Preparing wavepackets with short and long range entanglement

Roland Farrell

9/30/25

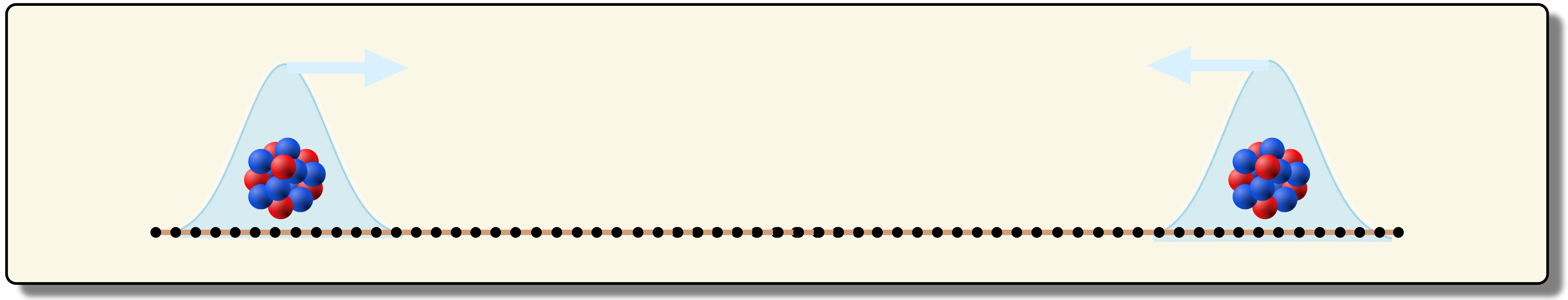
QuantHEP @ LBNL

BURKE INSTITUTE

CALIFORNIA INSTITUTE OF TECHNOLOGY 1891

IQIM

Initial states in high-energy collisions are wavepackets



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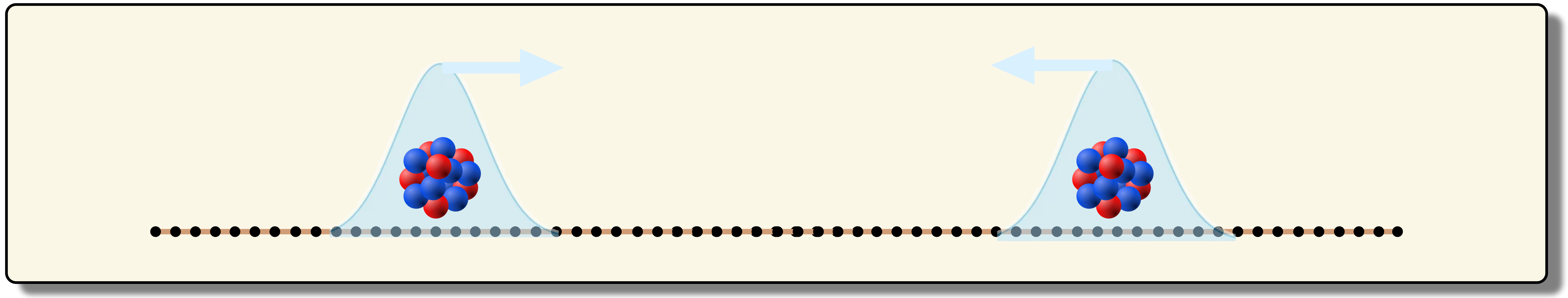

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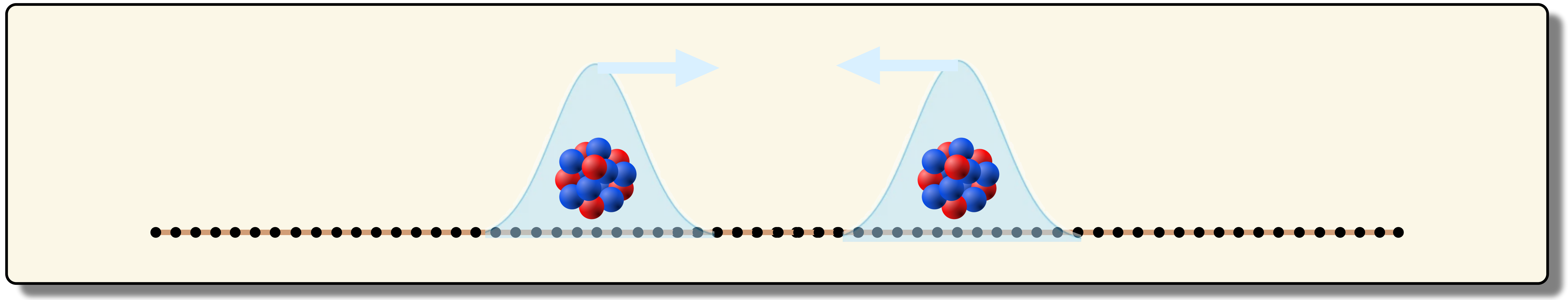

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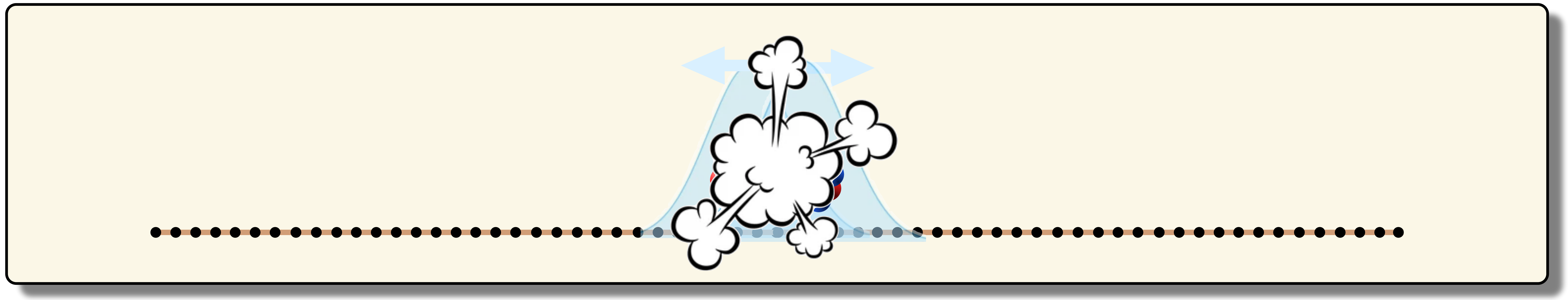

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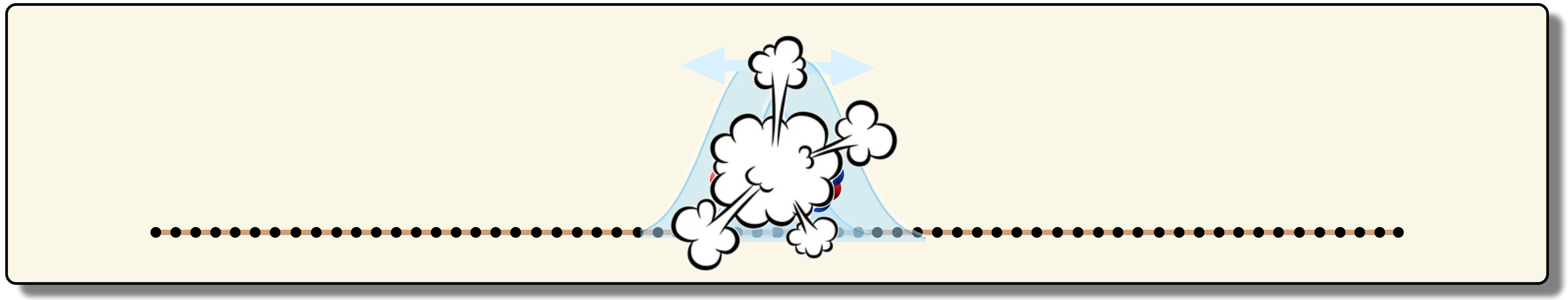
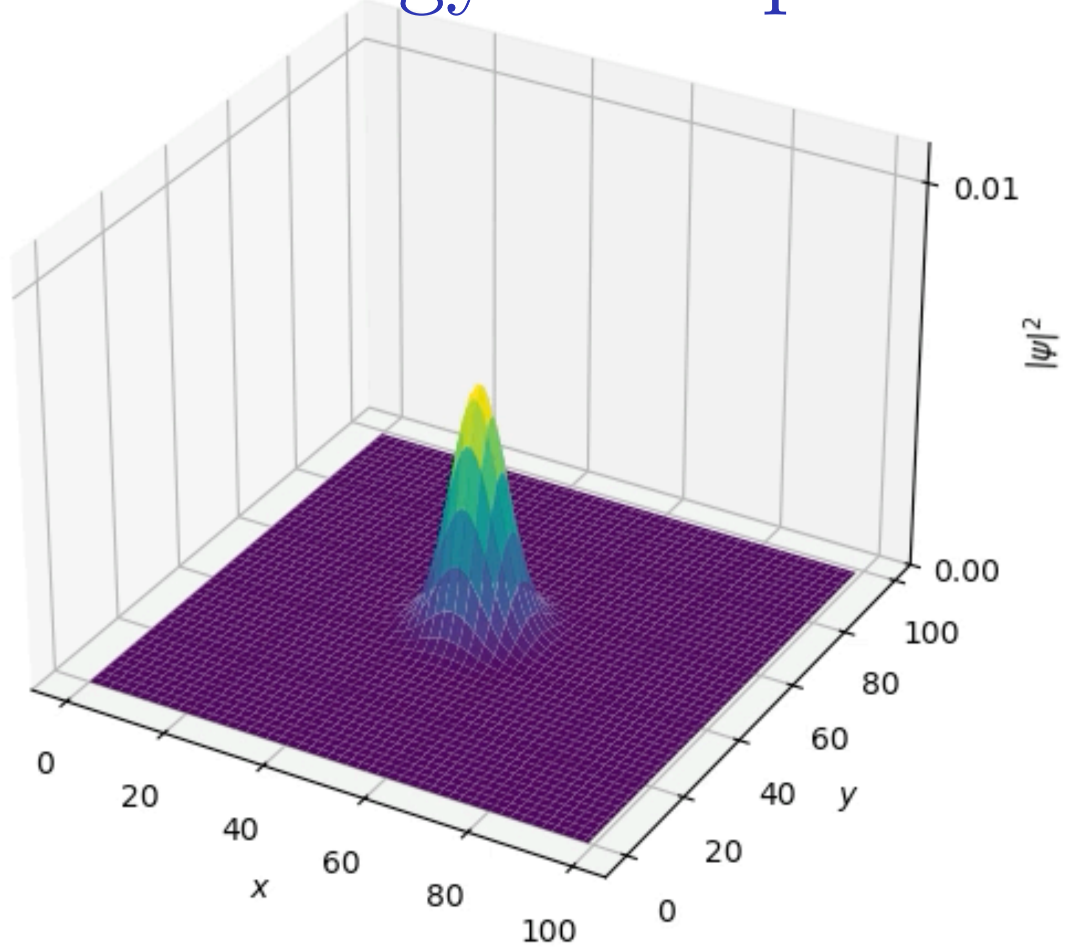
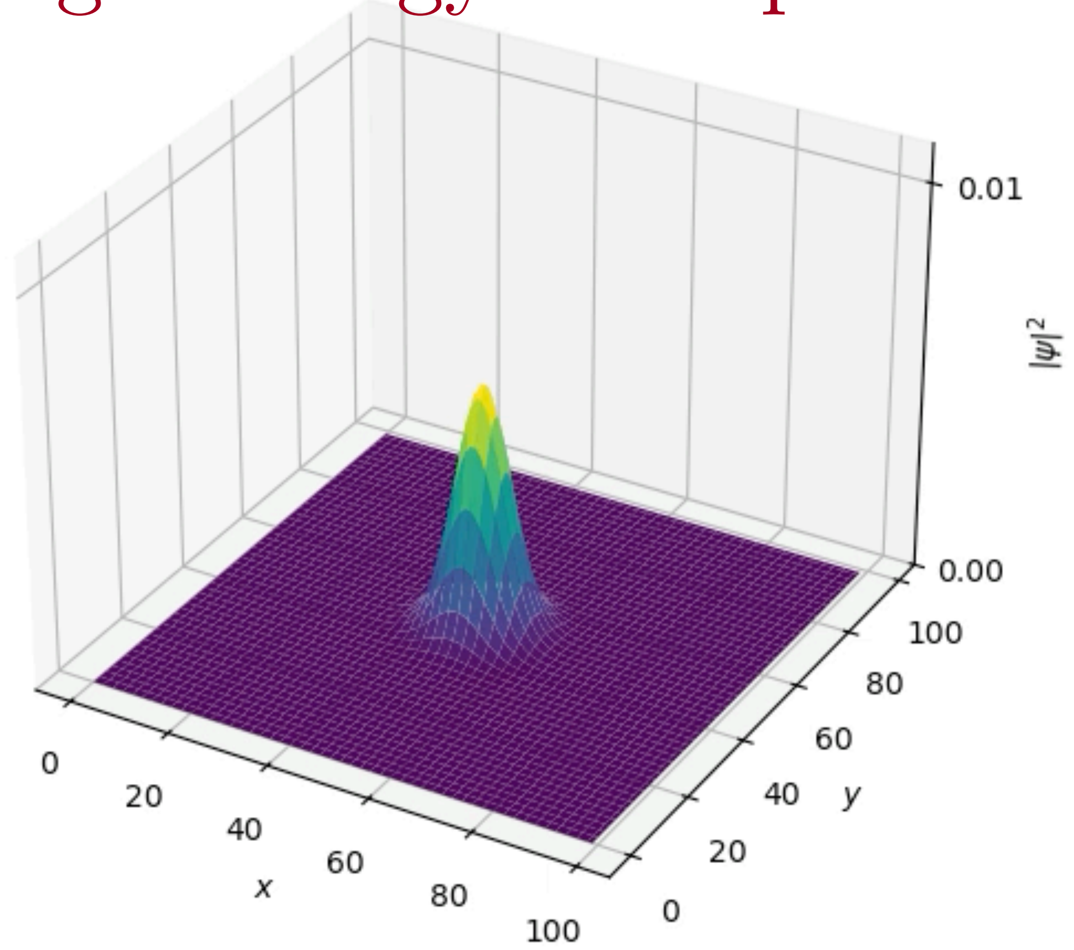


