Entanglement and thermalization in jet production in massive Schwinger model

David Frenklakh

Brookhaven National Laboratory

C²QA

Berkeley October 1, 2025

The goal

Real-time nonperturbative dynamics in QCD

The goal

Real-time nonperturbative dynamics in QCD

(if the future is bright)

The goal

Real-time nonperturbative dynamics in QCD

(if the future is bright)

Near-term

Relevant physics can already be learned with current methods:

- Entanglement in jet fragmentation
- Thermalization in high-energy processes

The goal

Real-time nonperturbative dynamics in QCD

(if the future is bright)

Near-term

Relevant physics can already be learned with current methods:

- Entanglement in jet fragmentation
- Thermalization in high-energy processes

Schwinger model

(1+1)D U(1) gauge theory

Image credit: ChatGPT

Image credit: ChatGPT

Tensor networks

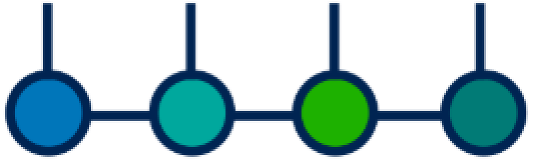
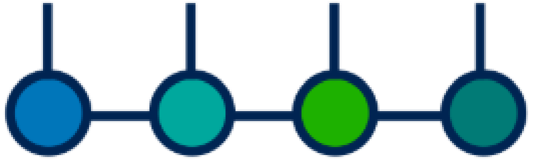
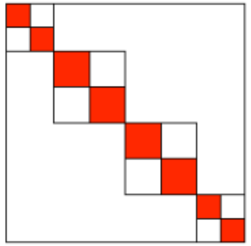


Image credit: ChatGPT

Tensor networks + Exact diagonalization





Schwinger model and jets: history

1974

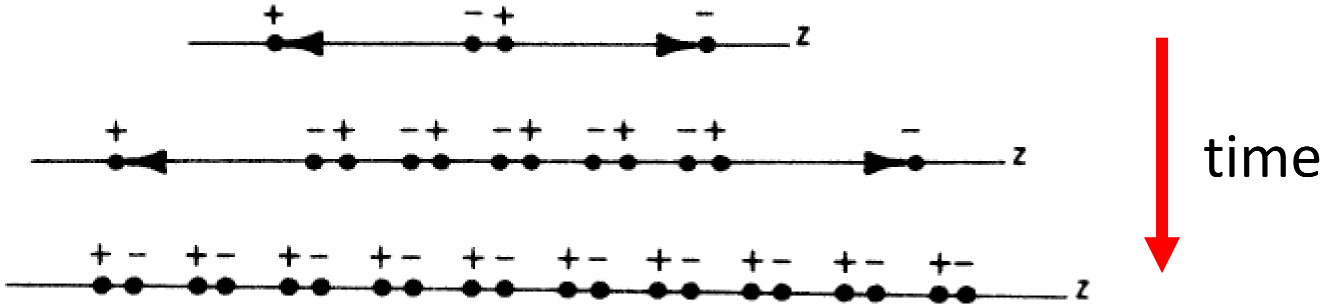
Vacuum polarization and the absence of free quarks

A. Casher,* J. Kogut,† and Leonard Susskind‡

Massless Schwinger model with external source:

$$j_0^{\text{ext}} = g\delta(z-t), \quad j_1^{\text{ext}} = g\delta(z-t) \qquad \text{for } z>0,$$

$$j_0^{\text{ext}} = -g\delta(z+t), \quad j_1^{\text{ext}} = g\delta(z+t) \qquad \text{for } z<0,$$



Schwinger model and jets: history

1974

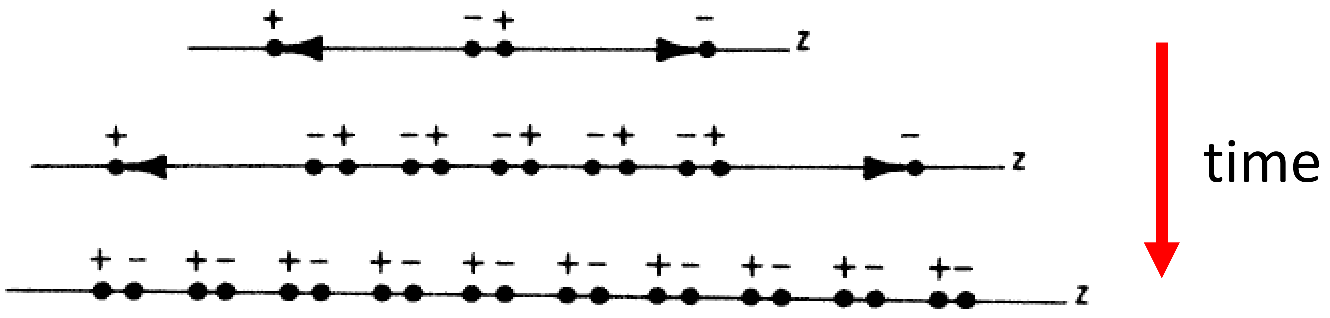
Vacuum polarization and the absence of free quarks

A. Casher,* J. Kogut,† and Leonard Susskind‡

Massless Schwinger model with external source:

$$j_0^{ext} = g\delta(z-t), \quad j_1^{ext} = g\delta(z-t) \qquad \text{for } z>0,$$

$$j_0^{ext} = -g\delta(z+t), \quad j_1^{ext} = g\delta(z+t) \qquad \text{for } z<0,$$



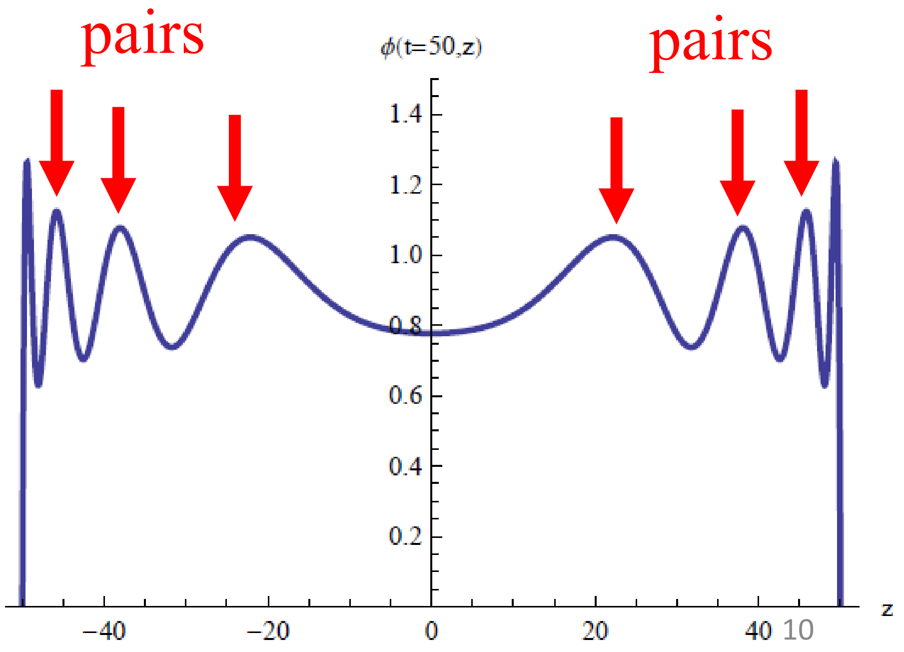
2012

Jet energy loss and fragmentation in heavy ion collisions

Dmitri E. Kharzeev[1,2] and Frashër Loshaj[1]

$$\phi(x) = \theta(t^2 - z^2)[1 - J_0(m\sqrt{t^2 - z^2})]$$

$$j^0 = \partial_z \phi$$



The setup

[Gauss, Jordan, Kogut, Law, Susskind, Wigner]

$$H^L = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1}+Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1} L_n^2$$

Kinetic energy

Mass term

Electric energy

$a = 1$

$g = 0.5$

$m = 0.25$

$$L_n = \sum_{i=1}^{n} q_i$$

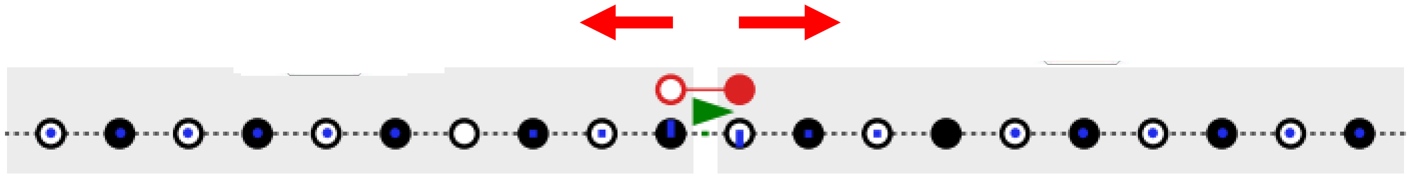
The setup

[Gauss, Jordan, Kogut, Law, Susskind, Wigner]

$$H^L = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1}+Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1} L_n^2$$

$a = 1$
$g = 0.5$
$m = 0.25$

Add the external charges (jets):



$$H^L(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1}+Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}(L_{\text{dyn},n}+L_{\text{ext},n}(t))^2.$$

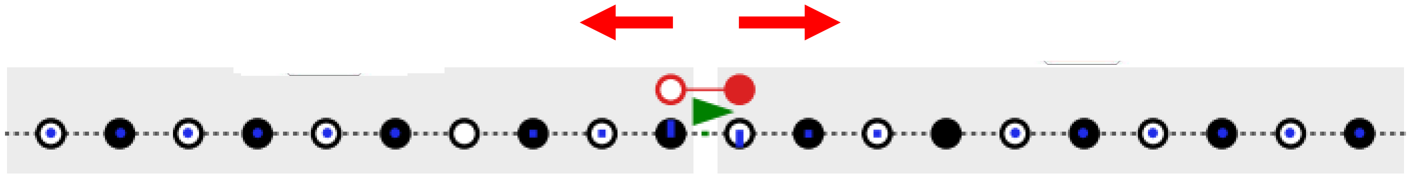
The setup

[Gauss, Jordan, Kogut, Law, Susskind, Wigner]

$$H^L = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1}+Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1} L_n^2$$

$a = 1$
$g = 0.5$
$m = 0.25$

Add the external charges (jets):

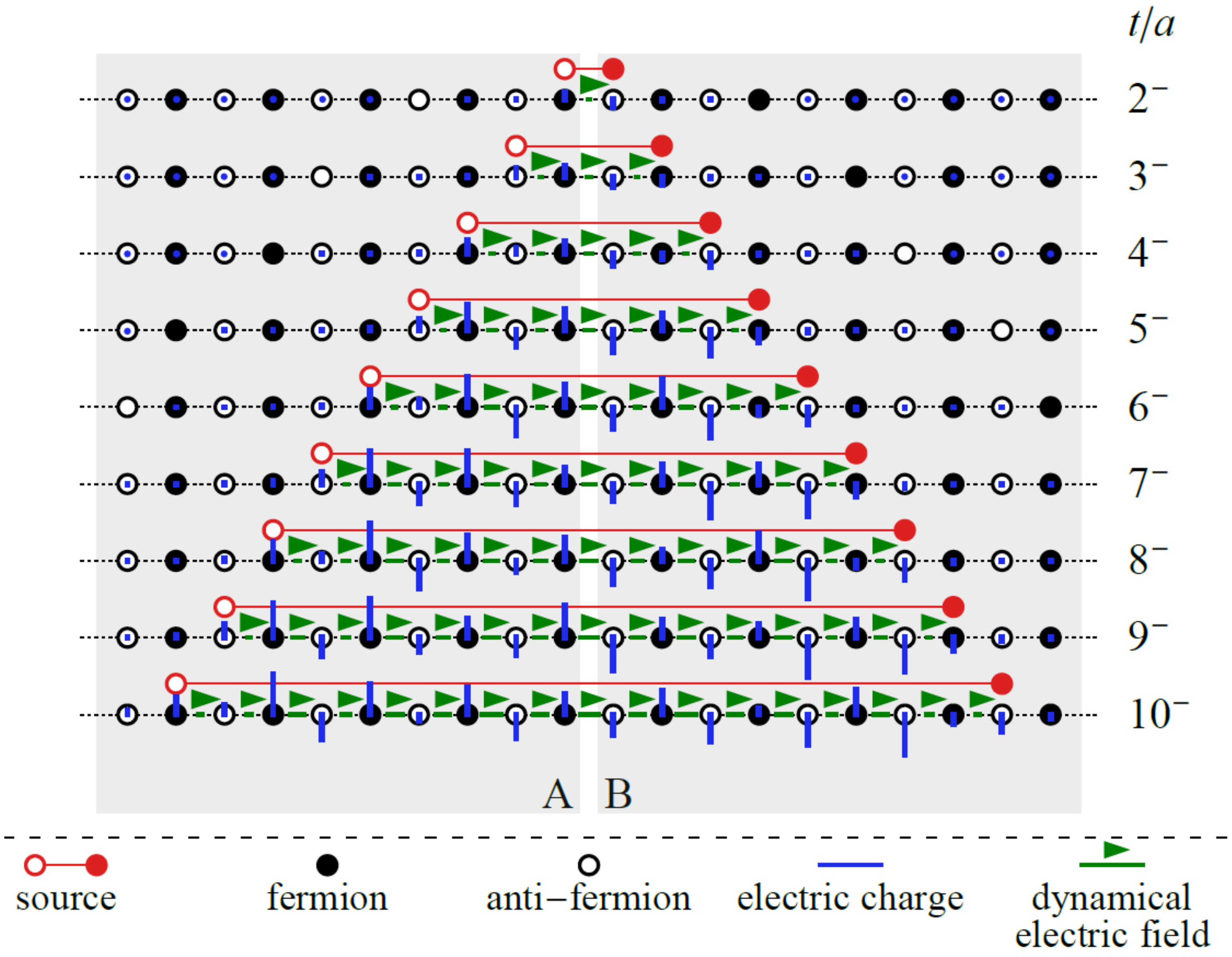


$$H^L(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_nX_{n+1}+Y_nY_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{ag^2}{2}\sum_{n=1}^{N-1}(L_{\mathrm{dyn},n} + L_{\mathrm{ext},n}(t))^2.$$

