Real-time Estimators for Scattering Observables (RESOs)


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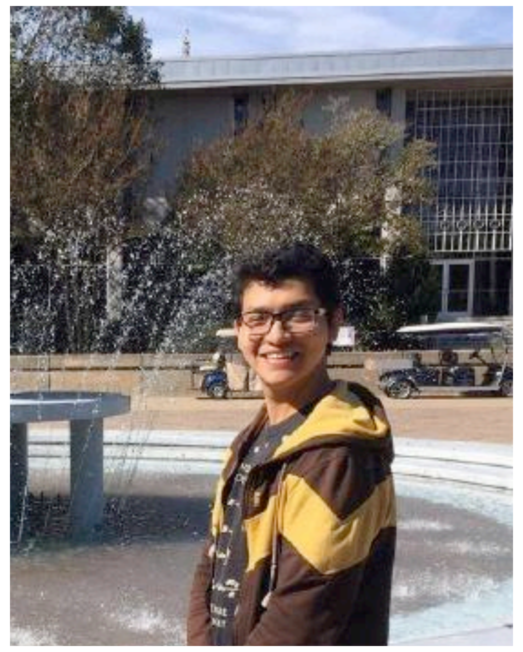
2506.06511


Marco Carrillo

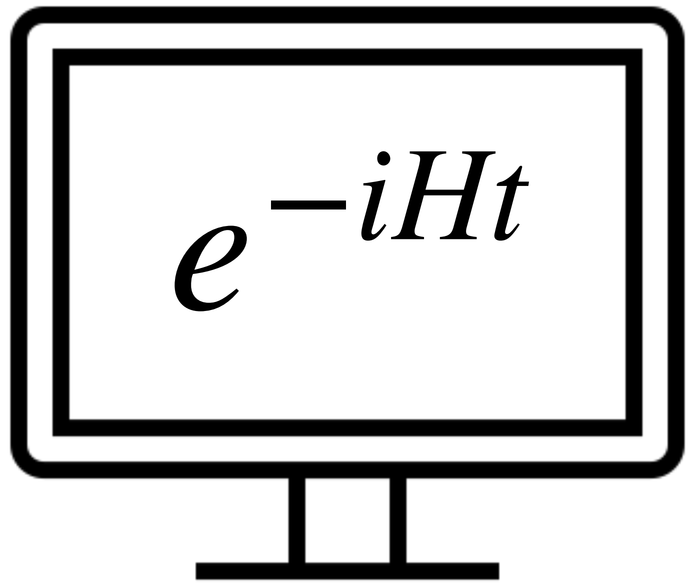

Rana Urek


Anthony Ciavarella

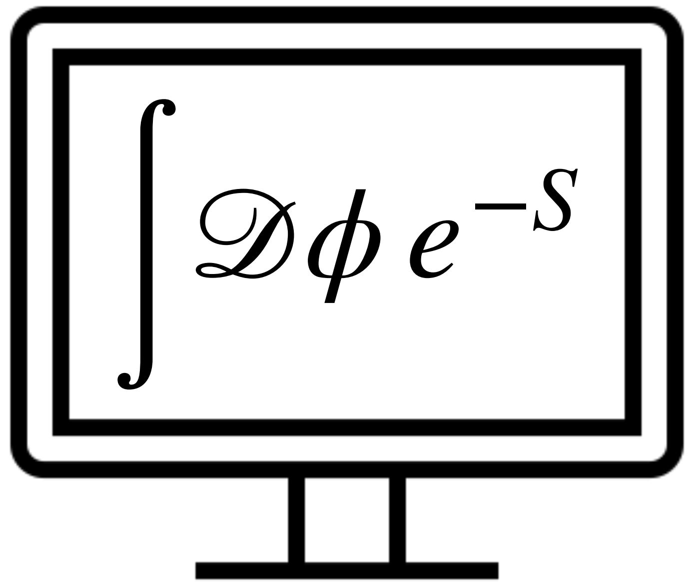

Raúl Briceño

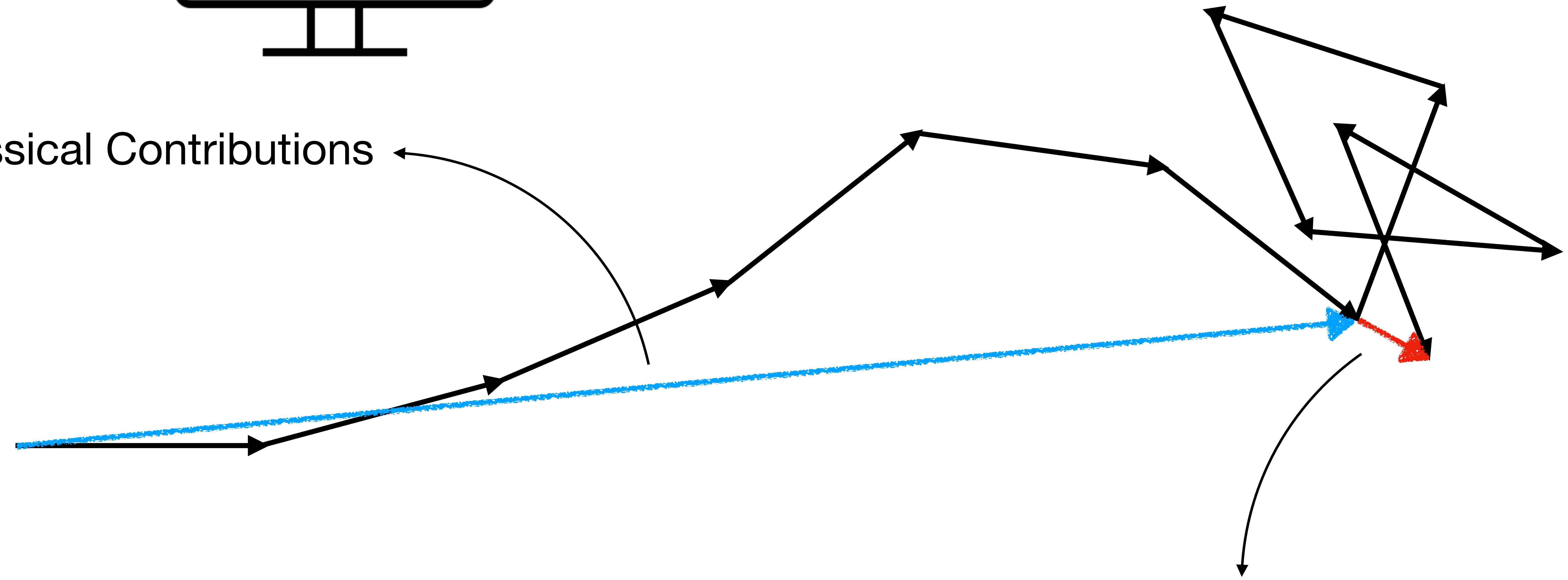
Sign problem

In Minkowski signature: $\int \mathscr{D}\phi \, e^{iI}$



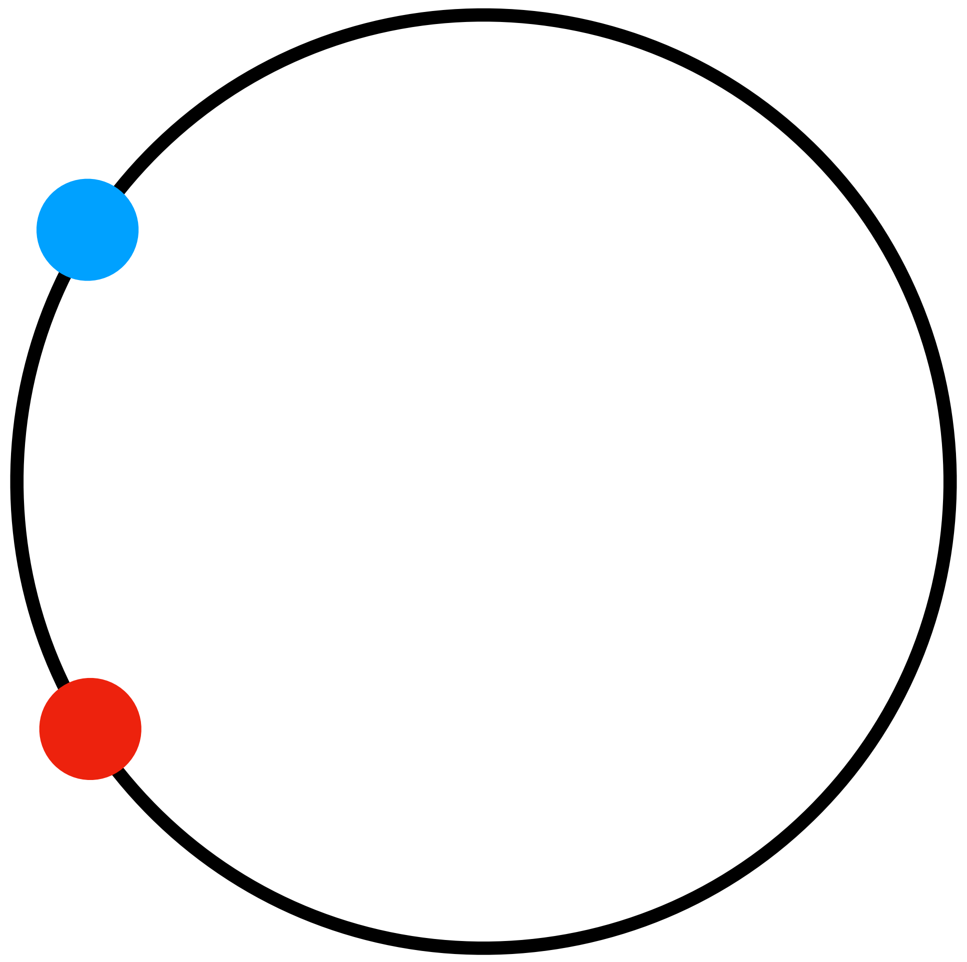