Wavepackets can be used to study transport properties*

Low energy wavepacket



High energy wavepacket



*Lee and Farrell in preparation

Initial states in high-energy collisions are wavepackets

Quantum advantage expected in simulations of real-time dynamics

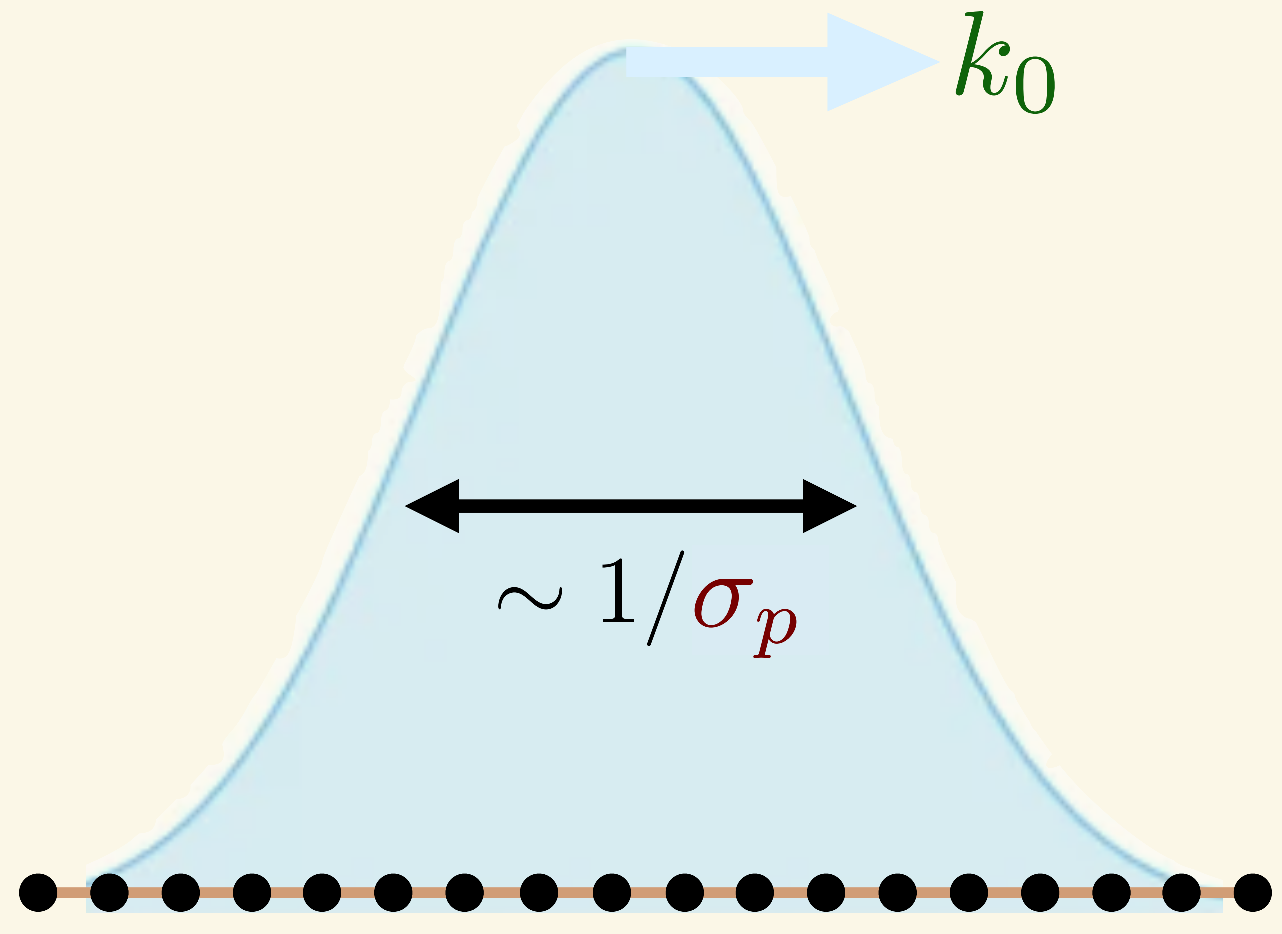
Low energy wavepacket

High energy wavepacket

*Lee and Farrell in preparation

Gaussian wavepackets saturate $\Delta x \Delta p \geq \hbar/2$

$$|\psi_{\mathrm{WP}}\rangle = \sum_k e^{-(k-k_0)^2/\sigma_p^2} |\psi_k\rangle$$



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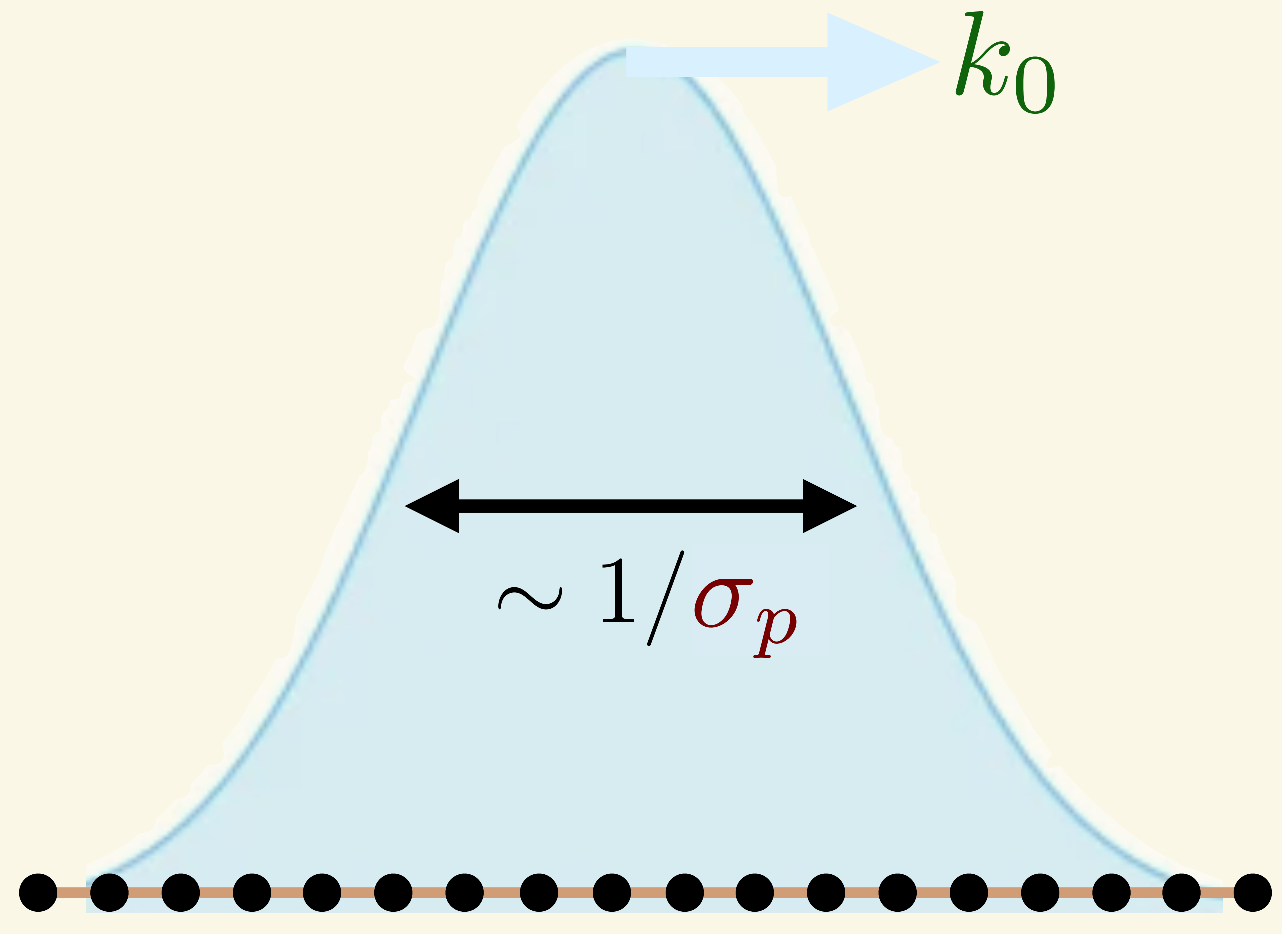
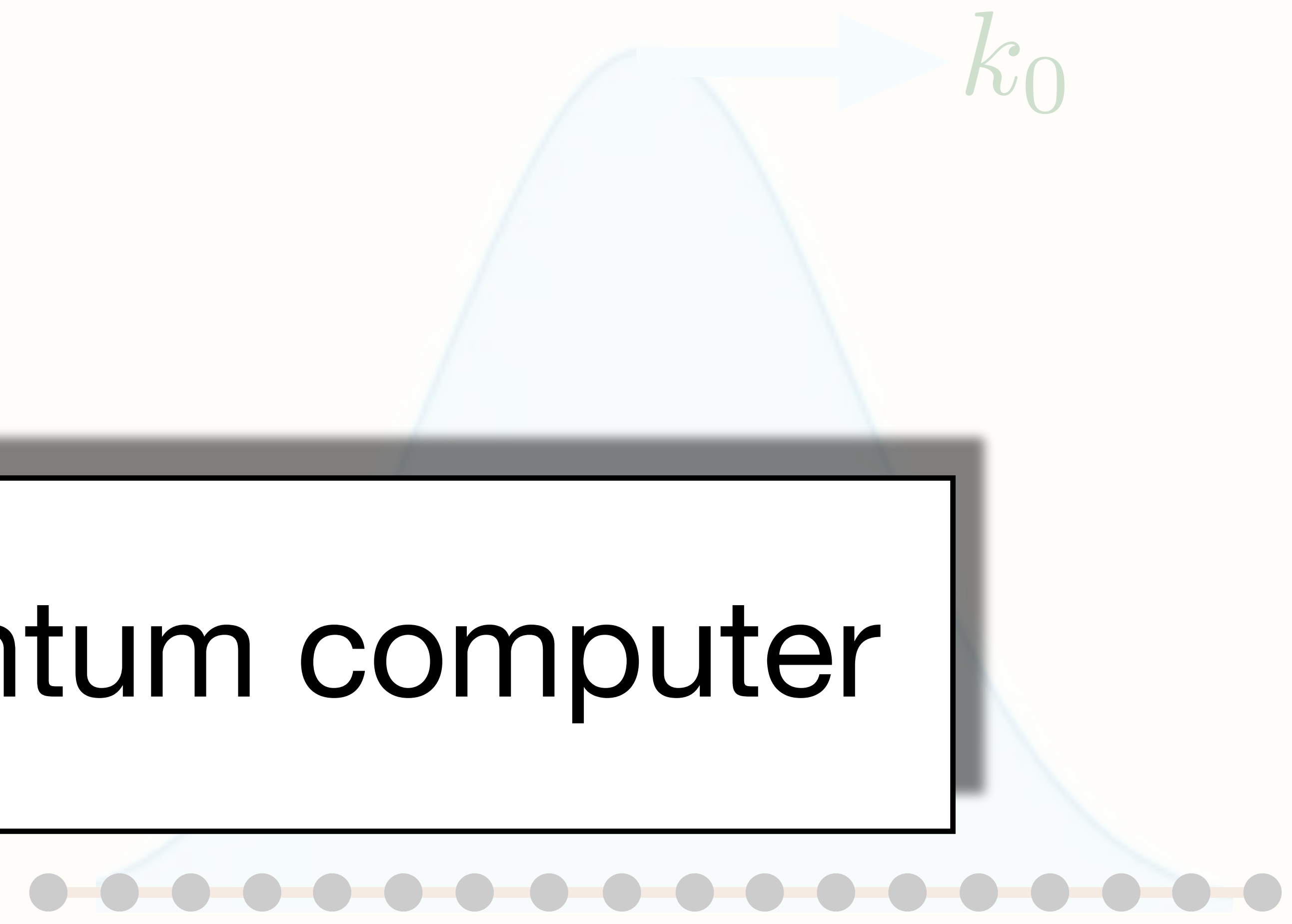


