Quantum thermodynamics of nonequilibrium processes in lattice gauge theories

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Based on
Quantum thermodynamics of nonequilibrium processes in lattice gauge theories,
Zohreh Davoudi, Christopher Jarzynski, Niklas Mueller, GO, Connor Powers, and Nicole Yunger Halpern
Phys. Rev. Lett. 133, 250402 (2024)
+ arXiv:2502.19418 [quant-ph] (2025)

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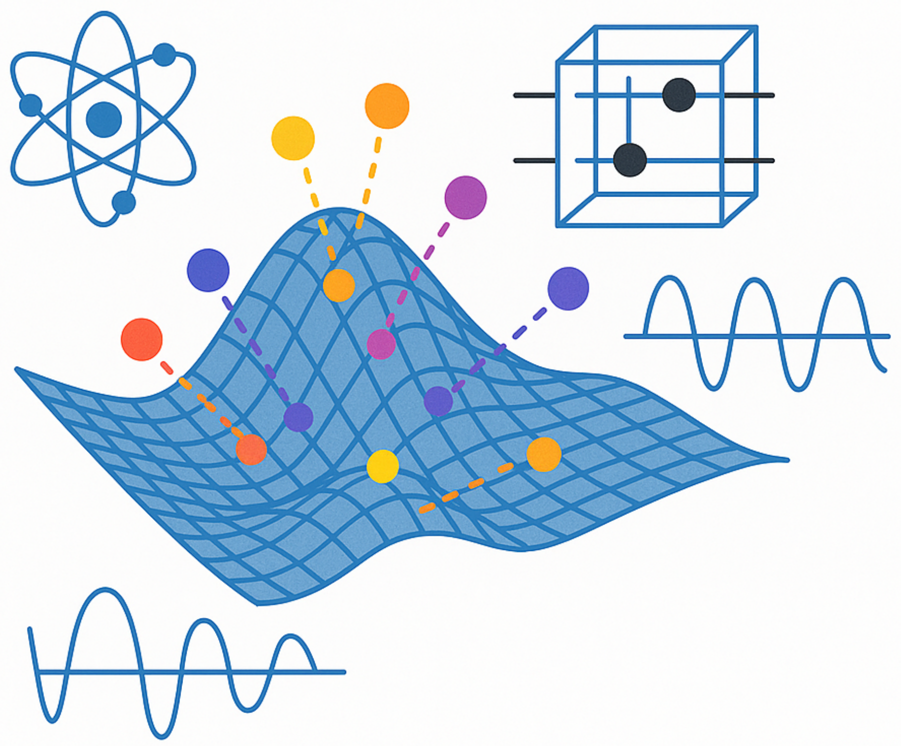
Are the three fields related?

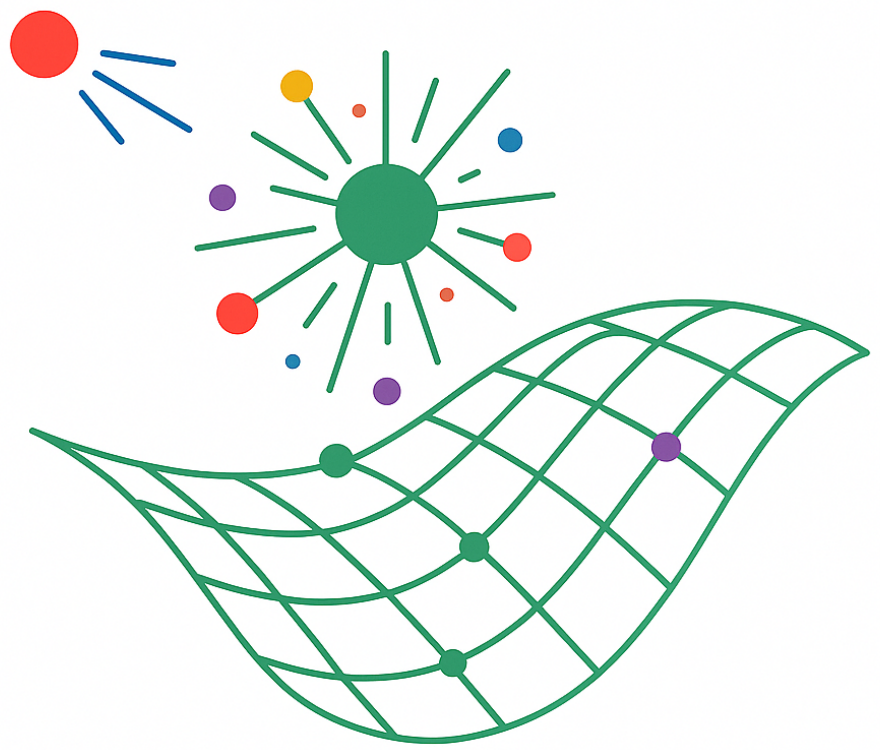
Quantum simulations



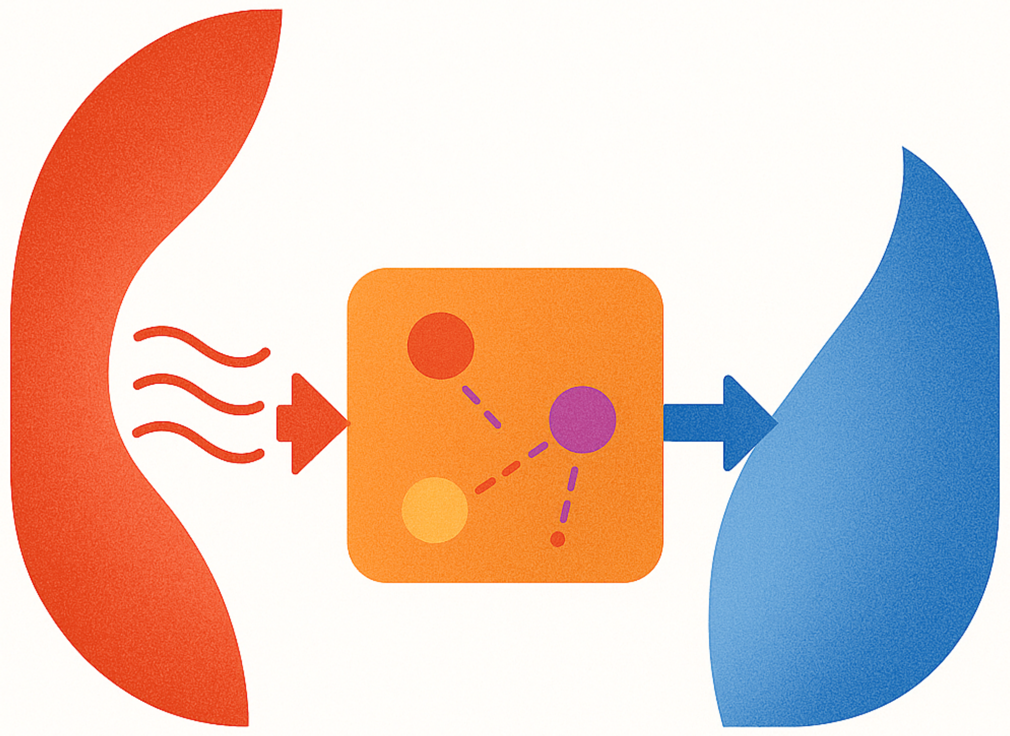
Bauer, *et al.*, Nature Reviews Physics , volume 5, 420–432 (2023)
Klco *et al.*, 2022 Rep. Prog. Phys. 85 064301
Bauer *et al.*, PRX Quantum 4, 027001 (2023)

Landi, *et al.*, Phys. Rev. A 101, 042106 (2020)
Li, *et al.*, Phys. Rev. B 103, 104306 (2021)
Aamir, *et al.*, Nat. Phys. (2025)

Lattice gauge theories



Quantum thermodynamics



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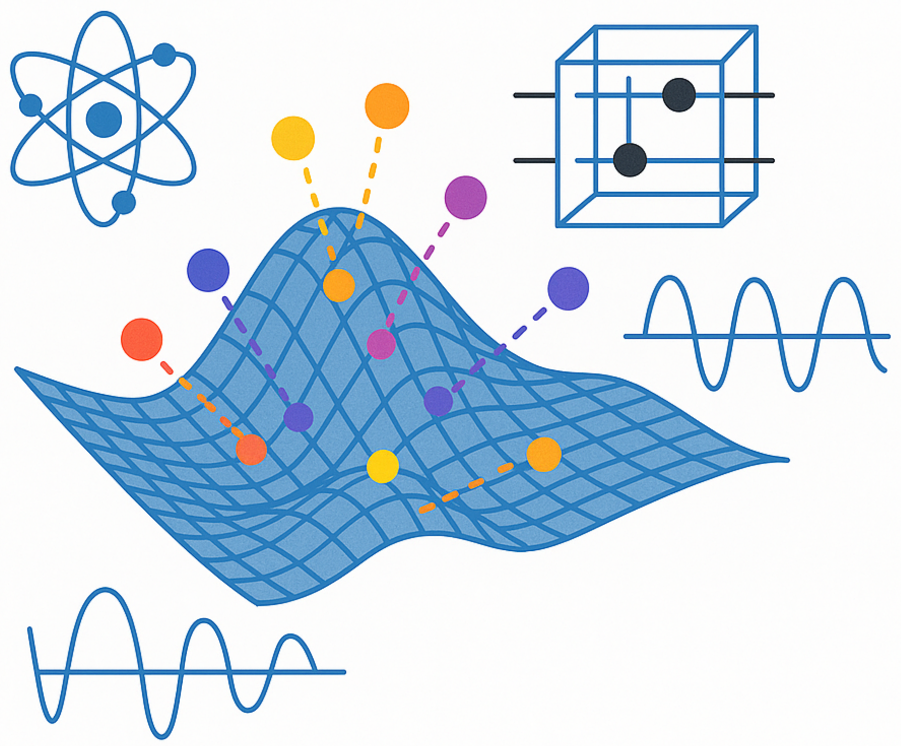

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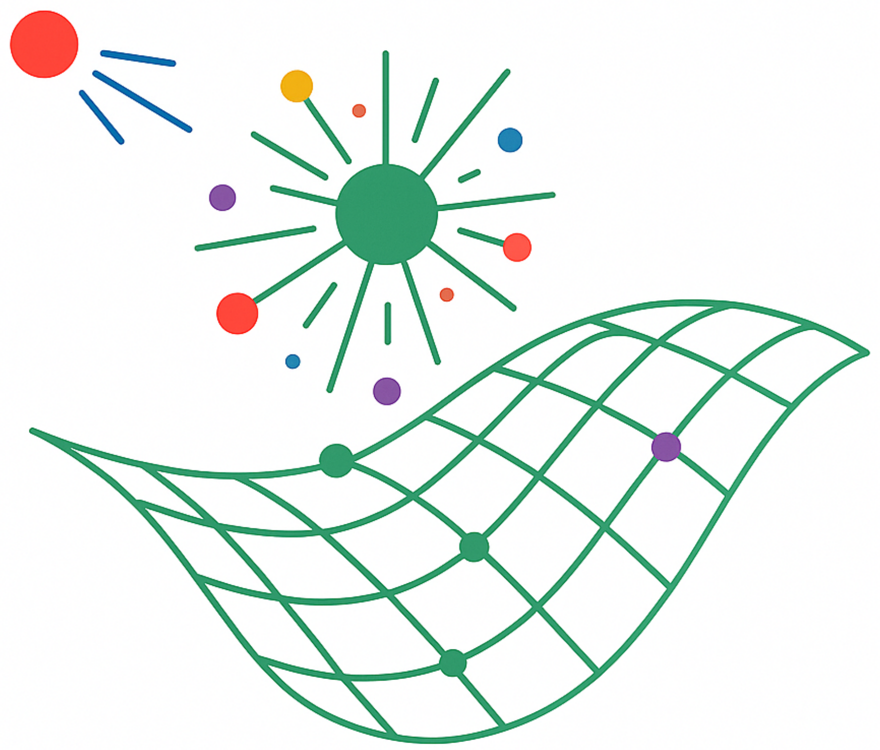