Euclidean





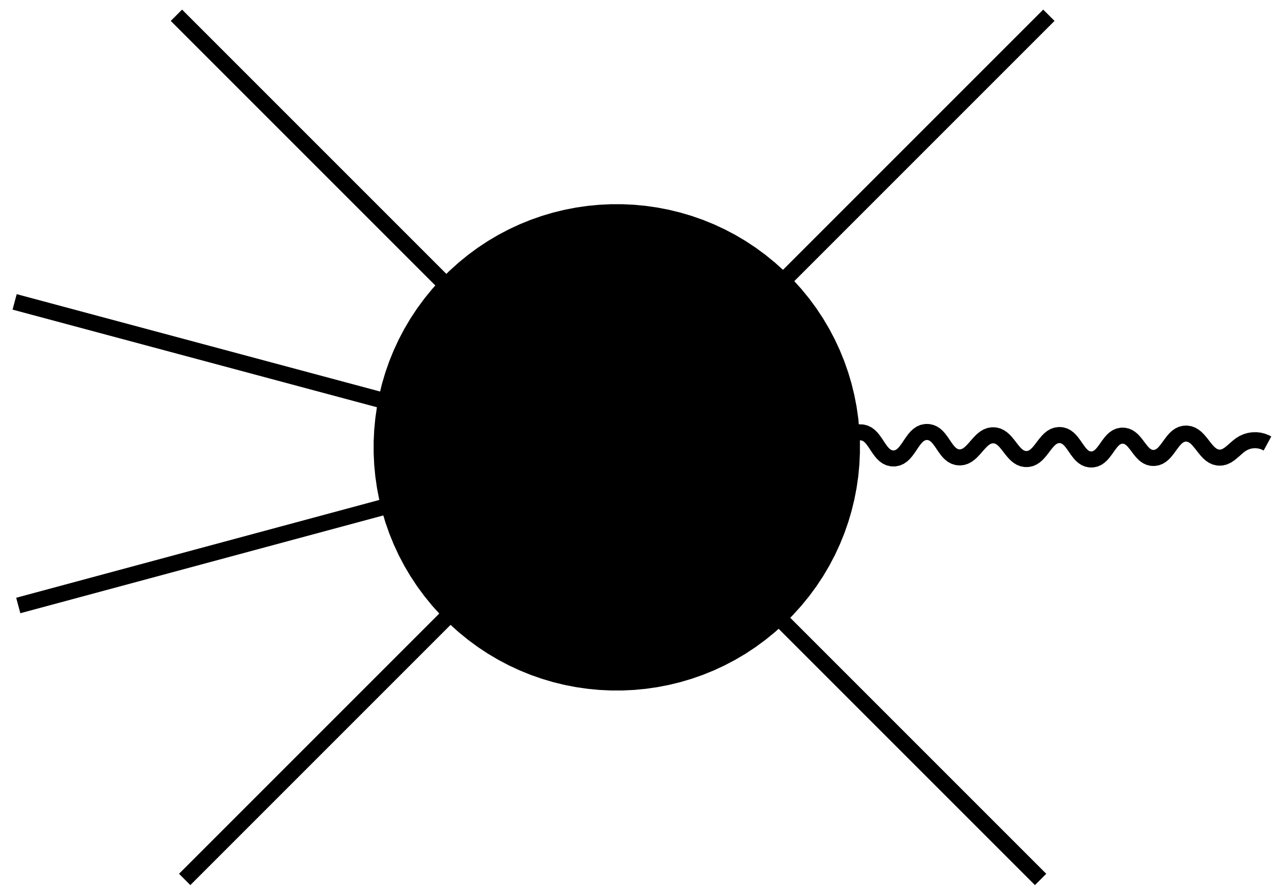
Finite volume

On a finite volume there are no asymptotic states!



Goal

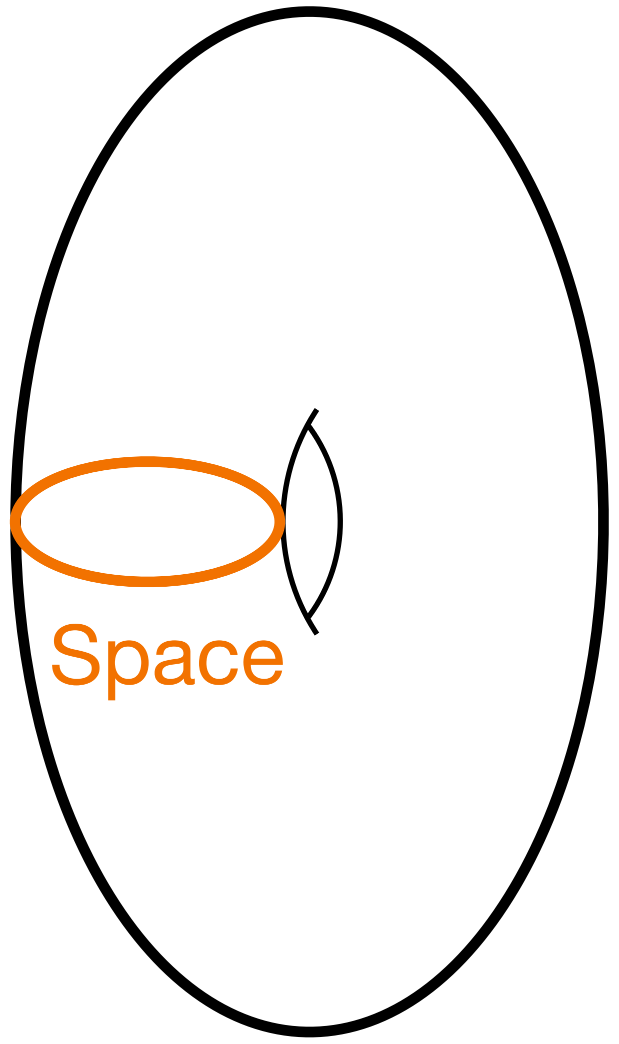
We will show that quantum computers can, in principle, rigorously constraint any reaction!



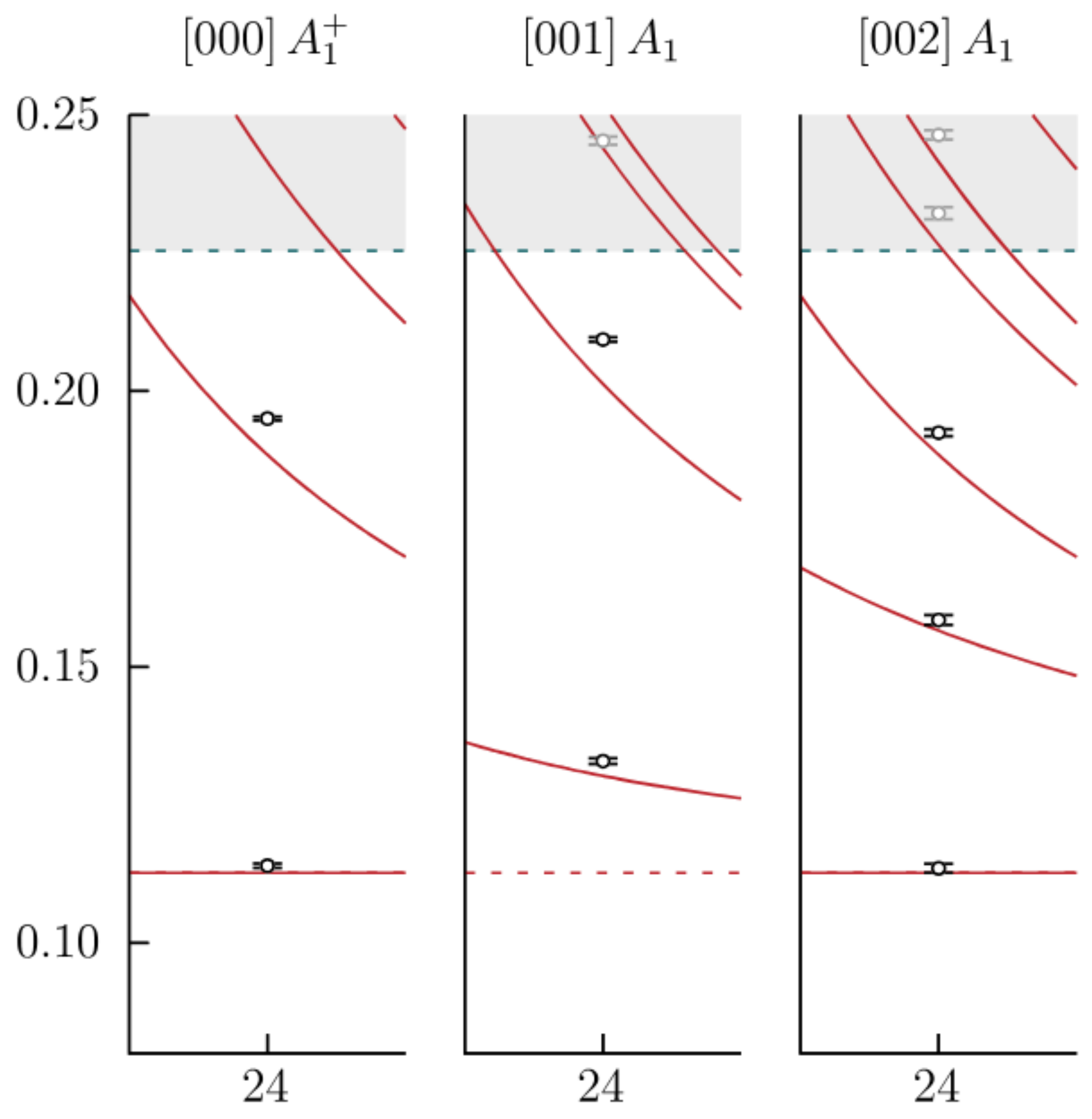

STEVEN SPIELBERG PRESENTS
BACK
TO
THE
EUCLIDEAN LQCD
A ROBERT ZEMECKIS FILM
OUTATIME

We can do a lot with Euclidean LQCD!

