Explorations of Fully Gauge-Fixed SU(2)

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They/Them

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Motivation

Studying the properties of strongly coupled theories from first principles is necessary to fully understand the Standard Model

Rich phenomena of non-perturbative quantum field theories is a profitable place to look for new answers to the big questions

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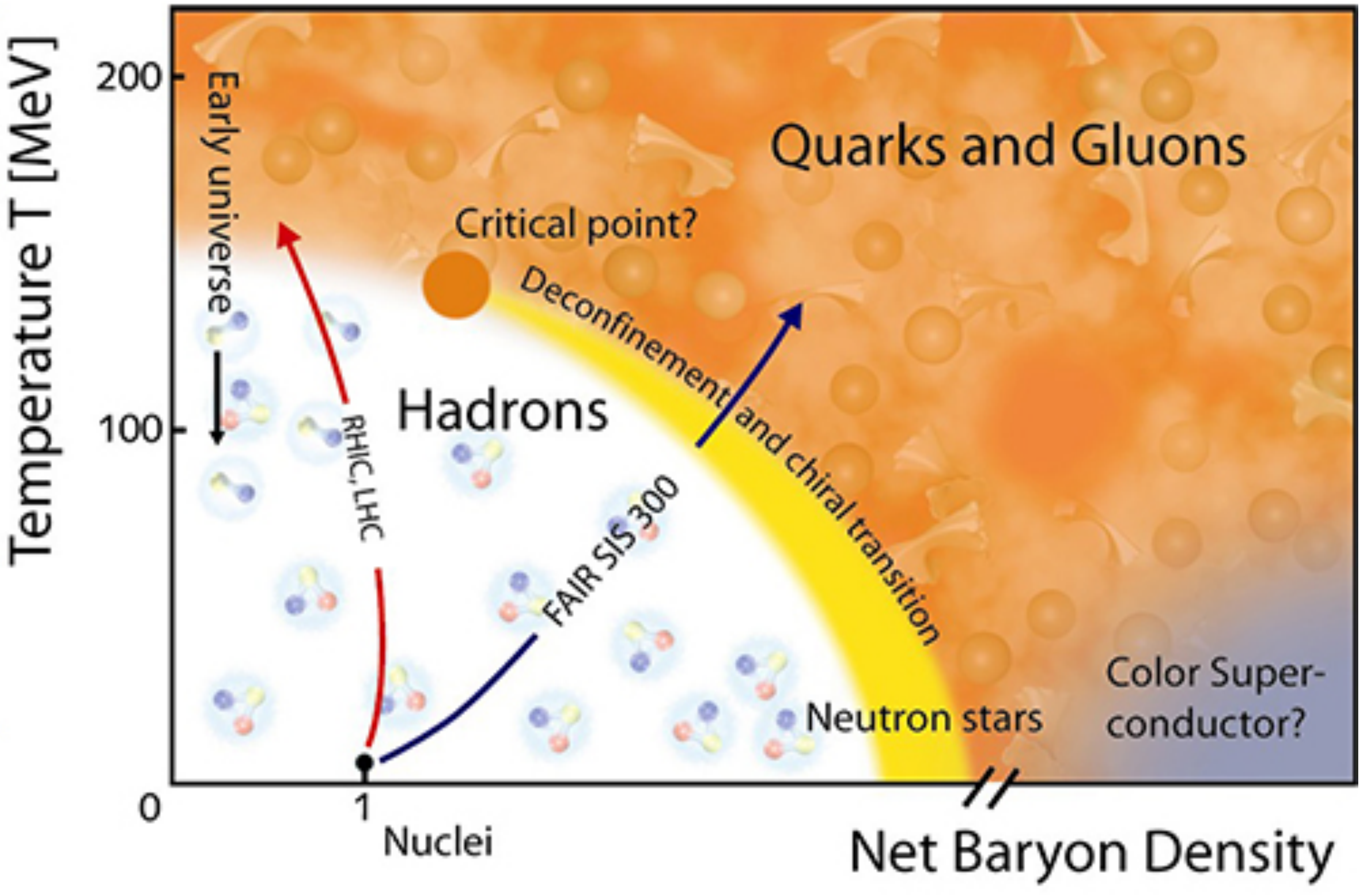

Proposed QCD Phase Diagram

Phys Rev D 11, 395 (1975)

Hamiltonian Lattice Gauge Theory, Abelian

Quantum simulations utilize Hamiltonian formulations

- Continuous time, but discrete space
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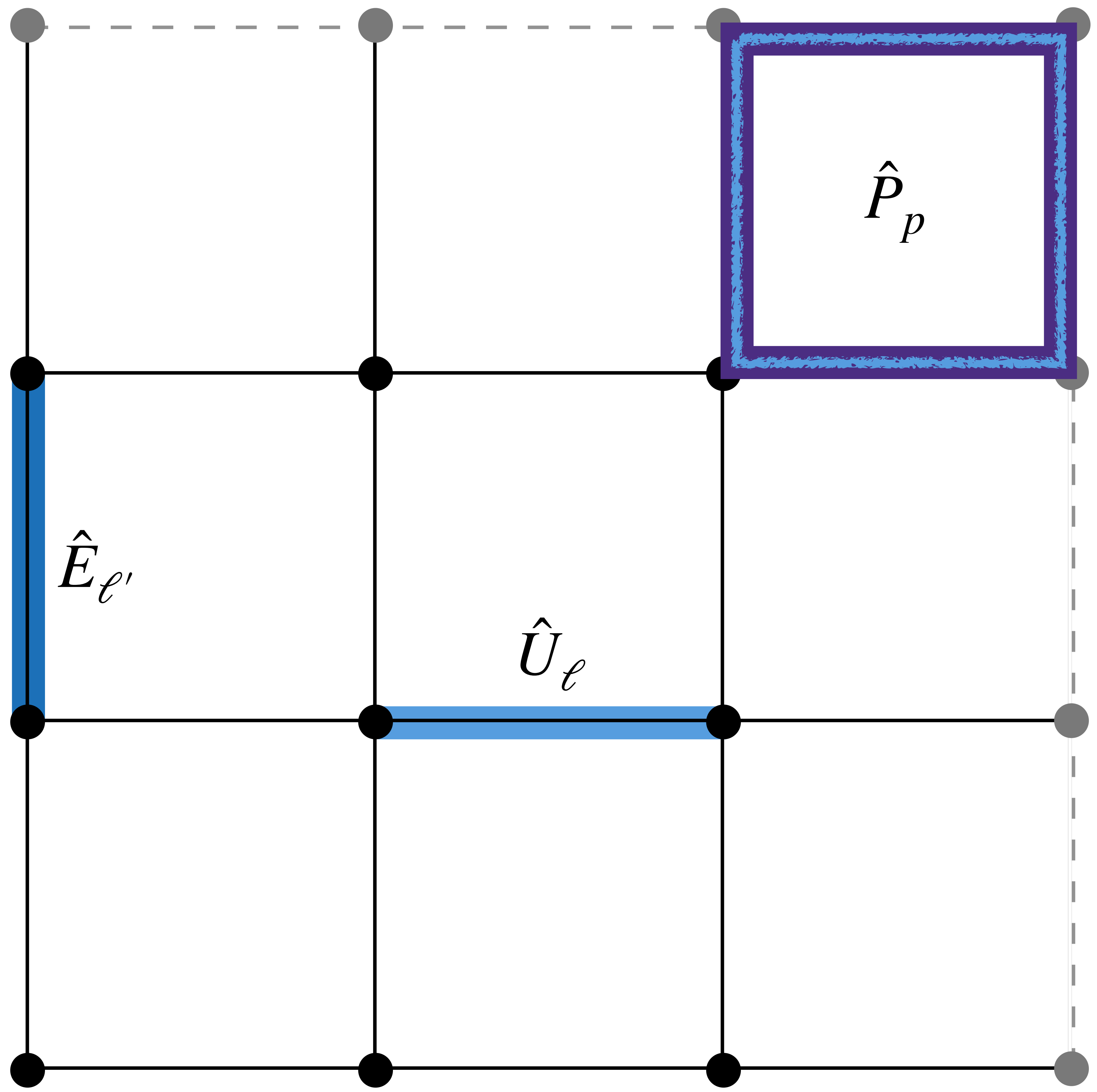