$|\psi_k\rangle$ is the lowest-energy state with momentum $k \neq 0$

Gaussian wavepackets saturate $\Delta x \Delta p \geq \hbar/2$

$|\psi_W$

First, consider preparing $|\psi_k\rangle$ on a quantum computer



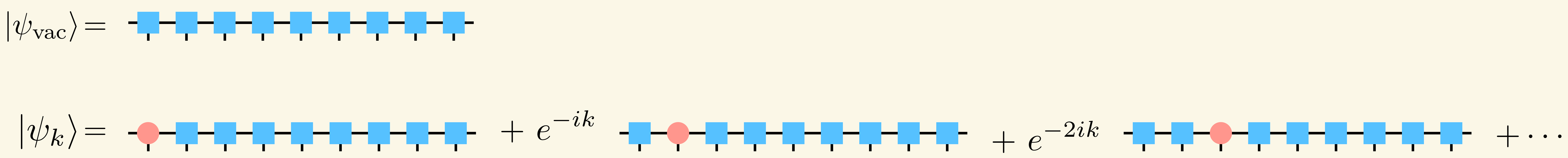
$|\psi_k\rangle$ is the lowest-energy state with momentum $k \neq 0$

$|\psi_k\rangle$ is always long-range entangled, even for gapped systems with a finite correlation length [1]

[1] Gioia and Wang PRX 2022
[2] Haegeman et. al PRB 2012

$|\psi_k\rangle$ is <u>always</u> long-range entangled, even for gapped systems with a finite correlation length [1]

Quasiparticle ansatz in MPS [2]



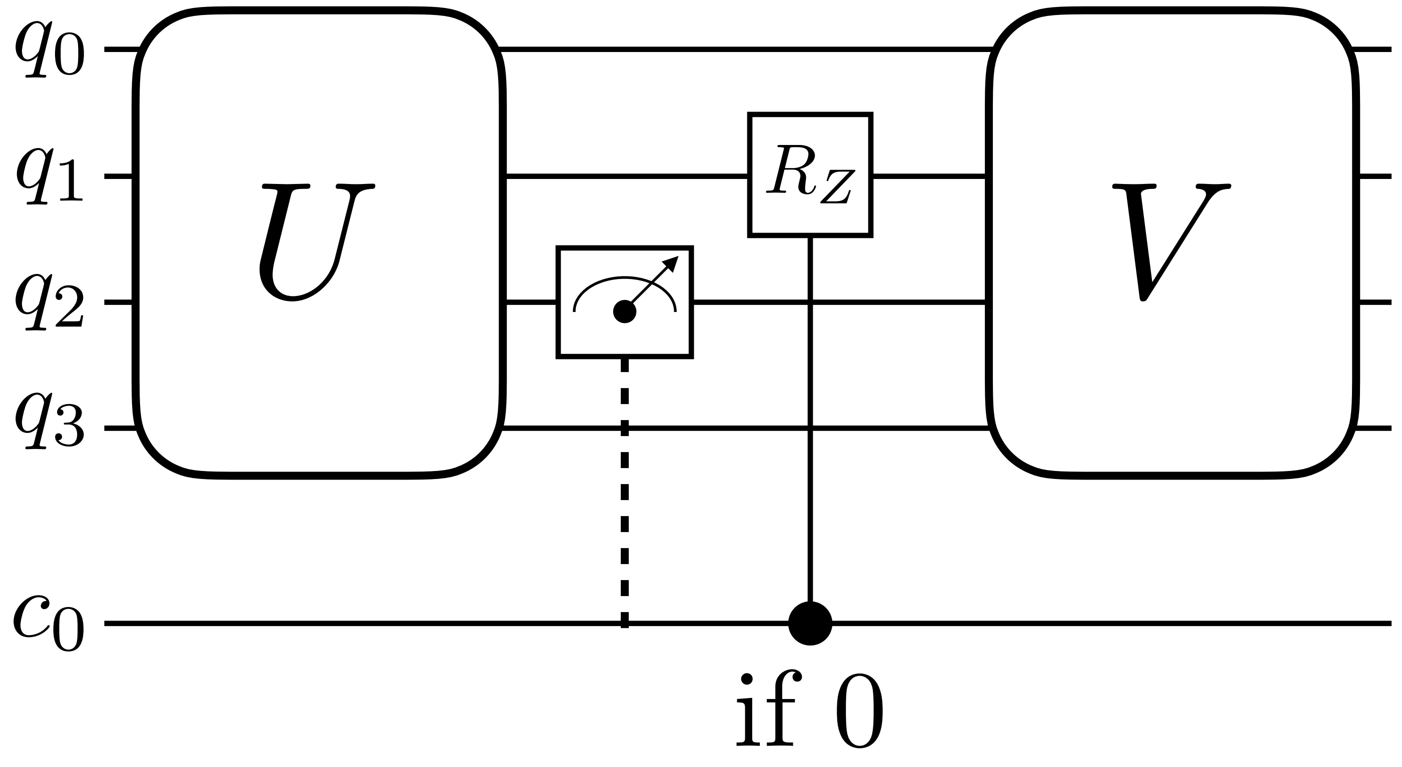
Plane-wave structures forces the position of excitations to be entangled

[1] Gioia and Wang PRX 2022
[2] Haegeman et. al PRB 2012

Strategy for preparing $|\psi_k\rangle$

1. Apply quantum circuits that build the long-range $O(L)$ entanglement

   - Constant-depth with mid-circuit measurement and feedforward (MCM-FF)



