ETH zürich

Institute for Theoretical Physics

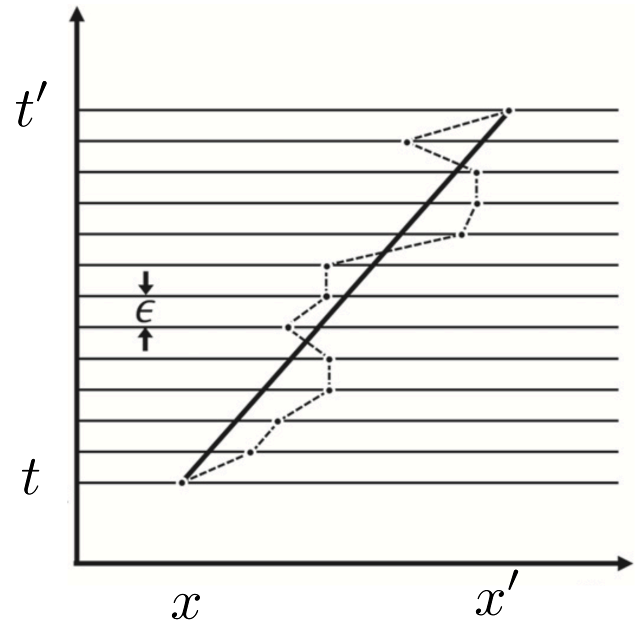
Quantum Many-Body Scars in 2+1D Gauge Theories

Marina Krstic Marinkovic marinama@ethz.ch

in collab. with **Thea Budde** and **Joao Pinto Barros**

QuantHEP 2025, 30 September, Lawrence Berkeley National Lab

Lattice Gauge Theories beyond Euclidean Monte Carlo Approach

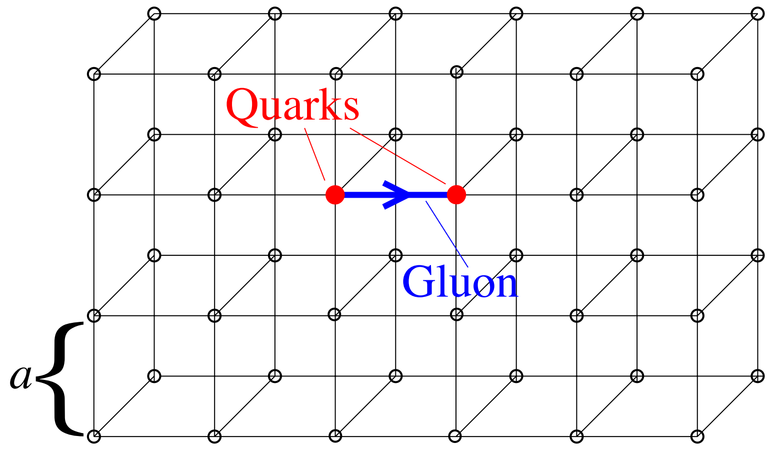


+

Euclidean space-time

$\epsilon \longrightarrow -i\, a$

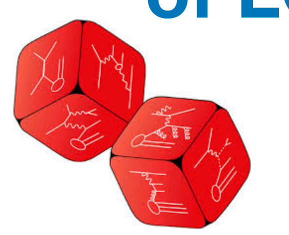
+



=

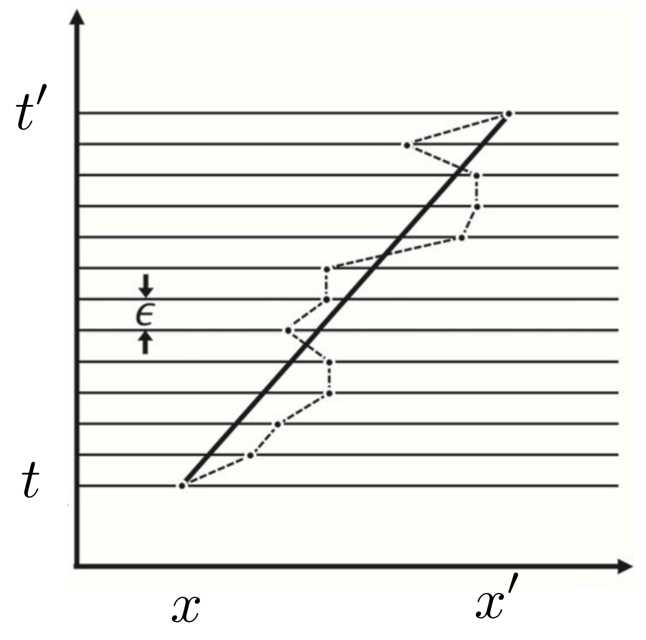
Lattice Gauge Theory (LGTs)

Monte Carlo (MC) Simulations of LGTs



$$\langle O[A,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int D[A]\,\det D[A]\, e^{-S_G[A]} O[A]$$

Lattice Gauge Theories beyond Euclidean Monte Carlo Approach

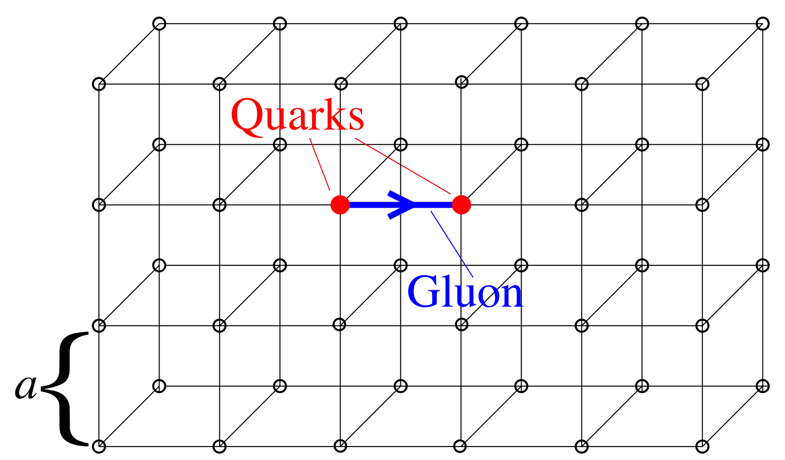


+

Euclidean space-time

$\epsilon \longrightarrow -i\,a$

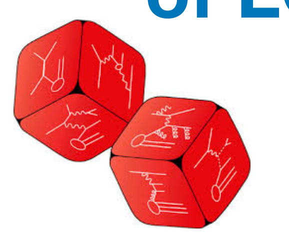
+



=

Lattice Gauge Theory (LGTs)

Monte Carlo (MC) Simulations of LGTs



$$\langle O[A,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int D[A]\,\det D[A]\,e^{-S_G[A]}\,O[A]$$

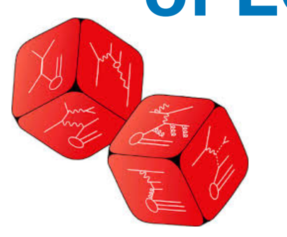
real Weight W

$$\langle O\rangle = \frac{\sum_C O_C \cdot W_C}{\sum_C W_C}$$

ETH zürich

Lattice Gauge Theories beyond Euclidean Monte Carlo Approach

Monte Carlo (MC) Simulations of LGTs



Inaccessible with conventional MC approach:

$$\langle O[A,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int D[A]\,\det D[A]\, e^{-S_G[A]}\, O[A]$$

Complex Weight W

[Troyer, Wiese PRL 94 170201(2005)]

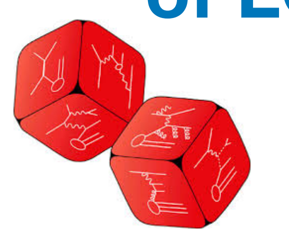
$$\langle O\rangle = \frac{\sum_C O_C \cdot W_C}{\sum_C W_C} = \frac{\sum_C O_C \cdot \mathrm{sign}(W_C)\cdot |W_C|}{\sum_C \mathrm{sign}(W_C)\cdot |W_C|}$$

$$= \frac{\langle \mathrm{sign}\cdot O\rangle_{|W|}}{\langle \mathrm{sign}\rangle_{|W|}}$$

Sign Problem

Lattice Gauge Theories beyond Euclidean Monte Carlo Approach

Monte Carlo (MC) Simulations of LGTs



[Troyer, Wiese PRL 94 170201(2005)]

$$\langle O \rangle = \frac{\sum_C O_C \cdot W_C}{\sum_C W_C} = \frac{\sum_C O_C \cdot \mathrm{sign}(W_C) \cdot |W_C|}{\sum_C \mathrm{sign}(W_C) \cdot |W_C|}$$

$$= \frac{\langle \mathrm{sign} \cdot O \rangle_{|W|}}{\langle \mathrm{sign} \rangle_{|W|}}$$

Inaccessible with conventional MC approach:

$$\langle O[A, \psi, \bar{\psi}] \rangle = \frac{1}{Z} \int D[A] \det D[A] e^{-S_G[A]}$$

Complex Weight W

Sign Problem

Quantum Simulations

- **Real time dynamics**
- **Topological θ-term**
- **Finite chemical potential**
- . . .

Real-time dynamics of $U(1)$ pure gauge theory in 2+1D

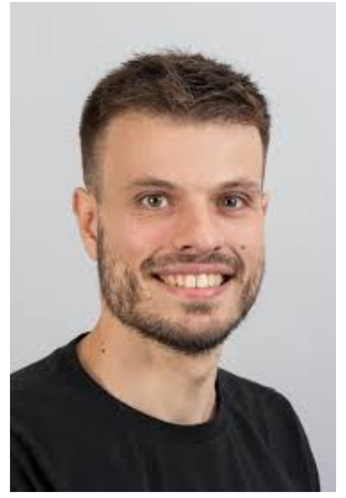
- Quantum systems with many degrees of freedom: fast/slow thermalization [von Neumann(1929), Deutsch. PRA (1991), Srednicki PRE (1994)]

- Under which conditions can thermalization be evaded?

Real-time dynamics of $U(1)$ pure gauge theory in 2+1D

- Quantum systems with many degrees of freedom: fast/slow thermalization [von Neumann(1929), Deutsch. PRA (1991), Srednicki PRE (1994)]
- Under which conditions can thermalization be evaded?
- Quantum Many-Body Scars in simple gauge/fermionic theories [Pakrouski et al. PRL 125 (2020), Mukherjee et al. PRB(2021), Banerjee, Sen PRL (2021), Surace et al. PRX (2021), Delacretaz et al. JHEP (2022), Desaules et al. PRB (2023), Srdinsek et al. PRL (2024), Calajo et al. (2024)…]
- Probed for small volumes and small spins due to severe sign problems in real-time dynamics simulations
- Analytic construction of ergodicity breaking states for arbitrary volumes —> U(1) gauge theory in 2+1D


Thea Budde


Joao Pinto Barros

[Budde, MKM, Pinto Barros, PRD 110, 094506 (2024) + work in preparation]

ETHzürich

Eigenstate Thermalization Hypothesis (ETH)

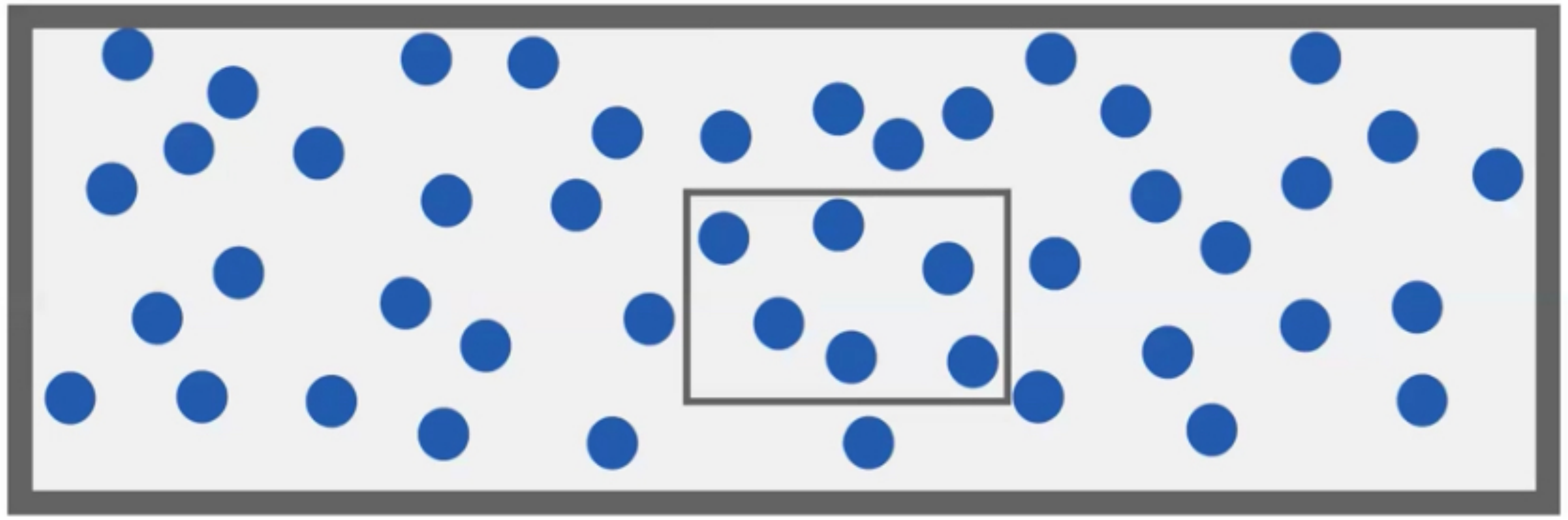
- Local observables are expected to thermalize

$$\mathcal{O}(t) = \langle \psi(t) | \mathcal{O} | \psi(t) \rangle = \sum_n |c_n|^2 \mathcal{O}_{nn} + \sum_{m \neq n} c_m^* c_n \, e^{i(E_m - E_n)t} \, \mathcal{O}_{mn}$$

- ETH [von Neumann(1929), Deutsch. PRA (1991), Srednicki PRE (1994)] bounds the fluctuations