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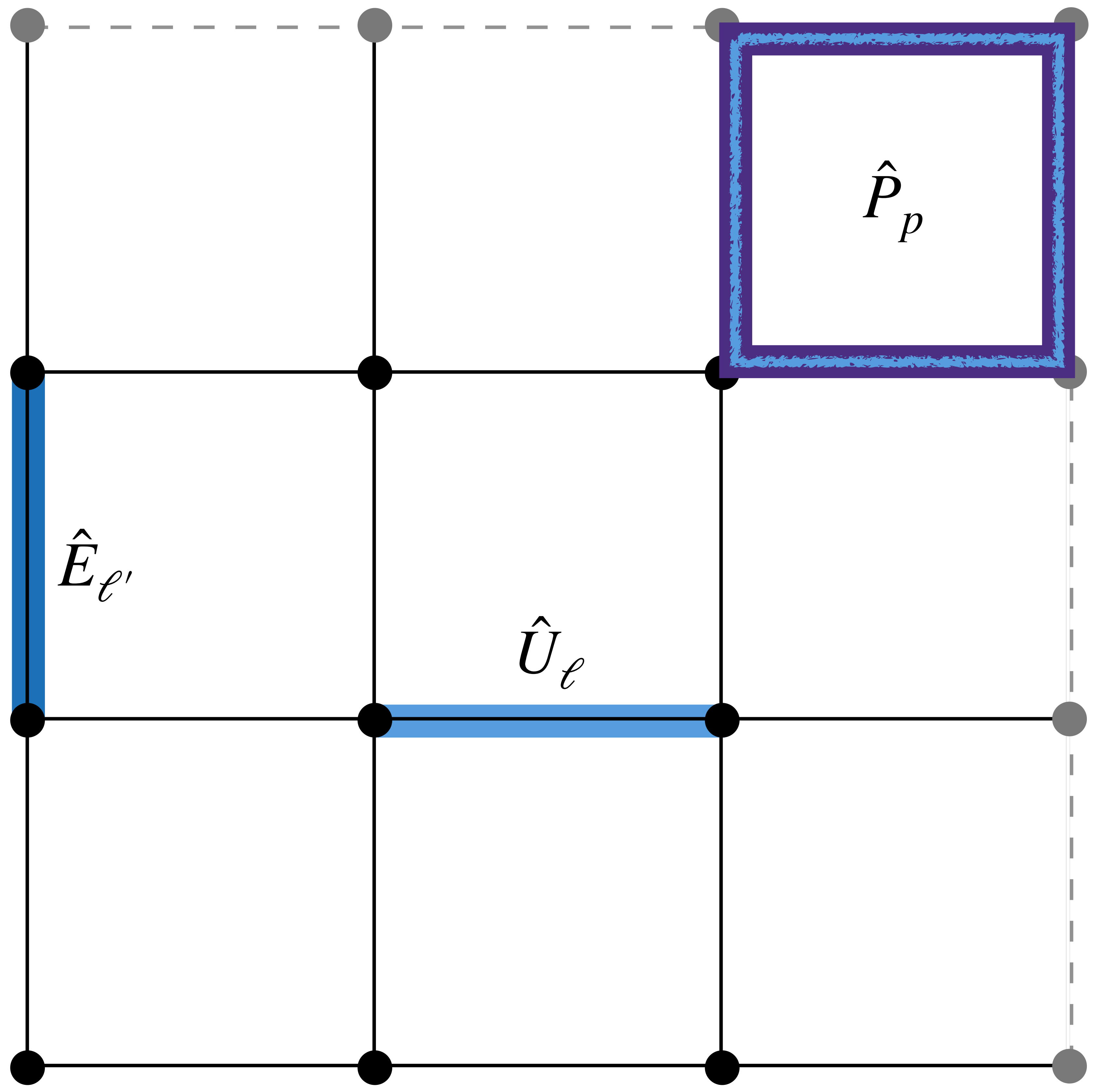

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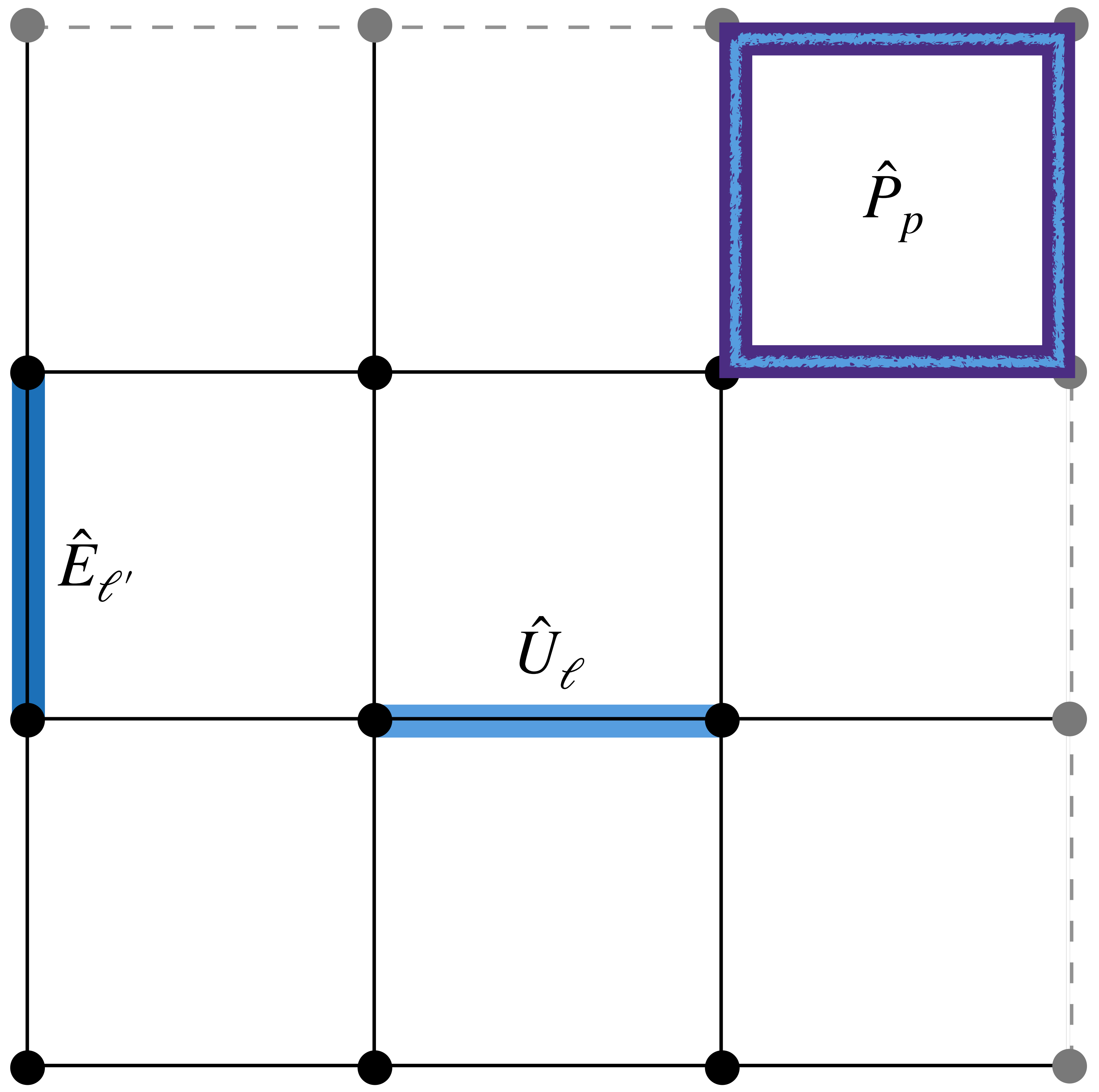