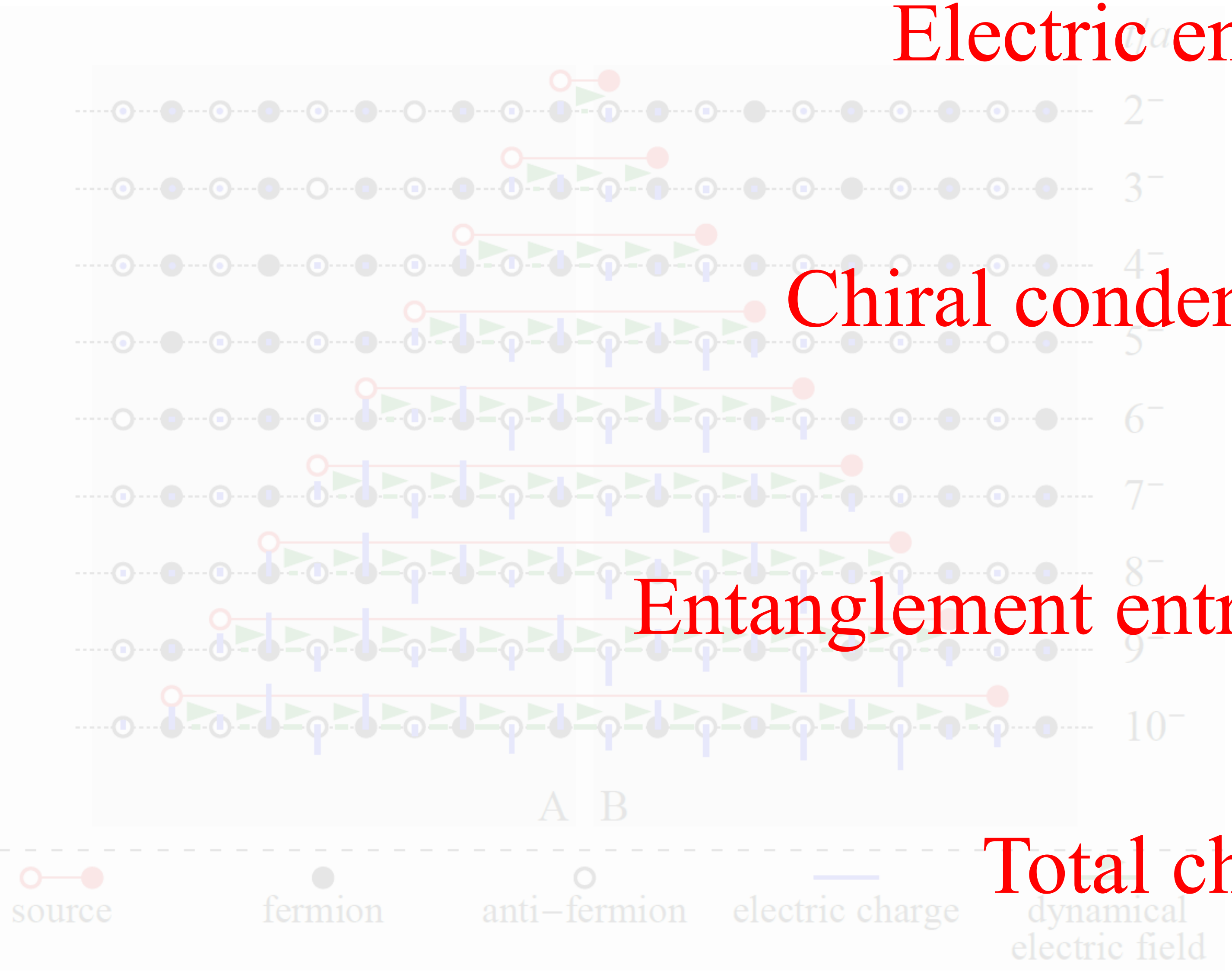
Vacuum at $t = 0$

$$|\Psi_t\rangle = \mathcal{T}e^{-i\int_0^t dt' H(t')}|\Psi_0\rangle$$

Screening, chiral condensate and entanglement



Screening, chiral condensate and entanglement

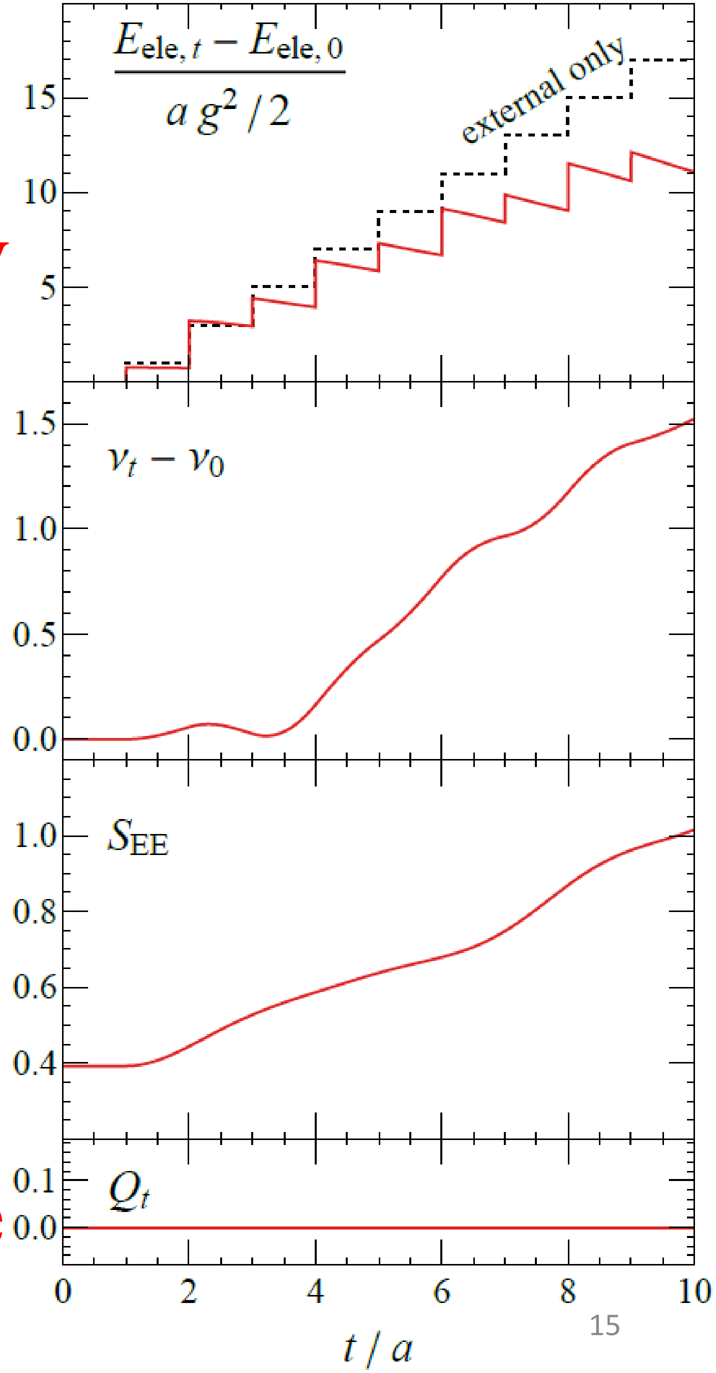


Electric energy

Chiral condensate

Entanglement entropy

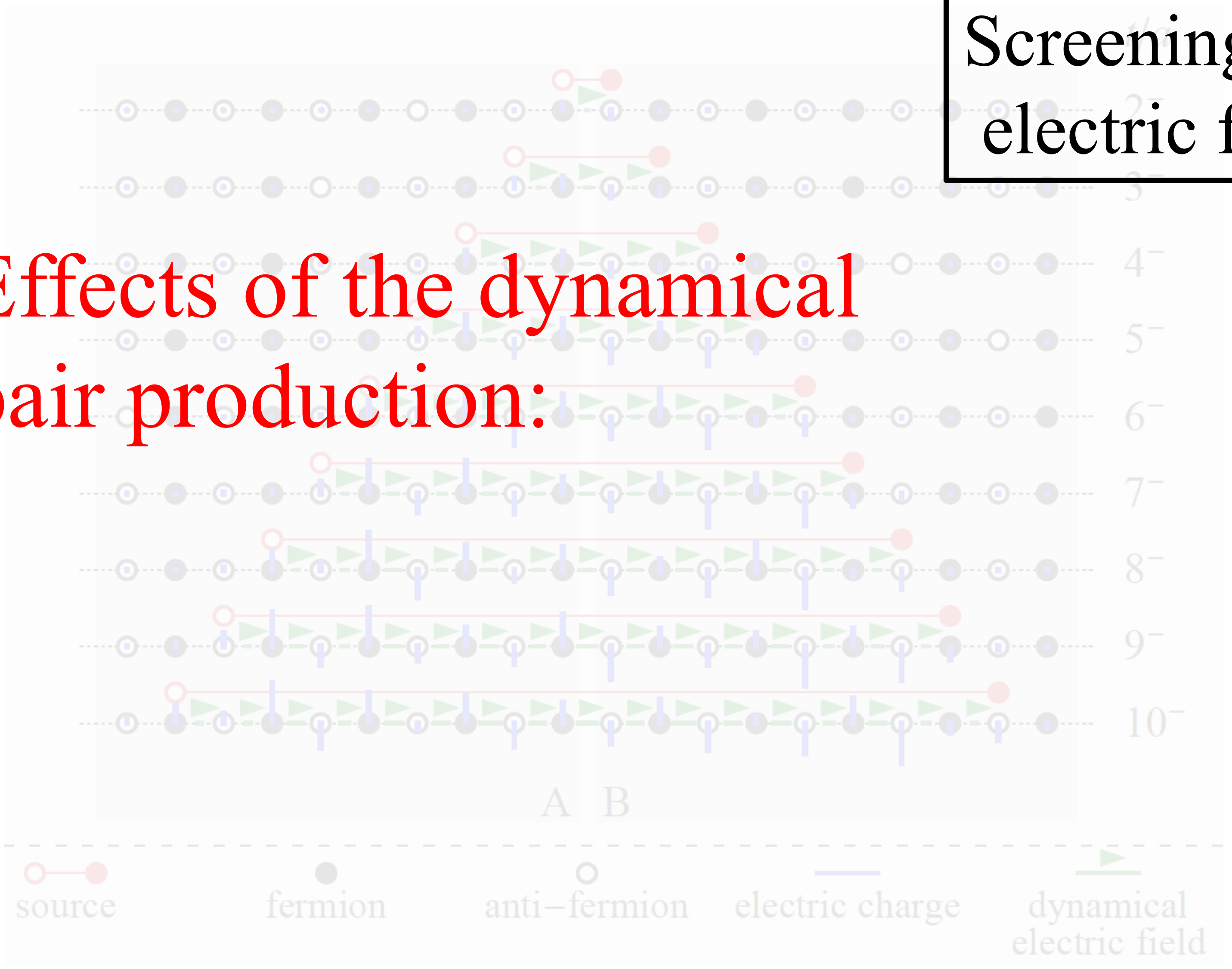
Total charge

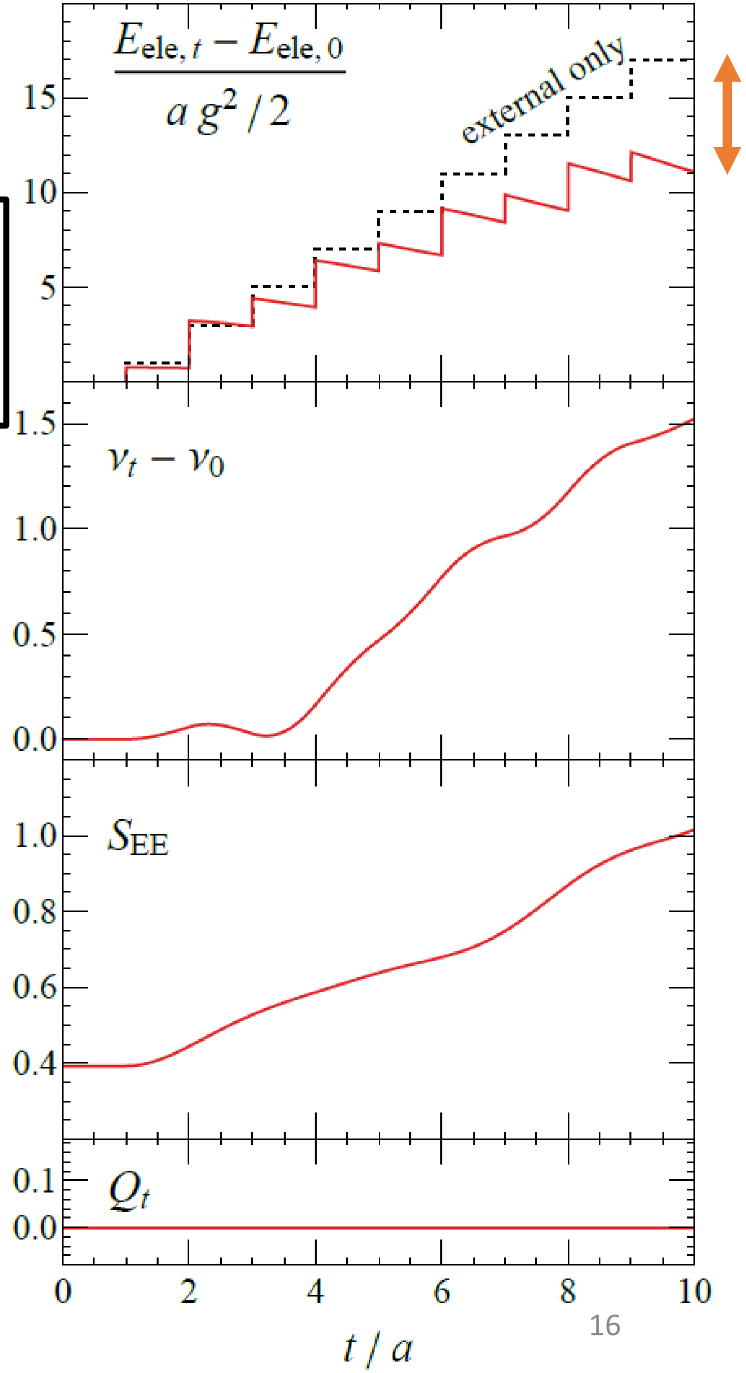


Screening, chiral condensate and entanglement

Screening the electric field

Effects of the dynamical pair production:



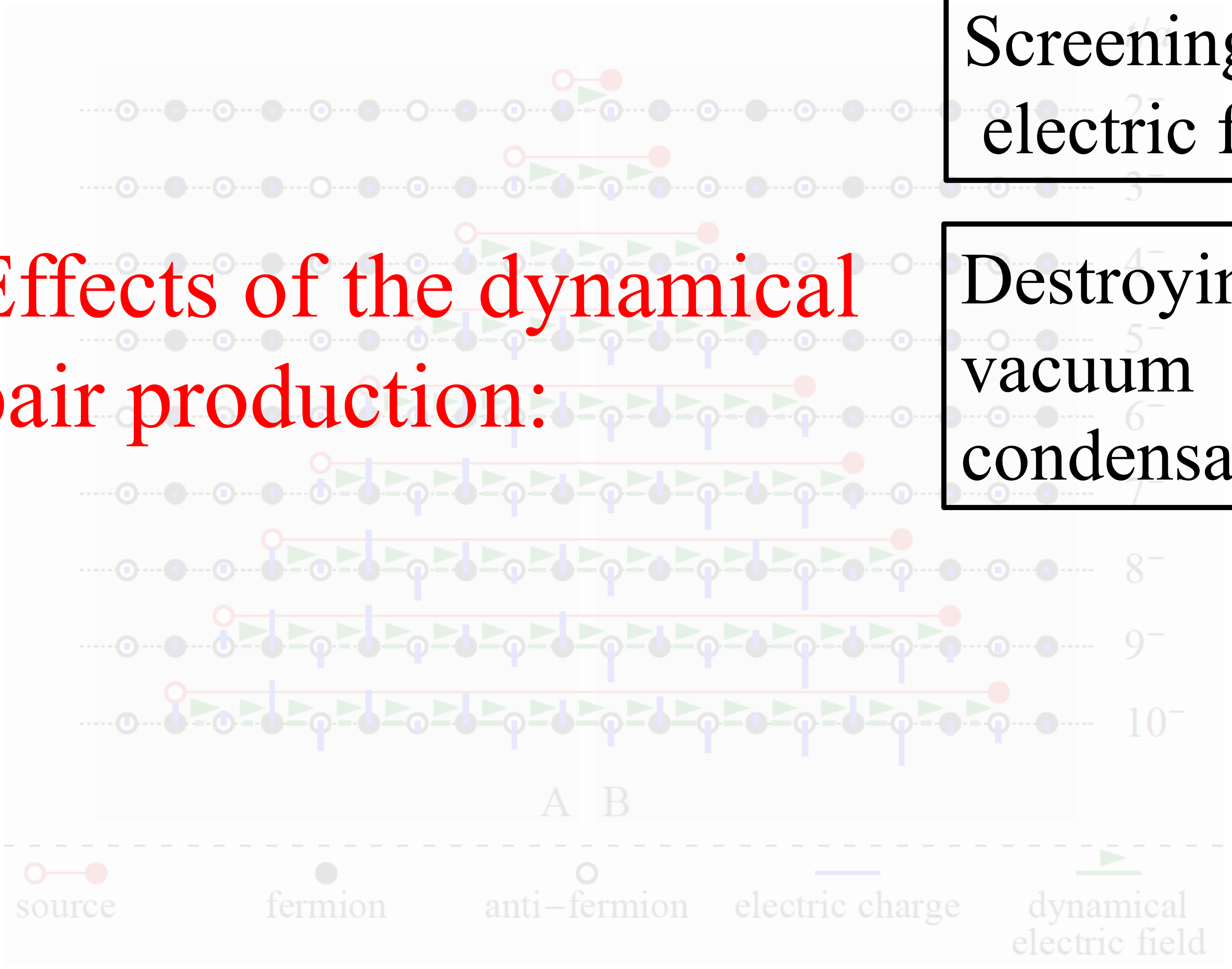


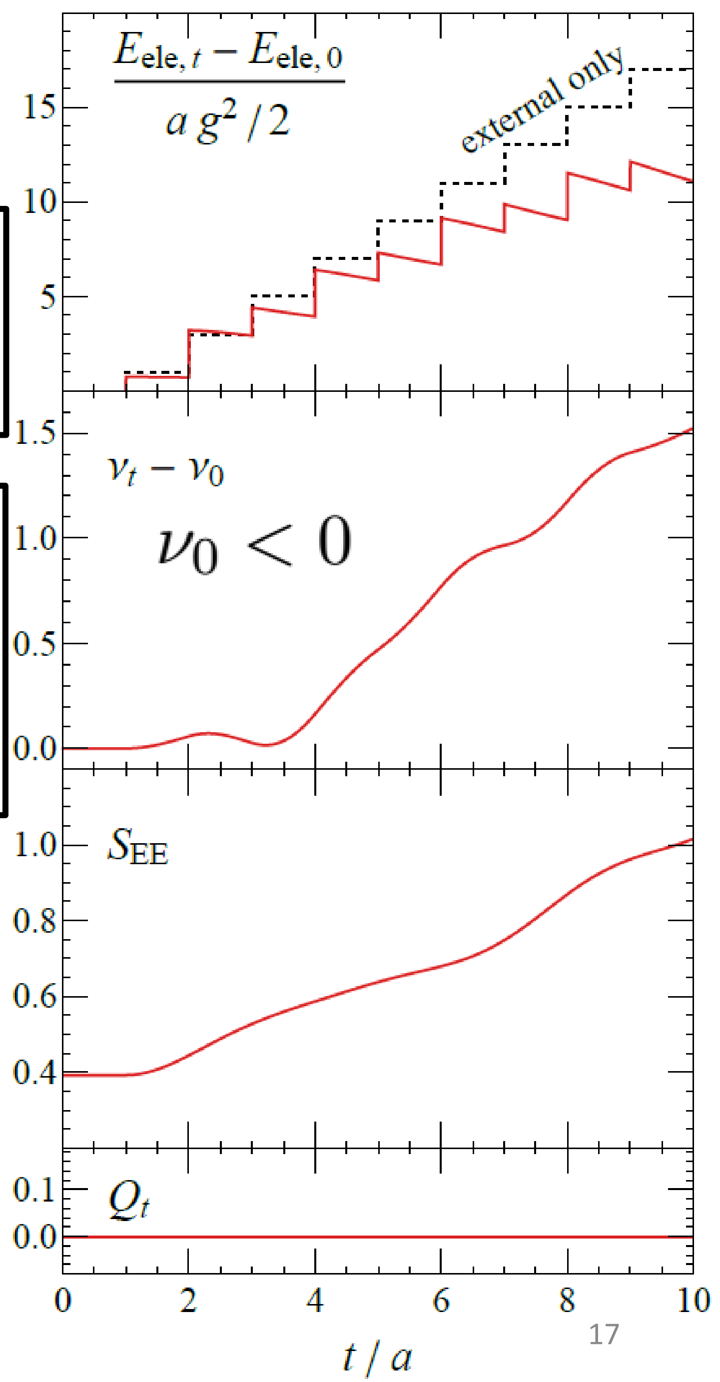
Screening, chiral condensate and entanglement

Effects of the dynamical pair production:

Screening the electric field

Destroying vacuum condensate





Screening, chiral condensate and entanglement

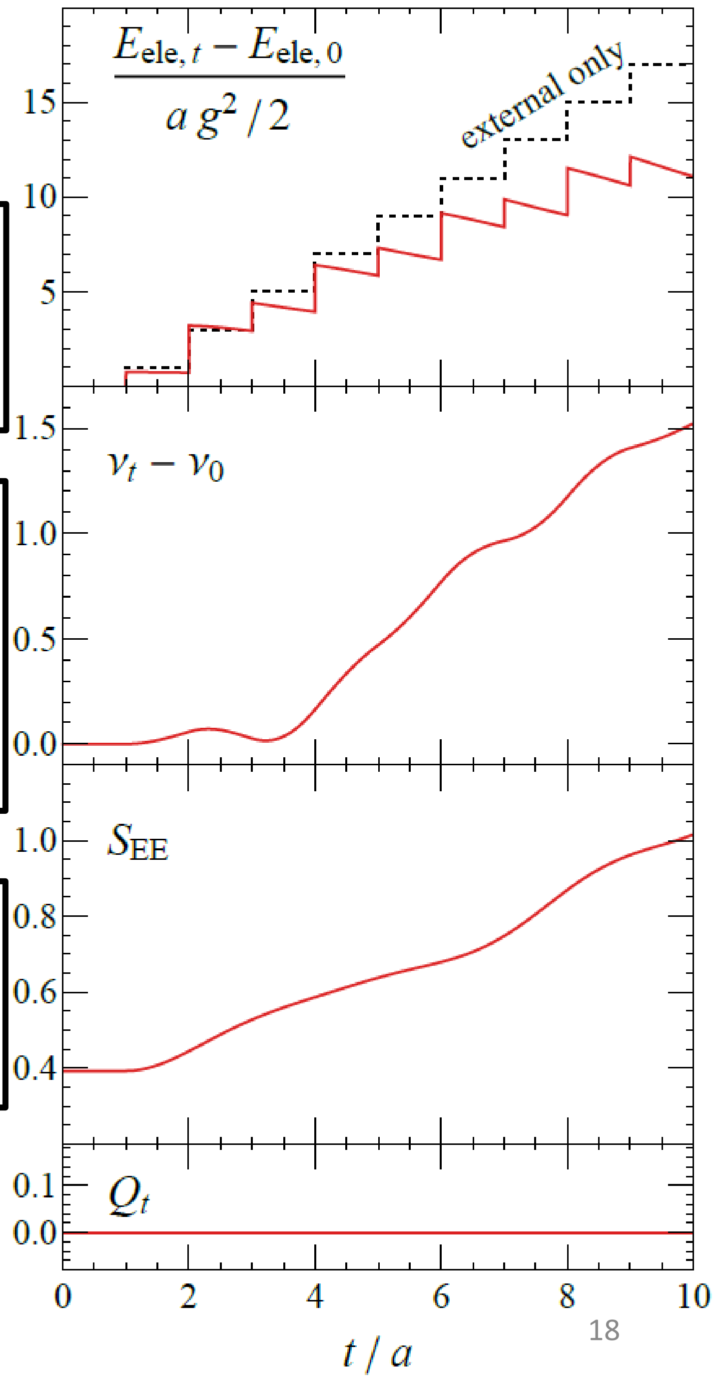
Effects of the dynamical pair production:

Screening the electric field

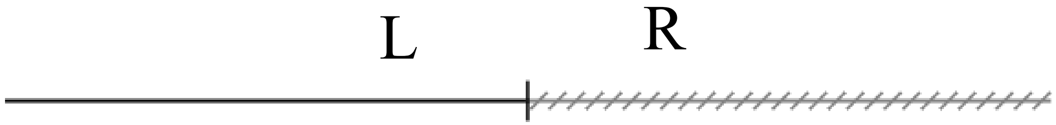
Destroying vacuum condensate

Entangling the jets

$$\rho_A = \mathrm{Tr}_B \rho, \qquad S_{EE} = -\mathrm{Tr}_A(\rho_A \log \rho_A)$$



Entanglement spectrum



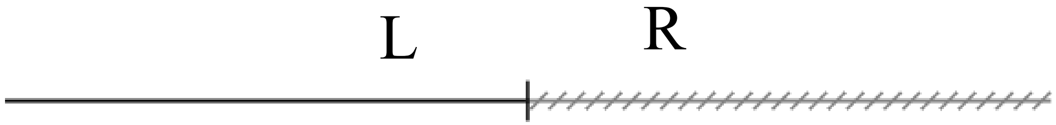
Schmidt spectrum

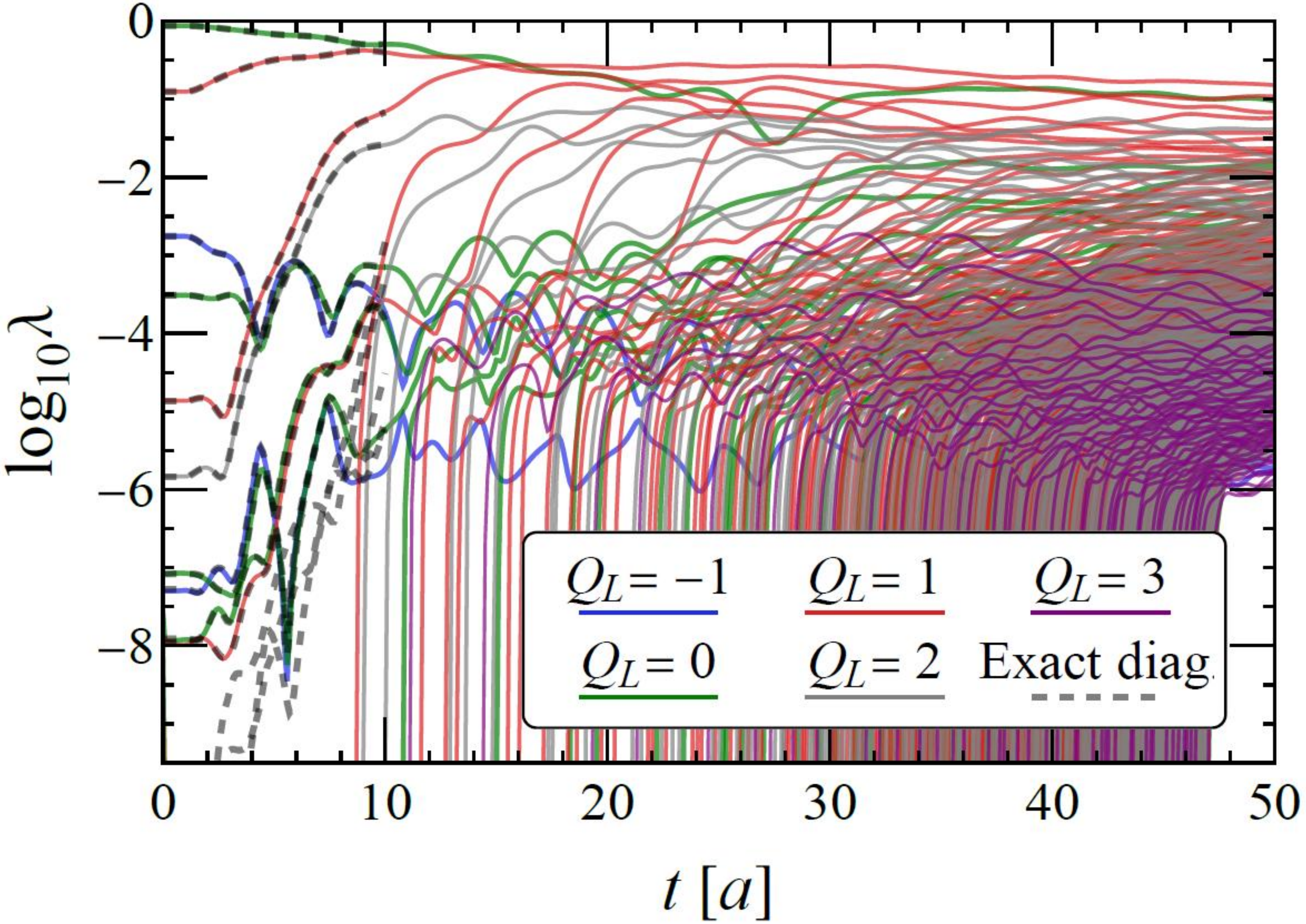
$$|\Psi(t)\rangle = \sum_{i=1}^{2^{N/2}} \sqrt{\lambda_i(t)}|\psi_i^L(t)\rangle \otimes |\psi_i^R(t)\rangle$$

$$\rho_L(t) = \sum_{i=1}^{2^{N/2}} \lambda_i(t)|\psi_i^L(t)\rangle\langle\psi_i^L(t)|$$

$$S_{EE}(t) = -\sum_{i=1}^{2^{N/2}} \lambda_i \ln \lambda_i$$

Entanglement spectrum





Schmidt spectrum

$$|\Psi(t)\rangle = \sum_{i=1}^{2^{N/2}} \sqrt{\lambda_i(t)}|\psi_i^L(t)\rangle \otimes |\psi_i^R(t)\rangle$$

$$\rho_L(t) = \sum_{i=1}^{2^{N/2}} \lambda_i(t)|\psi_i^L(t)\rangle\langle\psi_i^L(t)|$$

$$S_{EE}(t) = -\sum_{i=1}^{2^{N/2}} \lambda_i \ln \lambda_i$$

Towards maximally entangled state

Renyi α-th entropy

$$S_\alpha(t) \equiv \frac{\ln \mathrm{Tr}_L(\rho_L(t)^\alpha)}{1-\alpha} = \frac{\ln \sum_{i=1}^{2^{N/2}} \lambda_i^\alpha}{1-\alpha}$$