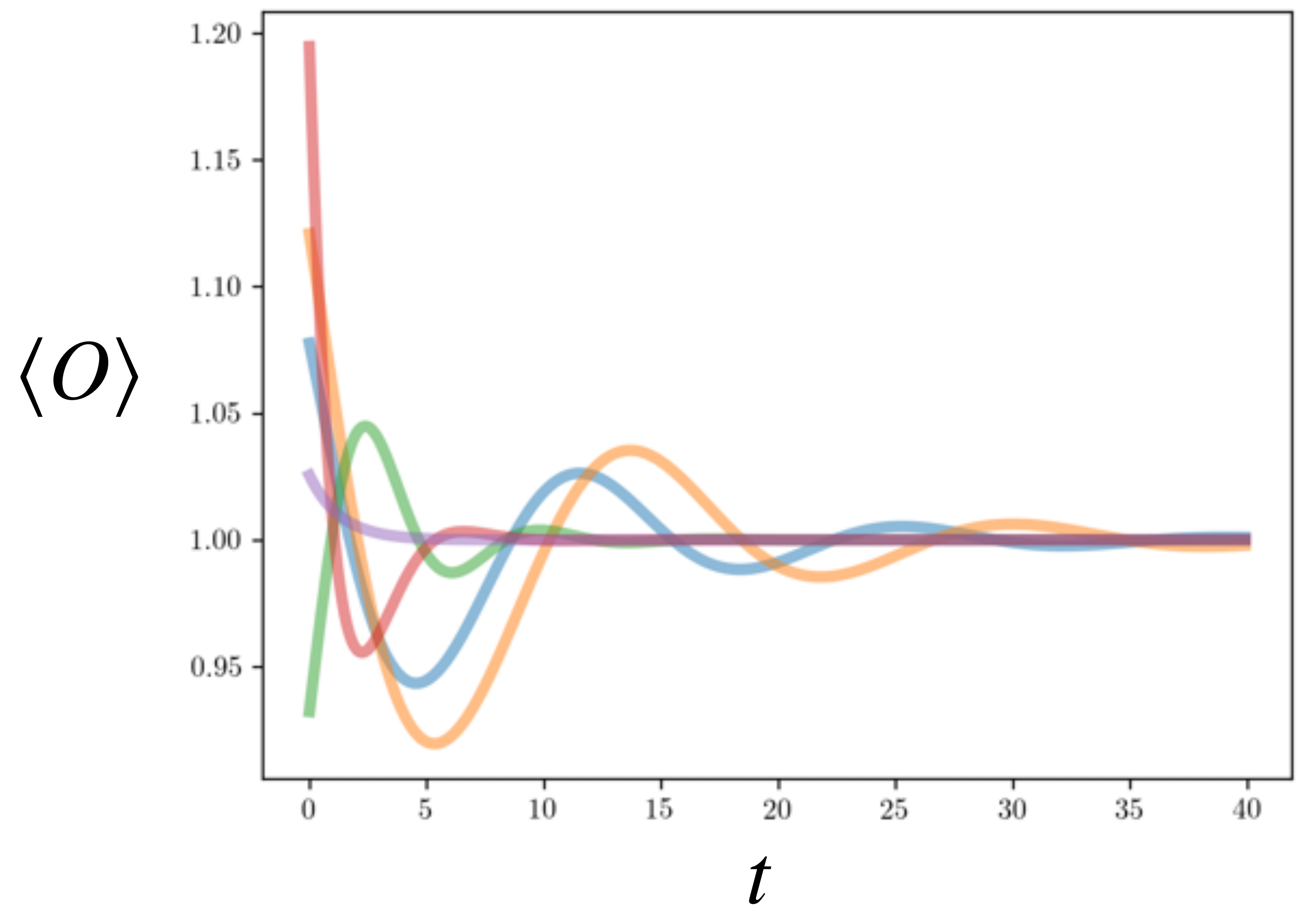
$$\mathcal{O}_{mn} = \mathcal{O}(\overline{E}) \, \delta_{mn} + e^{-S(\overline{E})/2} f_{\mathcal{O}}(\overline{E}, \omega) \, R_{mn}$$

$\overline{E} = (E_m + E_n)/2$; $\omega = E_m - E_n$; $f_{\mathcal{O}}$ — smooth function; R_{mn} — random variable



ETH zürich

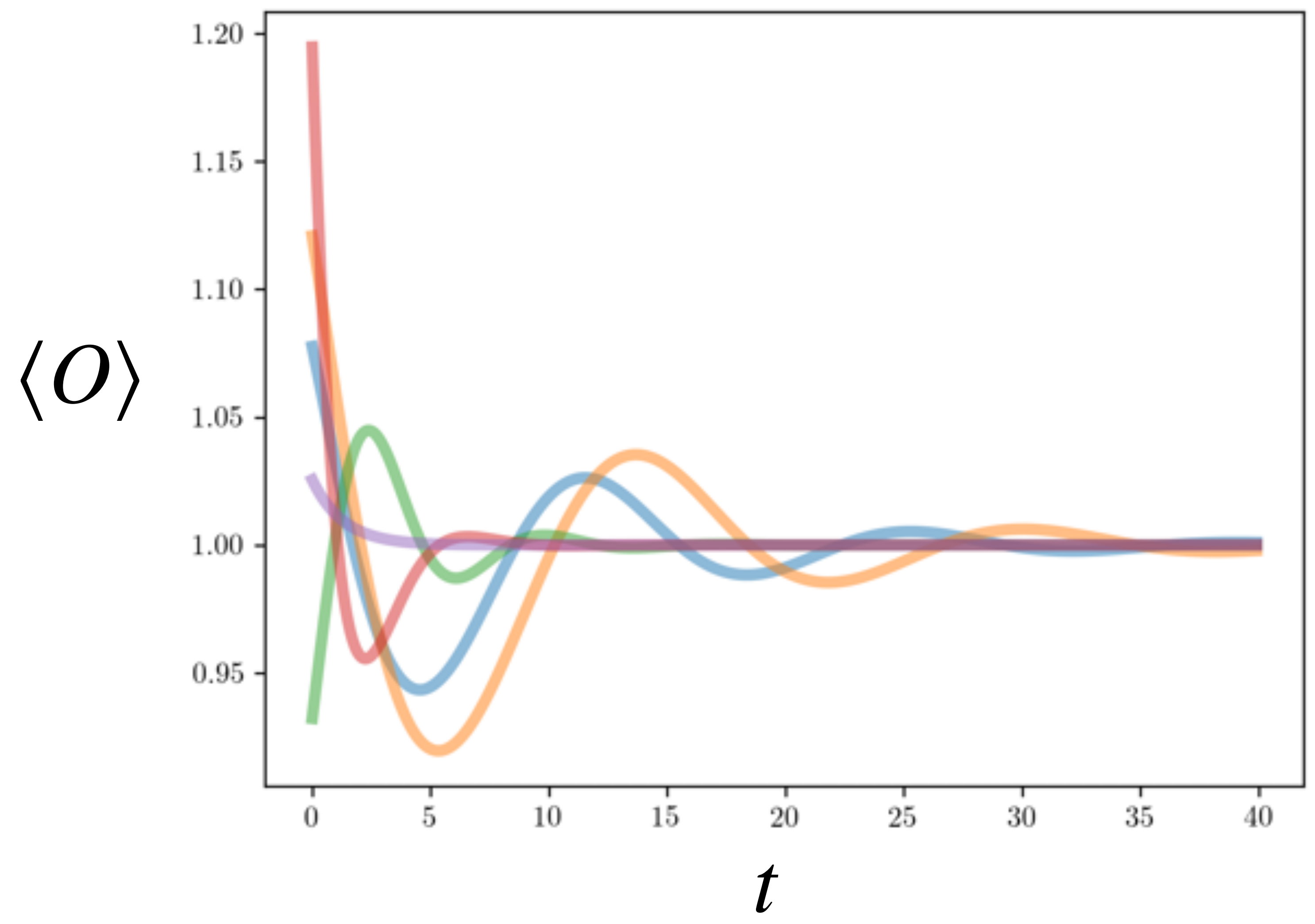
Thermalization of isolated quantum systems



Exceptions to thermalization:

- Integrable models
- Many Body Localization
- Fragmentation
- *Quantum Many-Body Scars*: special initial conditions avoid thermalization [Turner et al.,Nature Physics (2018))]

Thermalization of isolated quantum systems

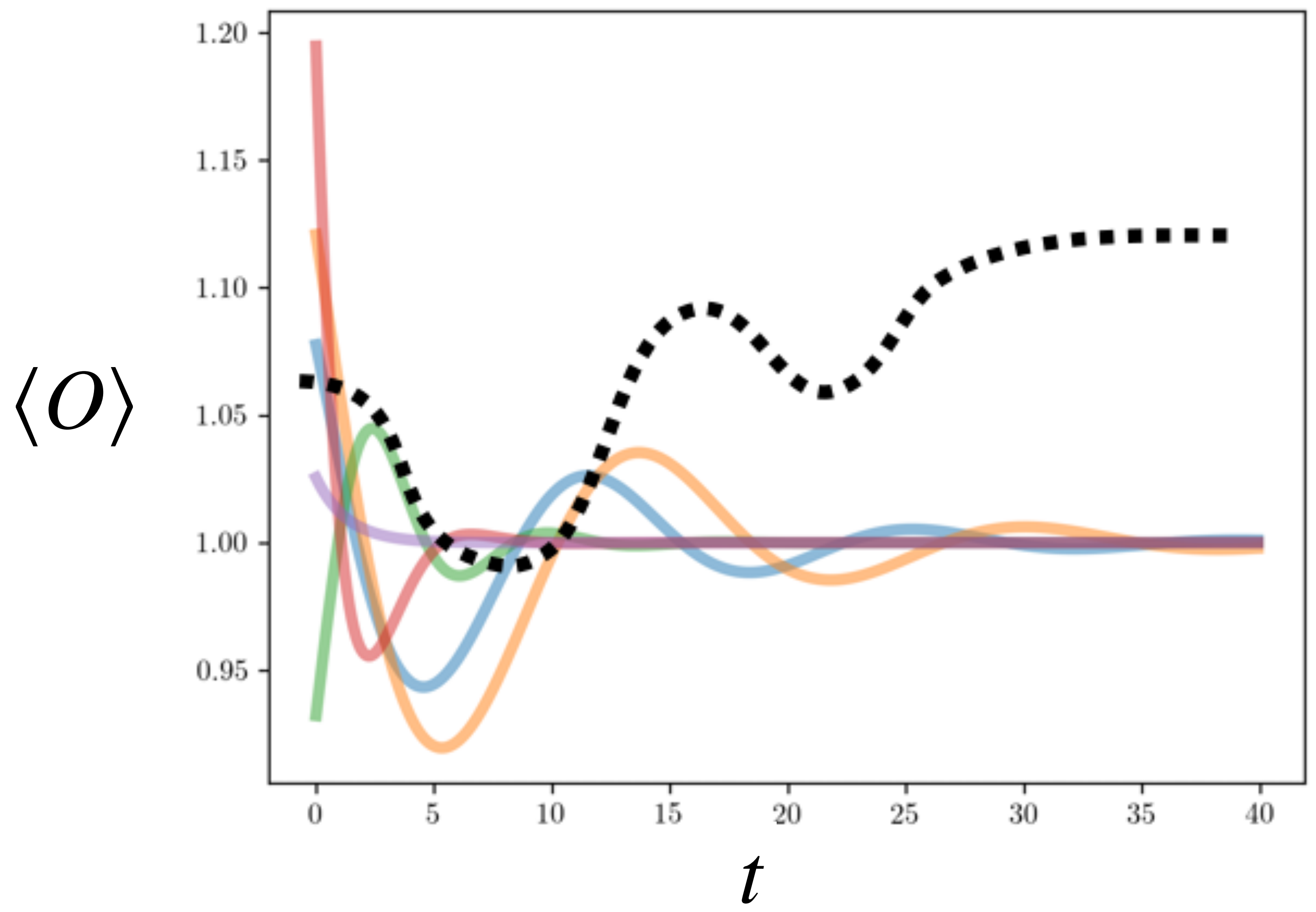


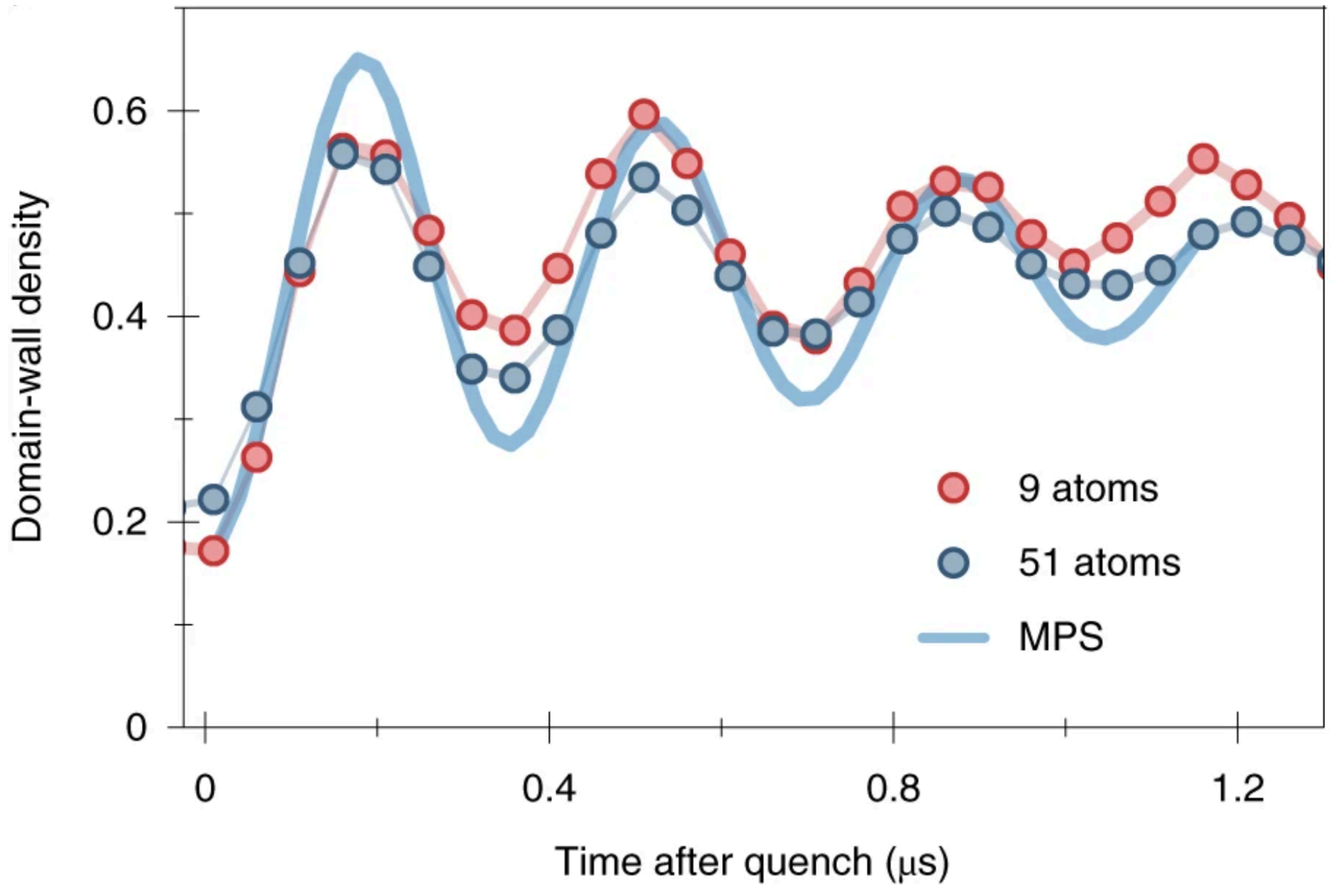
Exceptions to thermalization:

- Integrable models
- Many Body Localization
- Fragmentation
- *Quantum Many-Body Scars*: special initial conditions avoid thermalization [Turner et al.,Nature Physics (2018))]

- Ergodicity: single fragment Hilbert space; Hamiltonian connects all basis states
- Strong breaking of ETH: integrable models, disordered systems; exponentially many fragments; $\frac{n}{N_{tot}} \sim e^{-V}$
- QMBS: weak breaking of ETH, (many) anomalous high-energy states; $\frac{n}{N_{tot}} \to 1$ as $V \to \infty$.

