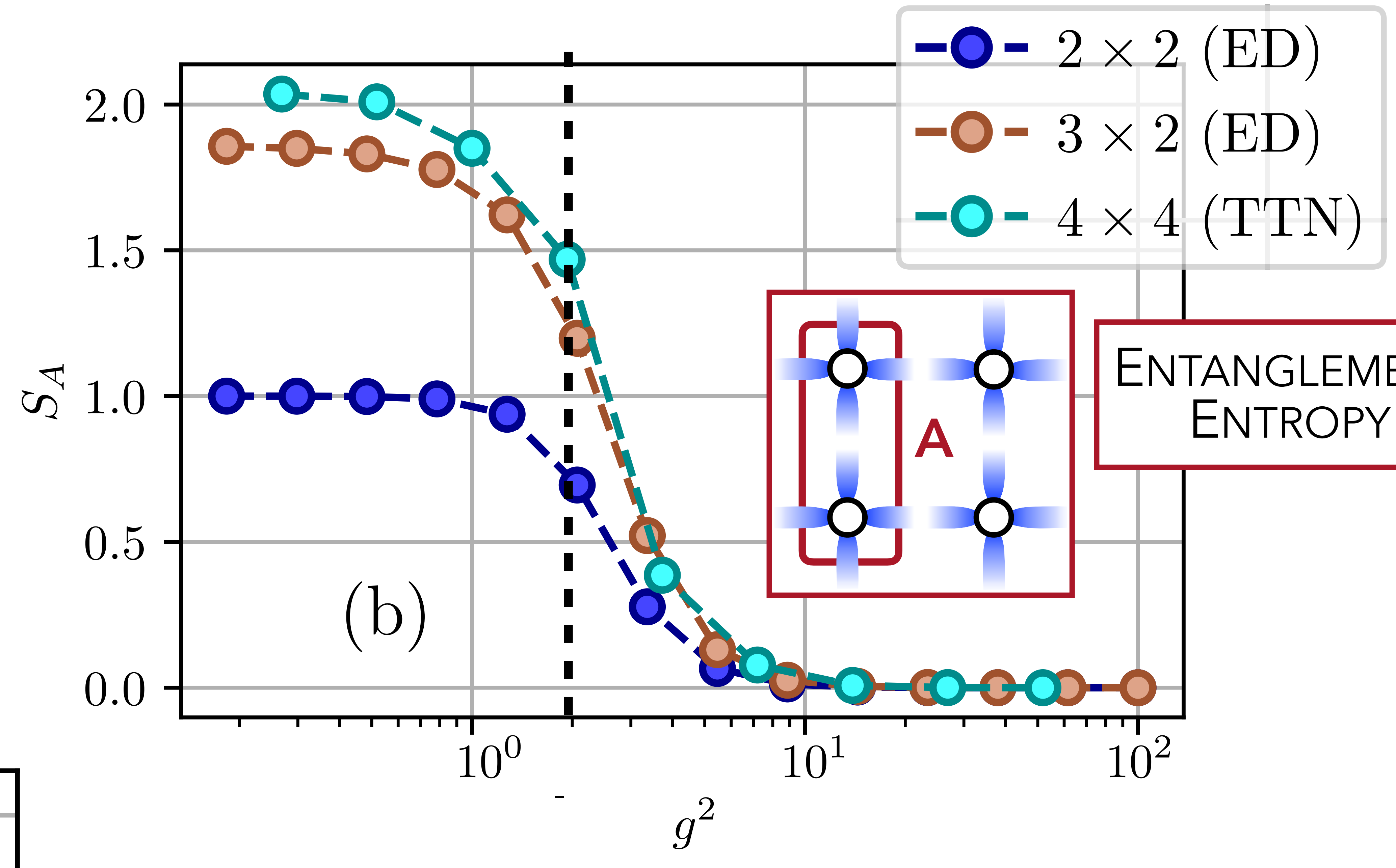


PURE GAUGE THEORY

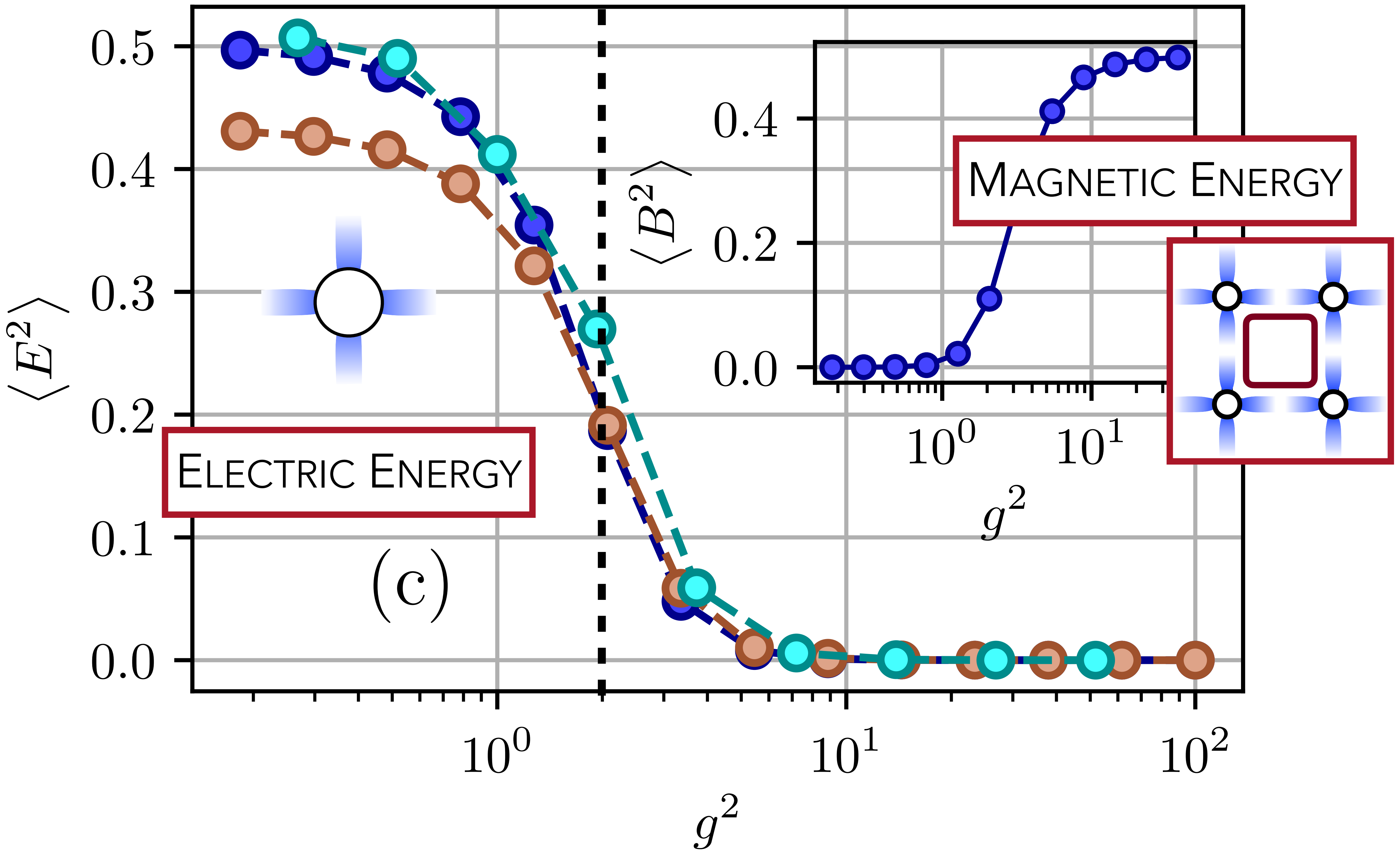
$$H_{\text{Pure}} = \frac{g^2}{2}\sum_{j,\mu} E^2_{j,j+\mu} - \frac{8}{g^2}\sum_{j}\sum_{\alpha,\beta,\gamma,\delta} \Re e \left(\begin{array}{ccc} \ulcorner & U^\dagger_{\gamma\delta} & \urcorner \\ U^\dagger_{\gamma\alpha} & & U_{\beta\gamma} \\ \llcorner & U_{\alpha\beta} & \lrcorner \end{array} \right)$$



NO MATTER



ENTANGLEMENT ENTROPY



ELECTRIC ENERGY

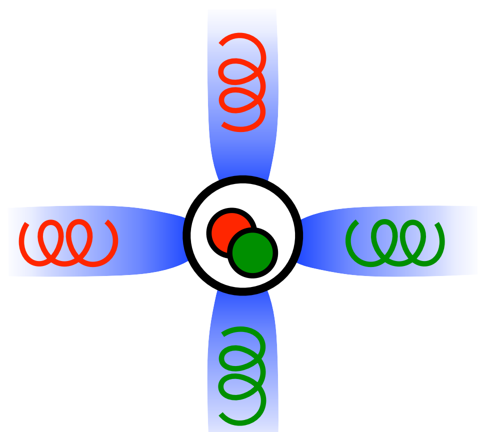
MAGNETIC ENERGY

WHAT IS ABOUT THE g-CROSSOVER/TRANSITION?

ANALOGIES WITH ROUGHENING TRANSITION?

DROUFFE & ZUBER, (1981)

FULL THEORY: ZERO & FINITE BARYON DENSITY

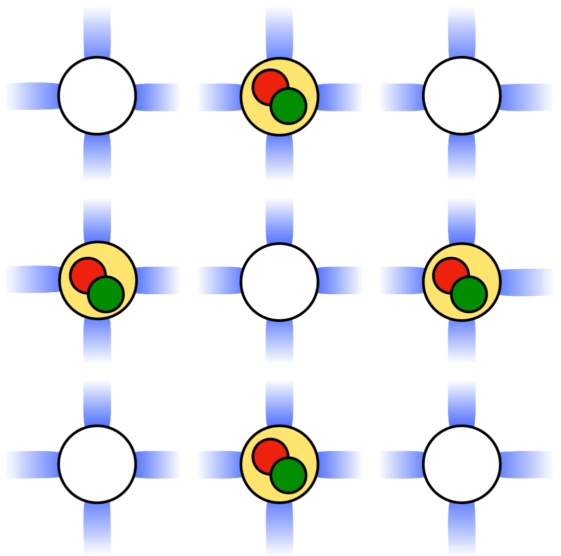


FIXED MASS
$m = 0.1$

STAGGERED FERMIONS

$\oplus$ EVEN

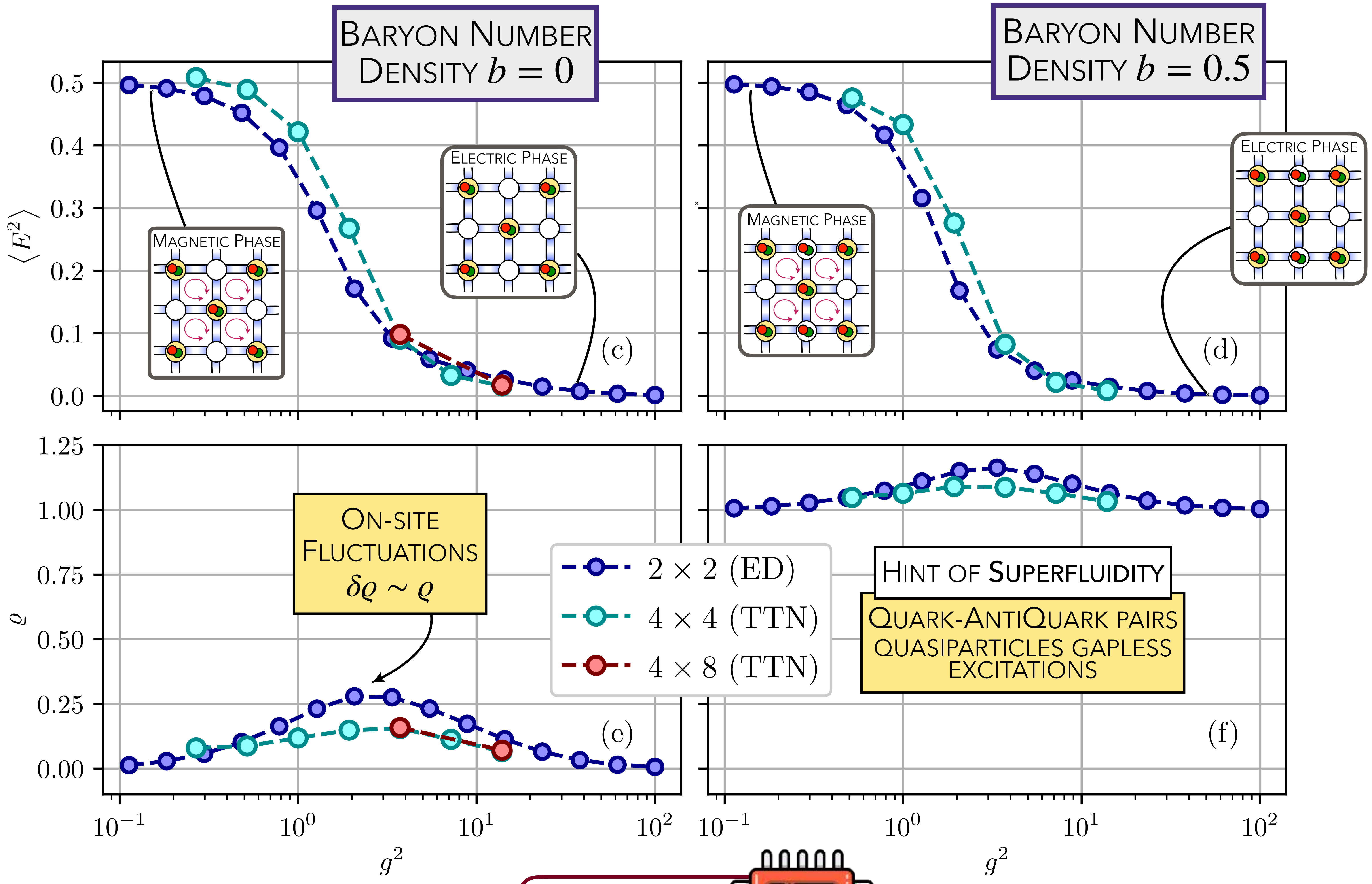
$\ominus$ ODD



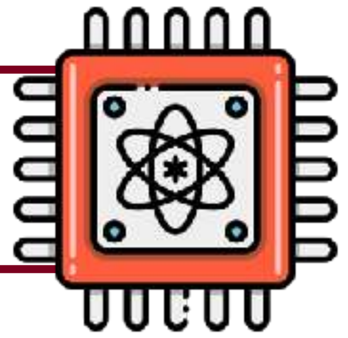
COLOR-ELECTRIC ENERGY DENSITY $\langle E^2 \rangle$

PARTICLE DENSITY (QUARK + ANTIQUARKS)
$\varrho = N_+ + (2 - N_-)$

BARYON NUMBER (QUARKS - ANTIQUARKS)
$b = \frac{1}{2}(N_+ - (2 - N_-))$



MORE THAN 160 QUBITS!



IN THE CONTINUUM LIMIT?

DIMENSIONAL ANALYSIS + SU(2) IN (2+1)D IS SUPER-RENORMALIZABLE

HAMER & IRVING (1985)

DIMENSIONLESS HAMILTONIAN: $(c = \hbar = 1)$ $\quad H \longrightarrow a \cdot H$

$$aH = \frac{1}{2}\sum_{\alpha,\beta}\sum_{j,\mu}\left[e^{ik(j,\mu)}\,\psi^\dagger_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m_0 a\sum_j(-1)^j\sum_\alpha \psi^\dagger_{j\alpha}\psi_{j\alpha}$$

$$+\frac{g^2}{2}\sum_{j,\mu}E^2_{j,j+\mu} - \frac{8}{g^2}\sum_j\sum_{\alpha,\beta,\gamma,\delta}\Re e\begin{pmatrix} & U^\dagger_{\gamma\delta} & \\ U^\dagger_{\gamma\alpha} & & U_{\beta\gamma} \\ & U_{\alpha\beta} & \end{pmatrix}$$

PARAMETERS
$m = m_0\, a$
$g^2 \propto q_c^2\, a$

CONTINUUM LIMIT: $g^2 = \alpha_c m \longrightarrow 0$

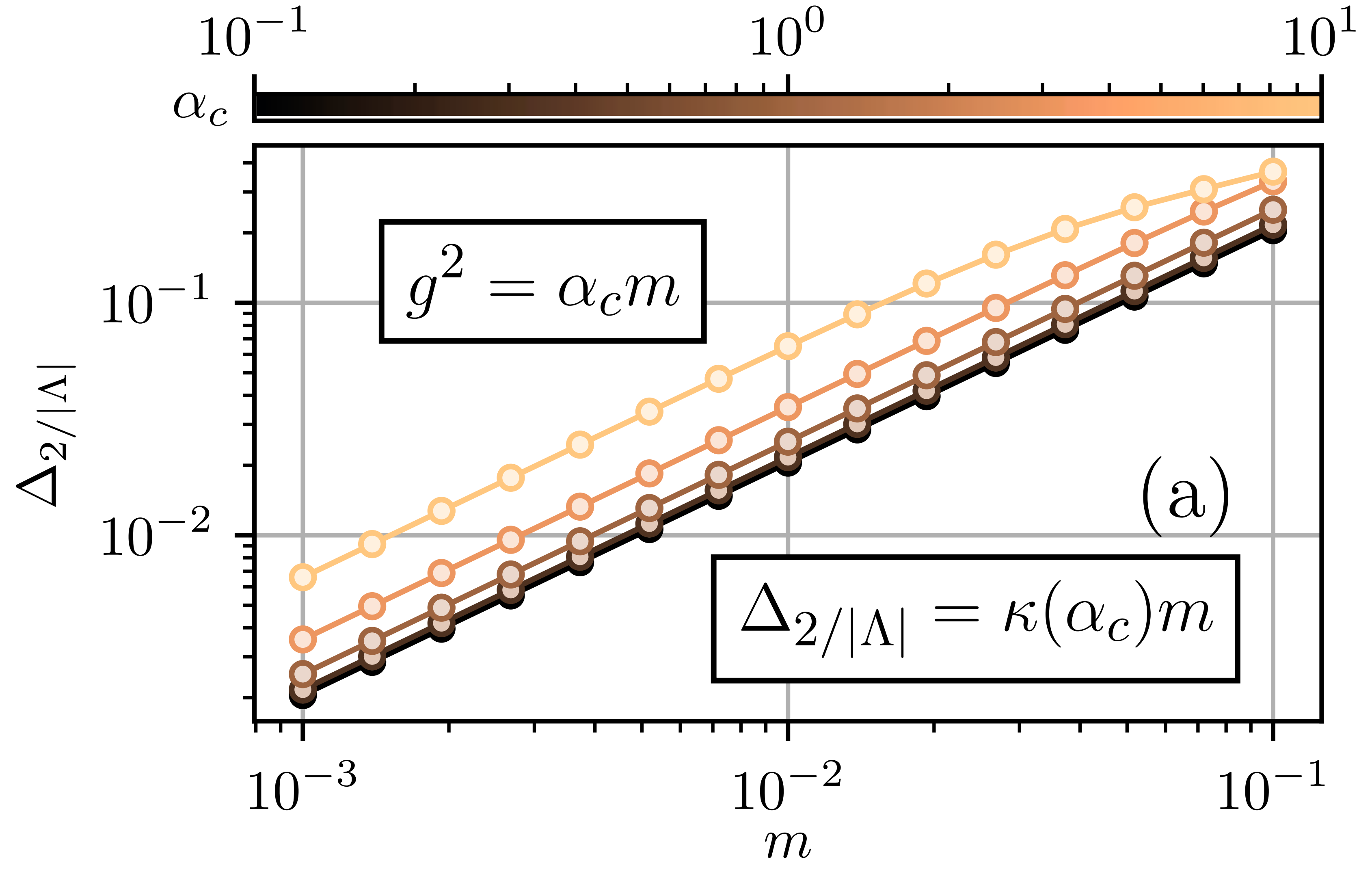
DIMENSIONLESS QUARK RATIO
$\alpha_c = g^2/m \propto q_c^2/m_0$
(INDEPENDENT OF THE SPACING a)

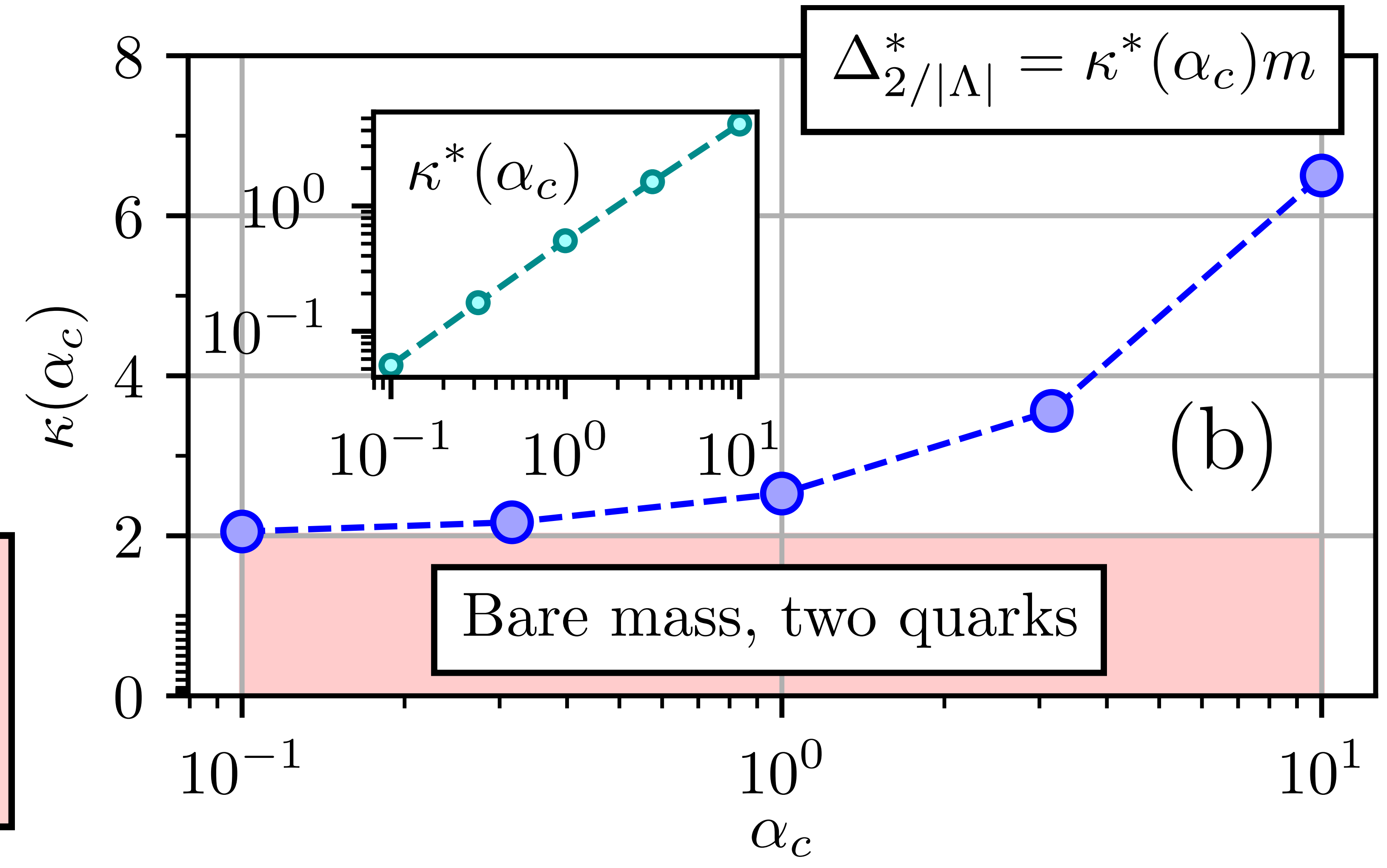
EFFECTIVE BARYON MASS
$m_b = |\Lambda|\, k(\alpha_c) m_0$
GREATER THAN THE MASS OF 2 QUARKS

INTERSECTOR GAP
$\Delta_{|b|} = \varepsilon_b - \varepsilon_0 = m|b| + \Delta^*_{|b|}$

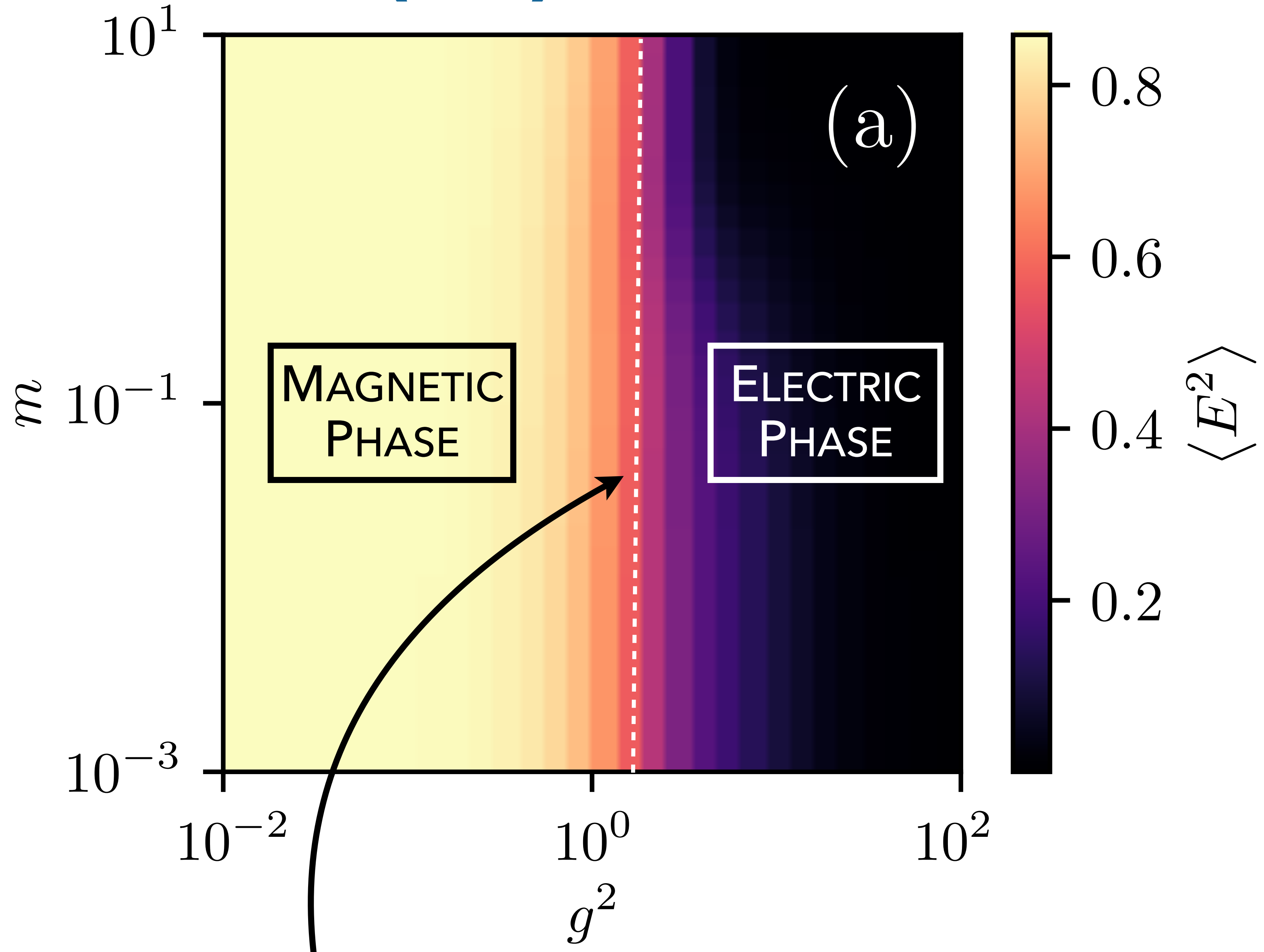
BINDING ENERGY DENSITY

1 BARYON SECTOR - VACUUM SECTOR

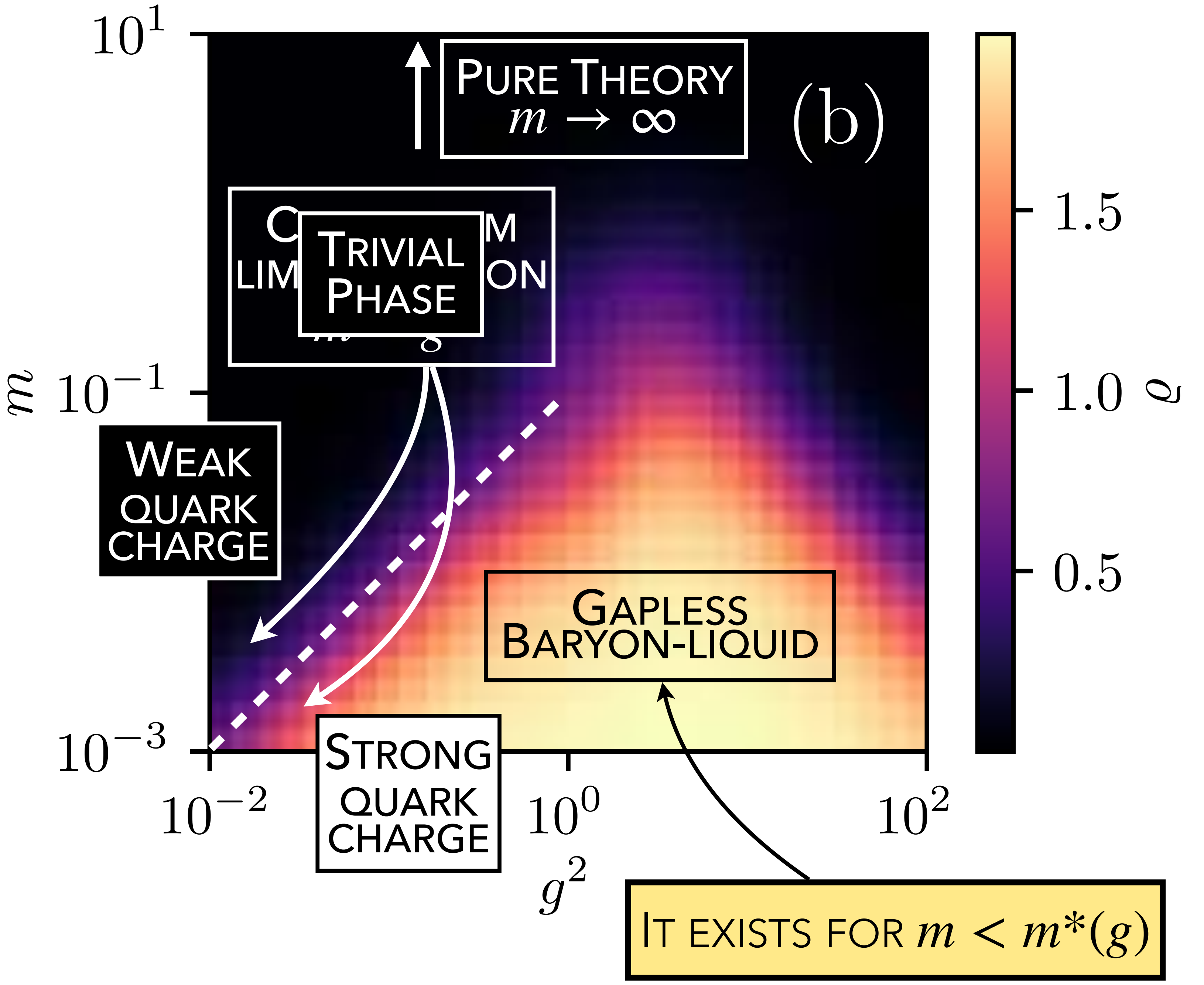




SU(2) FULL THEORY PHASE-DIAGRAM



Unaltered at finite masses m and finite Baryon Density b



It exists for $m < m^*(g)$

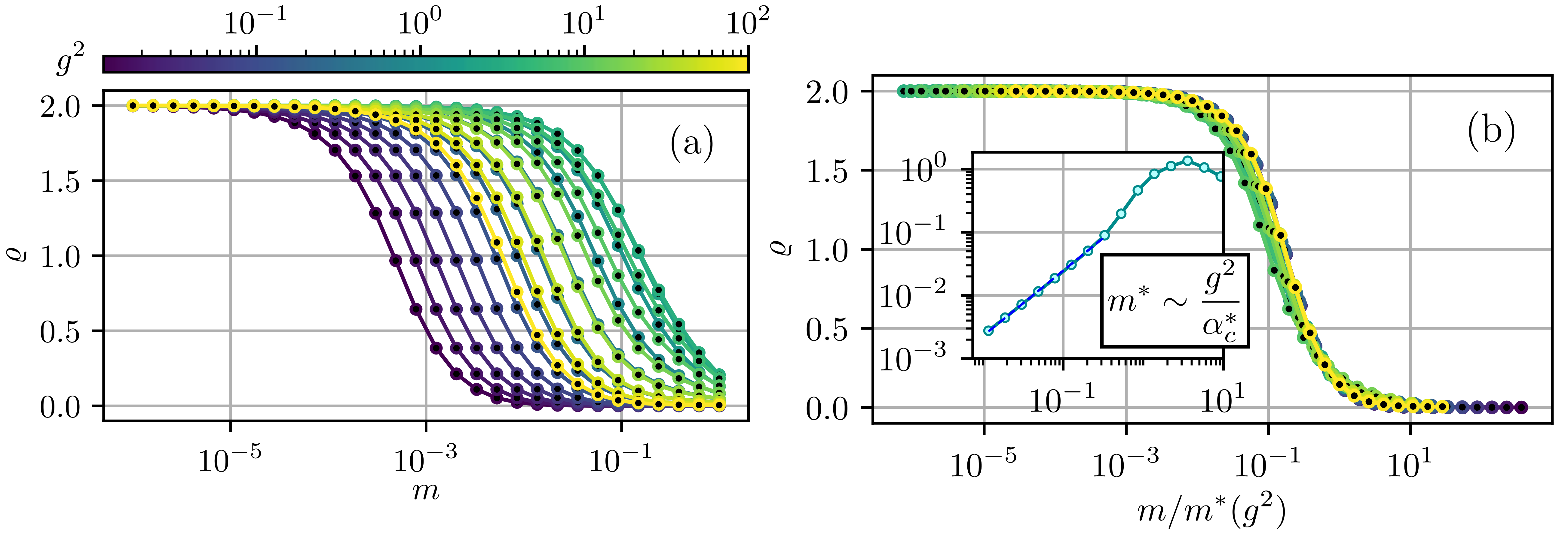
Strong Quark Charge $\alpha_c > \alpha_c^*$ $\xrightarrow{a \to 0}$ Gapless Baryon Fluid

Weak Quark Charge $\alpha_c < \alpha_c^*$ $\xrightarrow{a \to 0}$ Trivial Phase

$$m^*(g^2) \simeq 0.267(4) \cdot \left(g^2\right)^{1.03(2)}$$

Critical Quark Ratio $\alpha_c^* = 3.75(6)$

$$\alpha_c \propto \frac{q_c^2}{m_0}$$



TRAPPED ION QUANTUM SIMULATOR

(1+1)D SU(2) truncated model

1D SU(2) invariant Hamiltonian

$$H = \frac{1}{2a_0} \sum_n \sum_{a,b=\uparrow,\downarrow} \left[-i \hat{\psi}^\dagger_{na} \hat{U}^{ab}_{n,n+1} \hat{\psi}_{n+1,b} + \text{H.c.} \right] + m_0 \sum_{na} (-1)^n \hat{\psi}^\dagger_{na} \hat{\psi}_{na} + \frac{g_0^2}{2a_0} \sum_n \hat{E}^2_{n,n+1}$$

S. Chandrasekharan, U. J. Wiese, Phys. Rev B **492** 455 (1997); E. Zohar and M. Burrello, Phys. Rev D **91** 054506 (2015);

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5-dimensional Hilbert space

$(0,0)$-**IRREP**

$|00\rangle$

$\left(\frac{1}{2},\frac{1}{2}\right)$-**IRREP**

$|\uparrow\uparrow\rangle |\downarrow\downarrow\rangle |\downarrow\uparrow\rangle |\uparrow\downarrow\rangle$

*S. Chandrasekharan, U. J. Wiese, Phys. Rev B **492** 455 (1997); E. Zohar and M. Burrello, Phys. Rev D **91** 054506 (2015);*

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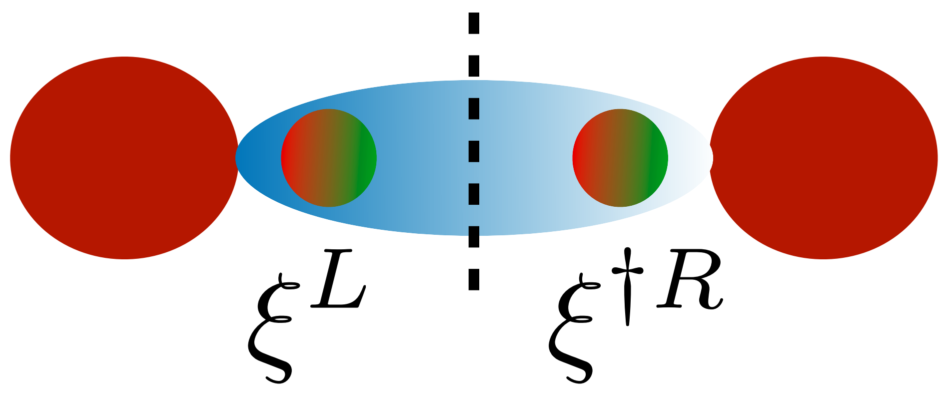
- *Decompose link in **pair of rishons***

$$\hat{U}^{ab}_{j,j+1} = \xi^L_{j,a} \xi^{\dagger R}_{j+1,b}$$

$\xi^{L/R}_{j,a}$ ***fermion operators***



5-dimensional Hilbert space

$(0,0)$**-IRREP**

$|00\rangle$

$\left(\frac{1}{2}, \frac{1}{2}\right)$**-IRREP**

$|\uparrow\uparrow\rangle |\downarrow\downarrow\rangle |\downarrow\uparrow\rangle |\uparrow\downarrow\rangle$

*S. Chandrasekharan, U. J. Wiese, Phys. Rev B **492** 455 (1997); E. Zohar and M. Burrello, Phys. Rev D **91** 054506 (2015);*

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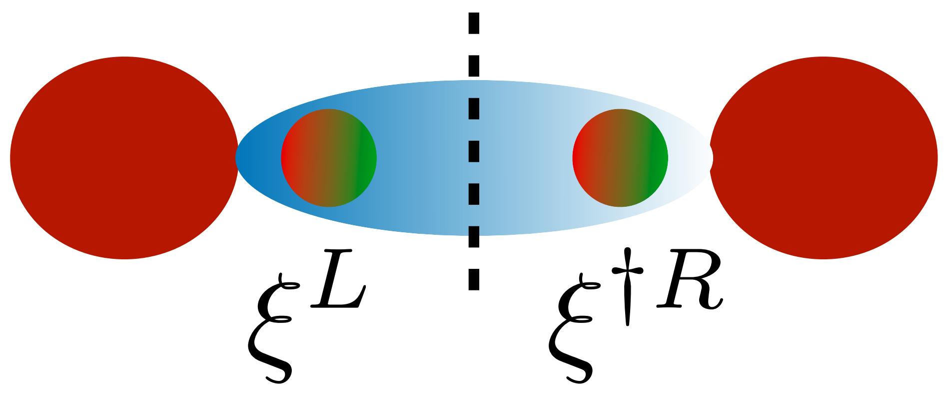
- *Decompose link in* ***pair of rishons***

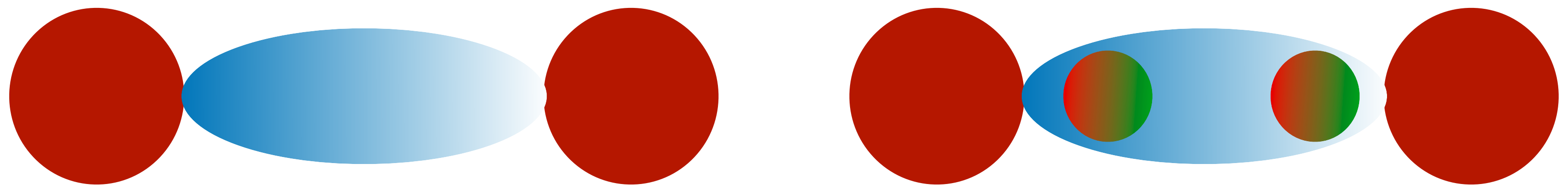
$$\hat{U}^{ab}_{j,j+1} = \xi^L_{j,a}\xi^{\dagger R}_{j+1,b} \qquad \xi^{L/R}_{j,a} \quad \textit{fermion operators}$$



5-dimensional Hilbert space

$(0,0)$-IRREP	$\left(\frac{1}{2},\frac{1}{2}\right)$-IRREP
$\|00\rangle$	$\|\uparrow\uparrow\rangle \|\downarrow\downarrow\rangle \|\downarrow\uparrow\rangle \|\uparrow\downarrow\rangle$

- ***Link parity constrain:***
  even number of rishon per link



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$$\hat{\psi}^\dagger_{j,a}\, \xi^L_{j,a} \,\Big|\, \xi^{\dagger R}_{j+1,b}\, \hat{\psi}_{j+1,b}$$