Entangleness

$$\mathcal{E} \equiv \frac{1-\mathrm{tr}\rho_L^2}{1-2^{-N/2}} = \frac{1-\sum_{i=1}^{2^{N/2}} \lambda^2}{1-2^{-N/2}}$$

Towards maximally entangled state

Renyi α-th entropy

$$S_\alpha(t) \equiv \frac{\ln \mathrm{Tr}_L(\rho_L(t)^\alpha)}{1-\alpha} = \frac{\ln \sum_{i=1}^{2^{N/2}} \lambda_i^\alpha}{1-\alpha}$$

Entangleness

$$\mathcal{E} \equiv \frac{1 - \mathrm{tr}\rho_L^2}{1-2^{-N/2}} = \frac{1 - \sum_{i=1}^{2^{N/2}} \lambda^2}{1-2^{-N/2}}$$

pure state (PS) *vs.*
maximally entangled state (MES)

$$S_\alpha[\mathrm{PS}] = 0 \ , \qquad \mathcal{E}[\mathrm{PS}] = 0$$

$$S_\alpha[\mathrm{MES}] = \frac{N \ln 2}{2} \ \forall \alpha \ , \qquad \mathcal{E}[\mathrm{MES}] = 1$$

Towards maximally entangled state

Renyi α-th entropy

$$S_\alpha(t) \equiv \frac{\ln \mathrm{Tr}_L(\rho_L(t)^\alpha)}{1-\alpha} = \frac{\ln \sum_{i=1}^{2^{N/2}} \lambda_i^\alpha}{1-\alpha}$$

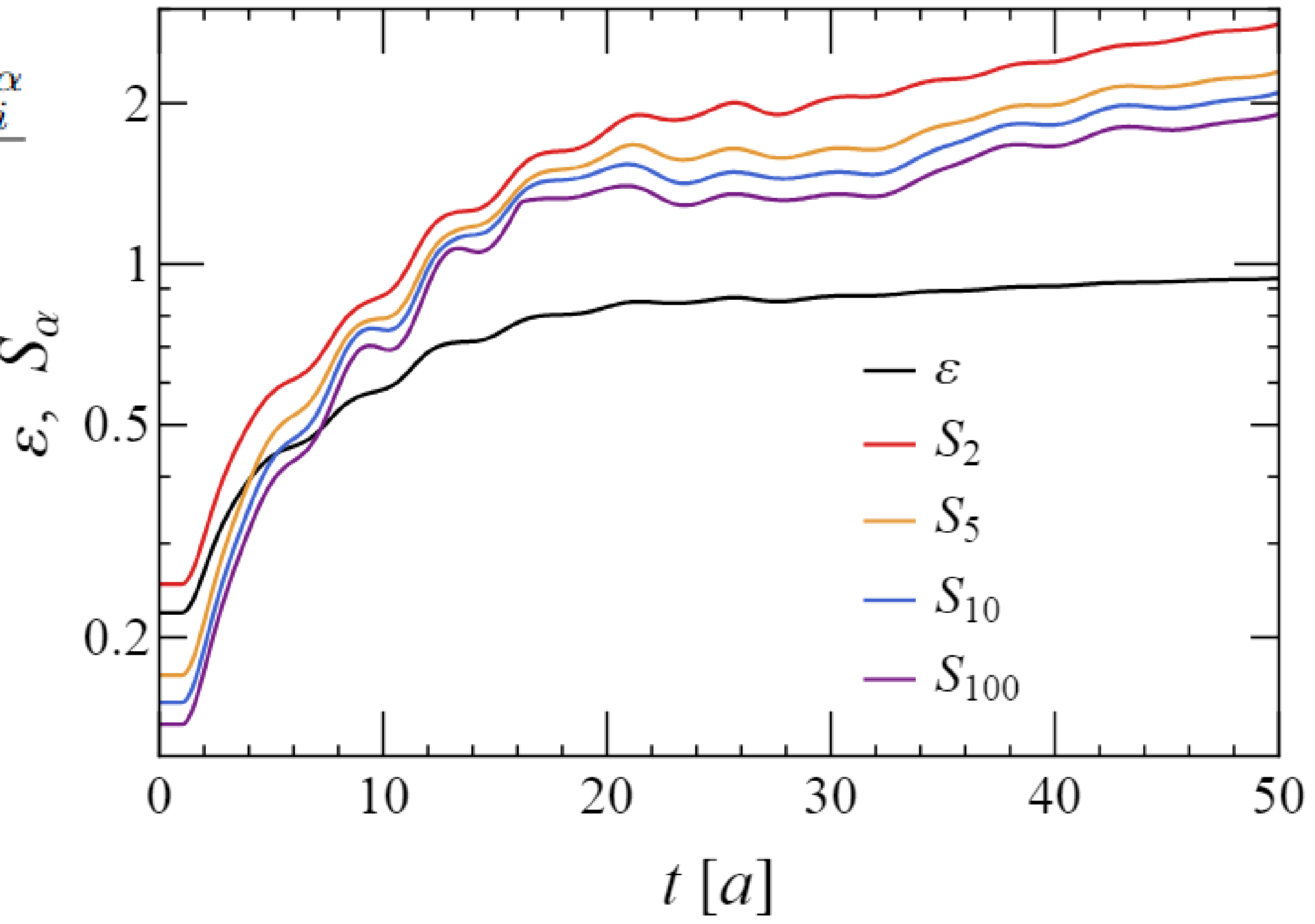
Entangleness

$$\mathcal{E} \equiv \frac{1-\mathrm{tr}\rho_L^2}{1-2^{-N/2}} = \frac{1-\sum_{i=1}^{2^{N/2}} \lambda^2}{1-2^{-N/2}}$$

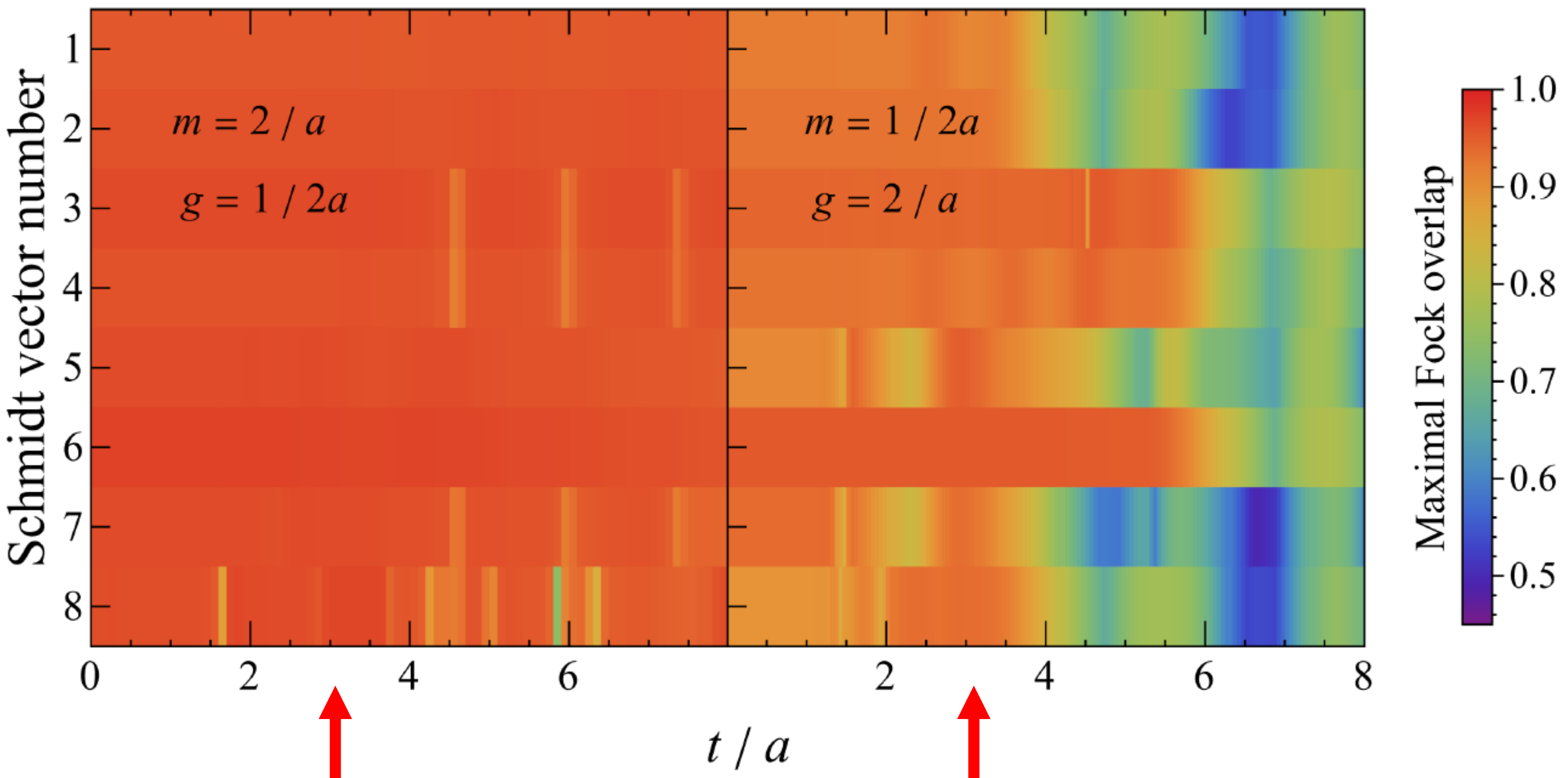
pure state (PS) *vs.*
maximally entangled state (MES)

$$S_\alpha[\mathrm{PS}] = 0\ , \qquad \mathcal{E}[\mathrm{PS}] = 0$$

$$S_\alpha[\mathrm{MES}] = \frac{N \ln 2}{2}\ \forall\alpha\ , \qquad \mathcal{E}[\mathrm{MES}] = 1$$



Hadronization in real time



computational basis state

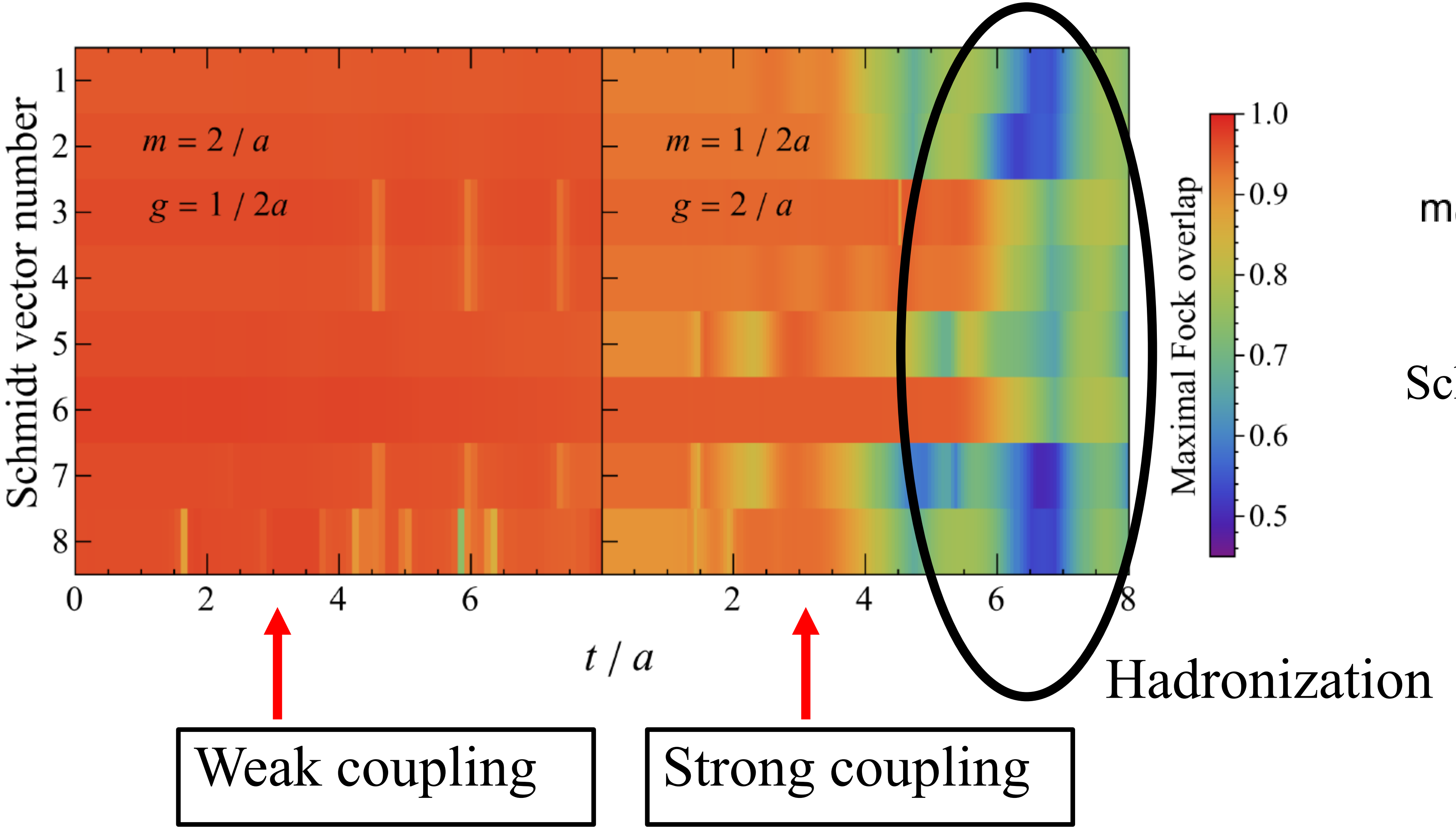
$$\max |\langle \psi_L^i(t) | jk \ldots \rangle|$$