Thermalization of isolated quantum systems

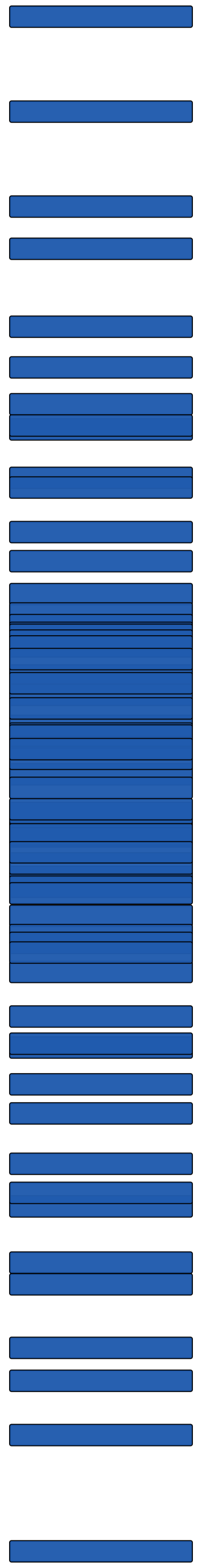




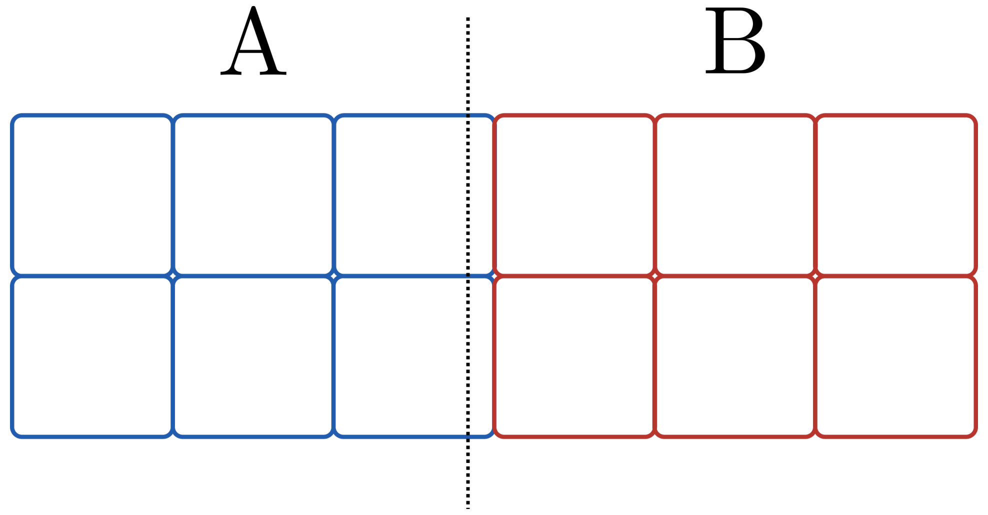
[Bernien et al. Nature 551 (2017)]

- Quantum Many-Body Scars: special initial conditions avoid thermalization [Turner et al.,Nature Physics (2018), Pakrouski et al. PRL 125, 230602 (2020)]
- Fundamental questions that might be answered on quantum simulators
- Real-time evolution; severe sign problems [Troyer, Wiese PRL 94 170201(2005)]
- Signatures found in analog quantum simulations with Rydberg atom arrays [Bernien et al. Nature (2017)]

Observables of interest: entanglement entropy

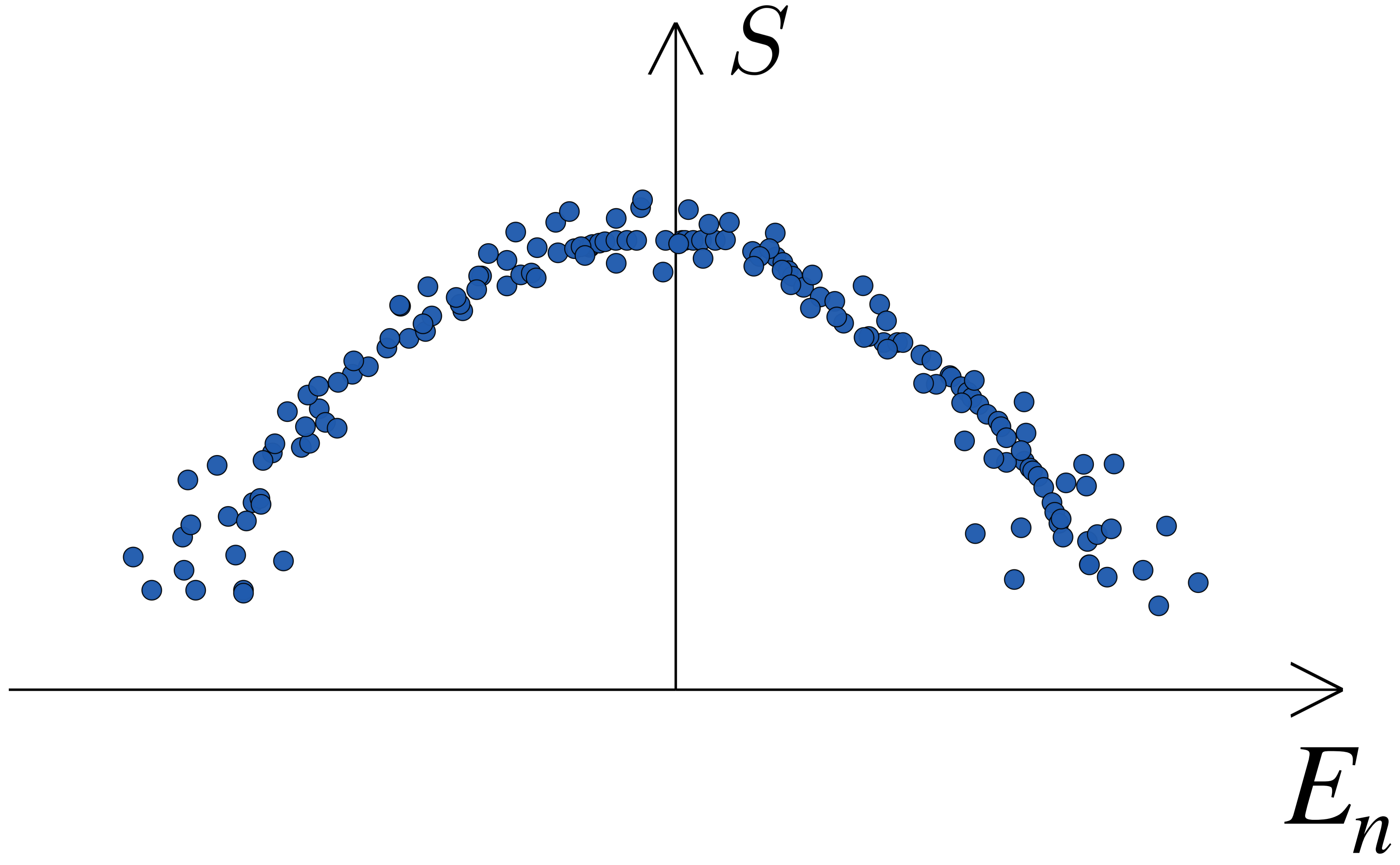


Bipartite entanglement entropy:

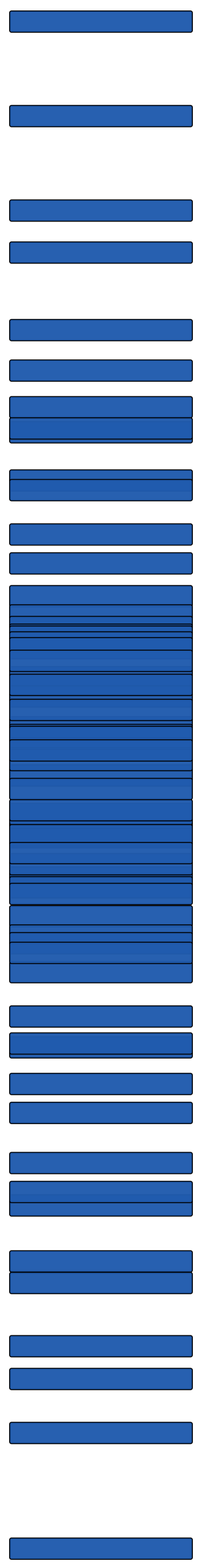


Compute the trace: $\rho_A = \mathrm{tr}_B \rho$

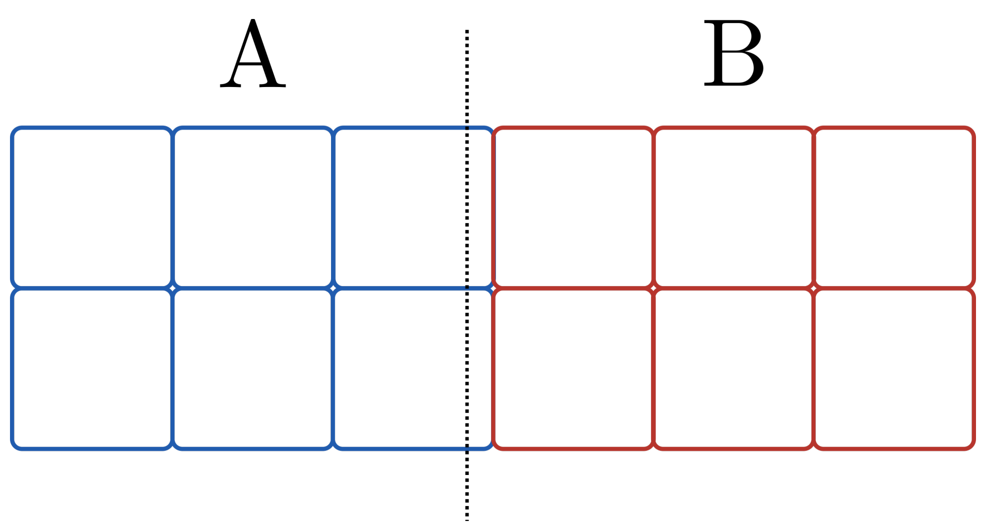
Compute entanglement entropy: $S = -\mathrm{tr} \rho_A \log \rho_A$