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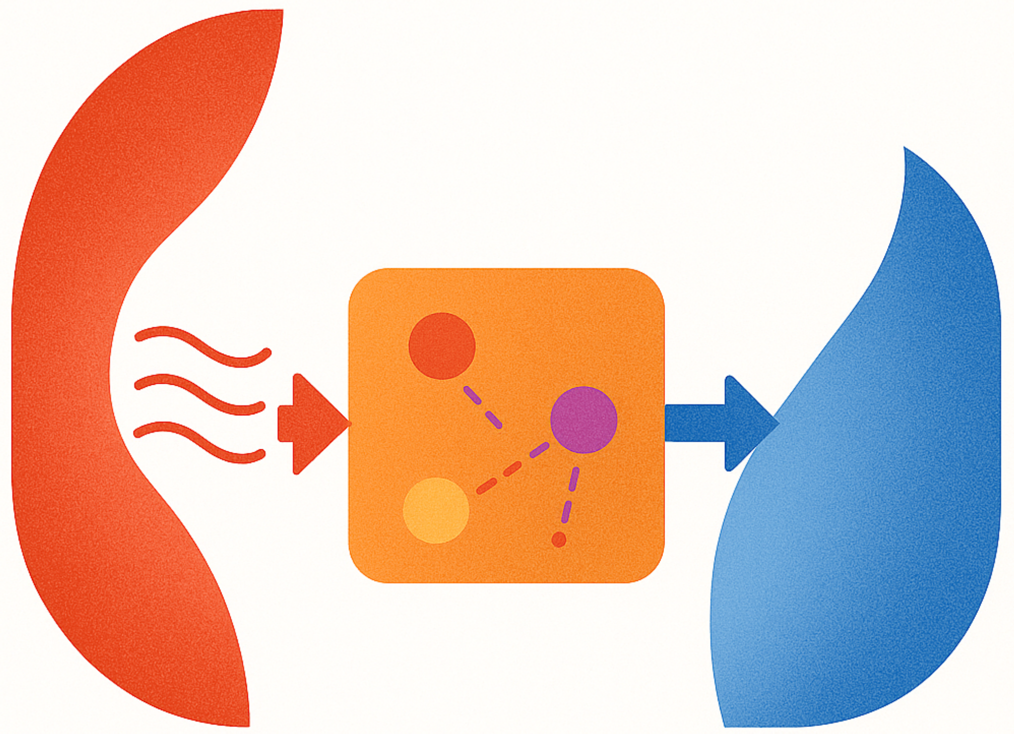

Outline

PART-1: Strong-coupling quantum thermodynamics and its relation to quantum information science

PART-2: Work, heat, and the second law for quantum quench processes of strongly coupled systems

PART-3: Applications to $\mathbb{Z}_2$ lattice gauge theory

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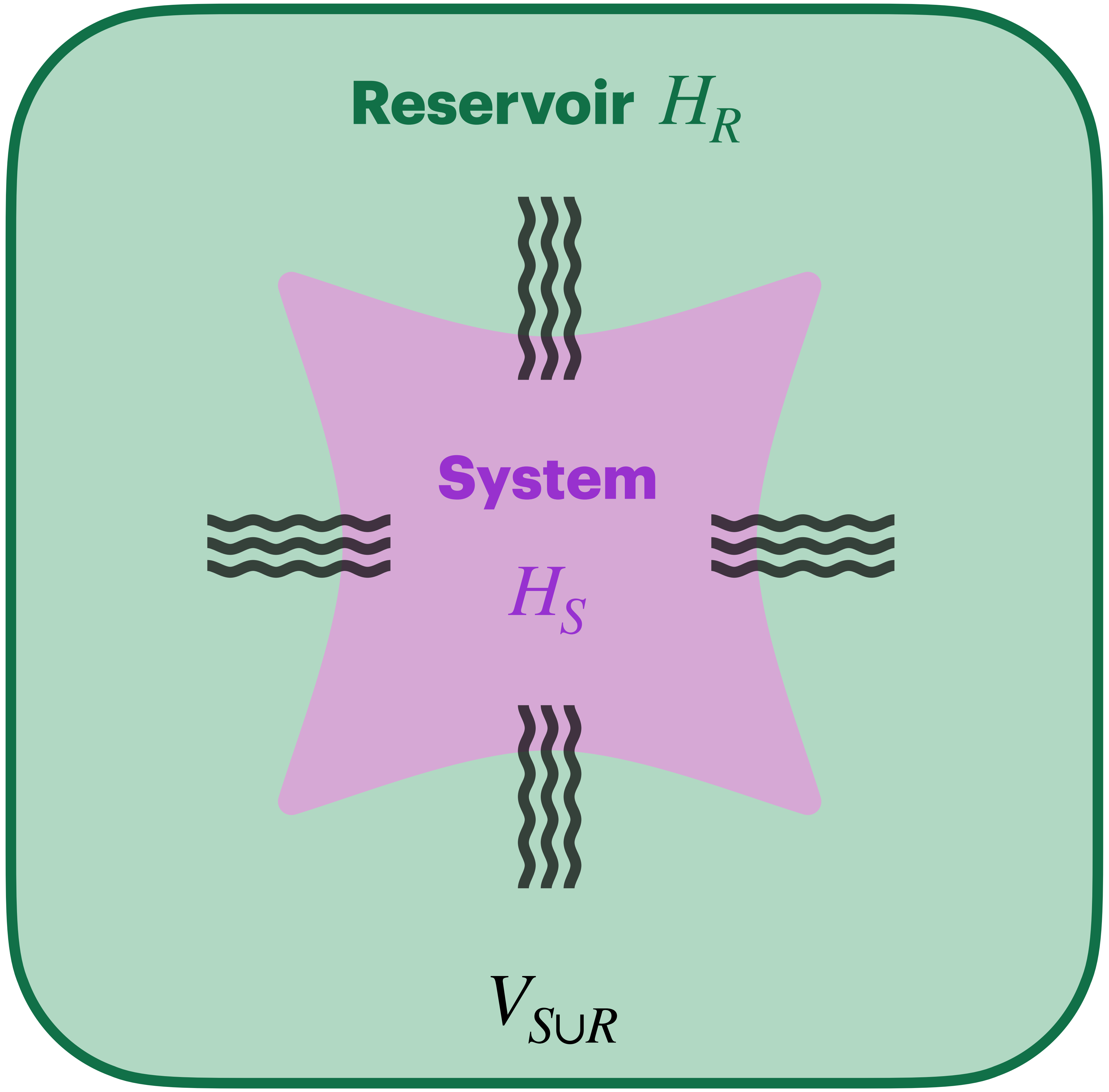
Setup

Hamiltonian: $H_{S\cup R} = H_S + H_R + V_{S\cup R}$

General state of $S \cup R$: $\rho_{S\cup R}$

General state of S : $\rho_S = \mathrm{Tr}_R\left[\rho_{S\cup R}\right]$



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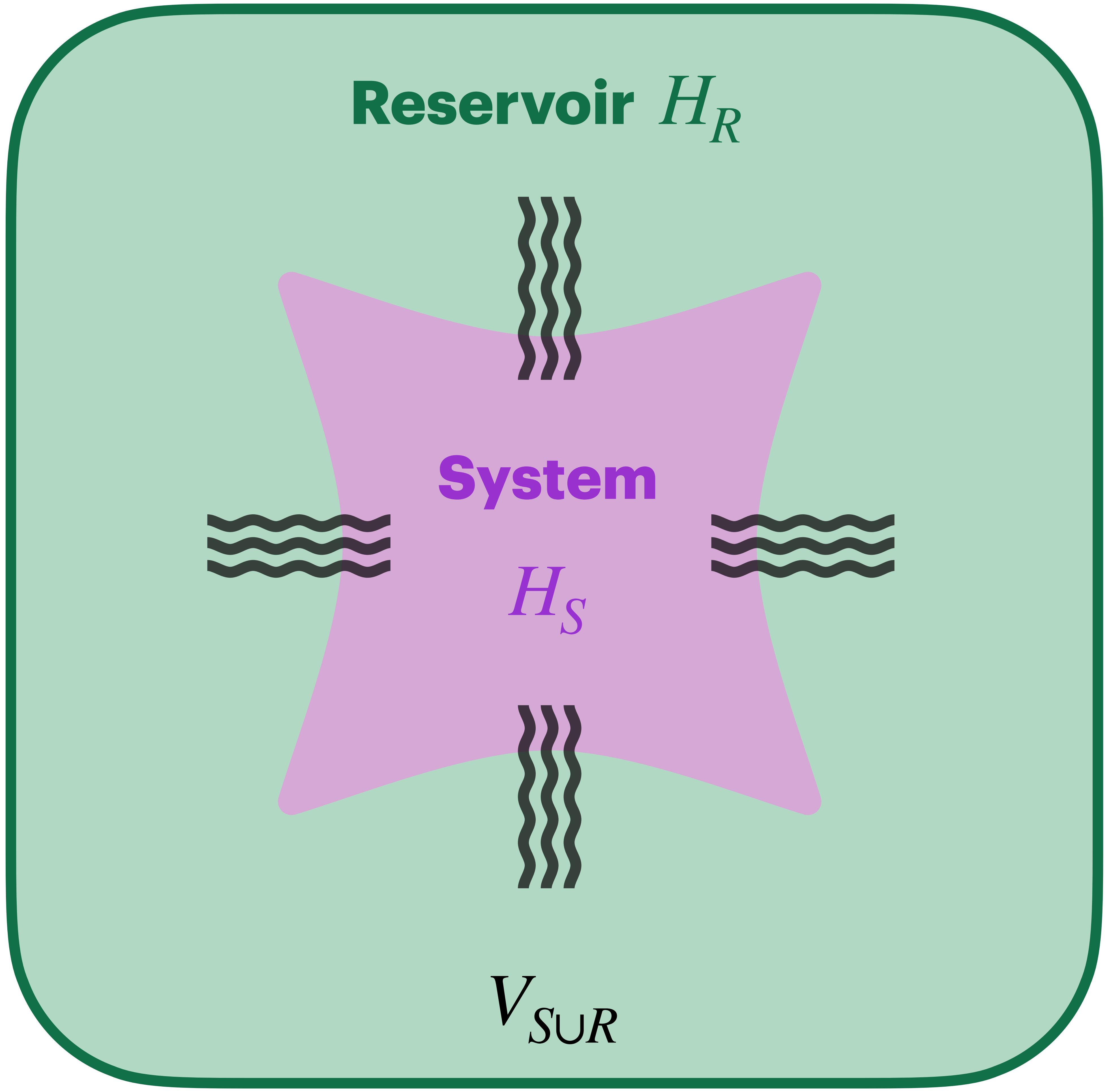
General state of S : $\rho_S = \mathrm{Tr}_R\left[\rho_{S\cup R}\right]$

Thermal equilibrium state of $S \cup R$: $\pi_{S\cup R} = \dfrac{e^{-\beta H_{S\cup R}}}{Z_{S\cup R}}$

Equilibrium state of S : $\pi_S = \mathrm{Tr}_R\left[\pi_{S\cup R}\right] \neq \dfrac{e^{-\beta H_S}}{Z_S}$



Weak-coupling vs strong-coupling thermodynamics

$$\hat{H}_{S \cup R} = \hat{H}_S + \hat{H}_R + \hat{V}_{S \cup R}$$

Weak Coupling[1,2]

- $\hat{V}_{S \cup R} \approx 0$
- $U_S = \mathrm{Tr}_S\, [\hat{H}_S\, \hat{\rho}_S],\ \pi_S^0 = \dfrac{e^{-\beta H_S}}{Z_S}$
- U_S defined for classical and quantum systems

1. Rivas, Phys. Rev. Lett. 124, 160601 (2020)
2. Strasberg & Esposito, Phys. Rev. E 99, 012120 (2019)

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- U_S defined for classical and quantum systems

Strong Coupling[2-8]

- $\langle \hat{V}_{S\cup R} \rangle \sim \mathrm{Tr}_S\,[\hat{H}_S\ \hat{\rho}_S]$
- $U_S \neq \mathrm{Tr}_S\,[\hat{H}_S\ \hat{\rho}_S], \pi_S = \mathrm{Tr}_R\,[\pi_{S\cup R}] \neq \dfrac{e^{-\beta H_S}}{Z_S}$
- U_S defined for classical systems (multiple frameworks)

1. Rivas, Phys. Rev. Lett. 124, 160601 (2020)
2. Strasberg & Esposito, Phys. Rev. E 99, 012120 (2019)
3. Anto-Sztrikacs et. al. PRX Quantum 4, 020307 (2023)
4. Miller & Anders Phys. Rev. E 95, 062123 (2017)
5. Jarzynski, Phys. Rev. X 7, 011008 (2017)
6. Seifert, *Phys.* Rev. Lett. 116, 020601 (2016)
7. Miller. & Anders. Phys. Rev. E 95, 062123 (2017)
8. Work and heat exchanged during sudden quenches of strongly coupled quantum systems, Davoudi, (G.O) *et al.* arXiv:2502.19418 [quant-ph] (2025)

Thermodynamics of strongly-coupled systems

$$\pi_S = \mathrm{Tr}_R\left[\pi_{S \cup R}\right] = \frac{e^{-\beta H_S^*}}{Z_S^*}$$

$$H_S^* := -\frac{1}{\beta} \ln \frac{\mathrm{Tr}_R[e^{-\beta H_{S \cup R}}]}{\mathrm{Tr}_R[e^{-\beta H_R}]}$$ Hamiltonian of mean force

H_S^* captures the effects of $V_{S \cup R}$ on the equilibrium state of S

Miller, H.J.D., Hamiltonian of Mean Force for Strongly-Coupled Systems. In 978-3-319-99046-0 vol 195. Springer, Cham. (2018)
For the classical case, see Jarzynski, J. Stat. Mech. (2004) P09005

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H_S^* captures the effects of $V_{S\cup R}$ on the equilibrium state of S

Internal energy of the system: $U_S := \mathrm{Tr}_S\left[H_S^*\,\rho_S\right]$

Defined for equilibrium and non-equilibrium states of S.