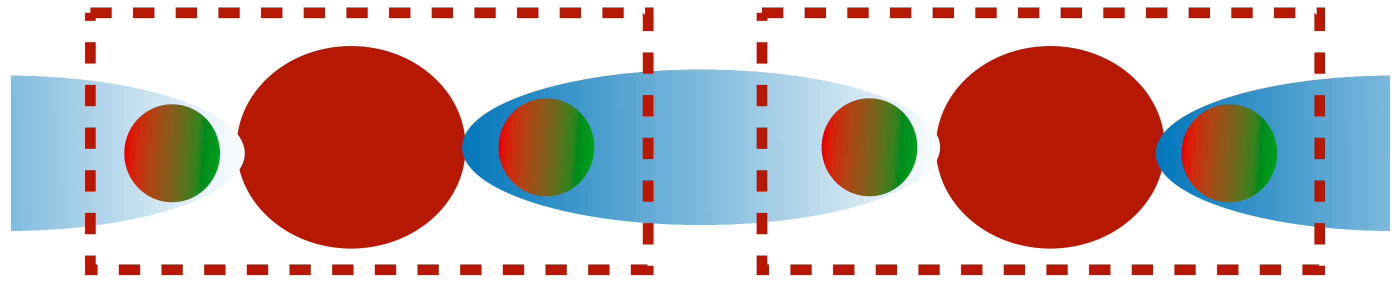
(1+1)D SU(2) truncated model

1D SU(2) invariant Hamiltonian

$$H = \frac{1}{2a_0}\sum_n \sum_{a,b=\uparrow,\downarrow}\left[-i\hat{\psi}^\dagger_{na}\hat{U}^{ab}_{n,n+1}\hat{\psi}_{n+1,b} + \text{H.c.}\right] + m_0\sum_{na}(-1)^n\hat{\psi}^\dagger_{na}\hat{\psi}_{na} + \frac{g_0^2}{2a_0}\sum_n \hat{E}^2_{n,n+1}$$

$$\hat{\psi}^\dagger_{j,a}\,\xi^L_{j,a}\,\xi^{\dagger R}_{j+1,b}\,\hat{\psi}_{j+1,b}$$

- ***Local dressed basis:***
  embed each rishon in adjacent site



- ***Gauss law:*** *each site on a color-singlet state*

(1+1)D SU(2) truncated model

1D SU(2) invariant Hamiltonian

$$H = \frac{1}{2a_0}\sum_n \sum_{a,b=\uparrow,\downarrow}\left[-i\hat{\psi}^\dagger_{na}\hat{U}^{ab}_{n,n+1}\hat{\psi}_{n+1,b} + \text{H.c.}\right] + m_0\sum_{na}(-1)^n\hat{\psi}^\dagger_{na}\hat{\psi}_{na} + \frac{g_0^2}{2a_0}\sum_n \hat{E}^2_{n,n+1}$$

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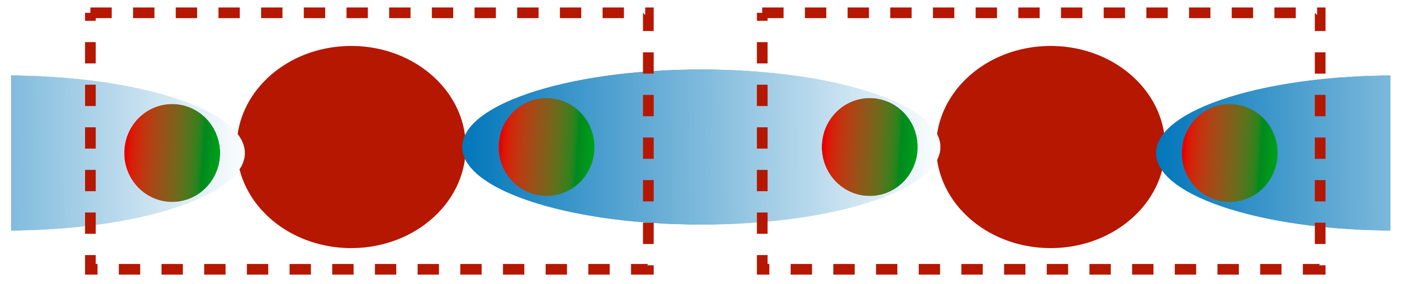
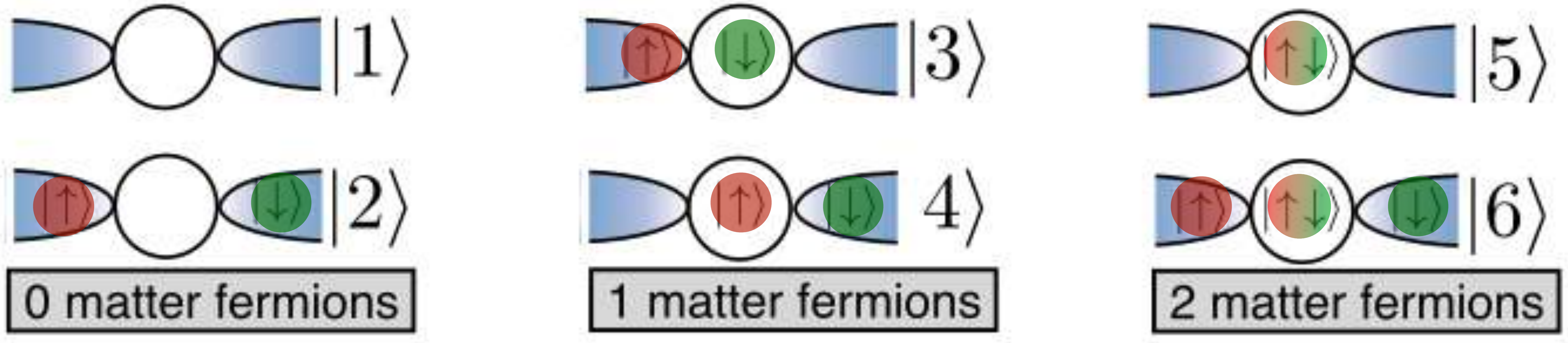


- ***Gauss law:** each site on a color-singlet state*

6 dimensional local Hilbert space



(1+1)D SU(2) truncated model

1D SU(2) invariant Hamiltonian

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$$\hat{\psi}^\dagger_{j,a}\,\xi^L_{j,a}\,\xi^{\dagger R}_{j+1,b}\,\hat{\psi}_{j+1,b}$$

Local dressed basis

Model with <u>local dimension 6</u>

$$H = \sum_j\left[\hat{A}^{(1)}_j\hat{B}^{(1)}_{j+1} + \hat{A}^{(2)}_j\hat{B}^{(2)}_{j+1}\right] + m\sum_j(-1)^j\hat{M}_j + g^2\sum_j\hat{C}_j$$

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1D SU(2) invariant Hamiltonian

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Mass term

Field term

Diagonal matrices!

$$\hat{M} = \begin{pmatrix} 0 &&&&& \\ & 0 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& 2 & \\ &&&&& 2 \end{pmatrix}$$

$$\hat{C} = \begin{pmatrix} 0 &&&&& \\ & 2 &&&& \\ && 1 &&& \\ &&& 1 && \\ &&&& 0 & \\ &&&&& 2 \end{pmatrix}$$

(1+1)D SU(2) truncated model

1D SU(2) invariant Hamiltonian

$$H = \frac{1}{2a_0}\sum_n \sum_{a,b=\uparrow,\downarrow}\left[-i\hat{\psi}^\dagger_{na}\hat{U}^{ab}_{n,n+1}\hat{\psi}_{n+1,b} + \text{H.c.}\right] + m_0\sum_{na}(-1)^n\hat{\psi}^\dagger_{na}\hat{\psi}_{na} + \frac{g_0^2}{2a_0}\sum_n \hat{E}^2_{n,n+1}$$

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Hopping terms

$$\hat{A}^{(1)} = \begin{pmatrix} 0 & & & \sqrt{2} & & \\ & 0 & 1 & & & \\ & 1 & 0 & & & 1 \\ \sqrt{2} & & & 0 & \sqrt{2} & \\ & & & \sqrt{2} & 0 & \\ & & 1 & & & 0 \end{pmatrix}$$

$$\hat{B}^{(1)} = \begin{pmatrix} 0 & & -\sqrt{2}i & & & \\ & 0 & & -i & & \\ \sqrt{2}i & & 0 & & -\sqrt{2}i & \\ & i & & 0 & & -i \\ & & \sqrt{2}i & & 0 & \\ & & & i & & 0 \end{pmatrix}$$

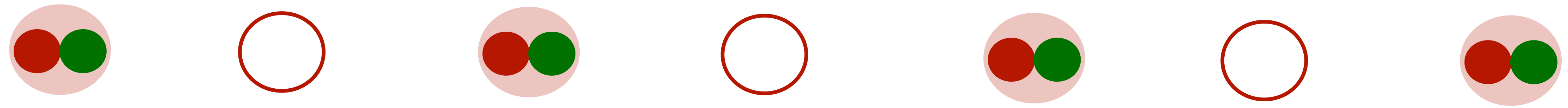
Sparse matrices!

(1+1)D SU(2) dynamics

Pairs production

$$H = \qquad m\sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

Dirac ground state

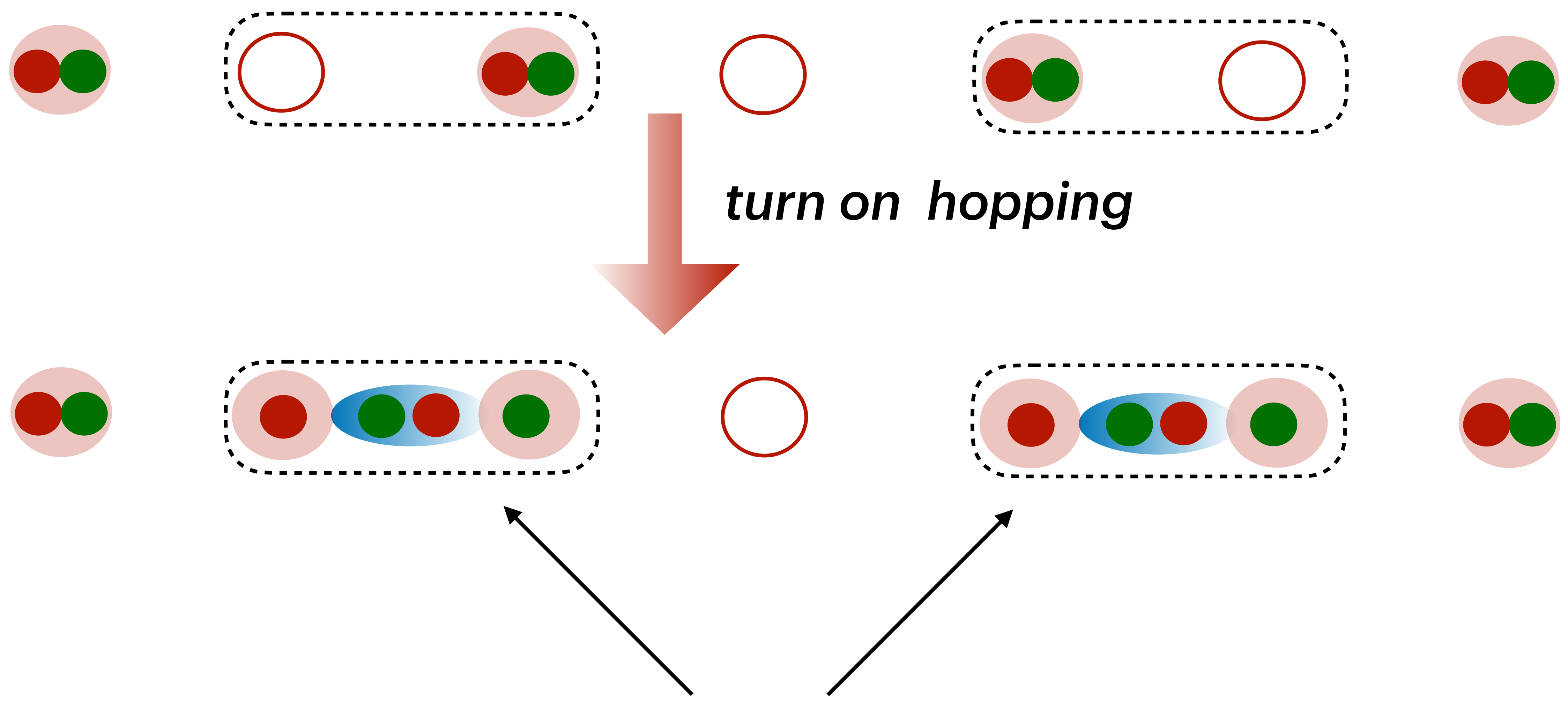


(1+1)D SU(2) dynamics

Pairs production

$$H = \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

Dirac ground state



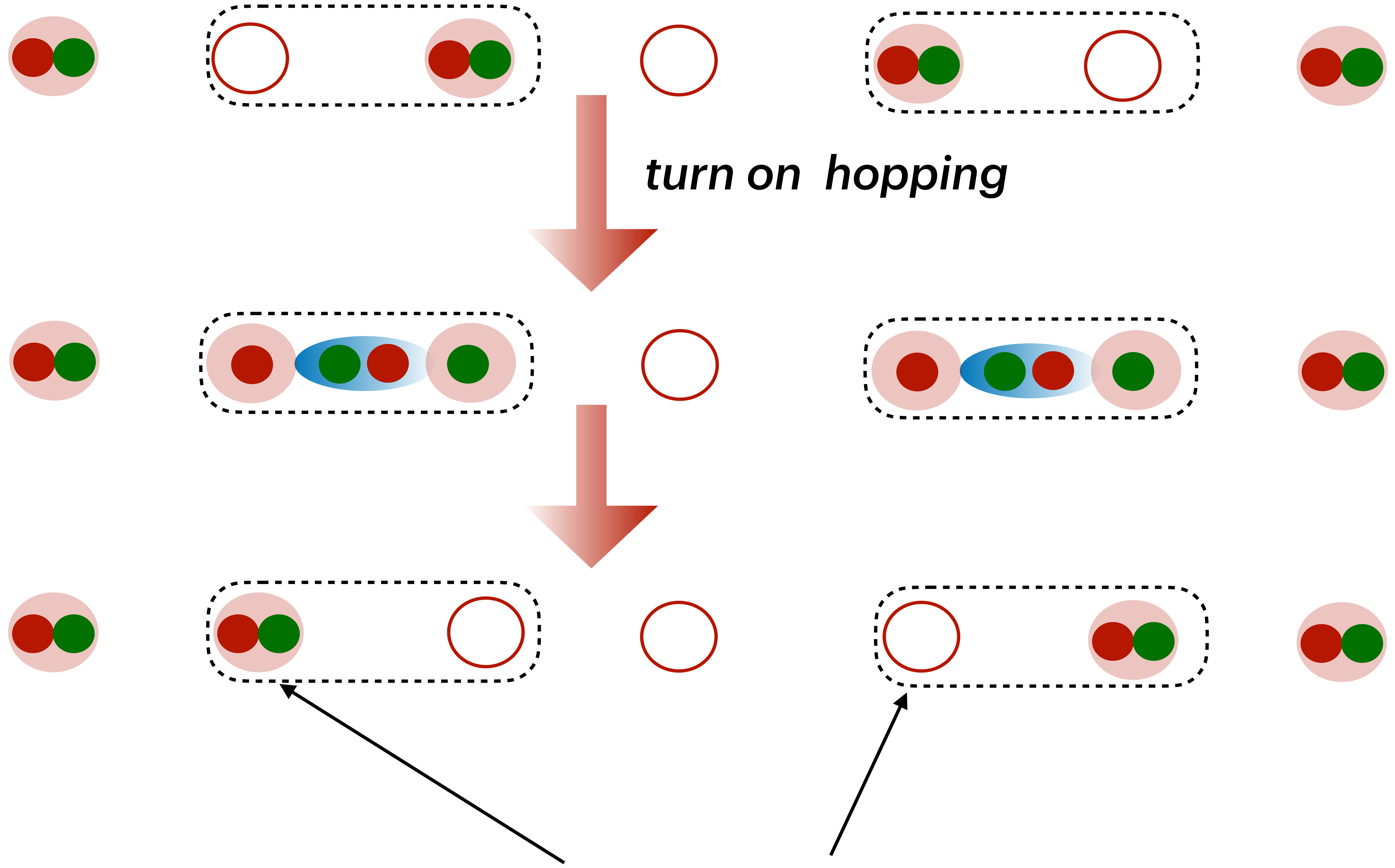
"Mesons" : matter—anti-matter pairs

(1+1)D SU(2) dynamics

Pairs production

$$H = \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

Dirac ground state

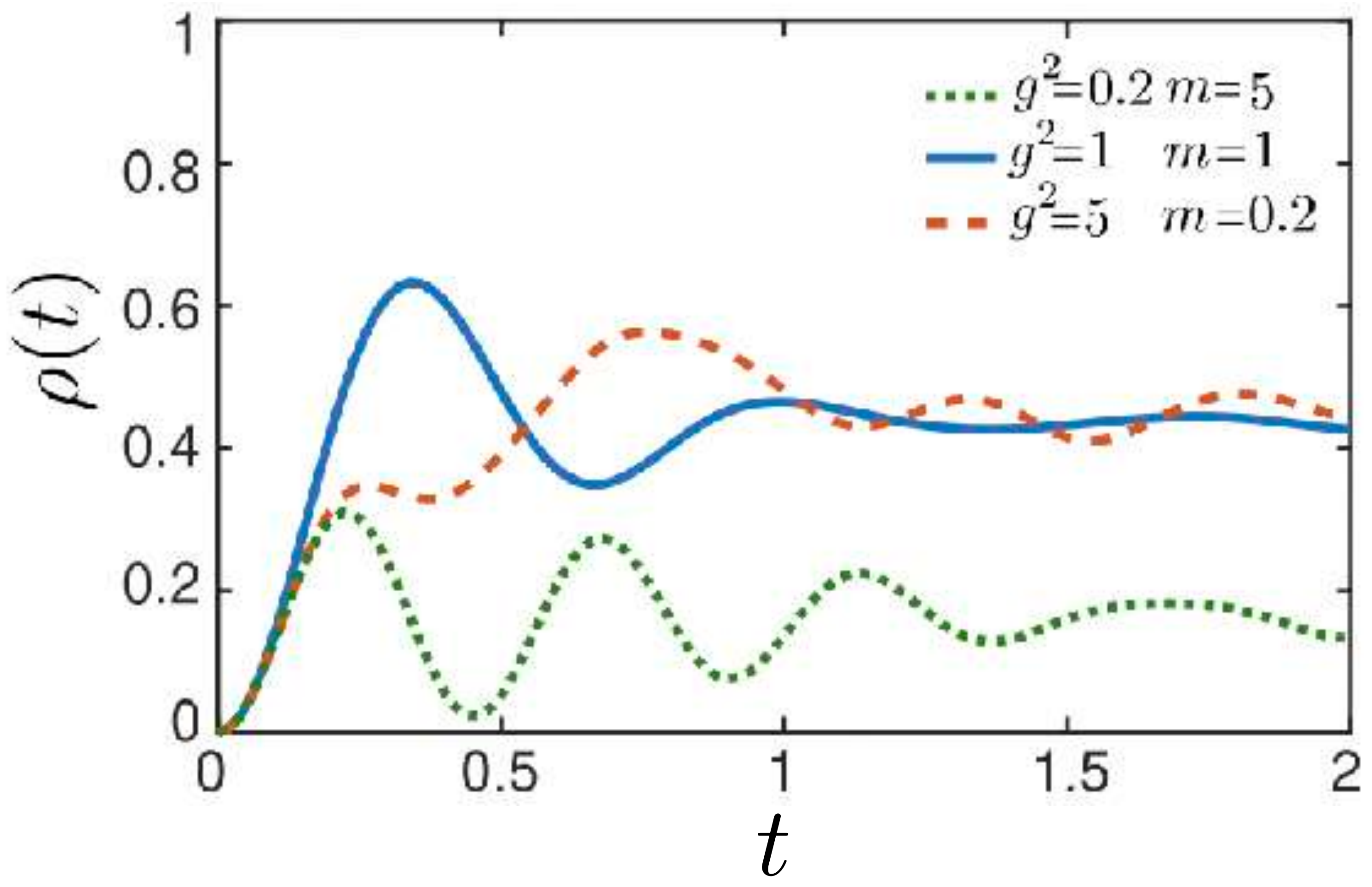


"Baryons and anti-baryons":
matter-matter and anti-matter—anti-matter pairs

(1+1)D SU(2) dynamics

Pairs production

$$H = \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$



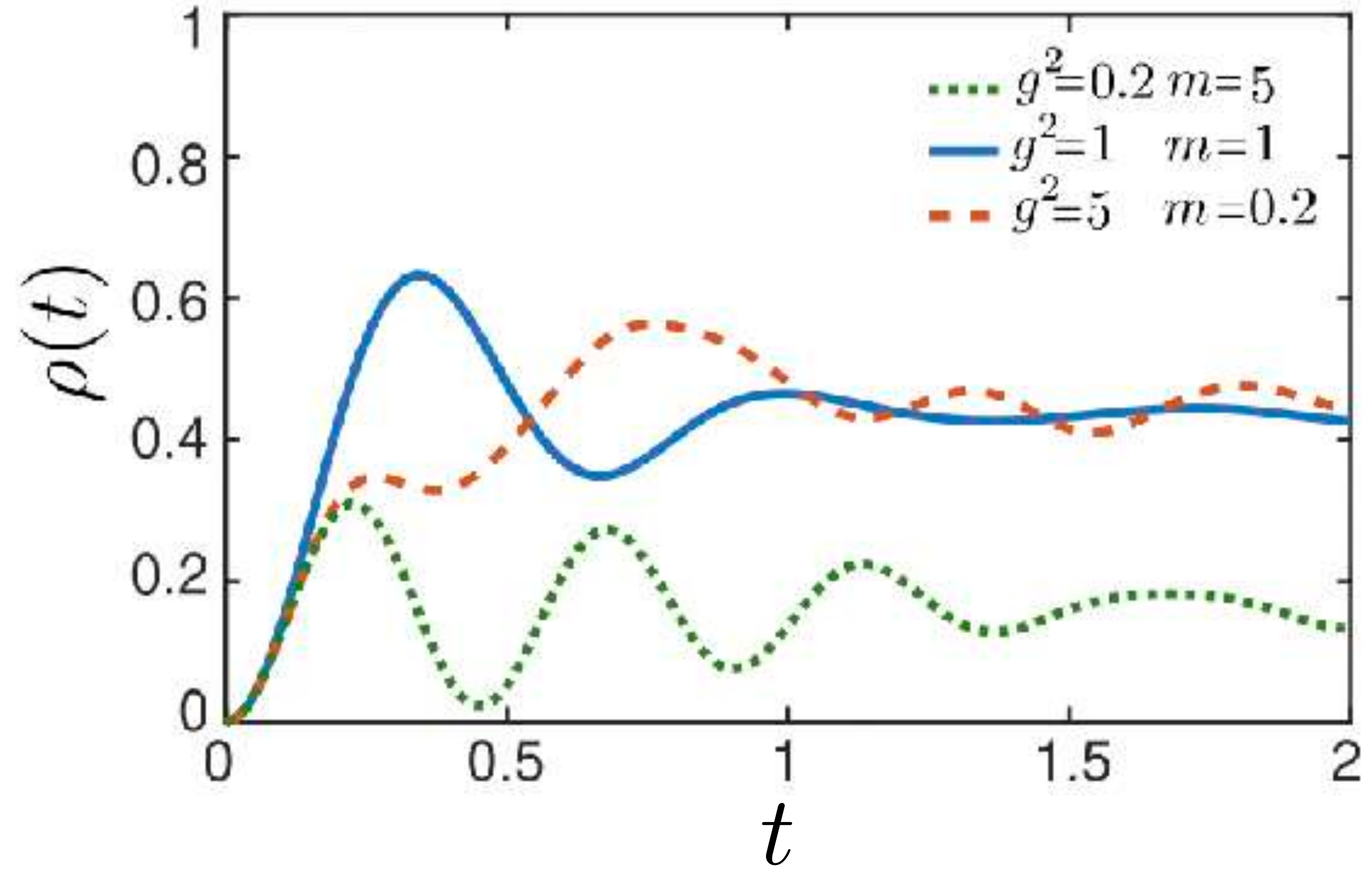
Particle density $\rho(t)$ grows in time

$$N = 20$$

(1+1)D SU(2) dynamics

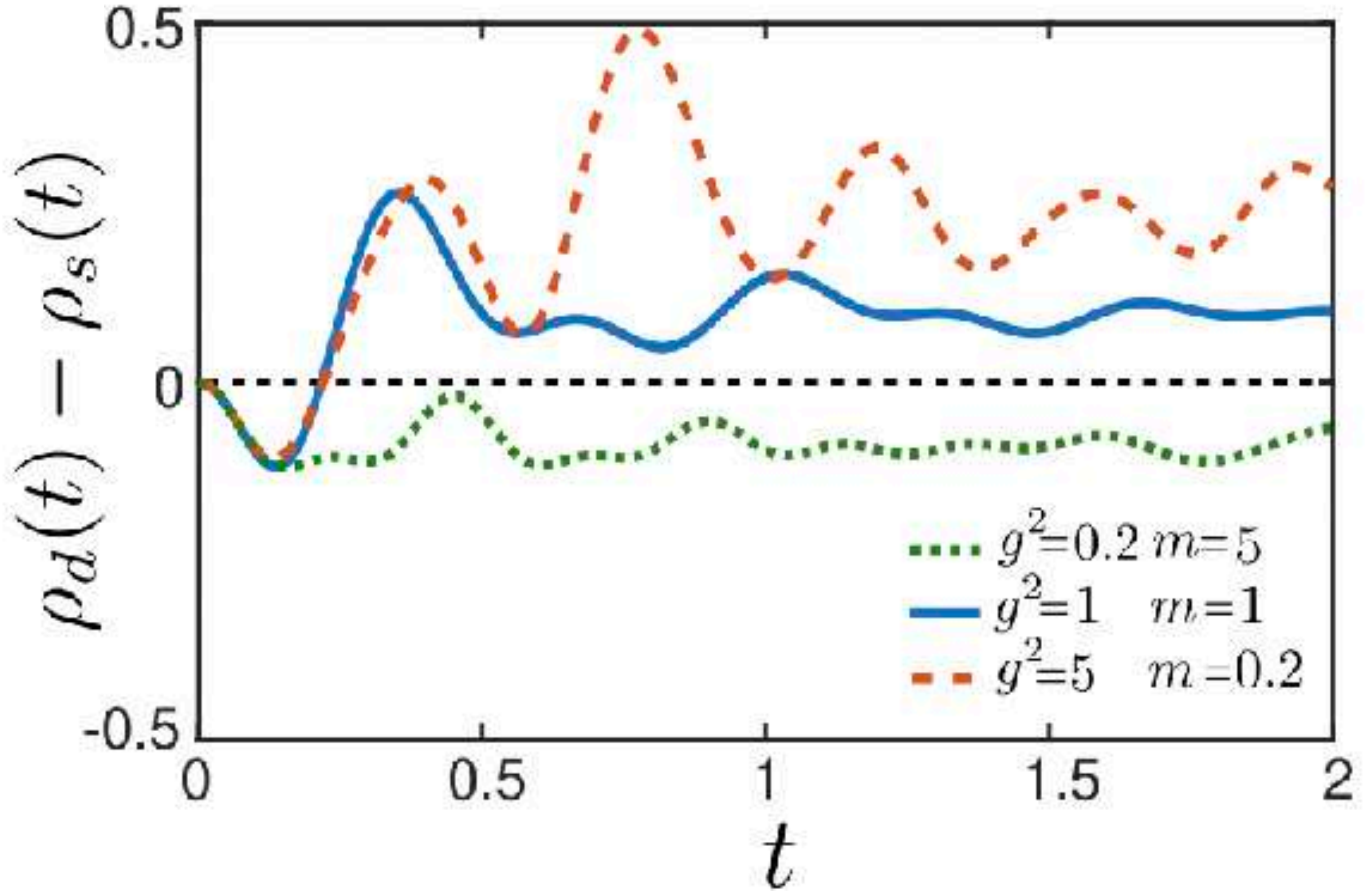
Pairs production

$$H = \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$



Particle density $\rho(t)$ *grows in time*

Difference between doubly and single occupied densities



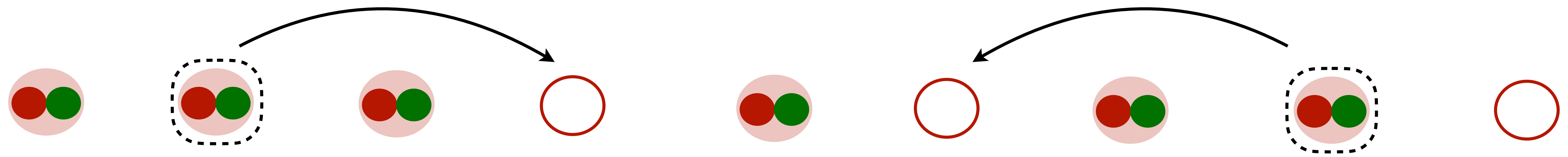
$$\rho(t) = \rho_s(t) + \rho_d(t)$$

Singly occupied site

Doubly occupied site

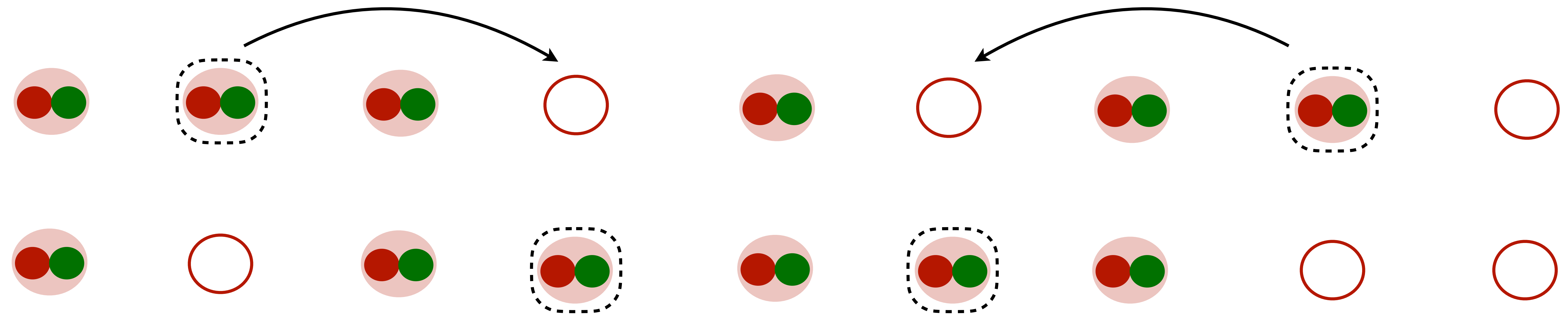
(1+1)D SU(2) dynamics

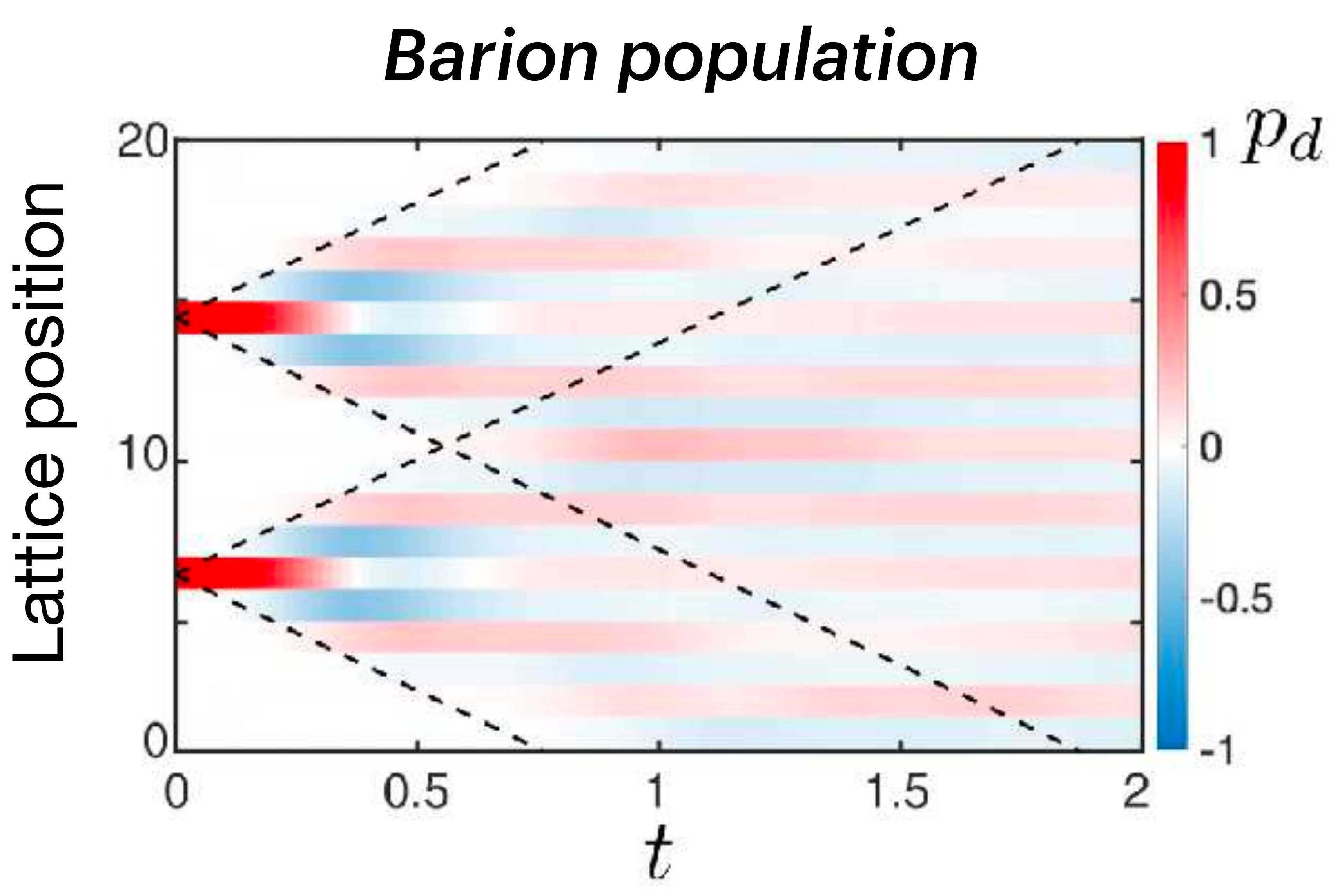
Barion diffusion

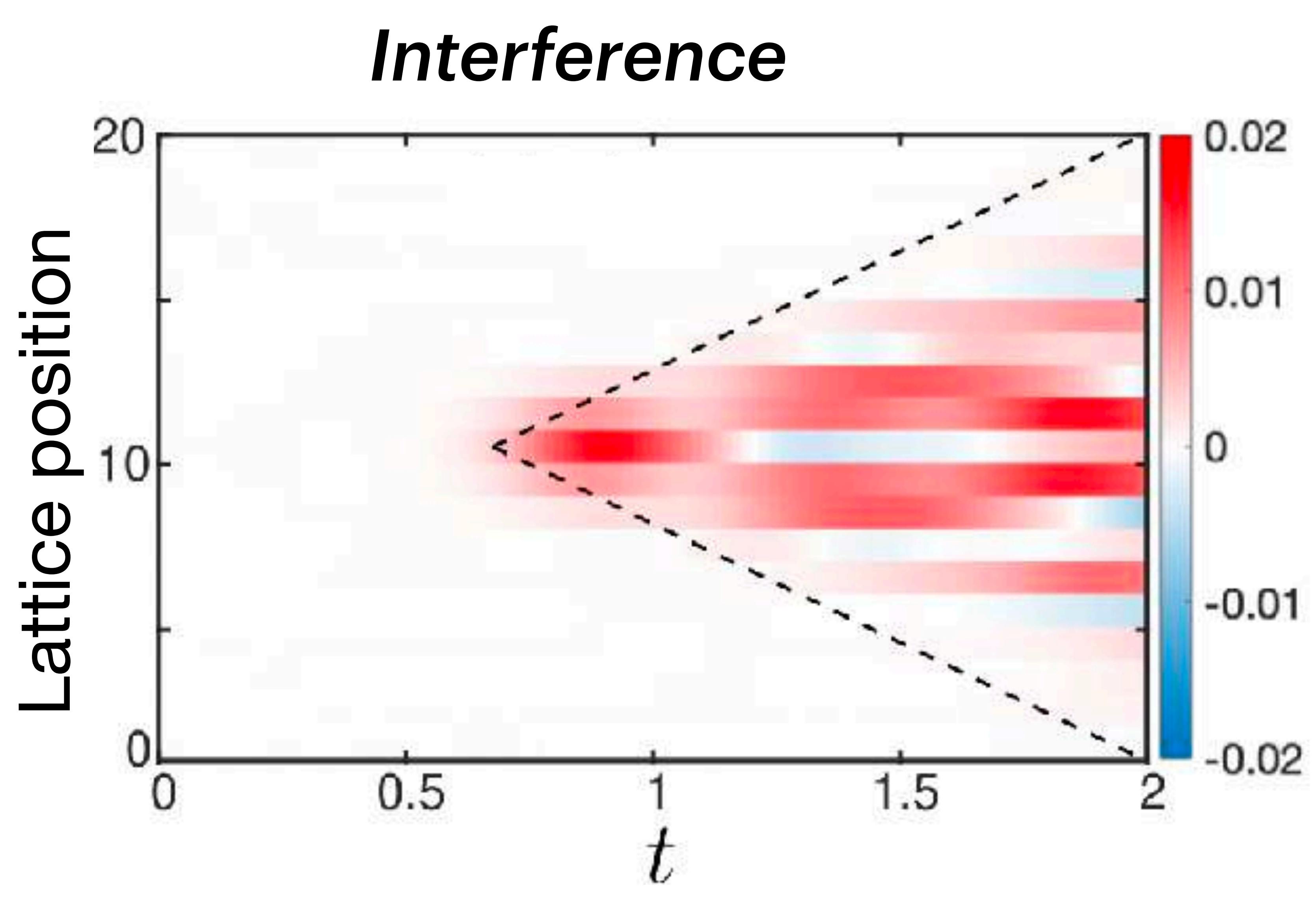


(1+1)D SU(2) dynamics

Barion diffusion



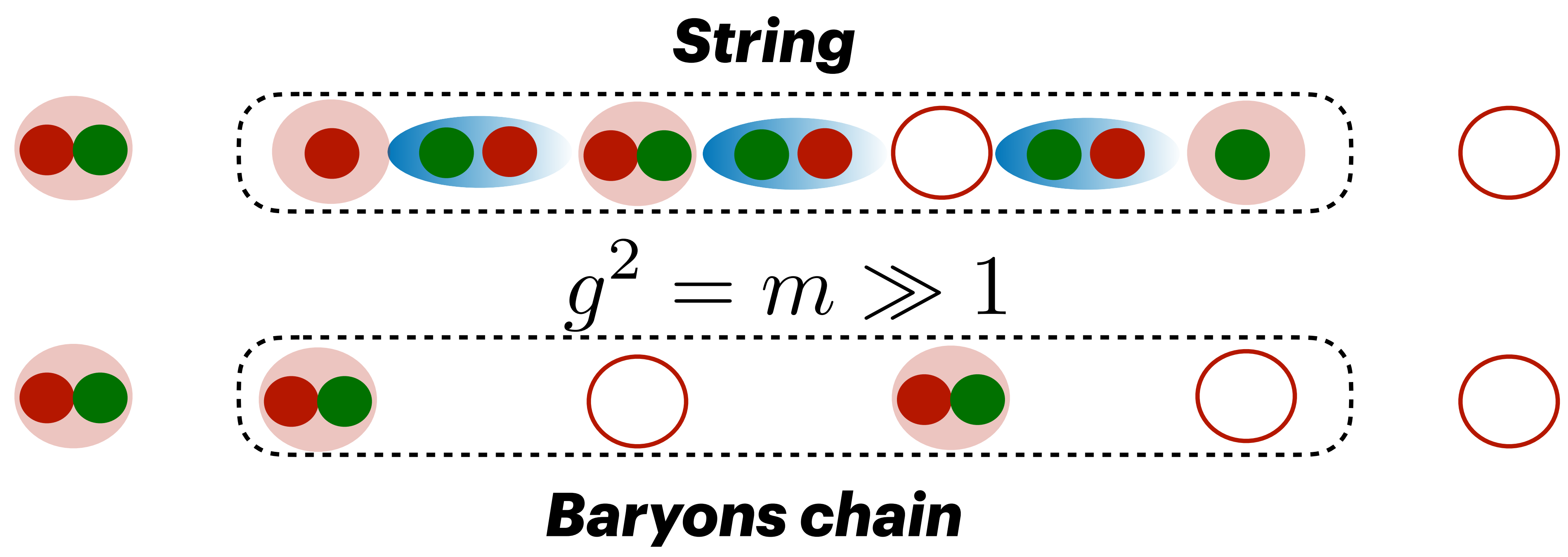




$N = 20$

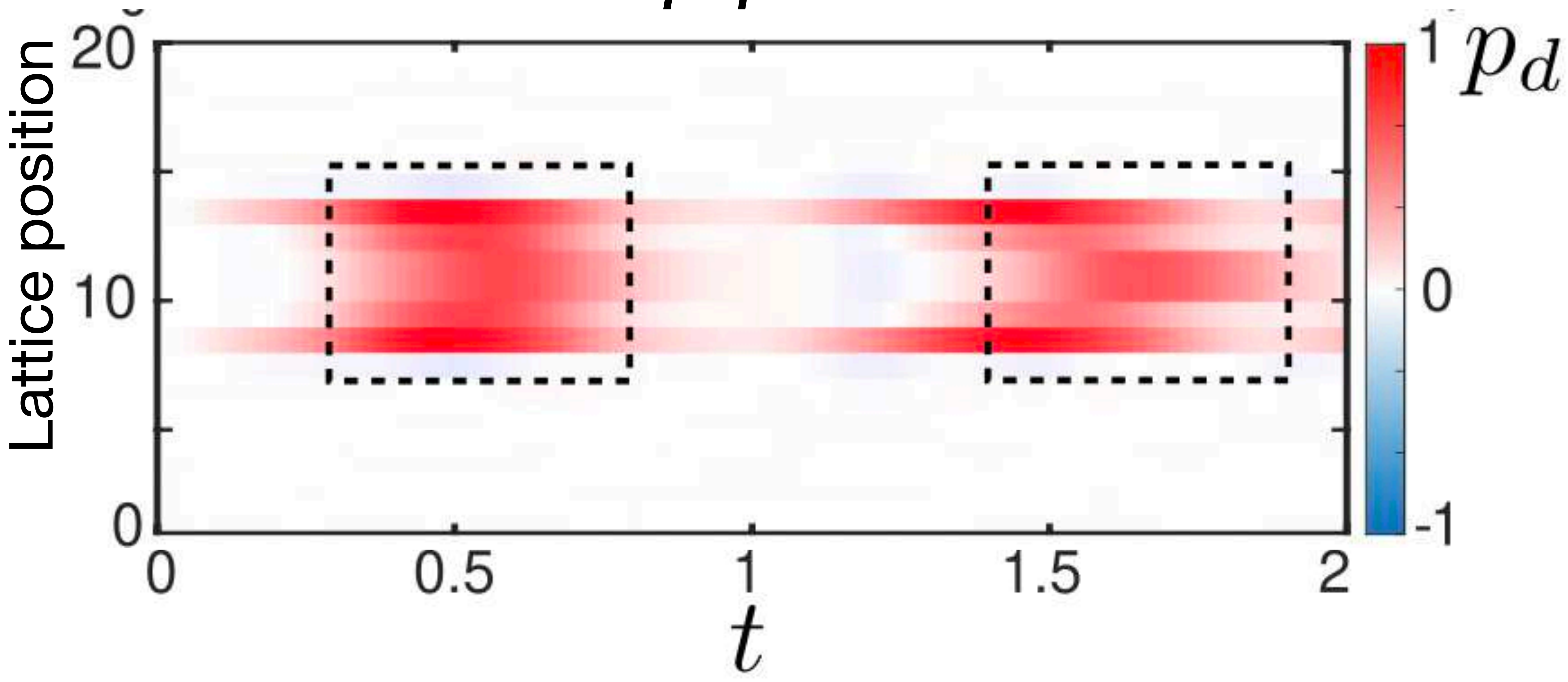
(1+1)D SU(2) dynamics

String dynamics



Resonant baryon excitation

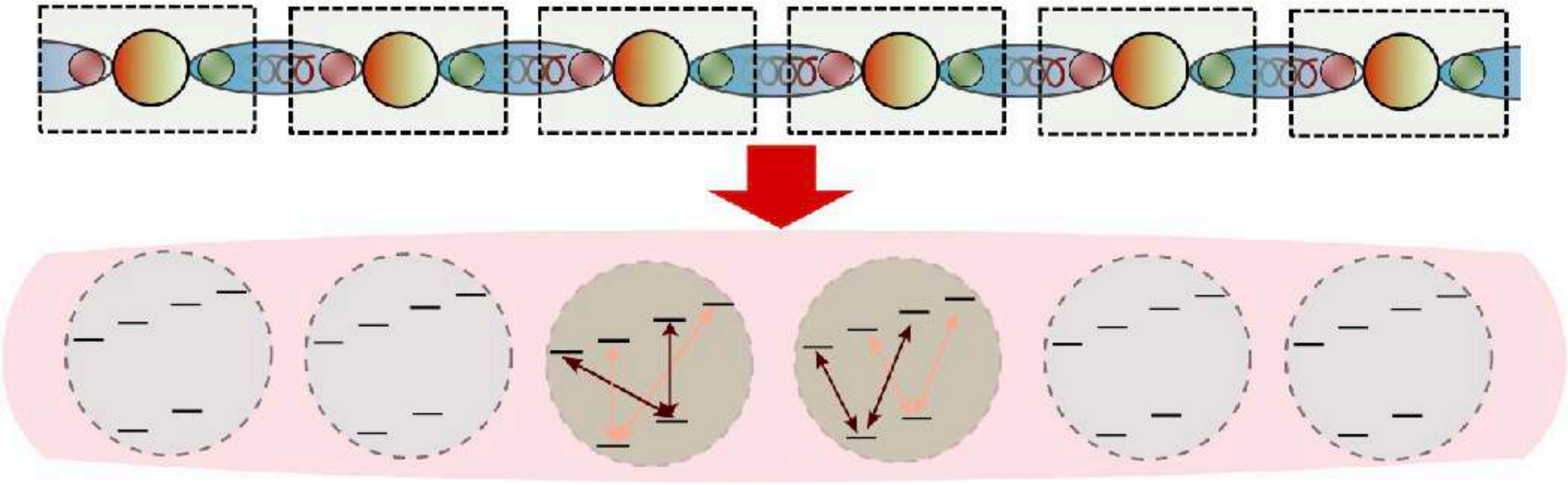
Barion population



$N = 20$

S. Kühn, E. Zohar, J. I. Cirac, and M. C. Bañuls, Journal of High Energy Physics **2015**, 1 (2015).

Encoding the model into **trapped ions qudits**



See also other cool ideas for simulating LGTs with qudits

*D. González-Cuadra, T. V. Zache, J. Carrasco, B. Kraus, and P. Zoller Phys. Rev. Lett. **129**, 160501 (2022);*
T. V. Zache, D. González-Cuadra, and P. Zoller, Quantum 7, 1140 (2023);