Observables of interest: entanglement entropy

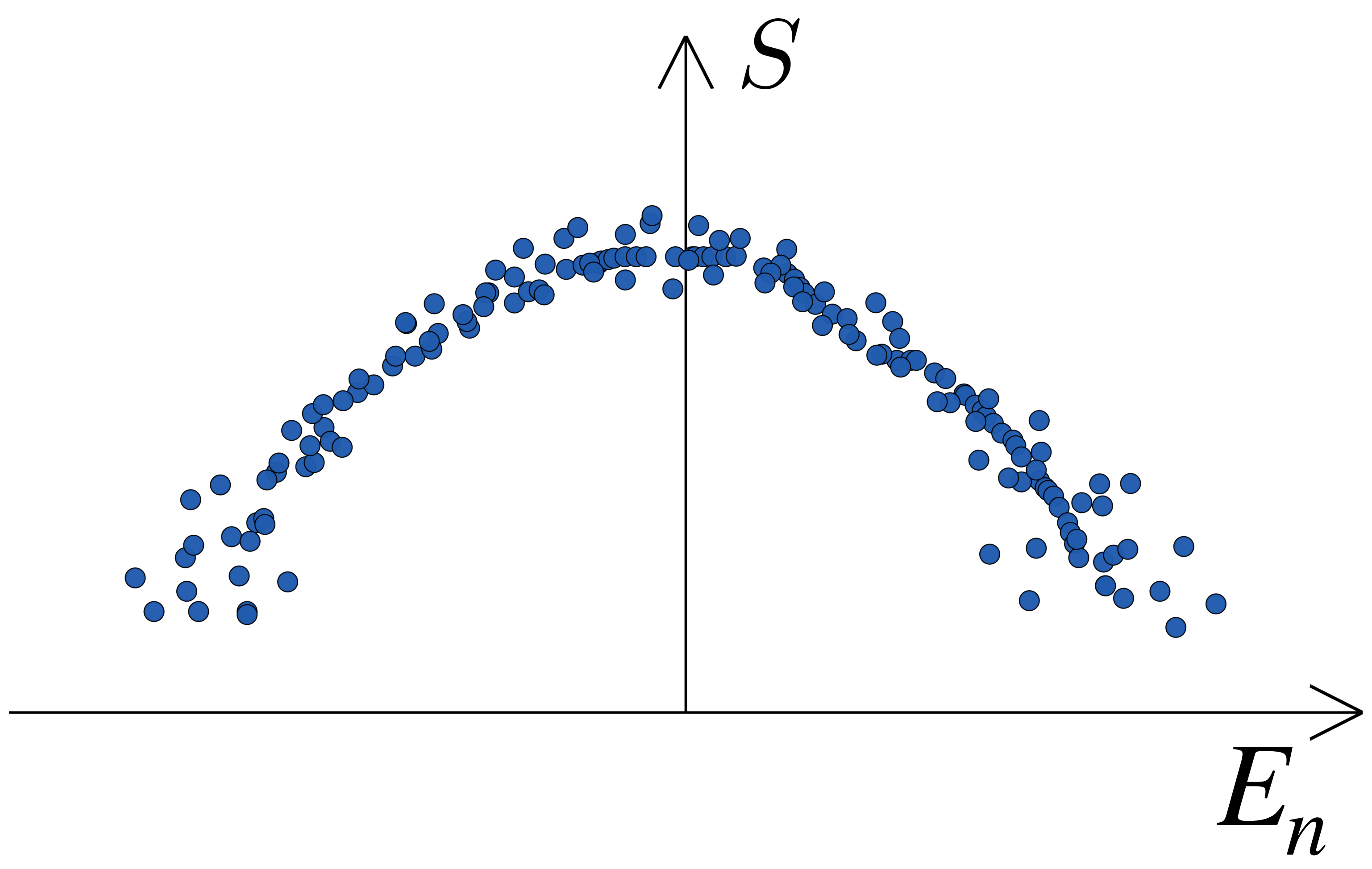


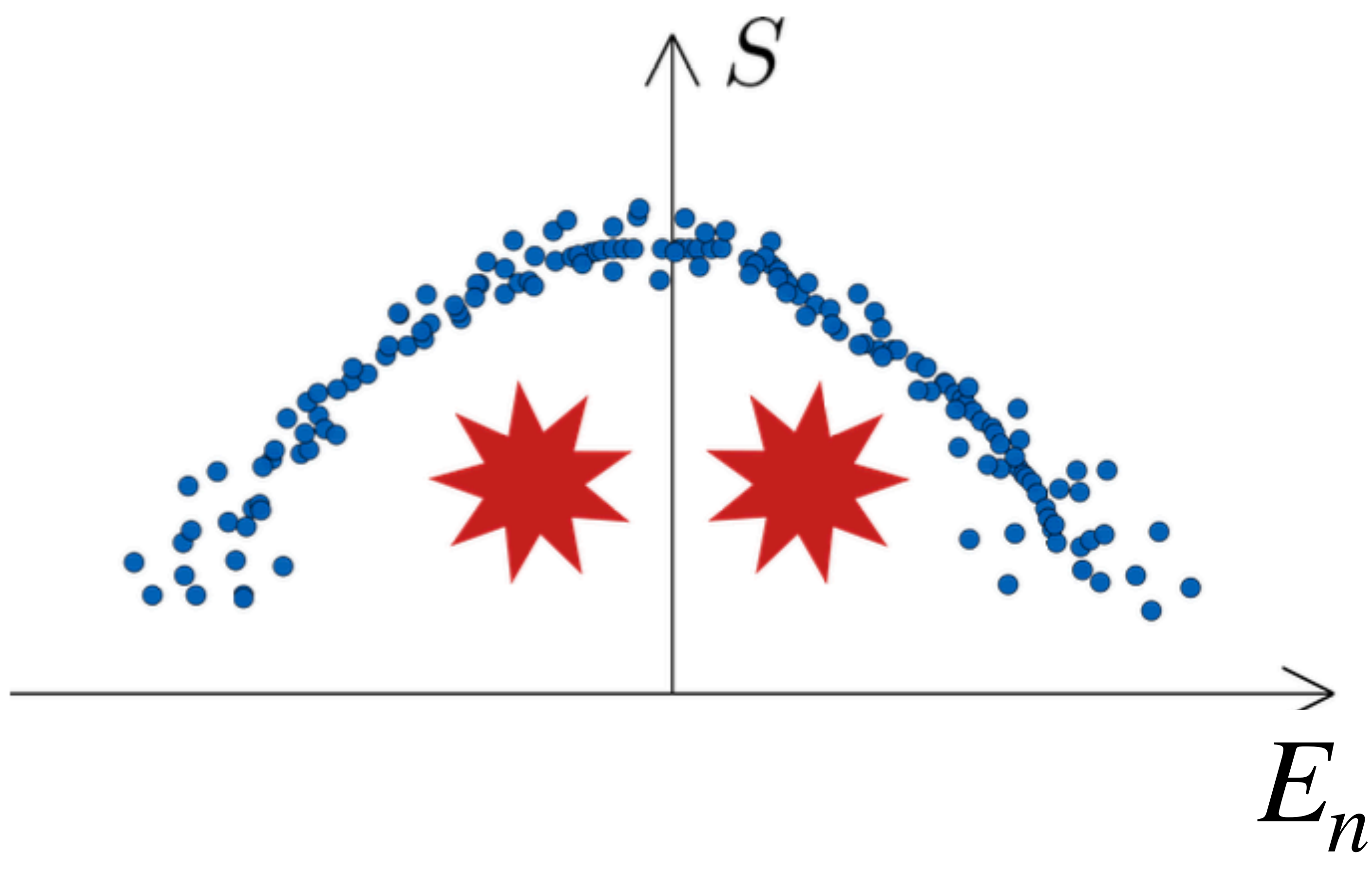
Bipartite entanglement entropy:



Compute the trace: $\rho_A = \mathrm{tr}_B \rho$

Compute entanglement entropy: $S = -\mathrm{tr} \rho_A \log \rho_A$





QMBS: entanglement measure shows anomalous behaviour

Quantum Many-Body Scars in Gauge Theories

- Observed experimentally in the PXP model (Rydberg atoms); [H. Bernien et al. Nature (2017)]
- PXP maps exactly to a U(1) gauge theory in 1+1D; [Surace et al. PRX (2020)]
- Quantum scars found in a variety of 1+1D gauge theories [Wang et al. PRL (2022), Desaules et al. PRX (2023), Halimeh et al. Quantum (2023), G. Calajo arXiv:2405.13112]
- Quantum scars predicted in 2+1D for $S = 1/2$ [Banerjee et al. PRL (2022), Biswas et al. SciPost. Phys. (2022), Sau et al. PRD (2024)]
- Quantum scars in 2+1D theory with fermions for $S = 1/2$ [Osborne et al. 2403.0885]
- Quantum scars for arbitrary S in 2+1D pure gauge theories [this work, Budde, MKM, Pinto Barros, PRD 110, 094506 (2024)]

Hamiltonian of U(1) gauge theory in 2+1D

- Quantum spins: continuous gauge symmetries with discrete link operators
- Finite Hilbert space at each link: (2S + 1)-dim.

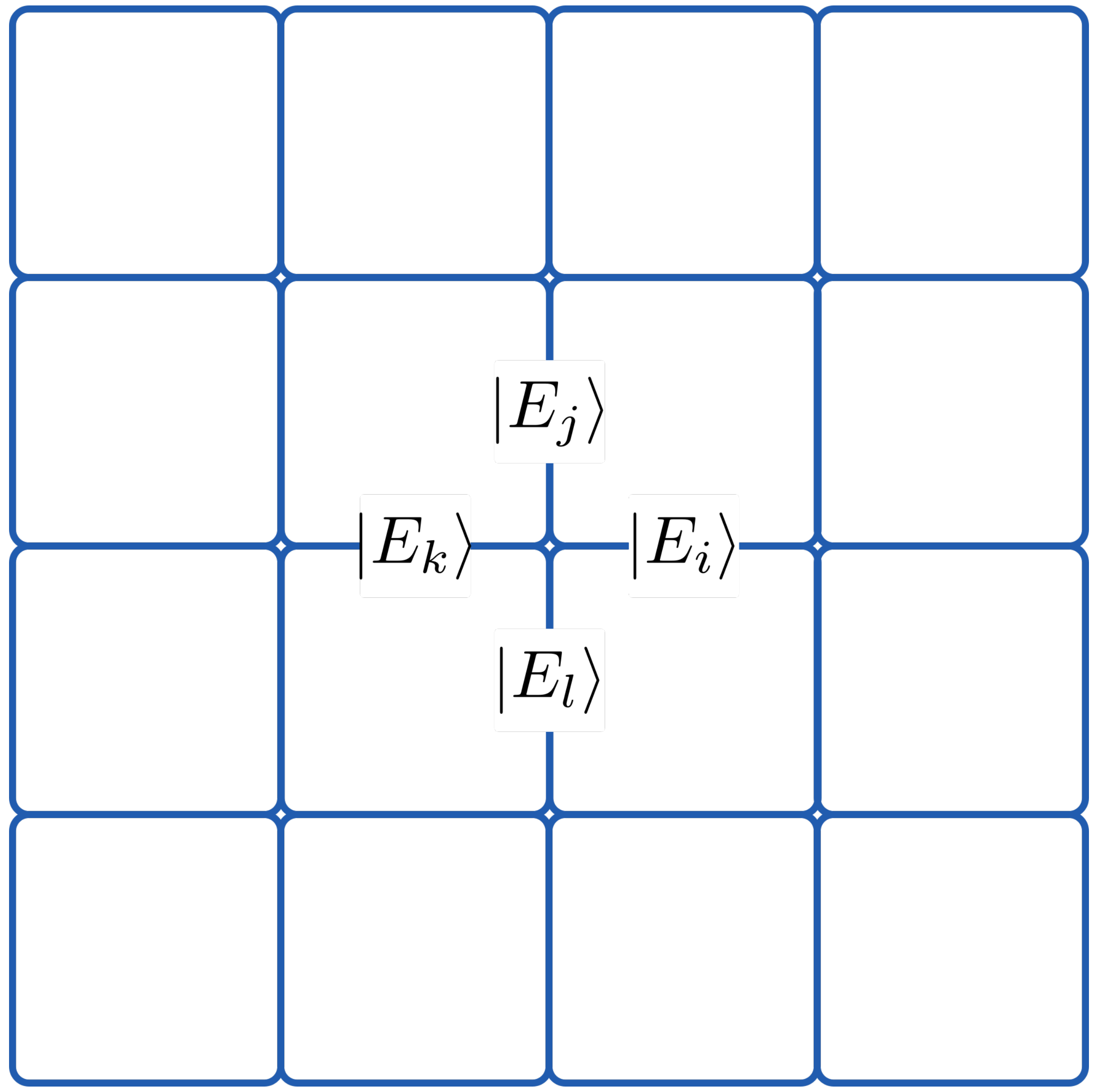
$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\Box} + \text{h.c.} + \kappa \sum_n E_n^2$$

ETH zürich

Hamiltonian of U(1) gauge theory in 2+1D

- Quantum spins: continuous gauge symmetries with discrete link operators
- Finite Hilbert space at each link: (2S + 1)-dim.

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$



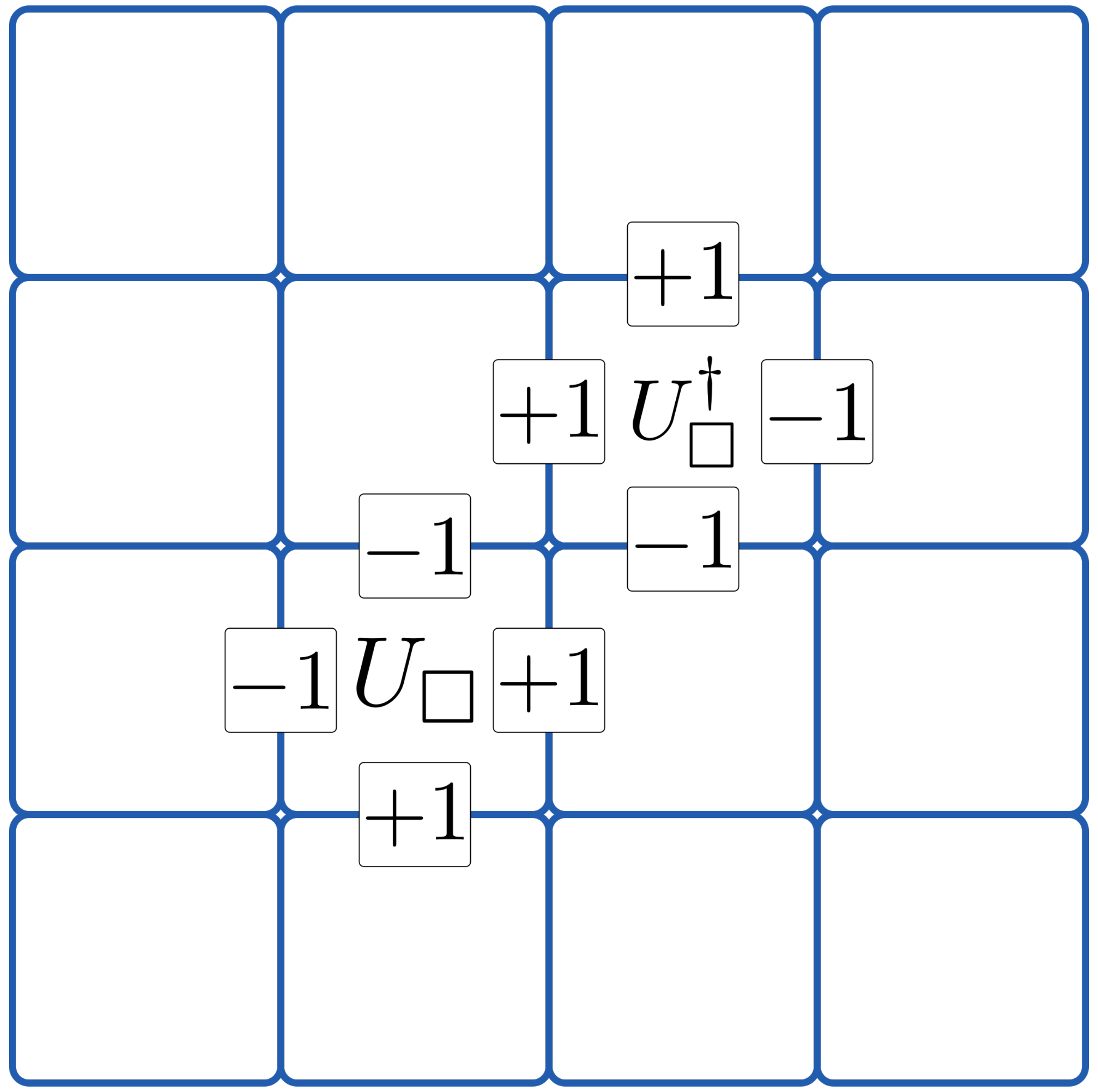
$E_n \in \mathbb{Z}$

U_n unitary raising operator

$U_i \, |E_i\rangle = |E_i + 1\rangle$

Gauss' Law

$E_i - E_k + E_j - E_l = 0$



Hamiltonian of U(1) gauge theory in 2+1D

- Quantum spins: continuous gauge symmetries with discrete link operators
- Finite Hilbert space at each link: $(2S+1)$-dim.