Schmidt vector

Weak coupling

Strong coupling

Hadronization in real time



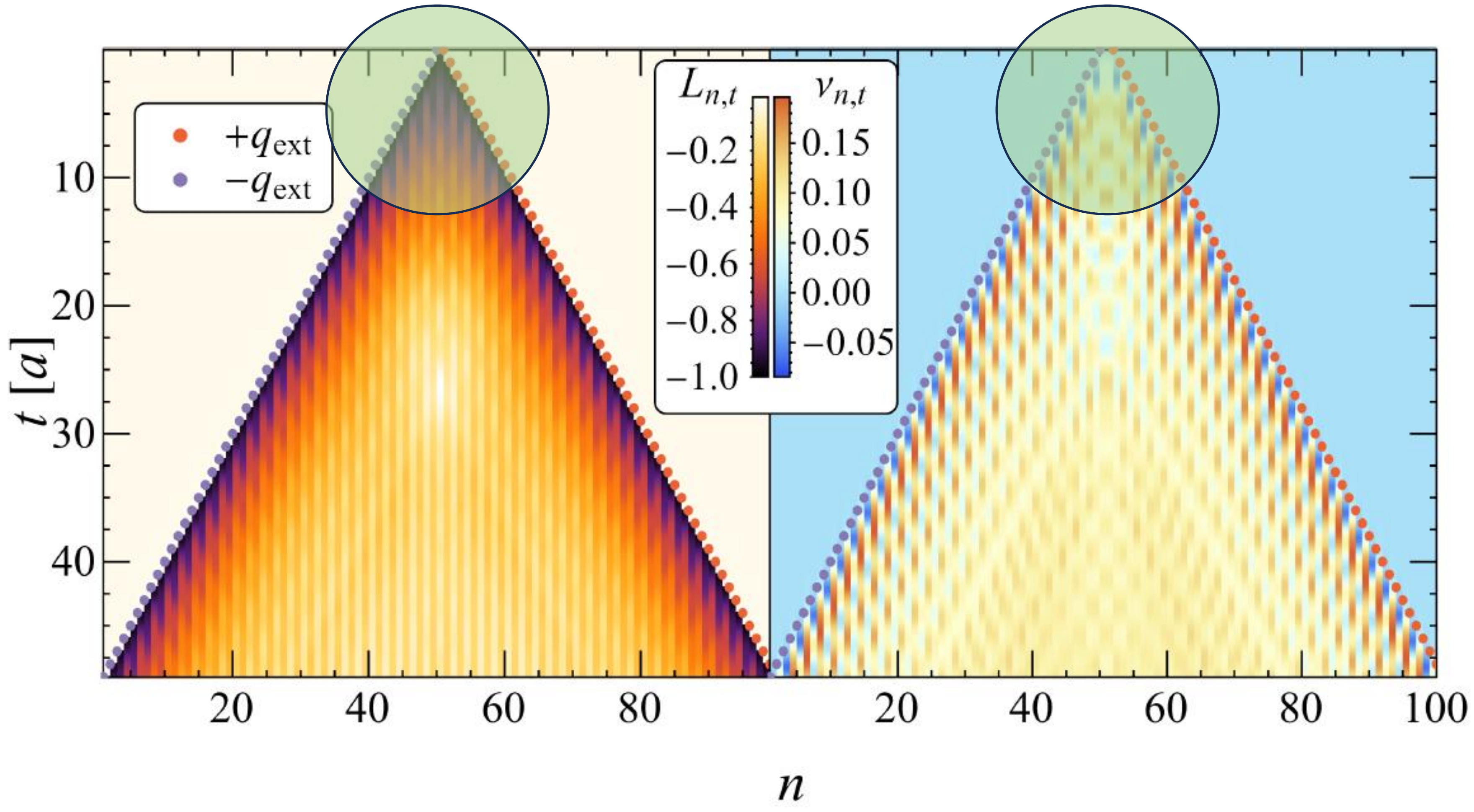
computational basis state

$\max |\langle \psi_L^i(t) | jk \ldots \rangle|$

Schmidt vector

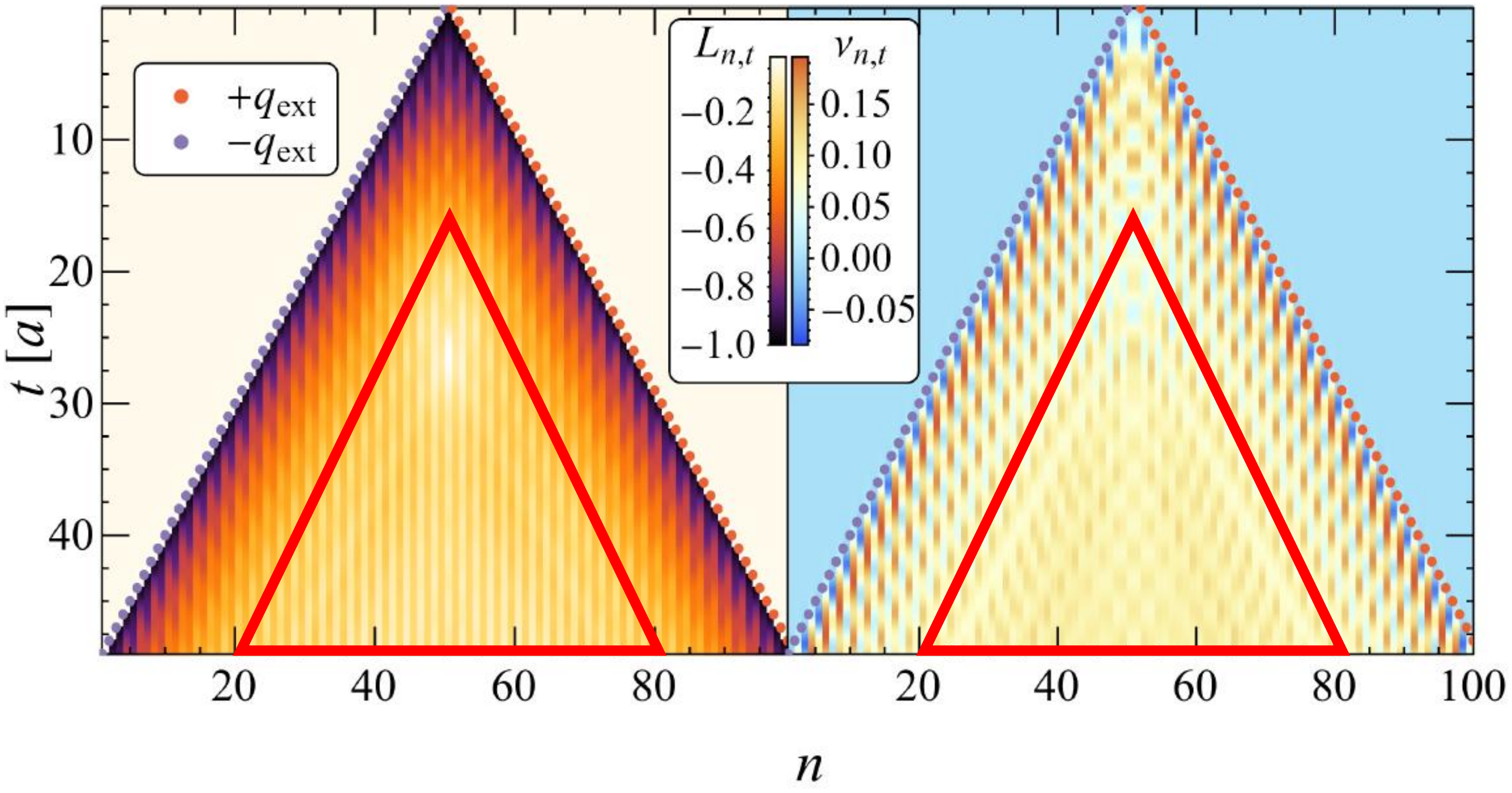
Towards thermalization

Accessible to exact diagonalization



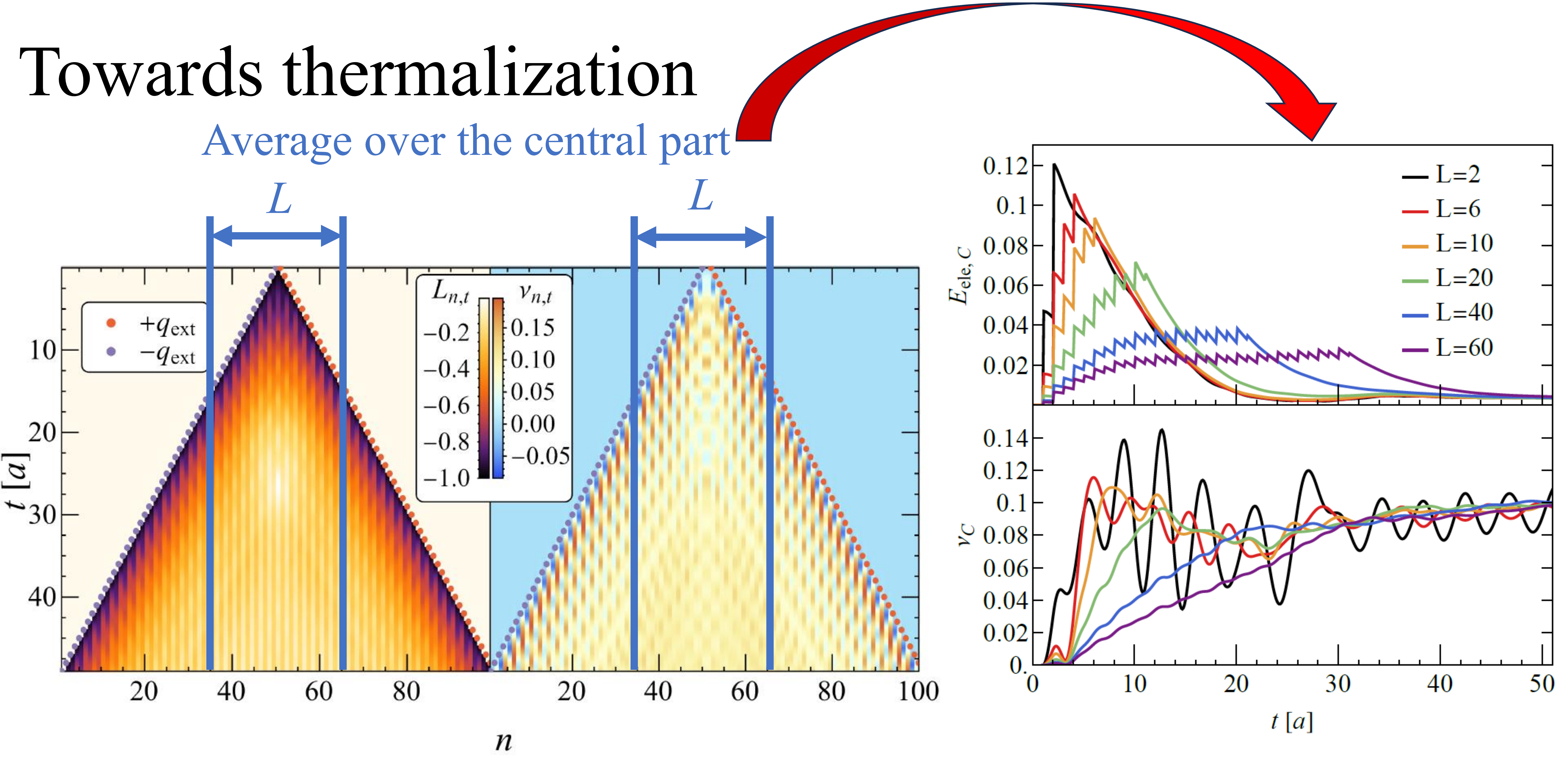
Tensor network methods allow studying much larger system

Towards thermalization



Equilibration towards late times

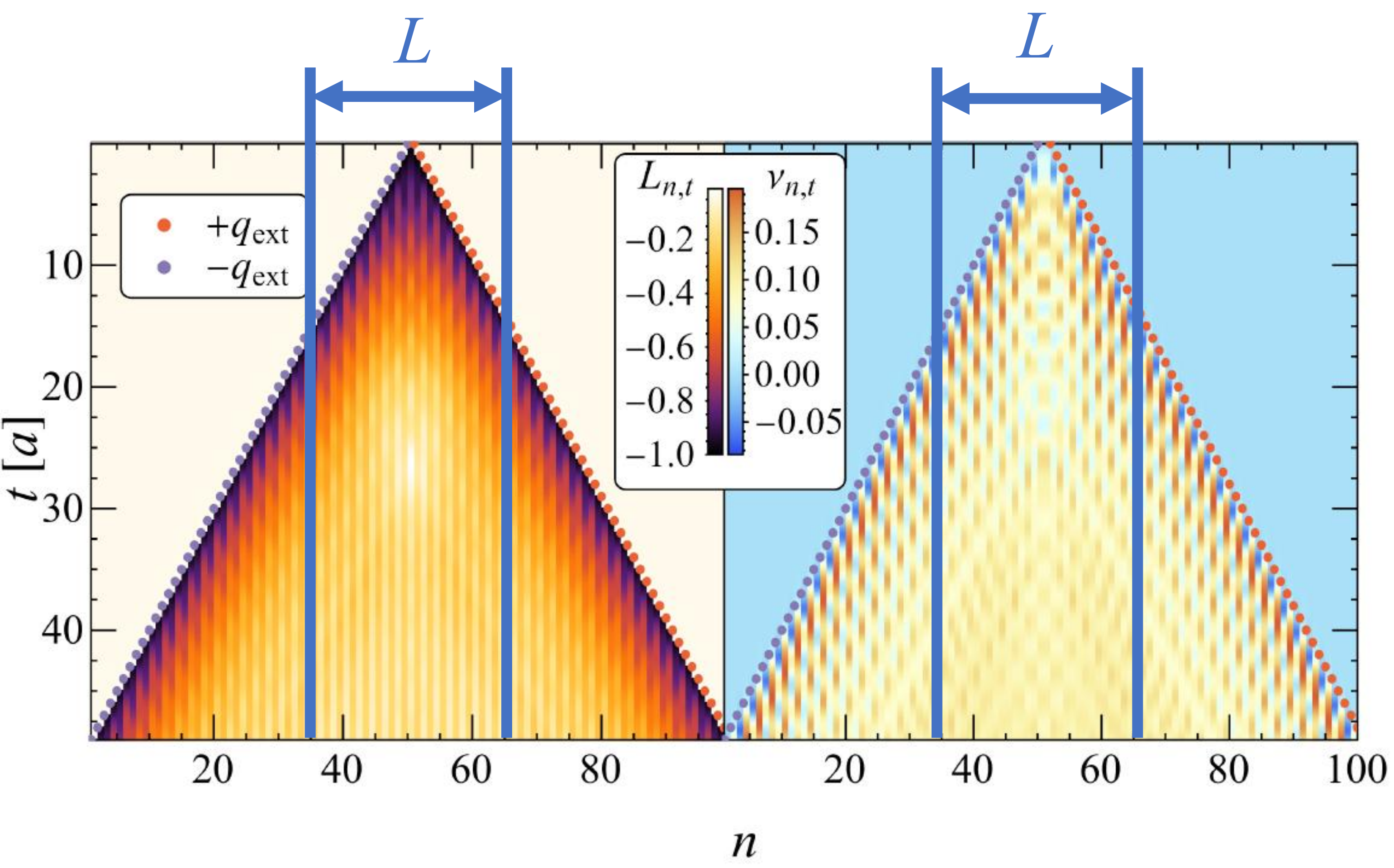
Towards thermalization

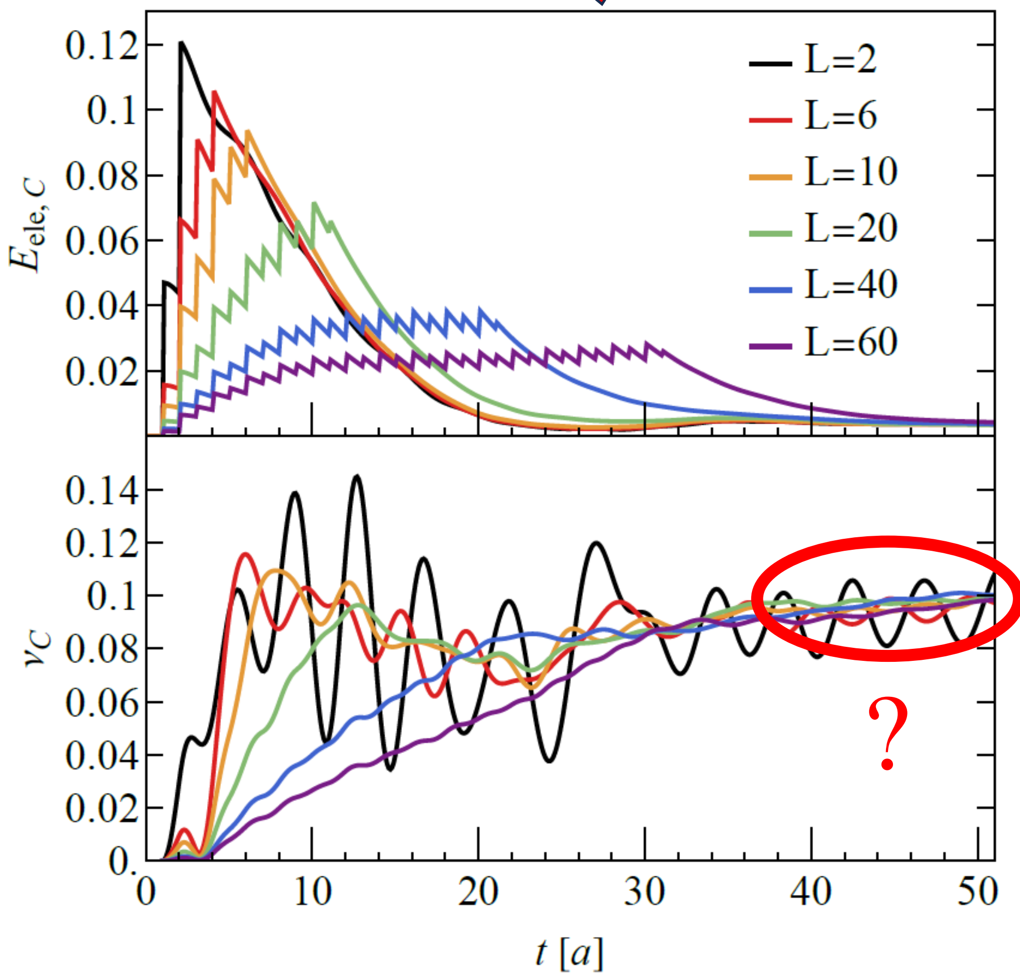


Equilibration towards late times

Towards thermalization

Average over the central part





Equilibration towards late times

Thermal expectation values

$$\langle \mathcal{O} \rangle_T = \frac{\sum_n e^{-E_n/T} \langle E_n | \mathcal{O} | E_n \rangle}{\sum_n e^{-E_n/T}}$$

Exact diagonalization: full diagonalization

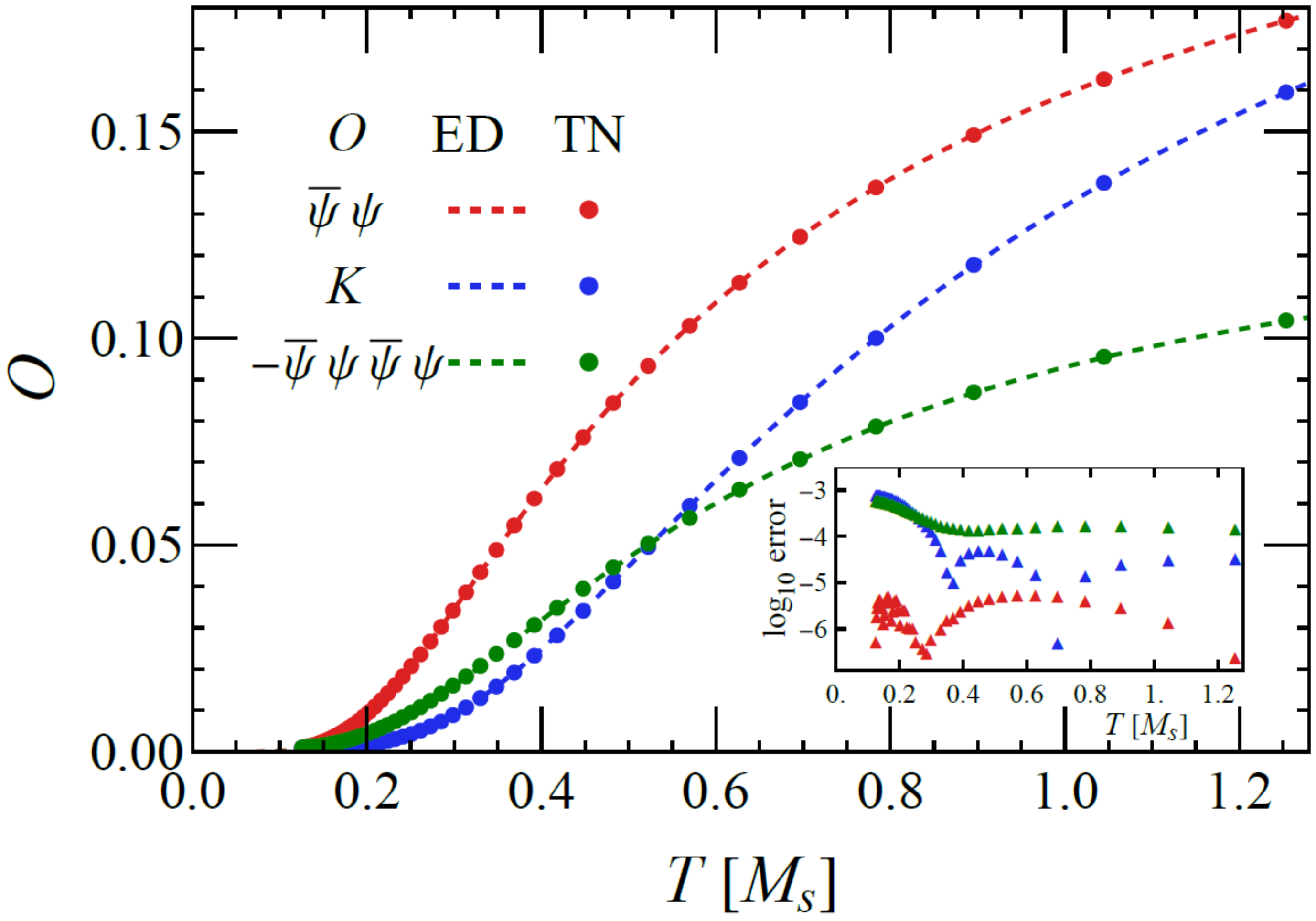
Tensor network: purification (requires ancillas, doubling system size)

Thermal expectation values

$$\langle \mathcal{O} \rangle_T = \frac{\sum_n e^{-E_n/T} \langle E_n | \mathcal{O} | E_n \rangle}{\sum_n e^{-E_n/T}}$$

Exact diagonalization: full diagonalization

Tensor network: purification (requires ancillas, doubling system size)