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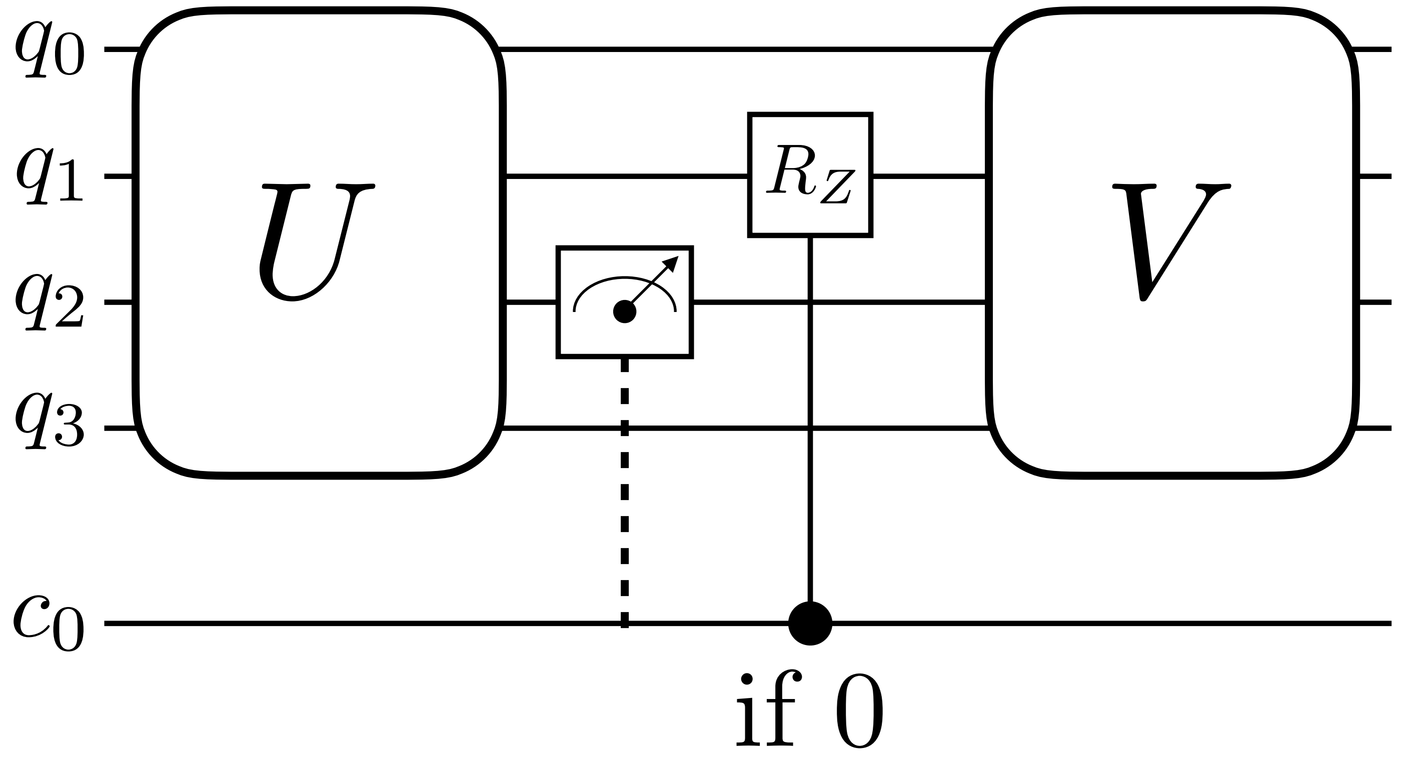
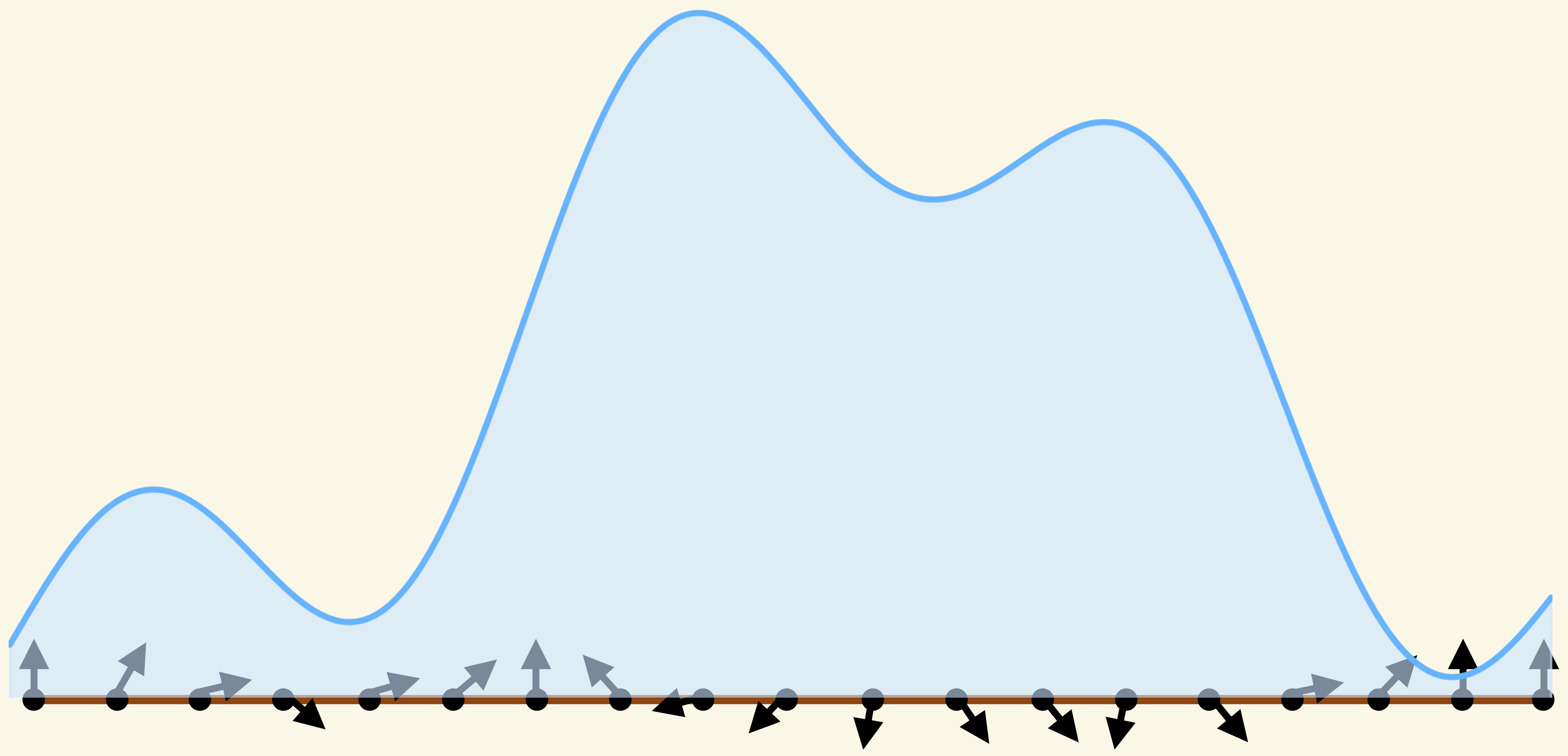


2. Apply quantum circuits that build the short-range $O(\xi)$ entanglement

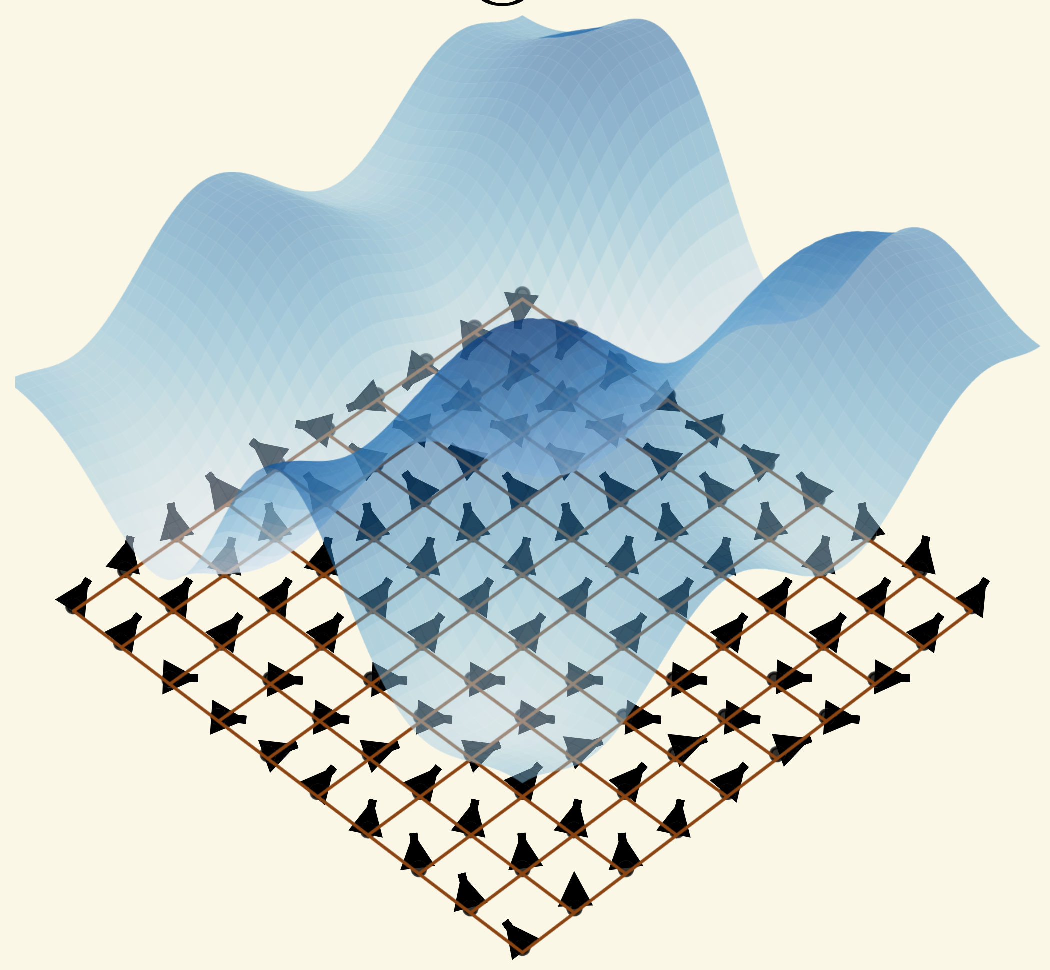
   - Reformulate as an energy minimization problem. Optimize parameterized quantum circuits

Our method is widely applicable

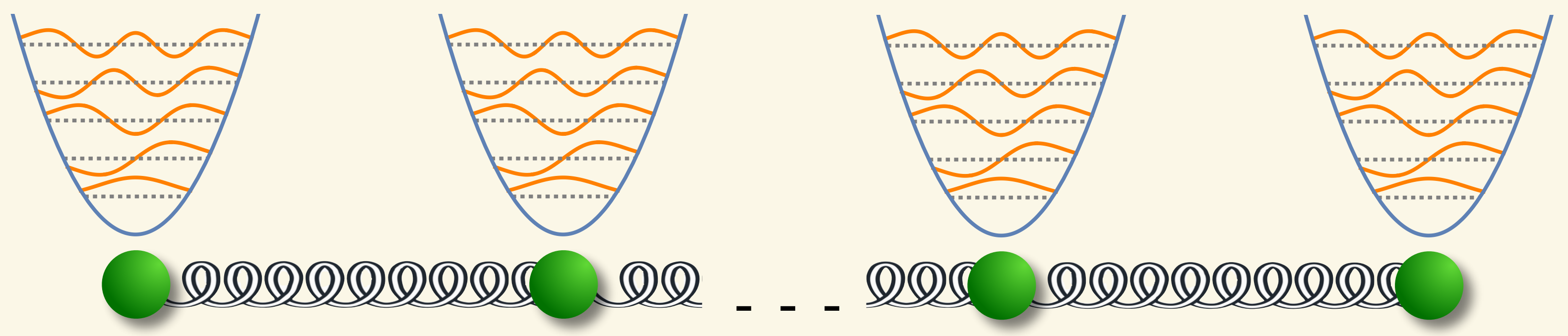
$1+1$D Ising field theory



$2+1$D Ising field theory



$1+1$D scalar field theory



1+1D $U(1)$ LGT

1+1D Luttinger liquid

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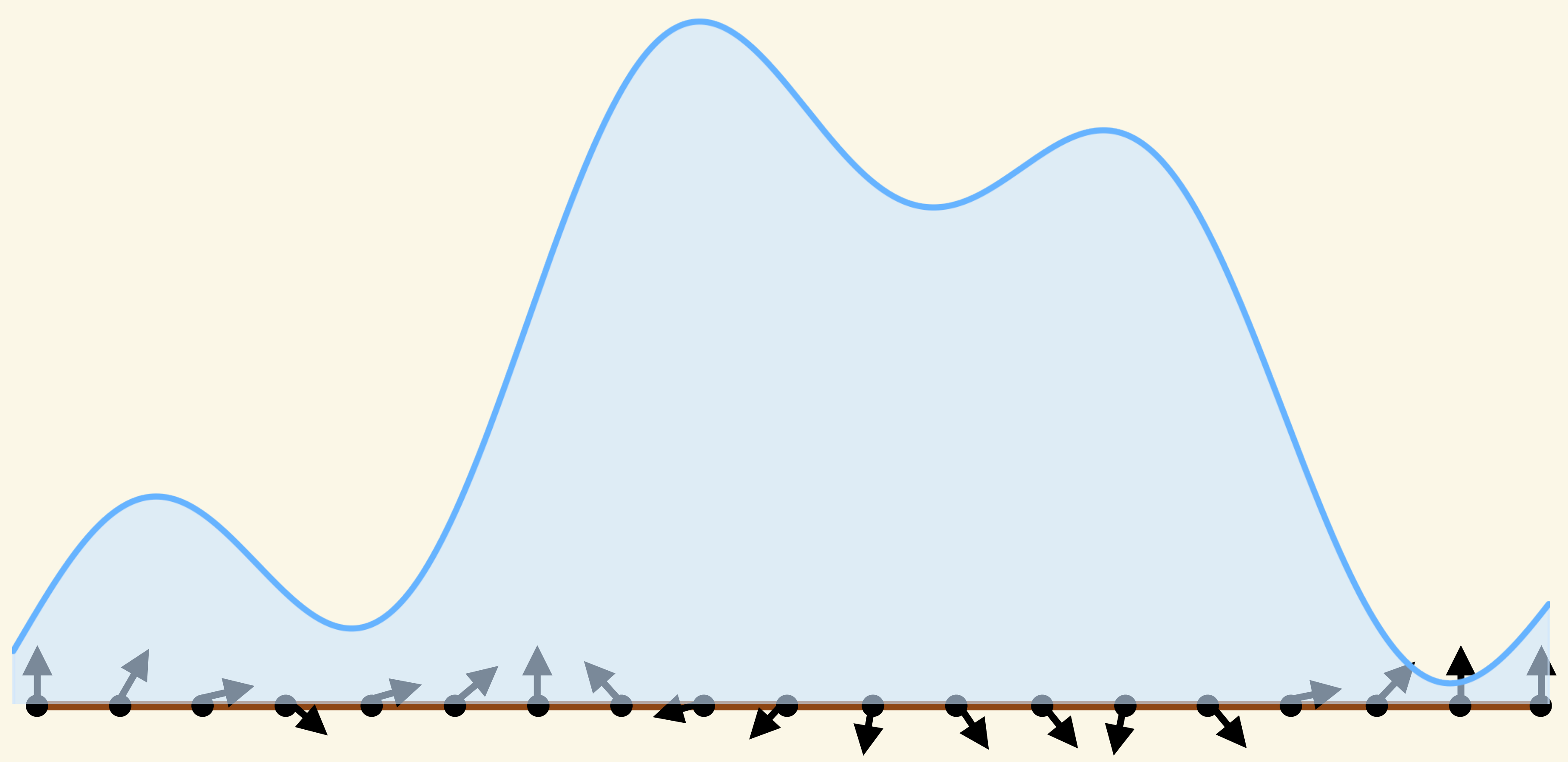