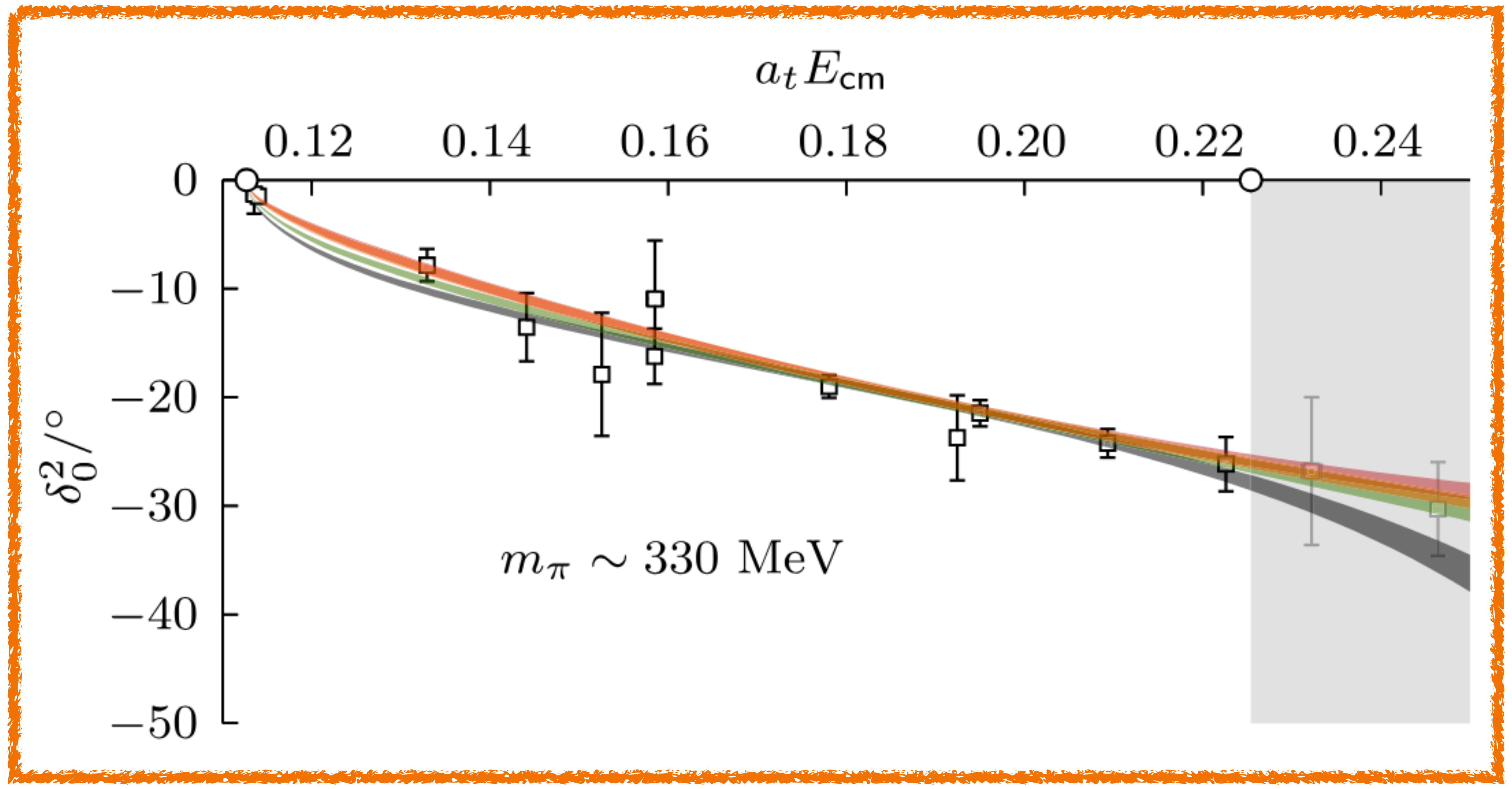
1) Extract finite-volume spectrum of Hamiltonian. This is Euclidean!



A. Rodas, J. Dudek, R. Edwards, Phys.Rev.D 108 (2023) 3, 3

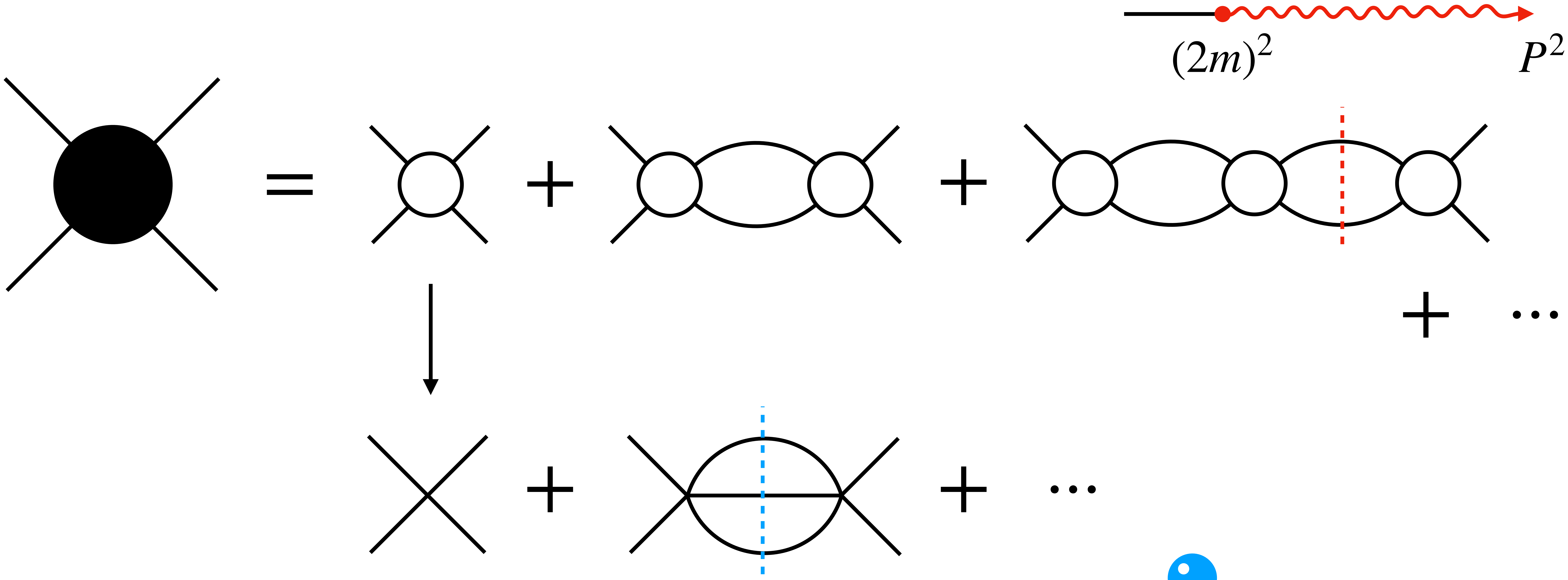


2) Apply dictionary (Lüscher quantization condition)



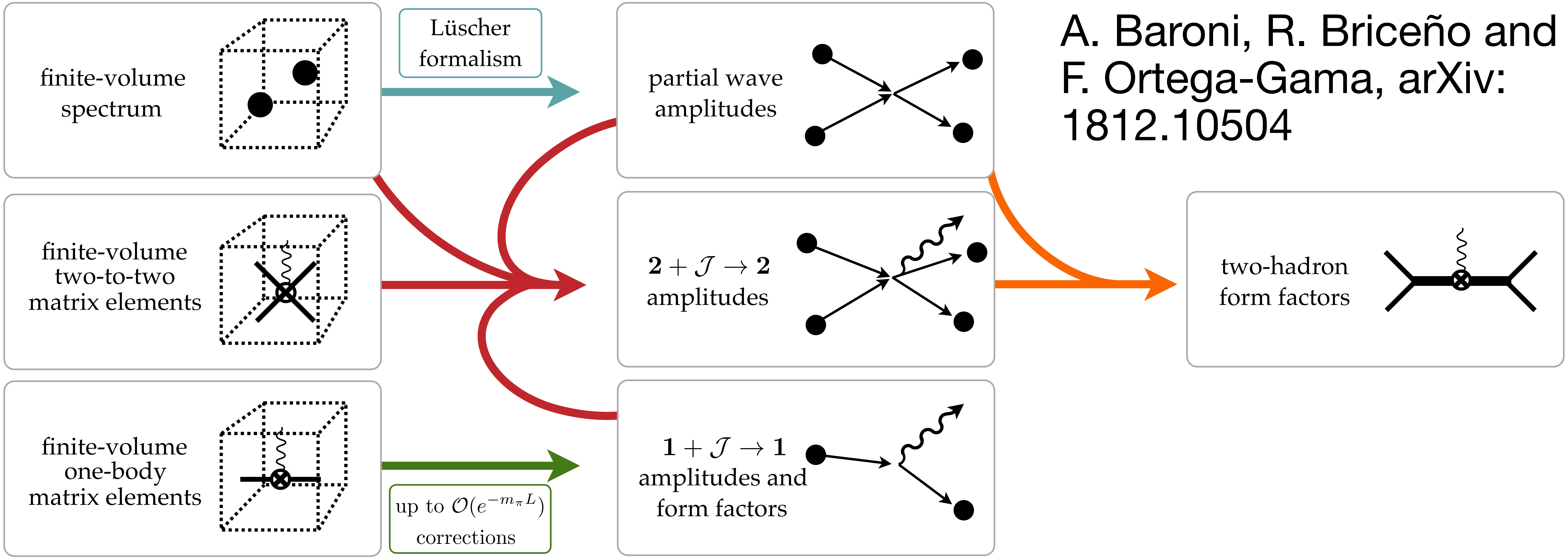
Kinematic Restrictions in the Traditional Approach

Can go on-shell



Can't go on-shell if $(2m)^2 \leq P^2 \leq (3m)^2$

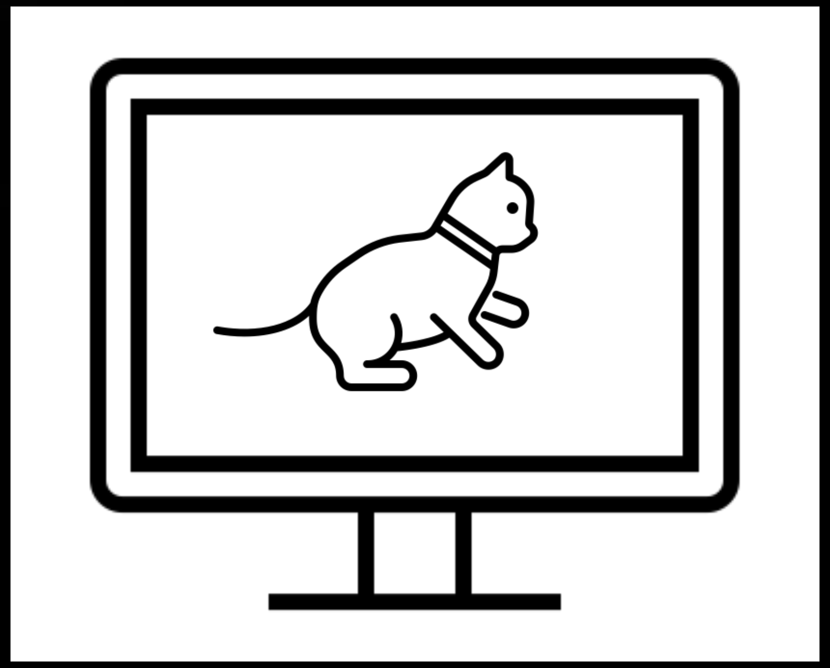
Many dictionaries have been developed!