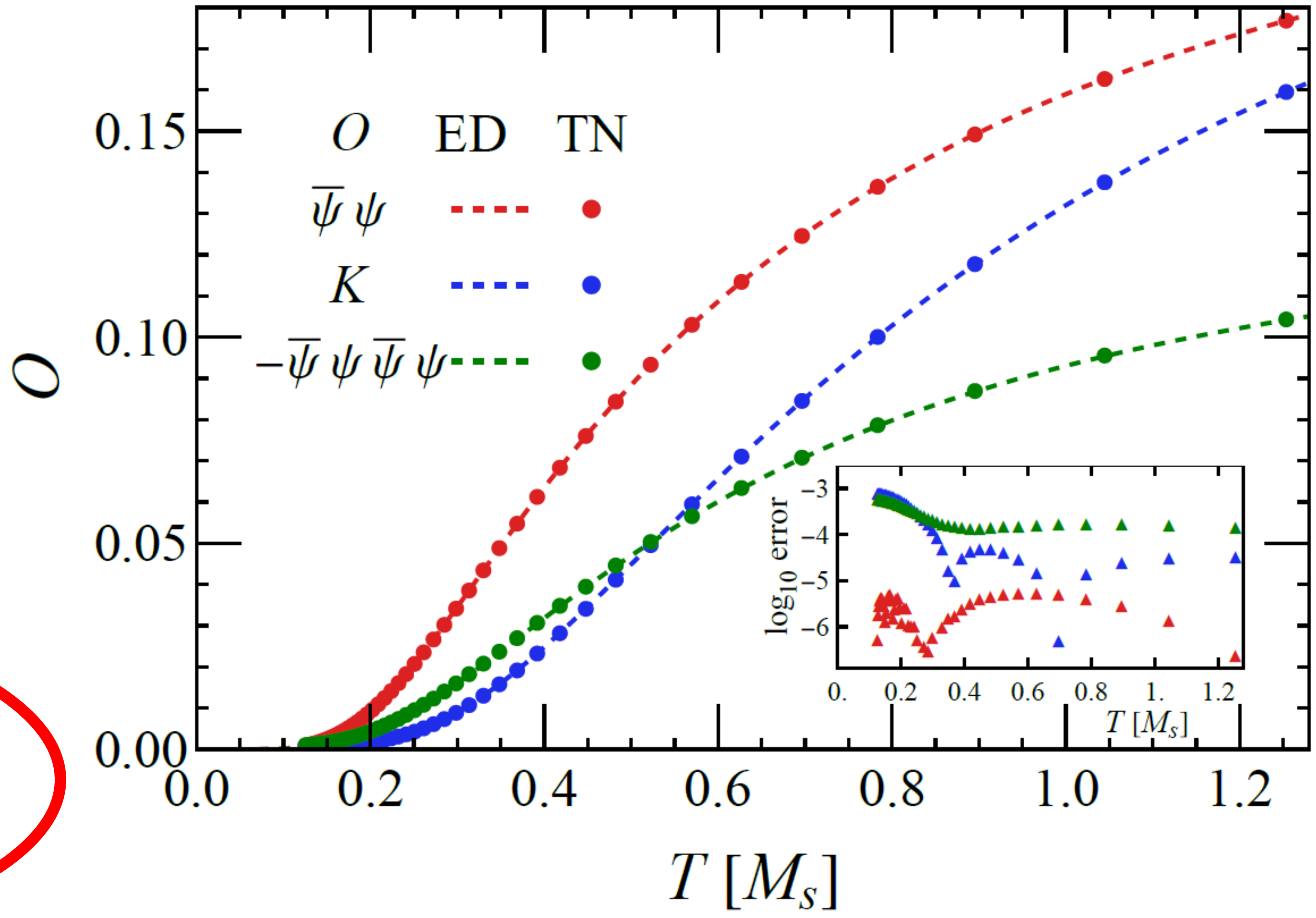


Thermal expectation values

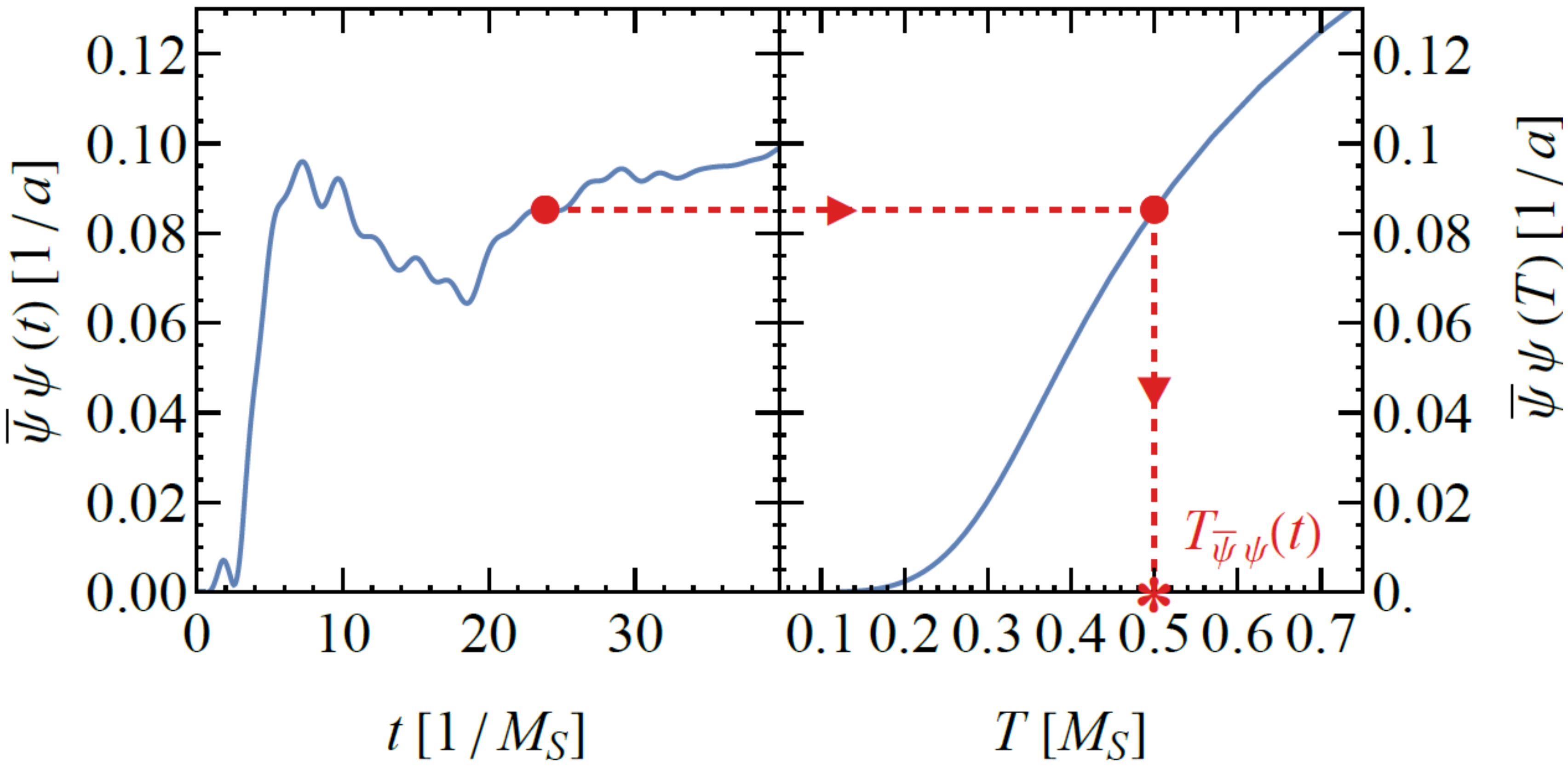
$$\langle \mathcal{O} \rangle_T = \frac{\sum_n e^{-E_n/T} \langle E_n | \mathcal{O} | E_n \rangle}{\sum_n e^{-E_n/T}}$$

Exact diagonalization: full diagonalization

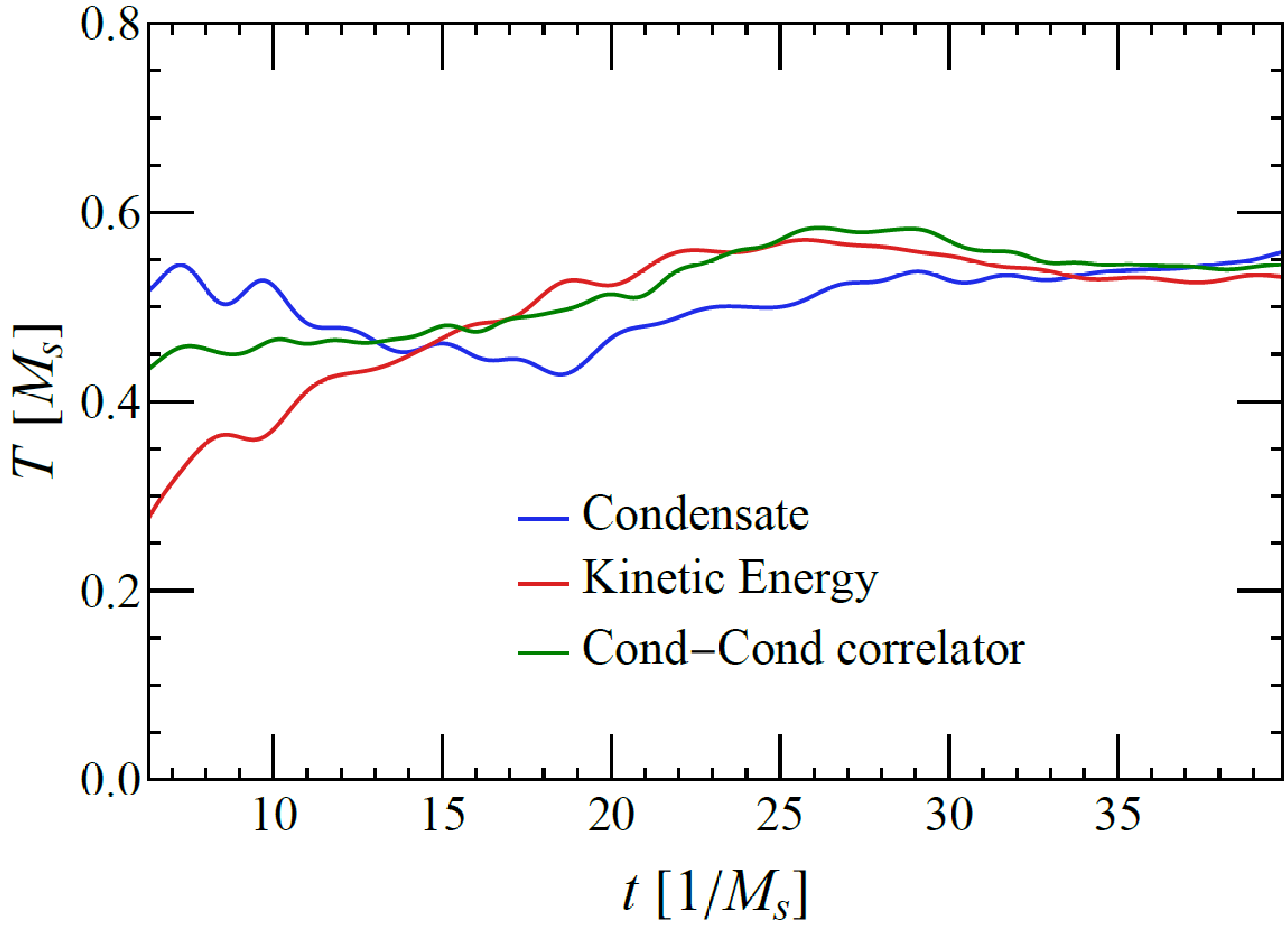
Tensor network: purification (requires ancillas, doubling system size)



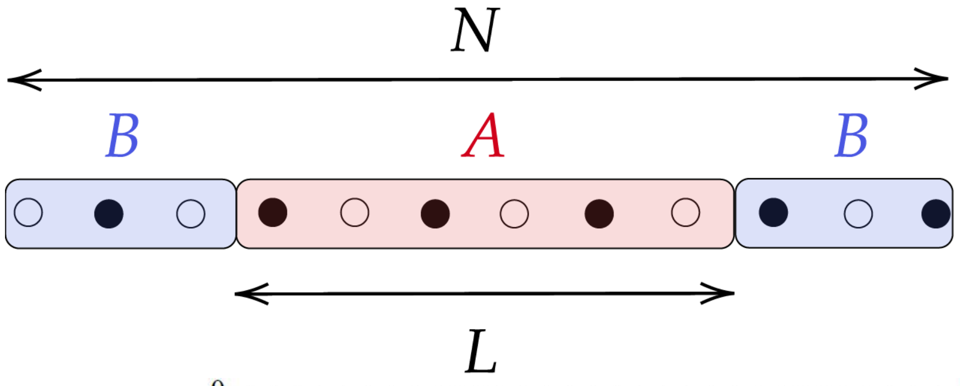
Temperature extraction



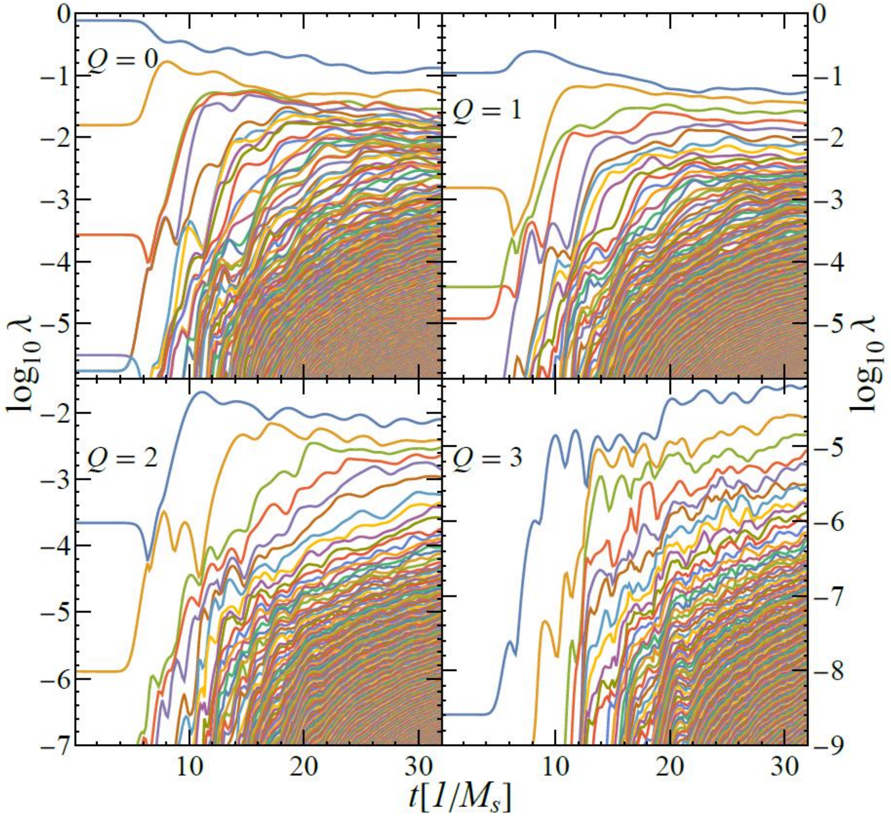
Local operators



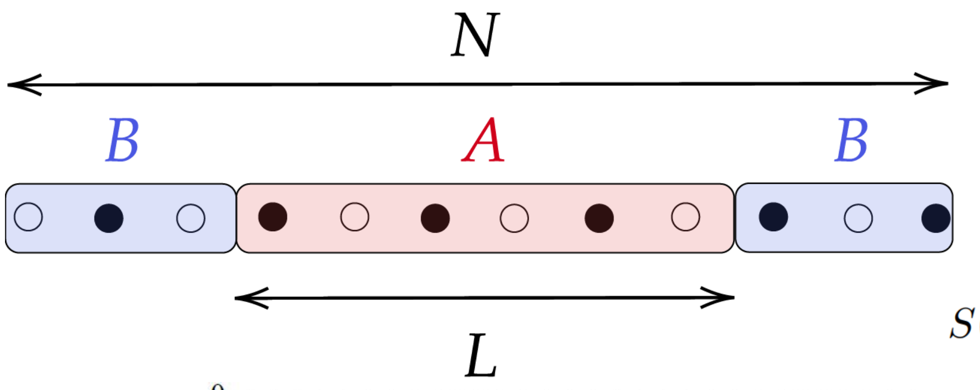
Beyond local operators: entropy



Schmidt spectrum:

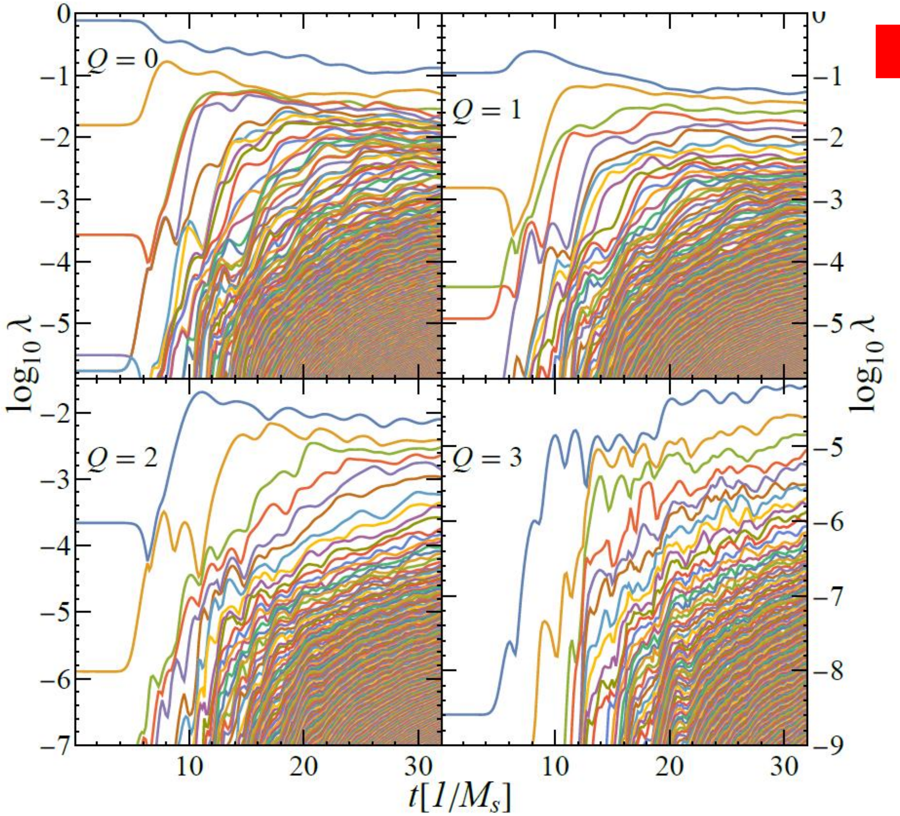


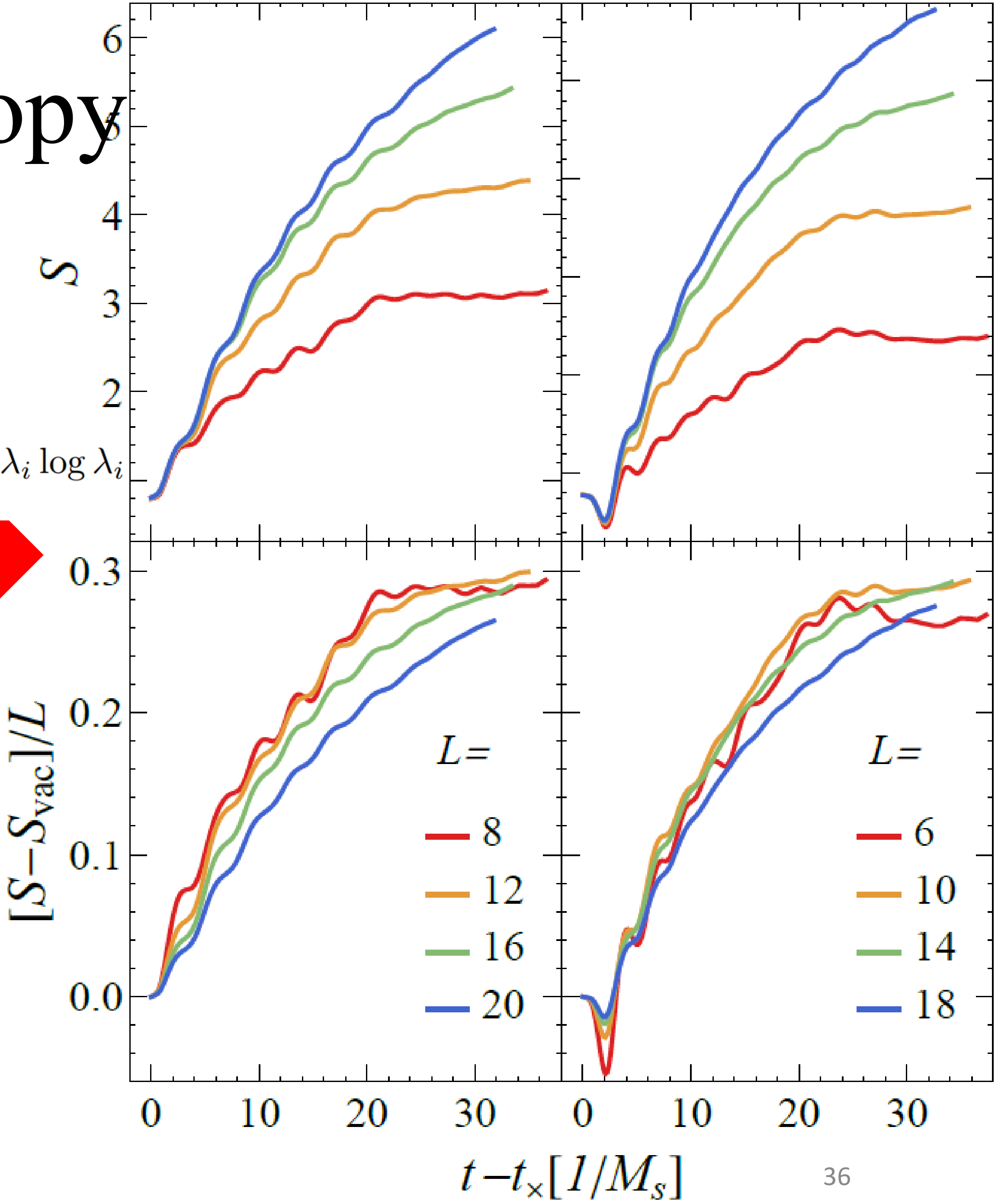
Beyond local operators: entropy



$$S(t) = -\sum_i \lambda_i \log \lambda_i$$

Schmidt spectrum:

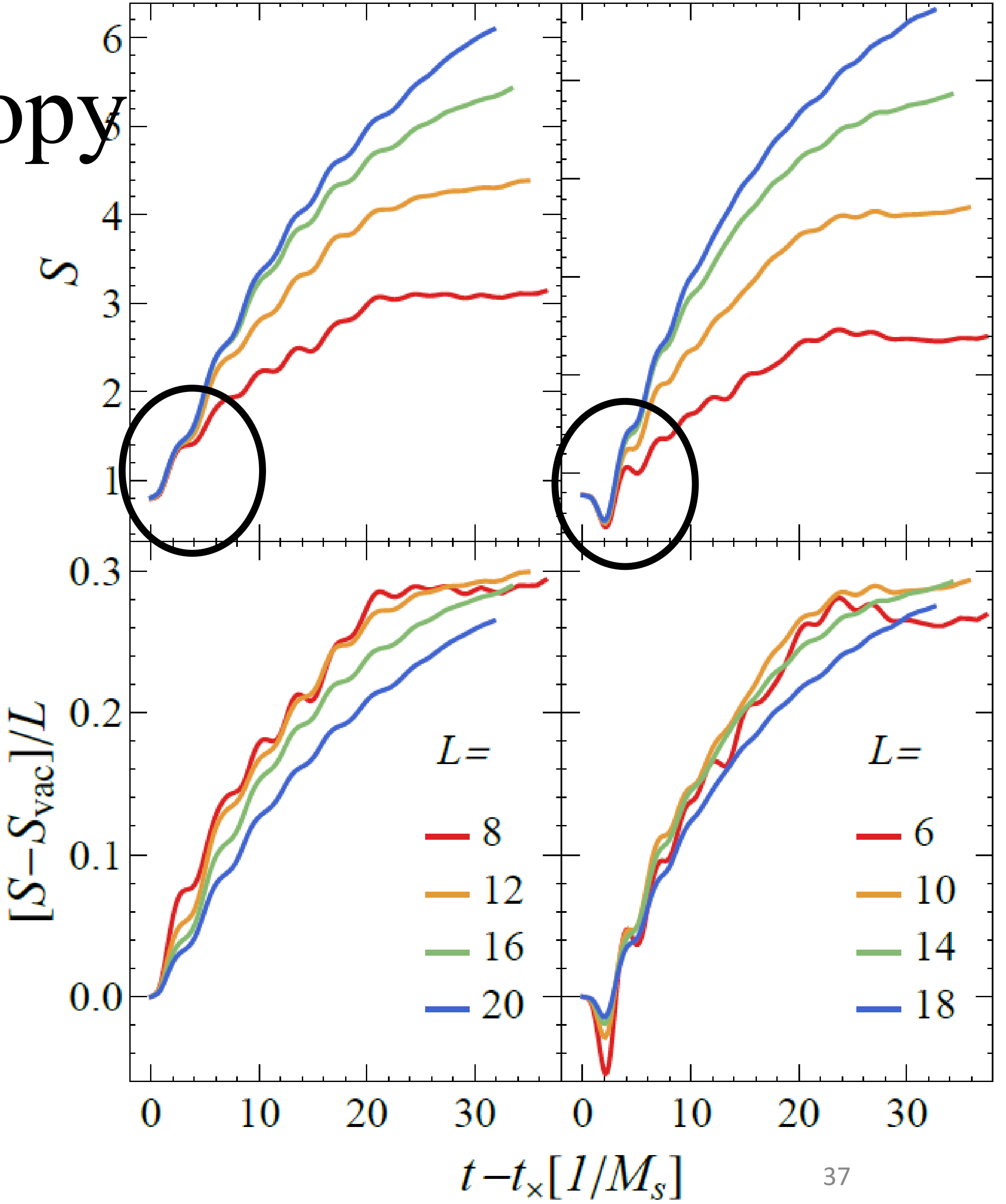




Beyond local operators: entropy

Adjust by the jet arrival time

area law at early times



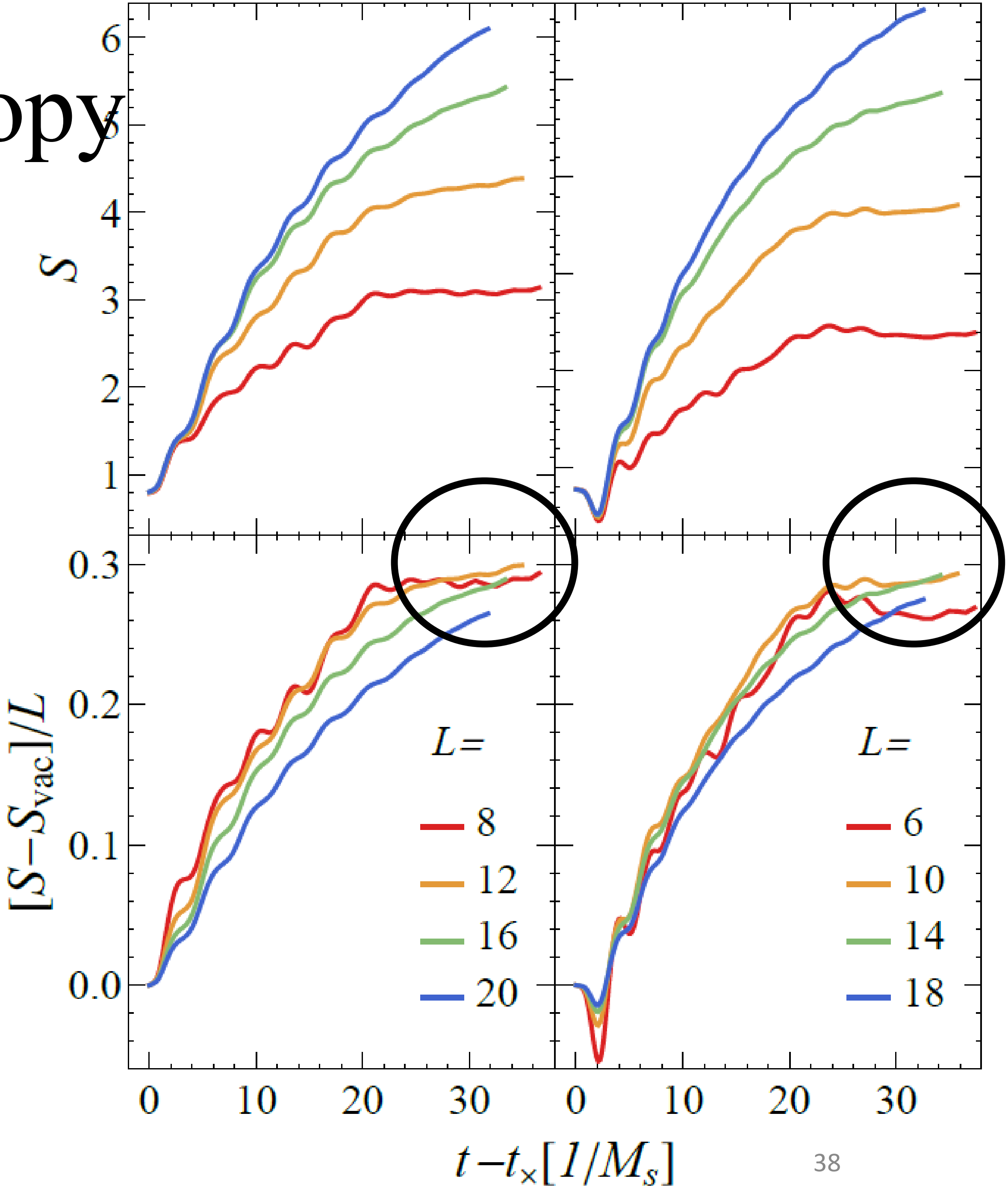
Beyond local operators: entropy

Adjust by the jet arrival time

area law at early times

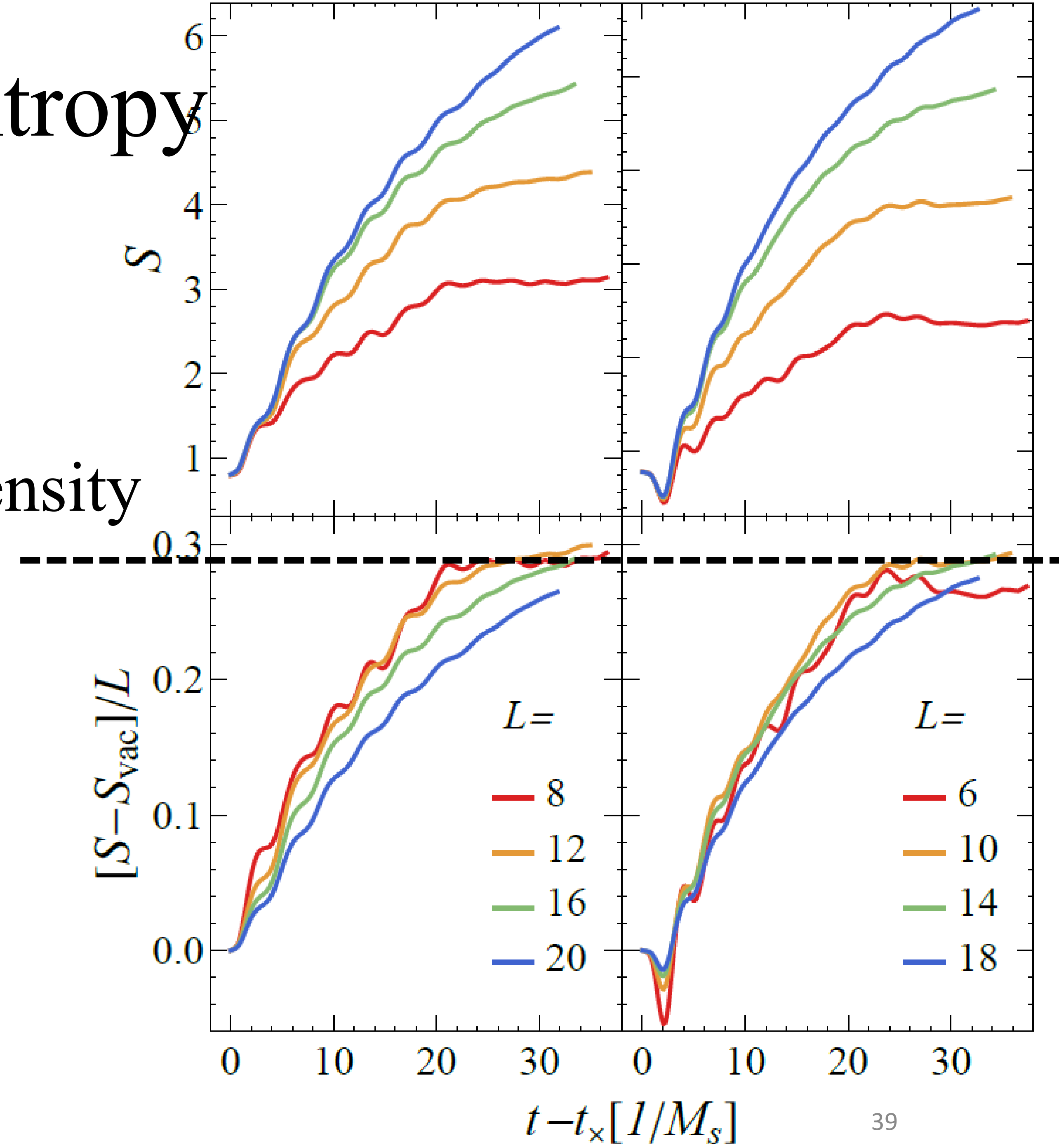
Rescale by the subsystem size

volume law at late times
(expected in thermal states)