2 + 1D Ising field theory

Focus on 1+1D Ising field theory

1+1D Luttinger liquid

Ising field theory

$$\hat{H} = -\sum \left(\hat{Z}_n \hat{Z}_{n+1} + g_x \hat{X}_n + g_z \hat{Z}_n \right)$$

Field theory limit: $g_x \to 1,\ g_z \to 0$ but $\dfrac{g_x - 1}{|g_z|^{8/15}}$ is fixed

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Field theory limit: $g_x \to 1,\ g_z \to 0$ but $\dfrac{g_x - 1}{|g_z|^{8/15}}$ is fixed

$g_z = 0$ $\qquad$ $g_z/(g_x - 1) = \infty$

free fermion $\qquad$ E_8 theory*

*Zamolodchikov 1989

Building the long-range entanglement in $|\psi_k\rangle$

Initialize: $|k\rangle = |000...001\rangle + e^{ik}|000...010\rangle + e^{2ik}|000...100\rangle + ...$

Entanglement between positions of excitations in $|\psi_k\rangle$

↕

Entanglement between positions of the "1"s in $|k\rangle$

*Dur et. al 2000

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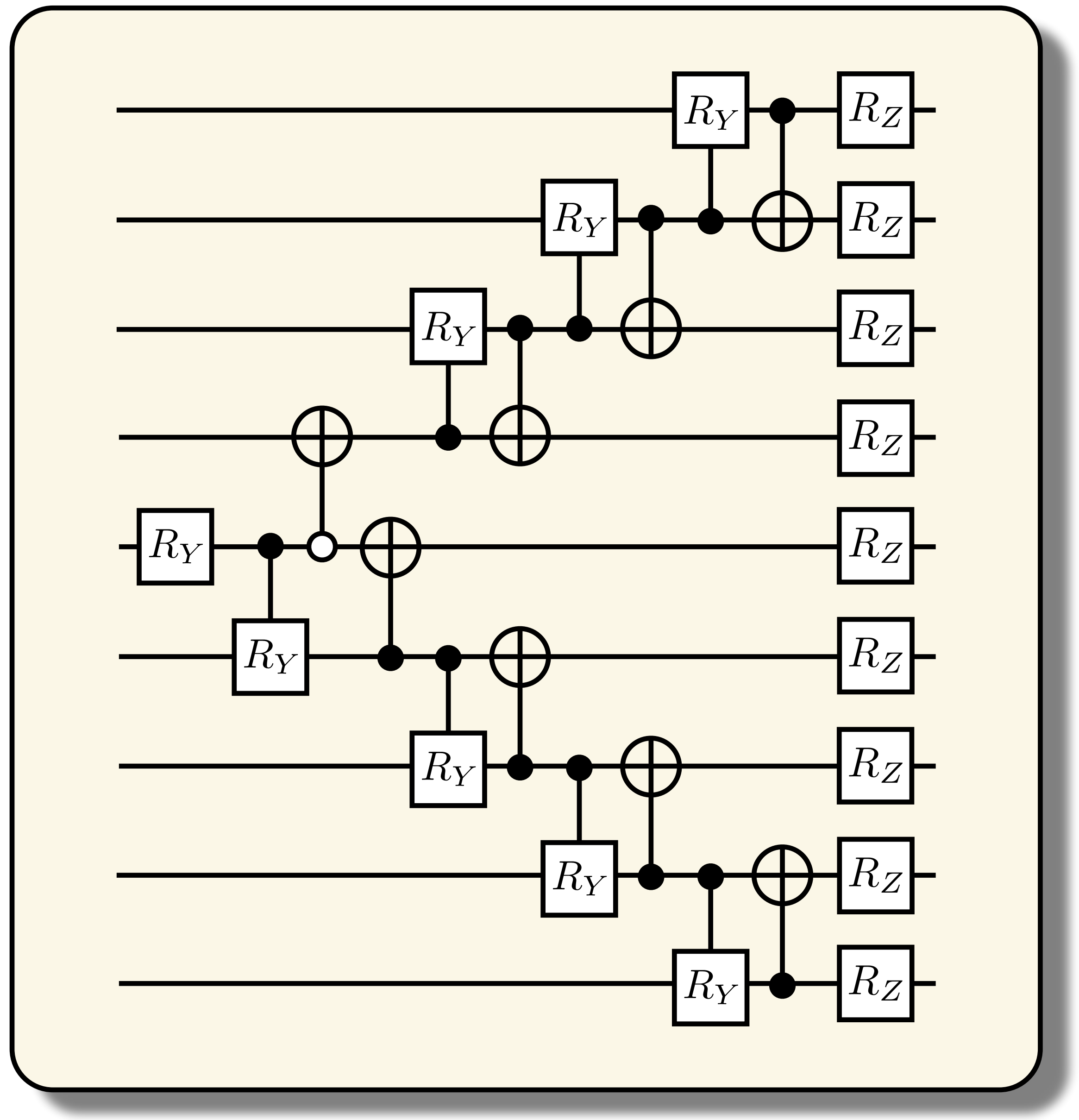
Entanglement between positions of the "1"s in $|k\rangle$

$|k\rangle$ has the same structure as the W-state that is well studied in QIS*

*Dur et. al 2000

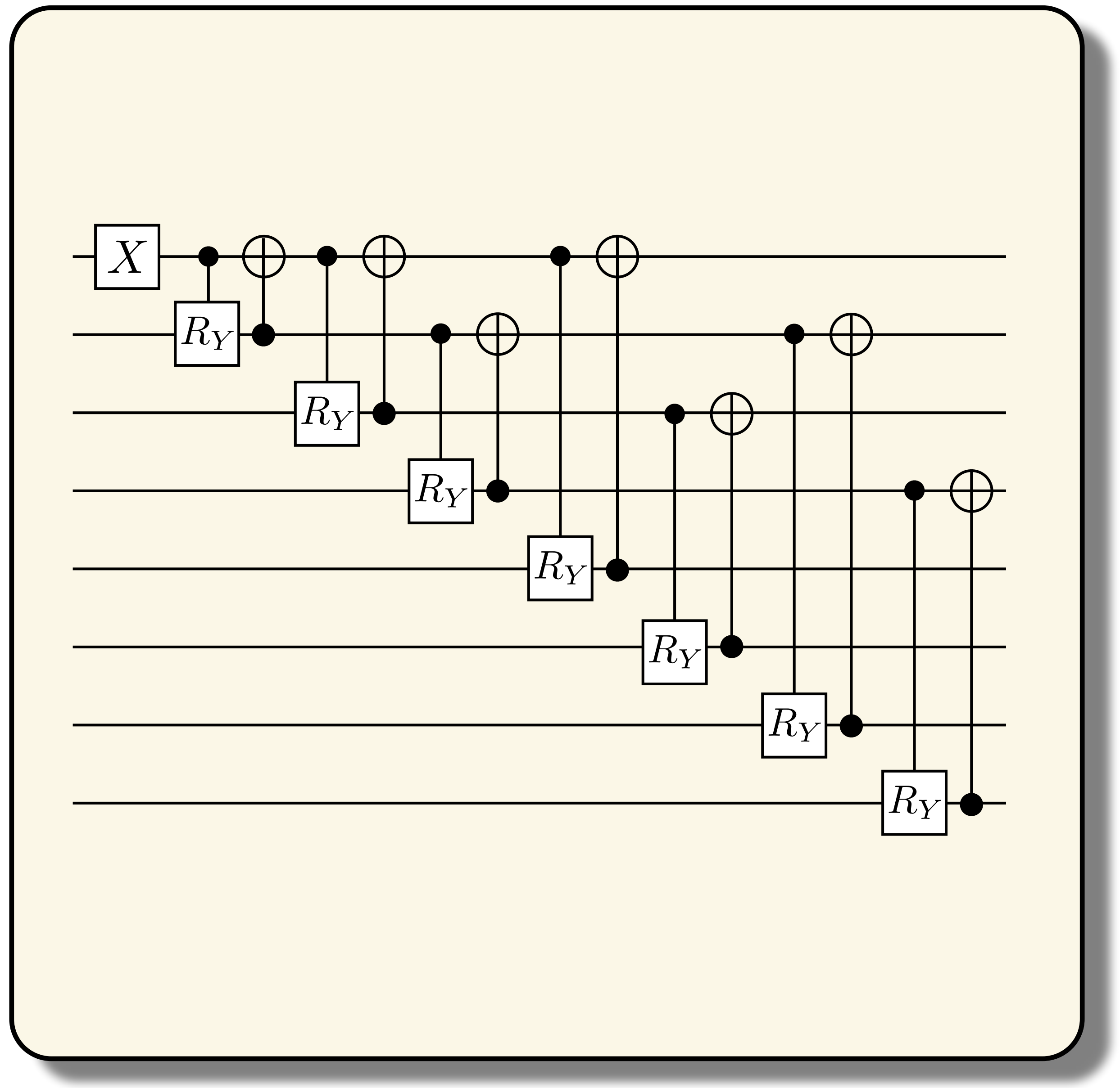
Generalize existing W-state preparation circuits to $|k\rangle = |000...001\rangle \ + \ e^{ik}|000...010\rangle \ + ...$

1D connectivity[1]