These define the theory and therefore the circuit

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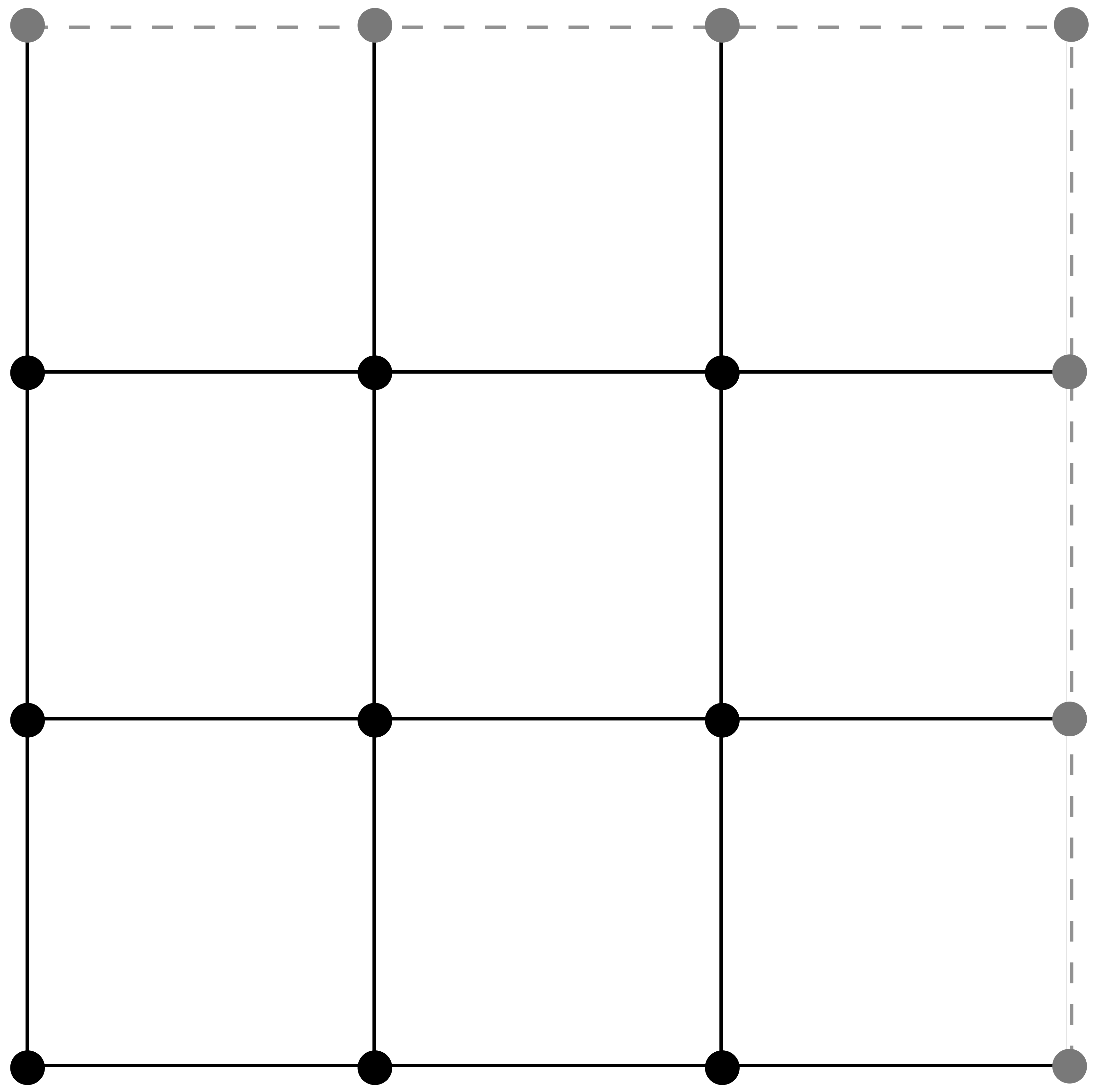