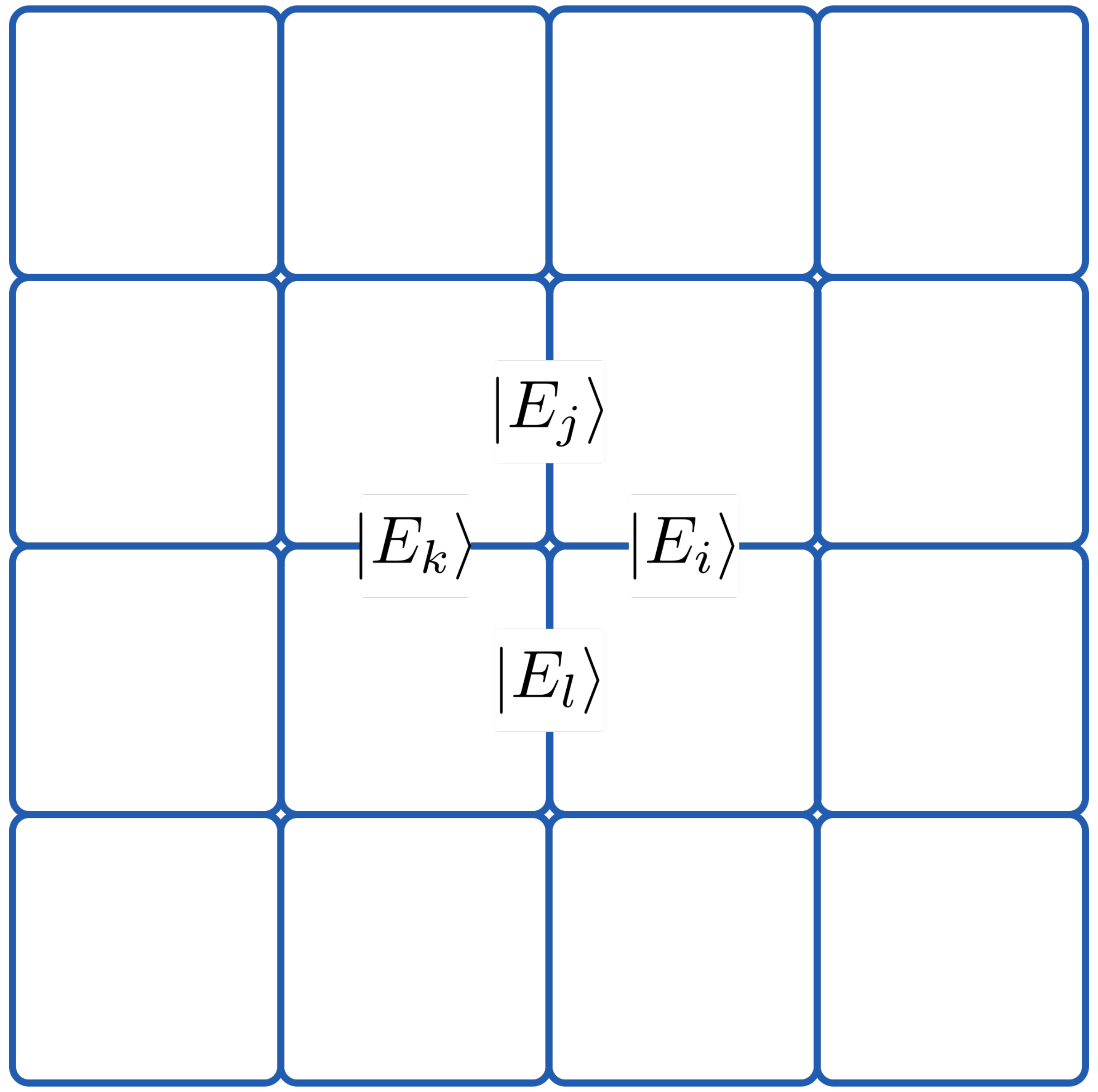
$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$

Quantum Link Models (QLM) [Horn PLB (1981); Orland, Rohrlich NPB, (1990); Wiese, Chandrasekharan NPB, (1997); Rokhsar, Kivelson PRL (1988)]

$$U_i |E_i\rangle = \sqrt{\frac{S(S+1) - E_i(E_i+1)}{S(S+1)}} |E_i + 1\rangle$$

Truncated Link Models (TLM) [Zohar, Cirac, Reznik PRL110 (2012); Desaules et al. PRB107 (2023), Popov et al. 2405.00745 PRD(2025)]

$$U_i |E_i\rangle = E_i |E_i + 1\rangle; \; U|S\rangle = 0; \, U^\dagger|-S\rangle = 0$$



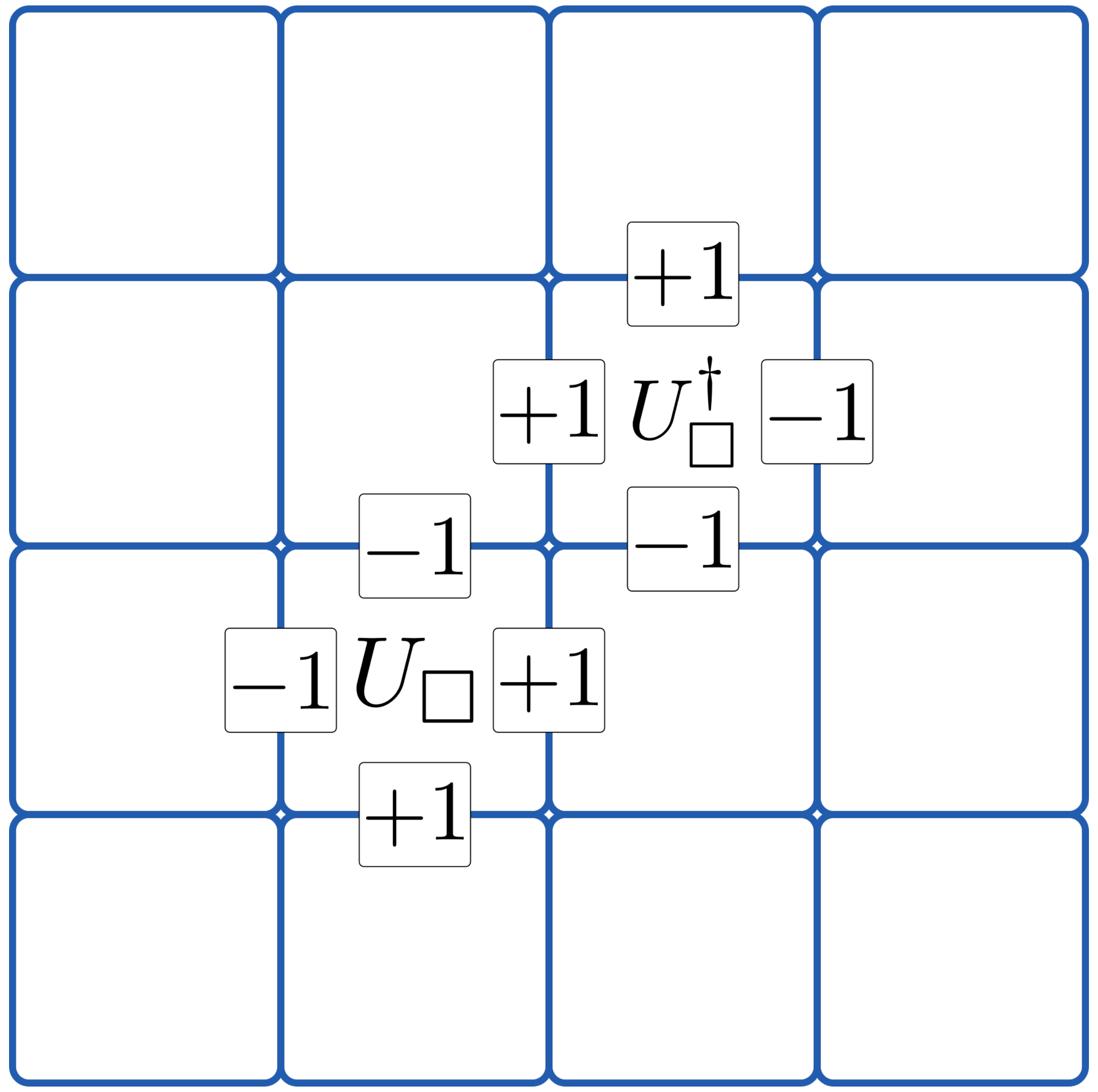
$E_n \in \mathbb{Z}$

U_n unitary raising operator

$U_i |E_i\rangle = |E_i + 1\rangle$

Gauss' Law

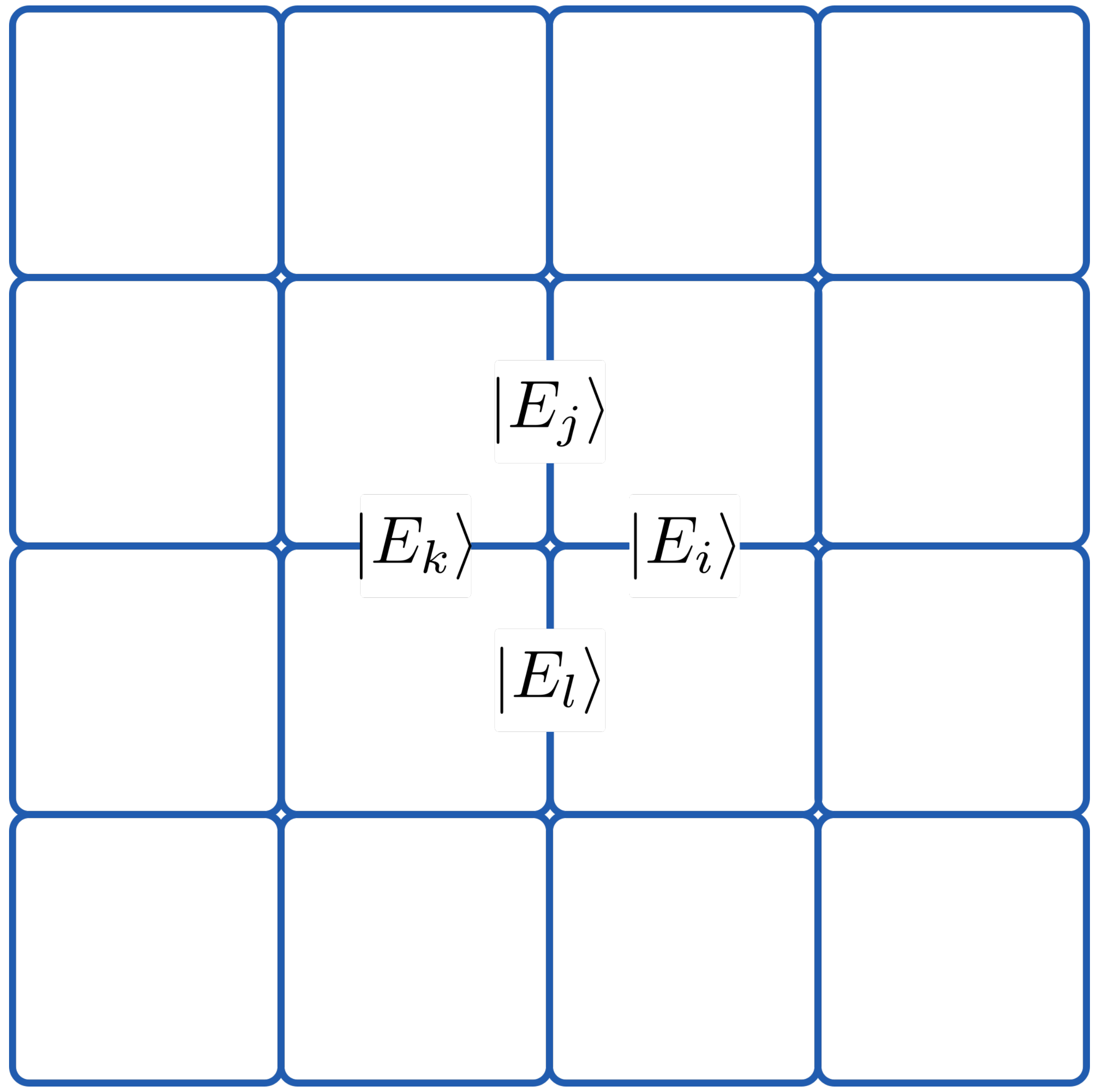
$E_i - E_k + E_j - E_l = 0$



Truncated Hamiltonian of U(1) gauge theory in 2+1D

$$E_n \in \{-S, \dots, S\}$$

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$



U_n **non**-unitary raising operator

$$U_i |E_i\rangle = |E_i + 1\rangle$$
$$U_n |S\rangle = 0$$

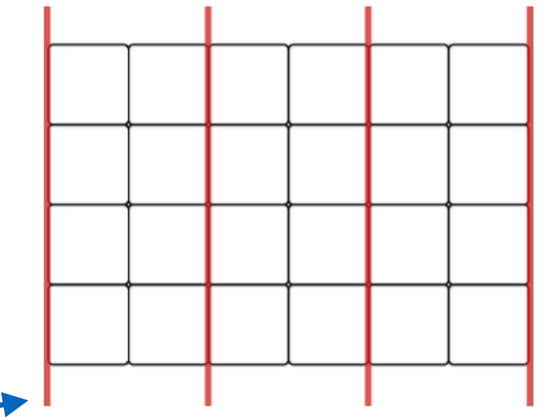
Gauss' Law

$$E_i - E_k + E_j - E_l = 0$$

- So far: $S = 1/2$; $S = 1$ spin models; both Quantum Link Models (**QLMs**), Truncated Link Models (**TLMs**)
- Gauge theory recovered in $S \to \infty$ limit
- In practice: S "large enough"
- In this work: S is arbitrary integer
- Note about **QLMs**: not constructed with $S \to \infty$ limit in mind

Index theorem for U(1) gauge theory in 2+1D

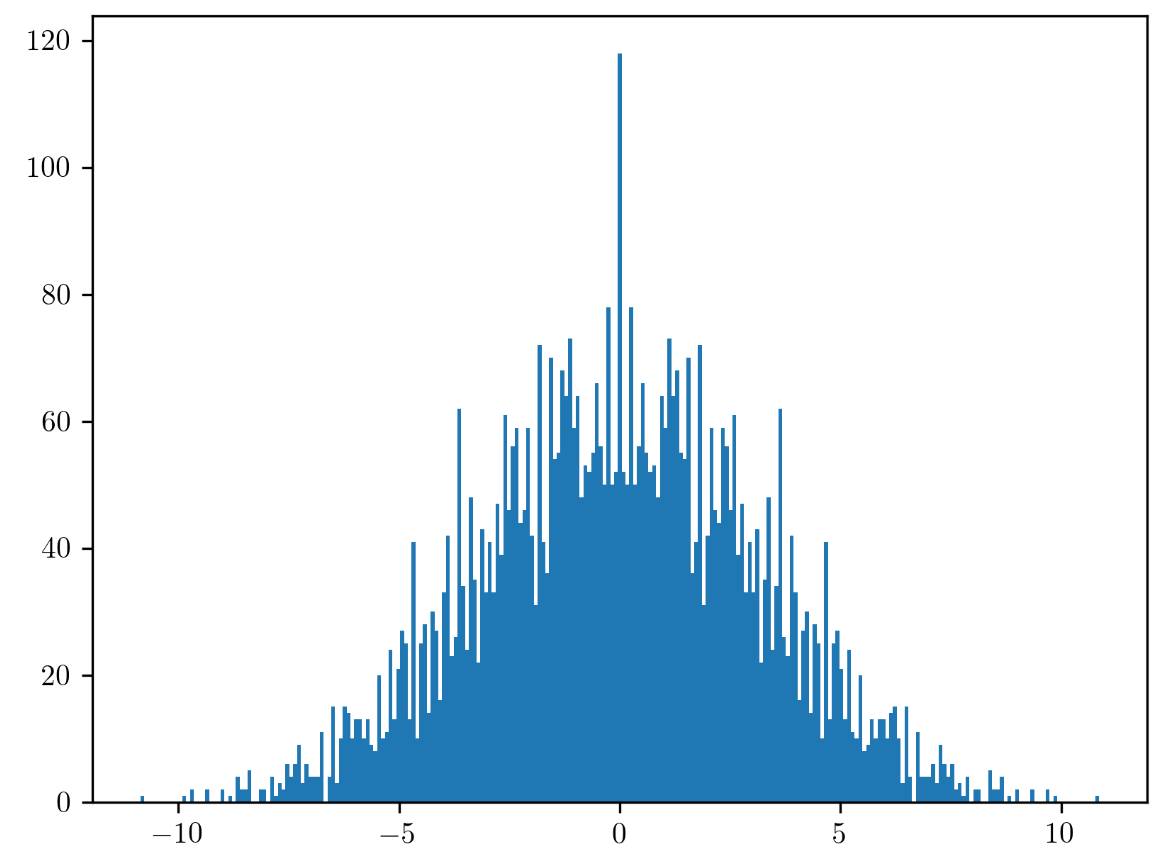
- Point group symmetries: lattice translation, rotations, reflections



- Charge conjugation: $(U, U^\dagger, E) \to (U, U^\dagger, -E)$

- For $\kappa = 0$ holds $\{H, \mathbb{C}\} = 0$; $\mathbb{C} = \prod_{xy} E_{xy} \to$ $x(y)$ even; $\mathbb{C}|E\rangle = |-E\rangle$

Density of States



- Zero-modes are protected by an index theorem and have a $\mathbb{C}$-charge [Schechter, Iadecola PRB 98, 035139 (2018)]

- # of zero-modes grows exponentially with volume in non-integrable spin chains

- Can we use the zero-modes to build low-entropy states for gauge theories?