P. Popov, M. Meth, M. Lewenstein, P. Hauke, M. Ringbauer, E. Zohar, and V. Kasper, arXiv:2307.15173 (2023).

M. Meth, et al., arXiv:2310.12110 (2023);

*M. A. Kruckenhauser, R. van Bijnen, T. V. Zache, M. Di Liberto, and P. Zoller ,QST **8**, 015020 (2022);*

nature physics

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https://doi.org/10.1038/s41567-022-01658-0

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A universal qudit quantum processor with trapped ions

Martin Ringbauer[1]✉, Michael Meth[1], Lukas Postler[1], Roman Stricker[1], Rainer Blatt[1,2,3], Philipp Schindler[1] and Thomas Monz[1,3]

Optical qudit in $^{40}Ca^{+}$ trapped ions

nature physics

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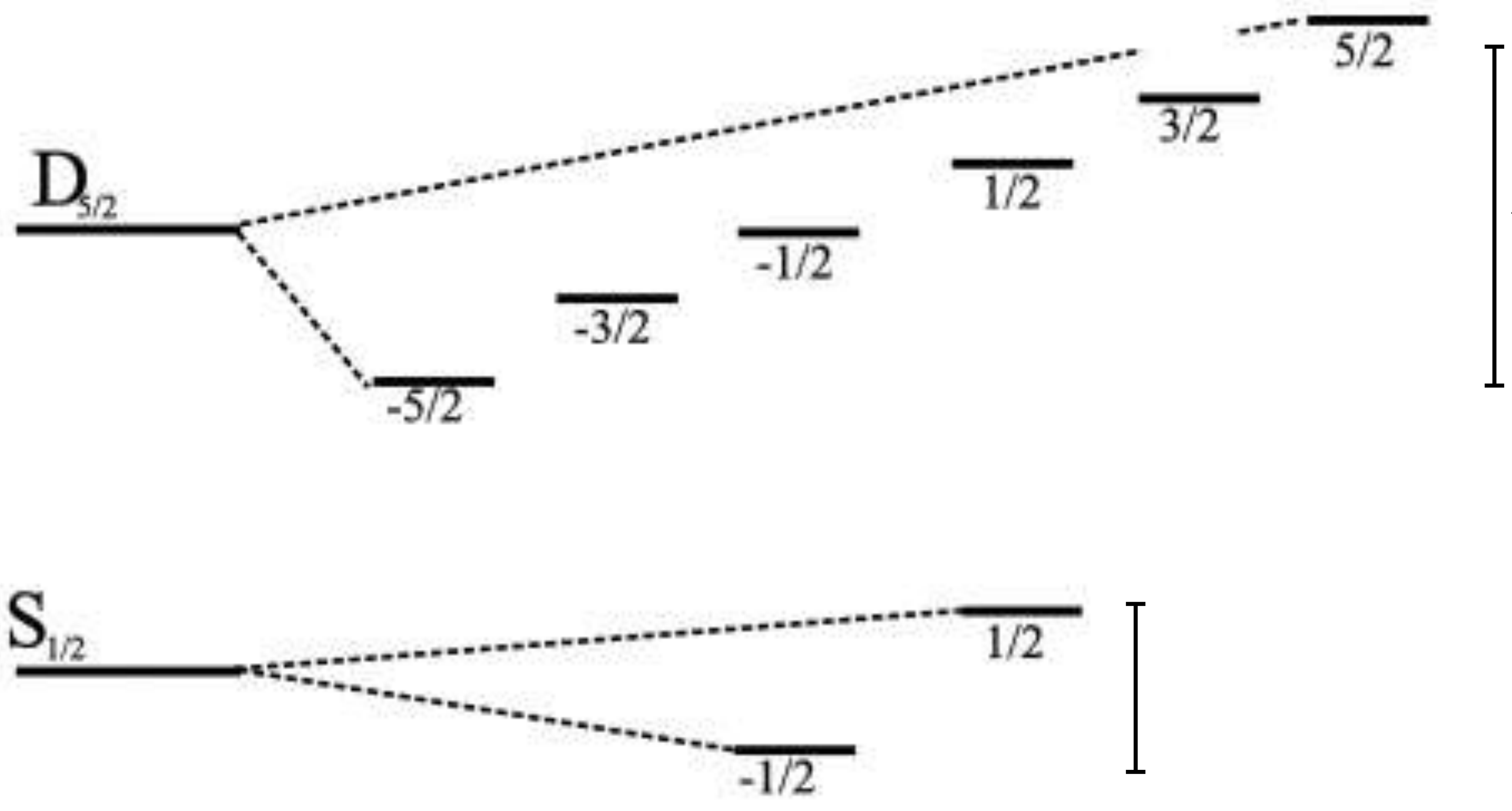

Zeeman splitted levels

nature physics

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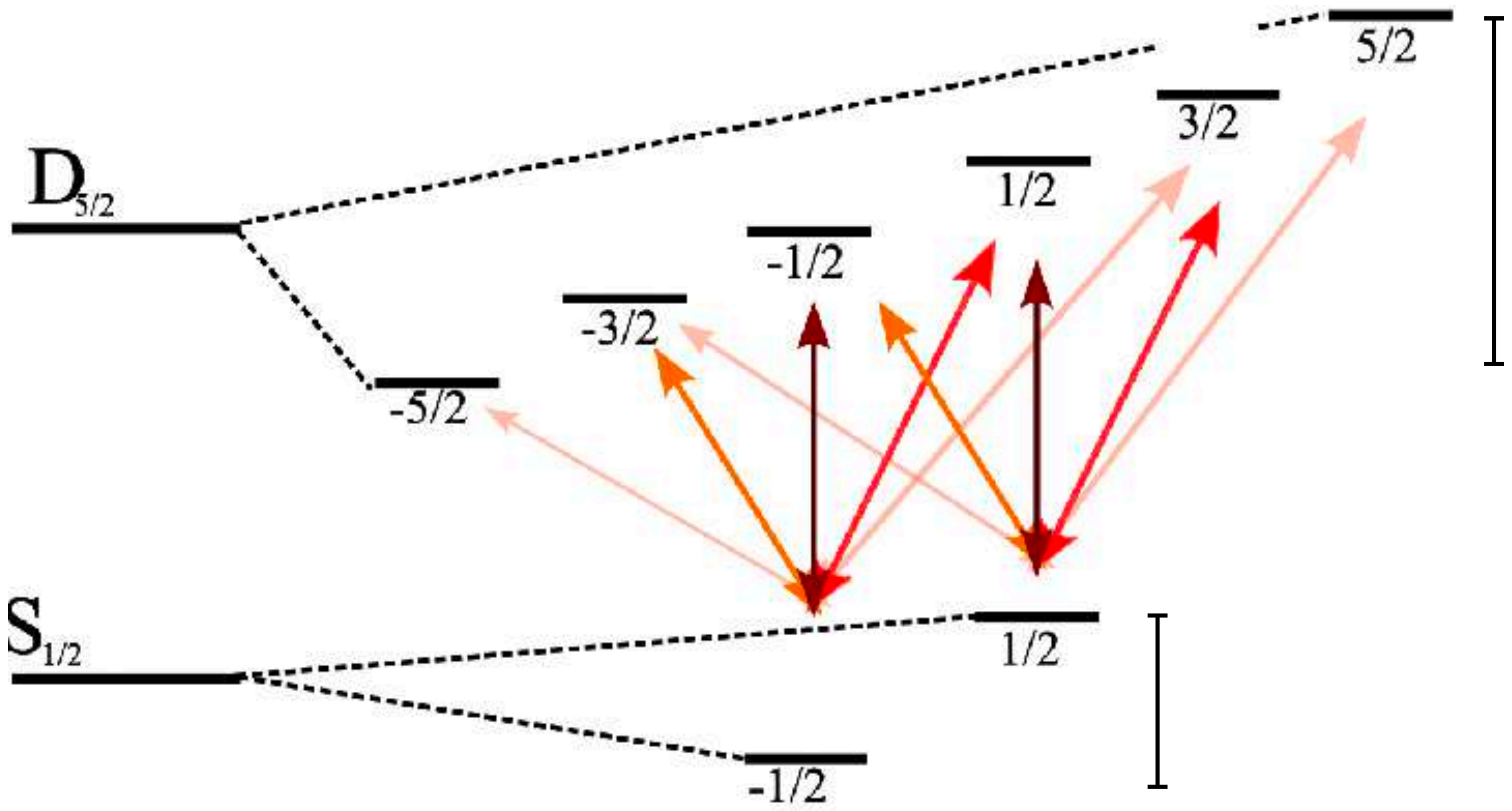

Zeeman splitted levels

$$\Delta m = 0, \pm 1, \pm 2$$

Quadrupole allowed transitions

8 levels fully connected qudit

nature physics

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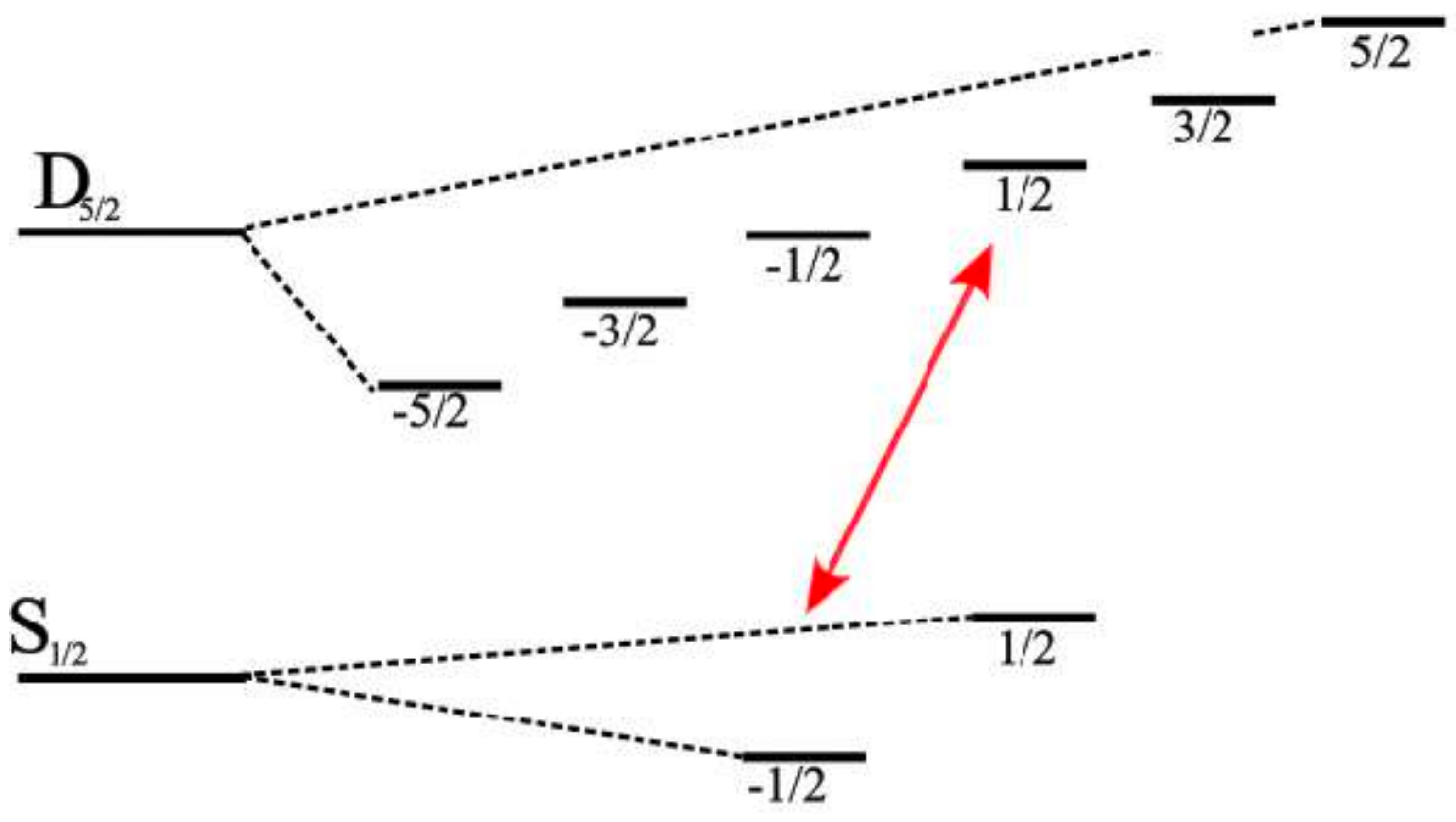

Single qudit operations:
decomposition in rotations on pairs of levels

$$R(\theta, \phi) = e^{-i\theta\hat{\sigma}_\phi/2}$$

High fidelities

nature physics

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A universal qudit quantum processor with trapped ions

Martin Ringbauer[1] ✉, Michael Meth[1], Lukas Postler[1], Roman Stricker[1], Rainer Blatt[1,2,3], Philipp Schindler[1] and Thomas Monz[1,3]

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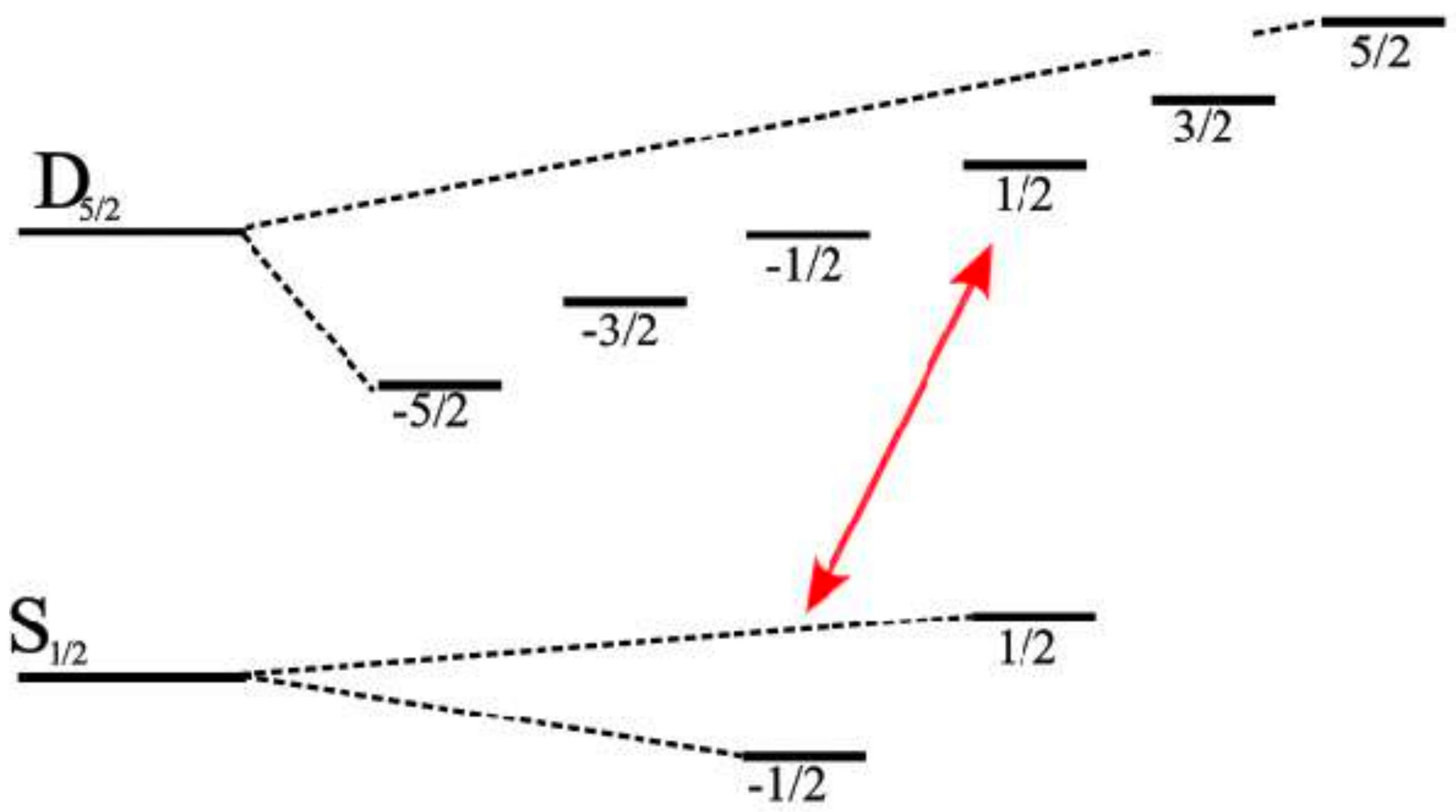

Single qudit operations:

decomposition in rotations on pairs of levels

$$R(\theta,\phi) = e^{-i\theta\hat{\sigma}_\phi/2}$$

High fidelities

Two-qudit operations:

decomposition in Mølmer Sørensen gates based on single driven transitions

$$\exp\left[i\theta\left(|e_\star\rangle\langle g_\star|_1 + |e_\star\rangle\langle g_\star|_2 + \text{h.c.}\right)^2\right]$$

Encoding the model into qudits

$$H = \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

Encoding the model into qudits

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Diagonal matrices:
single qudit operations

Encoding the model into qudits

$$H = \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

Diagonal matrices:
single qudit operations

Hopping terms

$$\hat{A}^{(1)} = \begin{pmatrix} 0 & & & \sqrt{2} & & \\ & 0 & 1 & & & \\ & 1 & 0 & & & 1 \\ \sqrt{2} & & & 0 & \sqrt{2} & \\ & & & \sqrt{2} & 0 & \\ & & 1 & & & 0 \end{pmatrix} \quad \hat{B}^{(1)} = \begin{pmatrix} 0 & & -\sqrt{2}i & & & \\ & 0 & & -i & & \\ \sqrt{2}i & & 0 & & -\sqrt{2}i & \\ & i & & 0 & & -i \\ & & \sqrt{2}i & & 0 & \\ & & & i & & 0 \end{pmatrix}$$

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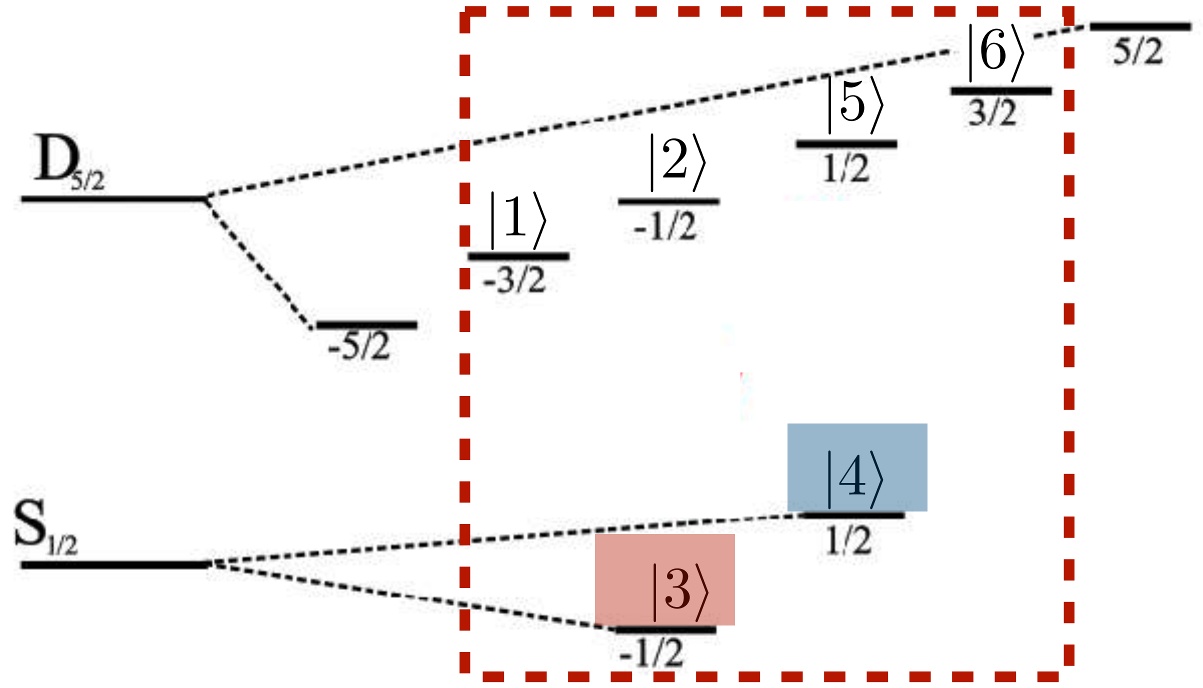
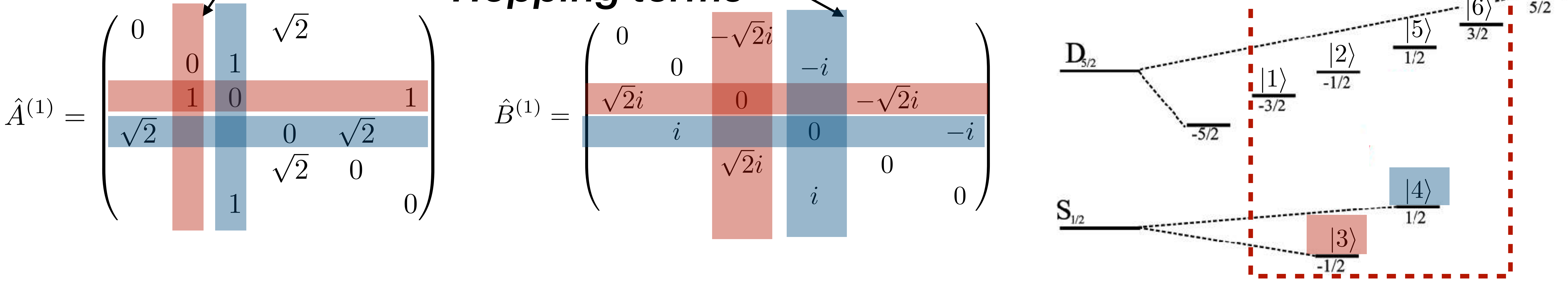


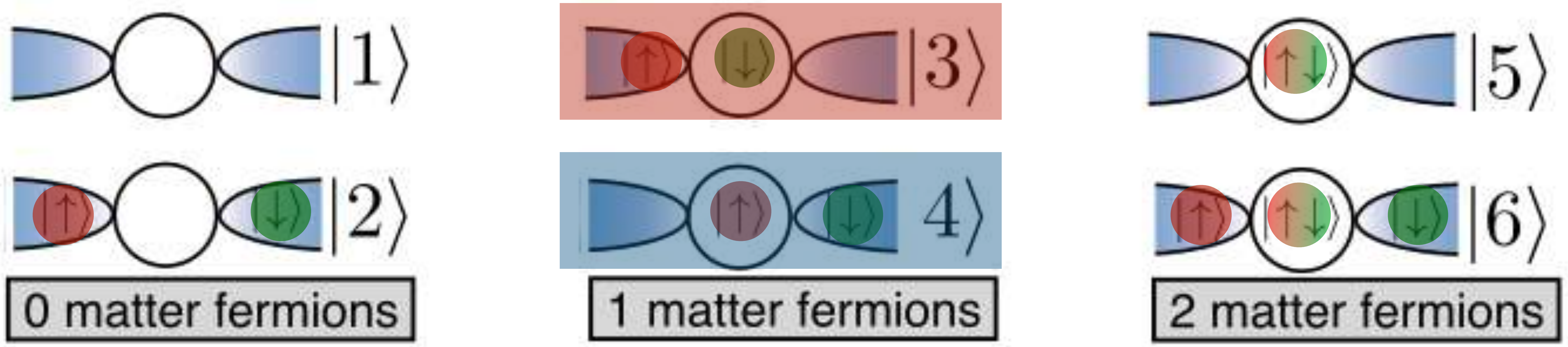
Encoding the model into qudits

$$H = \sum_j \left[\hat{A}^{(1)}_j \hat{B}^{(1)}_{j+1} + \hat{A}^{(2)}_j \hat{B}^{(2)}_{j+1} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

Diagonal matrices:
single qudit operations

Hopping terms





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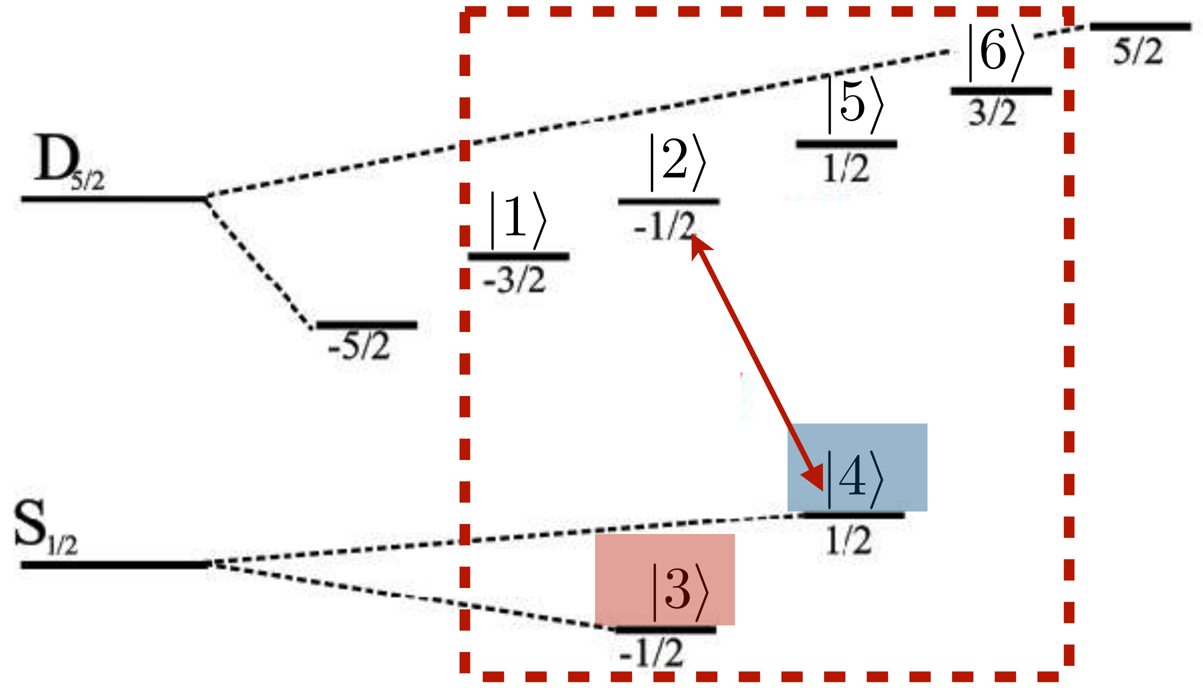
$$\hat{B}^{(1)} = \begin{pmatrix} 0 & & -\sqrt{2}i & & & \\ & 0 & & -i & & \\ \sqrt{2}i & & 0 & & -\sqrt{2}i & \\ & i & & 0 & & -i \\ & & \sqrt{2}i & & 0 & \\ & & & i & & 0 \end{pmatrix} \sim \hat{\sigma}_y^{n,m}$$

Diagonal matrices:
single qudit operations



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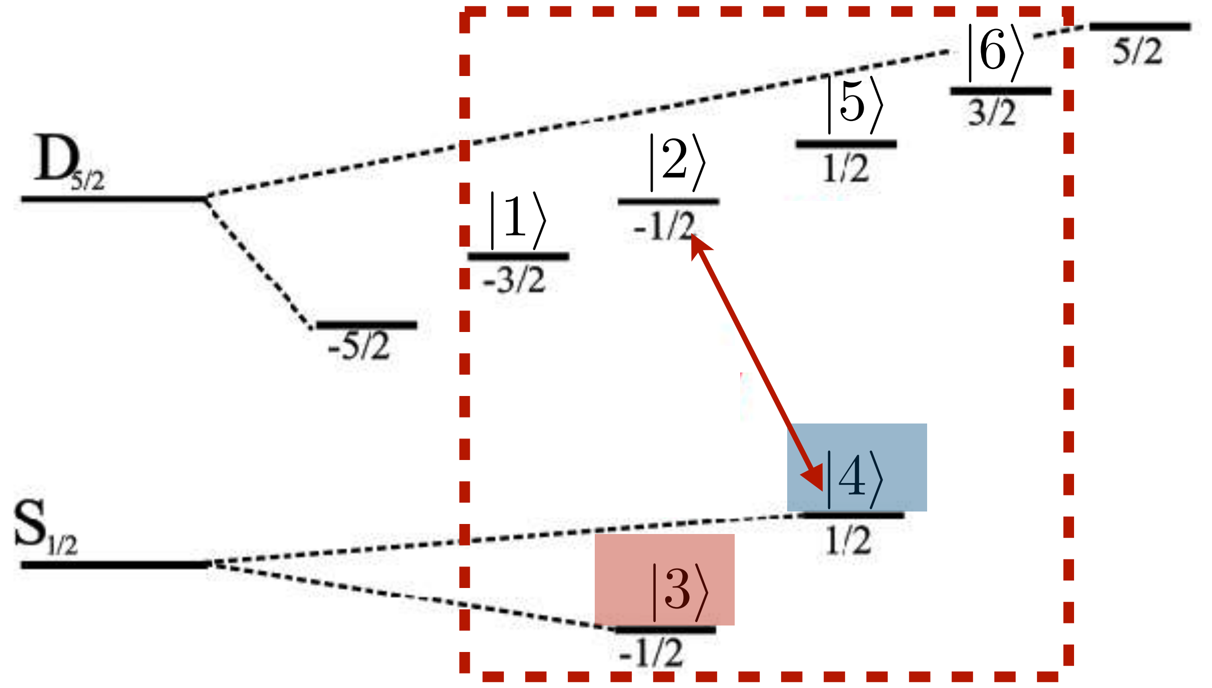
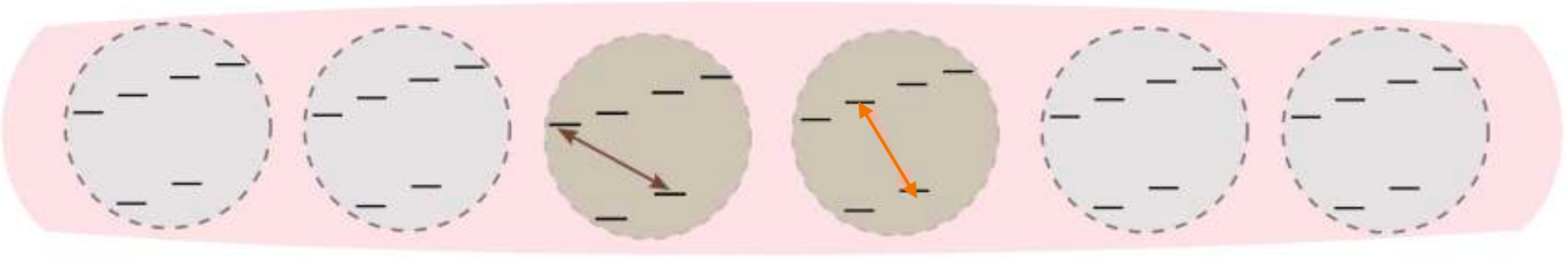


Decomposed in Mølmer Sørensen gates with <u>only direct transitions</u>



Encoding the model into qudits

$$H = \sum_j \left[\hat{A}_j^{(1)}\hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)}\hat{B}_{j+1}^{(2)}\right] + m\sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

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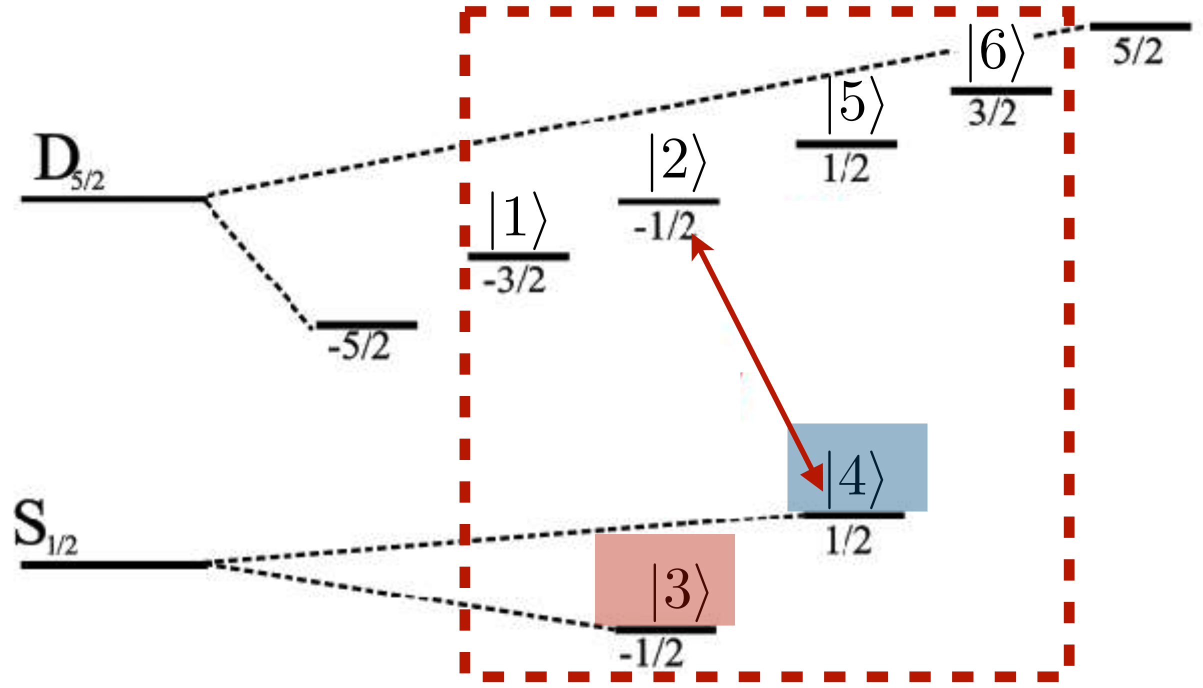
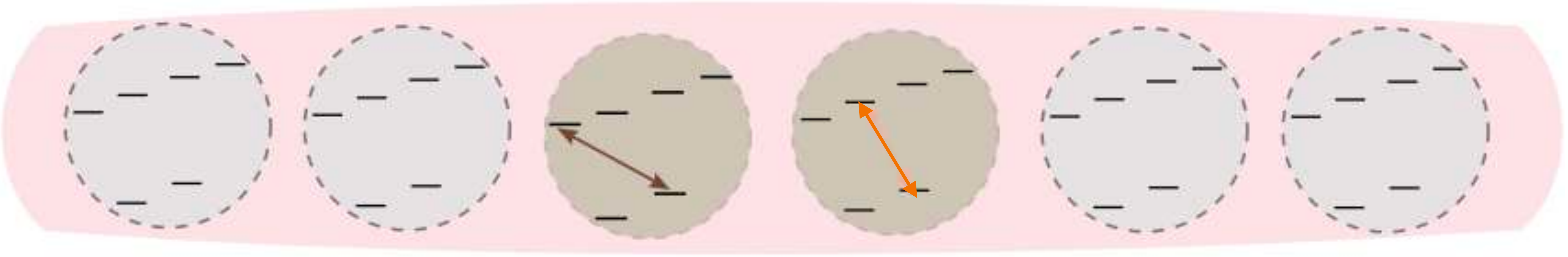
Diagonal matrices:
single qudit operations

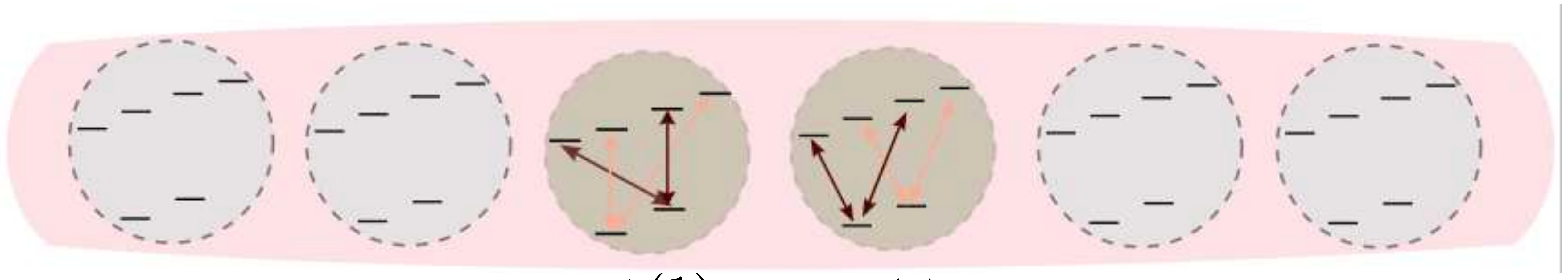


*Decomposed in Mølmer Sørensen gates with **only direct transitions***



64 MS gates necessary for one time step evolution of 3 sites

Qudit Mølmer Sørensen gate



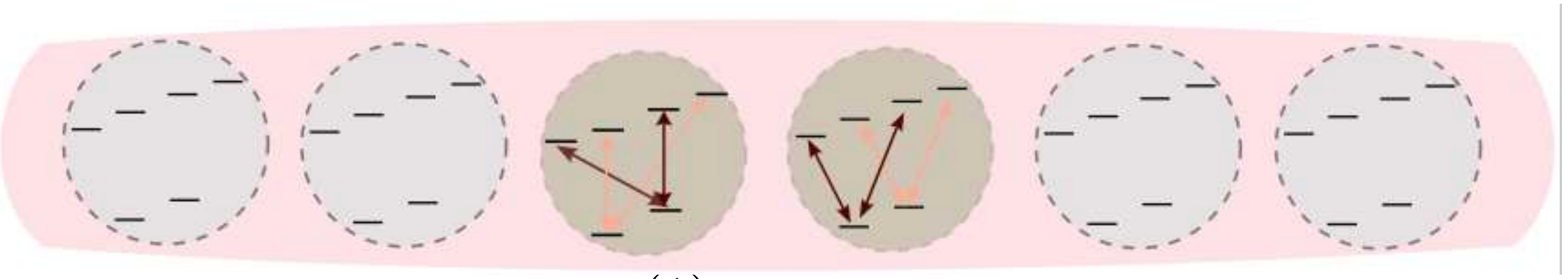
$\hat{A}_j^{(1)}$ $\hat{B}_{j+1}^{(1)}$

***Generalised MS gate for qudits:* simultaneously drive 4 transitions**

$$H = \sum_j \left[\hat{A}_j^{(1)} \hat{B}_{j+1}^{(1)} + \hat{A}_j^{(2)} \hat{B}_{j+1}^{(2)} \right] + m \sum_j (-1)^j \hat{M}_j + g^2 \sum_j \hat{C}_j$$

__For qudit gates see also:__ D. P.J. Low, B. M. White, A. A. Cox, M. L. Day, and C. Senko, Phys. Rev. Research __2__, 033128 (2020)

Qudit Mølmer Sørensen gate



$\hat{A}^{(1)}_j$ $\hat{B}^{(1)}_{j+1}$

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$$H_{\mathrm{MS}} \simeq \left[\hat{A}^{(1)}_j + \hat{B}^{(1)}_{j+1} \right]^2$$

Just one generalised MS gate for each hopping term!