Measured on system's degrees of freedom.

Thermodynamics of strongly-coupled systems

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For equilibrium states: $F_S = U_S - \beta^{-1}\mathcal{S}$

$$\mathcal{S} = -\,\mathrm{Tr}_S\,[\pi_S \ln \pi_S]$$

Bridge between QTD and QIS

Measure internal energy via the Hamiltonian of mean force: $H_S^* := -\frac{1}{\beta} \ln \frac{\mathrm{Tr}_R[e^{-\beta H_{S\cup R}}]}{\mathrm{Tr}_R[e^{-\beta H_R}]}$

Entanglement Hamiltonian: $H_{\mathrm{ent}} := -\ln \rho_S$

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- Characterizing topological order. See e.g. H. Li, F. D. M. Haldane, Phys. Rev. Lett. 101, 010504 (2008)
- As tool to investigate thermalization in LGTs. E.g. N. Mueller, T. V. Zache, R. Ott, Phys. Rev. Lett. 129, 011601 (2022)
- Used in quantum state tomography. E.g. M. Dalmonte , V. Eisler, M. Falconi, B. Vermersch, ANNALEN DER PHYSIK 2022, 534, 2200064.

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Sudden quench process

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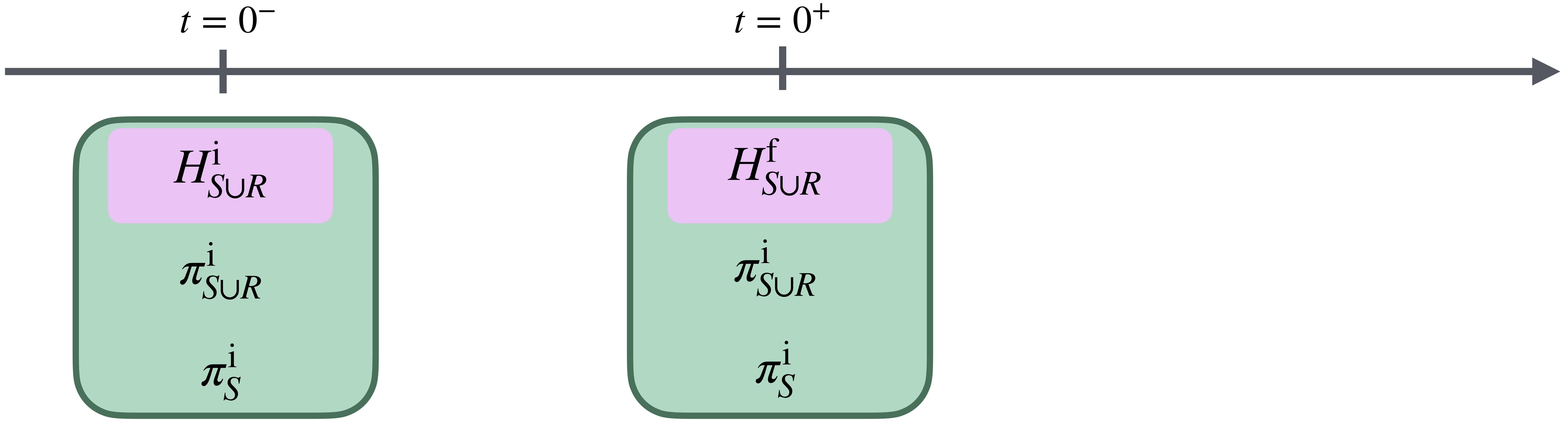

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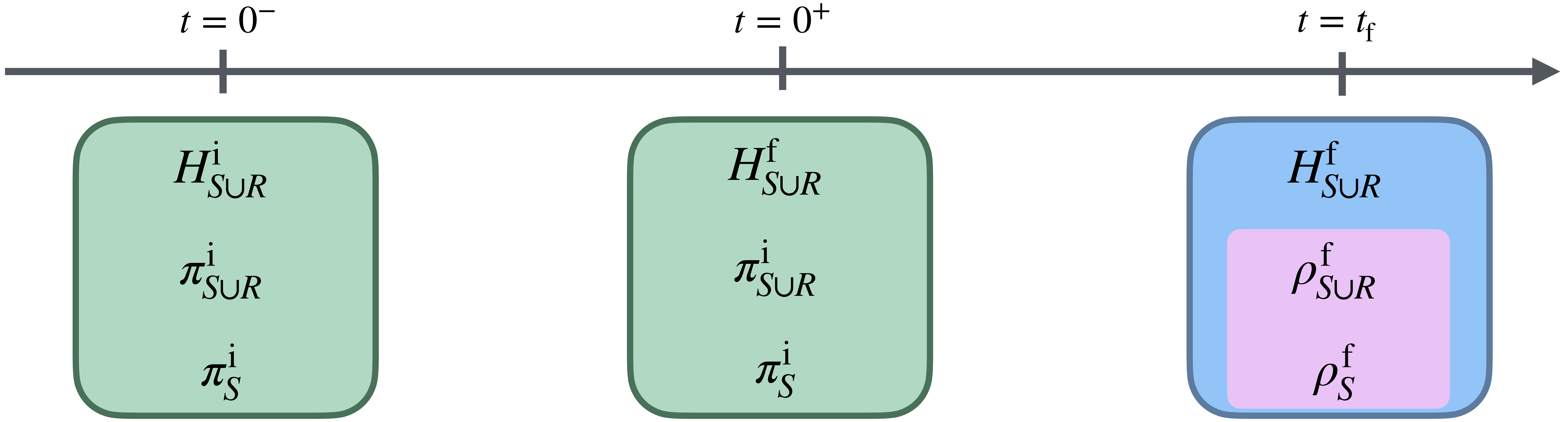

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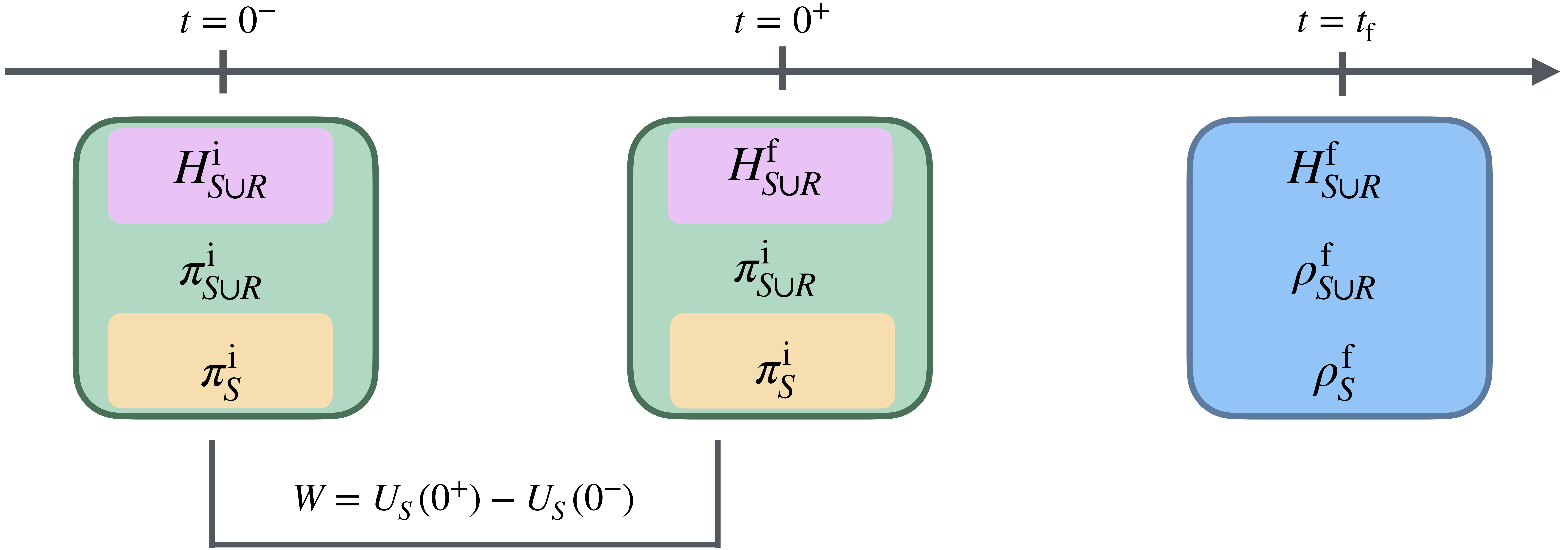

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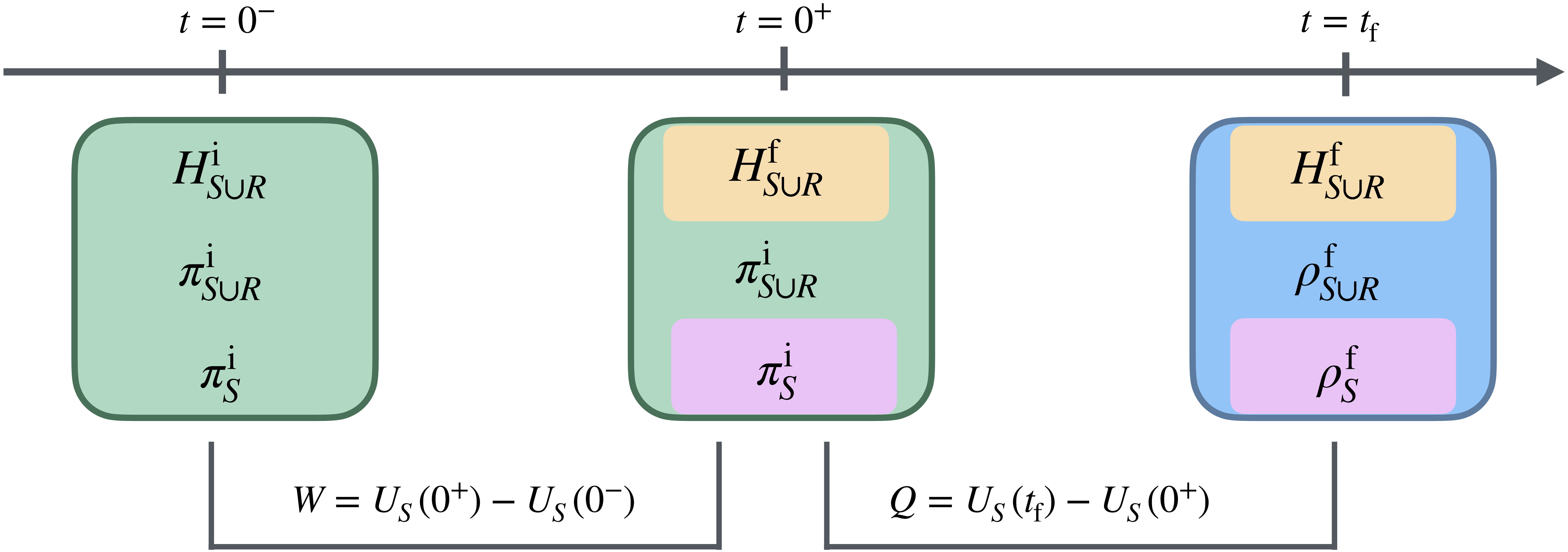
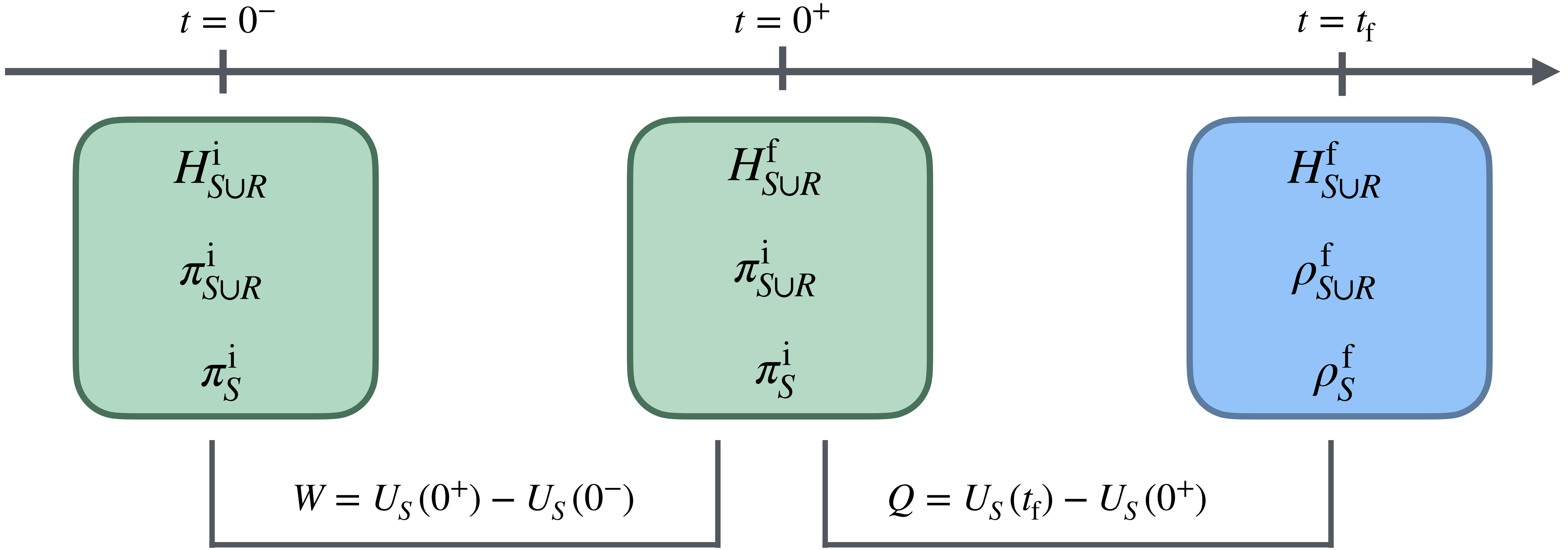