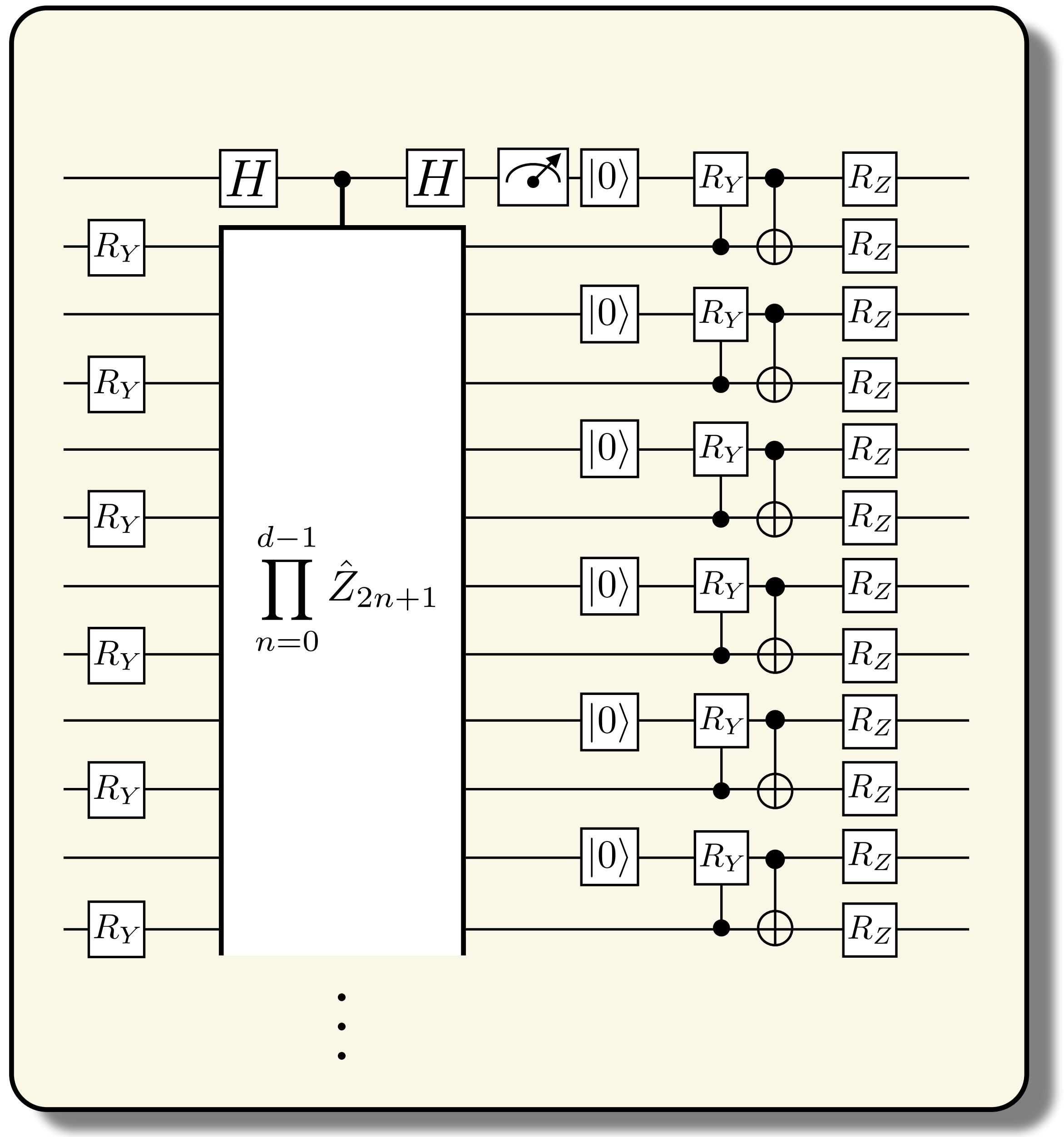
$\text{Depth} = L - 1$

All-to-all connectivity[1]



$\text{Depth} = 2\log(L)$

1D connectivity + MCM-FF[2]



$\text{Depth} = 13$

[1] Cruz et. al 2018
[2] Piroli et. al 2024

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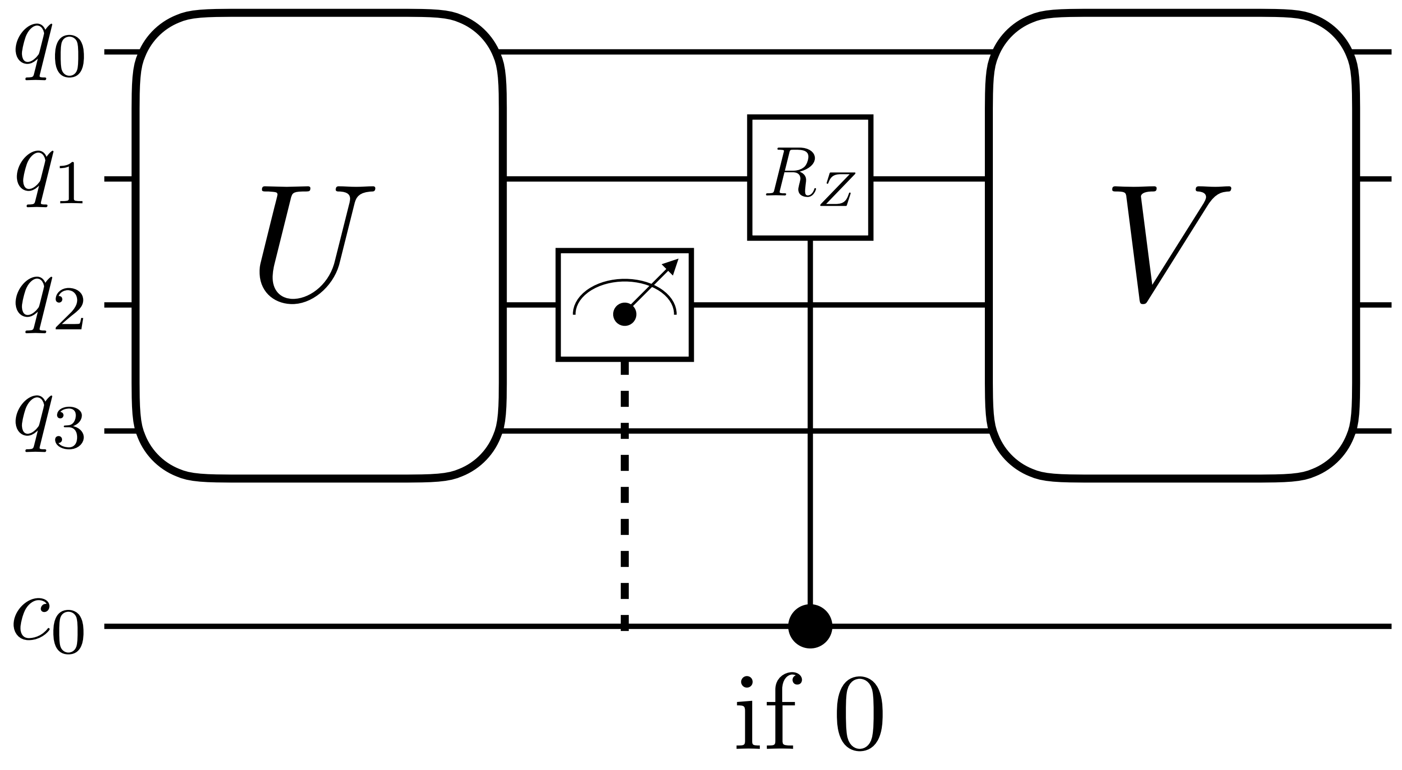


2. Apply quantum circuits that build the short-range $O(\xi)$ entanglement

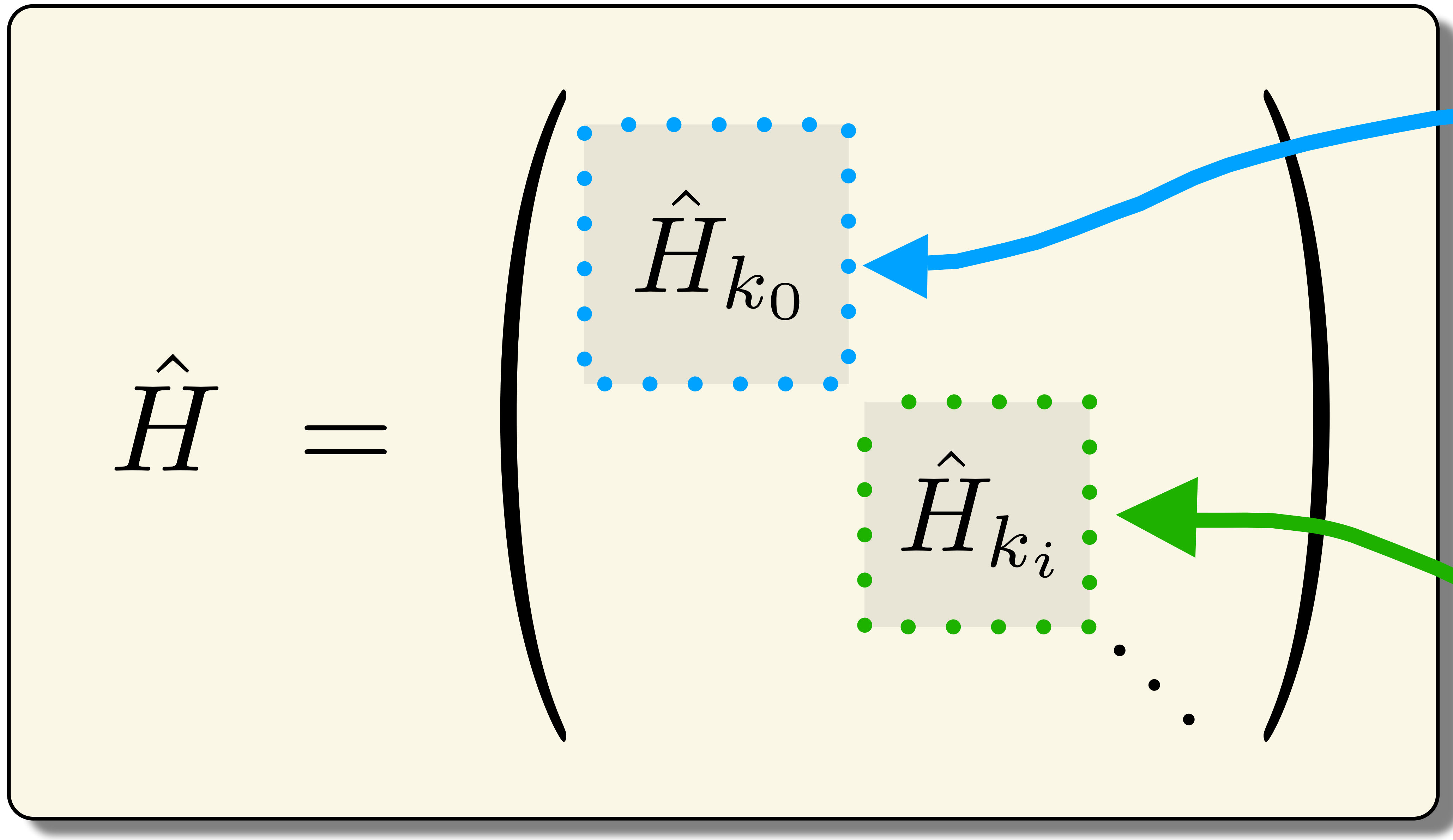
   - Reformulate as an energy minimization problem. Optimize parameterized quantum circuits

Structure of the Hamiltonian

$$\hat{H} = \begin{pmatrix} \hat{H}_{k_0} & & \\ & \hat{H}_{k_i} & \\ & & \ddots \end{pmatrix}$$

Translational invariance implies the Hamiltonian is block-diagonal in momentum space

Structure of the Hamiltonian



The vacuum is the ground state of the $k = 0$ block.