For qudit gates see also: *D. P.J. Low, B. M. White, A. A. Cox, M. L. Day, and C. Senko, Phys. Rev. Research* ***2****, 033128 (2020)*

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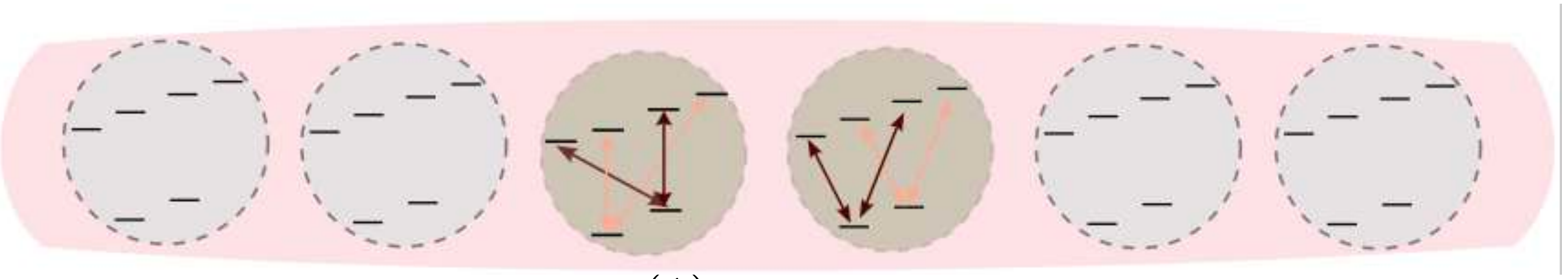

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Price to pay:
unwanted single qudit rotations $(\hat{A}_j^{(1)})^2$ $(\hat{B}_{j+1}^{(1)})^2$

__For qudit gates see also:__ D. P.J. Low, B. M. White, A. A. Cox, M. L. Day, and C. Senko, Phys. Rev. Research __2__, 033128 (2020)

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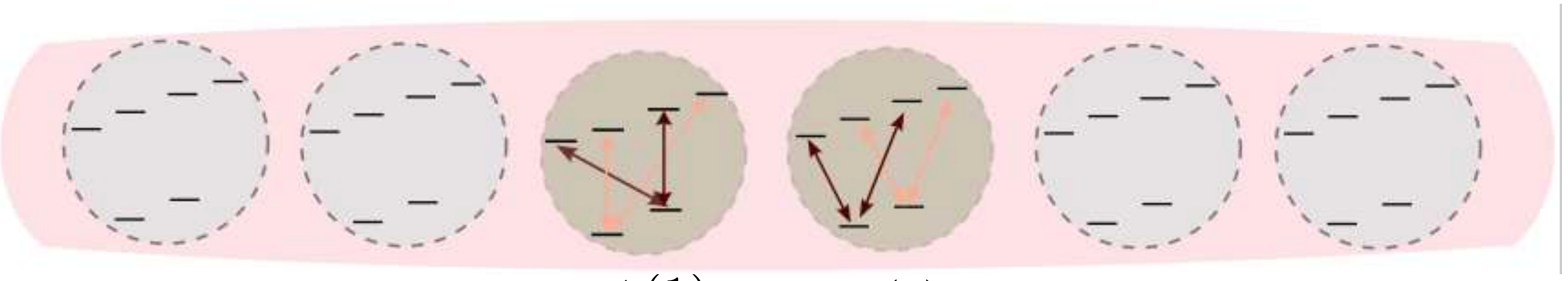

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Just one generalised MS gate for each hopping term!

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Just diagonal matrices!

__For qudit gates see also:__ D. P.J. Low, B. M. White, A. A. Cox, M. L. Day, and C. Senko, Phys. Rev. Research __2__, 033128 (2020)

Digital simulation of the model

- ***Suzuki-Trotter evolution*** $U(t) \simeq \left(\Pi_j e^{iH_j t_f/n}\right)^n$ n **Trotter steps**

Digital simulation of the model

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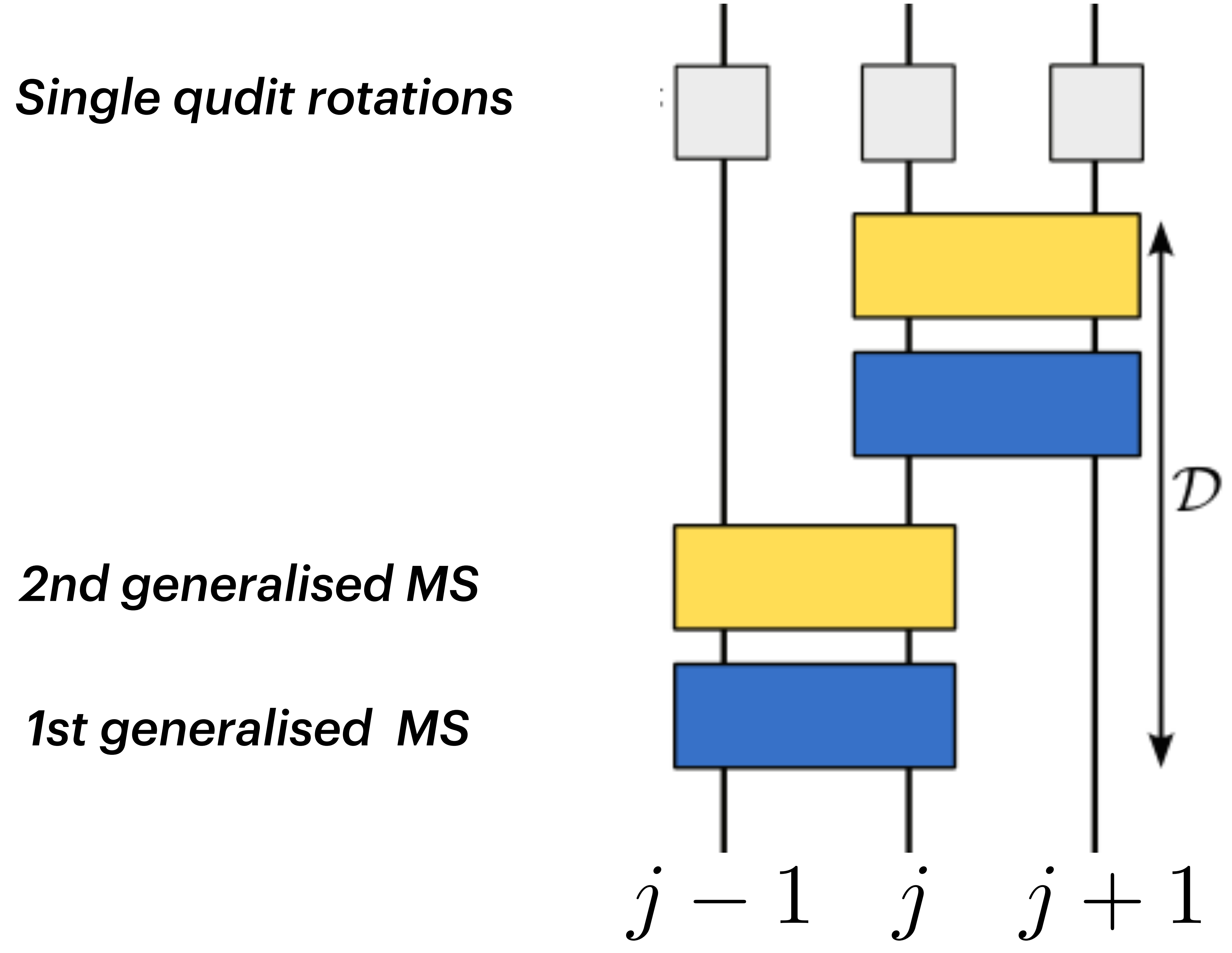
- **Circuit decomposition**

Circuit depth for only entangling operations

$$\mathcal{D} = 4$$



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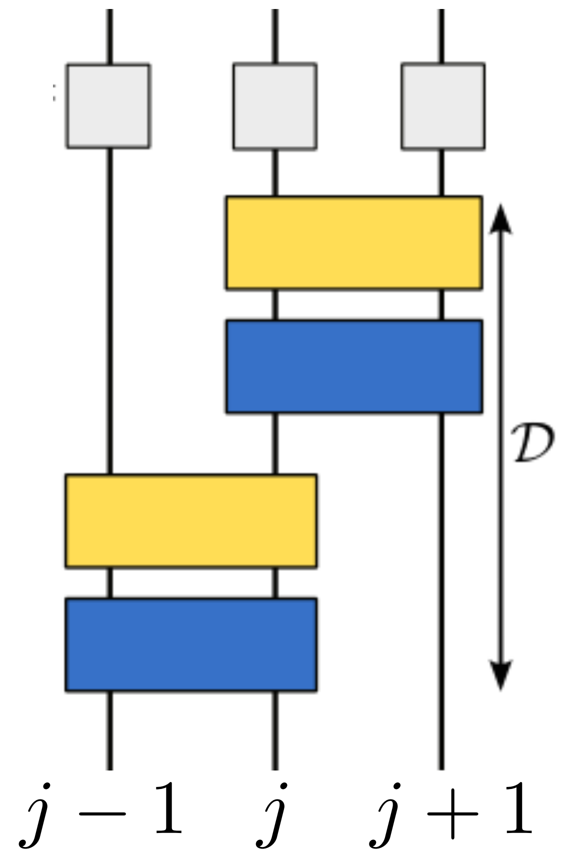
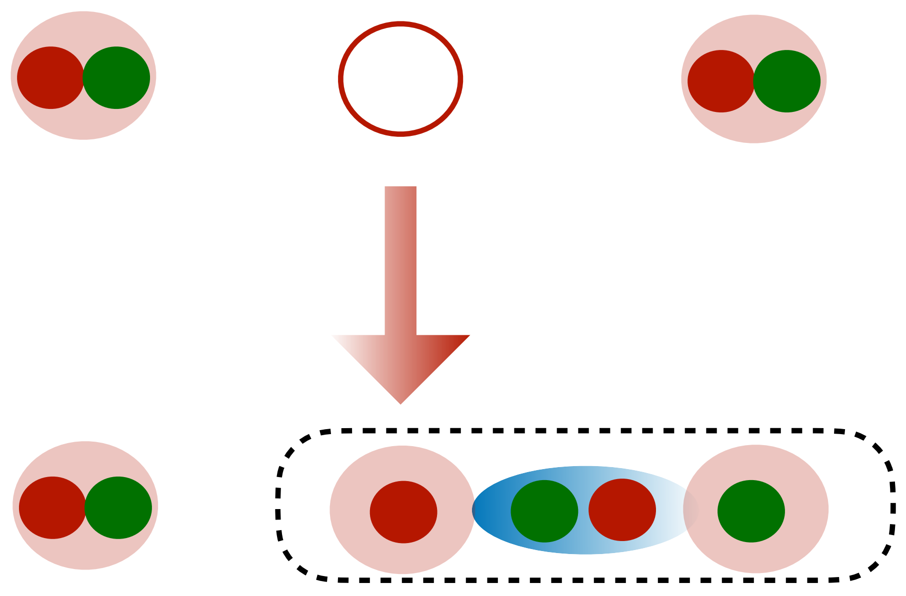
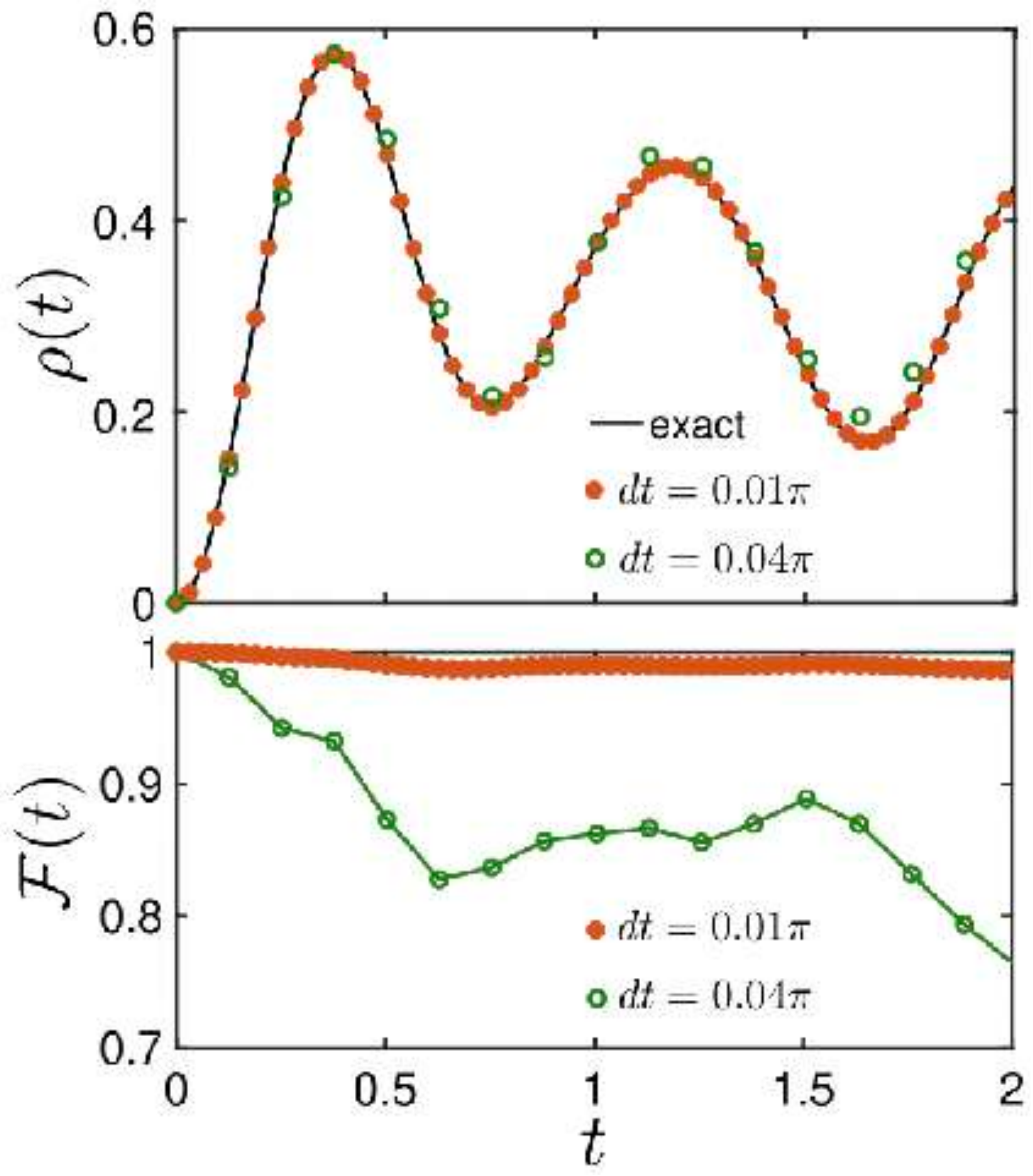


- ***Full simulation including the vibrational mode***

Particle production for 3 sites





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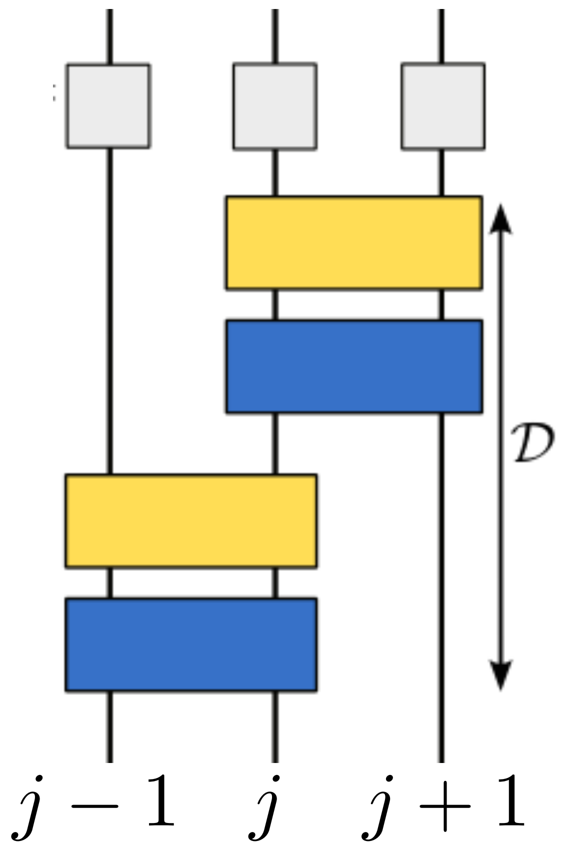

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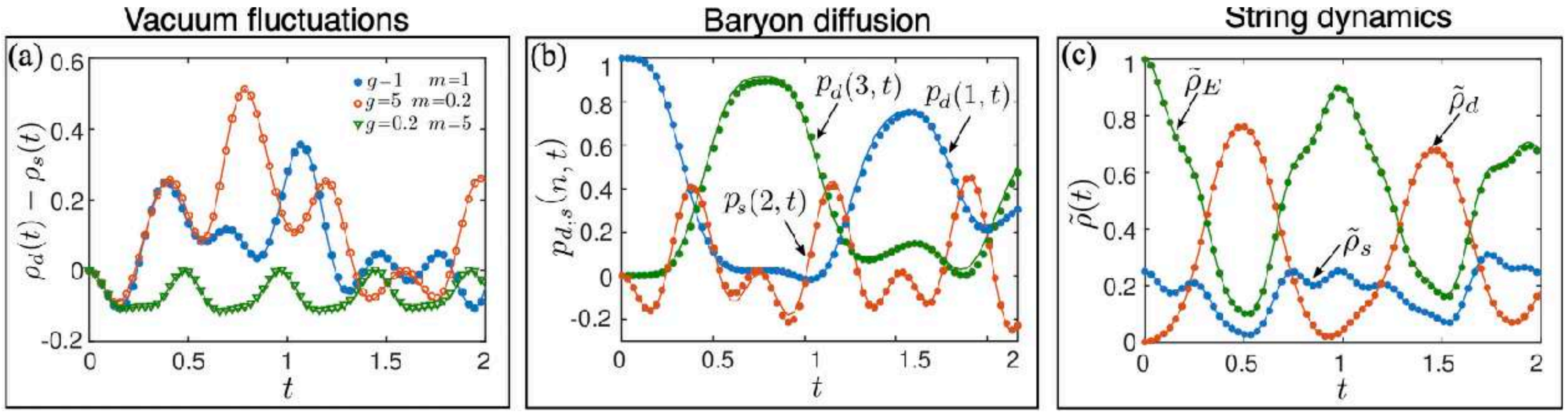
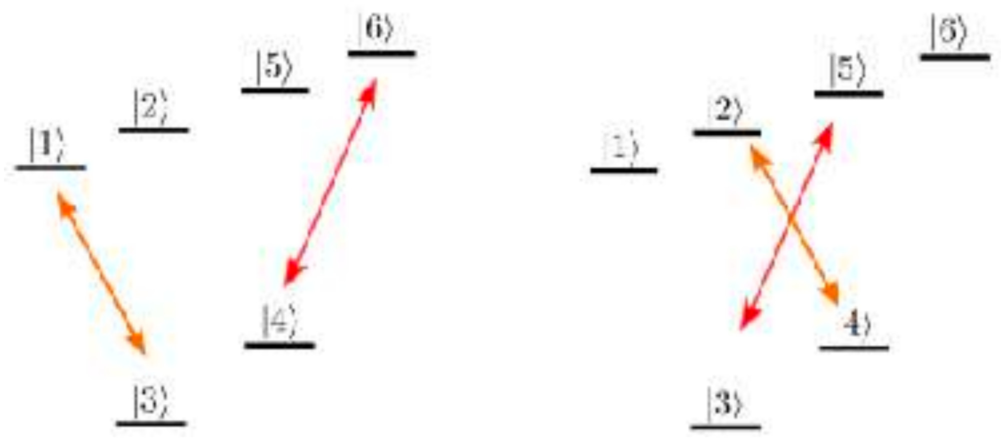


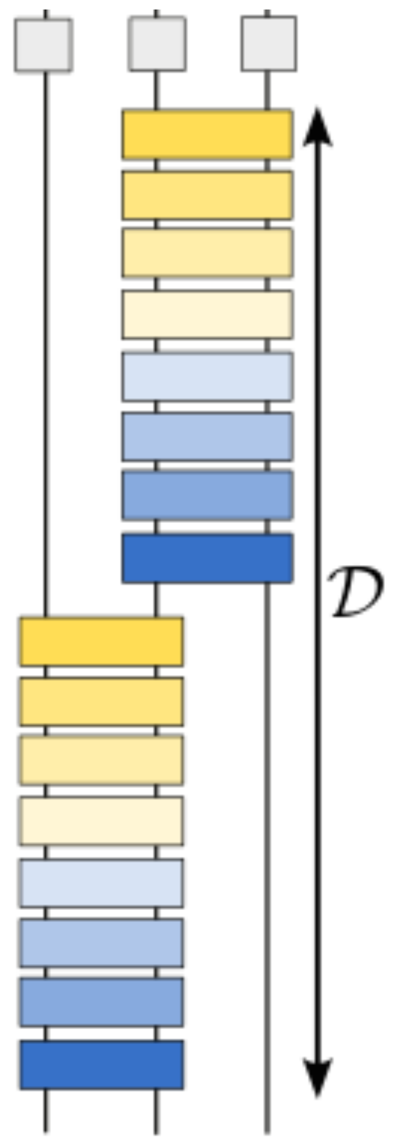
Experimental considerations

- ***Calibration challenges***

Intermediate protocol with two disjoint driven transitions



Larger circuit depth $\mathcal{D} = 16$



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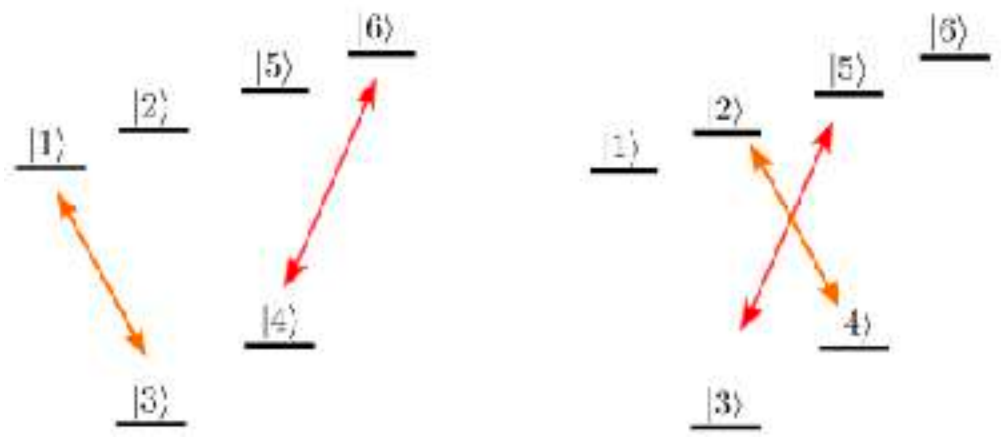

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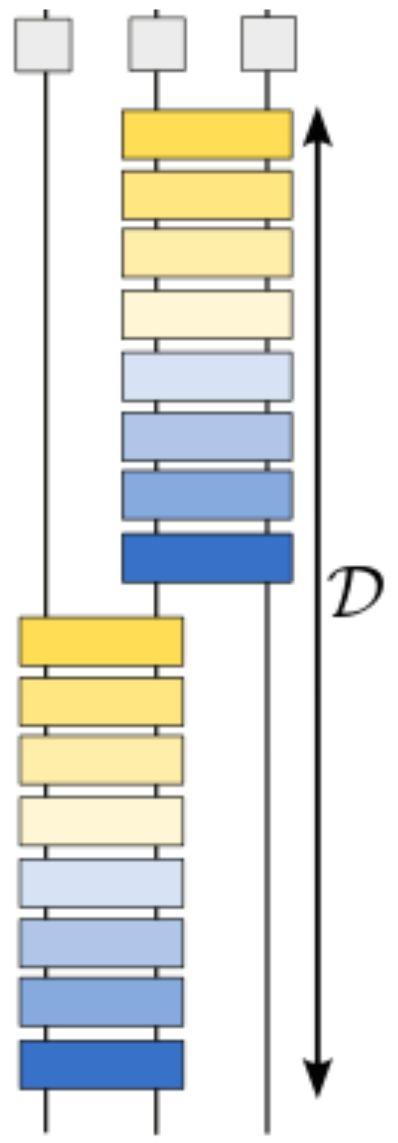
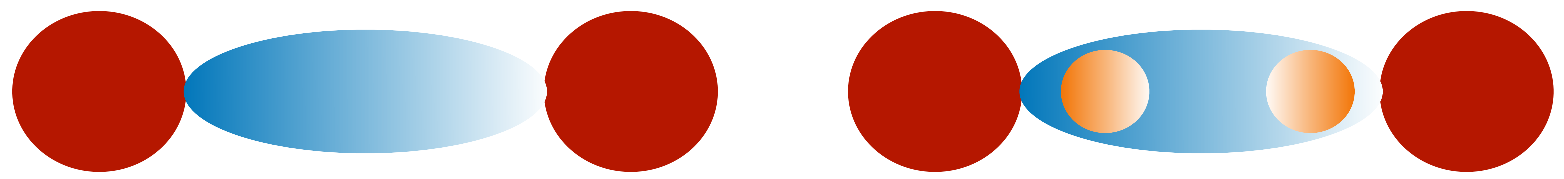


- ***Link parity constrain***

  Mitigated by post selection



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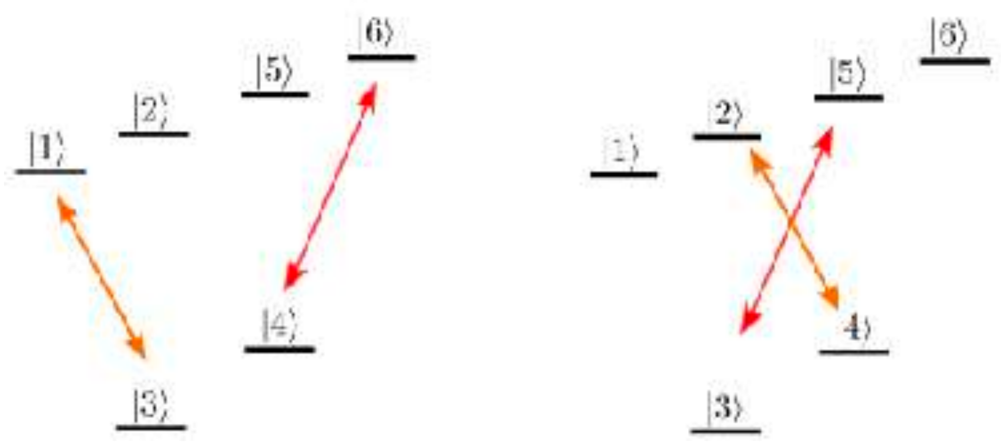

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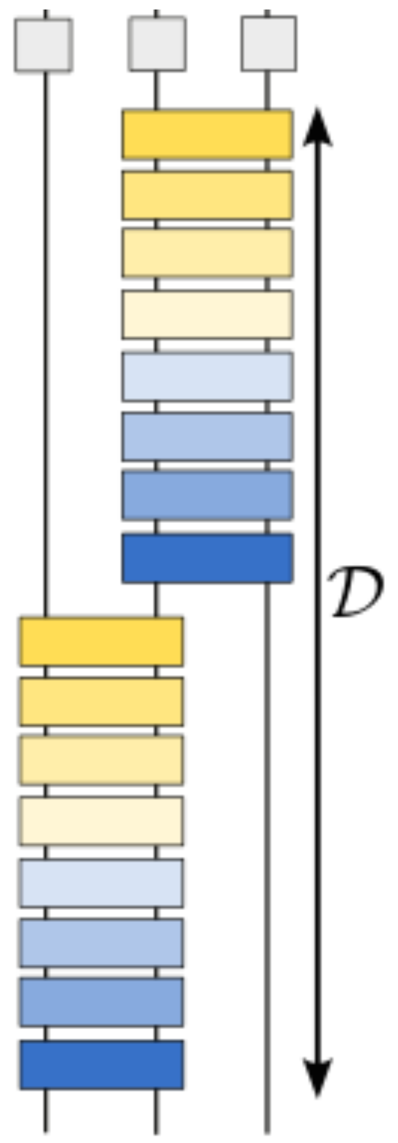

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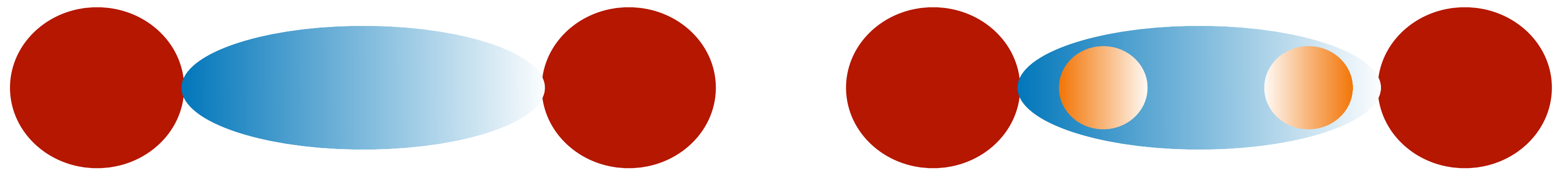

- ***Suzuki-Trotter error***

  Second order ST evolution $\mathcal{D} = 6$

Experimental considerations

- ***Calibration challenges***

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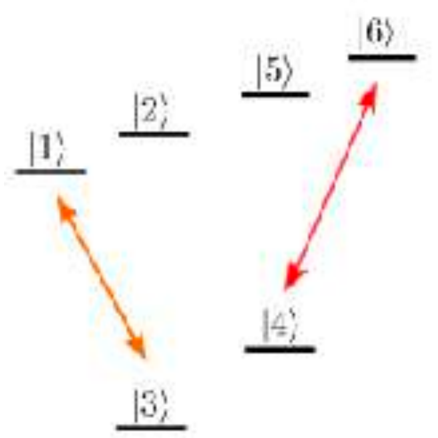

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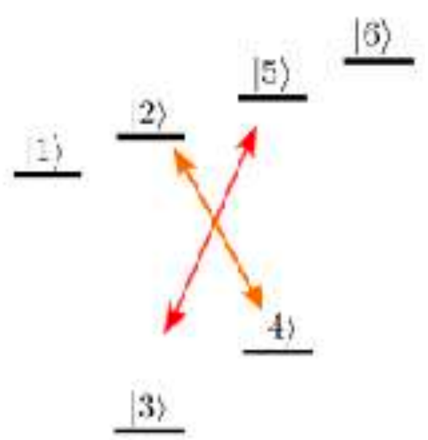

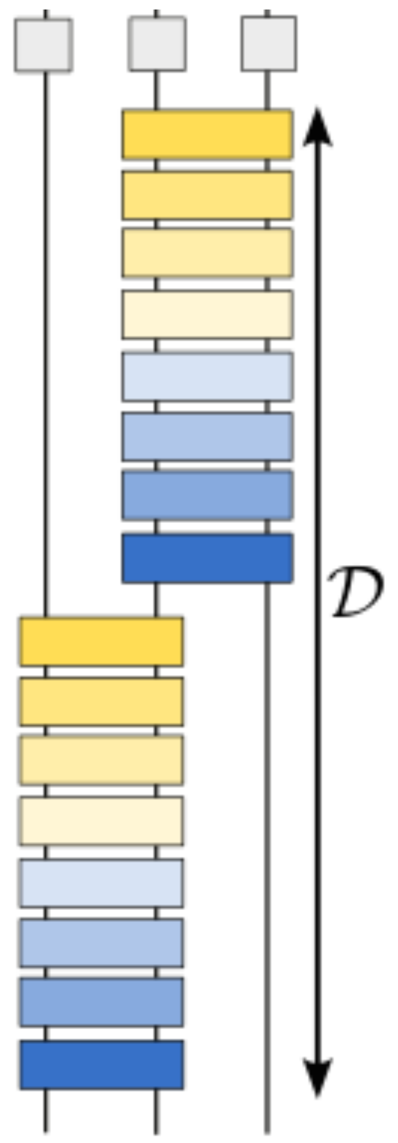

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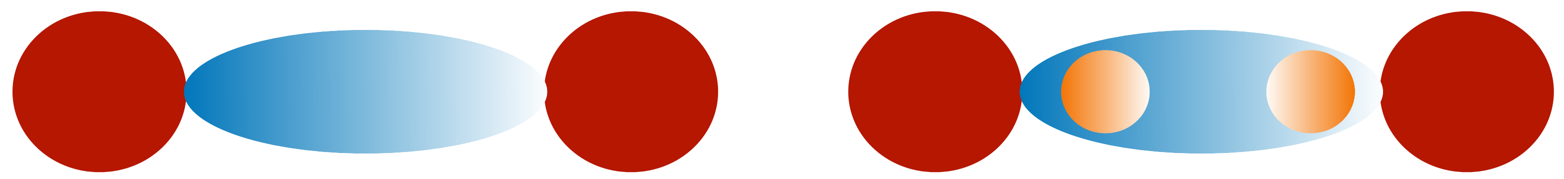


- ***Suzuki-Trotter error***

  Second order ST evolution $\mathcal{D} = 6$

- ***Impact of other errors on the gate fidelity***

  Error estimations

CONCLUSIONS

SUMMARY

- Tensor Network Simulation (2+1)D
- Quantum Sim. Design (1+1)D
- (Not Shown) Scarring Dynamics

OUTLOOK

- Larger truncation of SU(2) group
- Generalised MS-qudit-gates
- Scattering Dynamics
- Local Bases Optimisation

Dipartimento di Fisica e Astronomia Galileo Galilei

INFN

QUANTUM

T-NiSQ

BLANK

BACKUP SLIDES

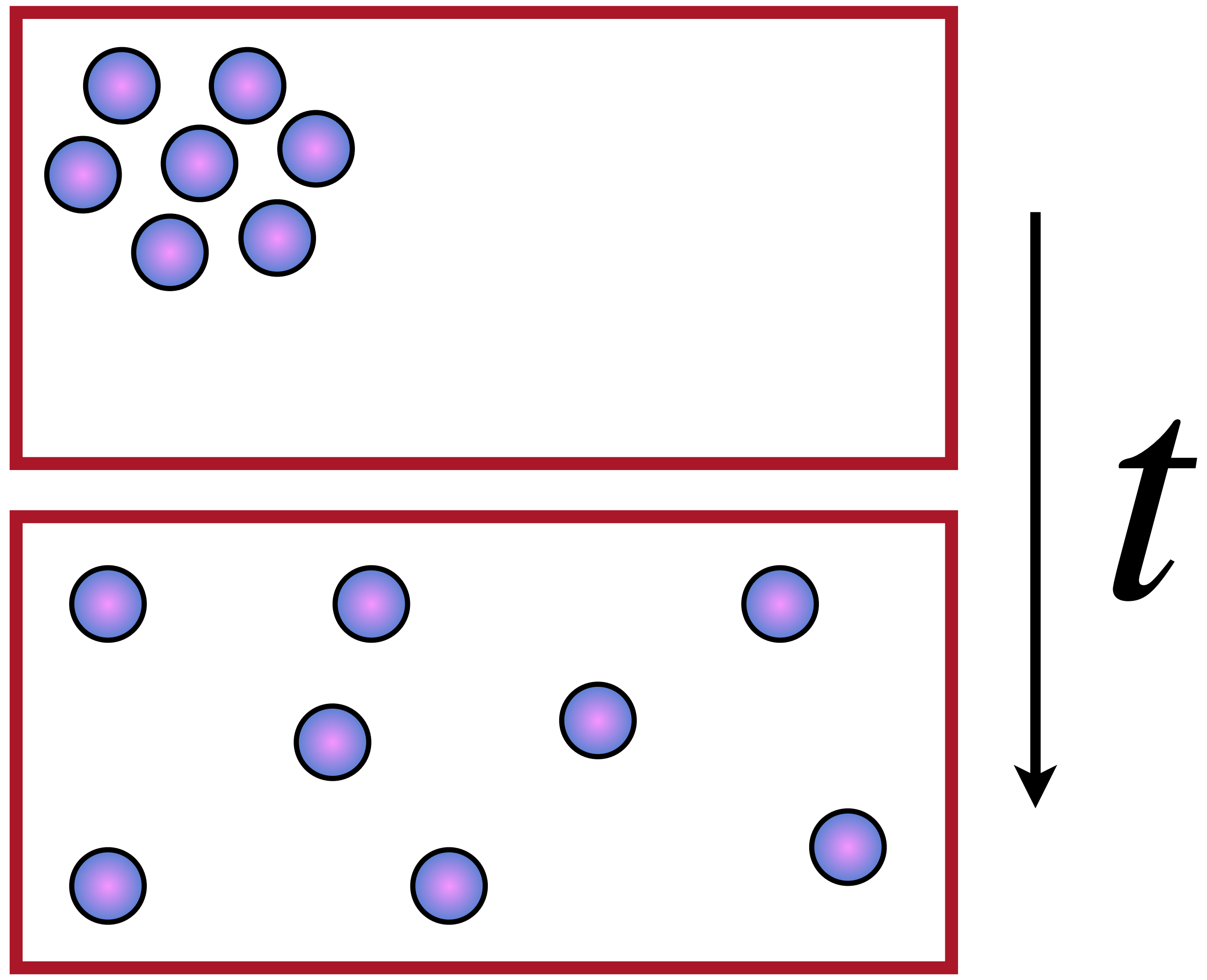
SCARRED DYNAMICS OF THE LATTICE SU(2)YM

THERMALIZATION IN CLASSICAL SYSTEMS

Environment

Isolated Classical System

Exchange of matter | Exchange of energy



Ergodic System

Model covering the whole phase space after a long enough time

Thermalization

Long Time Average

$$\langle O \rangle_t = \lim_{T\to\infty} \int_T dt O(\Gamma(t))$$

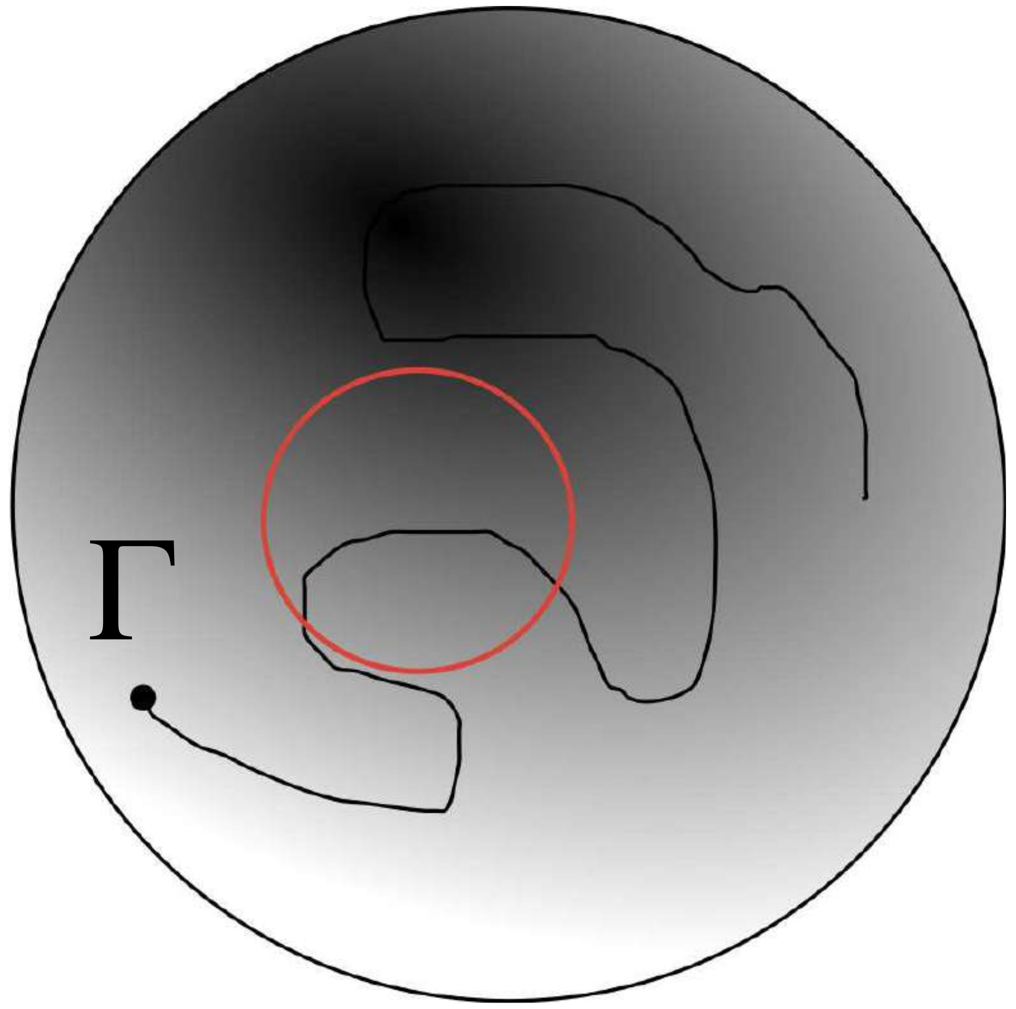
=

Microcanonical Ensemble

$$\langle O \rangle_t = \frac{\int_S O(\Gamma) d\Gamma}{\int_S d\Gamma}$$

Rep. Prog. Phys. 81 082001 (2018)

$$\Gamma = \{x_1, \ldots, x_N, p_1, \ldots, p_N\}$$



Phase space S at constant energy

Integrable System

Model with N invariants of motion displays periodicity. Phase space is partially covered

→ No Thermalization

THERMALIZATION IN QUANTUM SYSTEMS

ENVIRONMENT

ISOLATED QUANTUM SYSTEM

EXCHANGE OF MATTER

EXCHANGE OF ENERGY

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\Psi(0)\rangle = \sum_\alpha C_\alpha |\Phi_\alpha\rangle$$

SCHRODINGER EQUATION

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

$$\langle \hat{O} \rangle_t \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \langle \Psi(t) | \hat{O} | \Psi(t) \rangle$$

LONG TIME AVERAGE

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | e^{it\hat{H}} \hat{O} e^{-it\hat{H}} | \Phi_\beta \rangle$$

$$= \sum_{\alpha,\beta} C_\alpha^* C_\beta \langle \Phi_\alpha | \hat{O} | \Phi_\beta \rangle \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i(E_\alpha - E_\beta)t} dt$$

$$= \sum_\alpha |C_\alpha|^2 \langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle = \langle \hat{O} \rangle_{\mathrm{DE}}$$

DEPENDS ON THE INITIAL STATE!