First law of Thermodynamics $\Delta U_S = W + Q$

Sudden quench process



Second law: $W \geq \Delta F_S$

If $\rho^\mathrm{f}_S = \pi^\mathrm{f}_S$, then $Q \leq \beta^{-1}\Delta\mathcal{S}$

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Experimentally accessible thermodynamic quantity

For a system equilibrium state π_S, H_{ent} and H_S^* are related: $H_S^* = \frac{1}{\beta} H_{\text{ent}} + F_S$.

$$W = \text{Tr}\left[\pi_S^{\text{i}}\left(H_S^{*\text{f}} - H_S^{*\text{i}}\right)\right]$$

$$W = \frac{1}{\beta}\text{Tr}\left[\pi_S^{\text{i}}\left(H_{\text{ent},S}^{\text{f}} - H_{\text{ent},S}^{\text{i}}\right)\right] + \Delta F_S$$

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Measure $W_{\text{diss}} = W - \Delta F_S = \frac{1}{\beta}\text{Tr}\left[\pi_S^{\text{i}}\left(H_{\text{ent},S}^{\text{f}} - H_{\text{ent},S}^{\text{i}}\right)\right]$ to verify the second law.

This connection between QTD and QIS allows us to potentially verify QTD on quantum simulators.

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Lattice gauge theories

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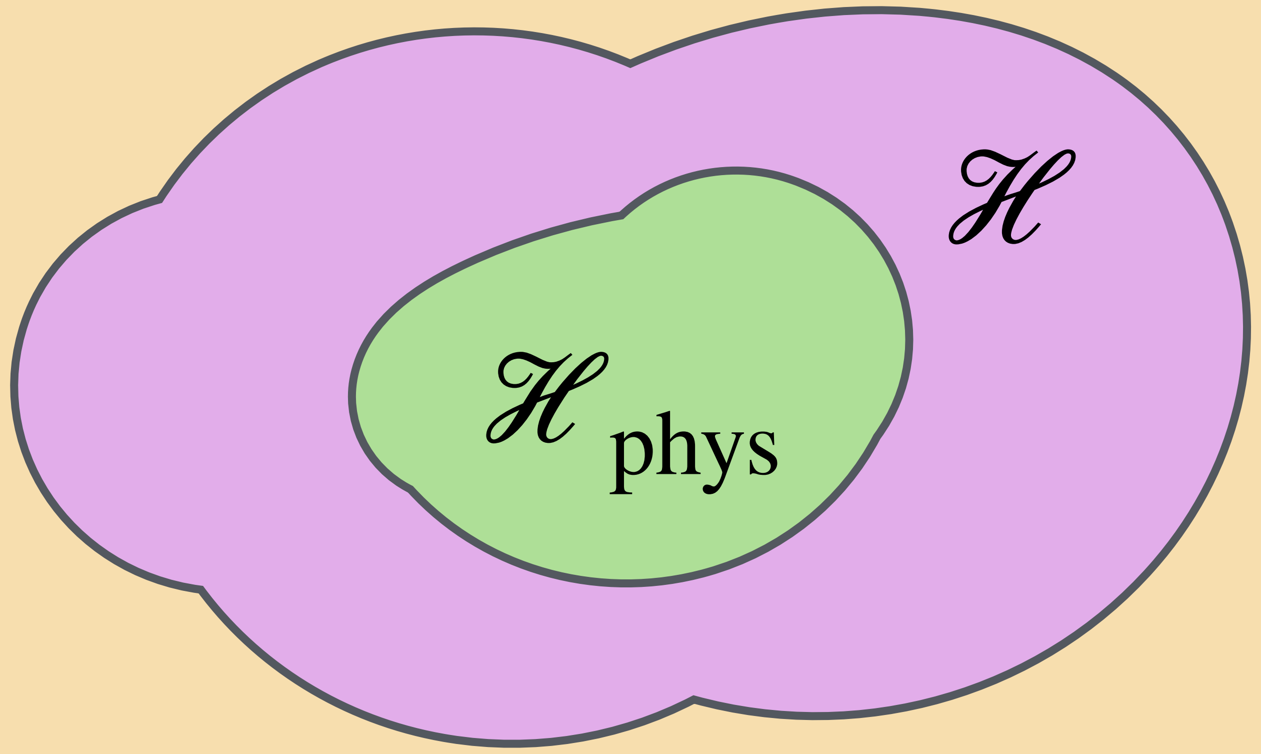
Hilbert space is split into two subspaces - the physical subspace ($\mathscr{H}_{\text{phys}}$) contains states that satisfy the Gauss law constraints.



Wilson, Phys. Rev. D 10, 2445 (1974).
Kogut & Susskind, Phys. Rev. D 11, 395 (1975).

Kogut, Rev. Mod. Phys. 51, 659 (1979).
Bauer, et al., Nat Rev. Phys. 5, 420-432 (2023).

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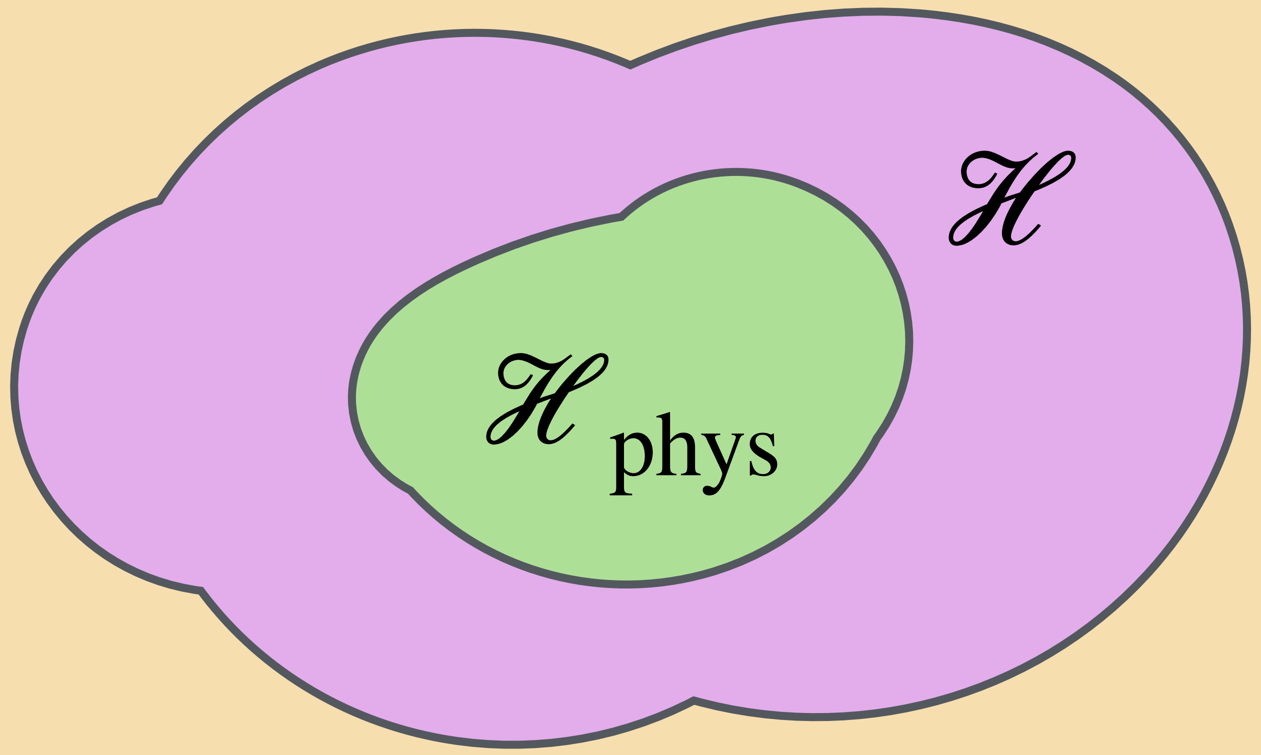


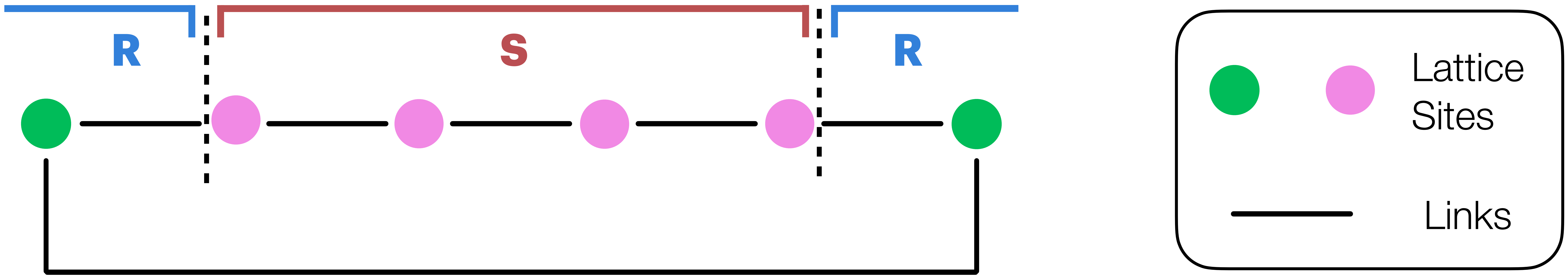
- Restrict dynamics by penalizing transitions to “unphysical states”. Leads to large $V_{S\cup R}$ in $H_S + H_R + V_{S\cup R}$.
- Large $V_{S\cup R} \implies$ Strong-coupling quantum thermodynamics
- Thermodynamic properties of LGTs have previously been computed in equilibrium and not under the framework of quantum thermodynamics. [Bazavov, *et. al.* arXiv:1904.09951]

Example: $\mathbb{Z}_2$ lattice gauge theory

$$H_{S\cup R} = -t\sum_{n=0}^{N-1}(\sigma_n^+\,\tilde{\sigma}_n^x\,\sigma_{n+1}^- + h.c.) - \epsilon\sum_{n=0}^{N-1}\tilde{\sigma}_n^z + m\sum_{n=0}^{N-1}(-1)^n\sigma_n^+\,\sigma_n^- - \mu\sum_{n=0}^{N-1}\sigma_n^+\,\sigma_n^-$$