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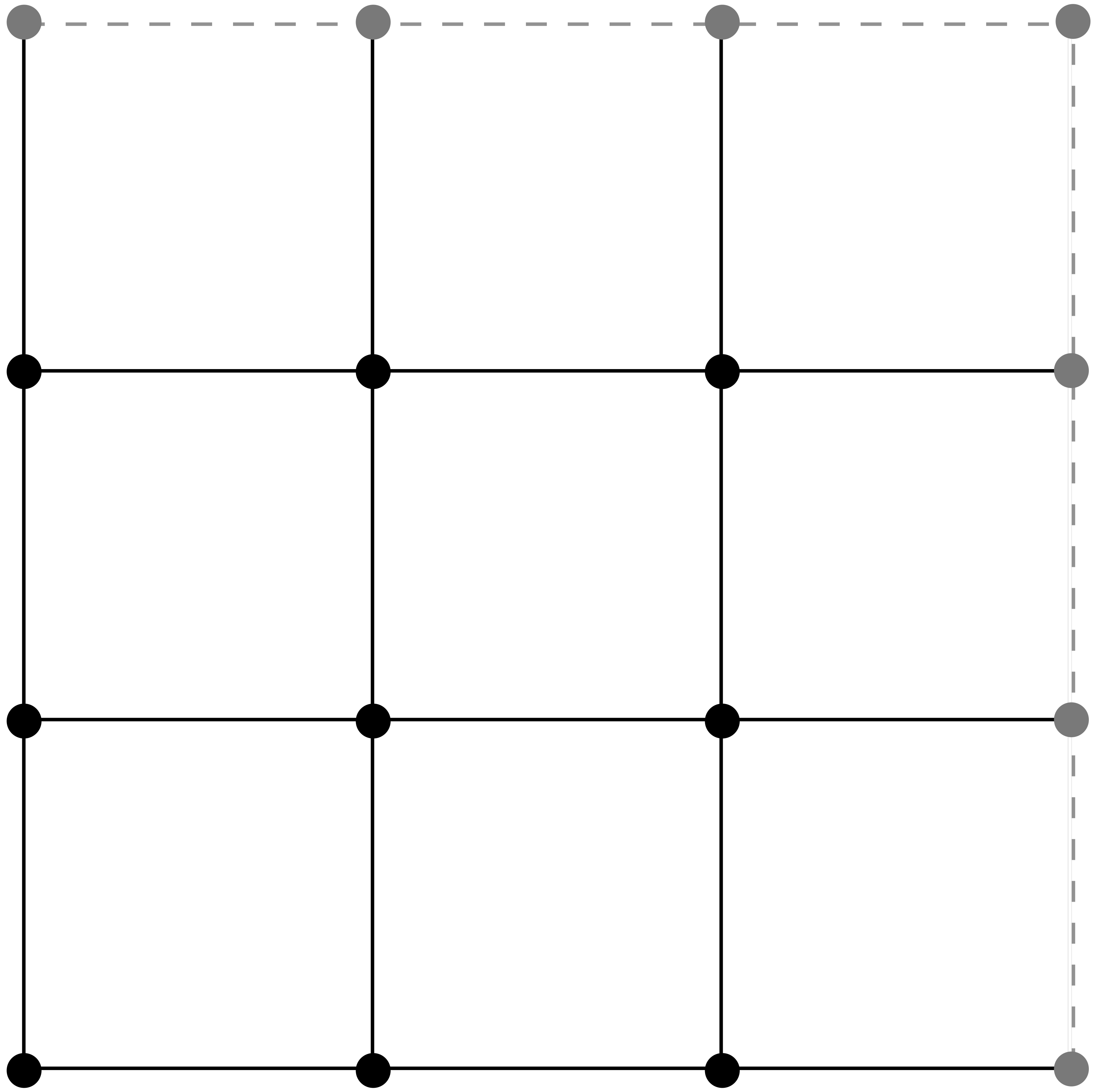

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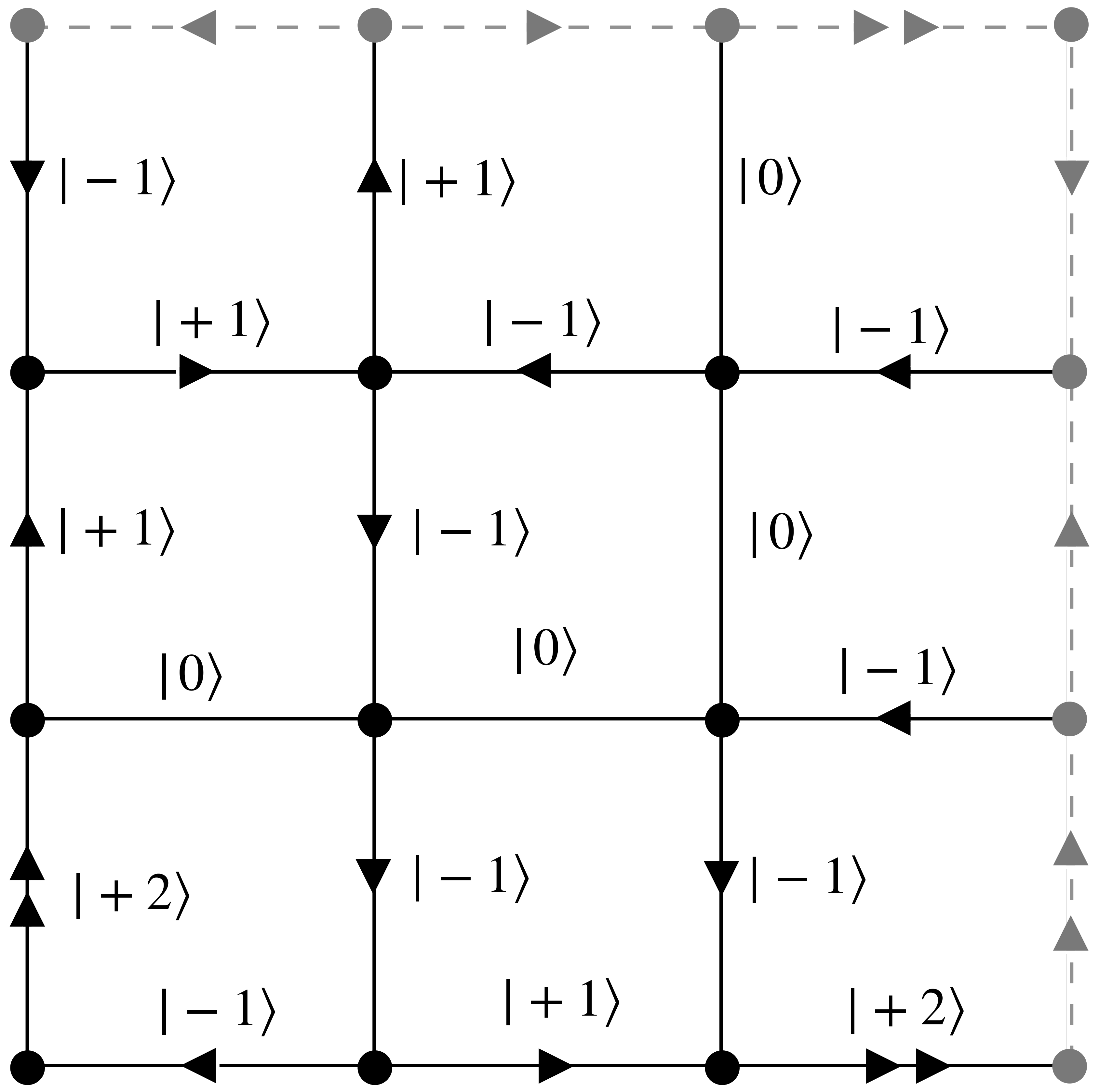