DIAGONAL ENSAMBLE AVERAGE

SMALL ENERGY SHELL

$$\delta E = \sqrt{\langle \Psi | \hat{H}^2 | \Psi \rangle - \langle \Psi | \hat{H} | \Psi \rangle^2}$$

STATISTICAL MECHANICS

$$\langle \hat{O} \rangle_{\mathrm{ME}} \equiv \mathrm{Tr}\{\hat{O}\hat{\varrho}_{\mathrm{ME}}\} = \sum_{\substack{\alpha \\ |E_\alpha - E| < \delta E}} \frac{\langle \Phi_\alpha | \hat{O} | \Phi_\alpha \rangle}{N_{E,\delta E}}$$

MICROCANONICAL ENSAMBLE AVERAGE

LONG-TIME AVERAGE $\neq$ STATISTICAL MECHANICS

BUT THERMALIZATION IS OBSERVED EXPERIMENTALLY IN ISOLATED COLD ATOMIC GASES FOR MANY INITIAL STATES!

REP. PROG. PHYS. 81 082001 (2018)

EIGENSTATE THERMALIZATION HYPOTHESIS

IDEA: INDIVIDUAL ENERGY EIGENSTATES BEHAVE LIKE THERMAL STATES

HYPOTHESIS I

N BODY SYSTEM WITH NON-DEGENERATE SPECTRUM

$$H = \sum_\alpha E_\alpha |\Phi_\alpha\rangle$$

$$|\psi(t)\rangle = \sum_\alpha C_\alpha e^{-iE_\alpha t} |\Phi_\alpha\rangle$$

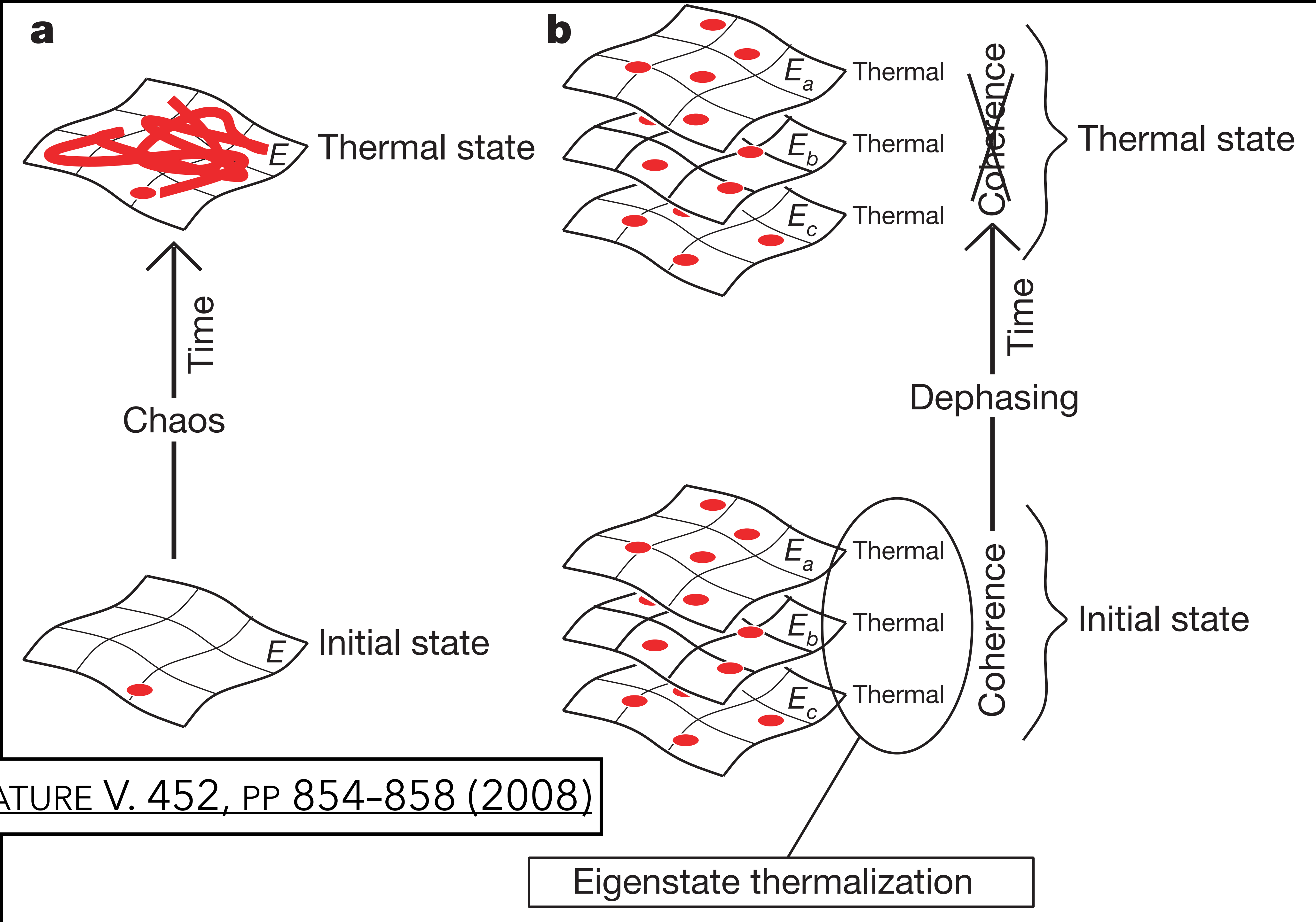
$$\langle H \rangle = \bar{E}$$

$$\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} \ll \bar{E}$$

HYPOTHESIS II

$O_{\alpha\alpha}$ IS THERMAL ~ CONSTANT VALUE IN A SMALL WINDOW $\forall \alpha \; |E_\alpha - \bar{E}| < \Delta E$

$$O_{\alpha\alpha} = \langle \Phi_\alpha | O | \Phi_\alpha \rangle = \langle O \rangle_{\text{thm}}$$



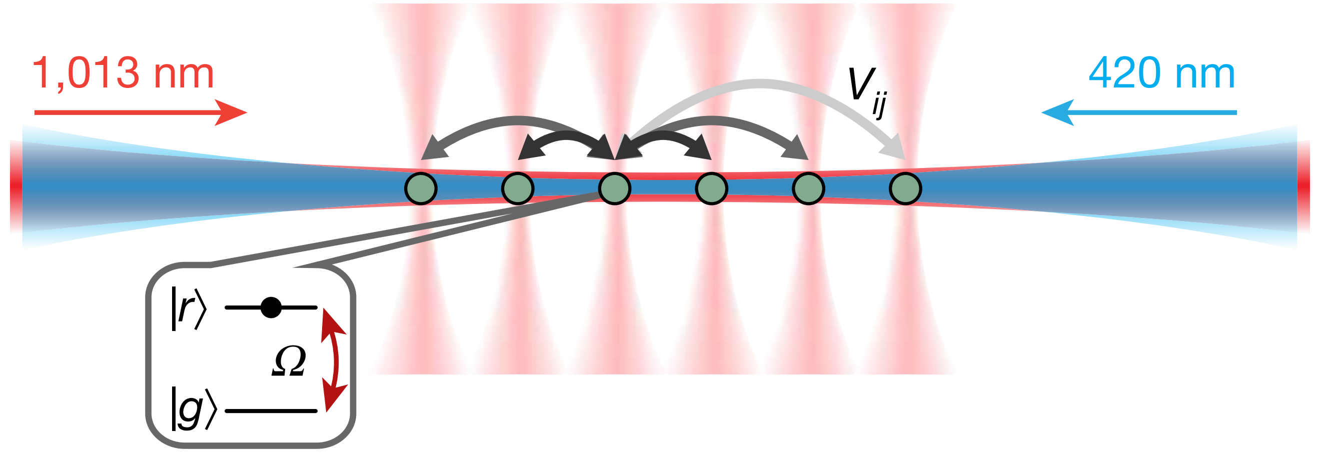
NATURE V. 452, PP 854-858 (2008)

CONSEQUENCE

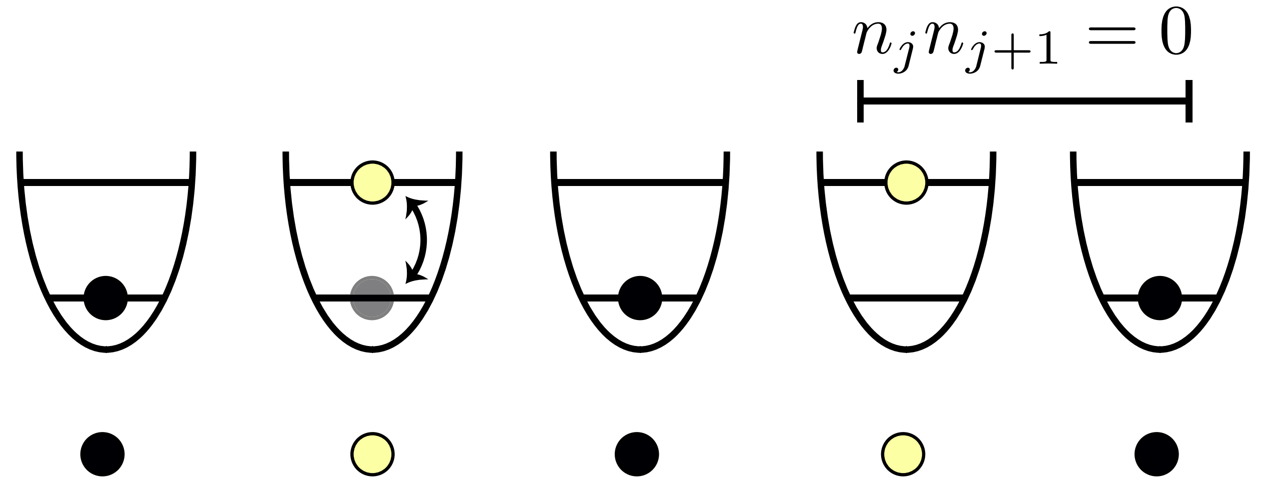
$$\langle O(t) \rangle = \sum_\alpha^N |c_\alpha|^2 O_{\alpha\alpha} \sim \langle O \rangle_{\text{thm}} \sim \sum_\alpha^{N_{E,\Delta E}} \frac{O_{\alpha\alpha}}{N_{E,\Delta E}} = \langle O \rangle_{\text{ME}}$$

DIAGONAL = MICROCANONICAL

QUANTUM MANYBODY SCARS

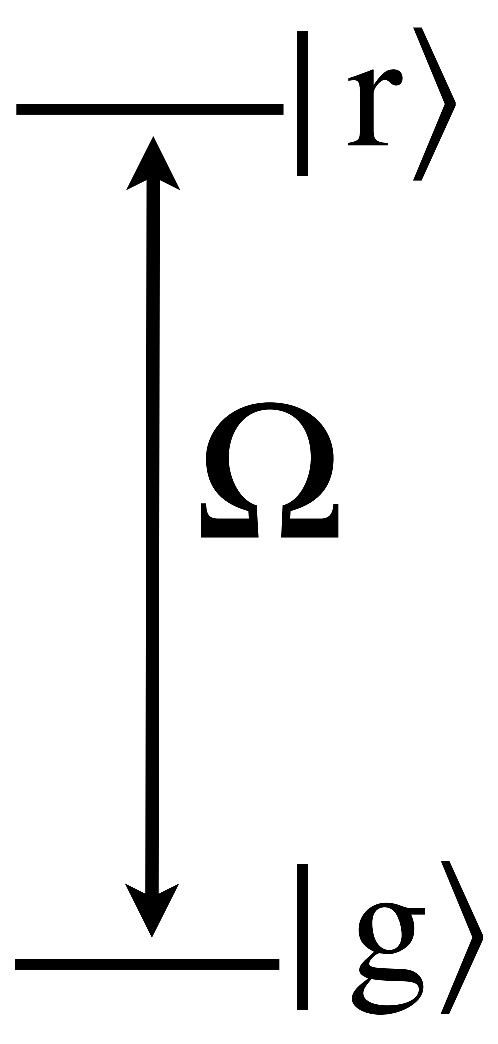


$$H_{\text{Ryd}} = \sum_j (\Omega\sigma_j^x + \Delta\sigma_j^z) + \sum_{j<k} V_{jk} n_j n_k$$

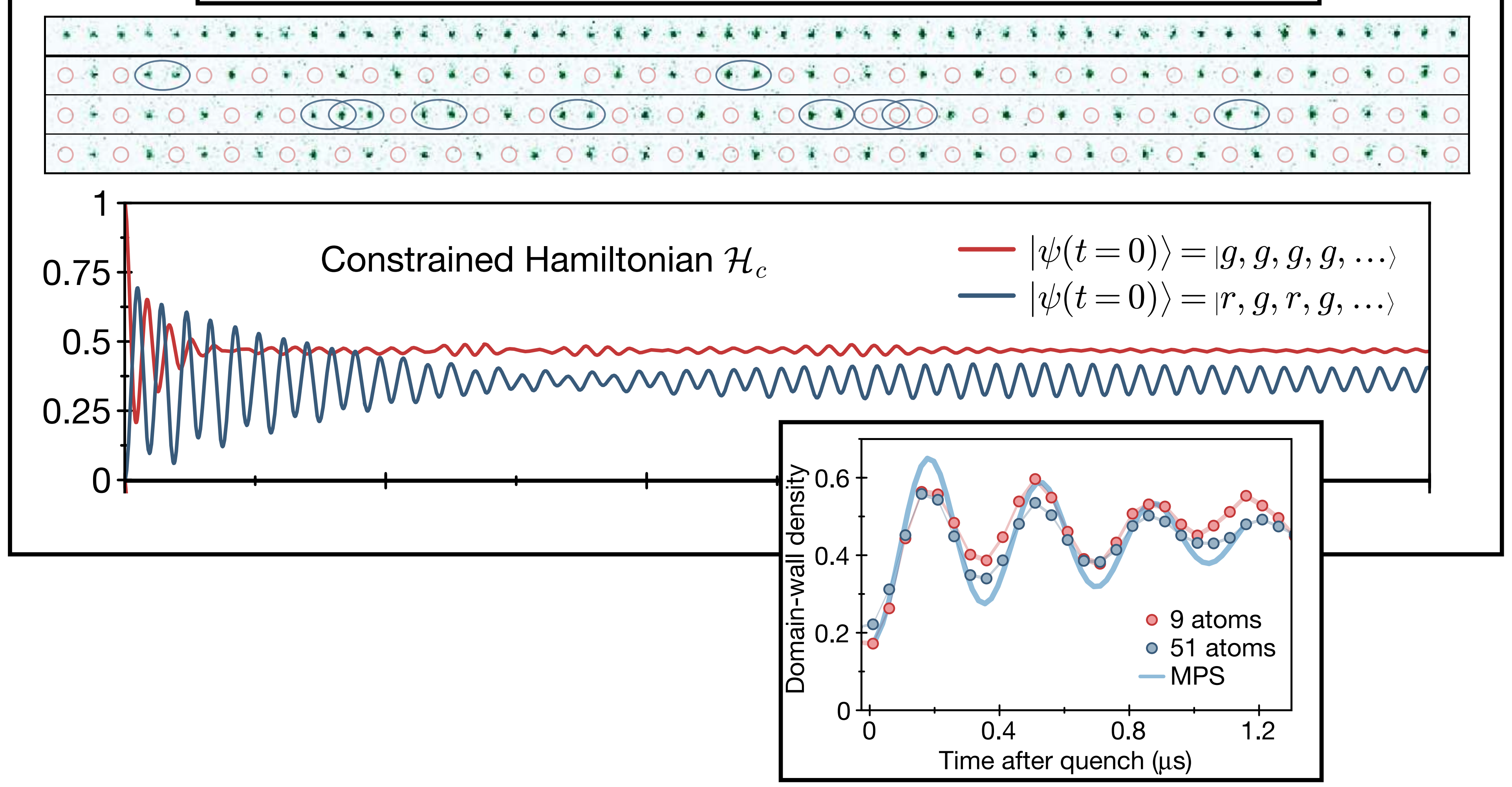


BLOCKADE: $V_{jk} \gg \Omega, \Delta$

$$H_{\text{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j)$$



BERNIEN ET AL, NATURE 551, PP 579-584 (2017)

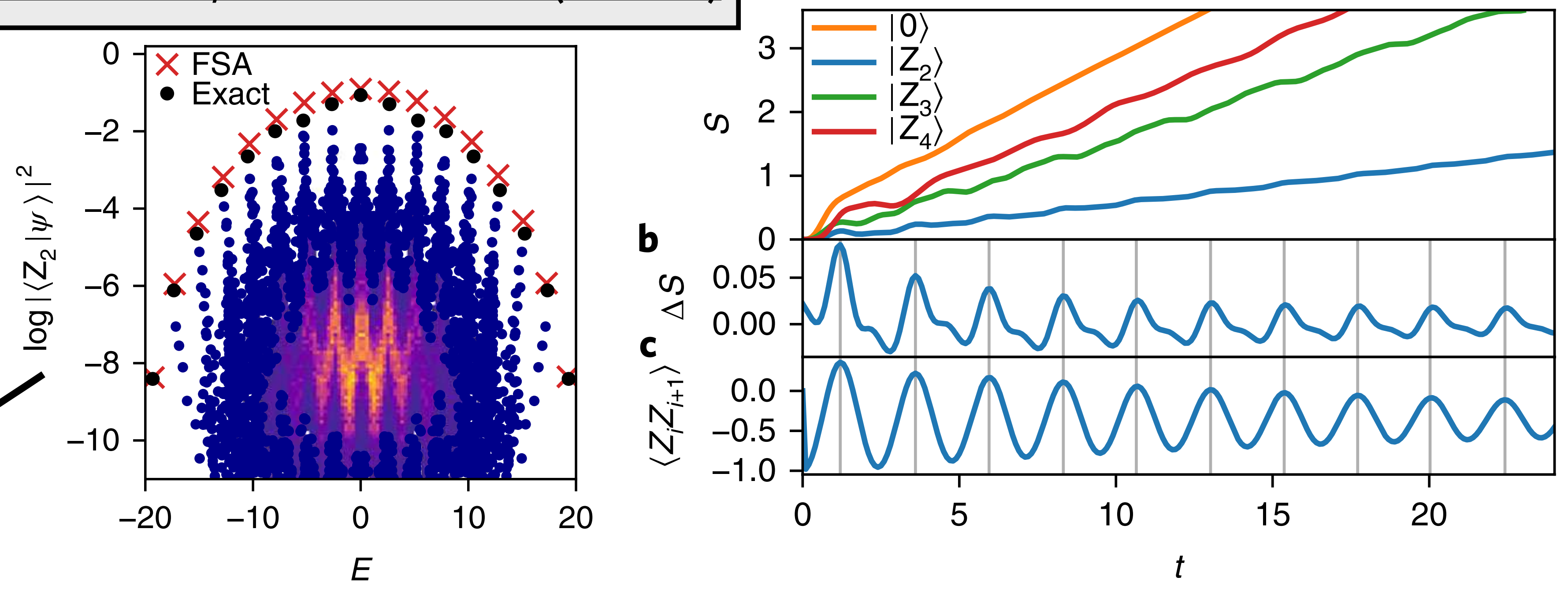


QUENCH DYNAMICS FROM
$|0\rangle = |\bullet\bullet\bullet\bullet\bullet\bullet\bullet\ldots\rangle$:
THERMALIZATION

QUENCH DYNAMICS FROM
$|\mathbb{Z}_2\rangle = |\bullet\circ\bullet\circ\bullet\circ\bullet\ldots\rangle$:
LONG-LIVED OSCILLATIONS

[1] PERSISTENT DYNAMICS
SPECIFIC INITIAL STATES
FROM AN HILBERT SUBSPACE

NAT. PHYS. 14, PP 745-749 (2018)



$$H = \sum_j P_j \sigma_{j+1}^x P_{j+2} \quad \text{with} \quad \mathrm{P} = (1-\sigma^z)/2$$

[3] TOWER OF STATES
IN THE EIGENSTATES
SPECTRUM

PHYS. REV. LETT. 122, 040603

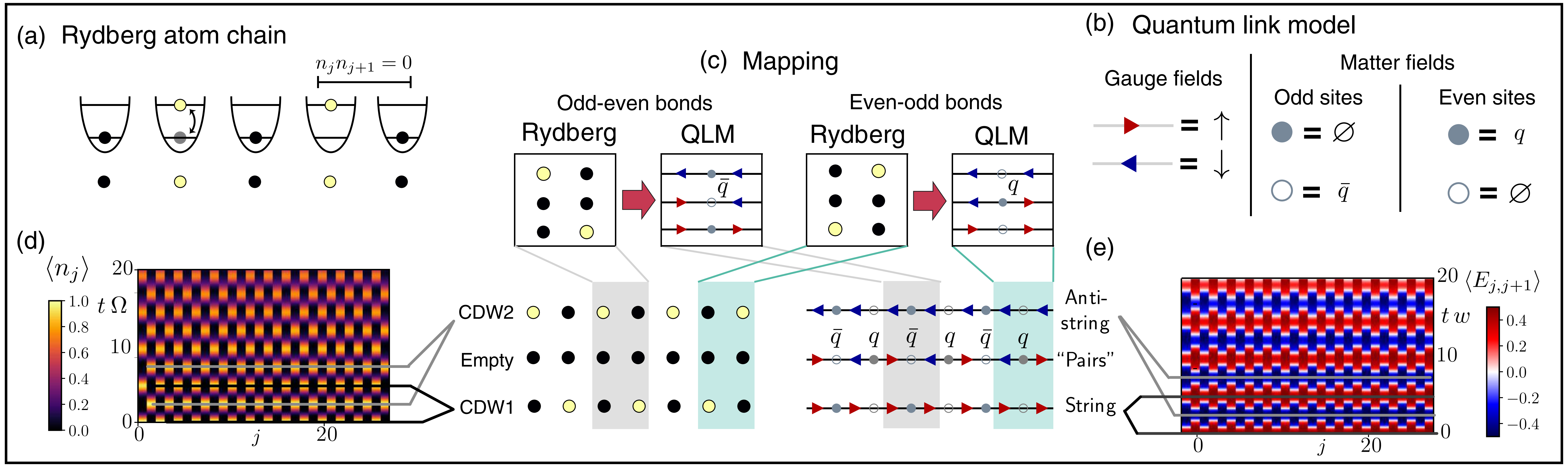
PHYS. REV. LETT. 122, 173401

[2] SLOW OSCILLATING
GROWTH OF
ENTANGLEMENT ENTROPY

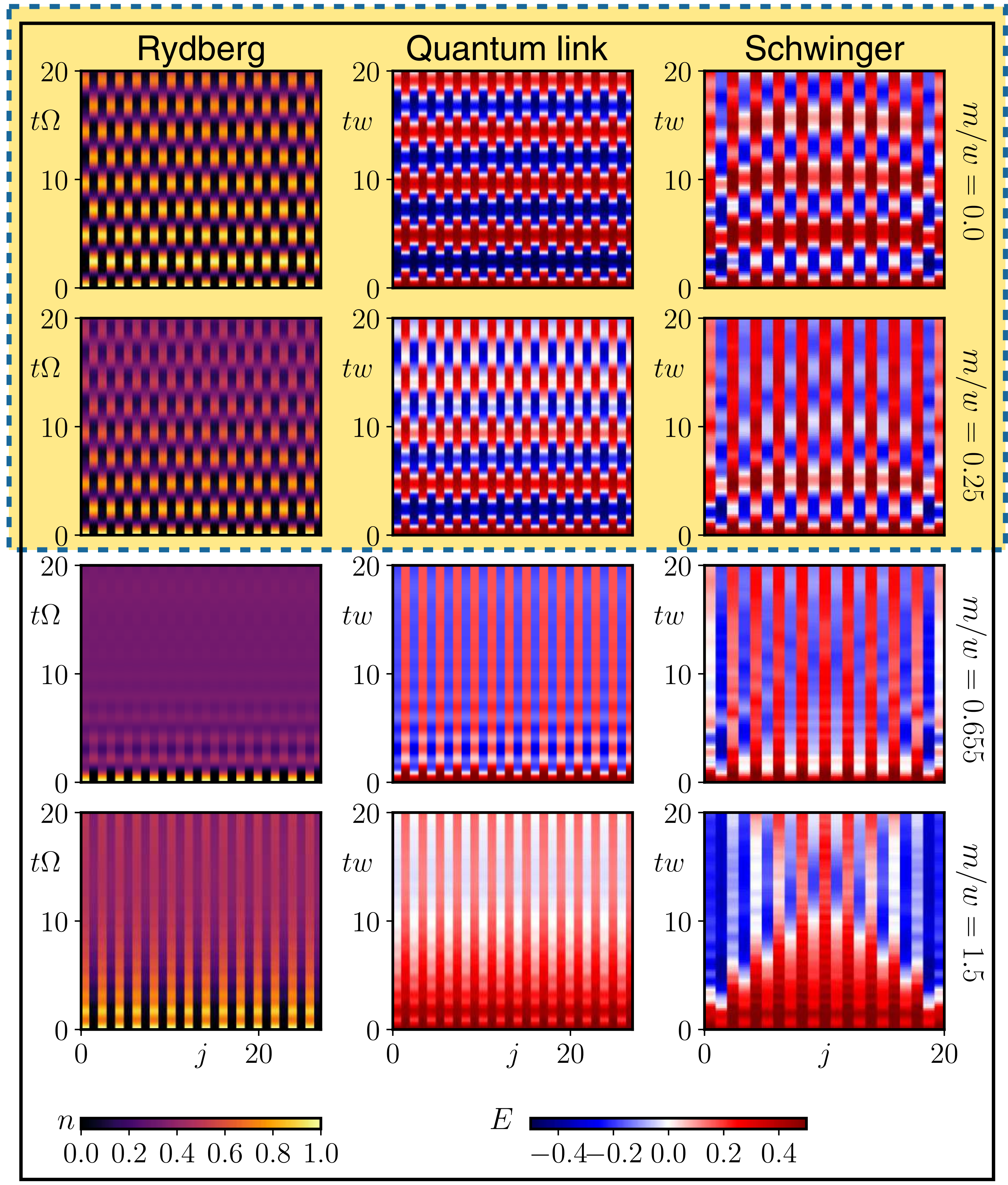
SCARS IN ABELIAN GAUGE THEORIES

$$H_{\mathrm{FSS}} = \sum_j (\Omega\sigma_j^x + 2\Delta n_j) \longleftrightarrow H_{\mathrm{QLM}}^{\mathrm{U(1)}} = -w\sum_{j,\mu}\left[\psi_j^\dagger U_{j,j+\mu}\psi_{j+\mu} + \mathrm{H.c.}\right] + m\sum_j (-1)^j\psi_j^\dagger\psi_j + J\sum_j E_{j,j+\mu}^2$$

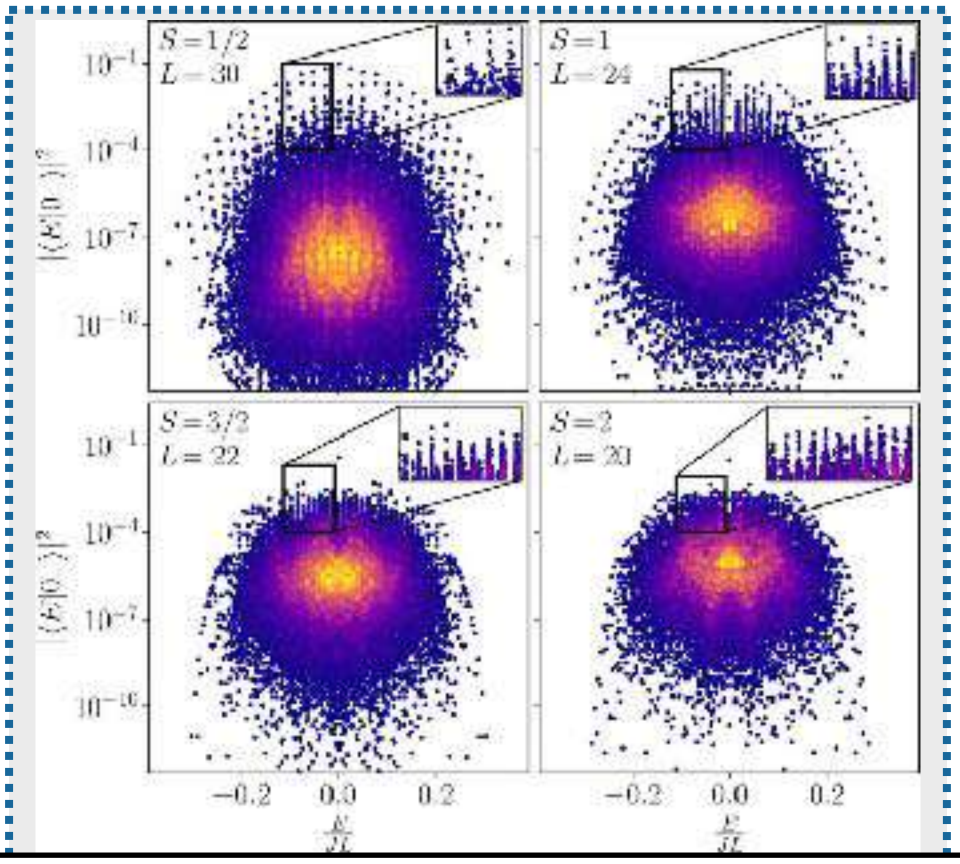
STRING INVERSION
THERMALIZATION SLOWDOWN



SURACE ET AL, PRX 10, 021041 (2020)

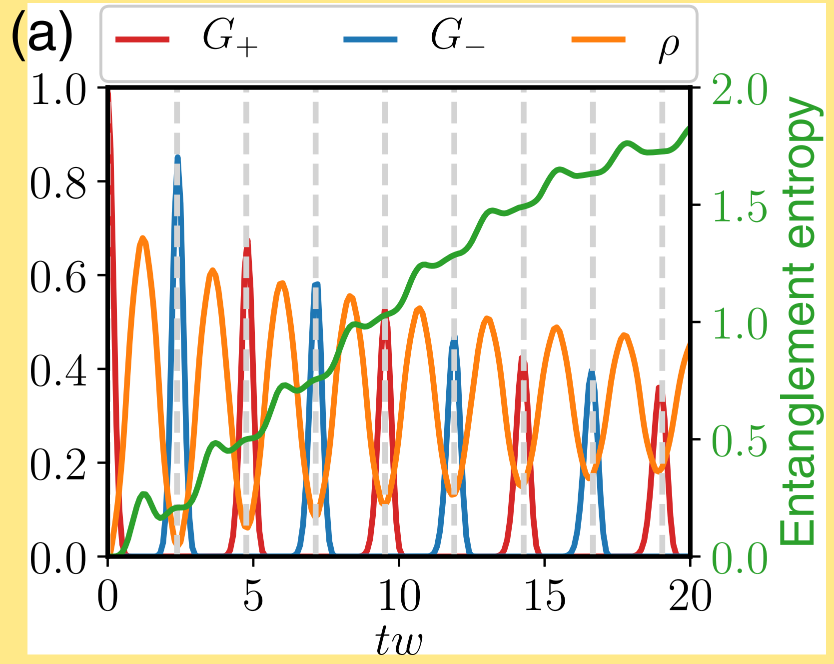


SCARS IN NONABELIAN LGTS?