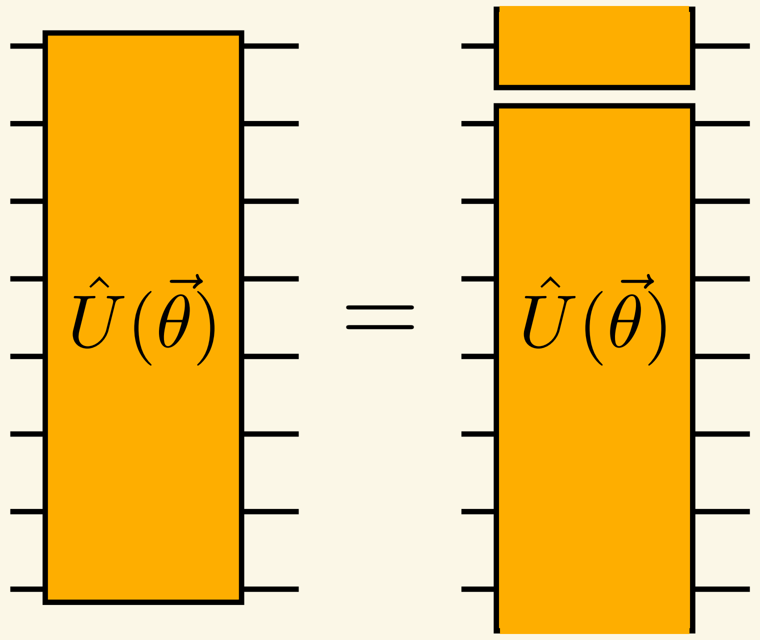
$|\psi_k\rangle$ are the lowest-energy states of the $k \neq 0$ blocks

Translational invariance implies the Hamiltonian is block-diagonal in momentum space

Minimizing energy within a momentum block

1. Initialize $|k\rangle = |000...001\rangle \; + \; e^{ik}|000...010\rangle \; + \; e^{2ik}|000...100\rangle \; + ...$

2. Minimize the energy using translationally invariant circuits



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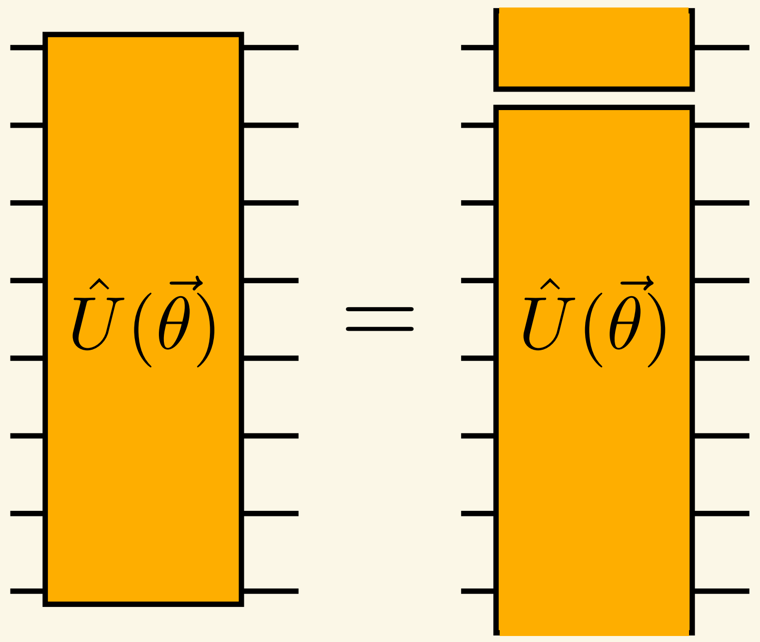

$$|\psi_k\rangle \;=\; U(\vec{\theta}_\star)|k\rangle$$

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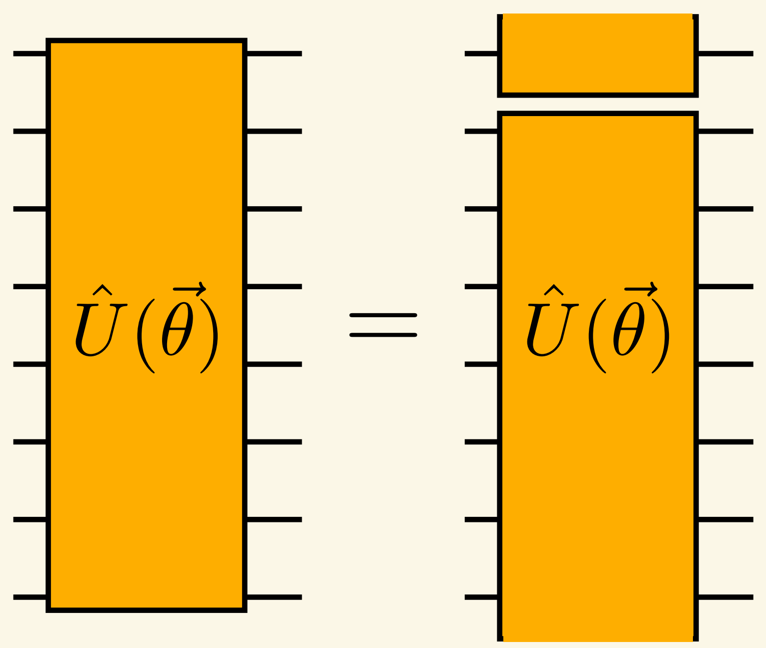

$$|\psi_k\rangle \ = \ U(\vec{\theta}_\star)|k\rangle$$

A similar strategy can be used to prepare wavepackets

Wavepacket preparation

1. Initialize $|W(k_0)\rangle \;=\; \sum_k e^{-(k-k_0)^2/\sigma_p^2}|k\rangle \;=\; c_0|00...01\rangle + c_1|00..10\rangle + \ldots$

Wavepacket preparation

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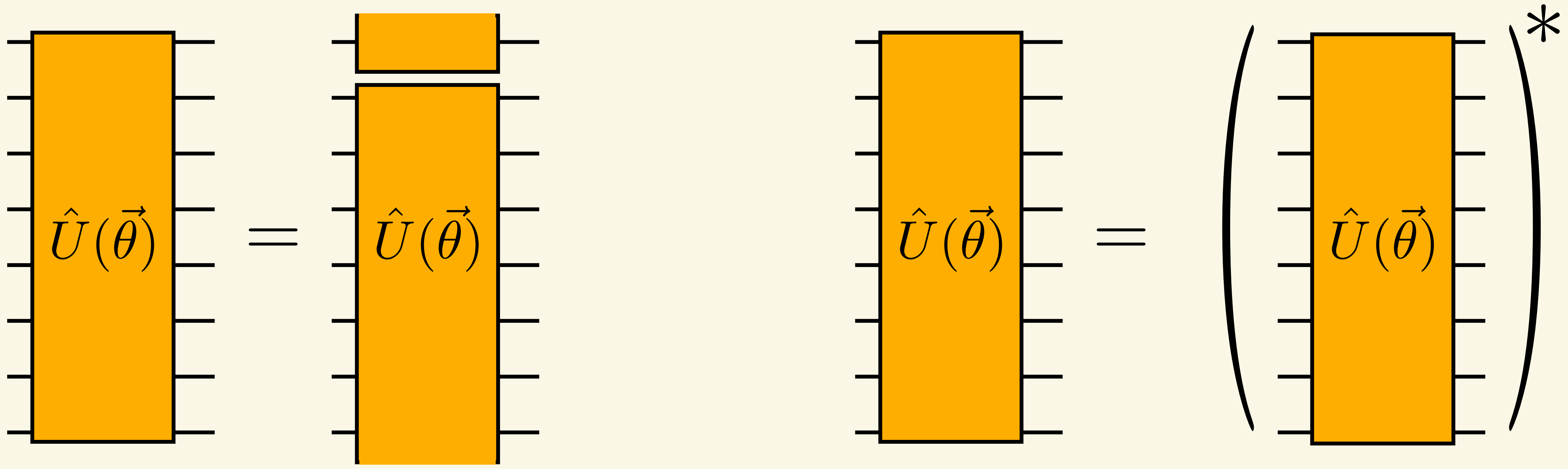
2. Minimize the energy using translationally invariant & real circuits



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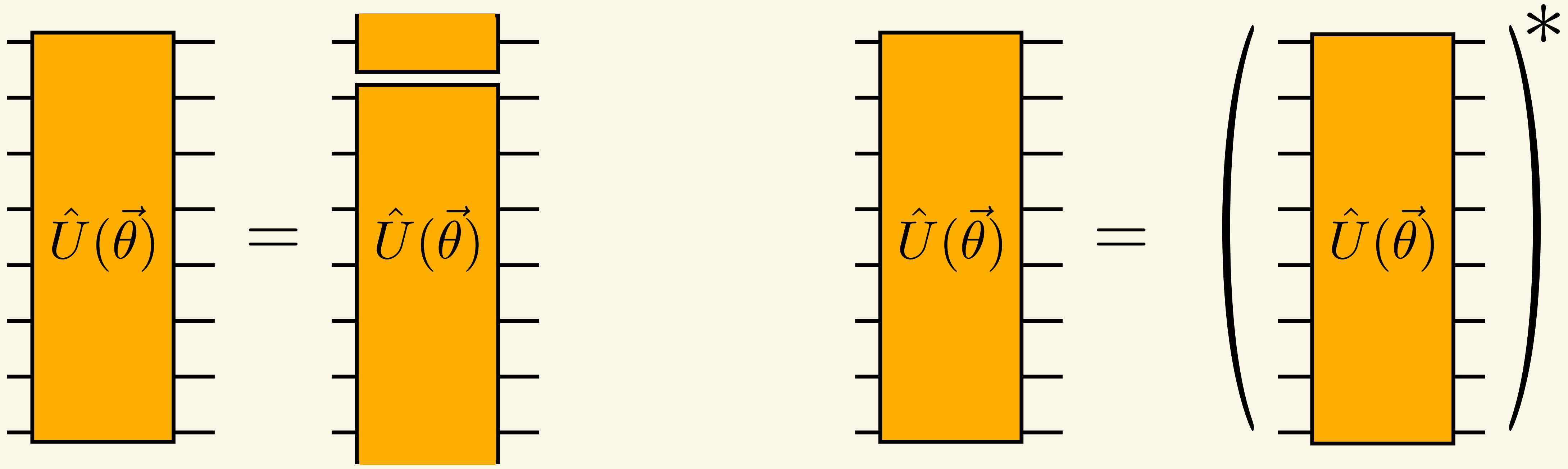
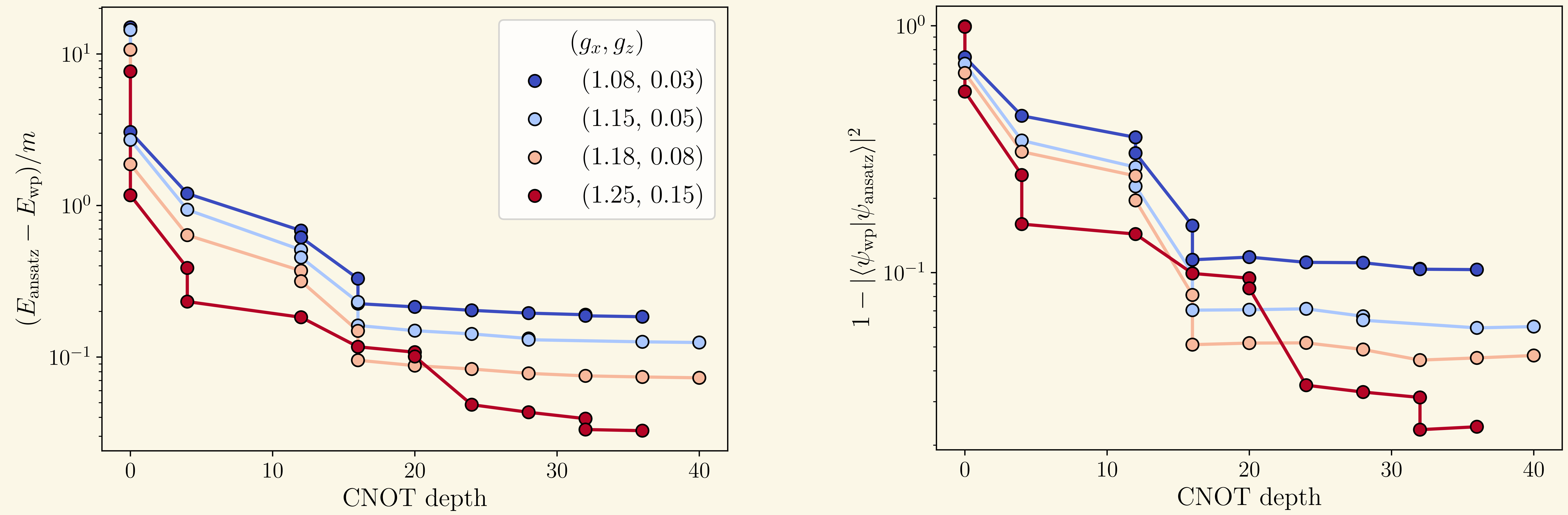