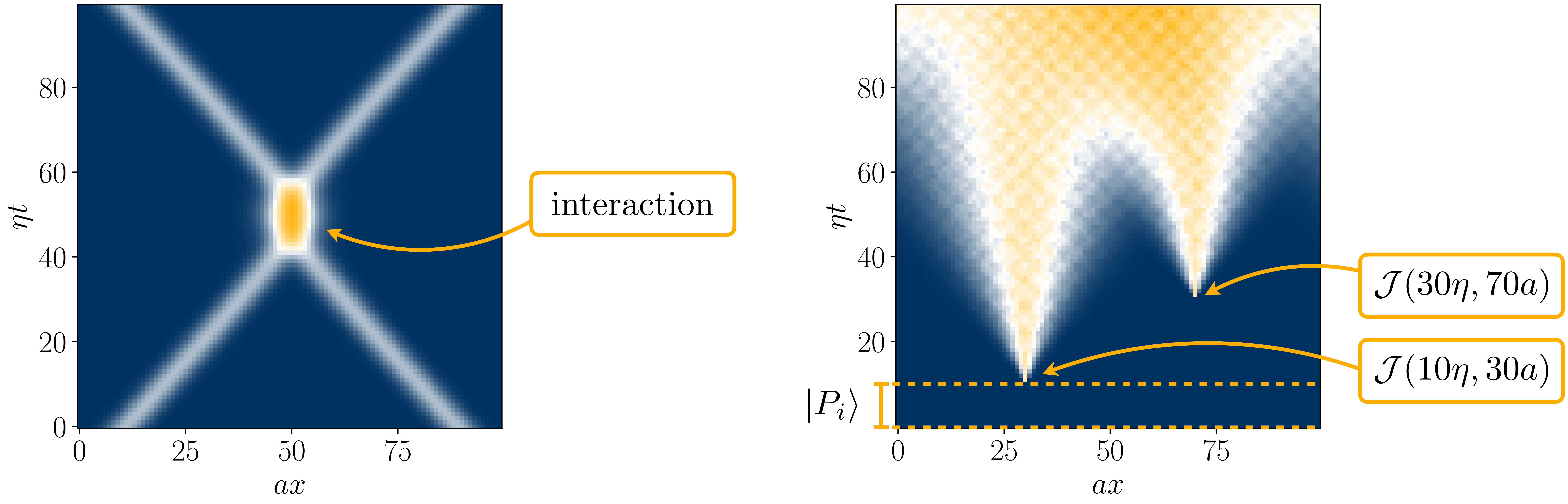
A. Baroni, R. Briceño and F. Ortega-Gama, arXiv: 1812.10504

AND THEY KEEP GETTING HARDER AND HARDER

Quantum Computing



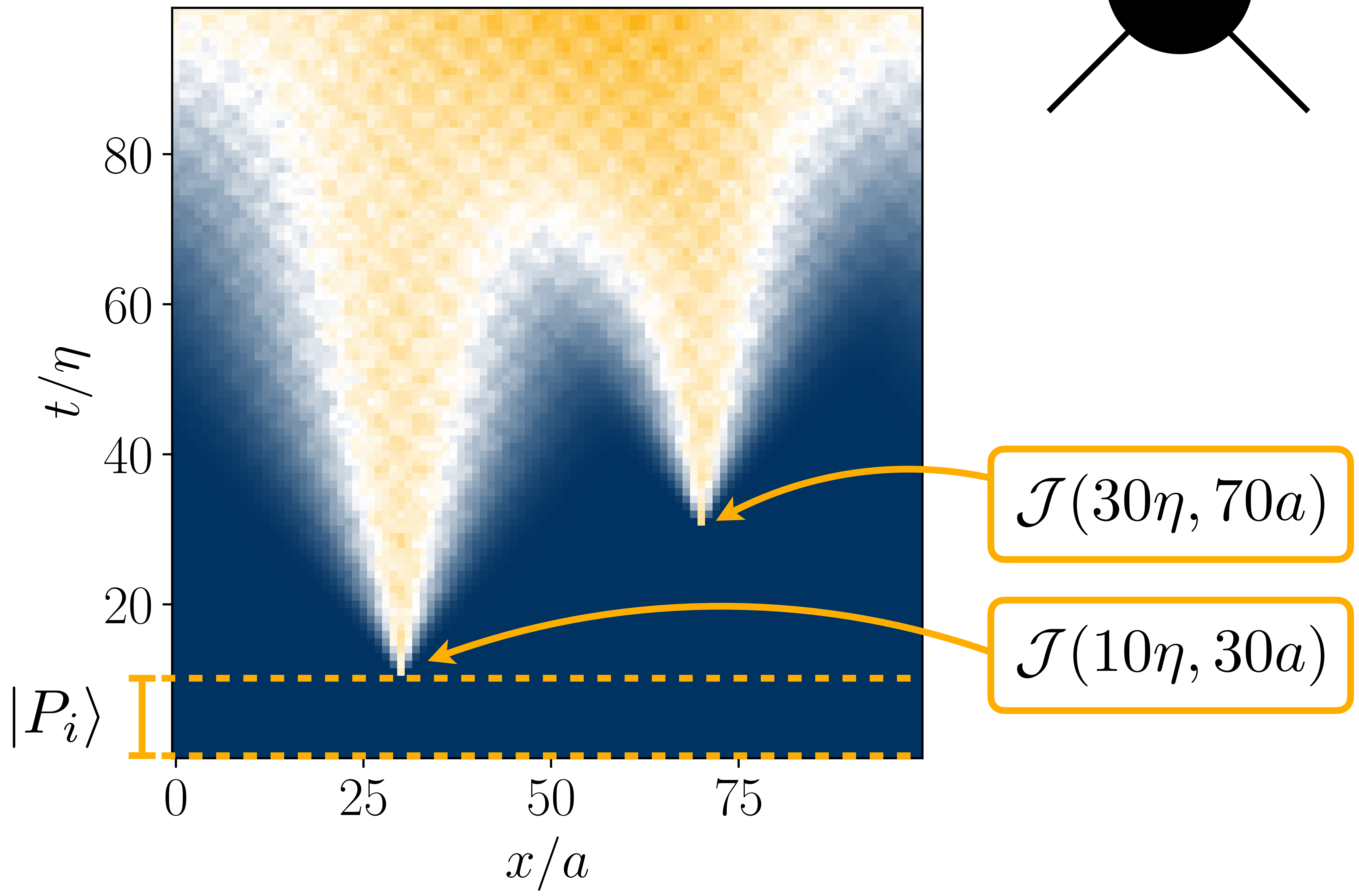
Lattice QCD on Quantum Computers Lifts These Restrictions!

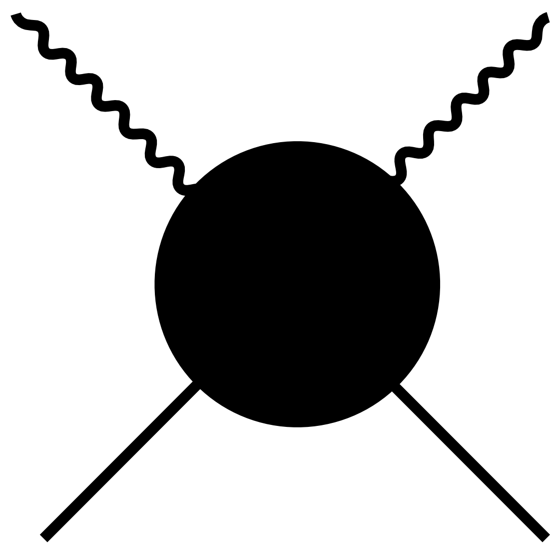


(a) (b)

Real-time Estimators for Scattering Observables

$$\langle P_f | T\mathcal{J}(t,\mathbf{x})\mathcal{J}(0)|P_i\rangle$$

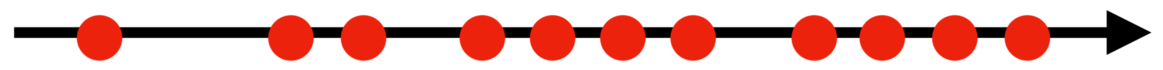


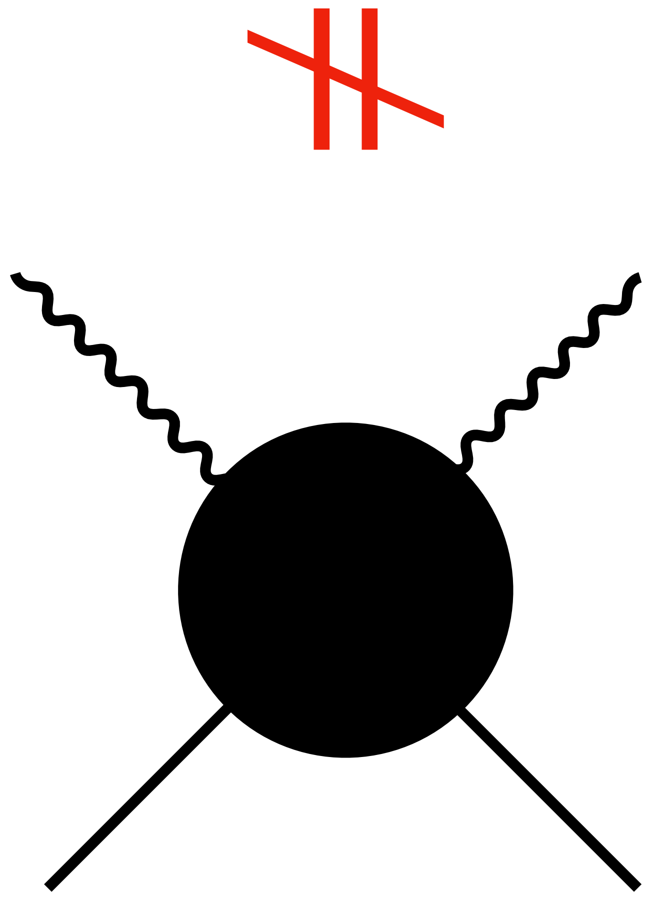


- R. Briceño, J. V. Guerrero, M. T. Hansen, A. Sturzu 2007.01155
- R. Briceño, M. A. Carrillo, J. V. Guerrero, M. T. Hansen, A. Sturzu 2112.01968
- R. Briceño, M. A. Carrillo, A. Sturzu 2406.06877

Extracting Compton-like Amplitudes

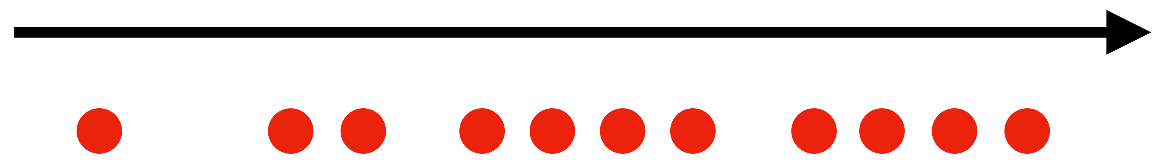
$$\int \mathrm{d}^D x\, e^{iq\cdot x} \qquad \langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle$$