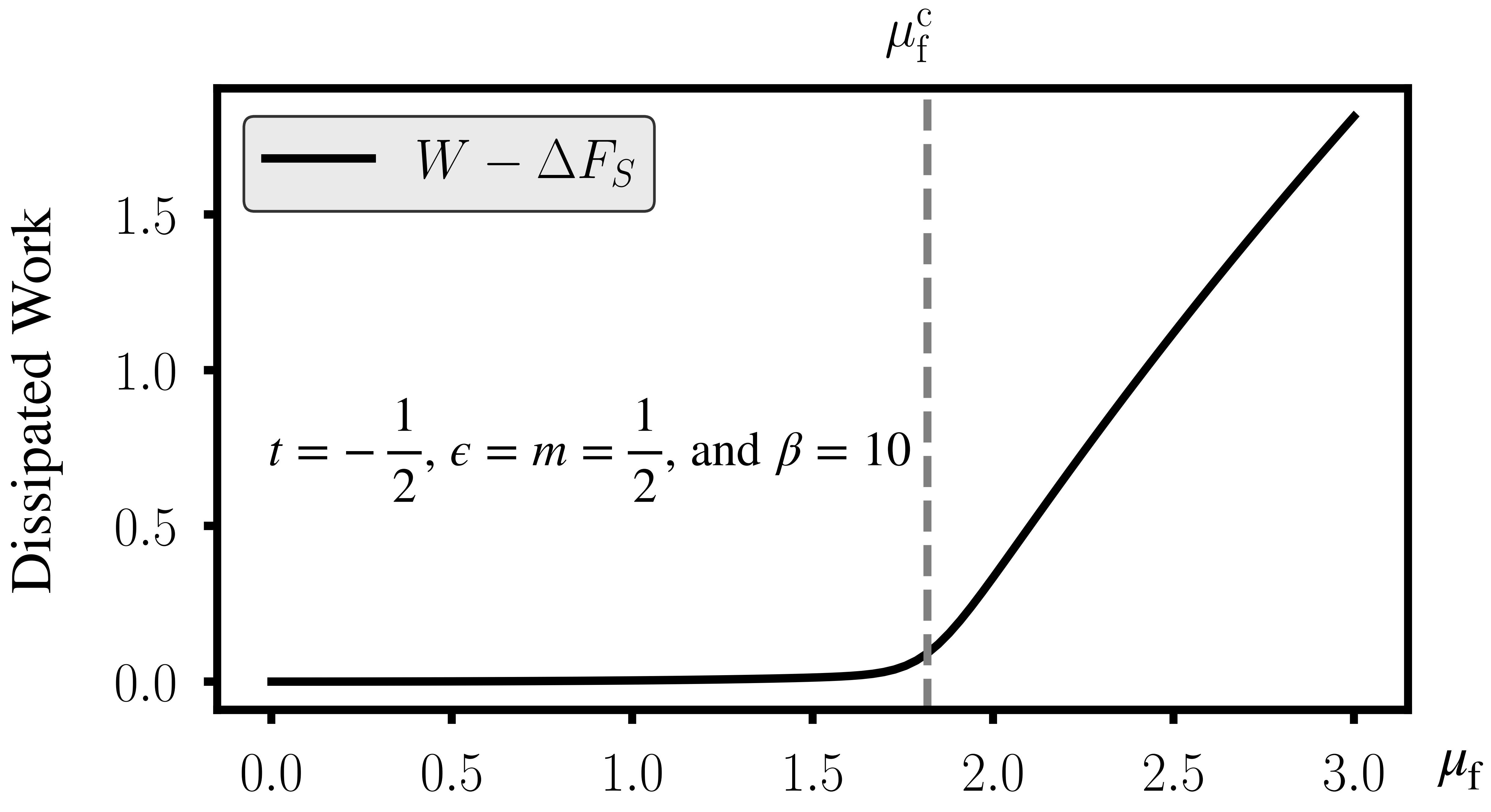
Matter hopping terms — Gauge field — Staggered mass — Chemical Potential

$$= H_S + H_R + V_{S\cup R}$$



Quench of $\mathbb{Z}_2$ lattice gauge theory: Dissipated work

$H_S(t) = H_{S,\,\text{hopping}} + H_{S,\,\text{g}} + H_{S,\,\text{m}} + H_{S,\,\mu}(t)$ $\quad \mu_i = 0 \to \mu_f$ within H_S at $t = 0$



$$W - \Delta F_S \geq 0 \implies W \geq \Delta F_S$$

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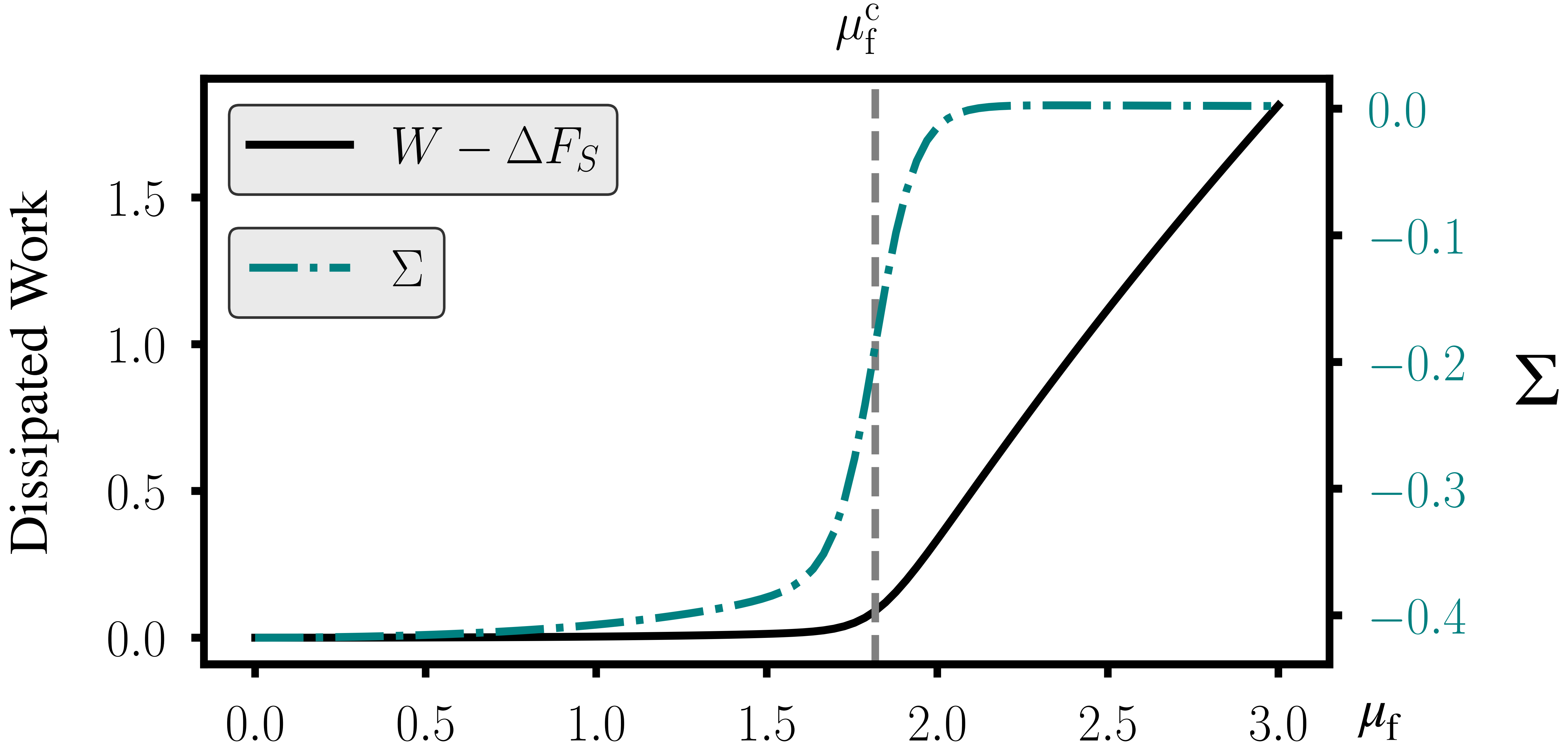
Phase transition in $\mathbb{Z}_2$ lattice gauge theory

Chiral condensate as an order parameter: $\Sigma = \langle \bar{\Psi}\Psi \rangle = \frac{1}{N_S} \sum_{n=0}^{N_S-1} (-1)^n \langle \sigma_j^+ \sigma_j^- \rangle$



Qualitative changes in the behavior of dissipated work around the chiral phase transition.

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Summary and outlook

- We defined thermodynamics quantities for a strongly-coupled open quantum system.
- For quench processes, work and heat analytically satisfy the first two laws of thermodynamics.
- A bridge between quantum information and quantum thermodynamics, potentially allows us to verify our framework on a quantum simulator.
- Lattice gauge theories can be cast in the language of strong-coupling quantum thermodynamics.
- We discovered a qualitative relationship between thermodynamic quantities and phase transitions for a $(1+1)$D $\mathbb{Z}_2$ LGT coupled to spin-$\frac{1}{2}$ hardcore bosons.

- Experimentally measure work and heat for LGTs on a quantum simulator.
- Extend the framework to other non-quench, non-equilibrium process in high-energy physics.

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Appendix

Weak-coupling vs strong-coupling thermodynamics

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Weak Coupling[1,2]

- $\hat{V}_{S\cup R} \approx 0$
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1. Rivas, Phys. Rev. Lett. 124, 160601 (2020)
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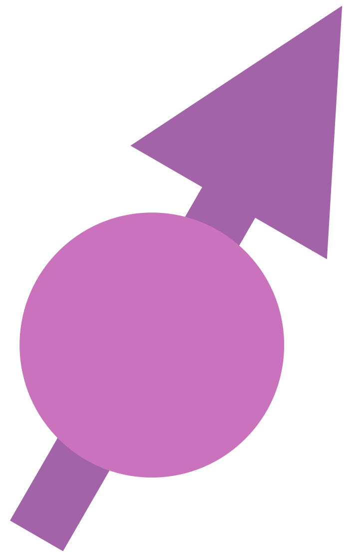
Gauss's laws in LGTs

$$V_{S\cup R} = k\,(I_{S\cup R} - \sigma_S^z \sigma_R^z) + \chi\,\sigma_S^x \sigma_R^x$$



$$H_S = \frac{\epsilon}{2}\sigma_S^z$$

$$H_{S\cup R} = H_S + H_R + V_{S\cup R}$$

$$H_R = \frac{\alpha}{2}\sigma_R^z$$

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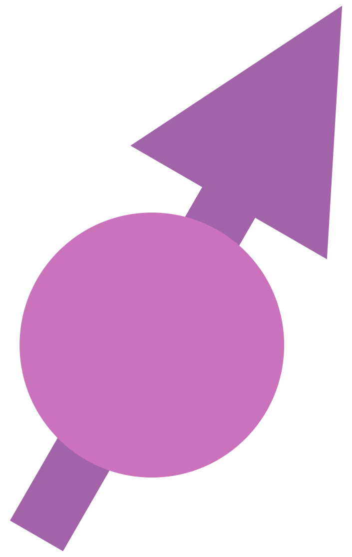

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Gauss's Law $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle \xrightarrow{\lim k \to \infty} |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$

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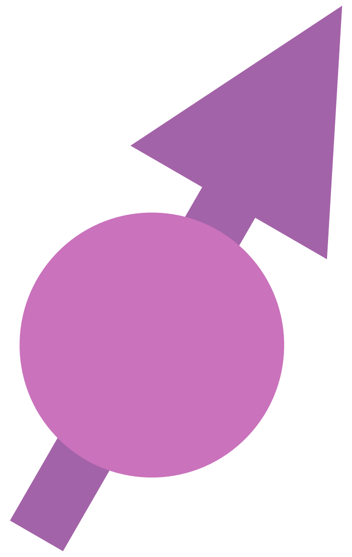

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Large $V_{S\cup R} \Longrightarrow \langle V_{S\cup R}\rangle \sim \langle H_S\rangle$, interactions are non-negligible.