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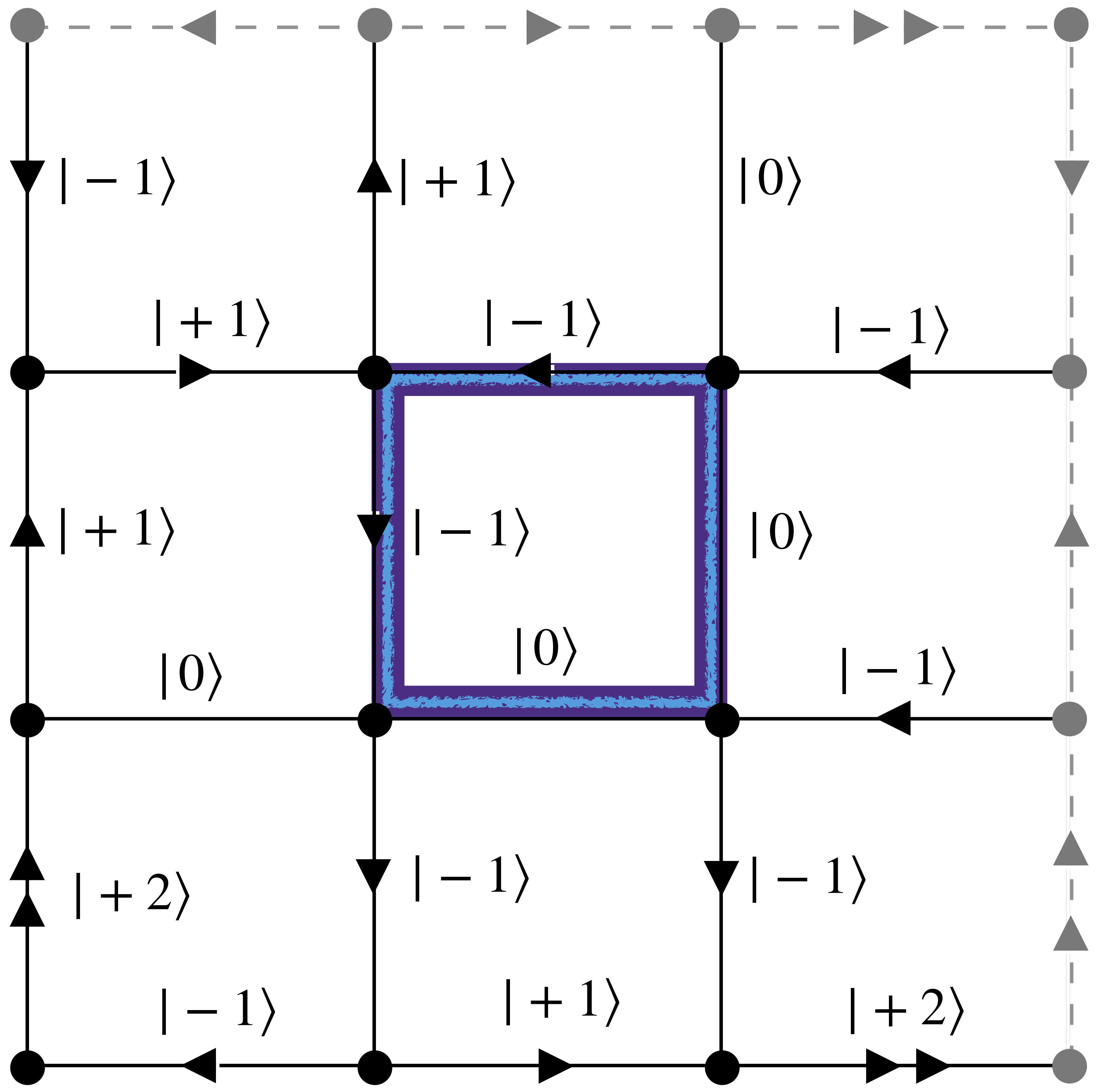

Action of plaquette on a given state

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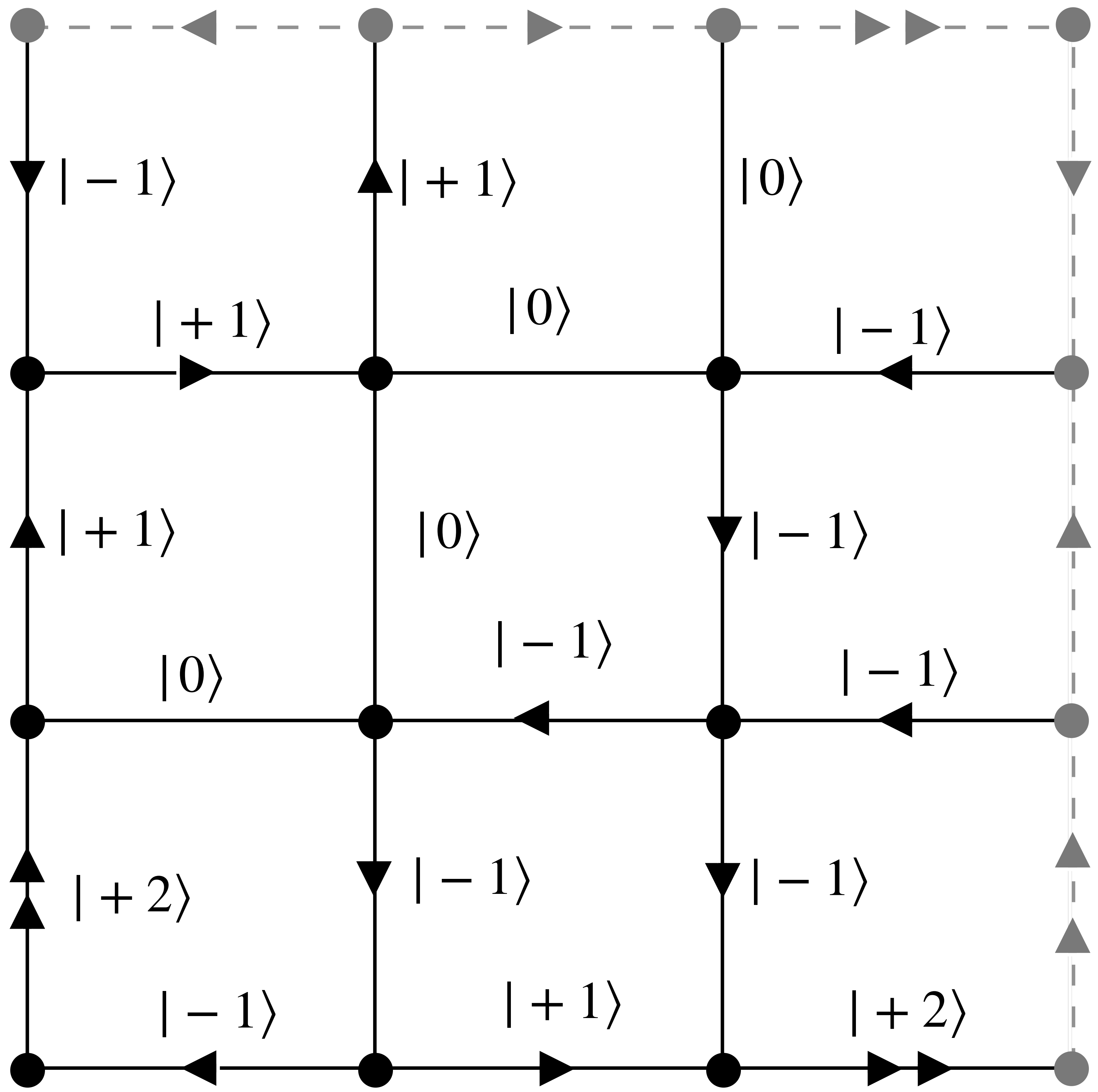

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General Idea: Similar to Abelian, but electric and gauge link operators carry color indices