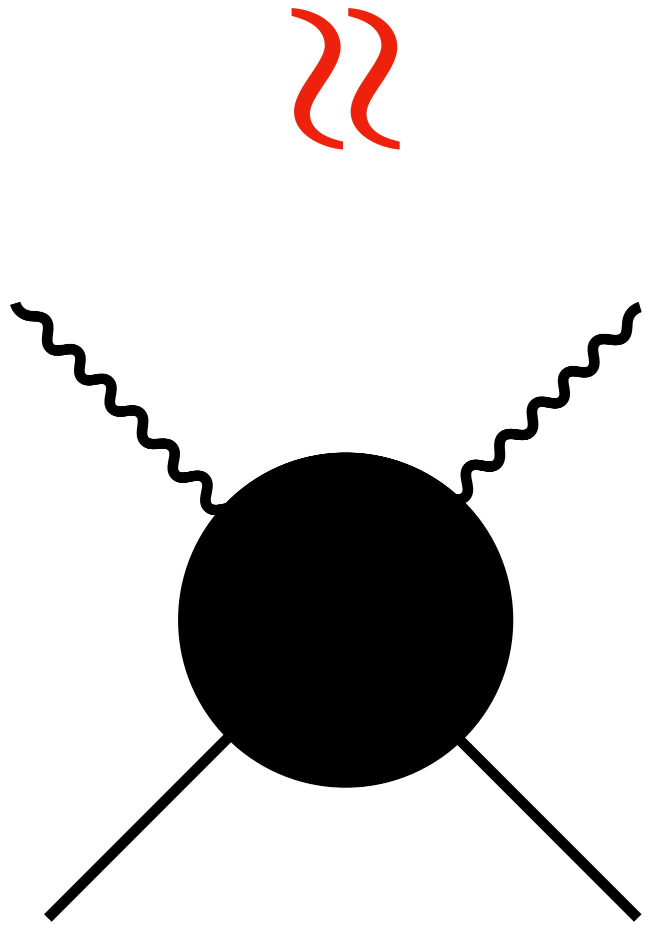




Extracting Compton-like Amplitudes

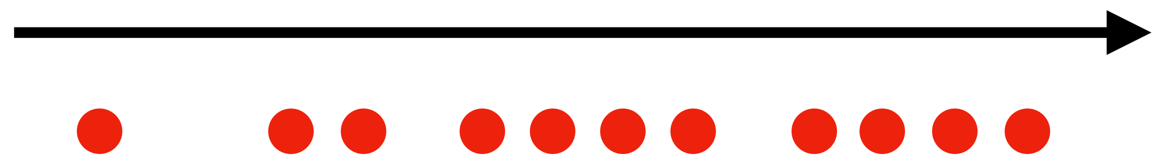
$$\int \mathrm{d}^D x \, e^{iq\cdot x - \epsilon|t|} \, \langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle$$





Extracting Compton-like Amplitudes

$$\left\langle \int \mathrm{d}^D x \, e^{iq\cdot x - \epsilon|t|} \, \langle P_f | T \mathcal{J}(t, \mathbf{x}) \mathcal{J}(0) | P_i \rangle \right\rangle_{P_i, P_f, q}$$



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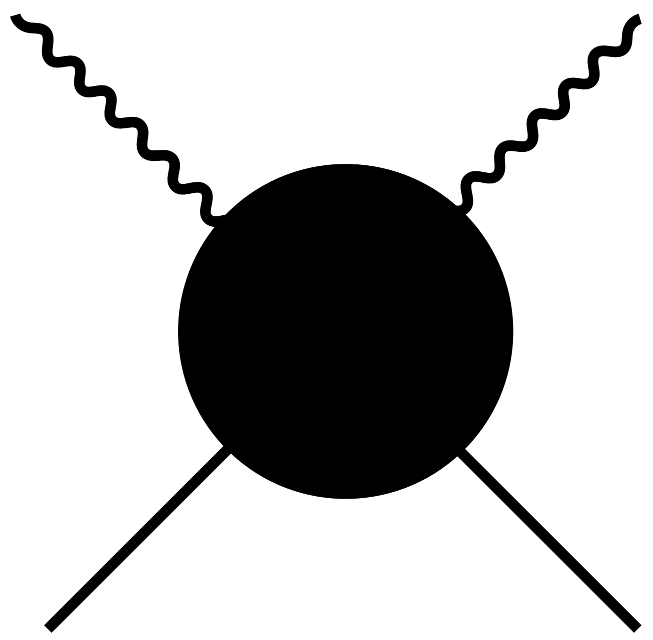
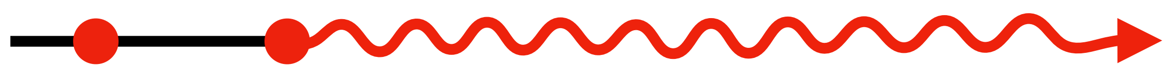
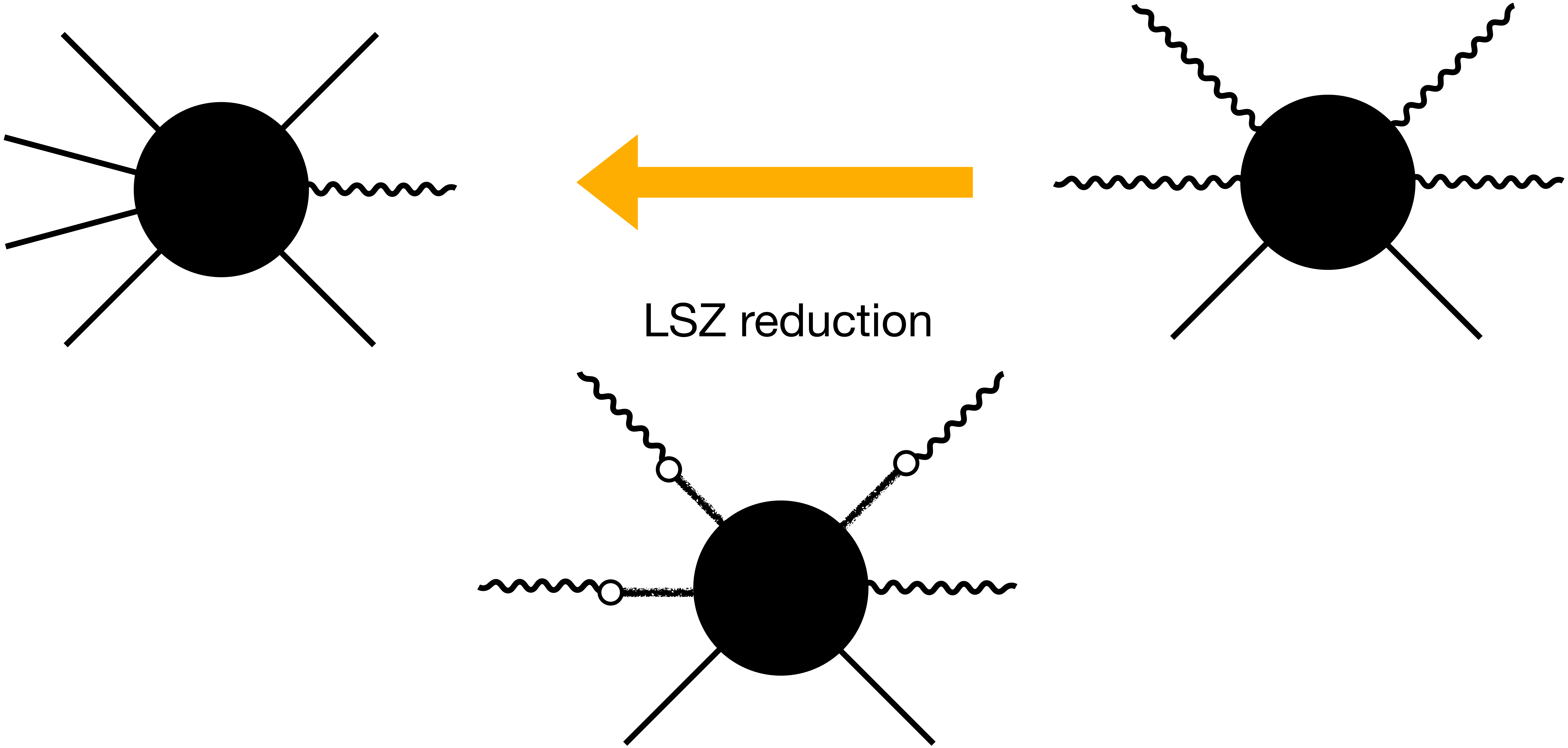