Example: $\mathbb{Z}_2$ lattice gauge theory

$$H_{S\cup R} = -t\sum_{n=0}^{N-1}(\sigma_n^+\,\tilde{\sigma}_n^x\,\sigma_{n+1}^- + h.c.) - \epsilon\sum_{n=0}^{N-1}\tilde{\sigma}_n^z + m\sum_{n=0}^{N-1}(-1)^n\sigma_n^+\,\sigma_n^- - \mu\sum_{n=0}^{N-1}\sigma_n^+\,\sigma_n^-$$

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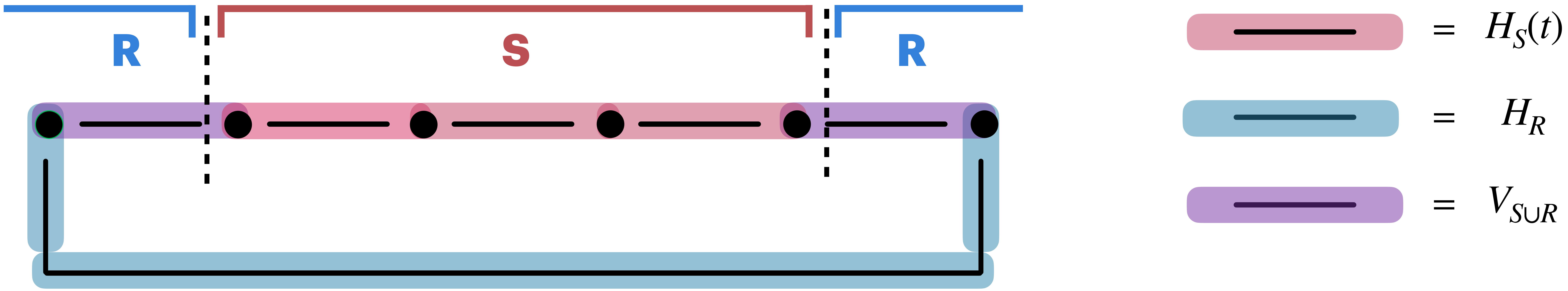

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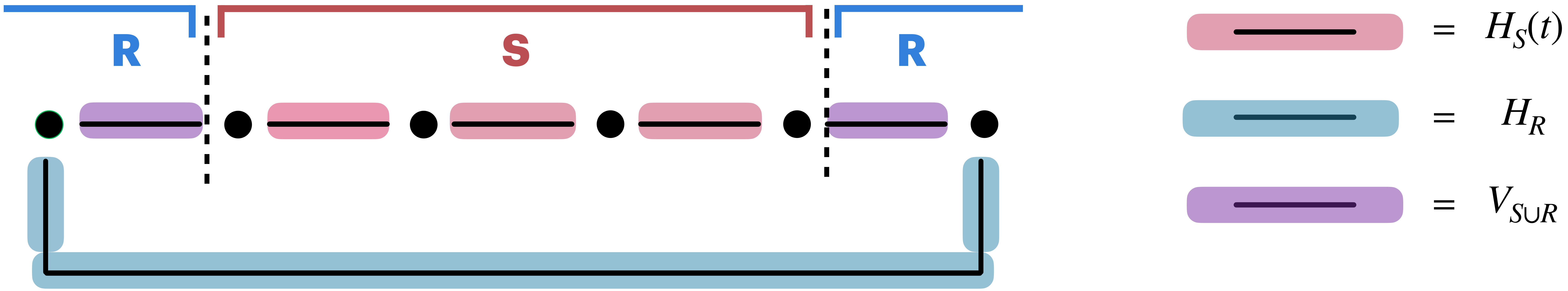
Gauge field

$$= H_S + H_R + V_{S\cup R}$$



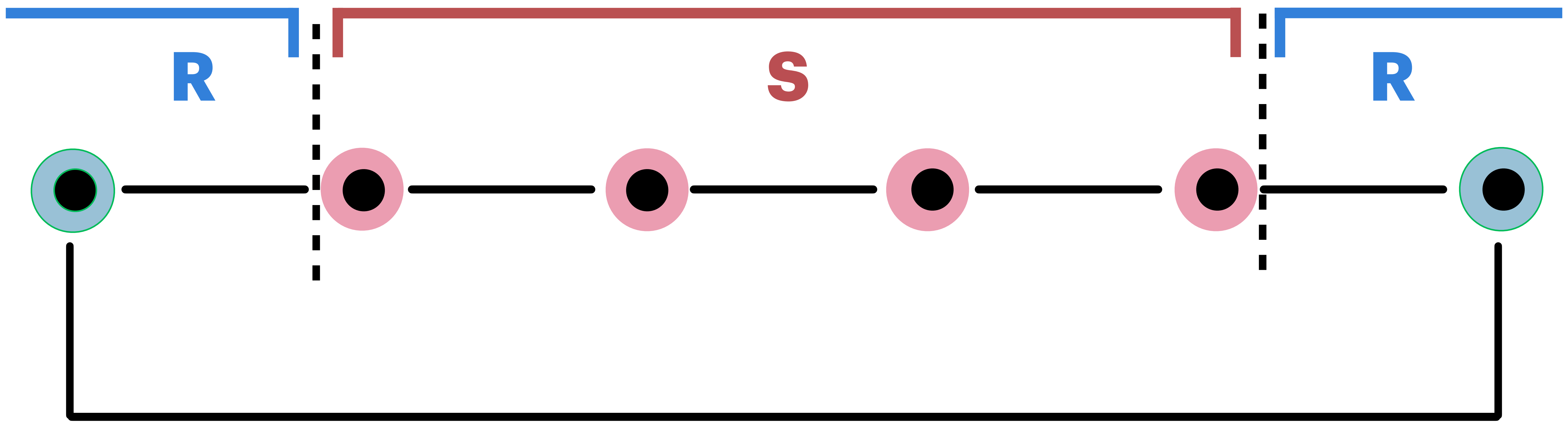
Example: $\mathbb{Z}_2$ lattice gauge theory

$$H_{S\cup R} = -t\sum_{n=0}^{N-1}\left(\sigma_n^+\,\tilde{\sigma}_n^x\,\sigma_{n+1}^- + \mathrm{h.c.}\right) - \epsilon\sum_{n=0}^{N-1}\tilde{\sigma}_n^z + m\sum_{n=0}^{N-1}(-1)^n\sigma_n^+\,\sigma_n^- - \mu\sum_{n=0}^{N-1}\sigma_n^+\,\sigma_n^-$$

Staggered mass

Chemical Potential

$$= H_S + H_R + V_{S\cup R}$$

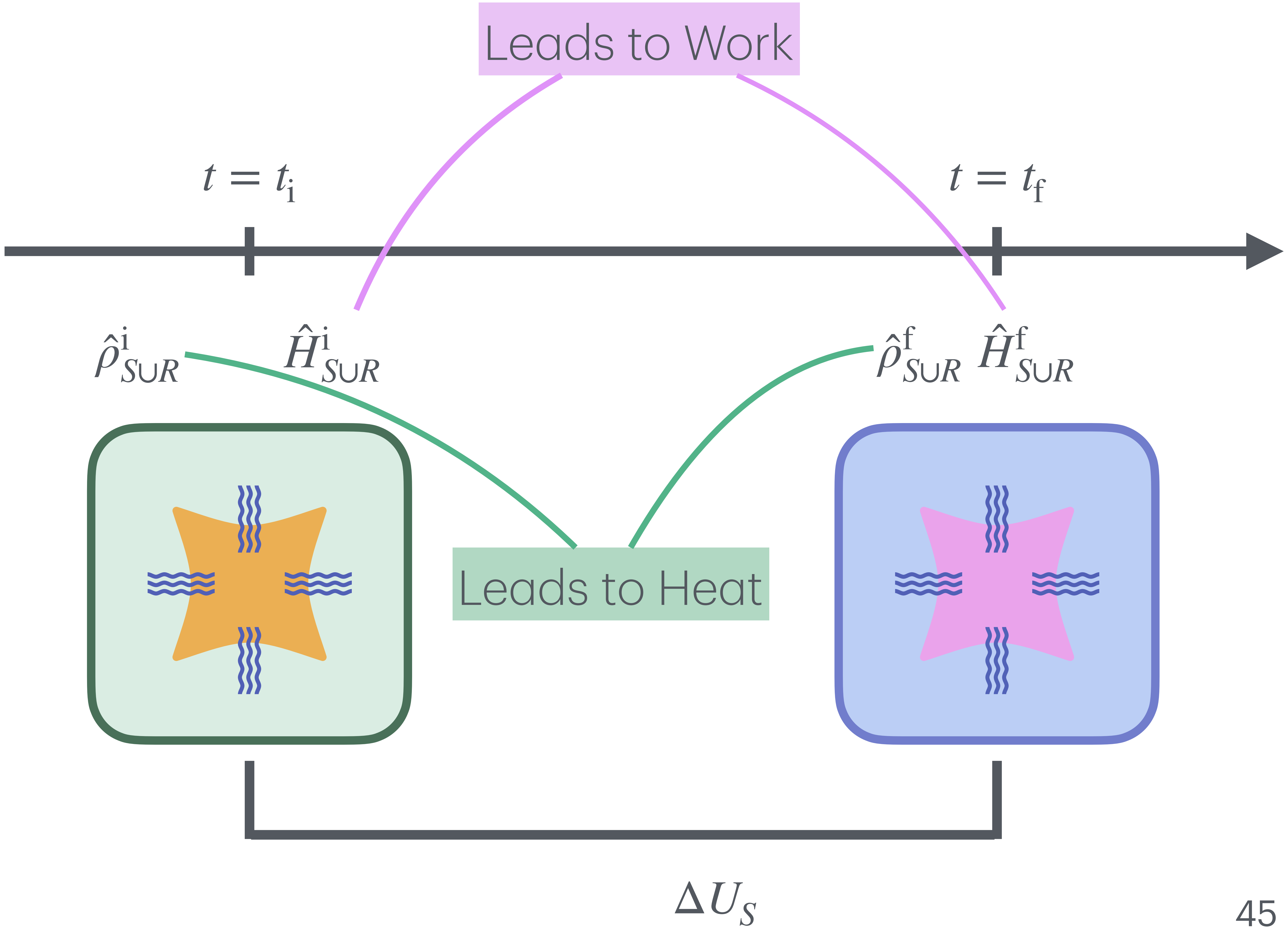


General thermodynamic process

Work: Change in internal energy by externally varying a parameter of $\hat{H}_{S \cup R}$

Heat: Change in internal energy associated with the change in S's state

First law: $\Delta U_S = W + Q$



Proof of Second Law

$$
\begin{aligned}
\beta(W - \Delta F_S) &= \beta \mathrm{Tr}_S\,[\hat{\pi}_S^{\mathrm{i}}\,(\hat{H}_S^{*\mathrm{f}} - \hat{H}_S^{*\mathrm{i}})] - (F_S^{\mathrm{f}} - F_S^{\mathrm{i}}) \\
&= \mathrm{Tr}_S\,[(\beta\,(\hat{H}_S^{*\mathrm{f}} - F_S^{\mathrm{f}}) - \beta\,(\hat{H}_S^{*\mathrm{i}} - F_S^{\mathrm{i}}))\,\hat{\pi}_S^{\mathrm{i}}] \\
&= \mathrm{Tr}_S\,[(\ln \hat{\pi}_S^{\mathrm{i}} - \ln \hat{\pi}_S^{\mathrm{f}})\,\hat{\pi}_S^{\mathrm{i}}] \\
&\geq 0
\end{aligned}
$$

Second Law in terms of Heat and Entropy

$$S = \beta(U_S - F_S)$$

$$\Delta S = \beta(\Delta U_S - \Delta F_S)$$

$$\Delta S = \beta(\Delta U_S - \Delta F_S)$$

$$\beta^{-1}\Delta S = \Delta U_S - \Delta F_S$$

$$\beta^{-1}\Delta S = W + Q - \Delta F_S$$

$$\beta^{-1}\Delta S - Q = W - \Delta F_S$$

$$\beta^{-1}\Delta S - Q \geq 0$$

$$Q \leq \beta^{-1}\Delta S$$

Different Internal Energy Definitions

U_{tot}

$$U_{\text{tot}} = U_{S\cup R} - U_R^0$$

Measured on $S \cup R$

Converges to $\text{Tr}_S\,[\hat{H}_S\,\hat{\pi}_S^0]$ when $S \cup R$ is in a Gibbs state and $\hat{V}_{S\cup R} \sim 0$