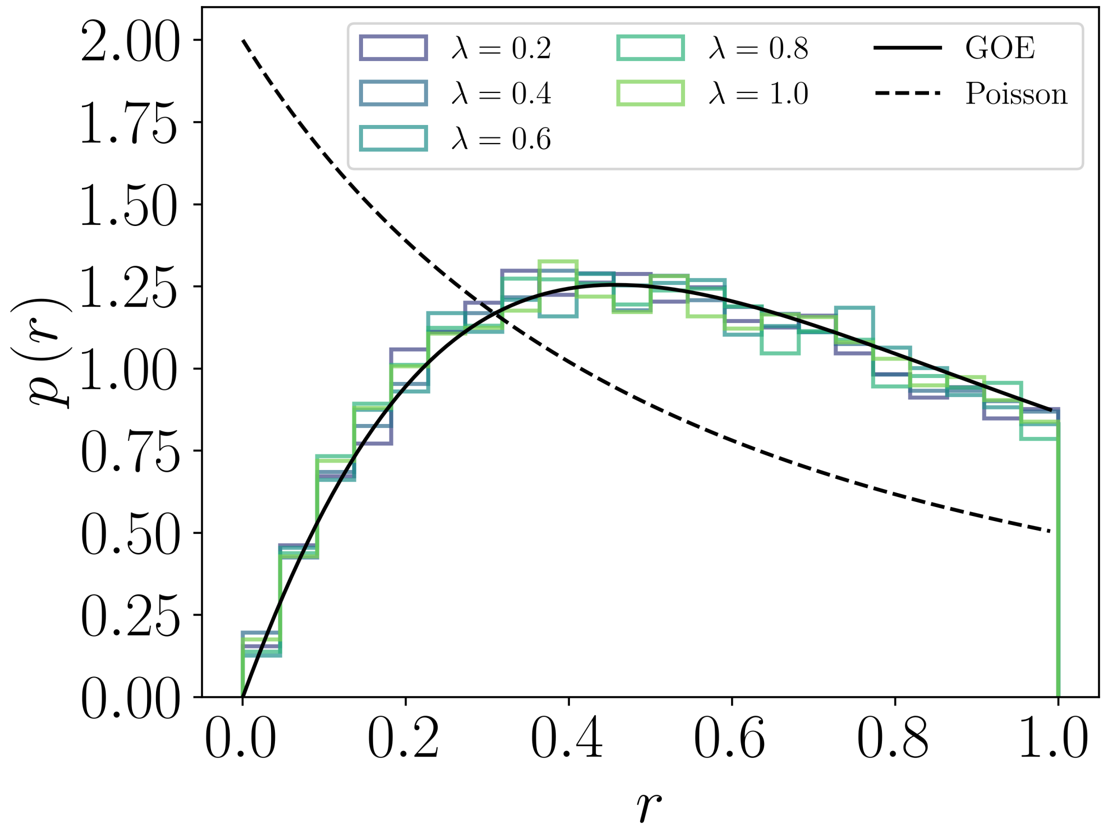
ETH zürich

Non-integrability of a 2+1D system with ladder geometry

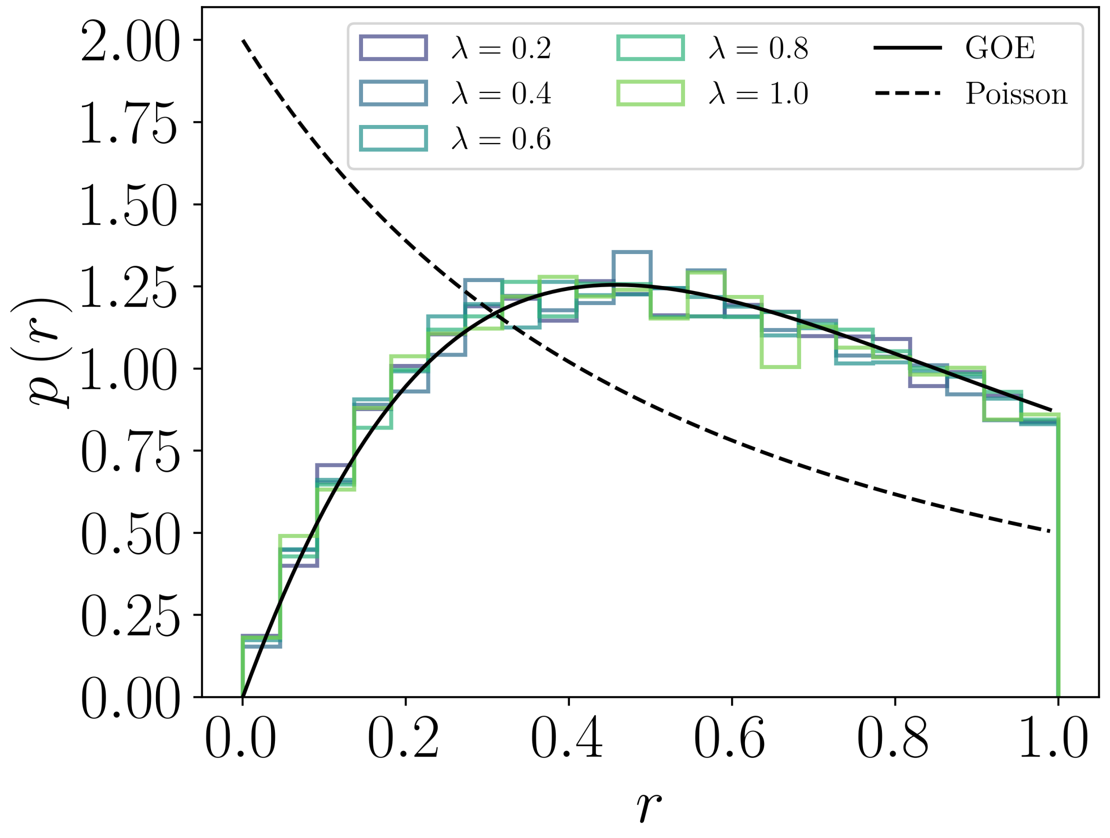
- Planar quantum (truncated) link models are strongly interacting, non-integrable physical systems

- Demonstrate non-integrability: resolve symmetries and compute level space distribution

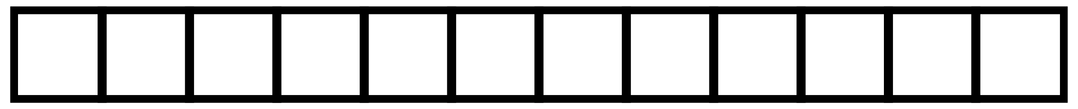
$-$ Parity Symmetry Sector



$+$ Parity Symmetry Sector



12×1 **lattice:**



$$p(r), \quad r_n = \min\left\{\frac{E_{n+1} - E_n}{E_n - E_{n-1}}, \frac{E_n - E_n - 1}{E_{n+1} - E_n}\right\}$$

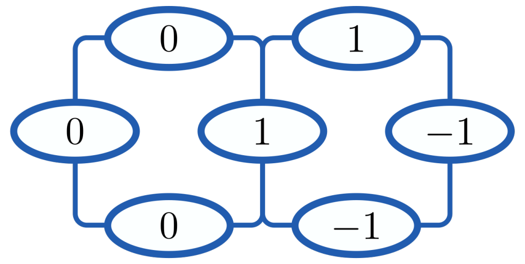
- Integrable systems: Poisson $p_{Poisson}(r) = \dfrac{1}{(1+r)^2}$

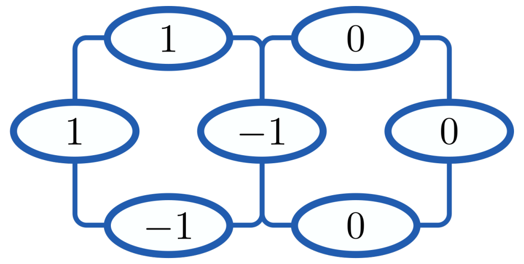
- Non-integrable: Gaussian Orthogonal Ensemble (GOE) $p_{GOE}(r) = \dfrac{27}{4}\dfrac{r + r^2}{(1 + r + r^2)^{5/2}}$

ETH zürich

$S = 1$ QLM: two-plaquette zero mode

- Explicit construction of a zero mode corresponding to $H_{kin} = \sum_n (U_\square + U_\square^\dagger)$ for $\square\square$ lattice:

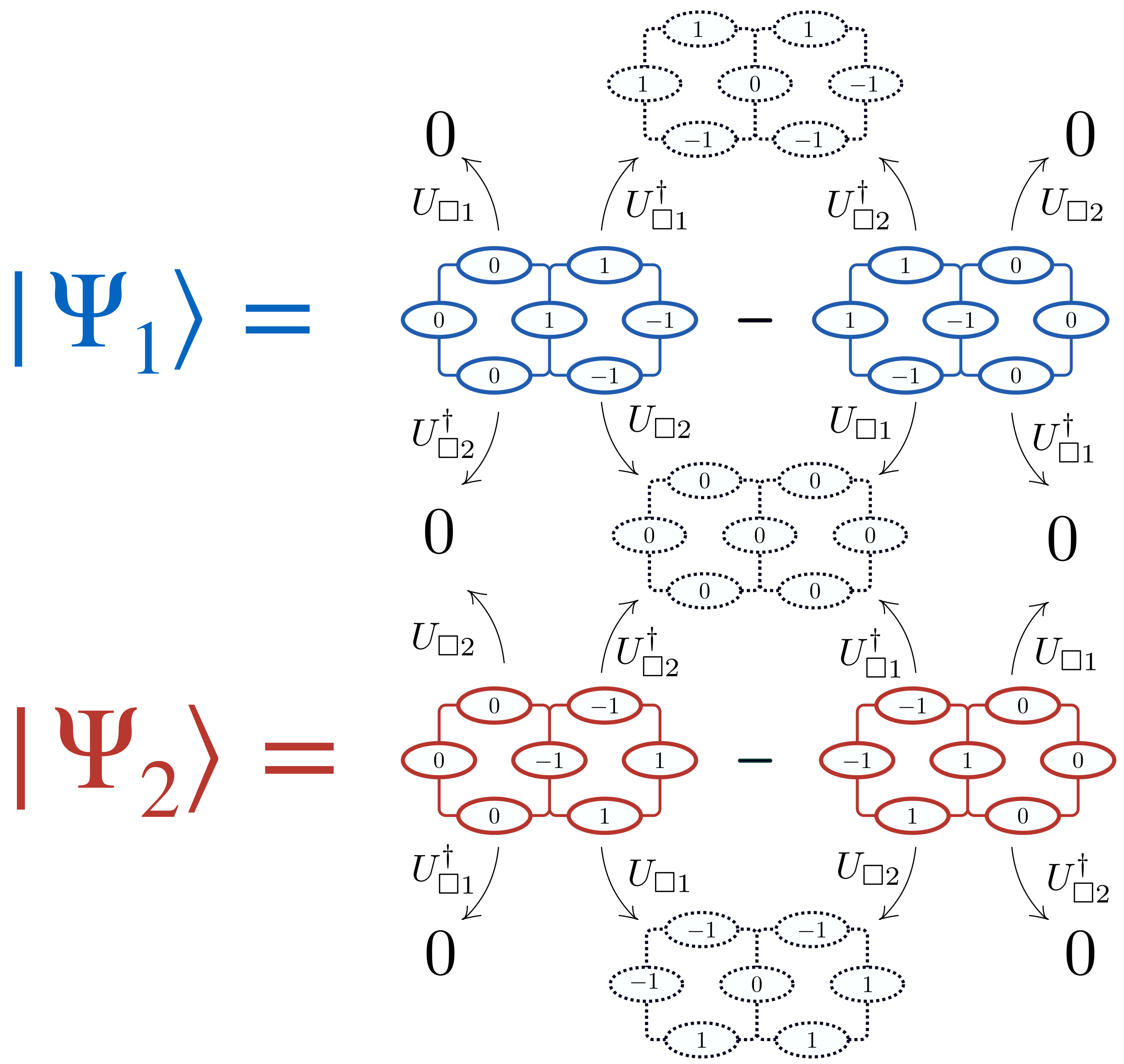
$|\Psi_1\rangle = \frac{1}{\sqrt{2}}$ 

− 

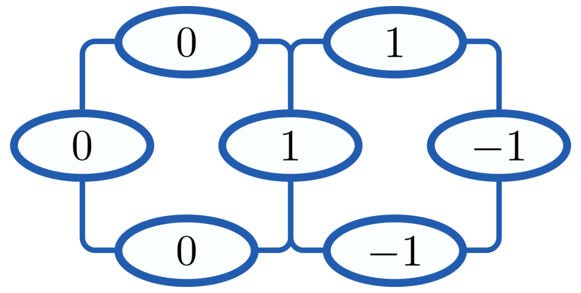
$$\sum_n \left(U_{n1}^\dagger U_{n+\hat{1}2}^\dagger U_{n2} U_{n+\hat{2}1} + \text{h.c.}\right) |\Psi_1\rangle = 0$$