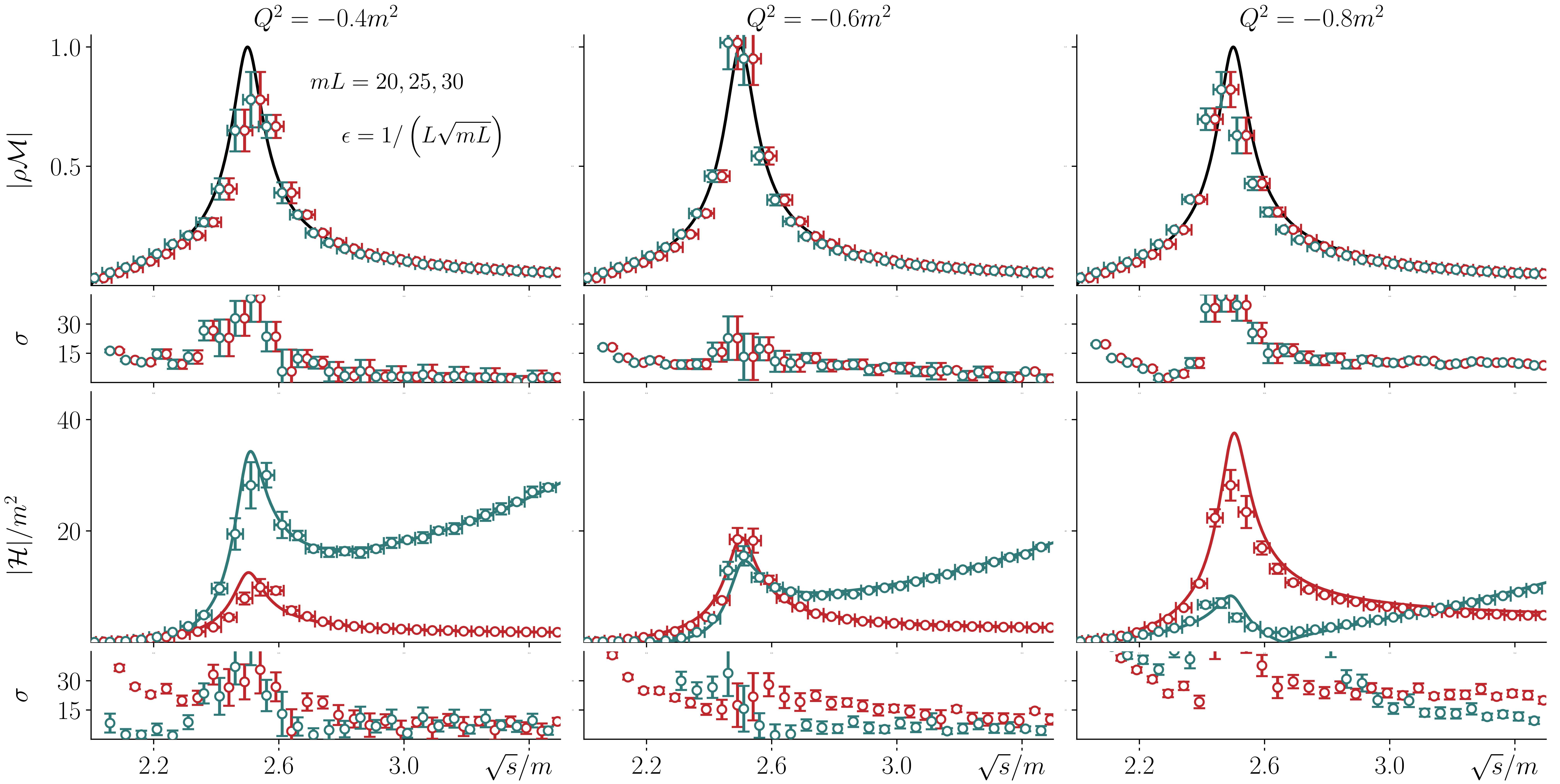


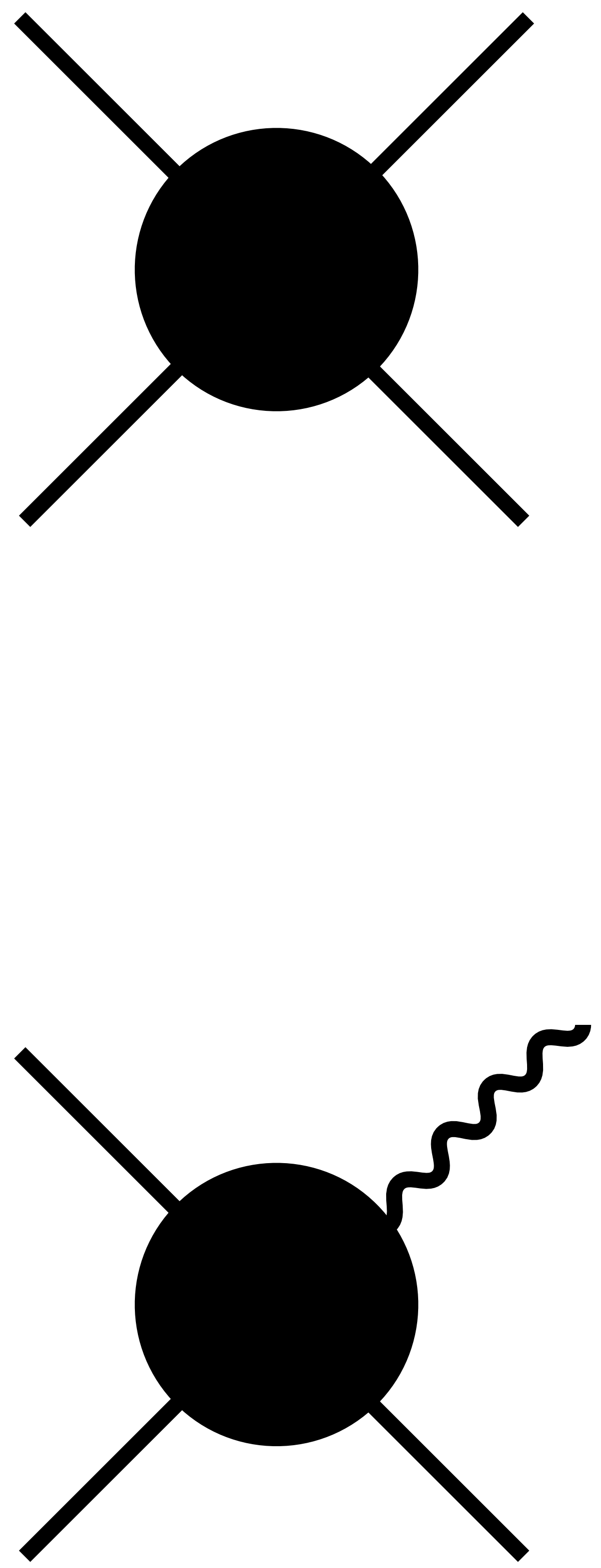
Extracting all amplitudes



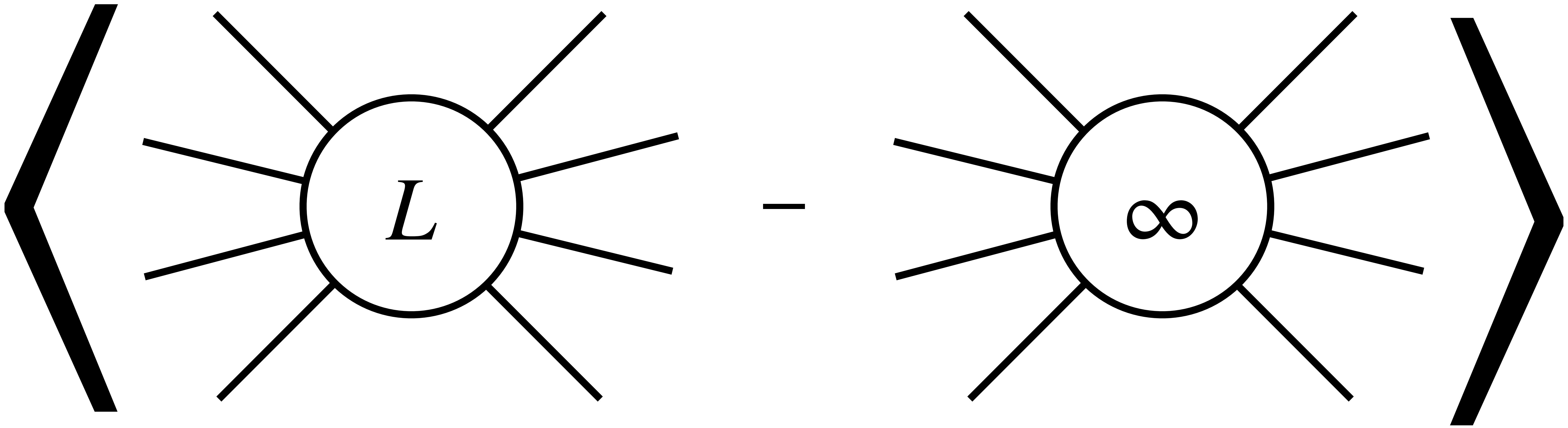
Numerical benchmarking

M. Carrillo, R. Briceño, A. Sturzu: arXiv: 2406.06877





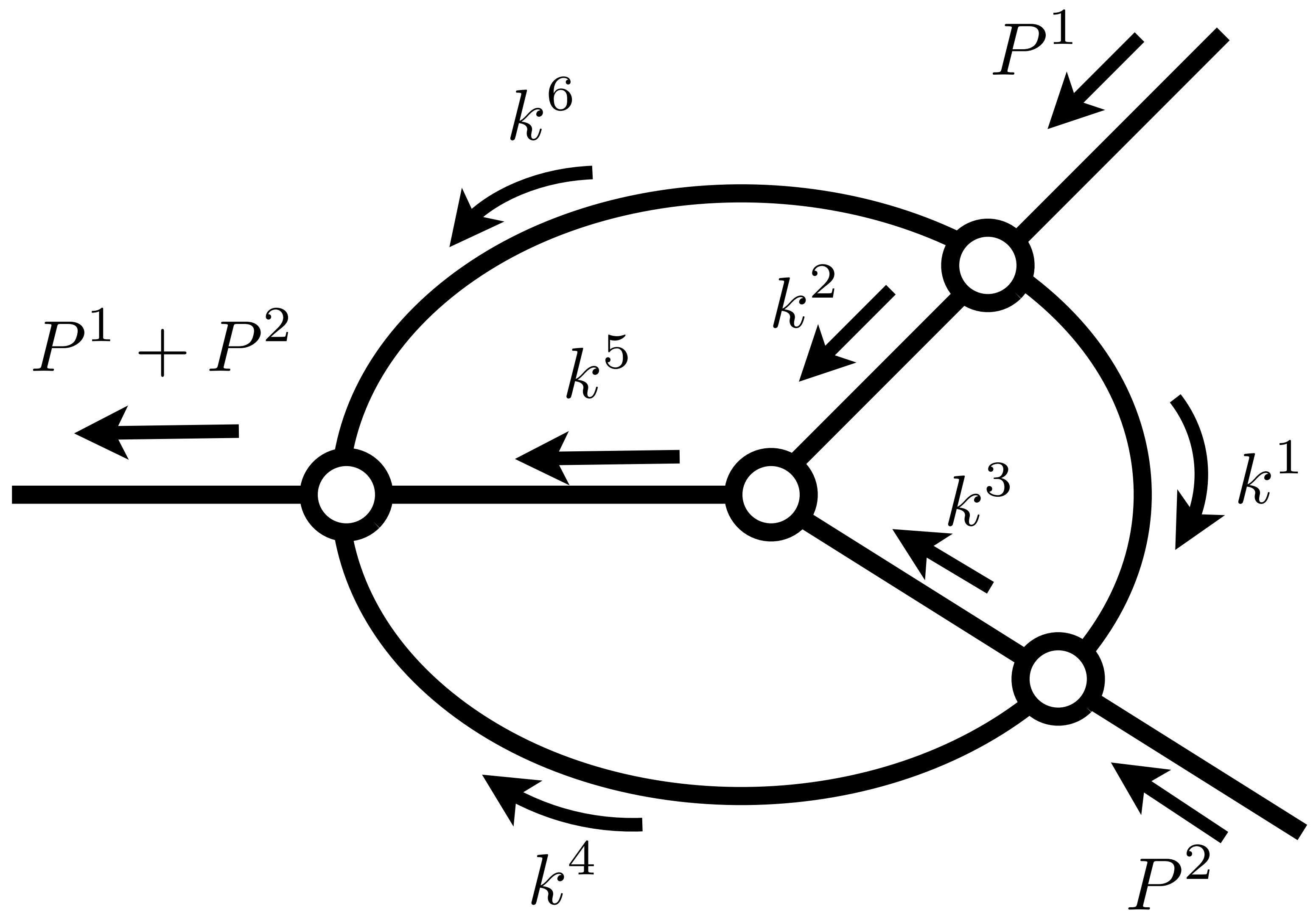
How well does this converge?



$$\sim e^{-\epsilon L} \left\langle e^{iPL} \right\rangle$$

Proof (without constants)

This is the only spacetime index that will appear throughout this talk!