U_{H*}

$$U_{H*} = \text{Tr}_S\,[\hat{H}_S^*\;\hat{\rho}_S]$$

Measured on S

Converges to $\text{Tr}_S\,[\hat{H}_S\,\rho_S]$ when $\hat{V}_{S\cup R} \sim 0$

U_{E*}

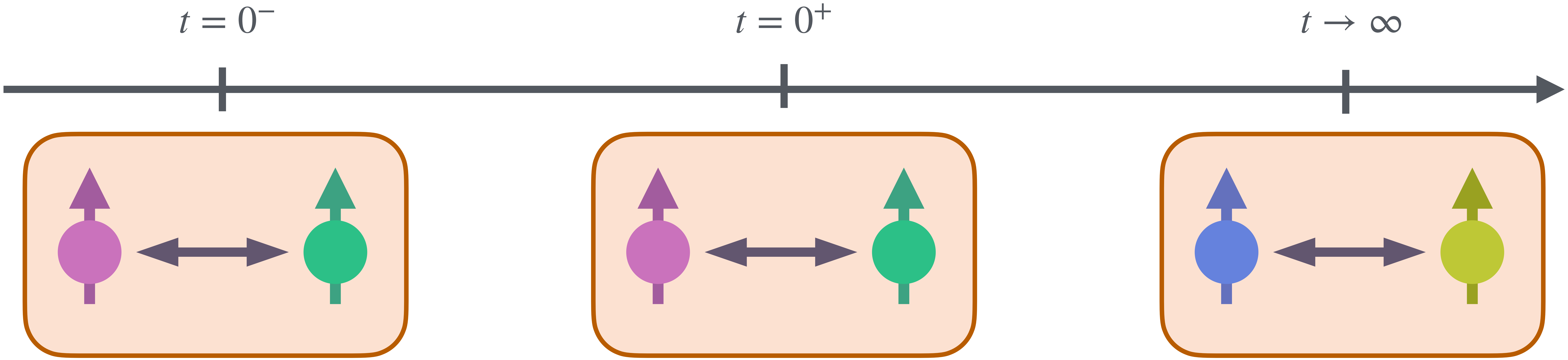
$$U_{E*} = \text{Tr}_S\,[\hat{E}_S^*\;\hat{\rho}_S]$$

Measured on S

Converges to $\text{Tr}_S\,[\hat{H}_S\,\rho_S]$ when $\hat{V}_{S\cup R} \sim 0$

Jarzynski, C. Stochastic and Macroscopic Thermodynamics of Strongly Coupled Systems. *Phys. Rev. X* **7**, 011008 (2017).
Seifert, U. First and Second Law of Thermodynamics at Strong Coupling. *Phys. Rev. Lett.* **116**, 020601 (2016).
Miller, H. J. D. & Anders, J. Entropy production and time asymmetry in the presence of strong interactions. *Phys. Rev. E* **95**, 062123 (2017).

Quench Process 1: *System Quench*



$\hat{H}^{\mathrm{i}}_{S\cup R} = \frac{\epsilon_{\mathrm{i}}}{2}\sigma^z_S + \frac{\alpha}{2}\sigma^z_R + \gamma\, \sigma^z_S\sigma^z_R + \chi\, \sigma^x_S\sigma^x_R$

$\hat{H}^{\mathrm{f}}_{S\cup R} = \frac{\epsilon_{\mathrm{f}}}{2}\sigma^z_S + \frac{\alpha}{2}\sigma^z_R + \gamma\, \sigma^z_S\sigma^z_R + \chi\, \sigma^x_S\sigma^x_R$

$\hat{H}^{\mathrm{f}}_{S\cup R} = \frac{\epsilon_{\mathrm{f}}}{2}\sigma^z_S + \frac{\alpha}{2}\sigma^z_R + \gamma\, \sigma^z_S\sigma^z_R + \chi\, \sigma^x_S\sigma^x_R$

State of S: $\hat{\pi}^{\mathrm{i}}_S$

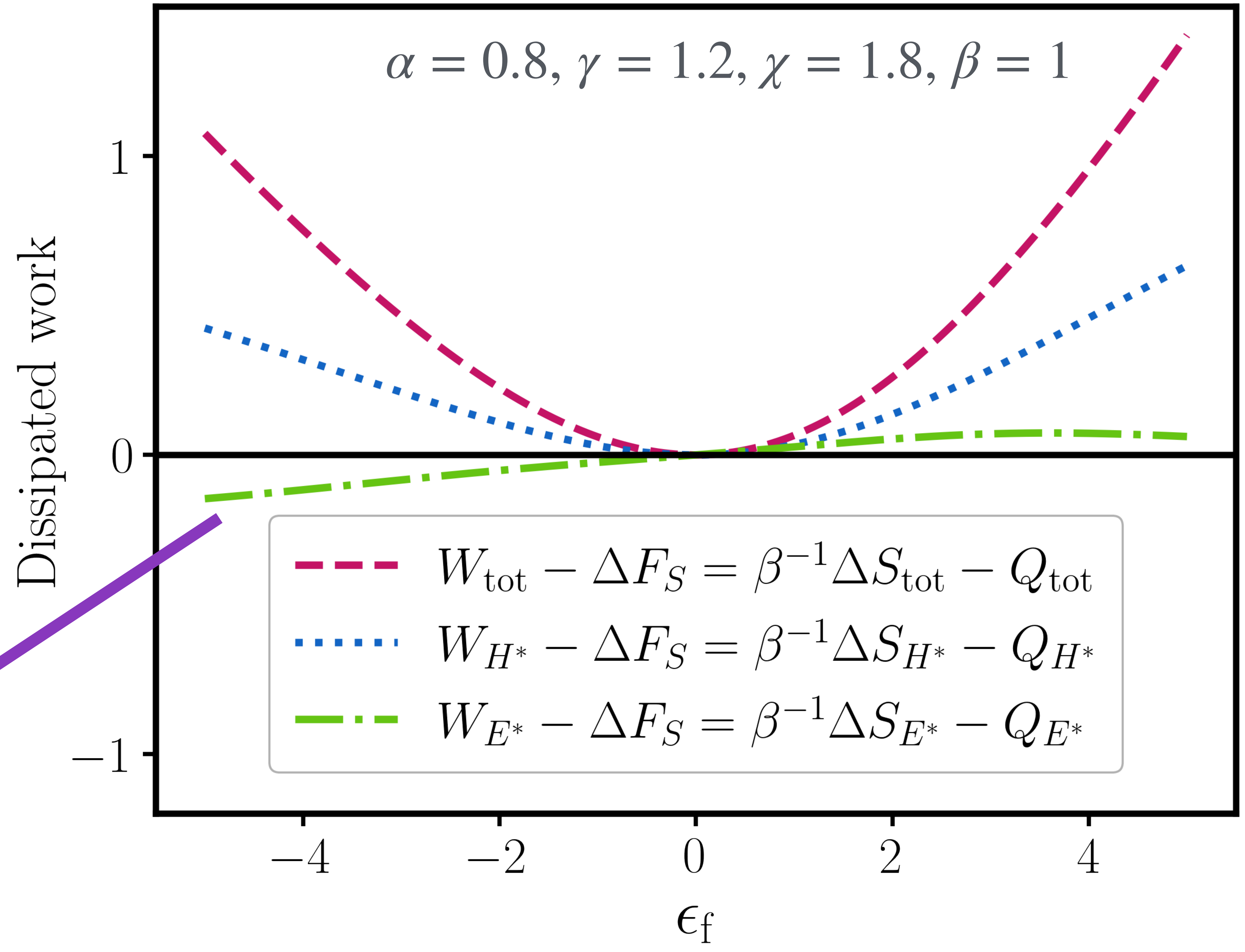
State of S: $\hat{\pi}^{\mathrm{i}}_S$

State of S: $\hat{\pi}^{\mathrm{f}}_S$

$W = U_S(0^+) - U_S(0^-)$

$Q = U_S(\infty) - U_S(0^+)$

Quench Process 1: *System Quench*



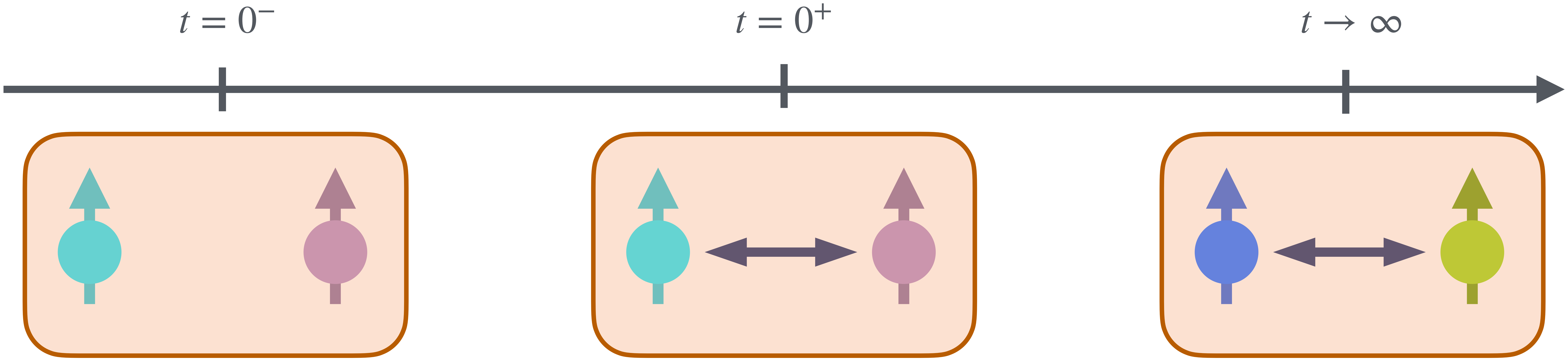
$$\hat{H}_{S\cup R} = \frac{\epsilon}{2}\sigma_S^z + \frac{\alpha}{2}\sigma_R^z + \gamma\, \sigma_S^z\sigma_R^z + \chi\, \sigma_S^x\sigma_R^x$$

Second law violation

$W_{\mathrm{tot}} \geq \Delta F_S$, $W_{H*} \geq \Delta F_S$, and $W_{E*} \ngeq \Delta F_S$

$Q_{\mathrm{tot}} \leq \beta^{-1}\Delta S_{\mathrm{tot}}$, $Q_{H*} \leq \beta^{-1}\Delta S_{H*}$, and, $Q_{E*} \nleq \Delta S_{E*}$

Quench Process 2: *Interaction Quench*



$$\hat{H}^{\mathrm{i}}_{SUR} = \frac{\epsilon}{2}\sigma^z_S + \frac{\alpha}{2}\sigma^z_R$$

$$\hat{H}^{\mathrm{f}}_{SUR} = \frac{\epsilon}{2}\sigma^z_S + \frac{\alpha}{2}\sigma^z_R + \gamma\, \sigma^z_S\sigma^z_R + \chi\, \sigma^x_S\sigma^x_R$$

$$\hat{H}^{\mathrm{f}}_{SUR} = \frac{\epsilon}{2}\sigma^z_S + \frac{\alpha}{2}\sigma^z_R + \gamma\, \sigma^z_S\sigma^z_R + \chi\, \sigma^x_S\sigma^x_R$$