HALIMEH ET AL, PRB 107, L201105 (2023)

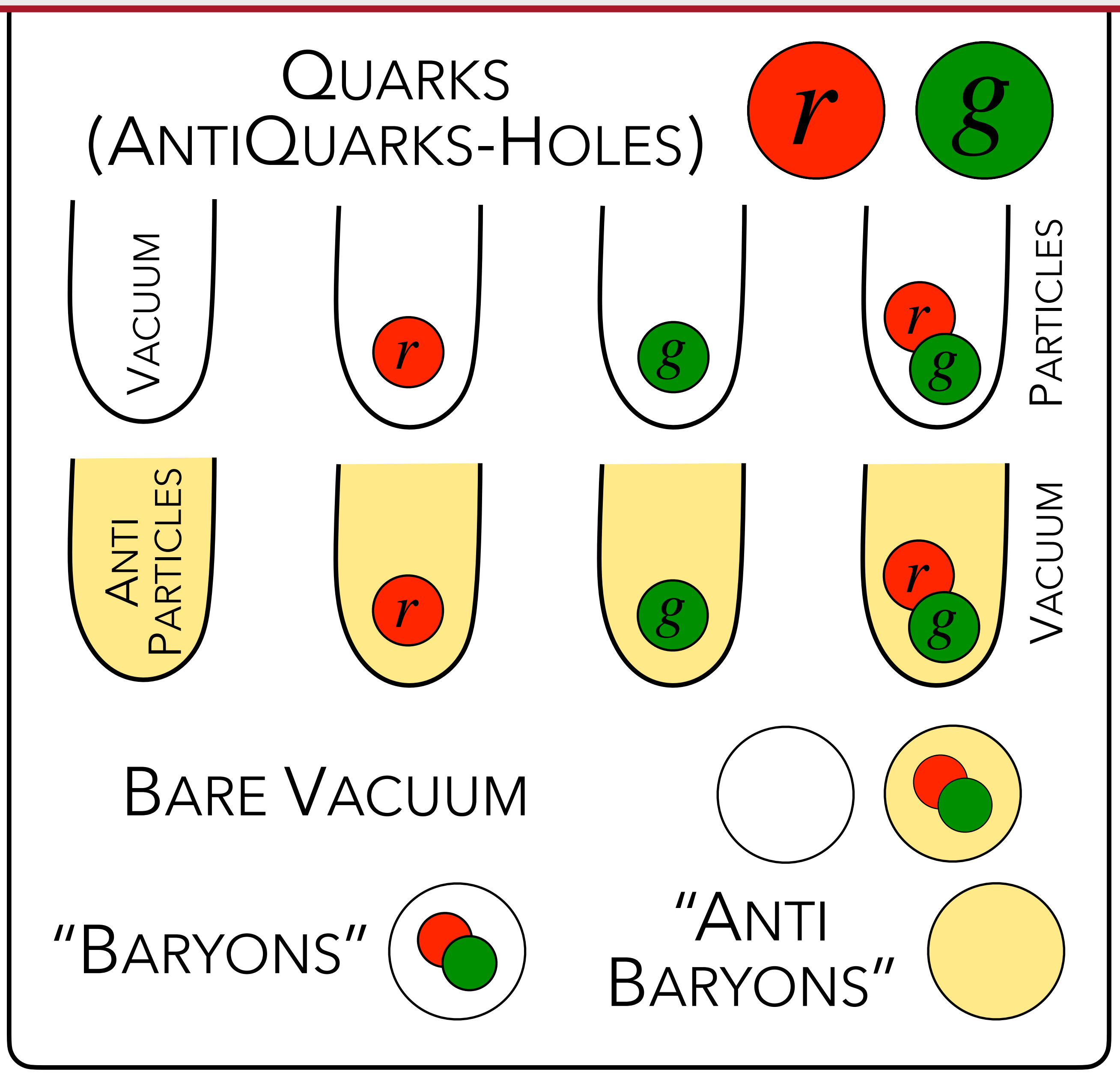
OSCILLATING FIDELITY & ENTANGLEMENT ENTROPY

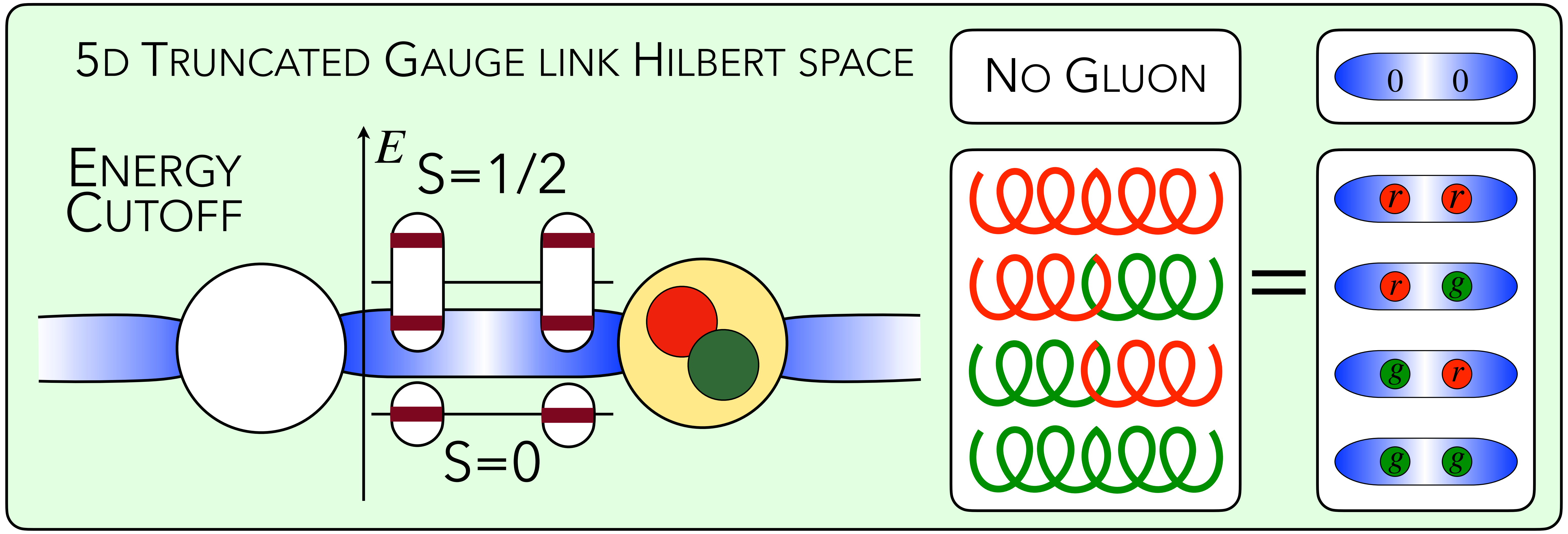


1D HARDCORE–GLUON SU(2) YANG MILLS LGT

$$H = \frac{1}{2a}\sum_{\alpha,\beta}\sum_{j,\mu}\left[\psi^{\dagger}_{j,\alpha}U^{\alpha\beta}_{j,j+\mu}\psi_{j+\mu,\beta} + \text{H.c.}\right] + m_0\sum_{j}(-1)^j\sum_{\alpha}\psi^{\dagger}_{j,\alpha}\psi_{j,\alpha} + \frac{ag^2}{2}\sum_{j}E^2_{j,j+\mu}$$

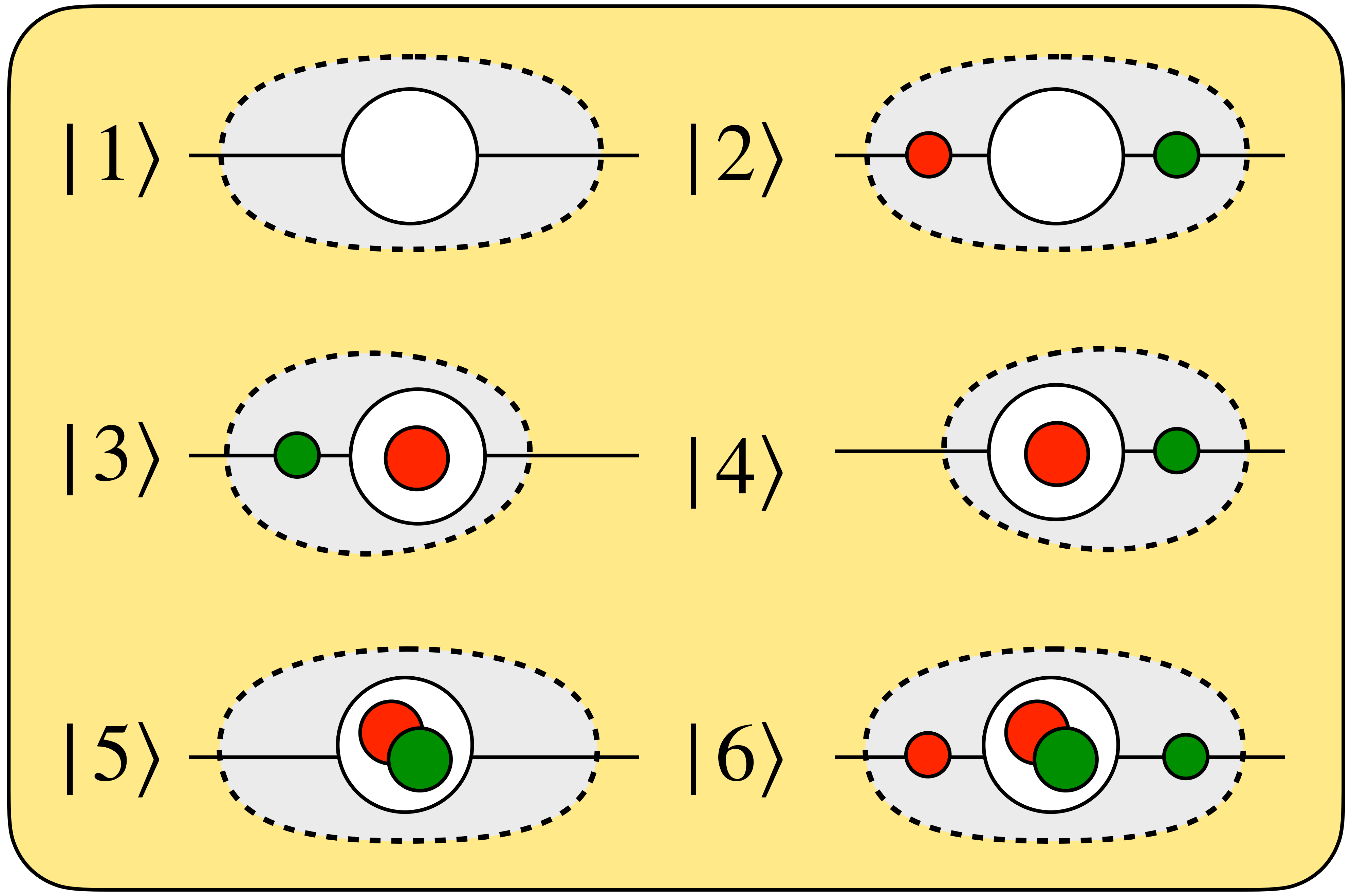
4D Matter Hilbert space
SU(2)-color 1/2 Flavorless Dirac Fermions

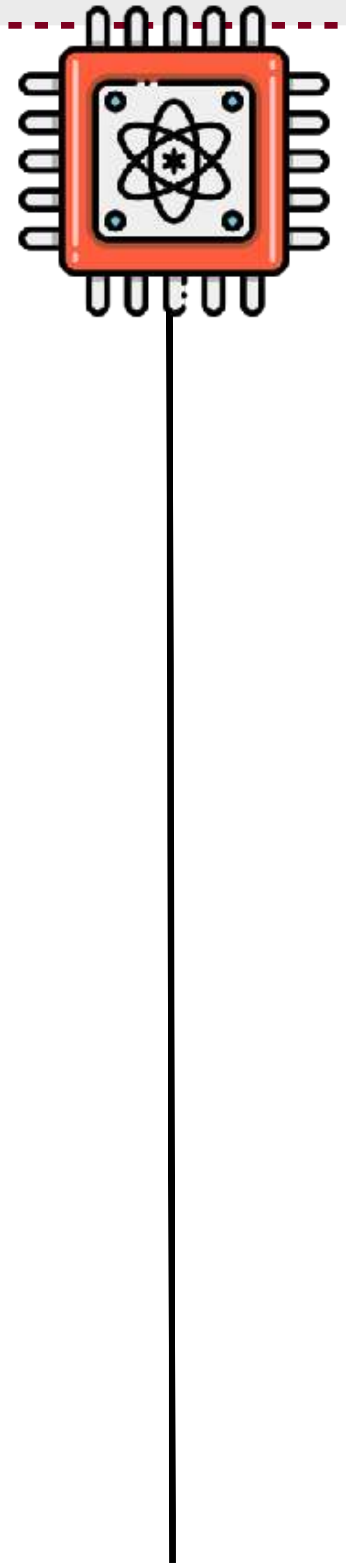




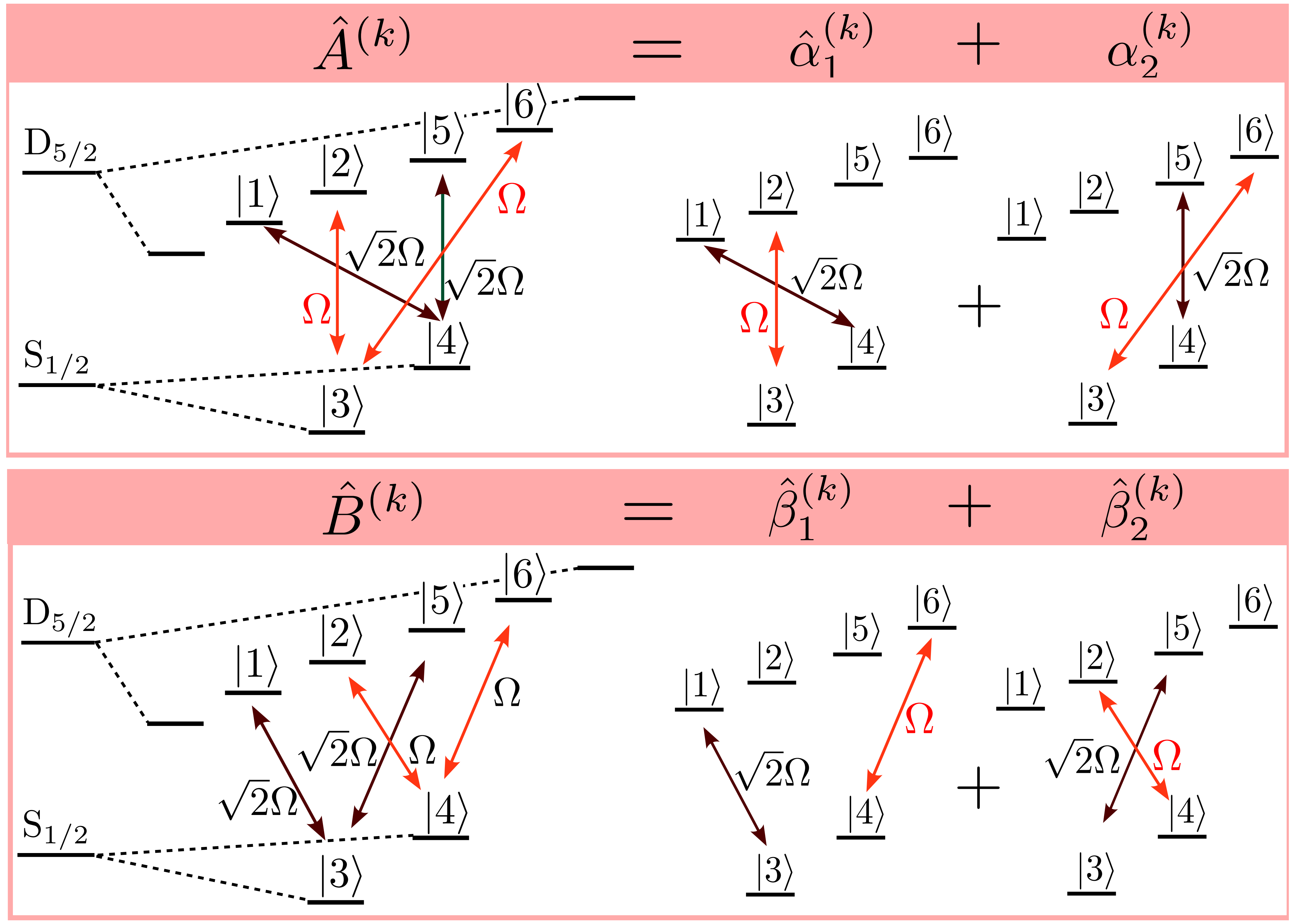
GAUGE INVARIANT QUDIT MODEL

6 SU(2) GAUGE INVARIANT STATES (COLOR SINGLETS)
6-QUDIT MODEL ON A QUANTUM COMPUTER



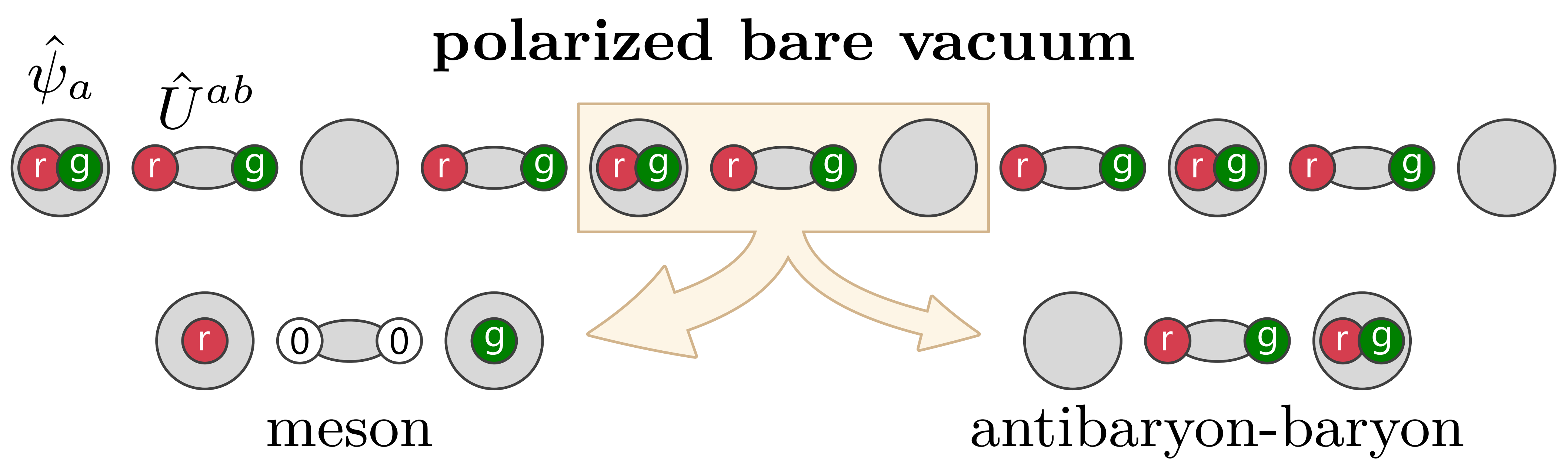


$^{40}Ca^{+}$



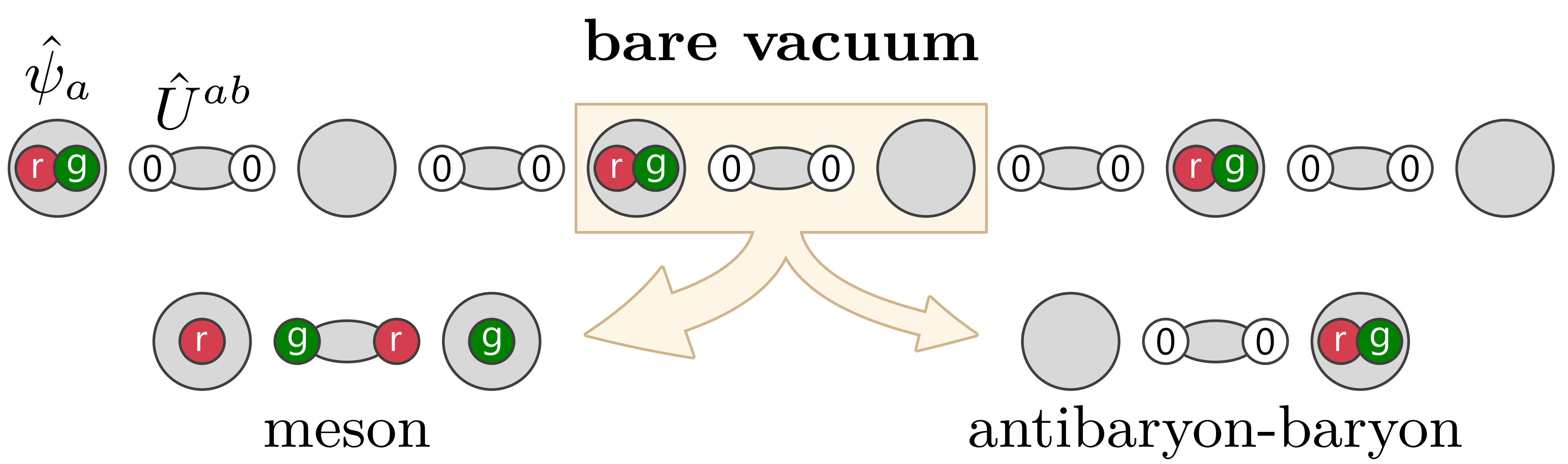
CALAJO' ET AL, ARXIV:2402.07987 (2024)

INITIAL STATE CANDIDATES FOR DYNAMICS



$$|\Psi_{\mathrm{PV}}\rangle = |\ldots 2020\ldots\rangle_{\mathrm{m}} \left| \cdots \frac{rg - gr}{\sqrt{2}} \cdots \frac{rg - gr}{\sqrt{2}} \cdots \right\rangle_{\mathrm{g}}$$

$$= |6\rangle|2\rangle\ldots|6\rangle|2\rangle\ldots|6\rangle|2\rangle$$



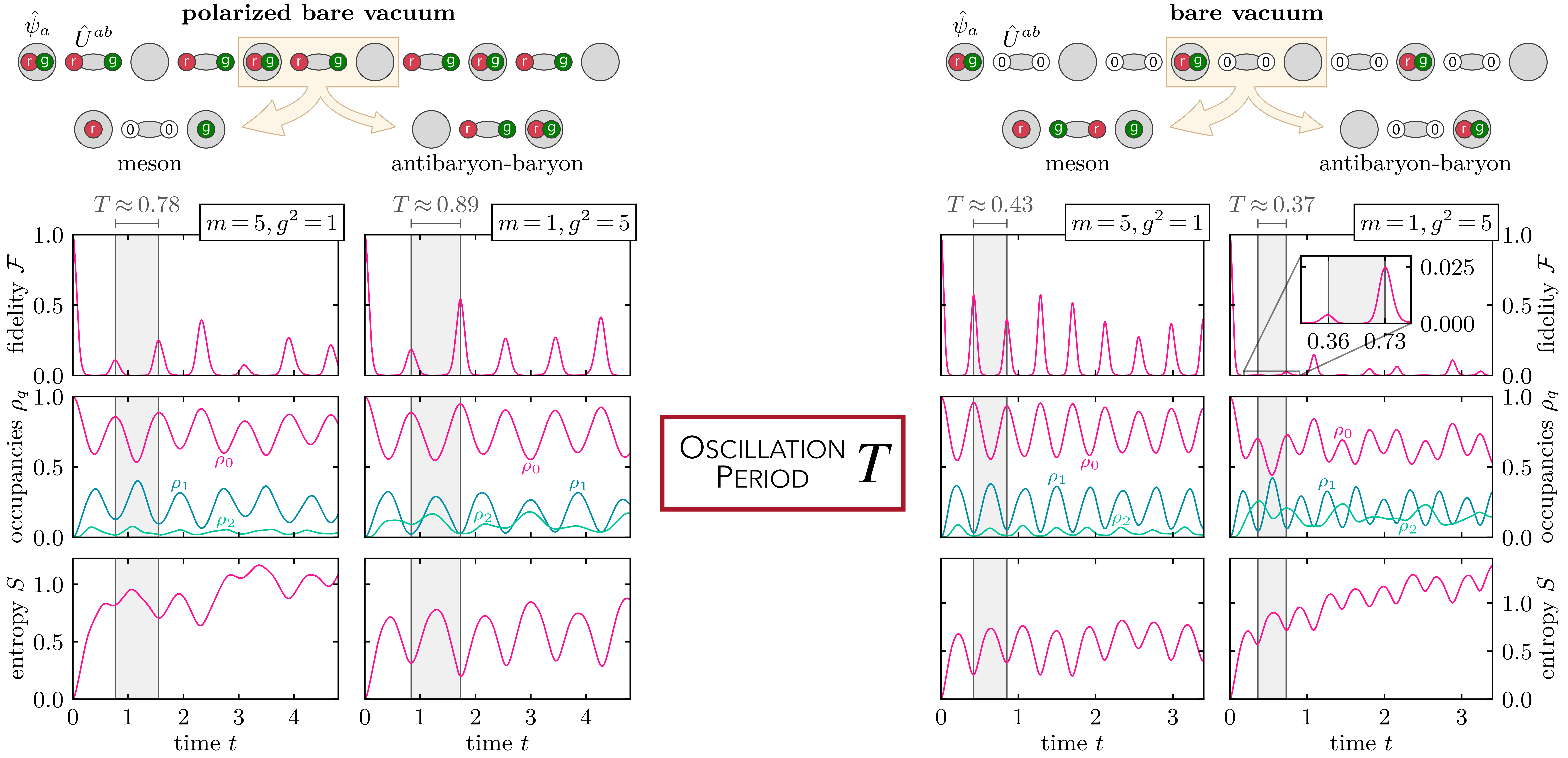
$$|\Psi_{\mathrm{V}}\rangle = |\ldots 2020\ldots\rangle_{\mathrm{m}} |\ldots 000\ldots\rangle_{\mathrm{g}}$$

$$= |5\rangle|1\rangle\ldots|5\rangle|1\rangle\ldots|5\rangle|1\rangle$$

Product States: easily implemented in TN and Quantum Simulators

SCARRING DYNAMICS

"UNIPD PEOPLE", JAD HALIMEH
ARXIV:2405.13112 (2024)



ERGODIC VS NON ERGODIC BEHAVIOR I

MICROCANONICAL ENSEMBLE (ME) STATE

$$|\Psi_{\mathrm{PV}}^{\mathrm{ME}}\rangle = \frac{1}{\sqrt{N_{E_{\mathrm{PV}},\delta E}}} \sum_{\substack{s \\ |E_s - E_{\mathrm{PV}}| < \delta E}} |\Phi_s\rangle$$

$\sqrt{N_{E_{\mathrm{PV}},\delta E}}$: NUMBER OF EIGENSTATES IN THE SHELL

$|E_s - E_{\mathrm{PV}}| < \delta E$: SMALL ENERGY SHELL AROUND E_{PV}

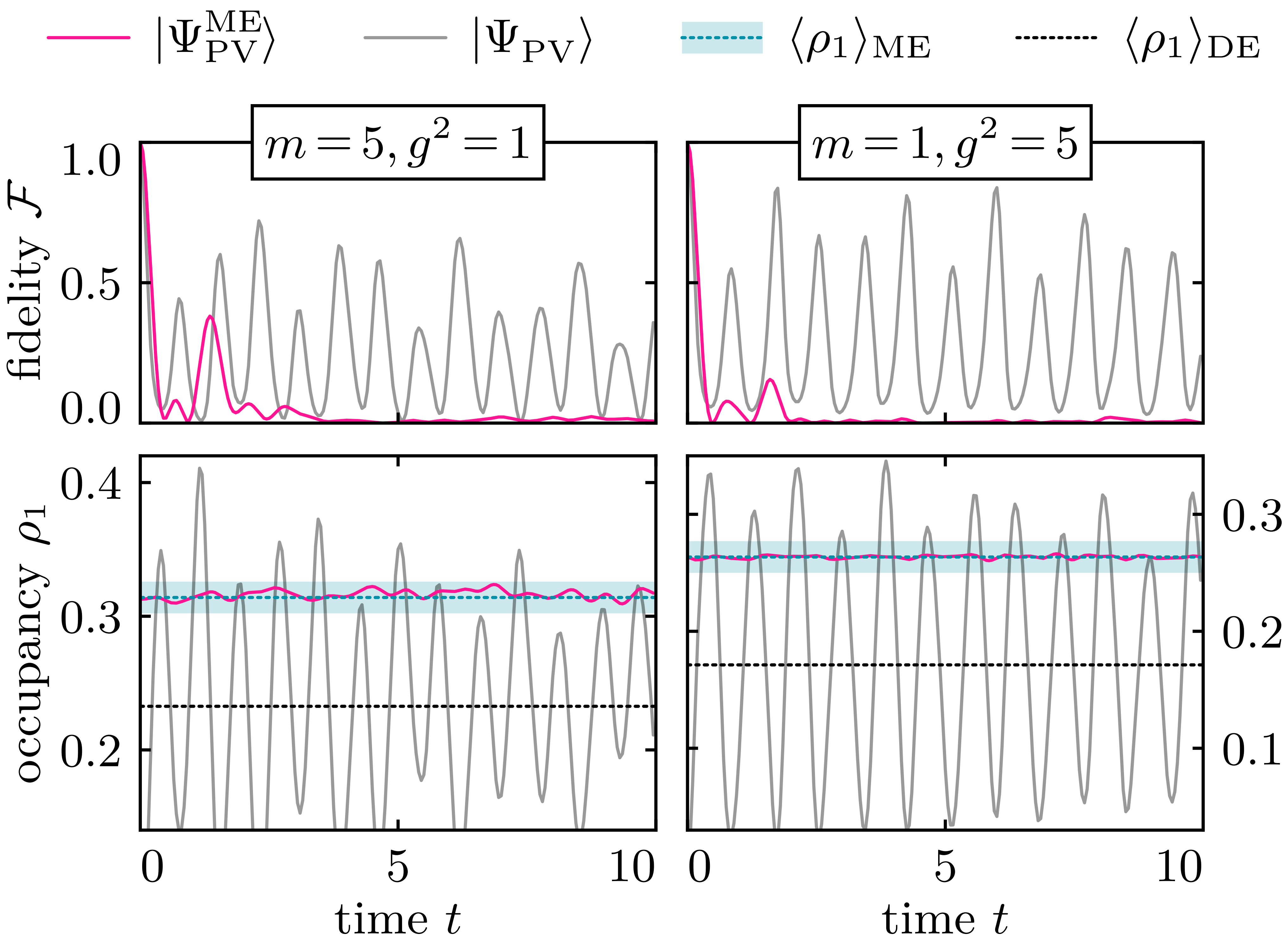
$$\delta E = \sqrt{\langle \Psi_{\mathrm{PV}} | \hat{H}^2 | \Psi_{\mathrm{PV}} \rangle - E_{\mathrm{PV}}^2}$$

$|\Phi_s\rangle$: SPECTRUM EIGENSTATES

SCARRING DYNAMICS VIOLATES ETH
(AND HENCE STATISTICAL MECHANICS)

NO SCARS

FOR OTHER INITIAL STATES
IN THE SAME PARAMETER REGIMES



Rishon representation

Truncated Link operator

$$\hat{U}^{ab} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cccc} 0 & +\delta_{ar}\delta_{bg} & -\delta_{ar}\delta_{br} & +\delta_{ag}\delta_{bg} & -\delta_{ag}\delta_{br} \\ \hline -\delta_{ag}\delta_{br} & 0 & & & \\ -\delta_{ag}\delta_{bg} & & 0 & & \\ +\delta_{ar}\delta_{br} & & & 0 & \\ +\delta_{ar}\delta_{bg} & & & & 0 \end{array} \right)$$

E. Zohar and M. Burrello, PRD 91 054506 (2015)

$$\left[\hat{L}^{(\nu)}_{n,n+1}, \hat{U}^{ab}_{n',n'+1}\right] = -\delta_{nn'} \sum_c \frac{\sigma^{(\nu)}_{ac}}{2} \hat{U}^{cb}_{n,n+1}$$

$$\left[\hat{R}^{(\nu)}_{n,n+1}, \hat{U}^{ab}_{n',n'+1}\right] = \delta_{nn'} \sum_c \hat{U}^{ac}_{n,n+1} \frac{\sigma^{(\nu)}_{cb}}{2}$$

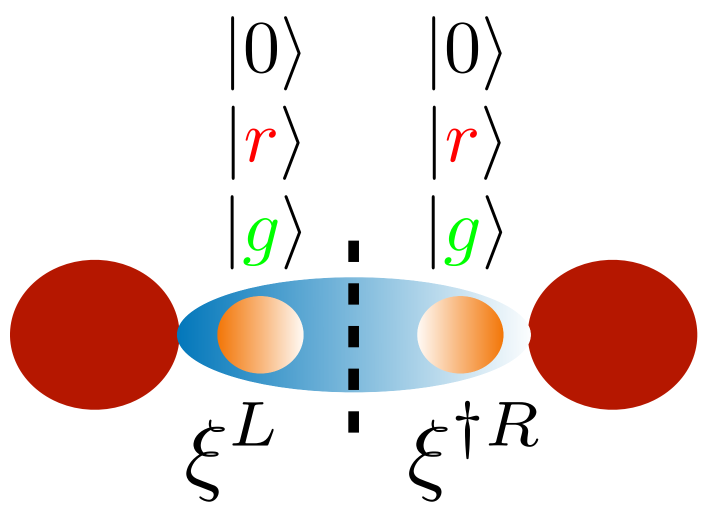
Left and right Generators of gauge transformation on the link

$$\hat{L}^{(\nu)}_{n,n+1} \qquad \hat{R}^{(\nu)}_{n,n+1}$$

chromoelectric basis $|j m_L m_R\rangle$

Rishon operators

$$\hat{\zeta}_r = \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right)_F \qquad \hat{\zeta}_g = \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)_F$$



Parity operator

$$P_\zeta = \left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & -1 & 0 \\ 0 & 0 & -1 \end{array} \right).$$

Rishon decomposition

$$\hat{U}^{ab}_{j,j+1} = \frac{1}{\sqrt{2}} \left(\zeta_a\right)_{jL} \left(\zeta_b^\dagger\right)_{j+1,R}$$

$(0,0)$

$|00\rangle$

$\left(\frac{1}{2}, \frac{1}{2}\right)$

$|\uparrow\uparrow\rangle |\downarrow\downarrow\rangle |\downarrow\uparrow\rangle |\uparrow\downarrow\rangle$

TOPOLOGY

$$P_{j,j+\mu} = \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{pmatrix}$$

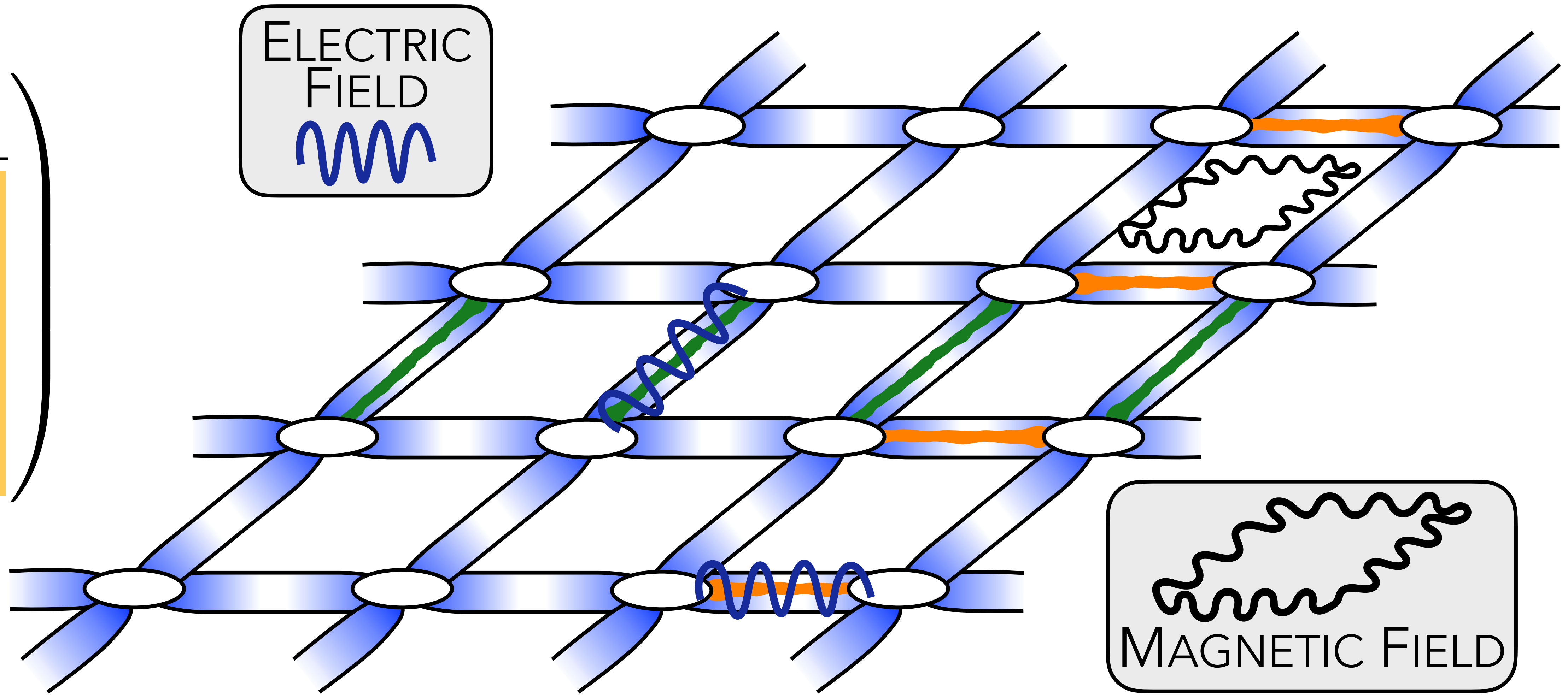
LINK PARITY

0 (INTEGER) SU(2) REPRESENTATION

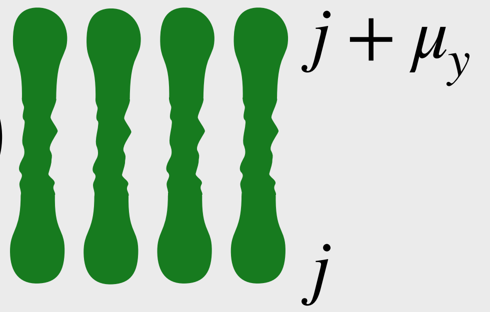
1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

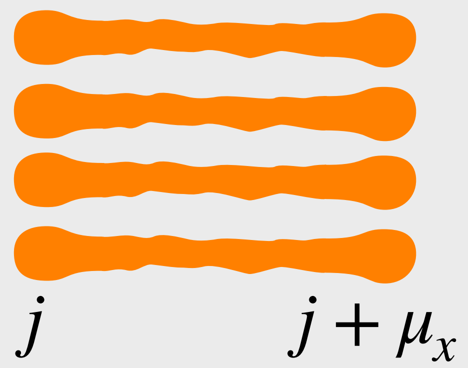
$$\left[P_{j,j+\mu}, E^2_{j,j+\mu}\right] = 0$$

$$\left\{P_{j,j+\mu}, U_{j,j+\mu}\right\} = 0$$

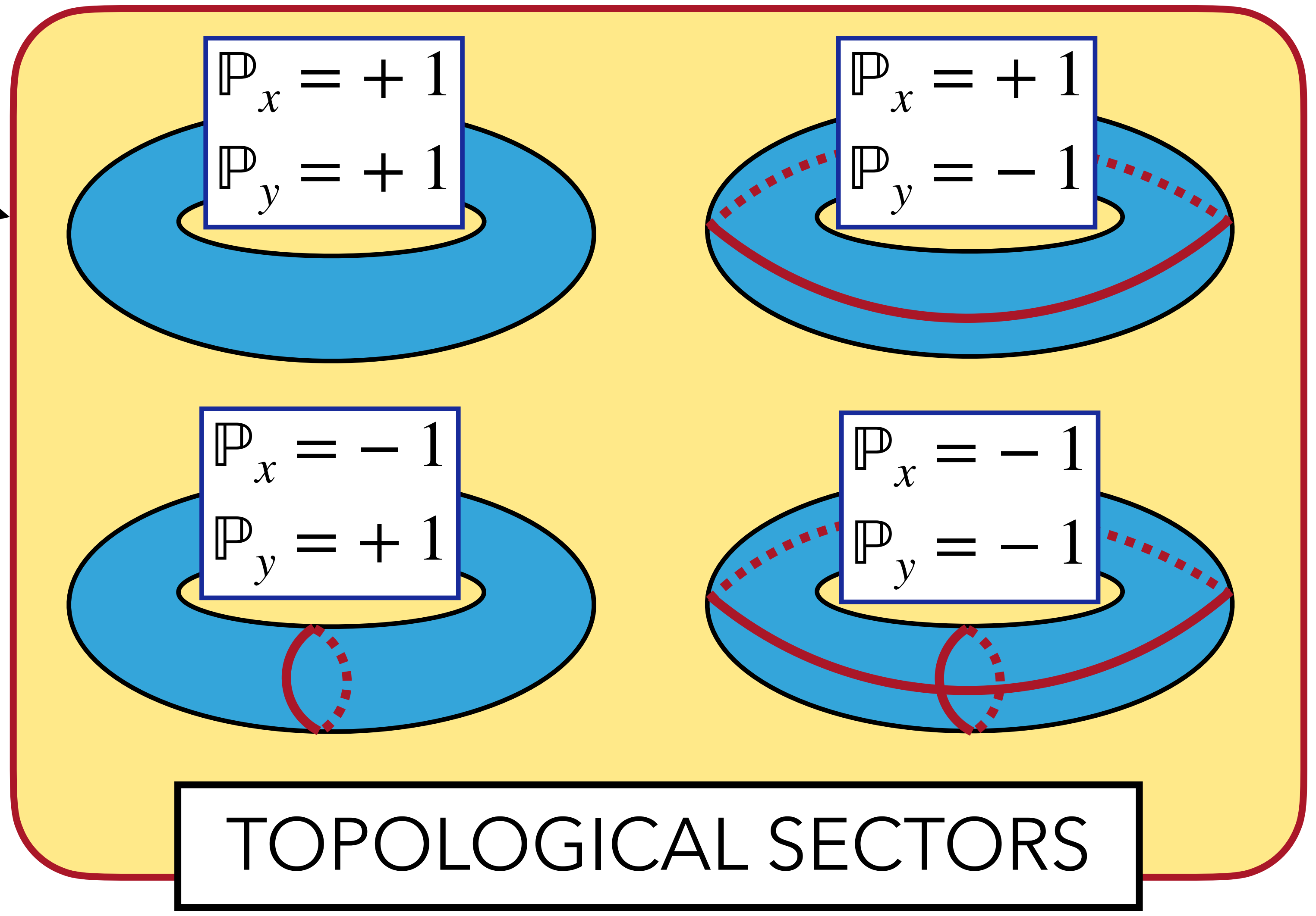


$$\mathbb{P}_x = \bigotimes_{k=1}^{|\Lambda_x|} P_{j+k\mu_x, j+k\mu_x+\mu_y} \qquad \mathbb{P}_y = \bigotimes_{k=1}^{|\Lambda_y|} P_{j+k\mu_y, j+k\mu_y+\mu_x}$$

$= \bigotimes$ 

$= \bigotimes$ 

TOPOLOGICAL INVARIANTS



TOPOLOGICAL SECTORS

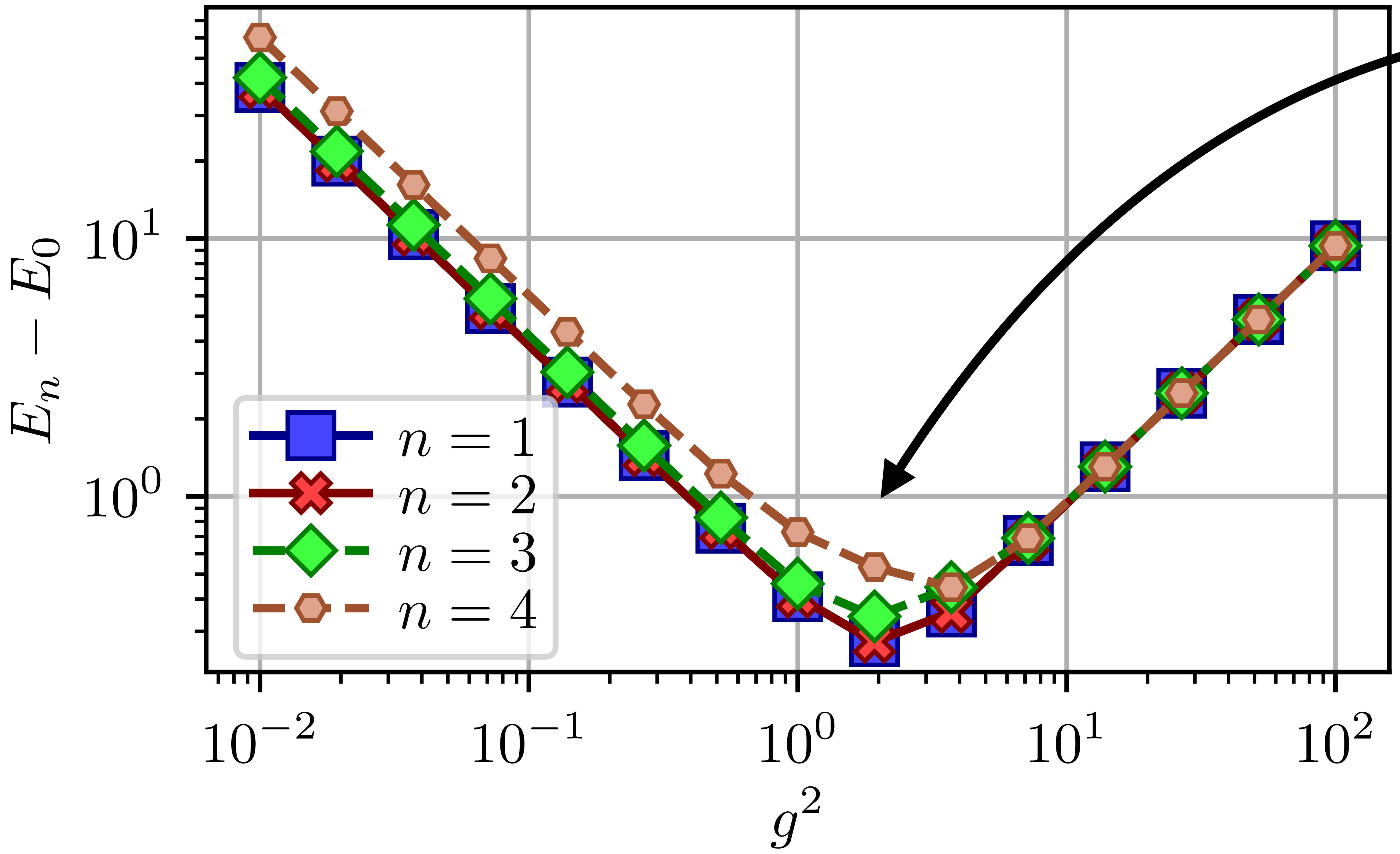
The 2D SU(2) pure LGT has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ topological symmetry

$$\left[\mathbb{P}_x, H_{\text{pure}}\right] = 0 \quad \left[\mathbb{P}_y, H_{\text{pure}}\right] = 0$$

are identified by the number of NON-REMOVABLE LOOP EXCITATIONS

TOPOLOGICAL SECTORS

Topological Sectors are separated in energy:



E_n	$\mathbb{P}_x$	$\mathbb{P}_y$
E_0	$+1$	$+1$
E_1	-1	$+1$
E_2	$+1$	-1
E_3	-1	-1
E_4	$+1$	$+1$

If there is a TOPOLOGICAL ORDER, it may survive ONLY at the critical point

SVETITSKY, YAFFE (1982)

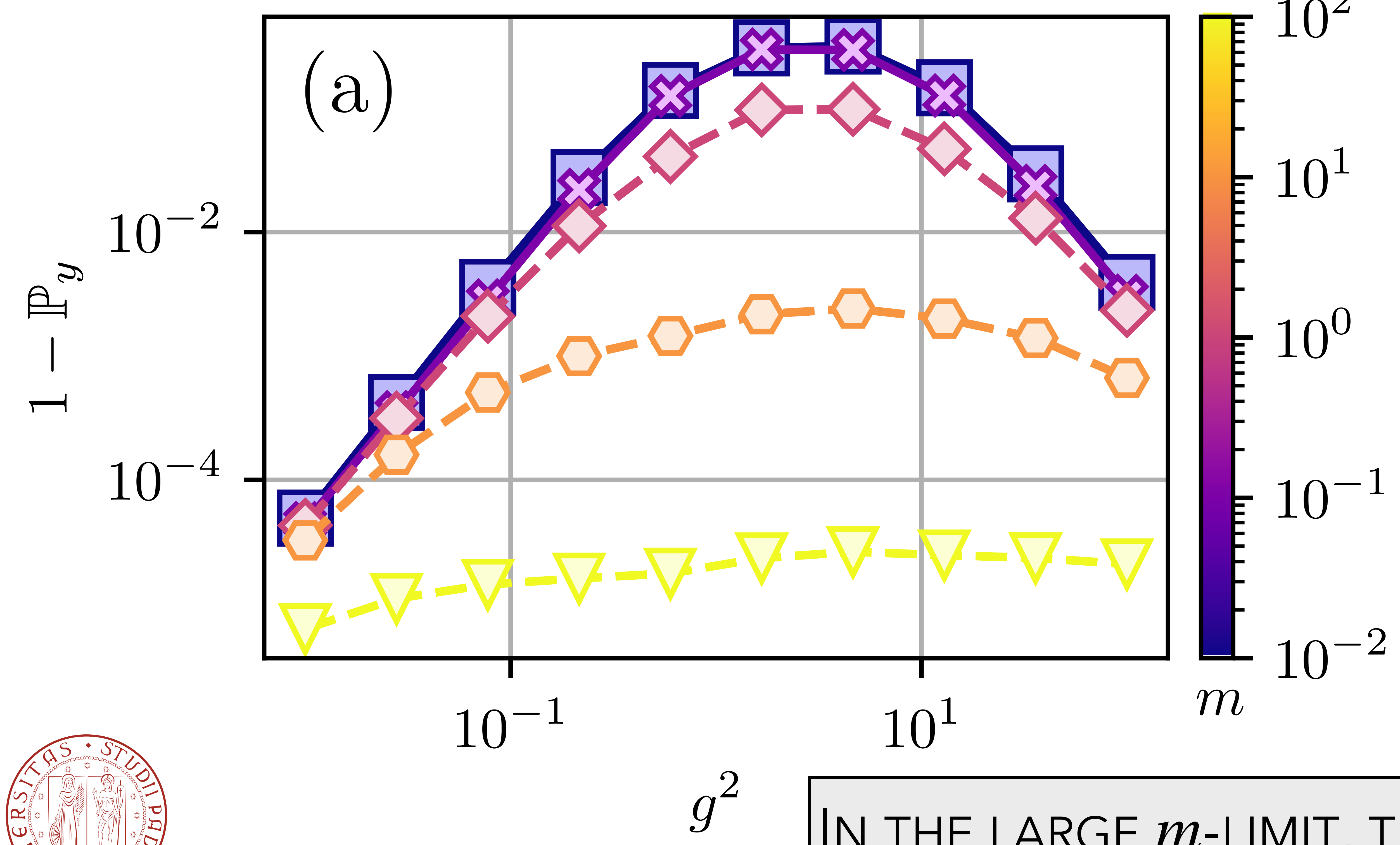
TAGLIACOZZO, VIDAL (2011)

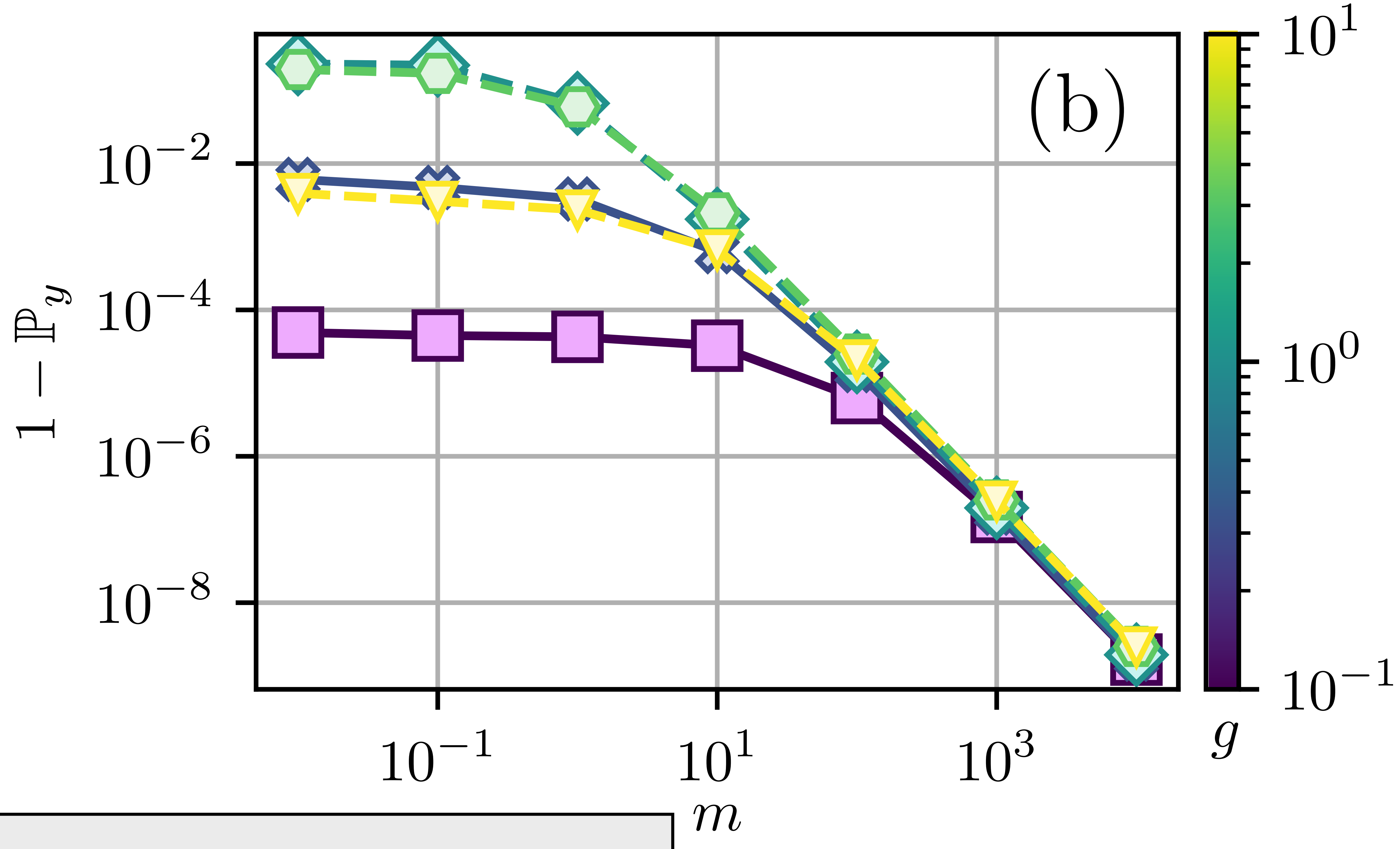
In the g-crossover, the *inter*-sector ($E_1 - E_0$) and the *intra*-sector ($E_4 - E_0$) gaps vanish, while opening far from the transition.

EFFECTS OF MATTER ON TOPOLOGY

In the full theory, the hopping term removes the topological symmetry because of the action of $\psi^{\dagger}_{j\alpha} U^{\alpha\beta}_{j,j+\mu} \psi_{j+\mu,\beta}$.

U flips the link parity $\{U, \mathbb{P}_x\} = \{U, \mathbb{P}_y\} = 0$





IN THE LARGE m-LIMIT, THE SYMMETRY IS RECOVERED

SU(2) GAUGE LINKS VIA RISHONS

ZOHAR, CIRAC (2018)

ZOHAR, CIRAC (2019)

EXPRESS THE PARALLEL TRANSPORTER WITH FERMIONIC RISHONS

$$U_{\alpha\beta} = \zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \zeta_{\alpha} \cdot P_{\zeta} \otimes \zeta^{\dagger}_{\beta}$$

$$\zeta_r = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)_F$$

$$\zeta_g = \frac{1}{\sqrt{2}} \left(\begin{array}{c|cc} 0 & 0 & 1 \\ \hline -1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)_F$$

SU(2) HAS A QUASI-REAL REPRESENTATION

RISHON PARITY

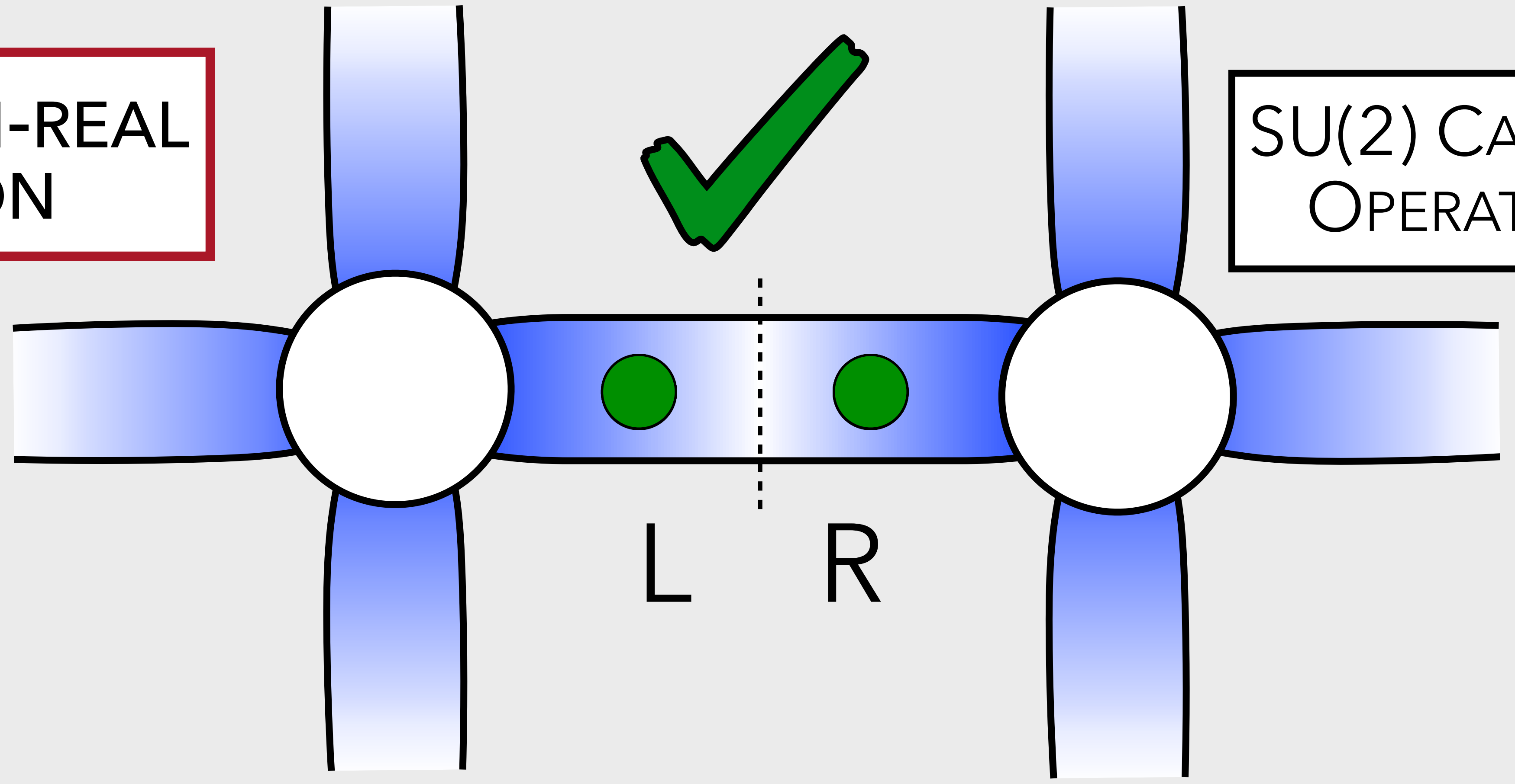
$$P_{\zeta} = \left(\begin{array}{c|cc} +1 & & \\ \hline & -1 & \\ & & -1 \end{array}\right)$$

0 (INTEGER) SU(2) REPRESENTATION

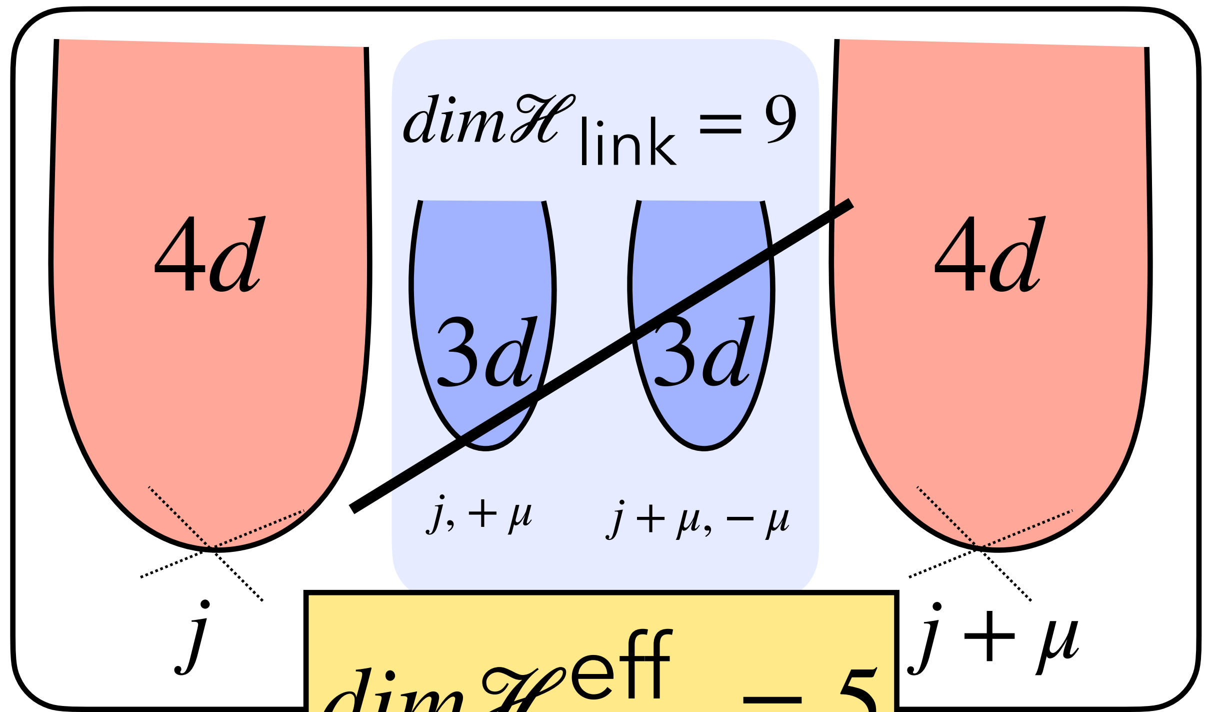
1/2 (SEMI-INTEGER) SU(2) REPRESENTATION

ALLOWED LINK CONFIGURATIONS SATISFY

$$S_L^2 = \left(S^2 \otimes 1\right) = \left(1 \otimes S^2\right) = S_R^2$$



SU(2) CASIMIR OPERATOR



$$dim\mathscr{H}^{\text{eff}}_{\text{link}} = 5$$

ζ Rishon-based Parallel Transport ✓

$$\zeta_{L,\alpha}\zeta^{\dagger}_{R,\beta} = \frac{1}{2}\begin{pmatrix} 0 & -\delta_{g,\alpha}\delta_{r,\beta} & +\delta_{g,\alpha}\delta_{\downarrow\beta} & -\delta_{r,\alpha}\delta_{r,\beta} & +\delta_{r,\alpha}\delta_{g,\beta} \\ +\delta_{r,\alpha}\delta_{g,\beta} & 0 & 0 & 0 & 0 \\ +\delta_{r,\alpha}\delta_{r,\beta} & & & & \\ -\delta_{g,\alpha}\delta_{g,\beta} & & & & \\ -\delta_{g,\alpha}\delta_{r,\beta} & 0 & 0 & 0 & 0 \end{pmatrix}$$

WE CAN GENERALIZE IT TO ANY SPIN-REPRESENTATION!

FERMIONIC QMB OPERATORS

HOW TO ENCODE FERMI-STATISTICS IN MATRIX NOTATION?

WHAT IS A FERMION?

BOSONS $\left[\hat{B}_i, \hat{B}_j\right] = 0$

FERMION $\left\{\hat{F}_i, \hat{F}_j\right\} = 0$

$$\hat{A}_j = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}_F$$

GETS **UNCHANGED** BY THE ACTION OF **BOSON** OPERATOR

$$[P, B] = 0$$

GETS **FLIPPED** BY THE ACTION OF **FERMION** OPERATOR

$$\{P, F\} = 0$$

PARITY OPERATOR

$$P = P^{-1} = P^T$$

$$P^2 = 1$$

DIRAC FERMION

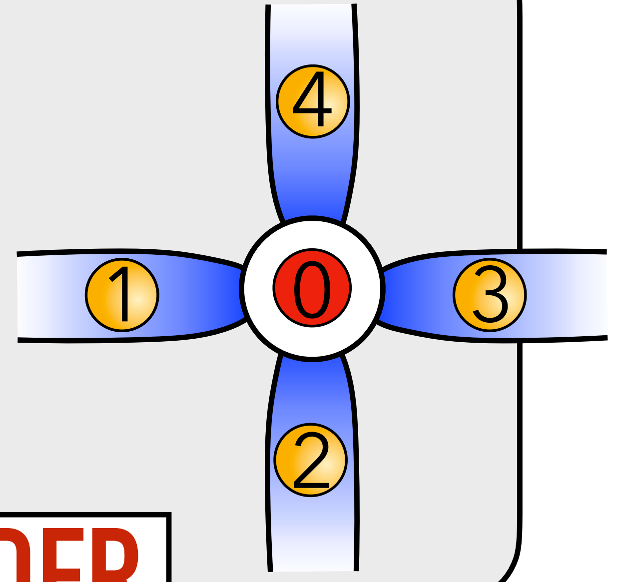
$$\Psi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}_F \qquad P = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$$

BOSONS $\hat{B}_j = \mathbb{1} \otimes \ldots \underset{j-1}{\mathbb{1}} \otimes \underset{j}{B} \otimes \underset{j+1}{\mathbb{1}} \otimes \cdots \otimes \underset{N}{\mathbb{1}}$

FERMION $\hat{F}_j = P \otimes \ldots \underset{j-1}{P} \otimes \underset{j}{F} \otimes \underset{j+1}{\mathbb{1}} \otimes \cdots \otimes \underset{N}{\mathbb{1}}$

THAT'S HOW A DRESSED SITE READS!

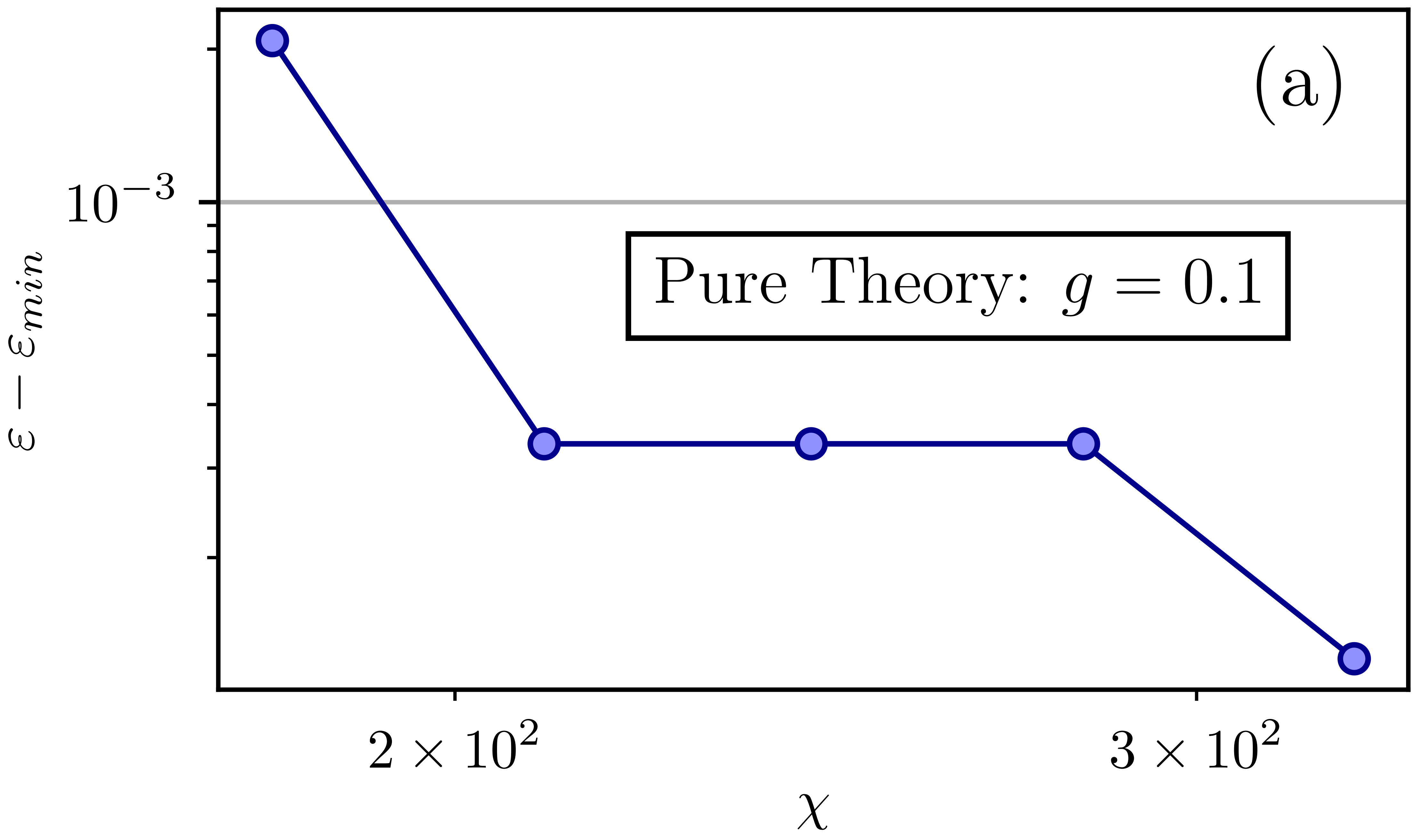
$$\left| \zeta^1_{-\mu_x} \; \begin{matrix} \zeta^4_{+\mu_y} \\ \psi^0 \\ \zeta^2_{-\mu_y} \end{matrix} \; \zeta^3_{+\mu_x} \right\rangle$$

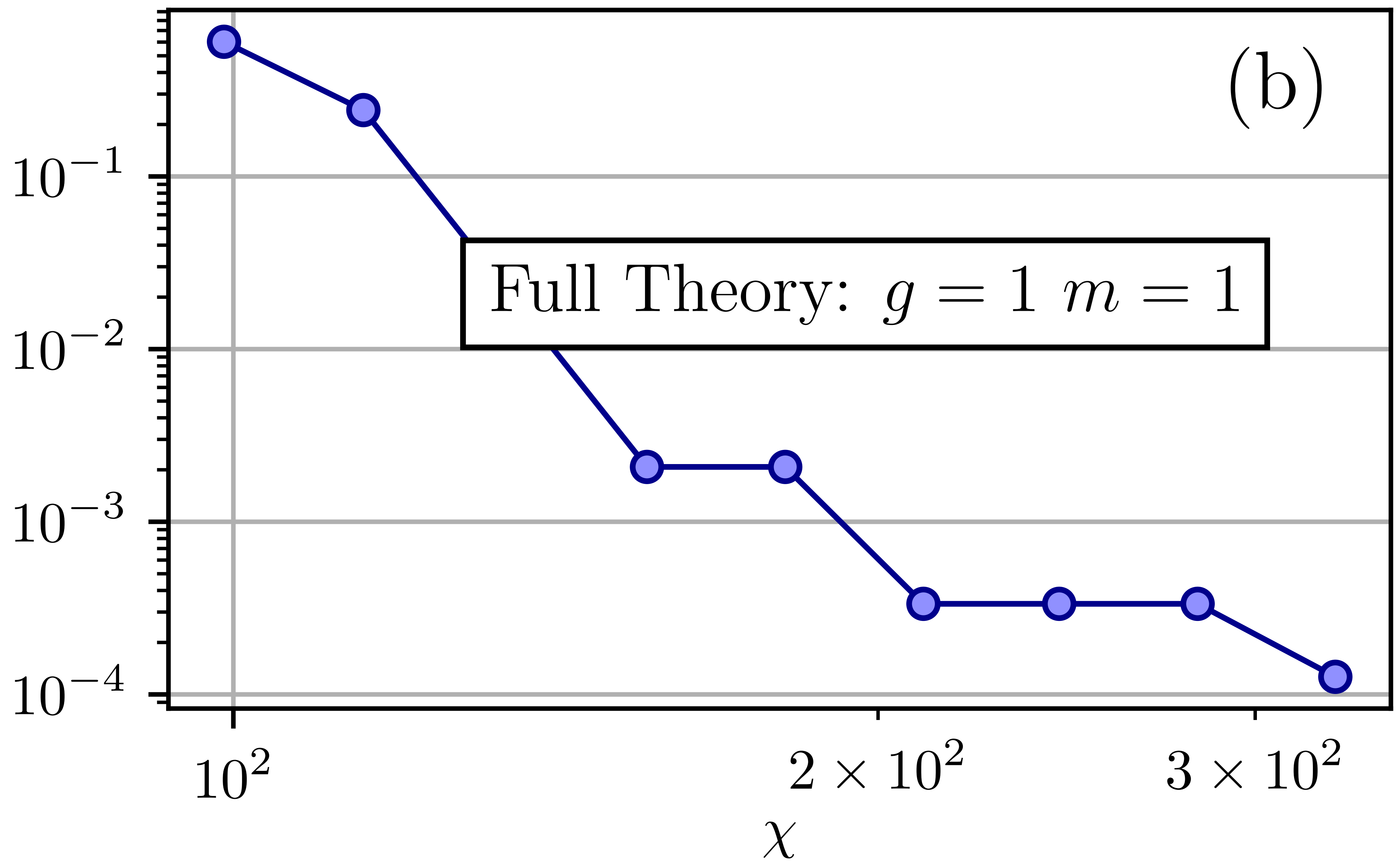


CHOOSE AN INTERNAL ORDER

TENSOR NETWORK CONVERGENCE

- Fix a bond dimension χ and perform a TTN simulation
- Obtain an estimate of ground state energy density $\varepsilon(\chi)$
- Increase $\chi \longrightarrow \chi'$ and obtain $\varepsilon' \leq \varepsilon$
- Stop at χ_{max} such that $\varepsilon(\chi) - \varepsilon(\chi_{max}) < 10^{-3}$
- The reference value is $\varepsilon_{min} = \varepsilon(\chi_{max})$





DIMENSIONAL ANALYSIS FOR CONTINUUM LIMIT

Gauge Theory in the Continuum

Electric Energy $H_{\text{elec}} = \frac{\epsilon_c}{2} \int \mathscr{E}^2(x) d^D x$

Color Vacuum Permittivity $[\epsilon_c] = \frac{\text{charge}^2 \cdot \text{length}^{2-D}}{\text{energy}}$

Color-Electric Field $[\mathscr{E}] = \frac{\text{energy}}{\text{charge} \cdot \text{length}}$

Hamiltonian Discretization

$$\int (dx)^D \longrightarrow a^D \sum_{j,\mu}$$

$$\mathscr{E}^2(x) \longrightarrow \frac{q_c^2 a^{2-2D}}{\epsilon_c^2} E^2_{j,\mu}$$

The way to go back $0 \longleftarrow a$

Lattice Gauge Theory

$$H_{\text{elec}} = \frac{q_c^2 a^{2-D}}{2\epsilon_c} \sum_{j,\mu} E^2_{j,\mu} = g^2 \frac{c\hbar}{2a} \sum_{j,\mu} E^2_{j,\mu}$$

Gauge Coupling $g = q_c \frac{a^{\frac{3-D}{2}}}{\sqrt{\hbar c \epsilon_c}}$

D=2 $g = \sqrt{\frac{q_c a^2}{2\hbar c \epsilon_c}}$

Quark Ratio $\alpha_c = \frac{q_c^2}{2 m_0 c^2 \epsilon_c} \propto \frac{q_c^2}{m_0}$

Continuum Limit at fixed α_c $g^2 = \alpha_c m \longrightarrow 0$

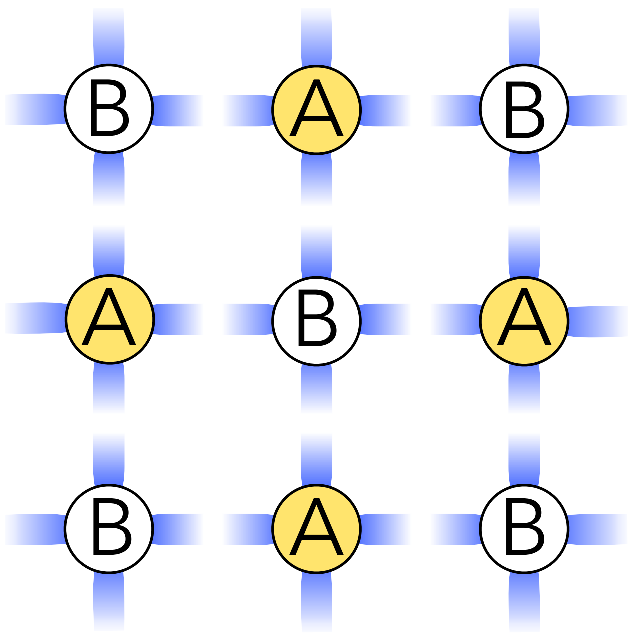
$$a \cdot H^{2D}_{SU(2)} = \frac{1}{2} \sum_{\alpha,\beta} \sum_{j,\mu} \left[e^{ik(j,\mu)} \psi^\dagger_{j,\alpha} U^{\alpha\beta}_{j,j+\mu} \psi_{j+\mu,\beta} + \text{H.c.} \right] + m \sum_j (-1)^{j_x+j_y} \sum_\alpha \psi^\dagger_{j\alpha}\psi_{j\alpha}$$

$$+ \frac{g^2}{2} \sum_j \left(E^2_{j,j+\mu_x} + E^2_{j,j+\mu_y} \right) - \frac{8}{g^2} \sum_j \sum_{\alpha,\beta,\gamma,\delta} \mathfrak{Re} \begin{pmatrix} & U^\dagger_{\gamma\delta} & \\ U^\dagger_{\gamma\alpha} & & U_{\beta\gamma} \\ & U_{\alpha\beta} & \end{pmatrix}$$

2D STAGGERED FERMIONS

Writing γ matrices in (2+1)D

$$(\gamma^0, \gamma^0\gamma^1, \gamma^0\gamma^2) \longrightarrow (\sigma^z, \sigma^x, \pm\,\sigma^y)$$



IDEA: place Spinor components on sub lattices to halve the 1 Brillouin Zone

Free Fermions Dynamics with the Dirac Hamiltonian: $H_{\text{Dirac}} = c\gamma^0\vec{\gamma}\cdot\vec{p} + mc^2\gamma^0$

$$H^{2D}_{\text{Dirac}} = \begin{pmatrix} mc^2 & c(p_x - ip_y) \\ c(p_x + ip_y) & -mc^2 \end{pmatrix} = \begin{pmatrix} mc^2 & \hbar c(-i\partial_x + \partial_y) \\ c(-i\partial_x - \partial_y) & -mc^2 \end{pmatrix}$$

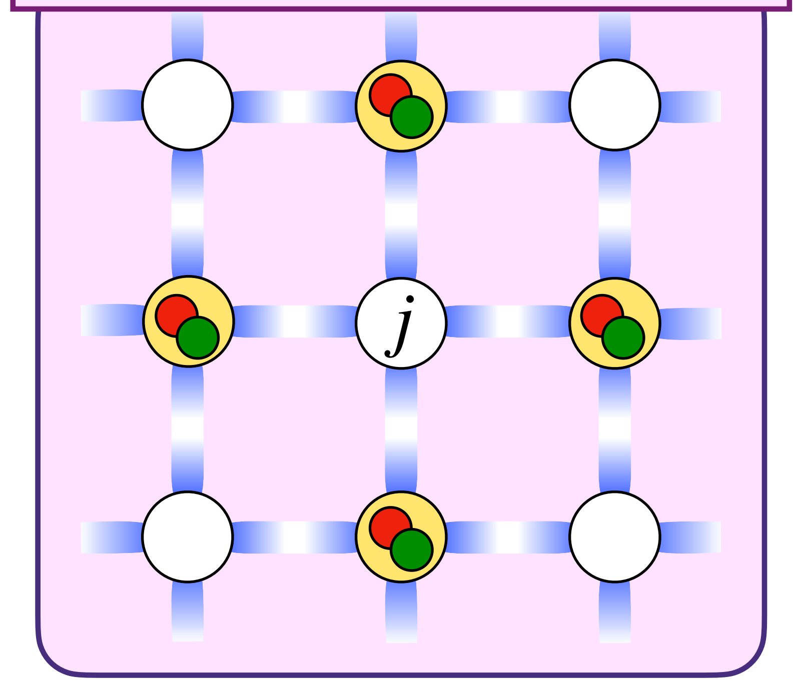
NIELSEN & NINOMIYA THEOREM

THE CHOICE OF THE SIGN BREAKS THE HAMILTONIAN CHIRALITY

NUCLEAR PHYSICS B. 185, 20-40

Place a 2-Spinor $\Psi = \begin{pmatrix} \psi_1 \\ \psi_4 \end{pmatrix}$ on the 2D lattice: $\psi = \begin{cases} \psi_1 \text{ on even sites } (j_x, j_y) \\ \psi_4 \text{ on odd sites } (j_x, j_y) \end{cases}$

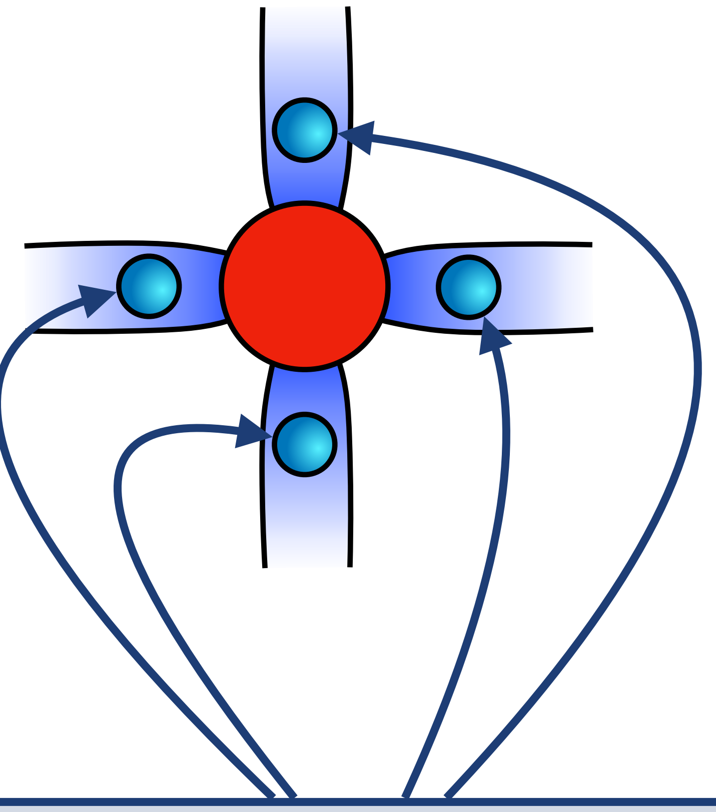
STAGGERING FACTORS



$$H^{2D-14}_{\text{Dirac}} = \frac{ch}{2a}\sum_{j\in\Lambda}\left[\text{-i}\,\psi_j^\dagger\psi_{j+\mu_x} - (-1)^{(j_x+j_y)}\psi_j^\dagger\psi_{j+\mu_y} + \text{H.c.}\right] + mc^2\sum_{j\in\Lambda}(-1)^{(j_x+j_y)}\psi_j^\dagger\psi_j$$

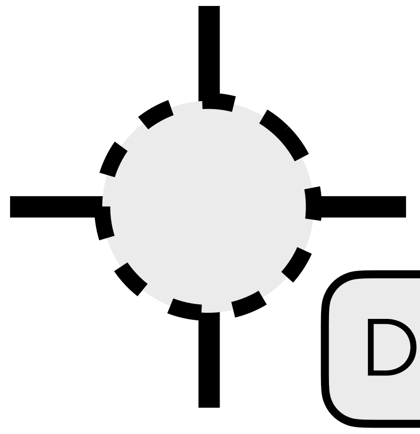
SUSSKIND HAMILTONIAN (PHYS. REV. D 16, 3031)

TOWARDS AN OPERATIVE HAMILTONIAN



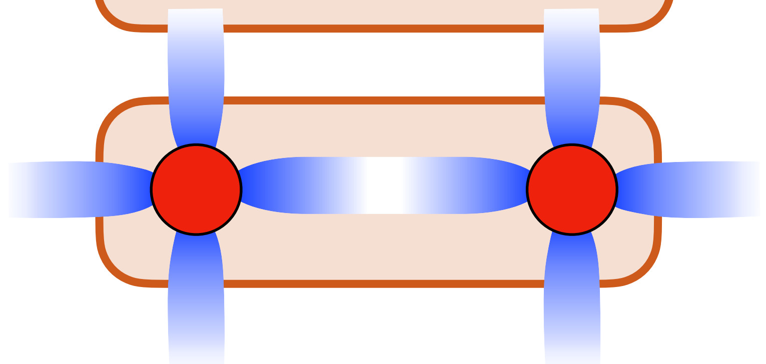
ELECTRIC SU(2) ENERGY

$$H^{2D}_{\text{LYM-SU(2)}} = \frac{1}{2}\sum_{j,\alpha,\beta}\left(-i\,\psi^{\dagger}_{j,\alpha}\,\zeta^{\alpha}_{j,\mu_x}\,Q^{\beta,\dagger}_{j+\mu_x,-\mu_x}\,\psi_{j+\mu_x,\beta} - (-1)^{j_x+j_y}\,\psi^{\dagger}_{j,\alpha}\,\zeta^{\alpha}_{j,\mu_y}\,Q^{\beta,\dagger}_{j+\mu_y,-\mu_y}\,\psi_{j+\mu_y,\beta} + \text{H.c.}\right) + m\sum_{j,\alpha}(-1)^{j_x+j_y}\,\psi^{\dagger}_{j,\alpha}\psi_{j,\alpha} + H_{\text{pure}} + H_{\text{penalty}}$$