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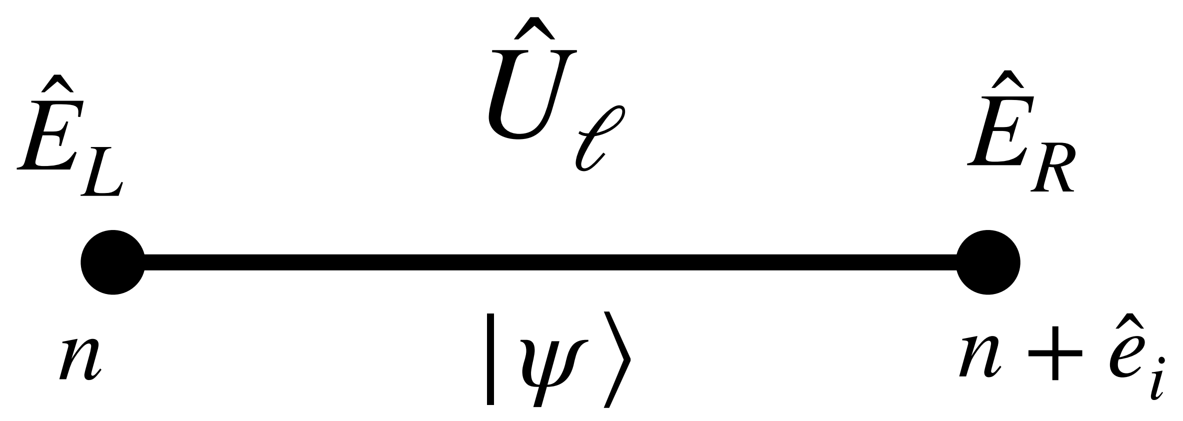

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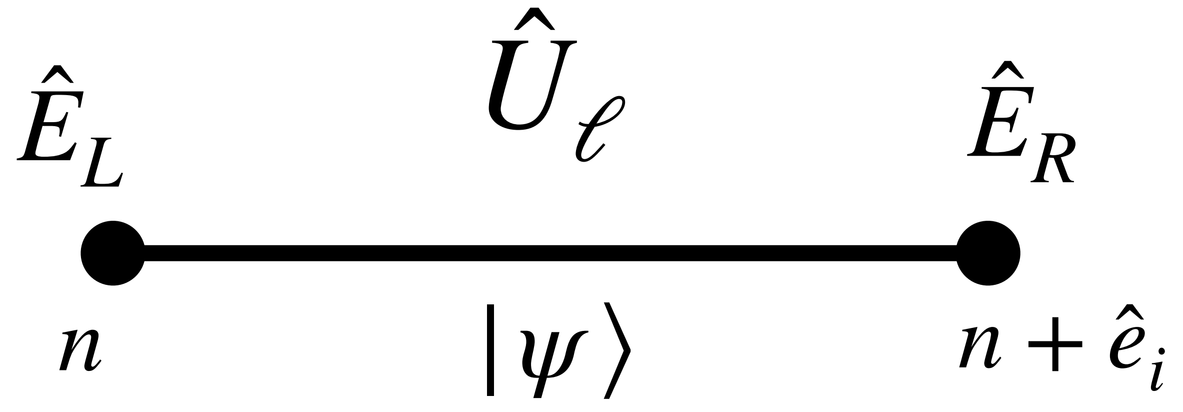

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So what is the obstacle to quantum simulation?

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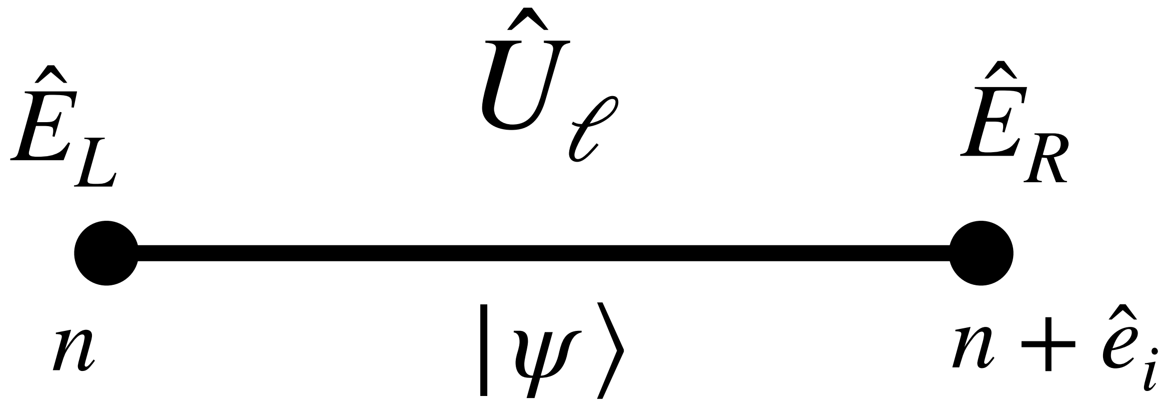

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- *Finite-dimensional Hamiltonian needs to faithfully capture desired physics*
- *Akin to UV regularization of Lagrangian methods*

B) Phenomenologically-relevant gauge groups are continuous

- *"Sampling" method needs to preserve gauge structure of the theory*

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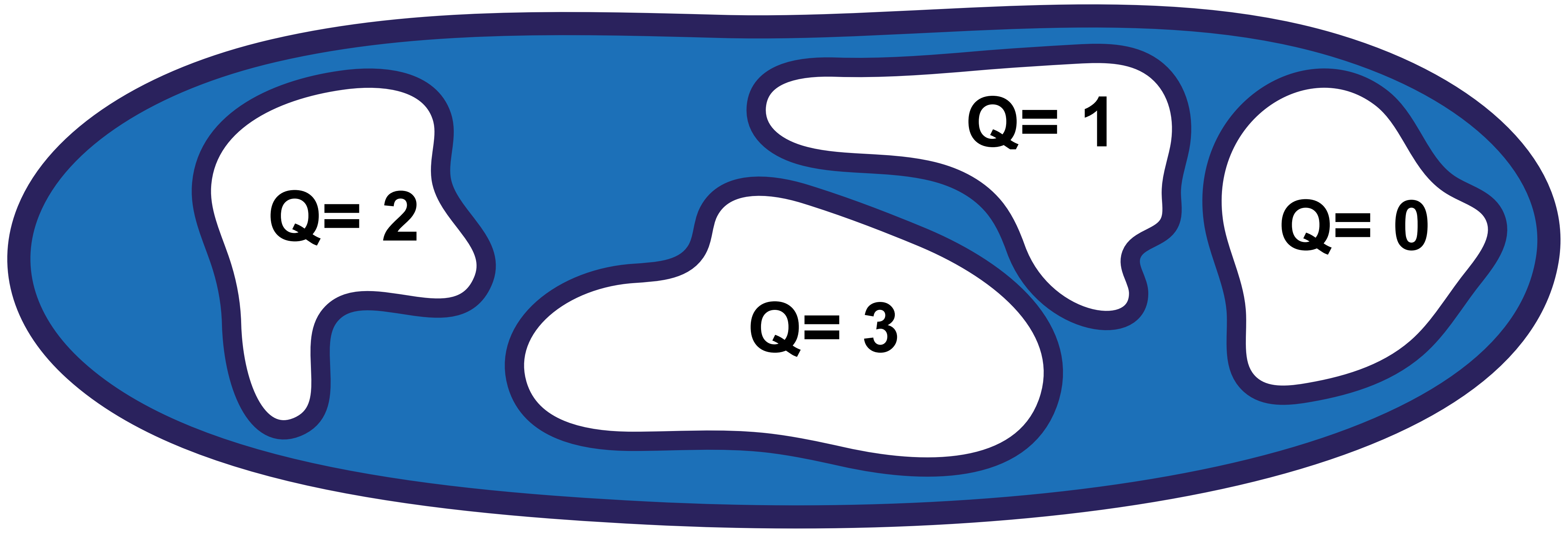
C) Gauss Law is not automatically satisfied