State of S: $\hat{\pi}^{\mathrm{i}}_S$

State of S: $\hat{\pi}^{\mathrm{i}}_S$

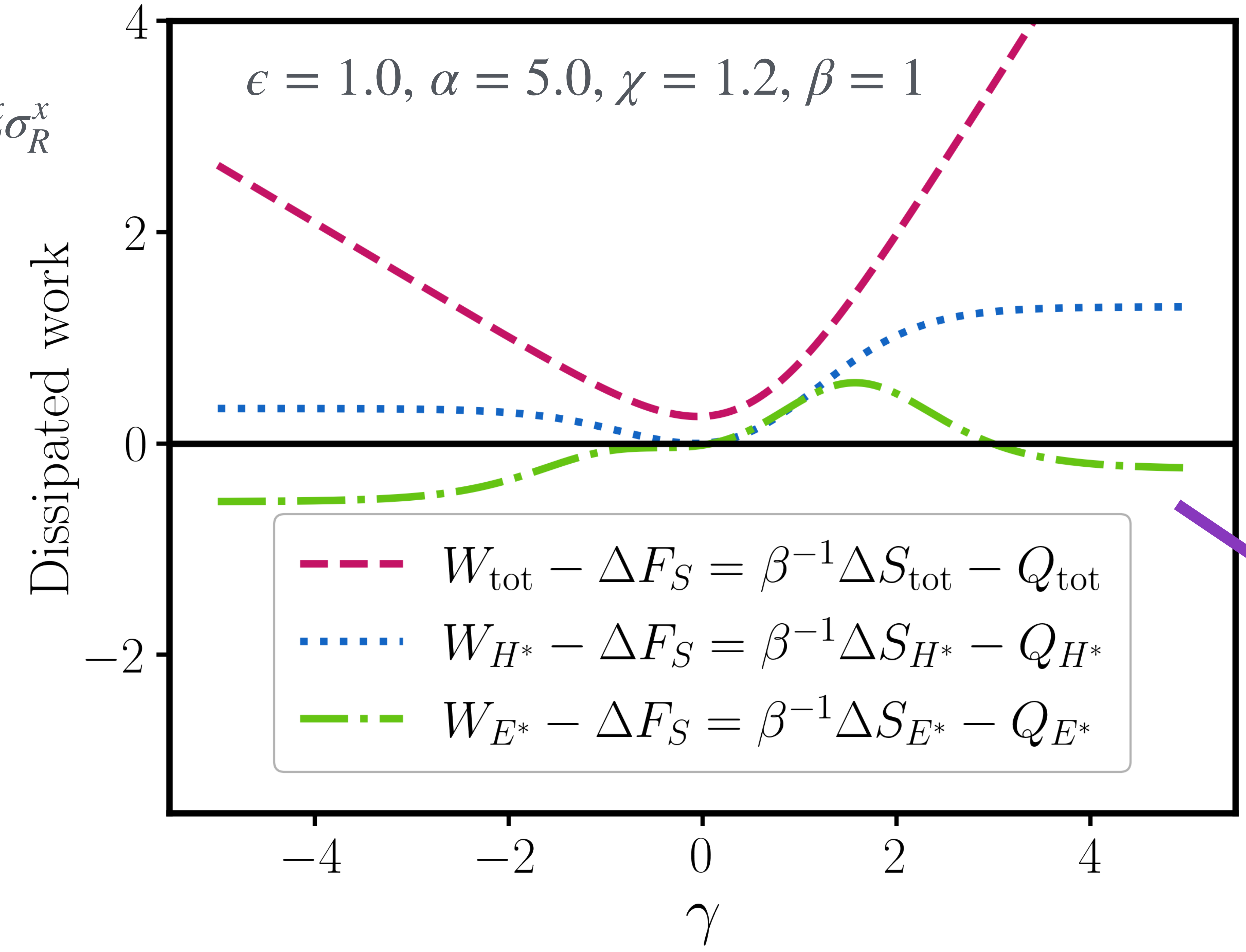
State of S: $\hat{\pi}^{\mathrm{f}}_S$

$$W = U_S(0^+) - U_S(0^-)$$

$$Q = U_S(\infty) - U_S(0^+)$$

Quench Process 2: *Interaction Quench*

$$\hat{H}_{S\cup R} = \frac{\epsilon}{2}\sigma_S^z + \frac{\alpha}{2}\sigma_R^z + \gamma\, \sigma_S^z\sigma_R^z + \chi\, \sigma_S^x\sigma_R^x$$



Second law violation

$W_{\text{tot}} \geq \Delta F_S$, $W_{H^*} \geq \Delta F_S$, and $W_{E^*} \ngeq \Delta F_S$

$Q_{\text{tot}} \leq \beta^{-1}\Delta S_{\text{tot}}$, $Q_{H^*} \leq \beta^{-1}\Delta S_{H^*}$, and, $Q_{E^*} \nleq \Delta S_{E^*}$

Proof of EH $\simeq$ HMF

$$
\begin{aligned}
H_S^* &= -\frac{1}{\beta} \ln \frac{\mathrm{Tr}_R[e^{-\beta H_{S\cup R}}]}{\mathrm{Tr}_R[e^{-\beta H_R}]} \\
&= -\frac{1}{\beta} \ln \pi_S \frac{Z_{S\cup R}}{Z_R} \\
&= \frac{1}{\beta} H_S^{\mathrm{ent}} + F_S
\end{aligned}
$$

$$
\begin{aligned}
W_{\mathrm{diss}} &= W - \Delta F_S \\
&= \frac{1}{\beta} \mathrm{Tr}\left[\rho_S^{\mathrm{i}} \left(H_{\mathrm{ent},S}^{\mathrm{f}} - H_{\mathrm{ent},S}^{\mathrm{i}} \right)\right]
\end{aligned}
$$

Quench of $\mathbb{Z}_2$ Lattice Gauge Theory: Work and Heat

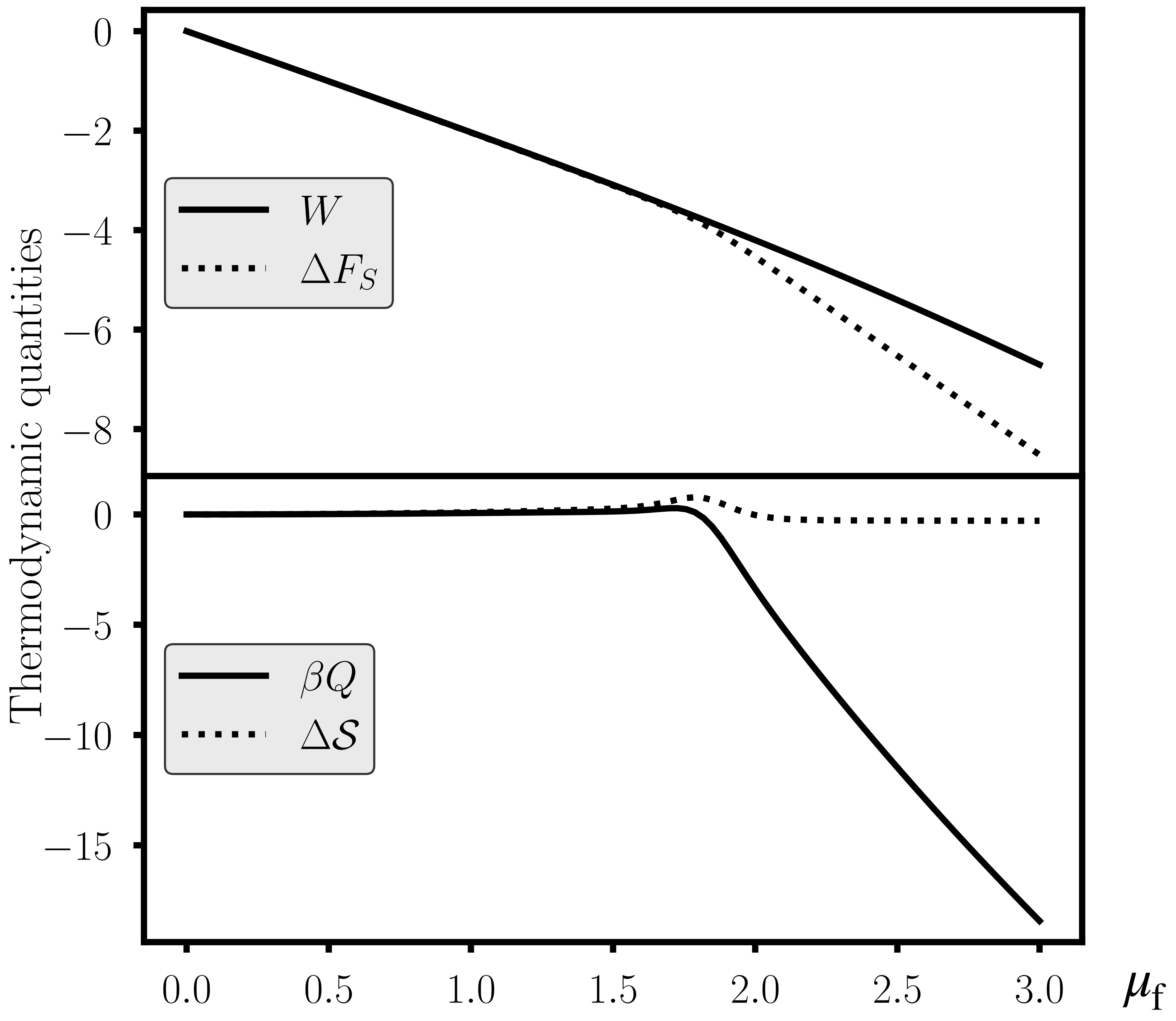
Hamiltonian of the system:

$$H_S(t) = H_{S,\,\text{hopping}} + H_{S,\,\text{g}} + H_{S,\,\text{m}} + H_{S,\,\mu}(t)$$

Chemical Potential quench: $\mu_i = 0 \rightarrow \mu_f$ within H_S at $t = 0$

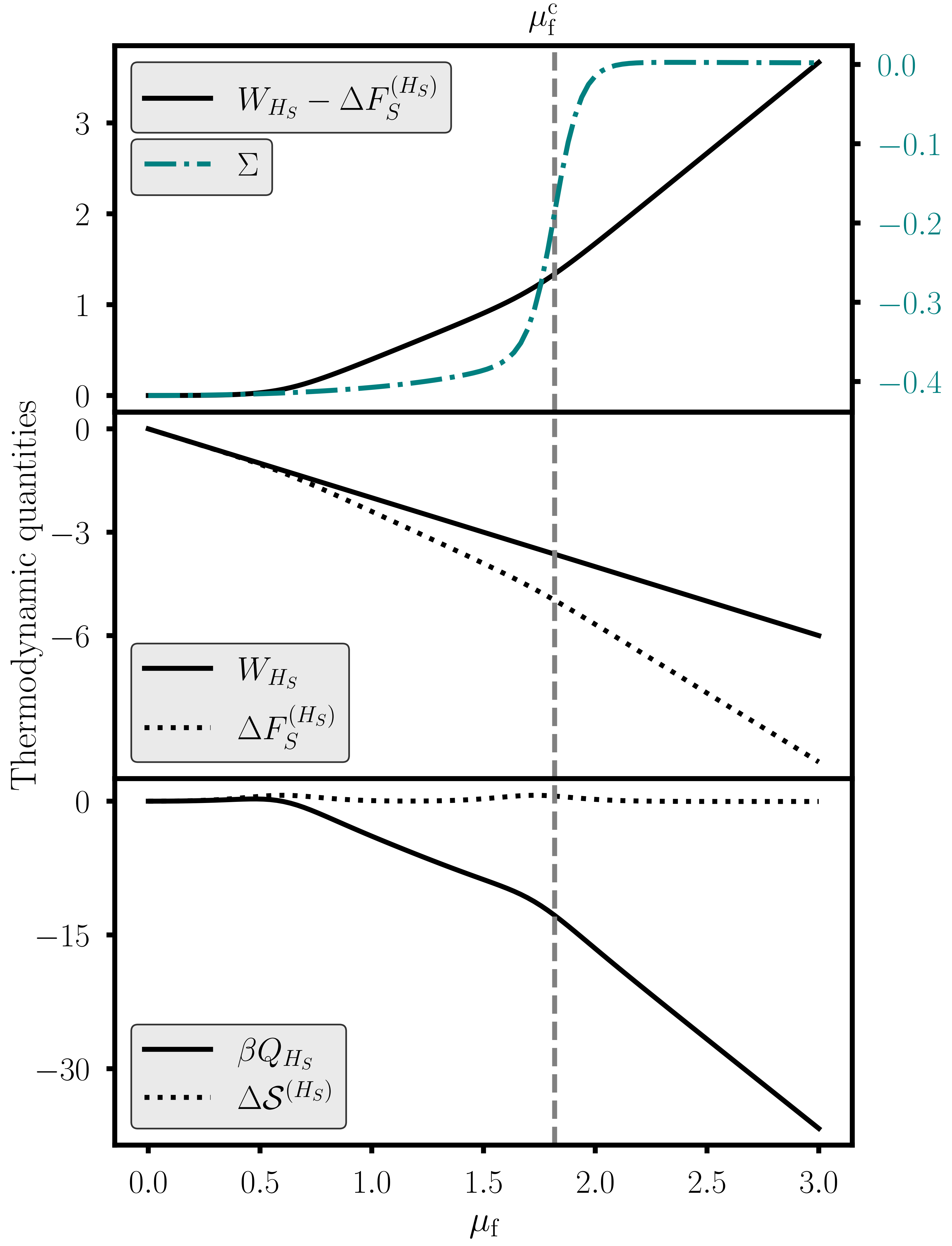
Calculate thermodynamic quantities: $W, Q, \Delta F_S$, and, $\Delta \mathcal{S}$

$t = -\frac{1}{2}, \epsilon = m = \frac{1}{2}$, and $\beta = 10$



Quantum thermodynamics of nonequilibrium processes in lattice gauge theories, Davoudi, (G.O) *et al.* Phys. Rev. Lett. 133, 250402 (2024)

What happens if we use weak-coupling thermodynamics?



Testing Numerical Stability

Real exponentials of Hamiltonians can lead to matrices with very large or very small absolute values of eigenvalues at sufficiently large β.

Smooth functions of β indicate lack of instabilities.