$$|\psi_{\mathrm{WP}}\rangle = U(\vec{\theta}_\star)|W(k_0)\rangle$$

Statevector simulations on $L = 28$ qubits

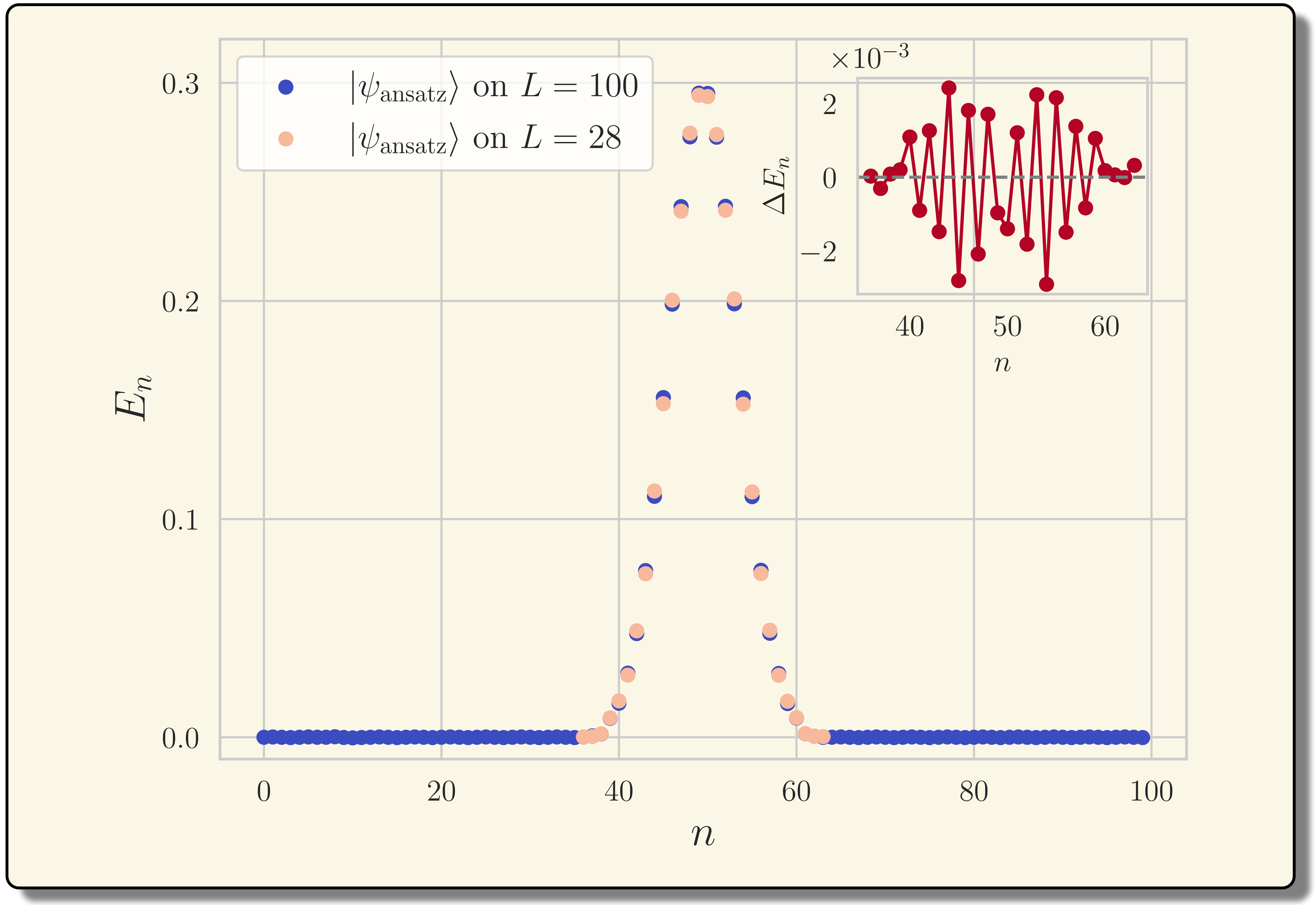
$$\hat{H} = -\sum \left(\hat{Z}_n \hat{Z}_{n+1} + g_x \hat{X}_n + g_z \hat{Z}_n \right)$$



Energy minimization using ADAPT-VQE* maximizes the overlap of $|\psi_{\mathrm{ansatz}}\rangle$ with $|\psi_{\mathrm{WP}}\rangle$

*Grimsely, Barnes, Economou, Mayhall (2019)

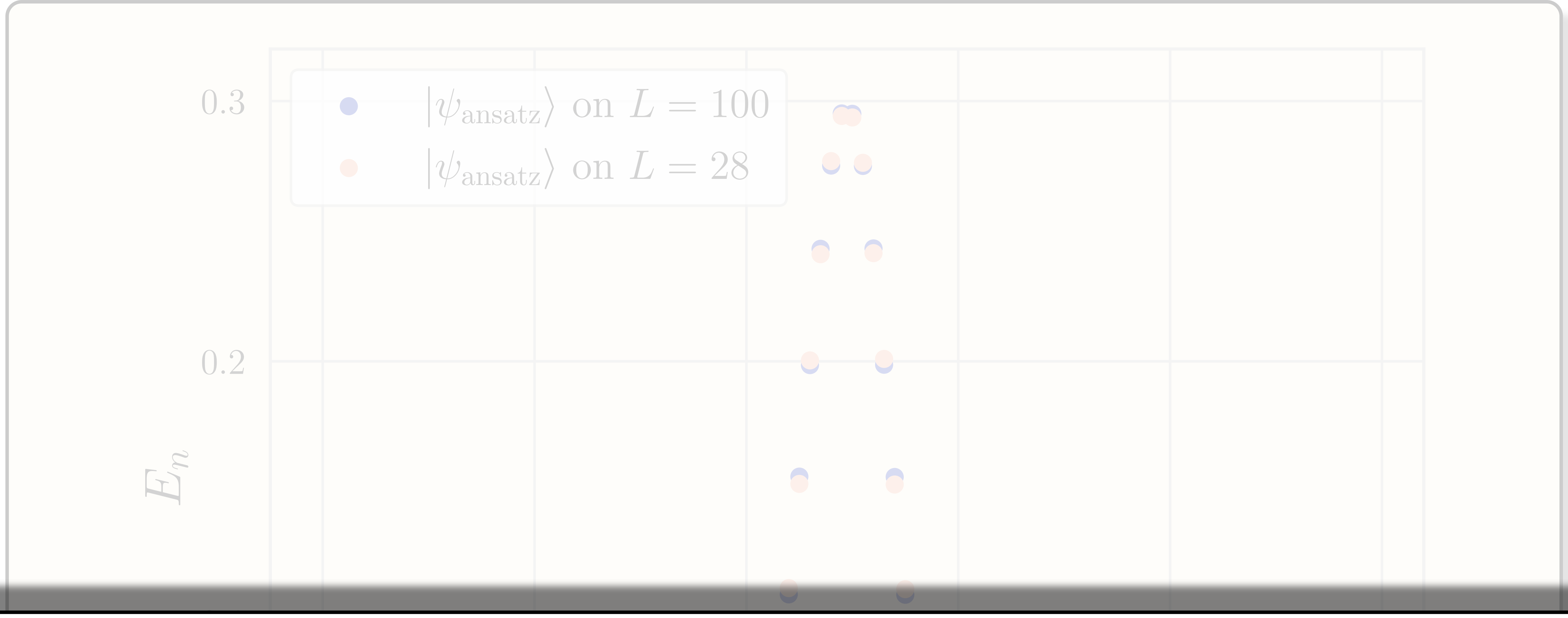
Wavepacket preparation on 100+ qubits



Circuits that minimize the energy and prepare wavepackets are determined on large lattices using a MPS circuit simulator

QUIMB

Wavepacket preparation on 100+ qubits



Next talk, Nikita will use these circuits to prepare wavepackets and simulate scattering on 104 qubits of IBM's quantum computers

Circuits that minimize the energy and prepare wavepackets are determined on large lattices using a MPS circuit simulator

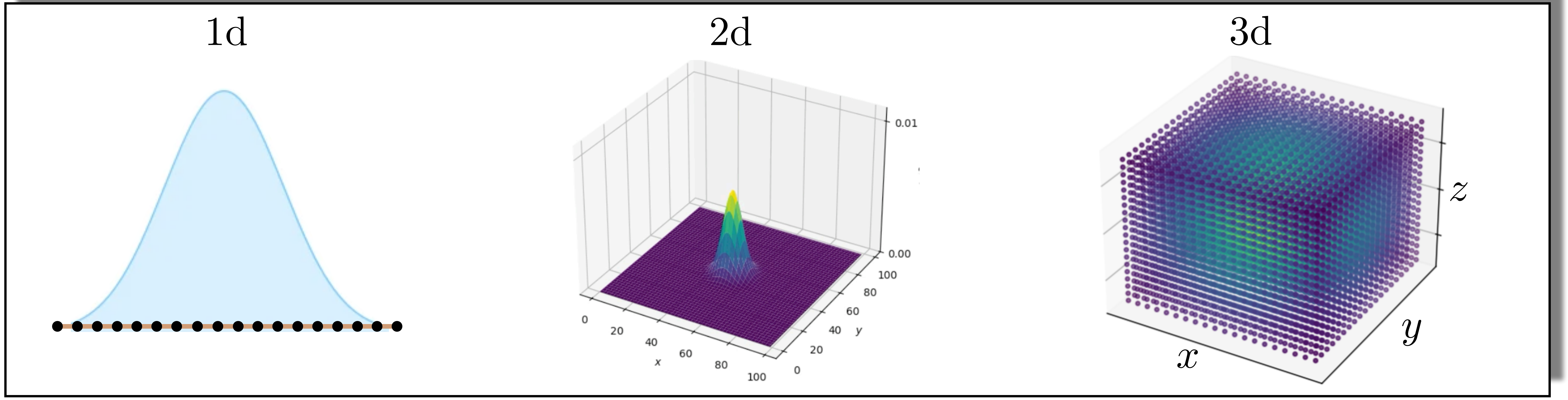
Summary

- Momentum eigenstates have long-range entanglement that must be considered when preparing wavepackets on a quantum computer
  - This entanglement is also present in the W state that can be prepared in constant depth using MCM-FF
- After building short-range correlations by optimizing variational quantum circuit to minimize the energy

Questions

Is the long-range entanglement of all gapped systems equivalent to the W state under local operations?

Will wavepacket preparation[1,2,3] ever be the bottlekneck?



Wavepacket size: $N_{\mathrm{WP}} \sim \left(\frac{1}{\sigma_p}\right)^d$ Max evolution time: $t_{\max} \approx \text{constant}$

[1] Chai et. al 2025
[2] Davoudi, Hsieh, Kadam 2025
[3] Jordan, Lee, Preskill 2011

Thank you for listening!

Nikita Zemlevskiy

Marc Illa

John Preskill

For more details see our paper on arxiv: 2505.03111