- *Gauss's law is the constraint associated with the* A_0 *Lagrange multiplier*
- *Naive Hilbert space is tensor product of different charge sectors*



Desired Properties of Formulations

Motivation: “Ideal” formulation has these three properties

Gauge Invariant

Systematically Improvable

Efficient for Fine Lattices

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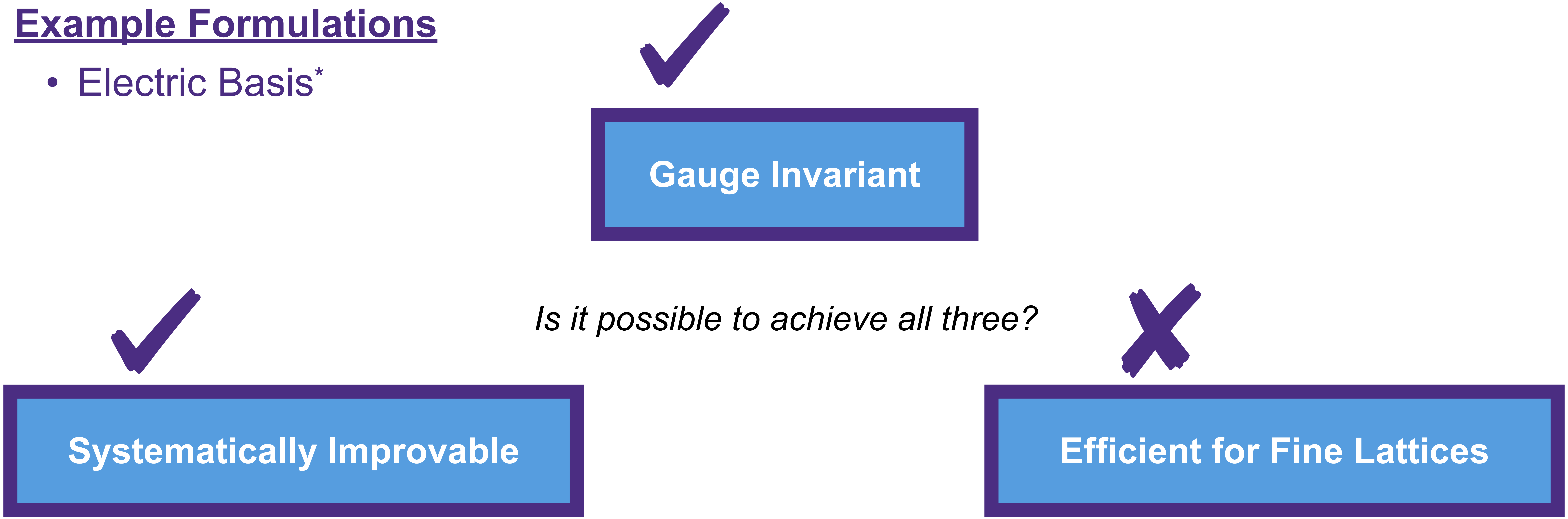

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** Phys. Rev. D 105, 114501 (2022) and subsequent work

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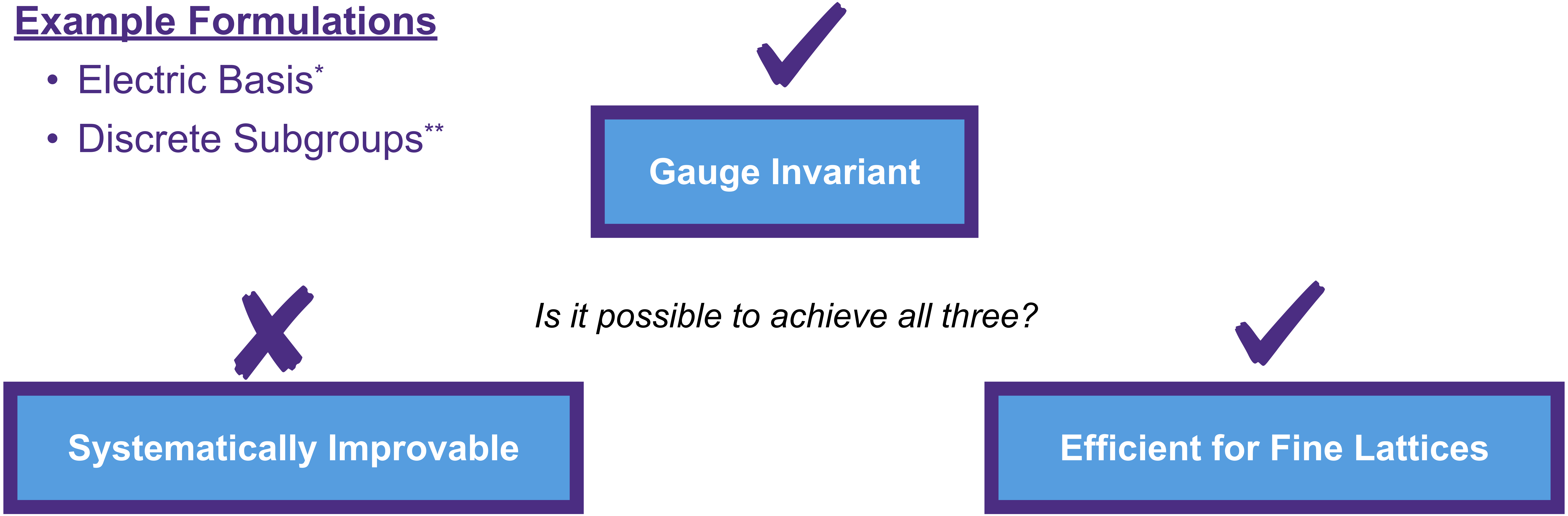