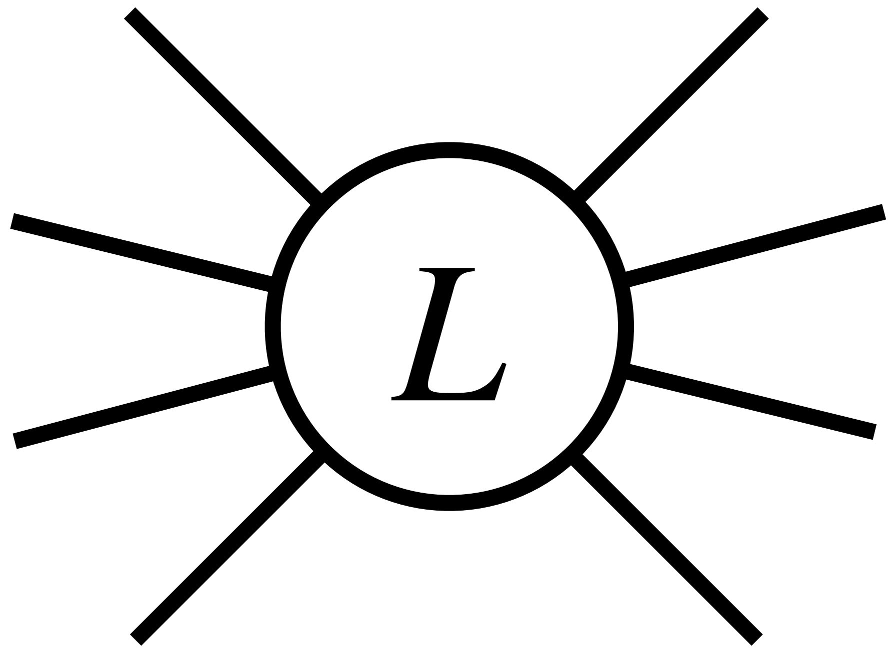
$$= \frac{1}{L^{D-1}} \sum_{\ell} \int \mathrm{d}^{N_L} \ell^0 \prod_i \frac{i}{k_i^2 - m^2 + i\epsilon}$$

loop momenta $\ell^a, \quad a = 1,\ldots,N_L$

external momenta P^A

internal momenta $k_i = f^i_a \ell^a + F^i_A P^A$

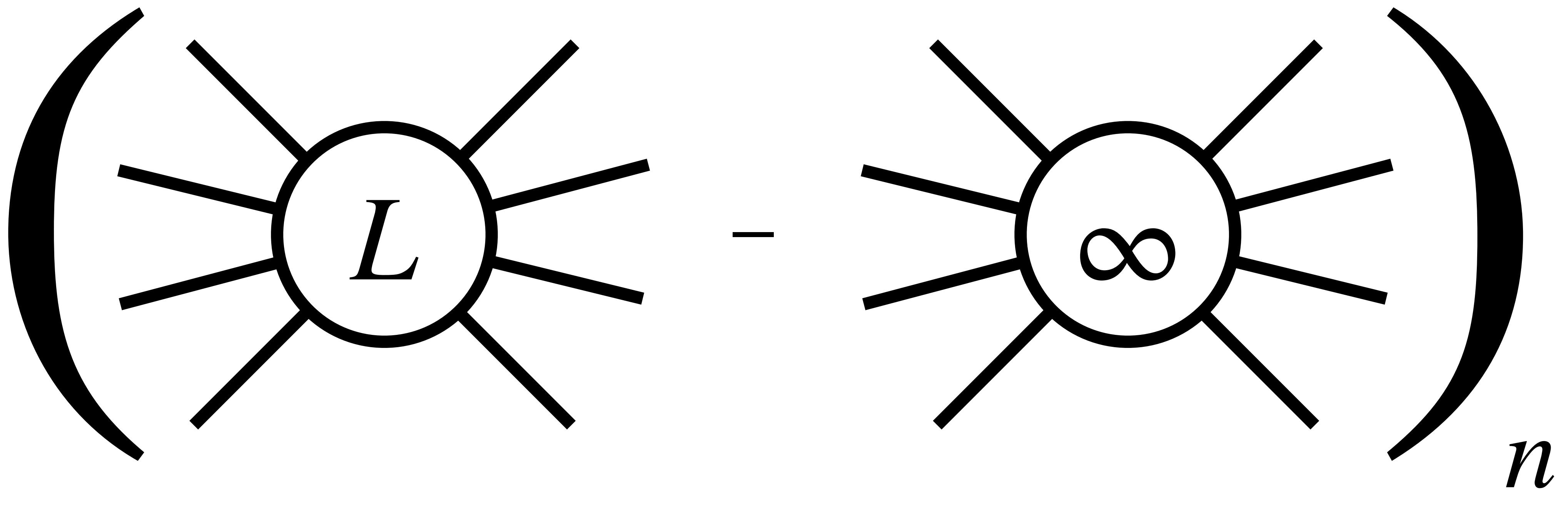
Proof (without constants)



$$= \sum_{\mathbf{n}} \int \mathrm{d}^{N_L D}\ell \, e^{-iLn_a\ell^a} \prod_i \frac{i}{k_i^2 - m^2 + i\epsilon}$$

- Poisson summation formula
- At $n = 0$ we have the $L = \infty$ diagram!
- As $\epsilon \to 0$, this series diverges (this has implications for Monte Carlo lattice you can ask me about 😊)

Proof (without constants)



$=$

$$\int_{[0,1]^{N_P}} \mathrm{d}^{N_P} u\, \delta\left(\sum_i u^i - 1\right) e^{iLn_a p^a} \int \mathrm{d}^{N_L D}\ell\, e^{-iLn_a \ell^a} \left(\frac{i}{g_{ab}\ell^a \ell^b - \Delta}\right)^{N_P}$$

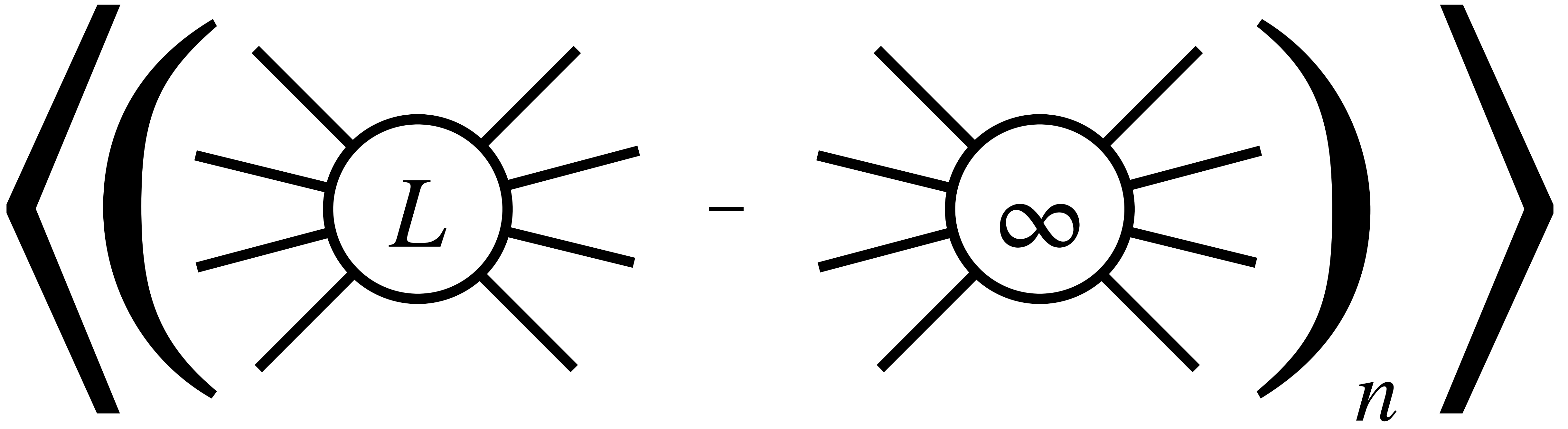
Feynman parameters

$$g_{ab} = \sum_i u^i f^i_a f^i_b$$

$$p_a = \sum_i u^i f^i_a F^i_A P^A$$

$$\Delta = m^2 + p^2 - \sum_i u^i (F^i_A P^A)^2 - i\epsilon$$

Proof (without constants)



$=$

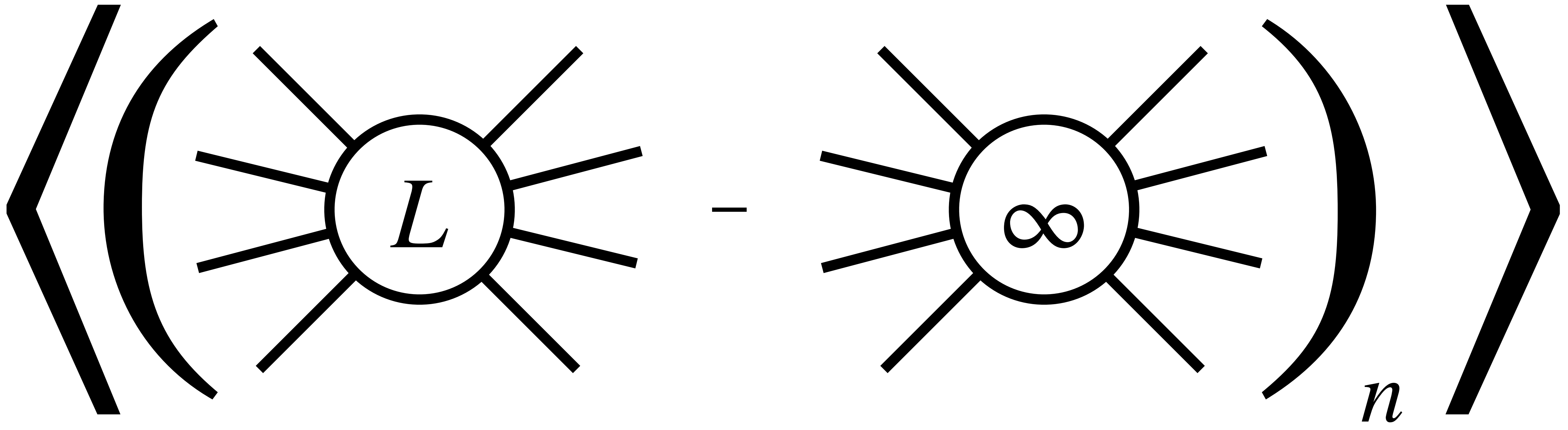
$$\int_{[0,1]^{N_P}} \mathrm{d}^{N_P}u\,\delta\left(\sum_i u^i - 1\right)\left\langle e^{iLn_ap^a}\right\rangle \int \mathrm{d}^{N_L D}\ell\, e^{-iLn_a\ell^a}\left(\frac{i}{g_{ab}\ell^a\ell^b-\Delta}\right)^{N_P}$$