Beyond local operators: entropy

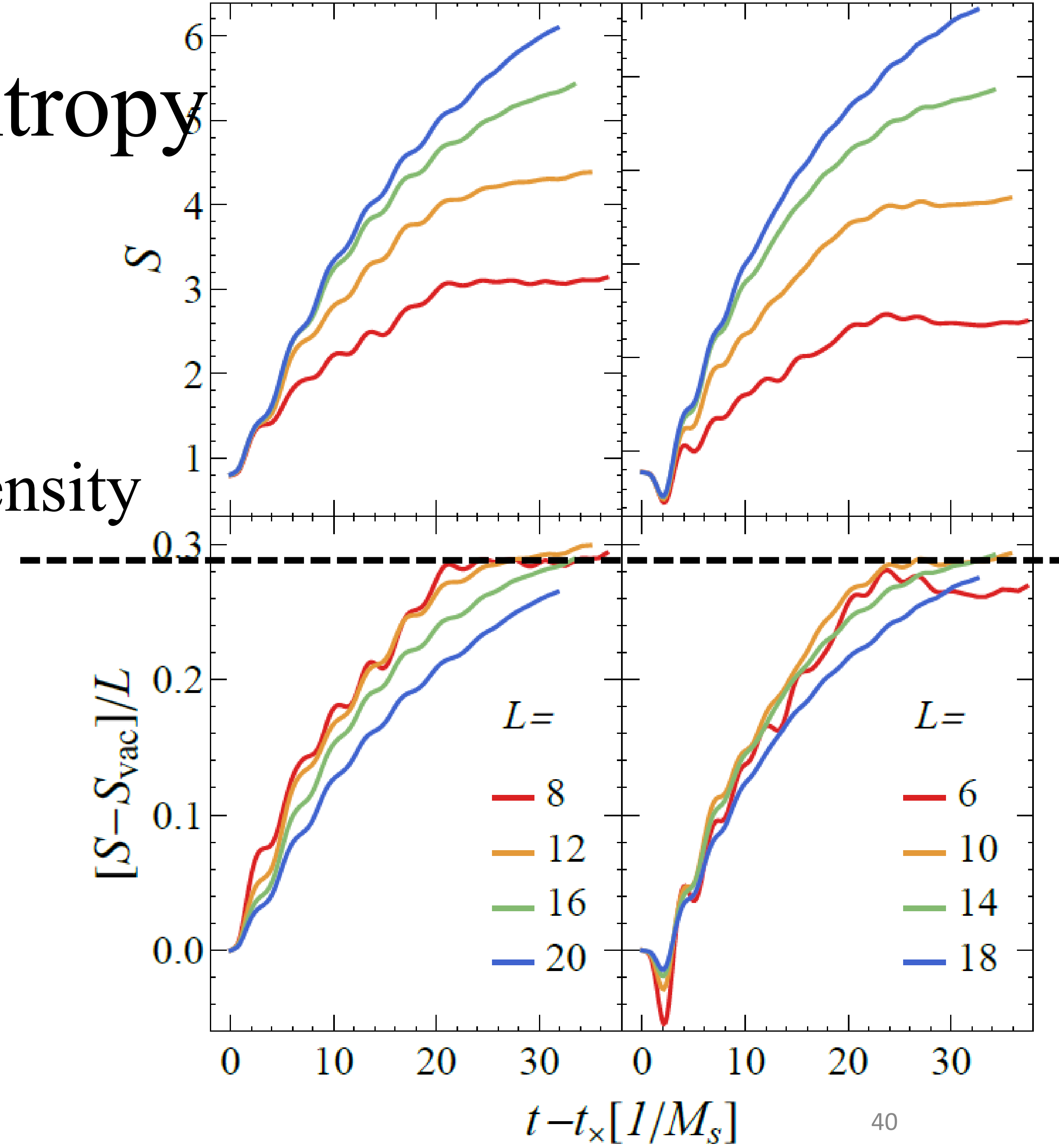
Equilibrium entropy density



Beyond local operators: entropy

Equilibrium entropy density

Compare to thermal entropy density



Gibbs entropy

$$S(T) = -\sum_n p_n(T) \log p_n(T)$$

↑

$$e^{-E_n/T}/Z$$

Requires full diagonalization

↓

Exact diagonalization

↓

Finite volume effects

Gibbs entropy

$$S(T) = -\sum_n p_n(T) \log p_n(T)$$

$$p_n(T) = e^{-E_n/T}/Z$$

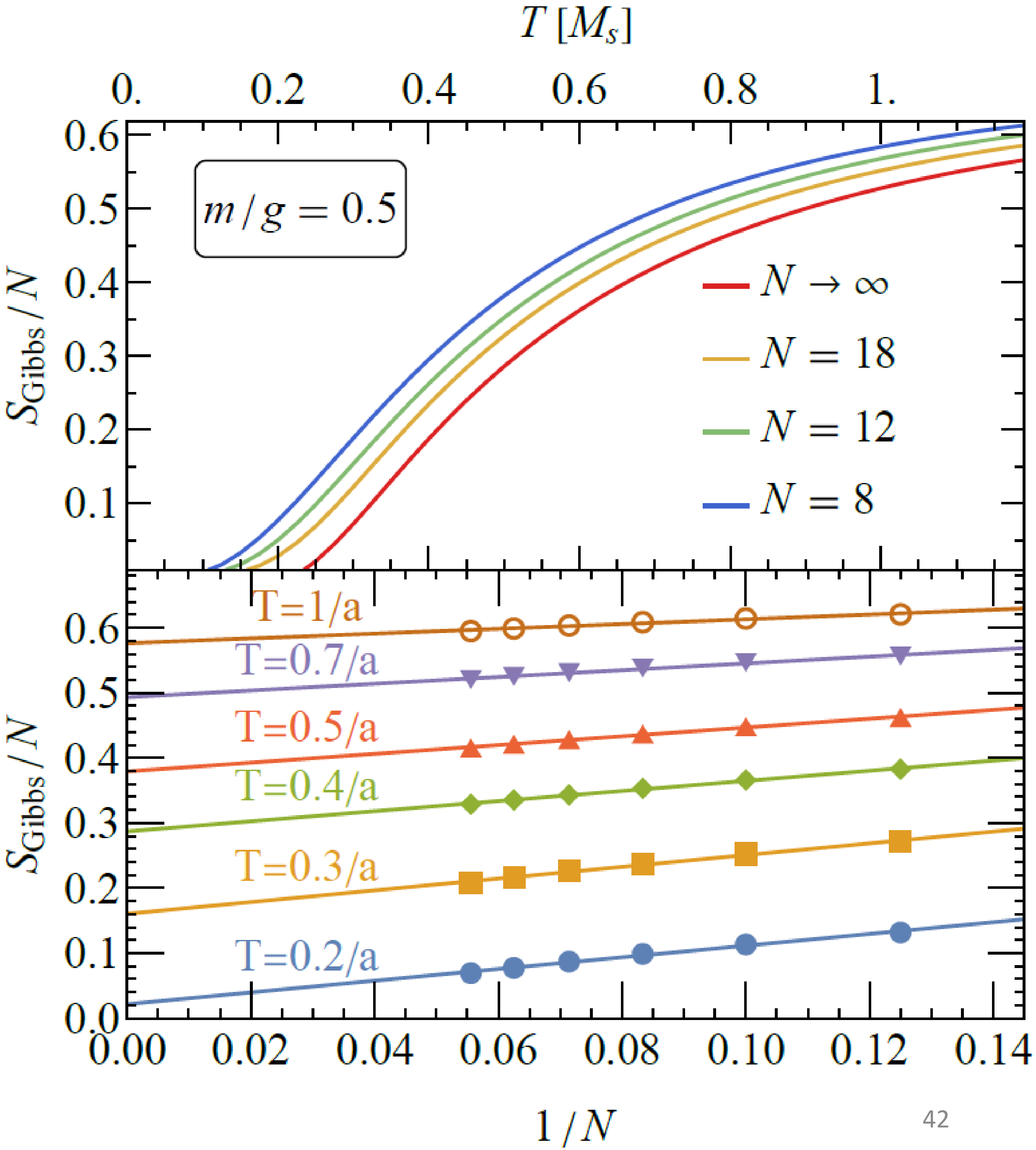
Requires full diagonalization

↓

Exact diagonalization

↓

Finite volume effects



Gibbs entropy

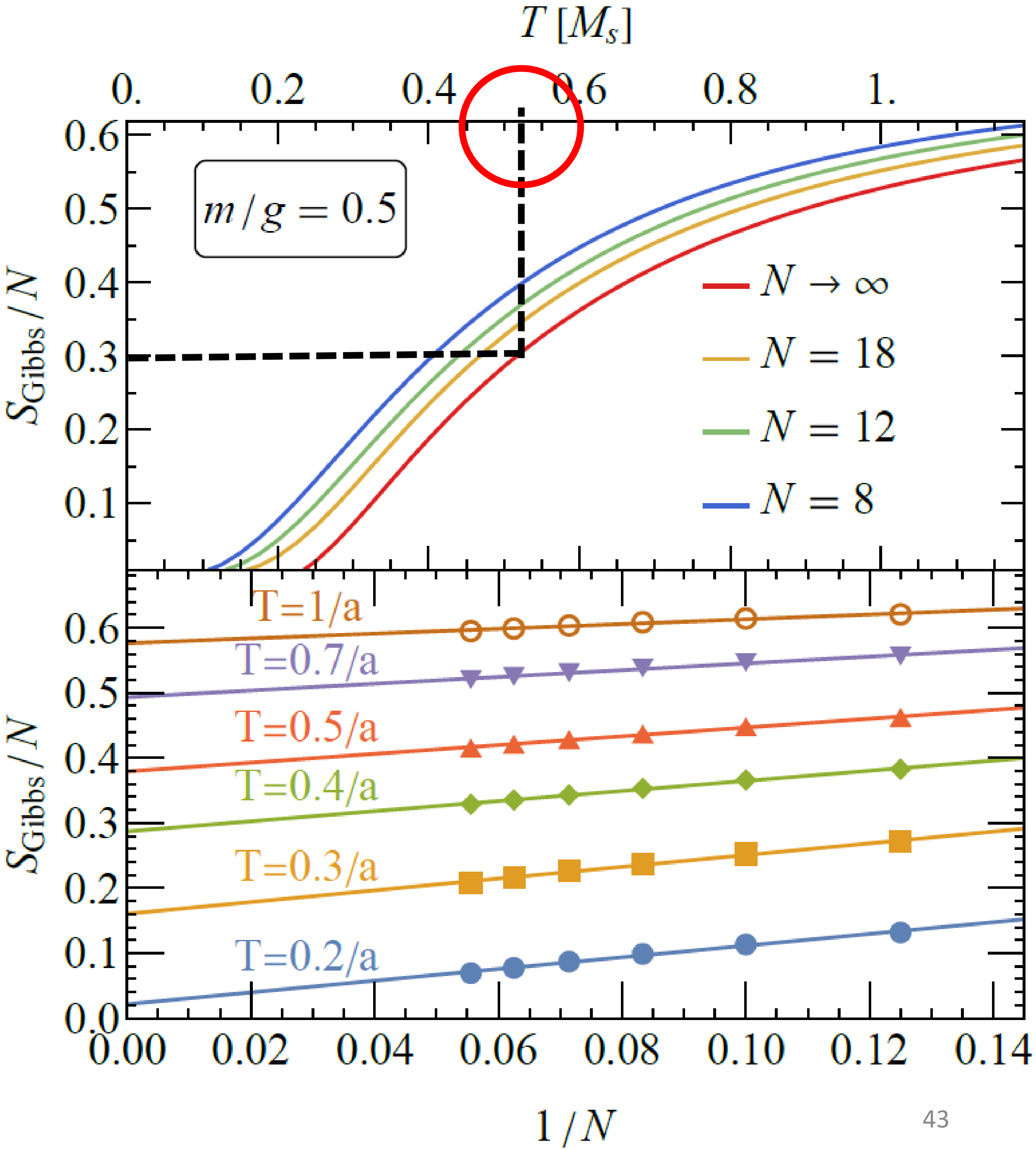
$$S(T) = -\sum_n p_n(T) \log p_n(T)$$

$$e^{-E_n/T}/Z$$

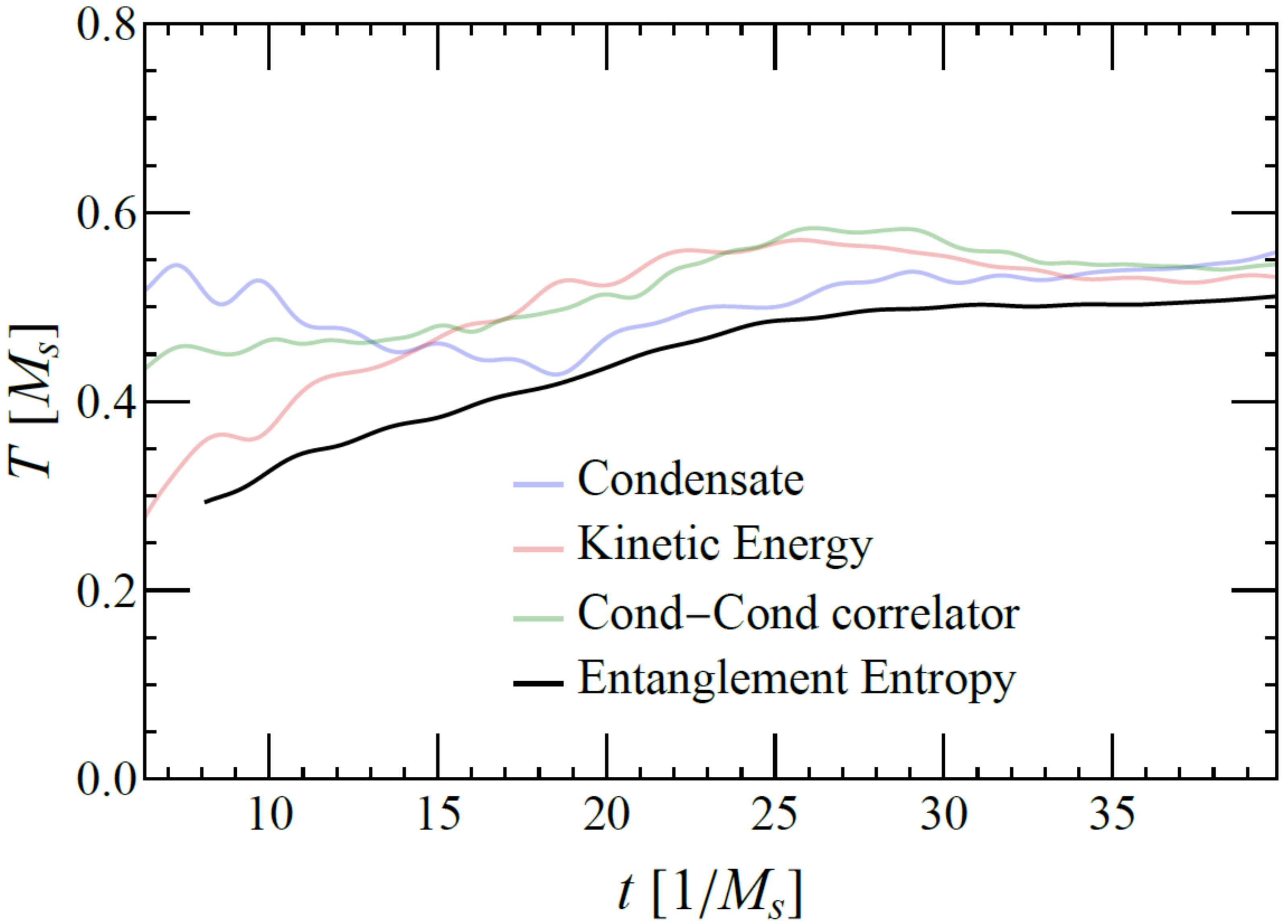
Requires full diagonalization

Exact diagonalization

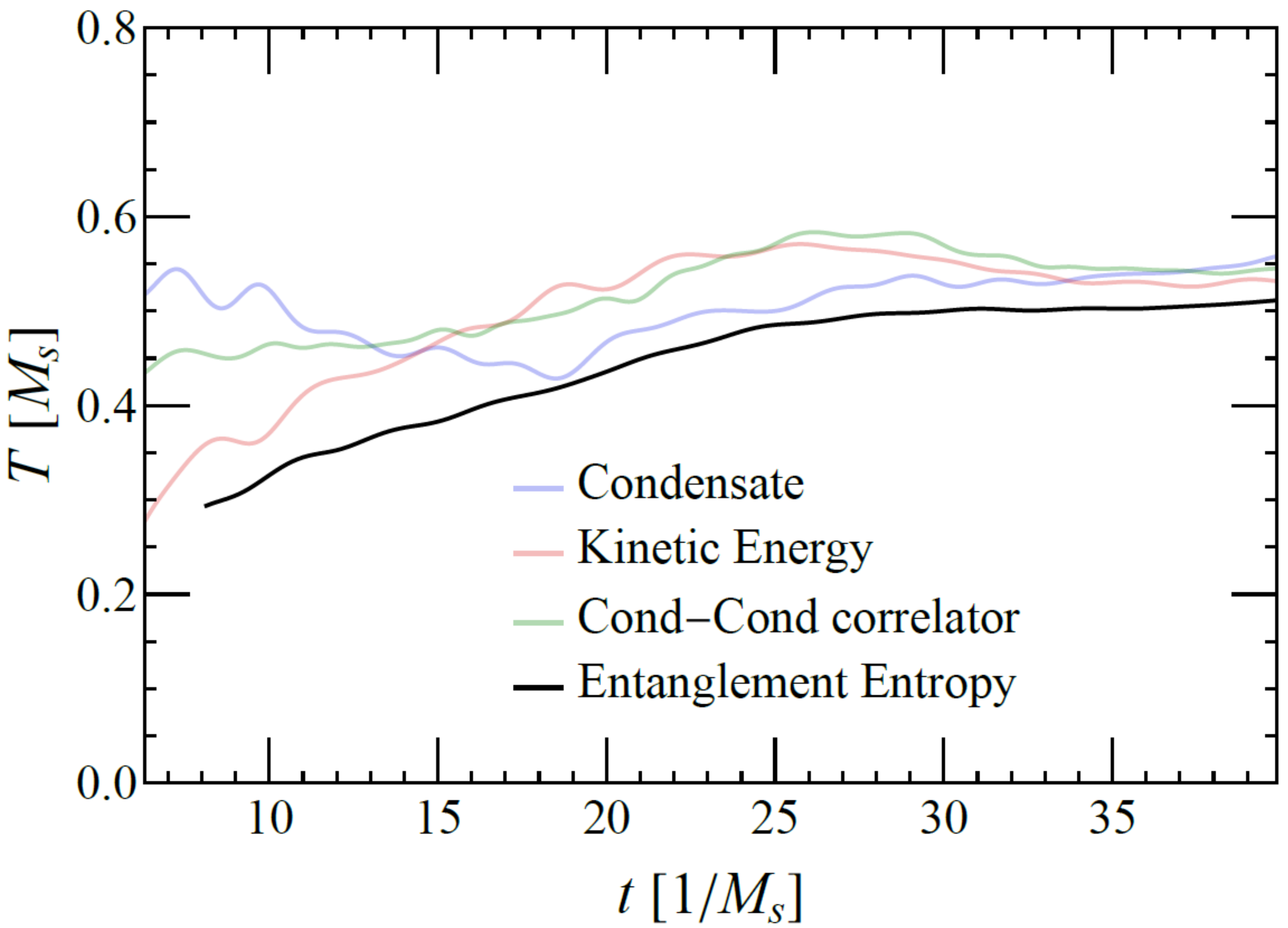
Finite volume effects



Temperature from entropy



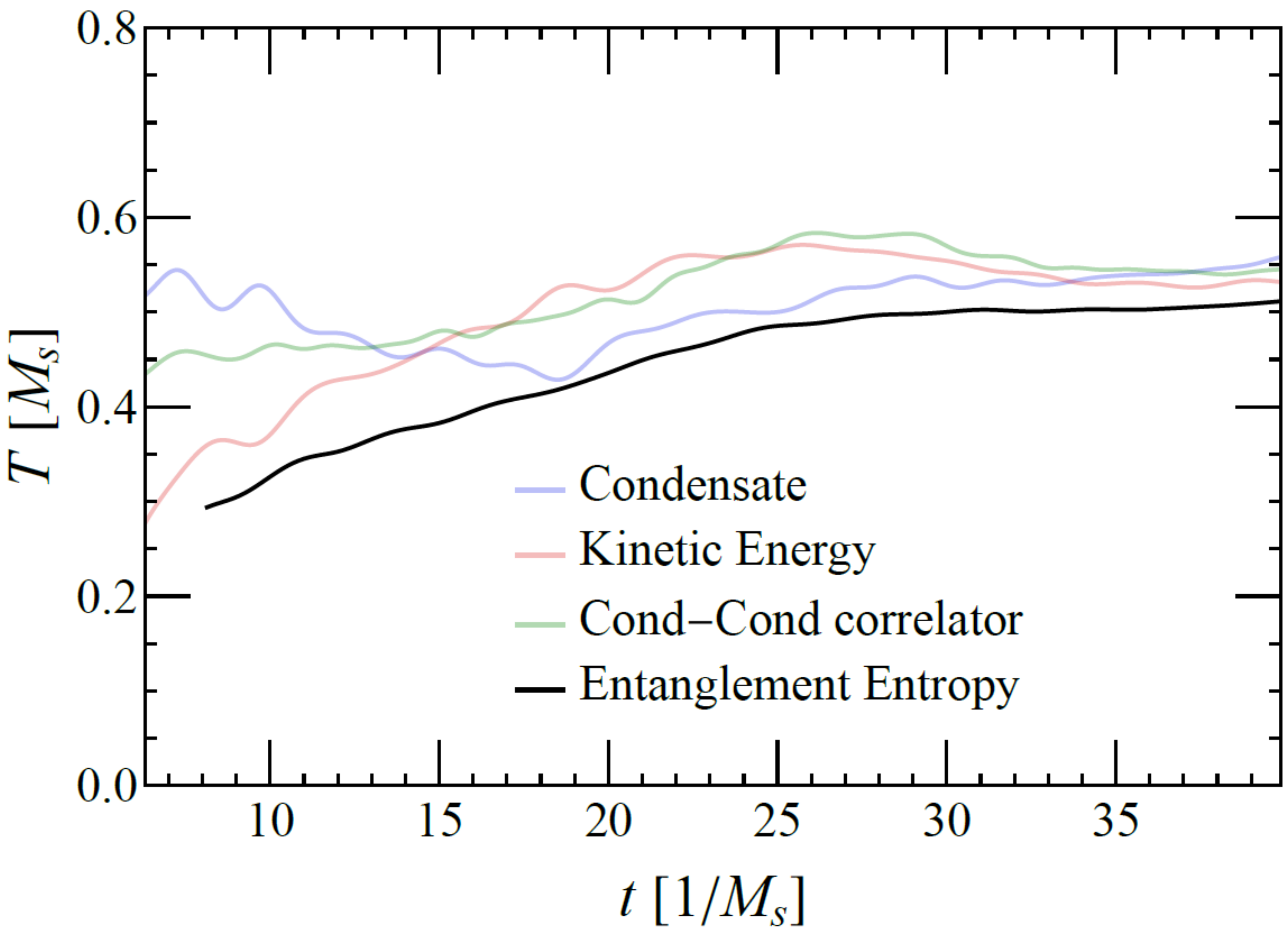
Temperature from entropy



Thermodynamical entropy

$$s = \frac{\epsilon + P}{T}$$

Temperature from entropy

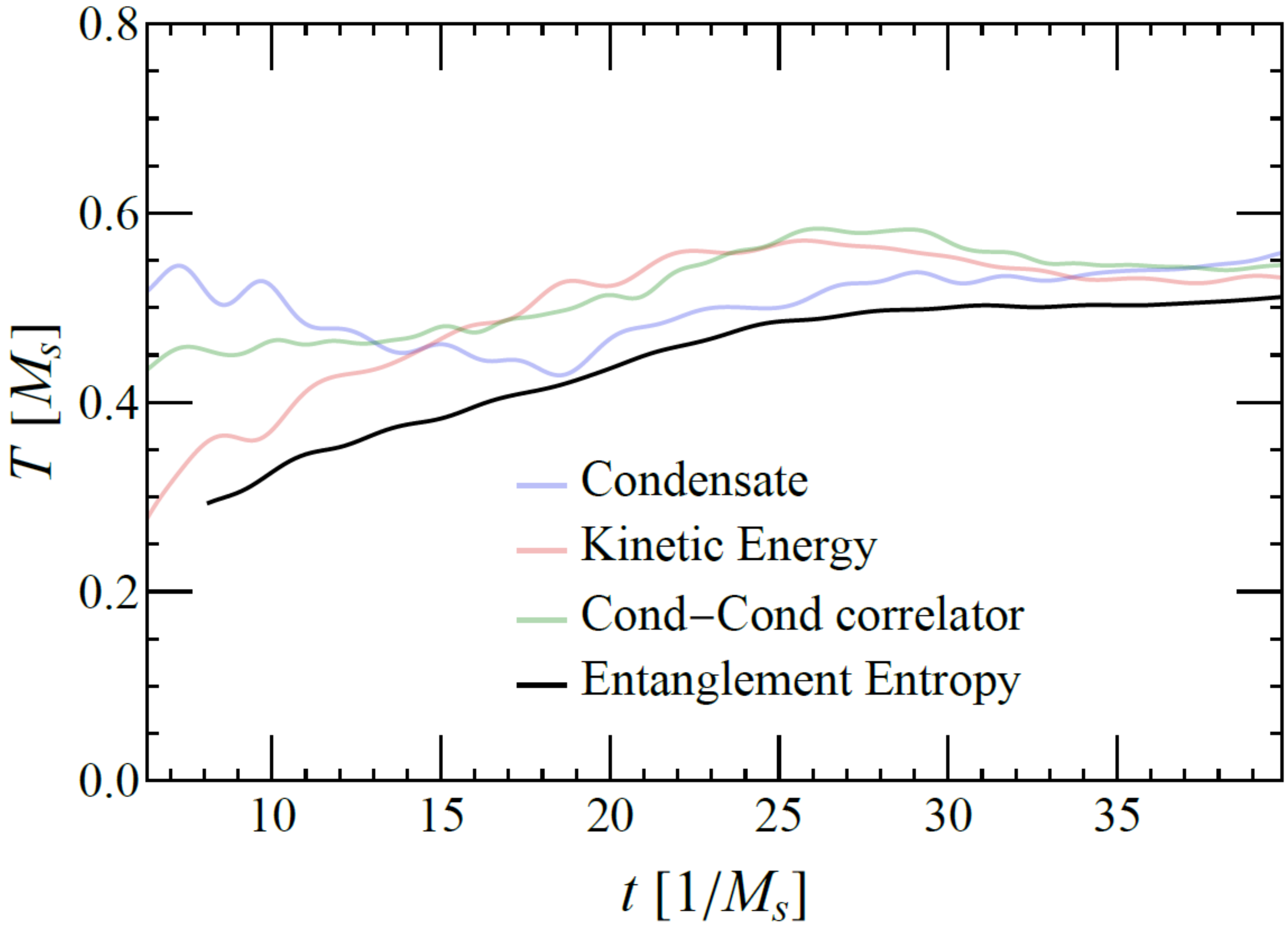


Thermodynamical entropy

measure

$$s = \frac{\epsilon + P}{T}$$

Temperature from entropy



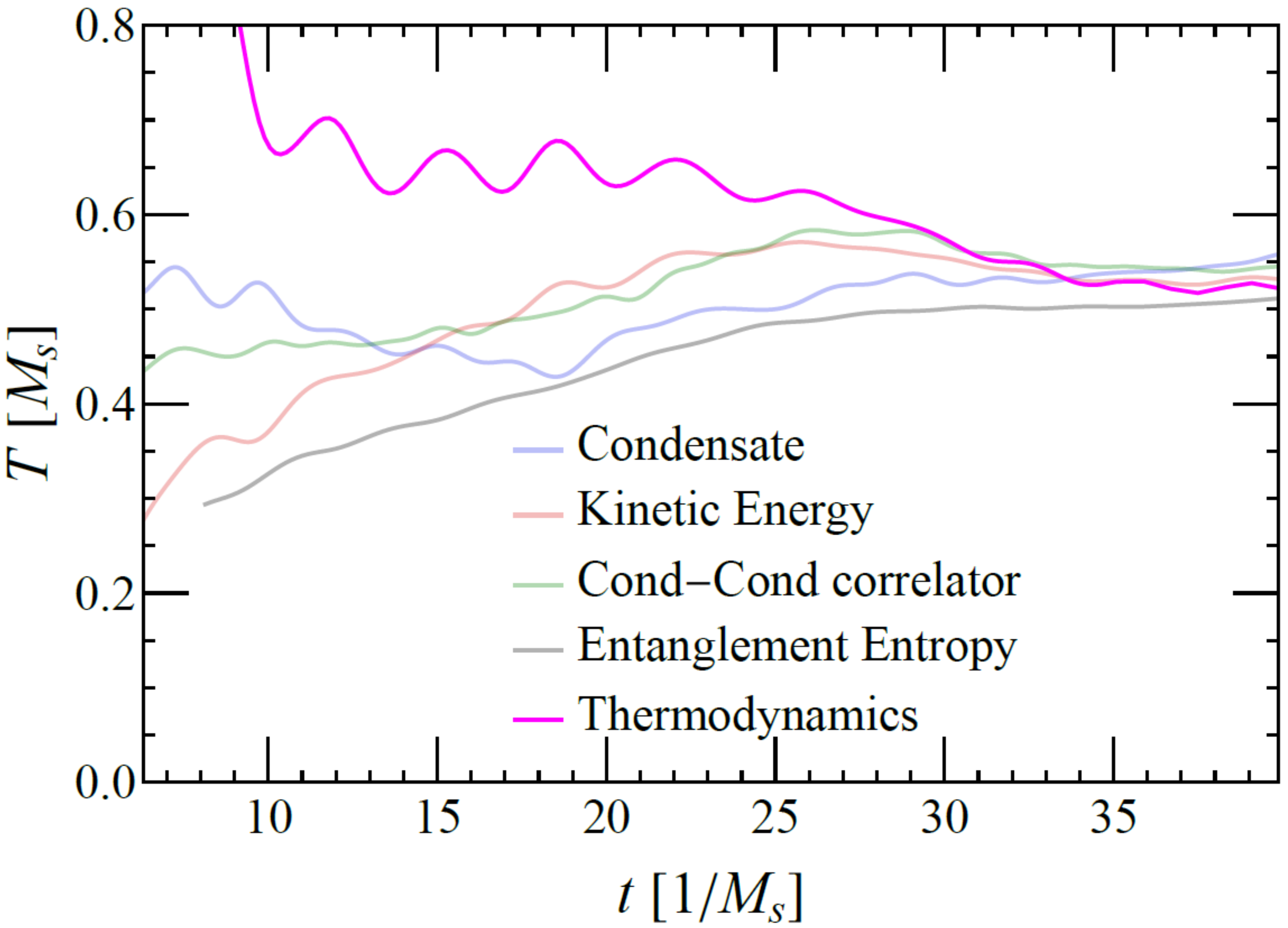
Thermodynamical entropy

measure

$$s = \frac{\epsilon + P}{T}$$

Identify with entanglement

Temperature from entropy



Thermodynamical entropy

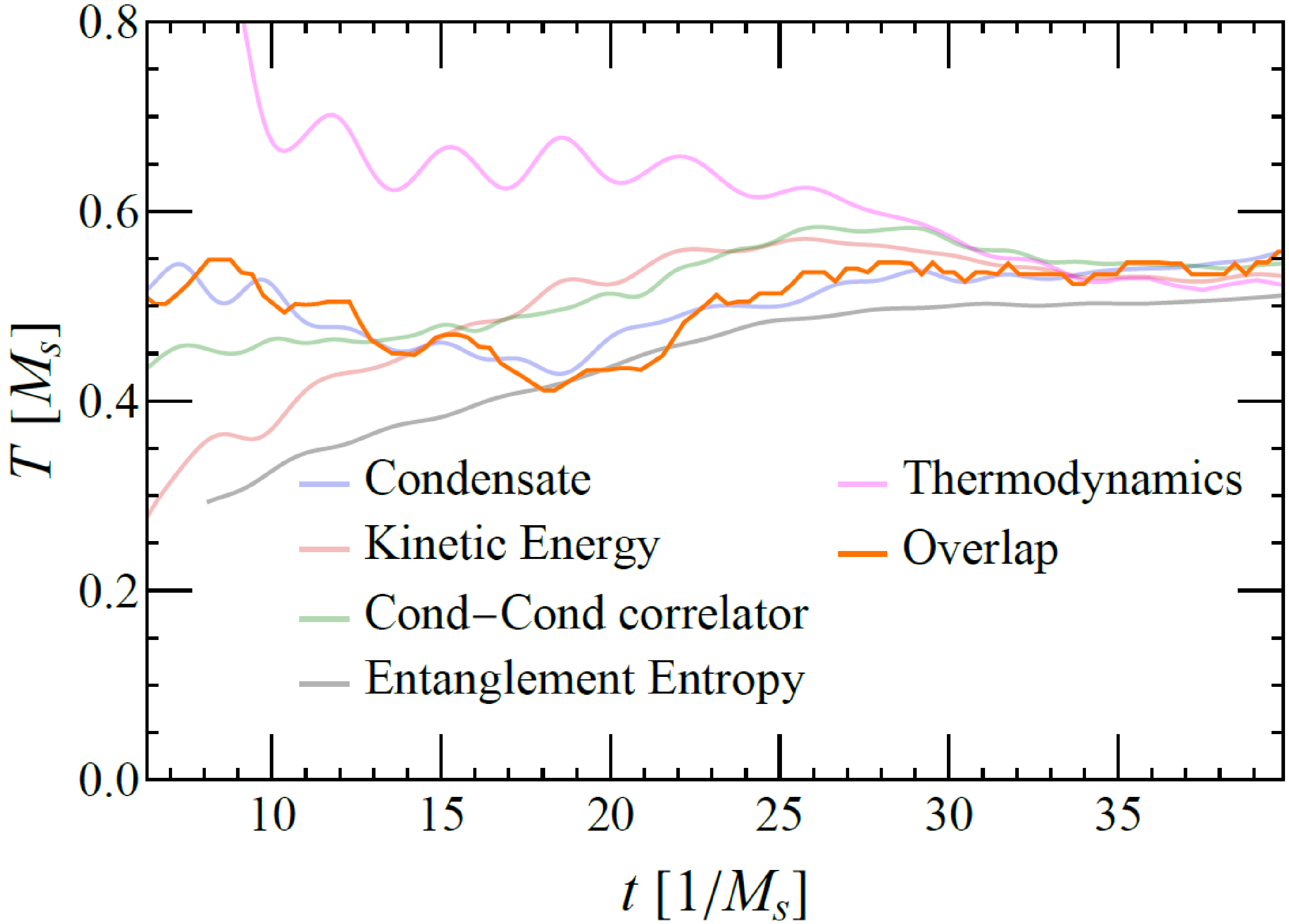
measure

$$s = \frac{\epsilon + P}{T}$$

Identify with entanglement

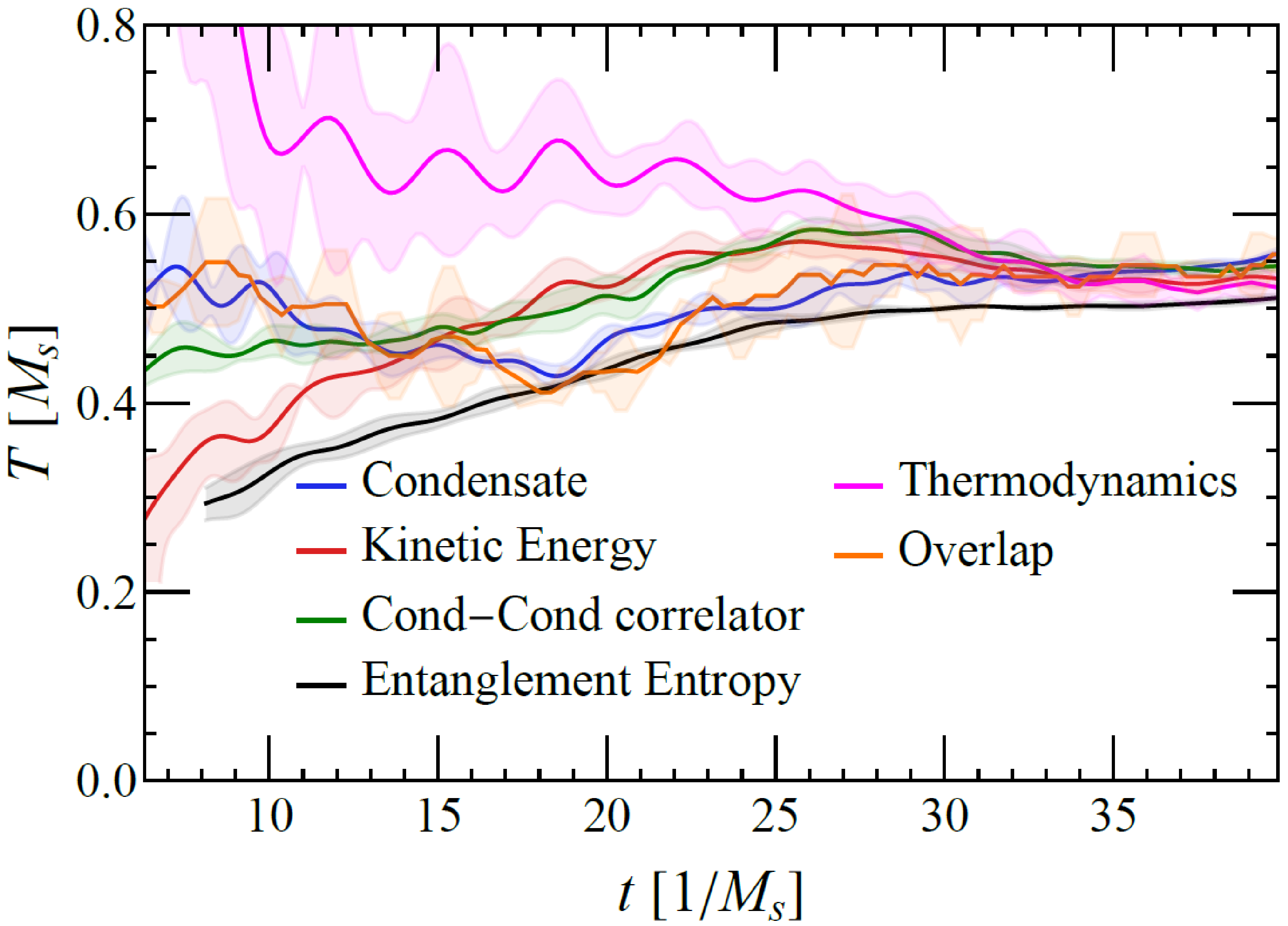
Density matrix comparison

$$``\langle \rho_{\text{system}} | \rho_{\text{thermal}} \rangle"$$



Maximize over T

More details in the talk by S. Grieninger tomorrow



0.8
0.6
0.4
0.2
0.0
10
15
20
25
30
35
$T\ [M_s]$
$t\ [1/M_s]$
Condensate
Kinetic Energy
Cond−Cond correlator
Entanglement Entropy
Thermodynamics
Overlap

Based on

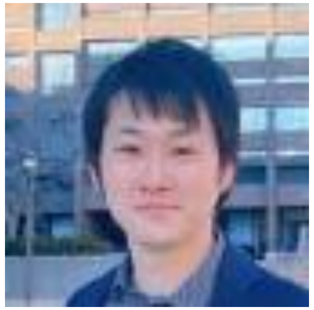