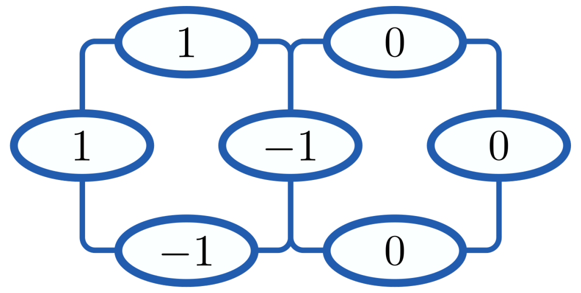
$S = 1$ QLM: two-plaquette zero mode

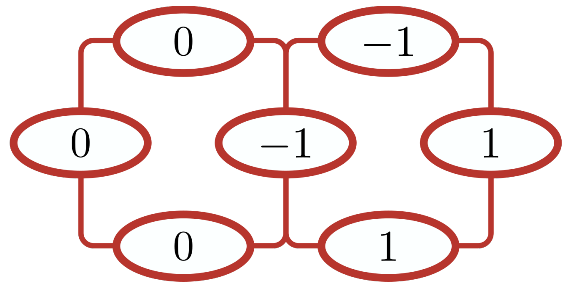
- Explicit construction of a zero mode corresponding to $H_{kin} = \sum_n (U_\square + U_\square^\dagger)$ for ◫ lattice:

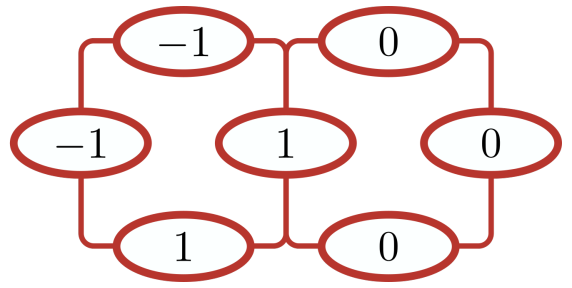


$S = 1$ QLM: two-plaquette zero mode

$|\Psi_1\rangle = \frac{1}{\sqrt{2}}$ 

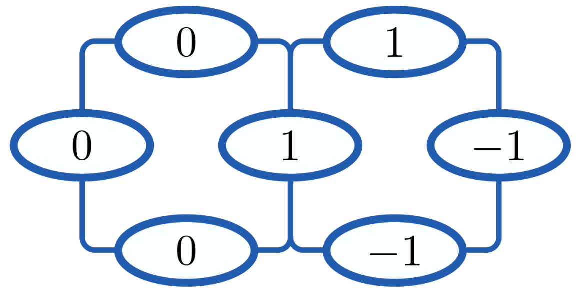
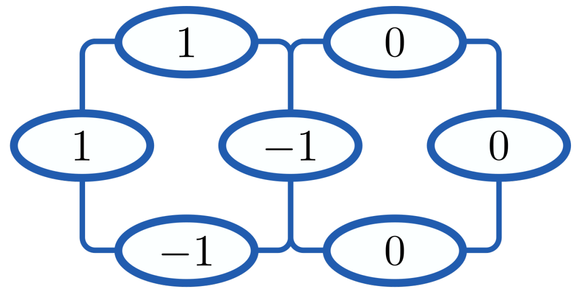
− 

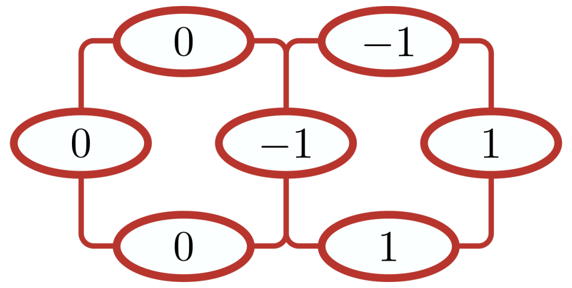
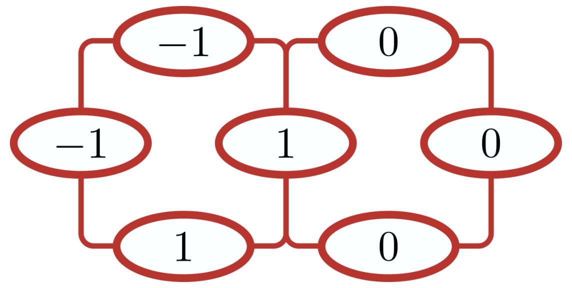
$|\Psi_2\rangle = \frac{1}{\sqrt{2}}$ 

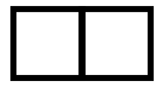
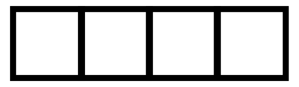
− 

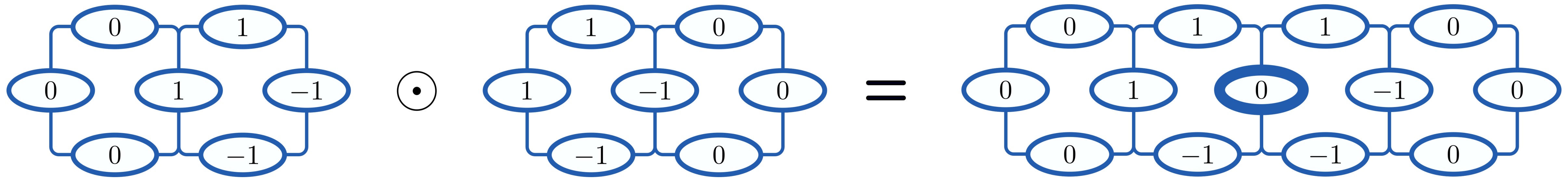
$$\sum_n \left(U_\square + U_\square^\dagger\right) |\Psi_1\rangle = \sum_n \left(U_\square + U_\square^\dagger\right) |\Psi_2\rangle = 0$$

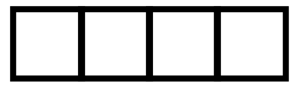
$S = 1$ QLM: two-plaquette zero mode

$|\Psi_1\rangle = \frac{1}{\sqrt{2}}$  $-$ 

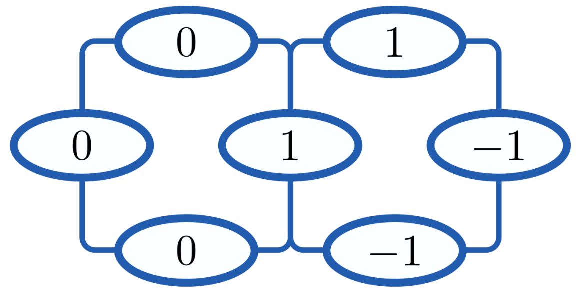
$|\Psi_2\rangle = \frac{1}{\sqrt{2}}$  $-$ 

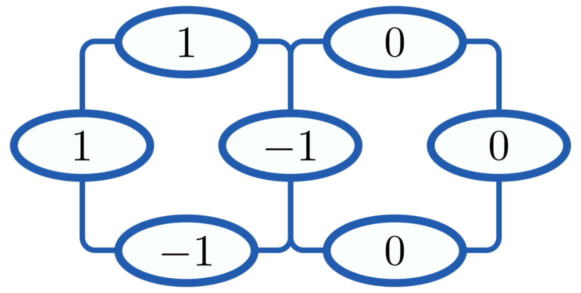
Tiling procedure:  $\longrightarrow$  :

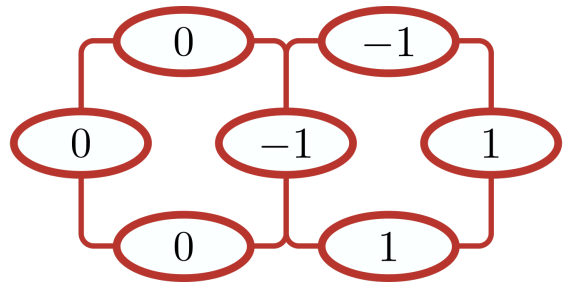


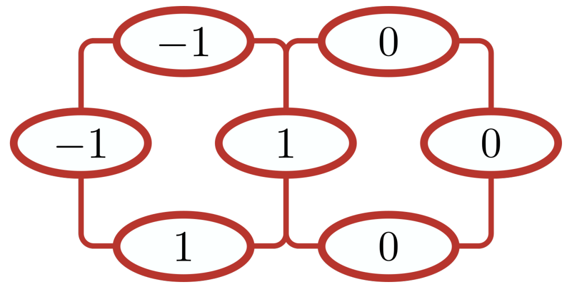
- Zero-energy state for H_{kin} on a  lattice

$S = 1$ QLM: two-plaquette zero mode

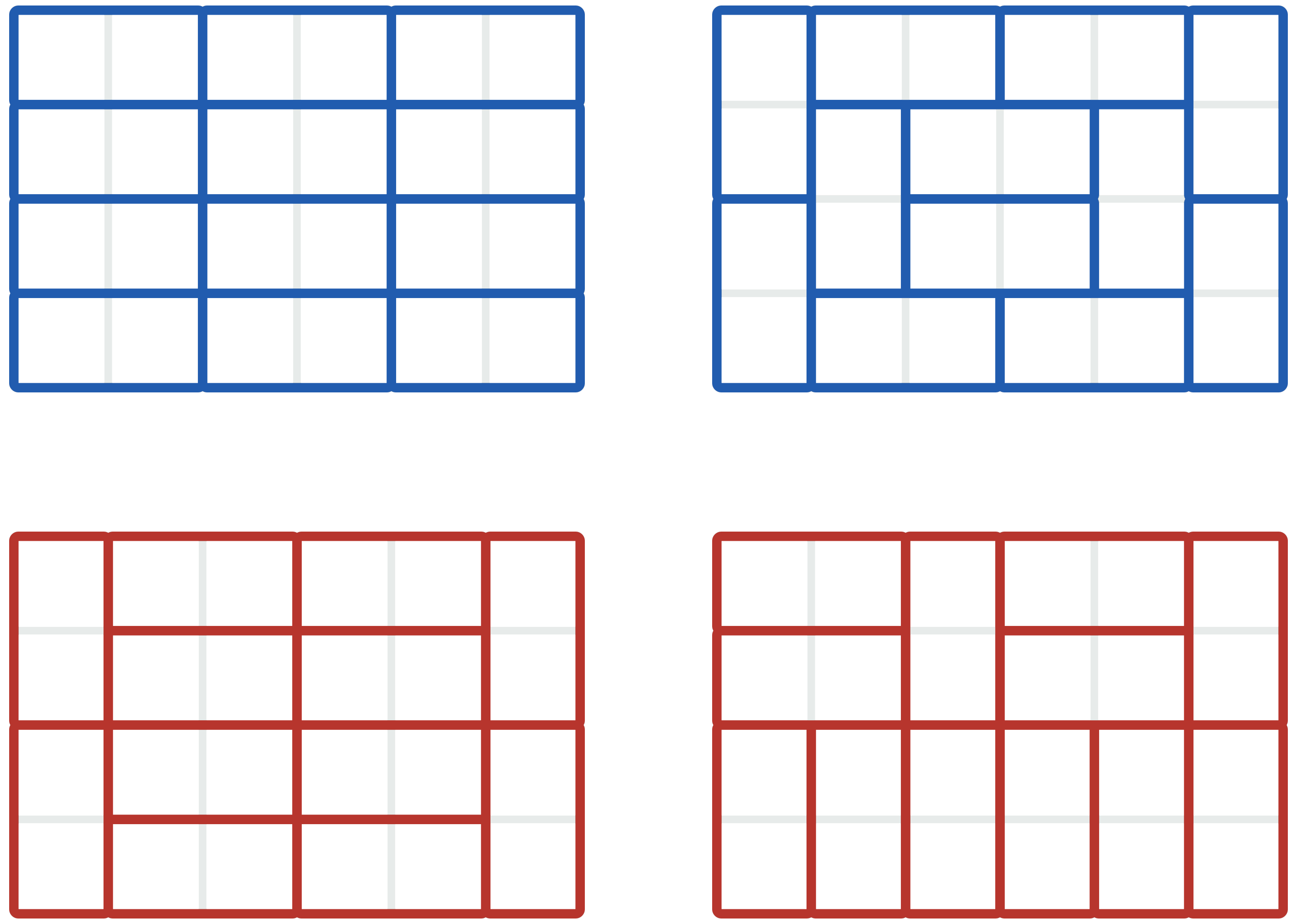
$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}$$ 

– 

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}$$ 

– 

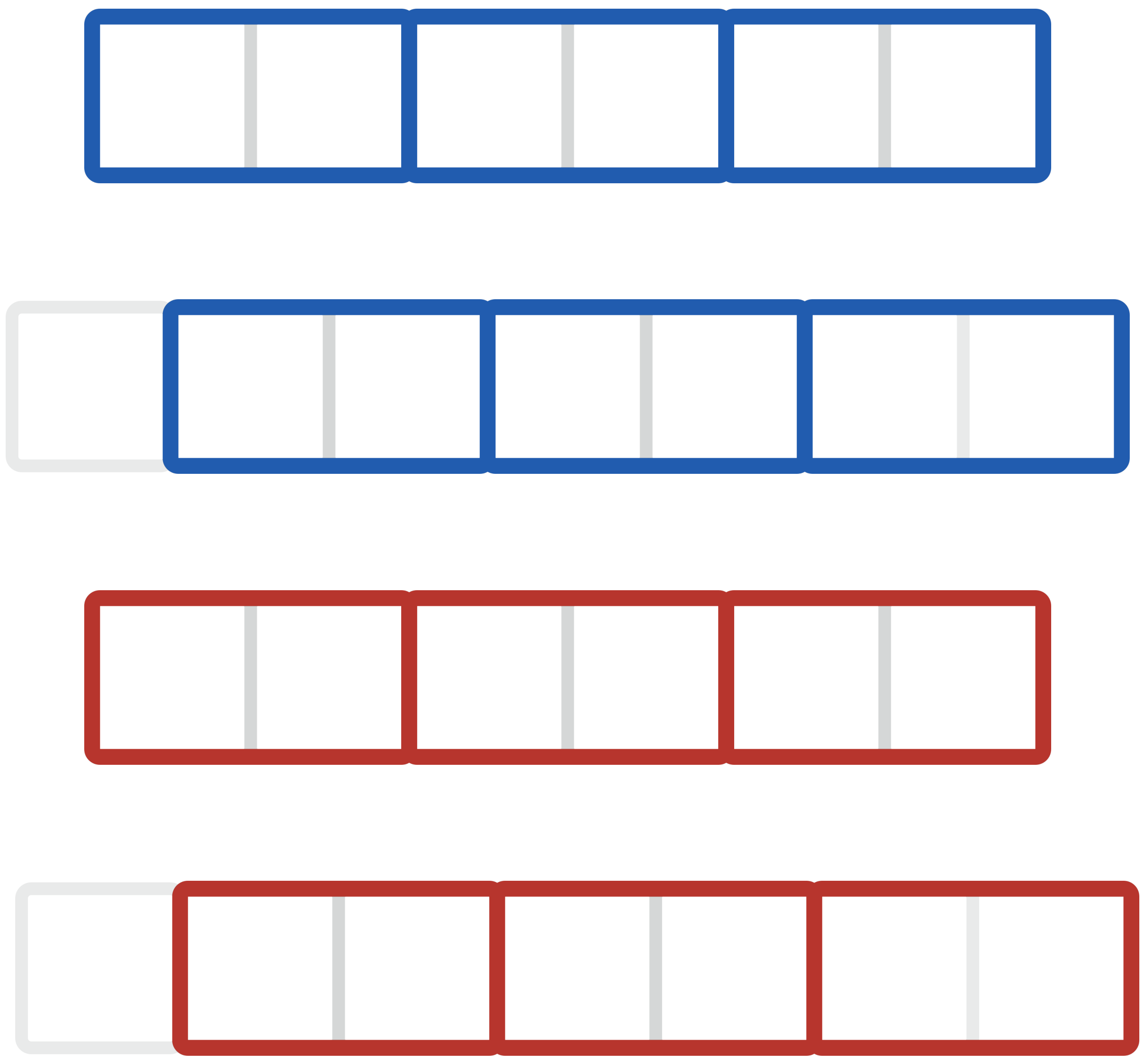
Construct many zero-mode states by tensoring appropriate zero-modes:

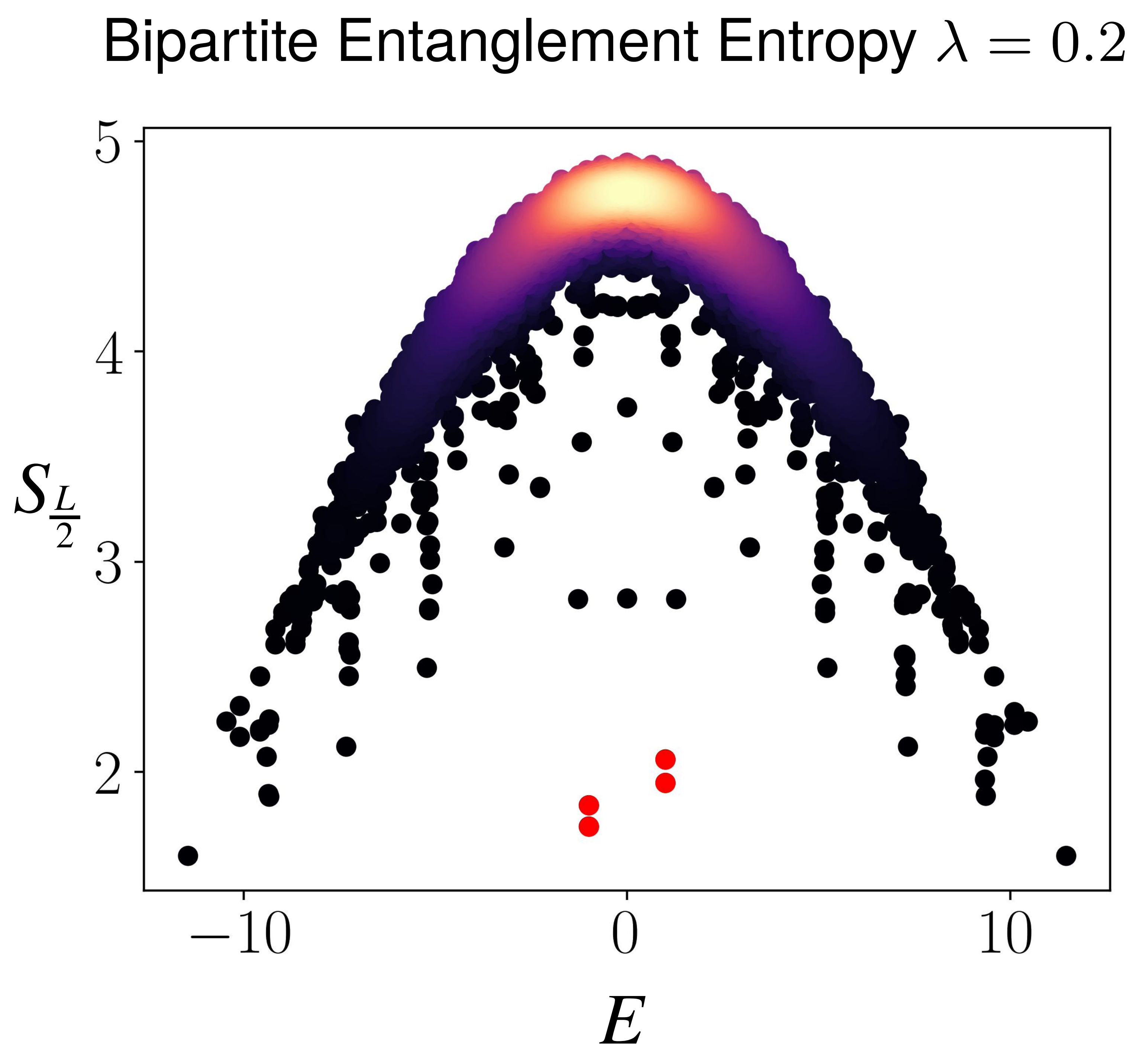


o o o

The Periodic Ladder

$$H = \sum_{n} \left(U_{\square} + U_{\square}^{\dagger} \right) + \lambda \sum_{m \text{ top row}} E_m$$

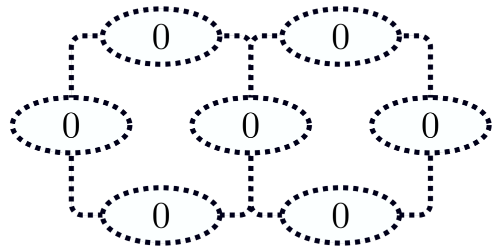


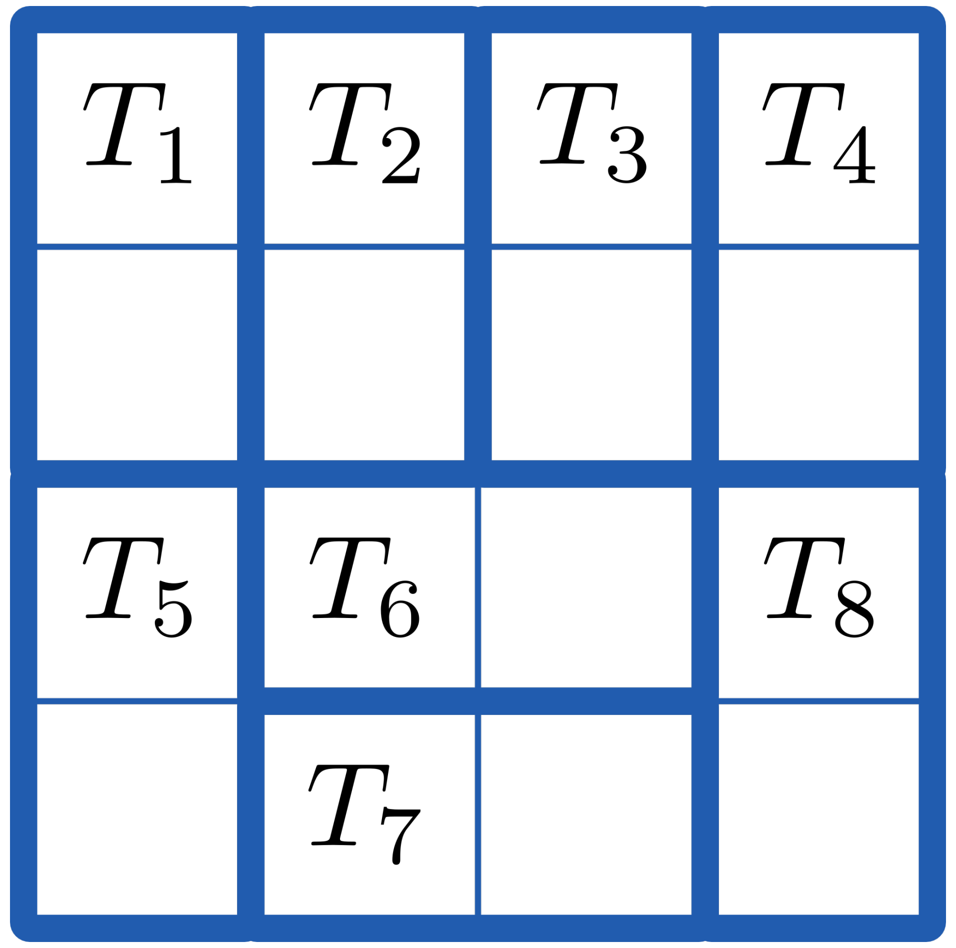


TLMs: Scars for arbitrary volumes and spins

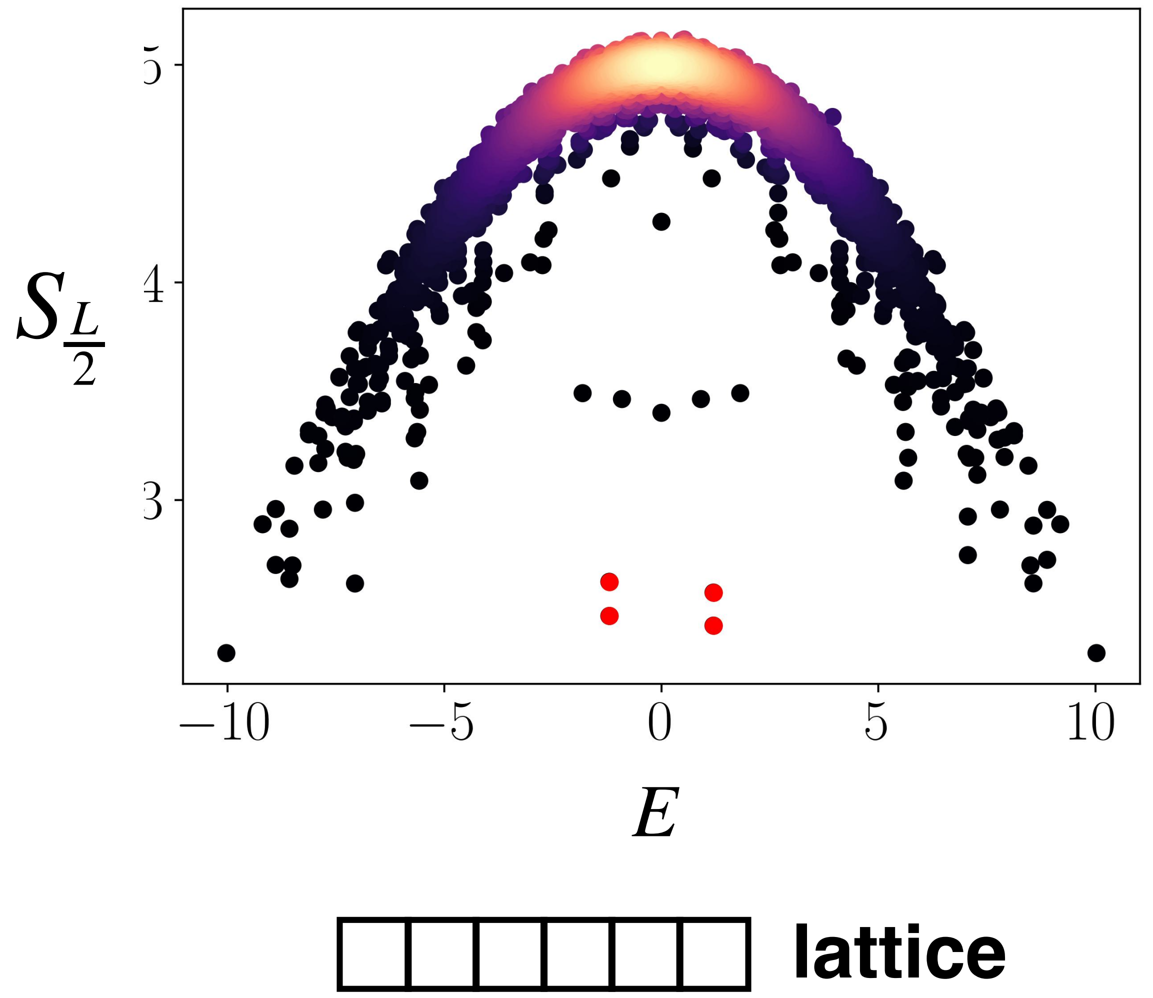
[Budde, MKM, Pinto Barros, PRD 110, 094506 (2024)]

$$|\psi_s^{(i,T)}\rangle = \frac{1}{(S+1)^{|T|/2}} \prod_{(n,n')\in T} \left(\sum_{k=0}^{S} (-1)^k (U_{\square n})^{i-S+k} (U_{\square n'})^{i-k} \right) |\mathbf{0}\rangle$$

$|\mathbf{0}\rangle \equiv$ 



Entanglement entropy for a $S = 2$ ladder



lattice

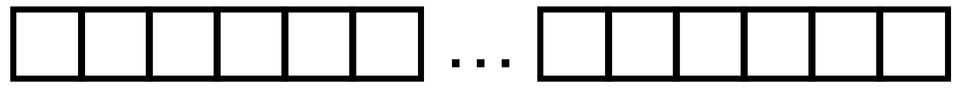
Summary & Outlook (I)

- Phase diagrams, real time dynamics of LGTs intractable from first principles: Hamiltonian formulations

- Quantum Many-Body Scars challenge foundational aspects of thermalization; initial-value problems in QFT

- Scars occur in many-body quantum systems including simple gauge theories; analytic construction for any spin S now possible [U(1) theory]

Summary & Outlook (II)

- Constructed scars in 2+1D amenable for experimental realization 
- Can the proposed mechanism be generalized for other models? [Osborne, McCulloch, Halimeh 2403.08858, Calajo et al. Phys. Rev. Research 7, 013322 (2025)]
- Are the scars present in the continuum limit? Fragile or protected by by special algebraic structures?
- Systematic way to construct efficient state preparation algorithms [Gottesman-Knill (1998), Bravyi, Kitaev, PRA 71, 022316 (2005), Chernyshev, Robin, Savage PRR7 (2025)
- 1. Analytic understanding; 2. Numerical confirmation; 3. Quantum simulation.

⟨S|Q|I⟩ Swiss Quantum Initiative

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

FNSNF

Google Research

QC QUANTUM CENTER

ETH zürich