Lorentz invariant!

$$\left(\frac{\sqrt{\Delta}}{L\|n\|}\right)^{\nu} K_\nu(L\|n\|\sqrt{\Delta})$$

dimensional analysis

Proof (without constants)



=

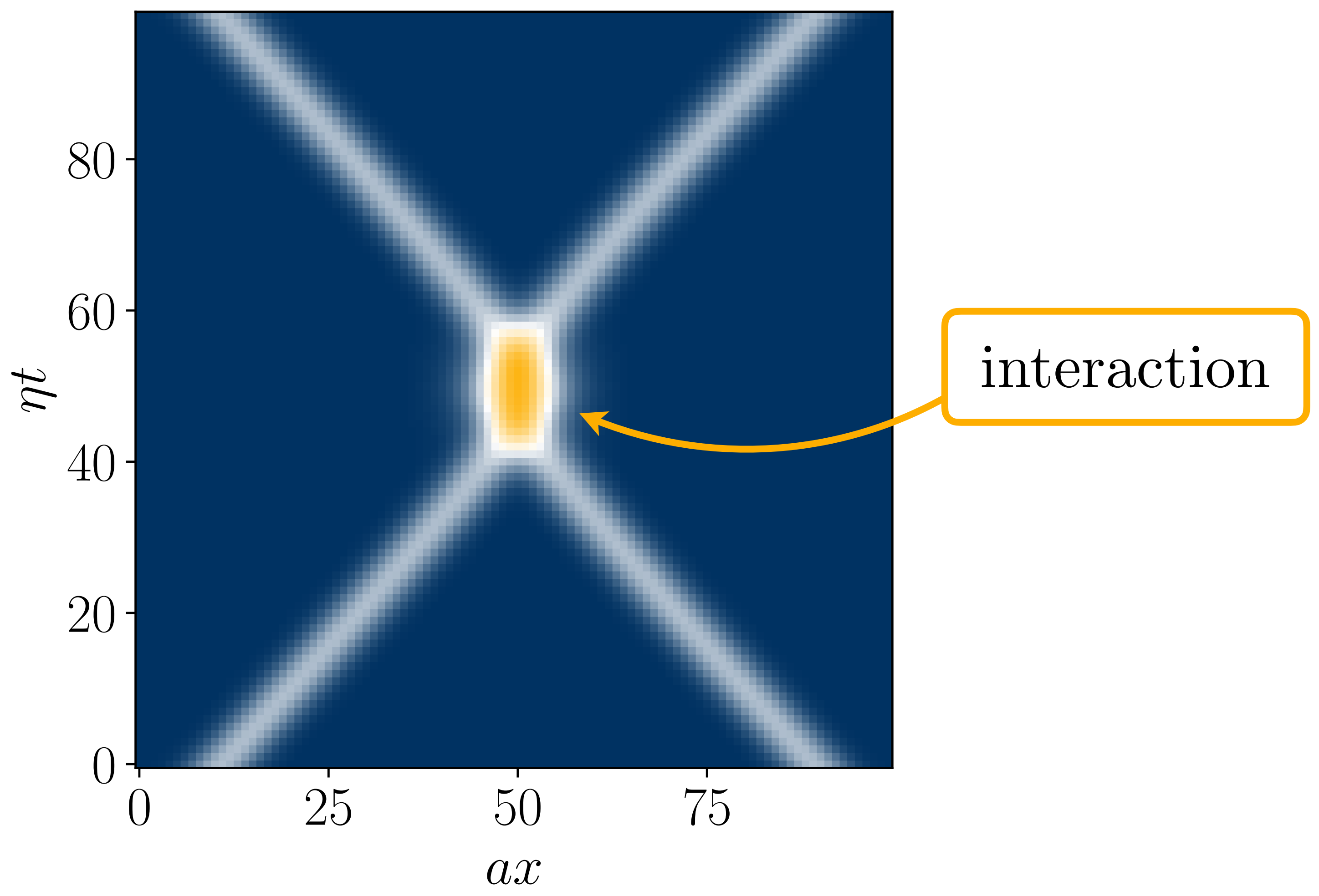
$$\int_{[0,1]^{N_P}} \mathrm{d}^{N_P}u\,\delta\left(\sum_i u^i - 1\right)\left\langle e^{iLn_ap^a}\right\rangle \int \mathrm{d}^{N_L D}\ell\, e^{-iLn_a\ell^a}\left(\frac{i}{g_{ab}\ell^a\ell^b-\Delta}\right)^{N_P}$$

$$\Delta\left(L\|n\|\sqrt{\Delta}\right)^{-\nu-1/2} e^{-L\|n\|\sqrt{\Delta}}$$

Wave packet approach

Boost averaging can help severely reduce finite-volume errors. This is key in the wave packet approach.

$$\left\langle e^{iLn_a p^a} \right\rangle \sim e^{-\frac{1}{2}L^2\sigma^2}$$

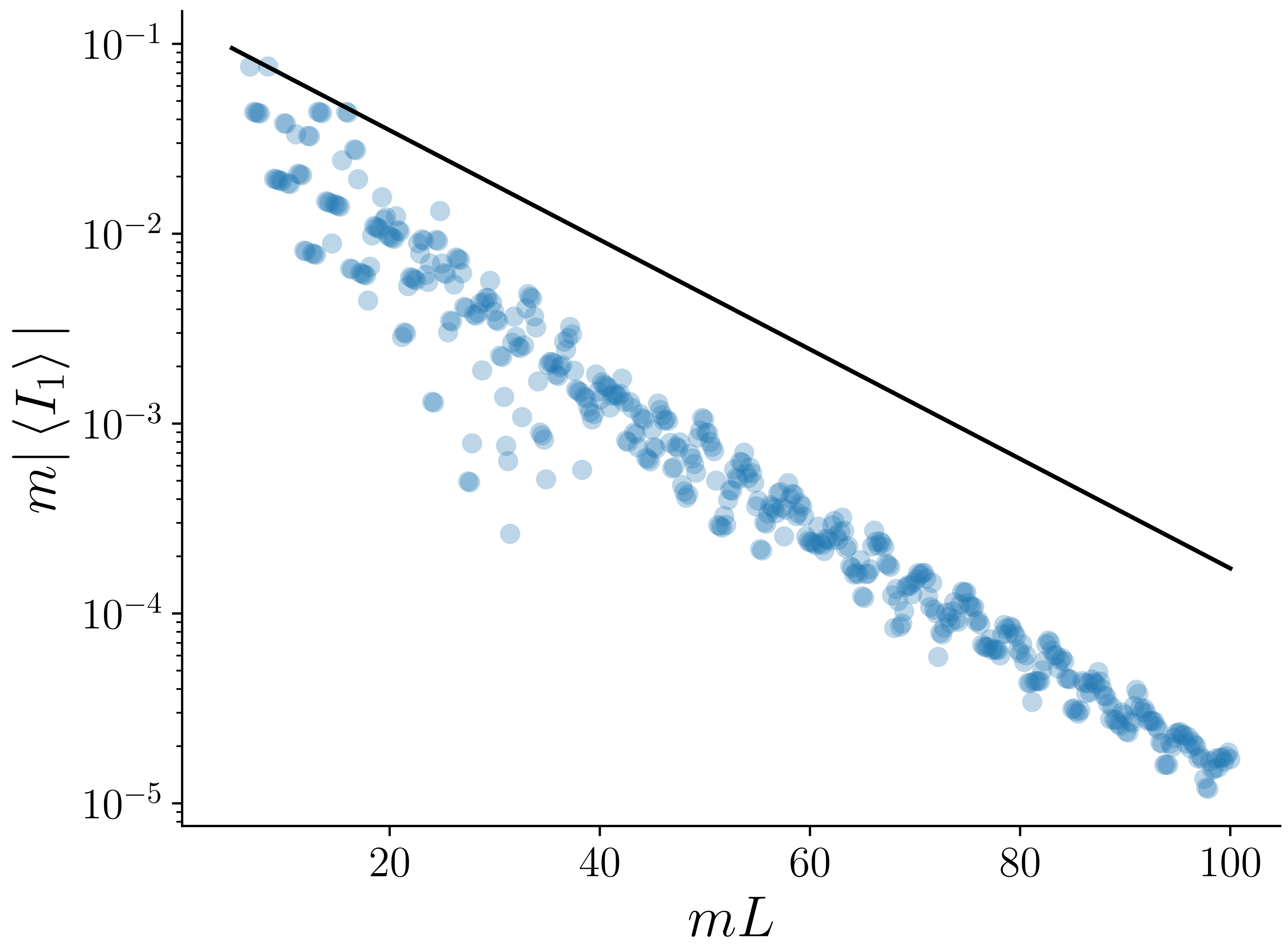


Conclusions

- Using input from quantum computers, there are workflows, uniform across all reactions and kinematics, that allow us to study general scattering processes.

- We have a simple to implement formula that allows us to estimate finite-volume errors due to arbitrary diagrams

$$\text{Error} = \sum_{\mathbf{n}} \frac{2}{(4\pi)^{N_L D/2}} \int_{[0,1]^{N_P}} \mathrm{d}^{N_P} u \, \delta\left(\sum_i u^i - 1\right) \times$$
$$g^{-D/2} \left\langle e^{iLn_a \cdot p^a} \right\rangle \left(\frac{2\sqrt{\Delta}}{L\|n\|}\right)^{\nu} K_\nu(L\|n\|\sqrt{\Delta})$$



Conclusions

- Depending on the kinematic region of interest, there can be different types of error suppression

$$K_\nu(L\|n\|\sqrt{\Delta}) \sim e^{-L\|n\|\sqrt{\Delta}} \sim \begin{cases} e^{-L\|n\|\sqrt{\mathrm{Re}\,\Delta}}, & \mathrm{Re}\,\Delta > 0, \\ e^{-L\|n\|\epsilon/\sqrt{|\,\mathrm{Re}\,\Delta|}}, & \mathrm{Re}\,\Delta < 0. \end{cases}$$

- Polynomial errors are also under control

$$L^{N_P-(N_L D+1)/2}$$

Bonus: This can also be used for Euclidean explorations!