- Real-Time Nonperturbative Dynamics of Jet Production in Schwinger Model: Quantum Entanglement and Vacuum Modification

  Phys. Rev. Lett. **131**, 021902 (2023)

- Quantum real-time evolution of entanglement and hadronization in jet production: Lessons from the massive Schwinger model

  Phys. Rev. D **110**, 094029 (2024)

A.Florio , DF, K.Ikeda, D.Kharzeev, V.Korepin, S.Shi, K.Yu



- Thermalization from quantum entanglement: jet simulations in the massive Schwinger model

  2506.14983 A.Florio, DF, S.Grieninger  ,D.Kharzeev, A.Palermo  , S.Shi

Outlook

Existing 1+1-dimensional methods

- Mimic real-time QCD processes and look through the prism of QIS
- Establish links between thermalization and entanglement

Goal: go to 2+1

- Angular structure
- Energy correlators
- Choice of tools – not so obvious

Let’s hope the future is bright!

BACKUP

Purification

$$\tilde{\mathcal{H}} = \mathcal{H} \otimes \mathcal{H}'$$

$$|\tilde{\Psi}(0)\rangle = \bigotimes_{i=1}^{N} \frac{|0\rangle_i |0\rangle_i' + |1\rangle_i |1\rangle_i'}{\sqrt{2}}$$

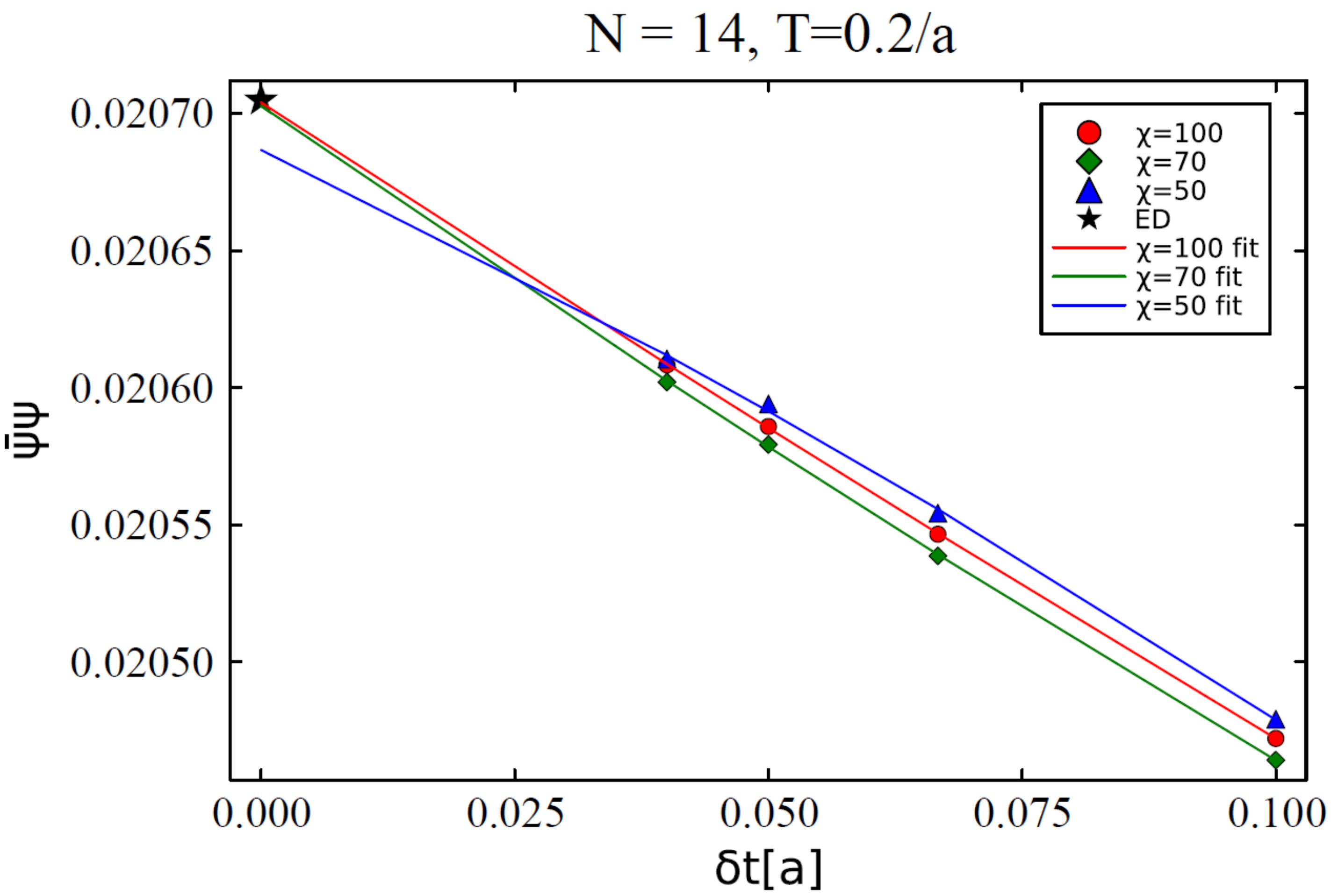
$$|\tilde{\Psi}(\beta)\rangle = \frac{1}{\sqrt{Z(\beta)}} e^{-\beta \widehat{\tilde{H}}/2} |\tilde{\Psi}(0)\rangle$$

$$\widehat{\tilde{H}}_{\tilde{\mathcal{H}}} \equiv \widehat{H}_{\mathcal{H}} \otimes I_{\mathcal{H}'}$$

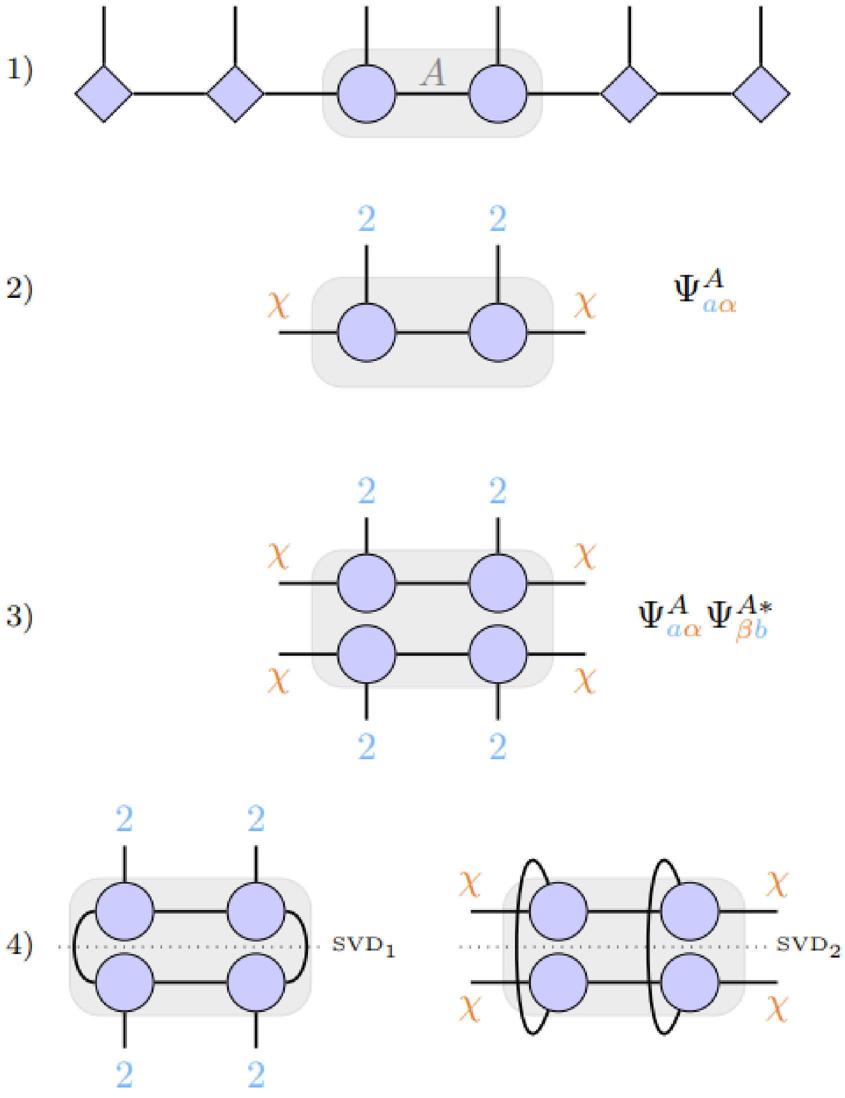
$$Z(\beta) \equiv \langle \tilde{\Psi}(0) | e^{-\beta \widehat{\tilde{H}}} | \tilde{\Psi}(0) \rangle$$

$$\langle \mathcal{O} \rangle_\beta = \langle \tilde{\Psi}(\beta) | \mathcal{O}_{\mathcal{H}} \otimes I_{\mathcal{H}'} | \tilde{\Psi}(\beta) \rangle$$

Trotter error control



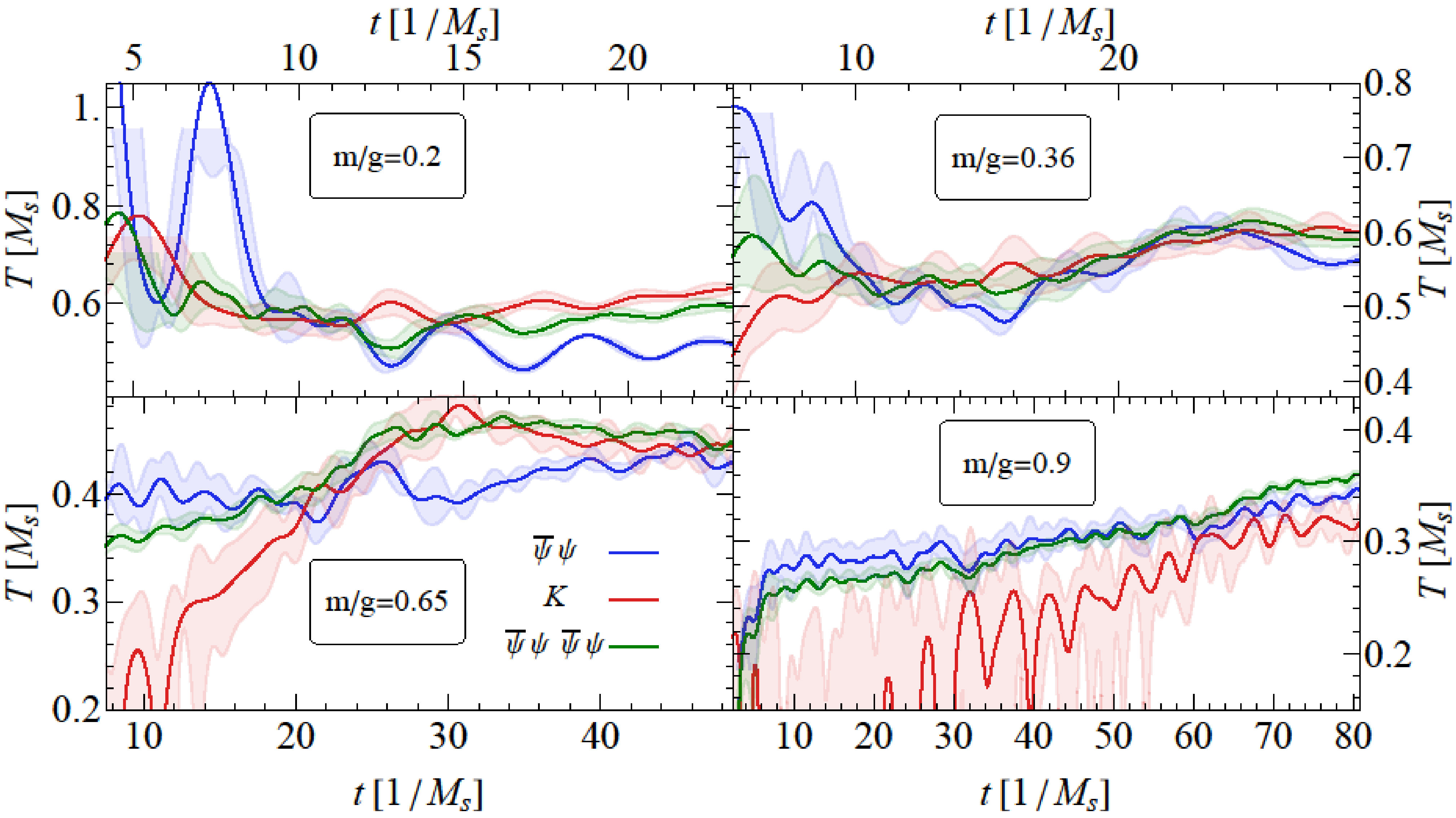
Entanglement spectrum of finite interval



$$\Psi^{A}_{a\alpha}\Psi^{A*}_{\alpha b} \equiv (\Psi\Psi^{\dagger})_{ab} \equiv (\rho^{A})_{ab}$$

$$\Psi^{A*}_{\alpha a}\Psi^{A}_{a\beta} \equiv (\Psi^{\dagger}\Psi)_{\alpha\beta}$$

Fermion mass effect



$a = 1$
$g = 0.5$