DRESSED SITE

MATTER-GAUGE INTERACTION

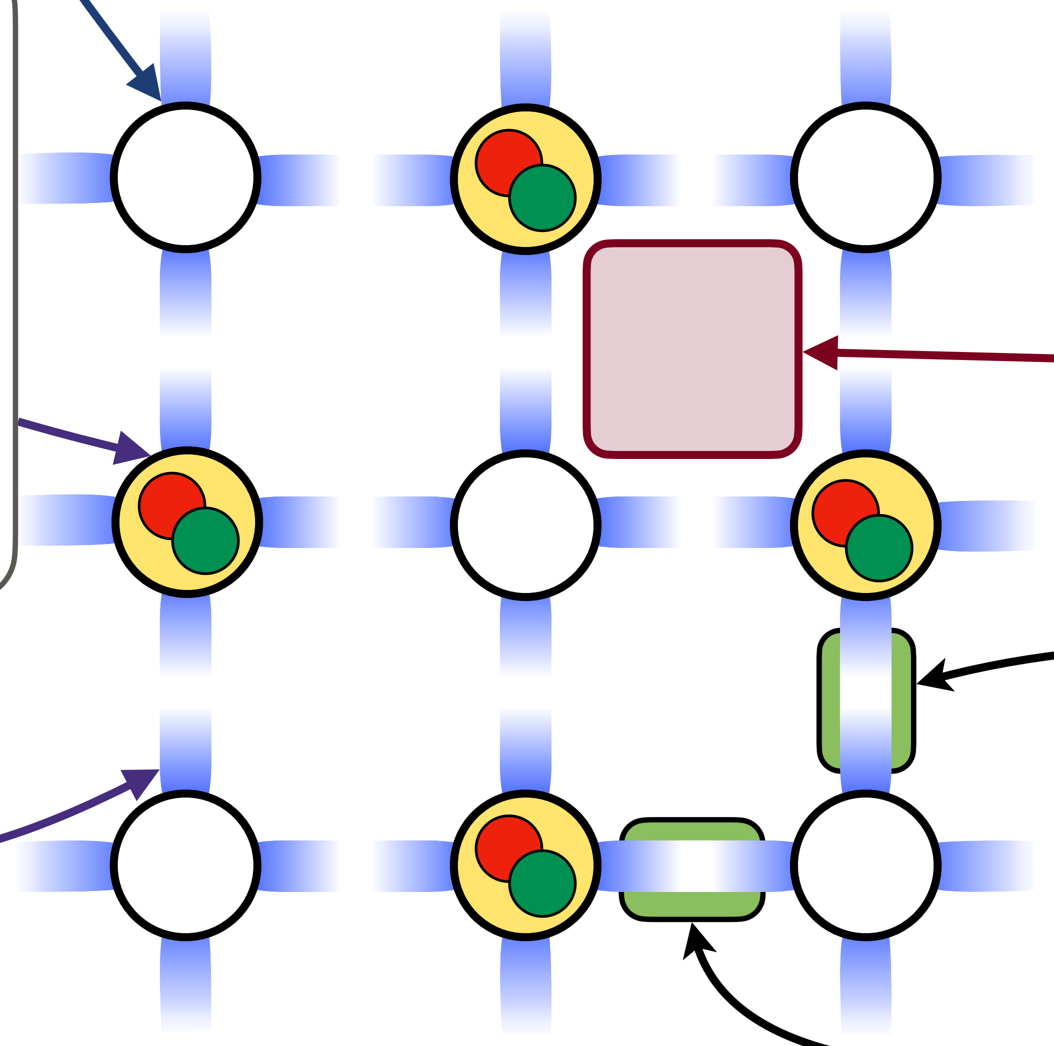


$$H_{\text{pure}} = \frac{3g^2}{16}\sum_{j,\mu} S^2_{j,j+\mu} + \frac{4}{g^2}\sum_{j}\sum_{\alpha,\beta,\gamma,\delta}\left(C_{\ulcorner}\ \zeta^{\gamma}\zeta^{\delta\dagger}\ {}_{\urcorner}C \;\; C_{\llcorner}\ \zeta^{\beta}\zeta^{\gamma\dagger}\ {}_{\lrcorner}C \;\; \zeta^{\delta}\zeta^{\alpha\dagger} \;\; \zeta^{\alpha}\zeta^{\beta\dagger} + \text{H.c.}\right)$$

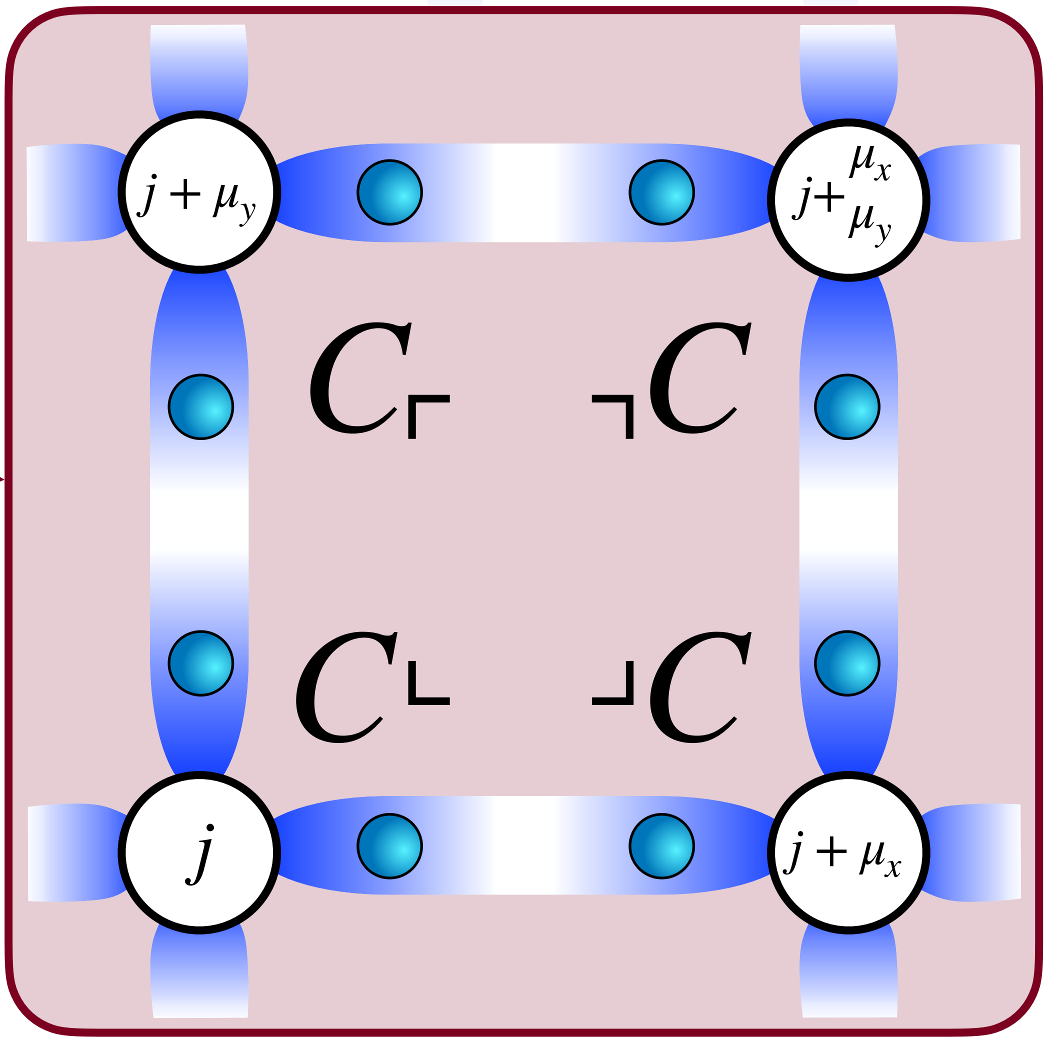
STAGGERING FACTORS

⊕ EVEN

⊖ ODD



PLAQUETTE MAGNETIC SU(2) INTERACTION $\propto B^2$



LINK CONSTRAINT

$$H_{\text{penalty}} = -\eta\sum_{j}\left[W_{j,+\mu_x}W_{j+\mu_x,-\mu_x} + W_{j,+\mu_y}W_{j+\mu_y,-\mu_y}\right]$$

USEFUL PROPERTIES OF SU(2) RISHONS

ζ-Rishons have a unique gauge transformation algebra, generated by $\vec{T} = \frac{1}{2}\sum_{a,b} \vec{\sigma}_{ab} c_a^\dagger c_b$

$\vec{T}$ is genuinely *local*
$\left[\vec{T}_1, \zeta_{2,a}\right] = \left[\vec{T}_1, \psi_{2,a}\right] = 0$

It is possible to check that ζ-Rishons transform covariantly under $\vec{T}$.

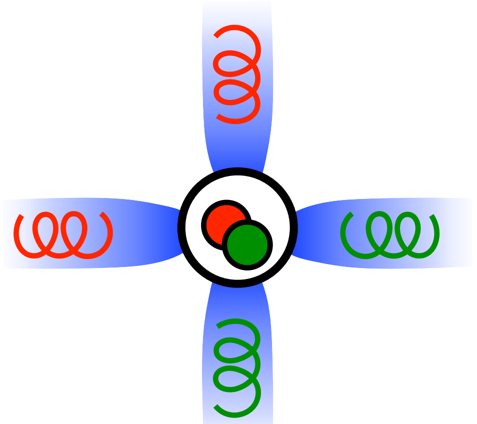
$$\left[\vec{T}, \zeta_a\right] = -\frac{1}{2}\sum_b \vec{\sigma}_{ab}\zeta_b \qquad \left[\vec{T}, \zeta_a^\dagger\right] = -\left[\vec{T}, \zeta_a\right]^\dagger = \frac{1}{2}\sum_b \zeta_b^\dagger \vec{\sigma}_{ba}$$

Left- and Right-generators of the gauge field at link $(j, j+\mu)$ can be expressed like

$$\vec{L}_{j,\mu} = \vec{T}_{j,+\mu} \otimes 1_{j+\mu,-\mu} \qquad \vec{R}_{j,\mu} = 1_{j,+\mu} \otimes \vec{T}_{j+\mu,-\mu}$$

$\zeta^a \zeta^{b\dagger}$ transforms like U^{ab} do with $\vec{L}$ and $\vec{R}$ generators!

EXACT RESULTS

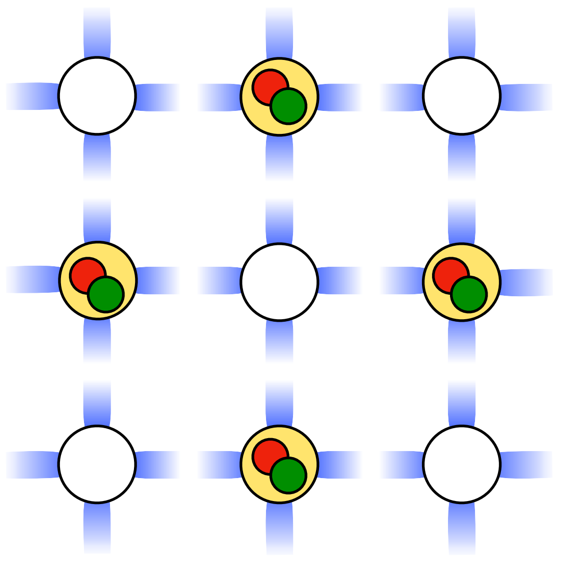


WITH MATTER

STAGGERED FERMIONS

$\oplus$ EVEN

$\ominus$ ODD

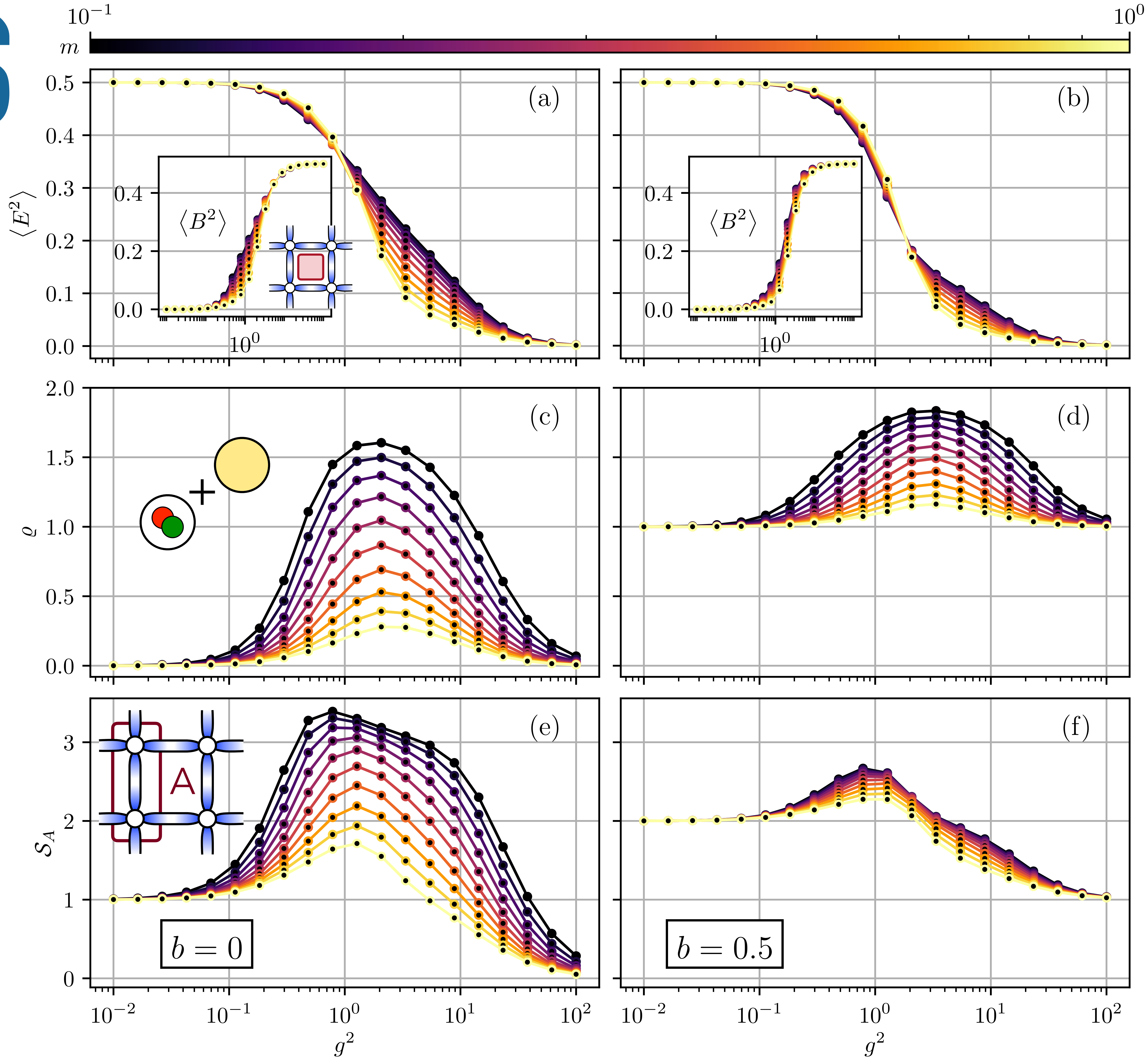


GROUND-STATE
ELECTRIC ENERGY $\langle E^2 \rangle$
MAGNETIC ENERGY $\langle B^2 \rangle$

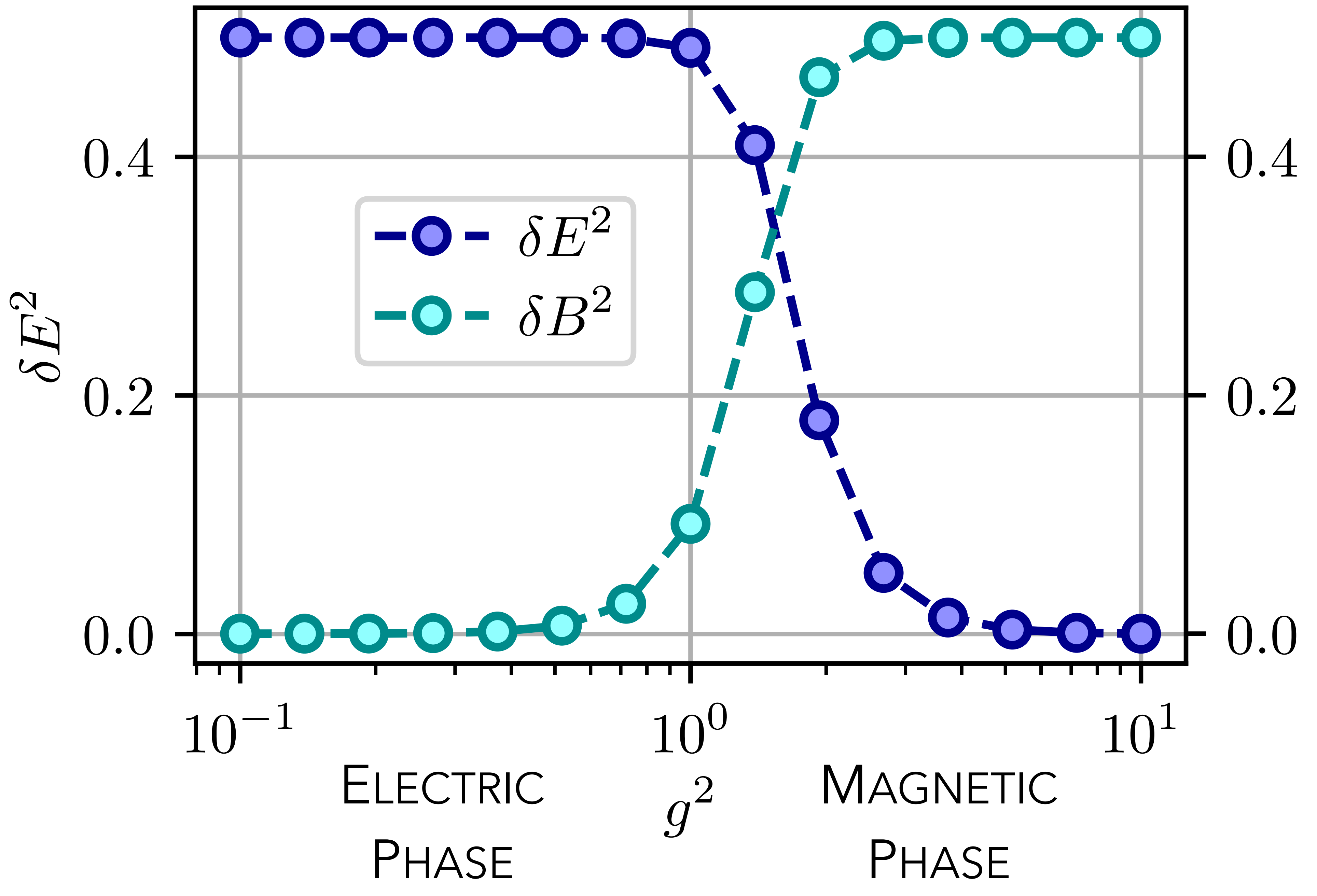
PARTICLE DENSITY
(QUARK + ANTIQUARKS)
$\varrho = N_+ + (2 - N_-)$

BARYON NUMBER
(QUARKS - ANTIQUARKS)
$b = \frac{1}{2}(N_+ - (2 - N_-))$

ENTANGLEMENT ENTROPY
$\mathcal{S}_A = -\,\mathrm{Tr}\rho_A \log_2 \rho_A$



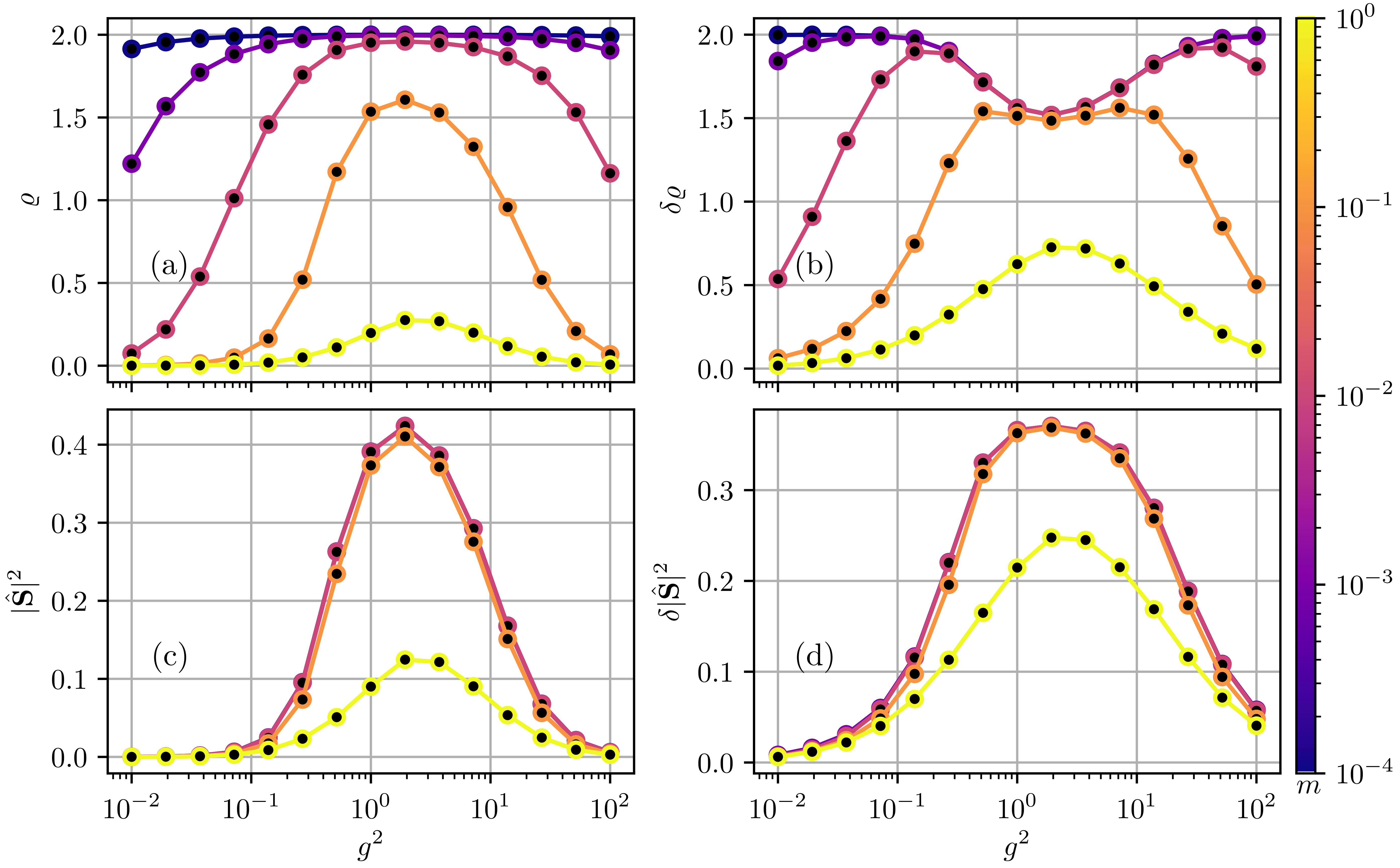
EXACT DIAGONALIZATION FLUCTUATIONS



QUANTUM FLUCTUATIONS

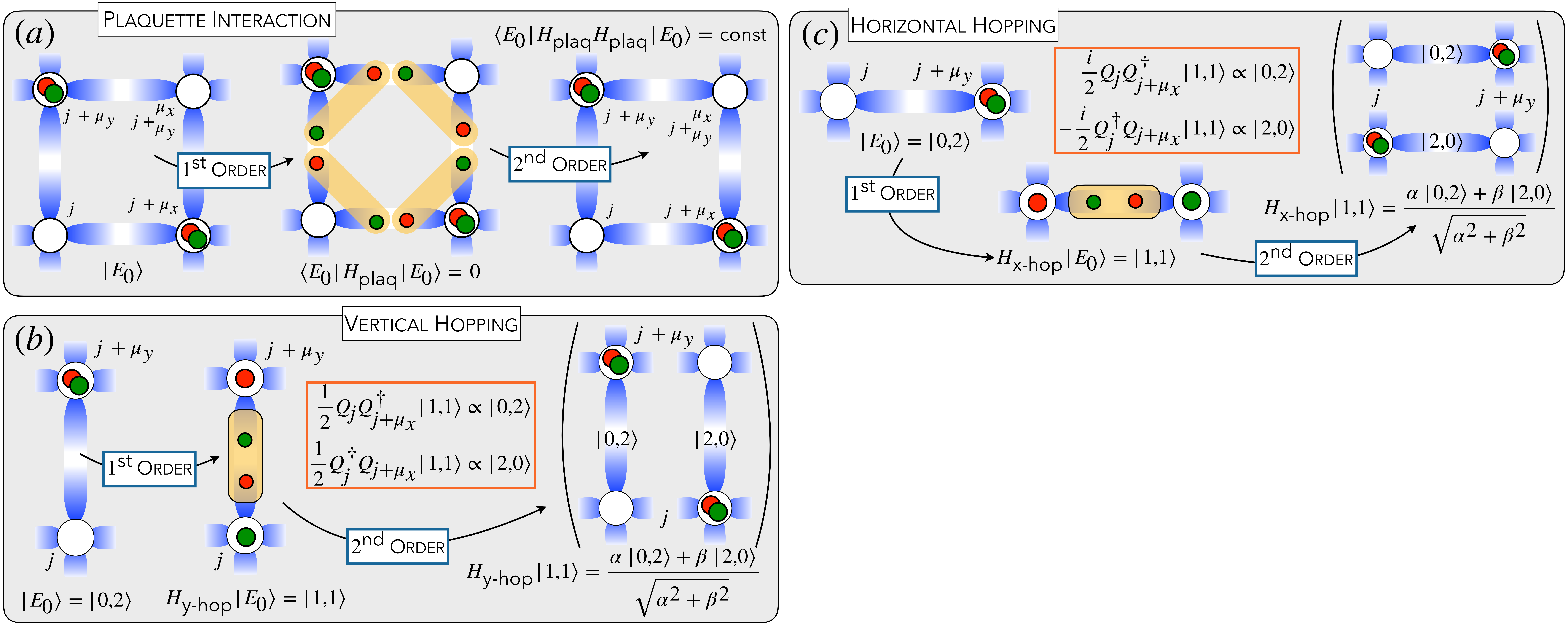
$$\delta E^2 = \sqrt{\langle E^4 \rangle - \langle E^2 \rangle^2} \qquad \delta B^2 = \sqrt{\langle B^4 \rangle - \langle B^2 \rangle^2}$$

FLUCTUATING PARTICLE DENSITY: IT SUGGESTS A BARYON LIQUID PHASE



FLUCTUATING MATTER COLOR DENSITY: THE ONLY TRACE FOR POSSIBLE DECONFINEMENT

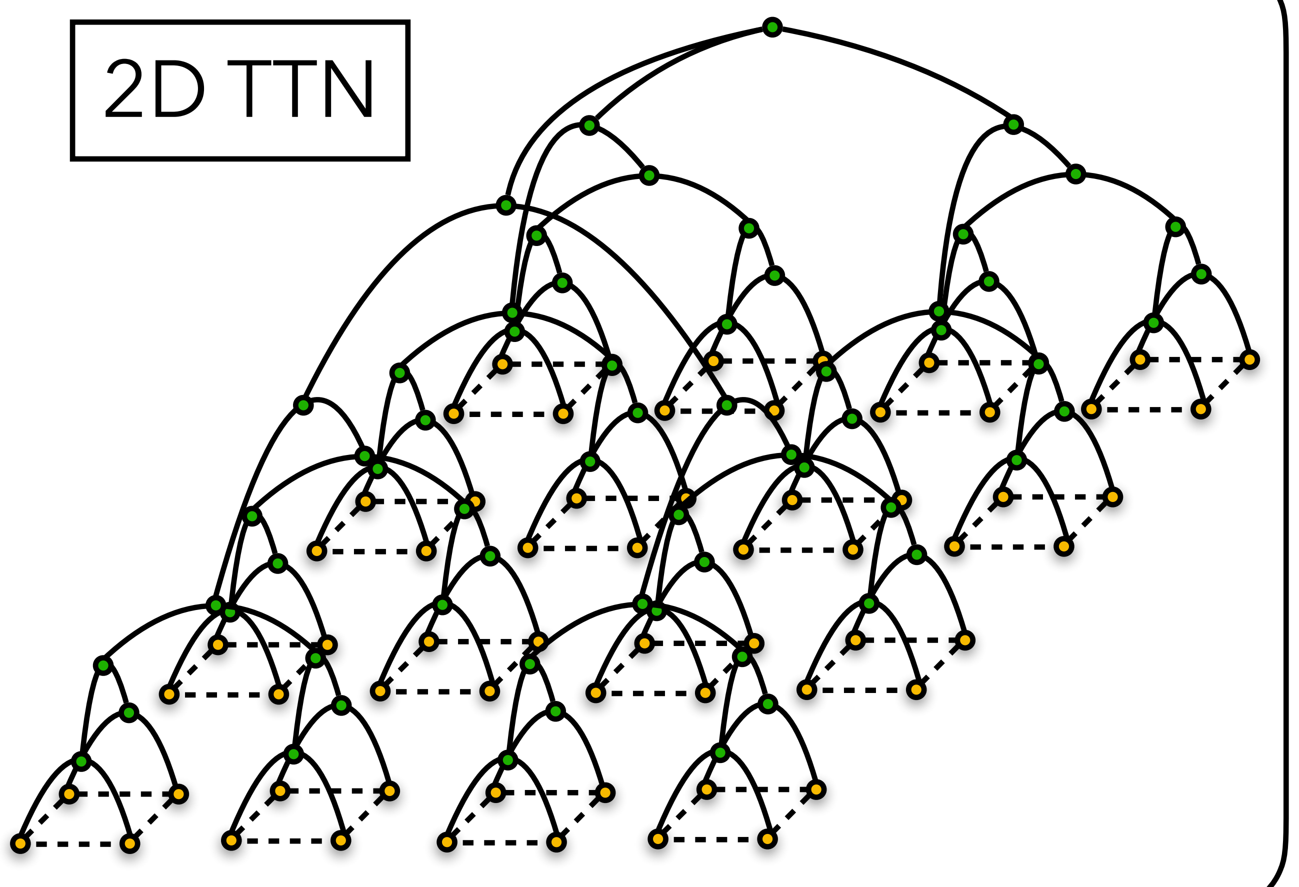
SU(2) PERTURBATION THEORY



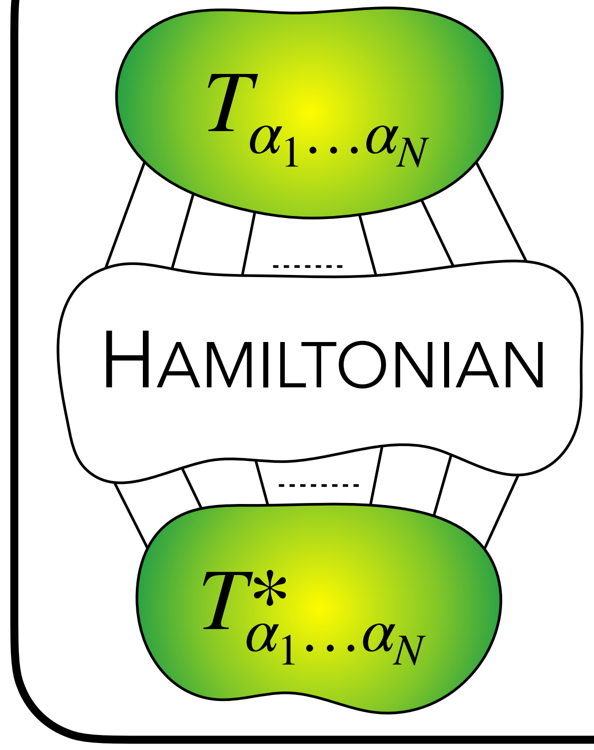
TENSOR NETWORK METHODS

The Hilbert space of N-Body Systems *GROWS EXPONENTIALLY*: $\dim \mathscr{H} = d^N$

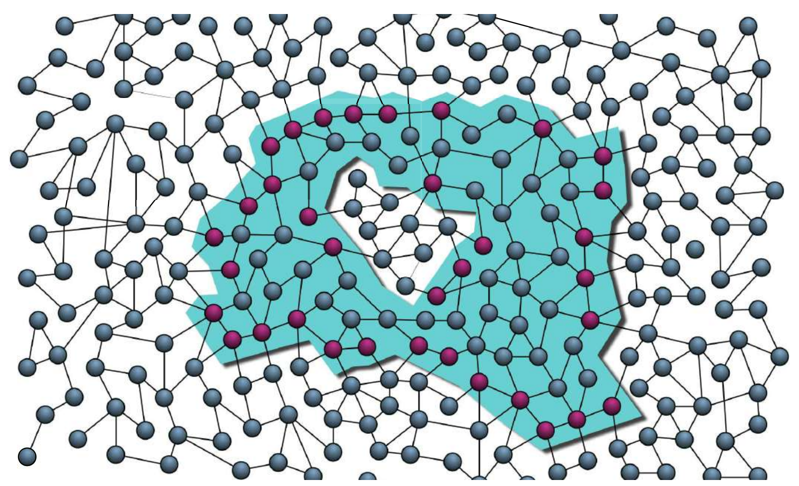
Exact Diagonalization (ED) is not sustainable for large N

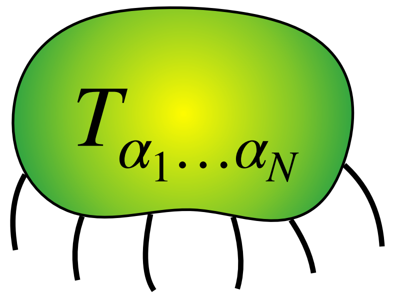


Ground State Energy Properties



Entanglement Area Law
Eisert et al. (2007)

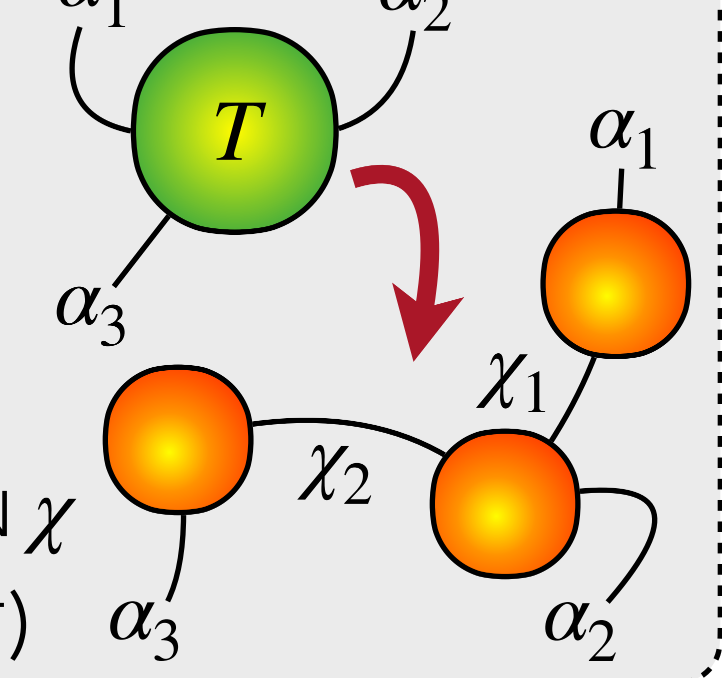




$$|\Psi_{QMB}\rangle = \sum_{\alpha_1\dots\alpha_N} T_{\alpha_1\dots\alpha_N} |\alpha_1\dots\alpha_N\rangle$$

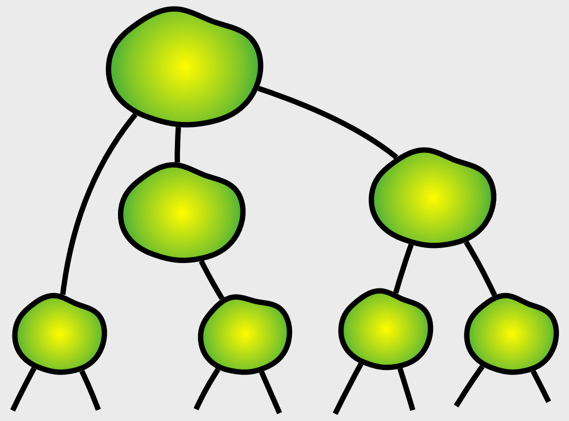
Polynomial Algorithmic Complexity: Manipulations scale $\sim \mathcal{O}(\chi^k)$

Physical links α_i

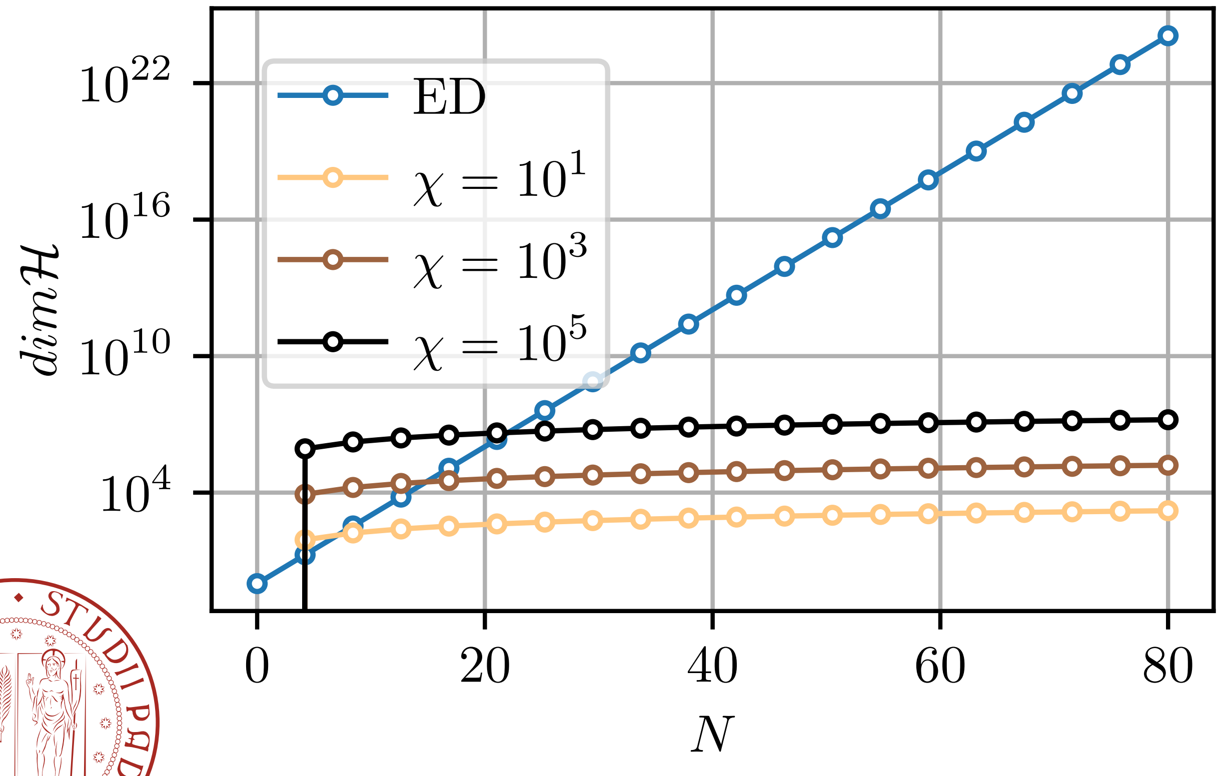


Bond Dimension χ (Entanglement)

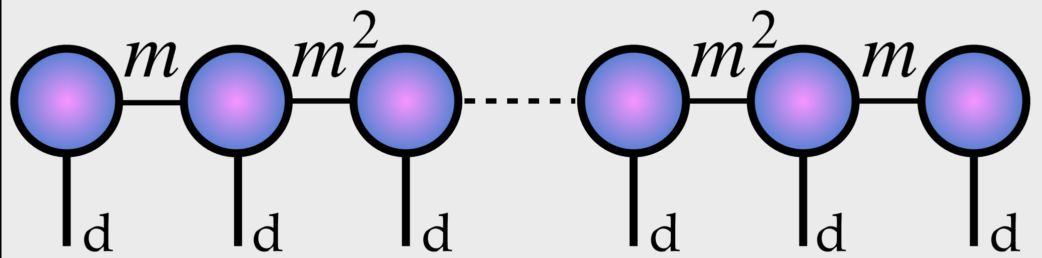
Tree Tensor Networks (TTN)



Not implementing Area Law
Low Algorithmic Complexity $\sim \mathcal{O}(\chi^4)$



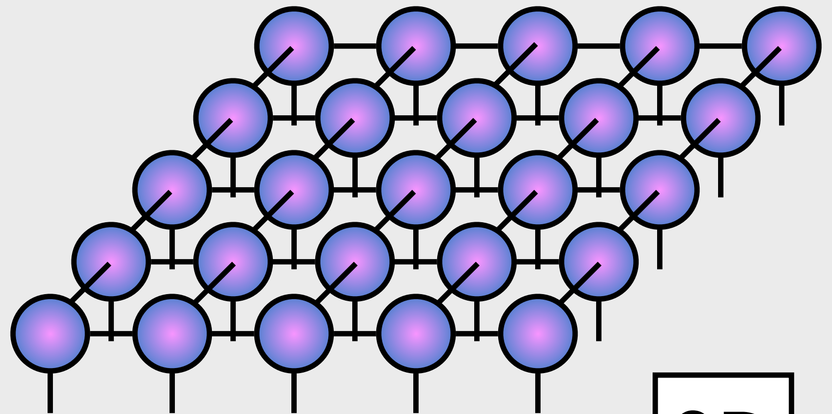
Matrix Product States (MPS)



1D

Implementing Area Law
Low Complexity $\sim \mathcal{O}(\chi^3)$

Projected Entangled Pair States (PEPS)



2D

Naturally implementing Entanglement Area Law
Too High Algorithmic Complexity $\sim \mathcal{O}(\chi^8)$