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Efficient for Fine Lattices

Unfortunately achieving this trifecta has proven quite challenging

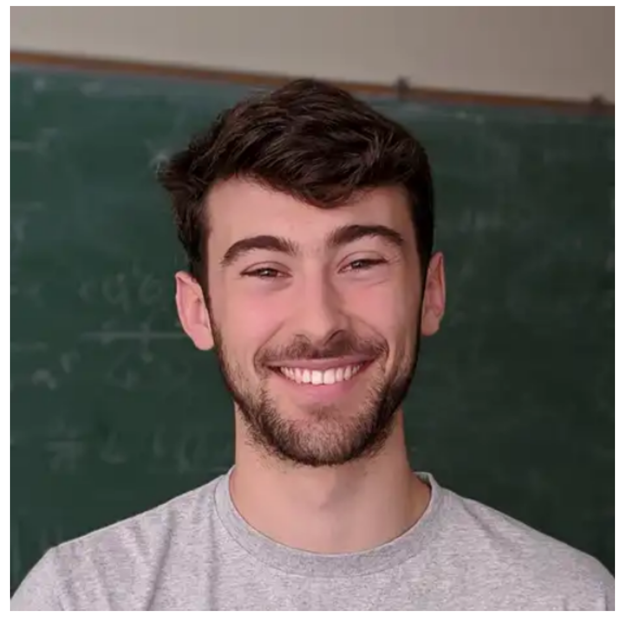
Fully Gauge-Fixing SU(2) in 2+1 and 3+1 Dimensions

Bauer, D'Andrea, Freytsis and **DMG**, *Phys.Rev.D 109 (2024) 7, 074501*

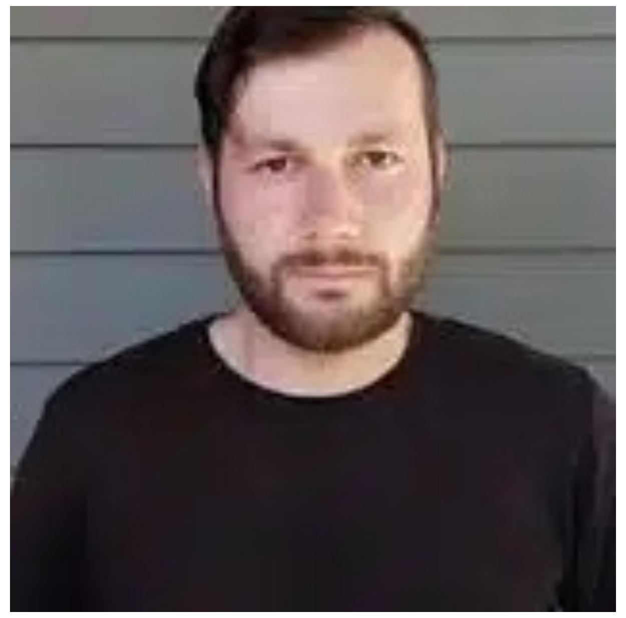
DMG, *Kane and Bauer, Phys.Rev.D 111 (2025) 11, 114516*

Christian Bauer

Irian D'Andrea



Chris Kane



Marat Freytsis

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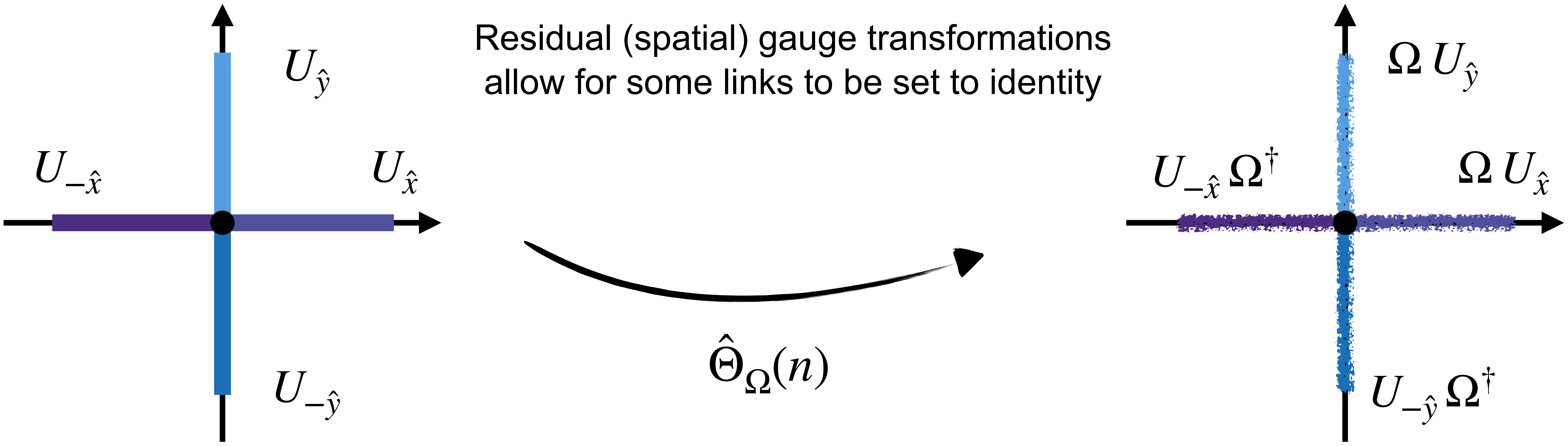
General Idea



Residual (spatial) gauge transformations allow for some links to be set to identity

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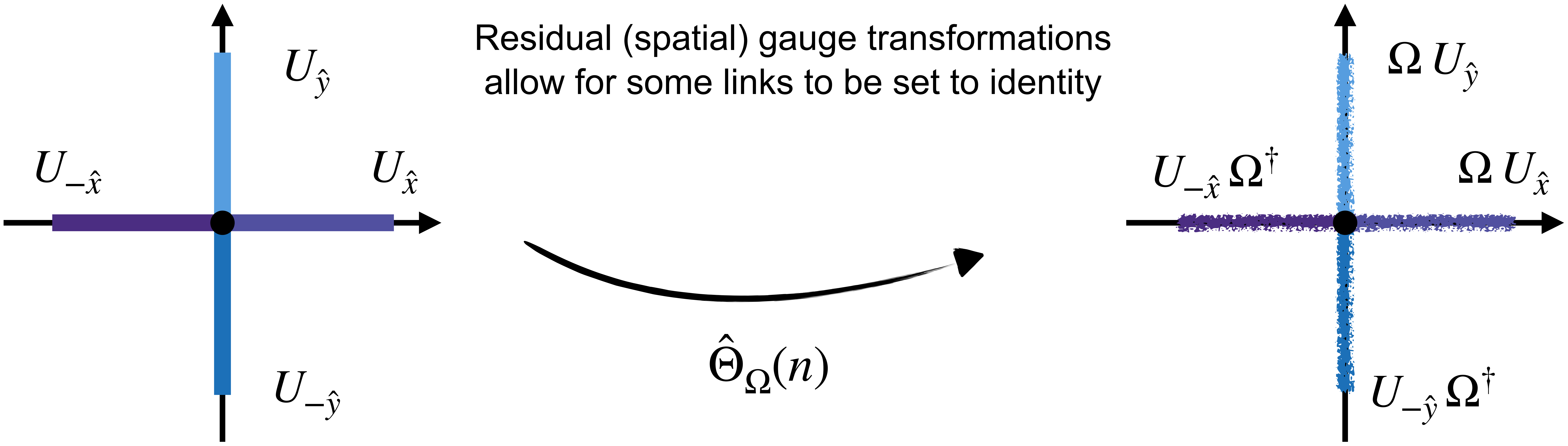


Residual (spatial) gauge transformations allow for some links to be set to identity

Not all gauge links can be set to the identity as gauge transformations affect neighboring links

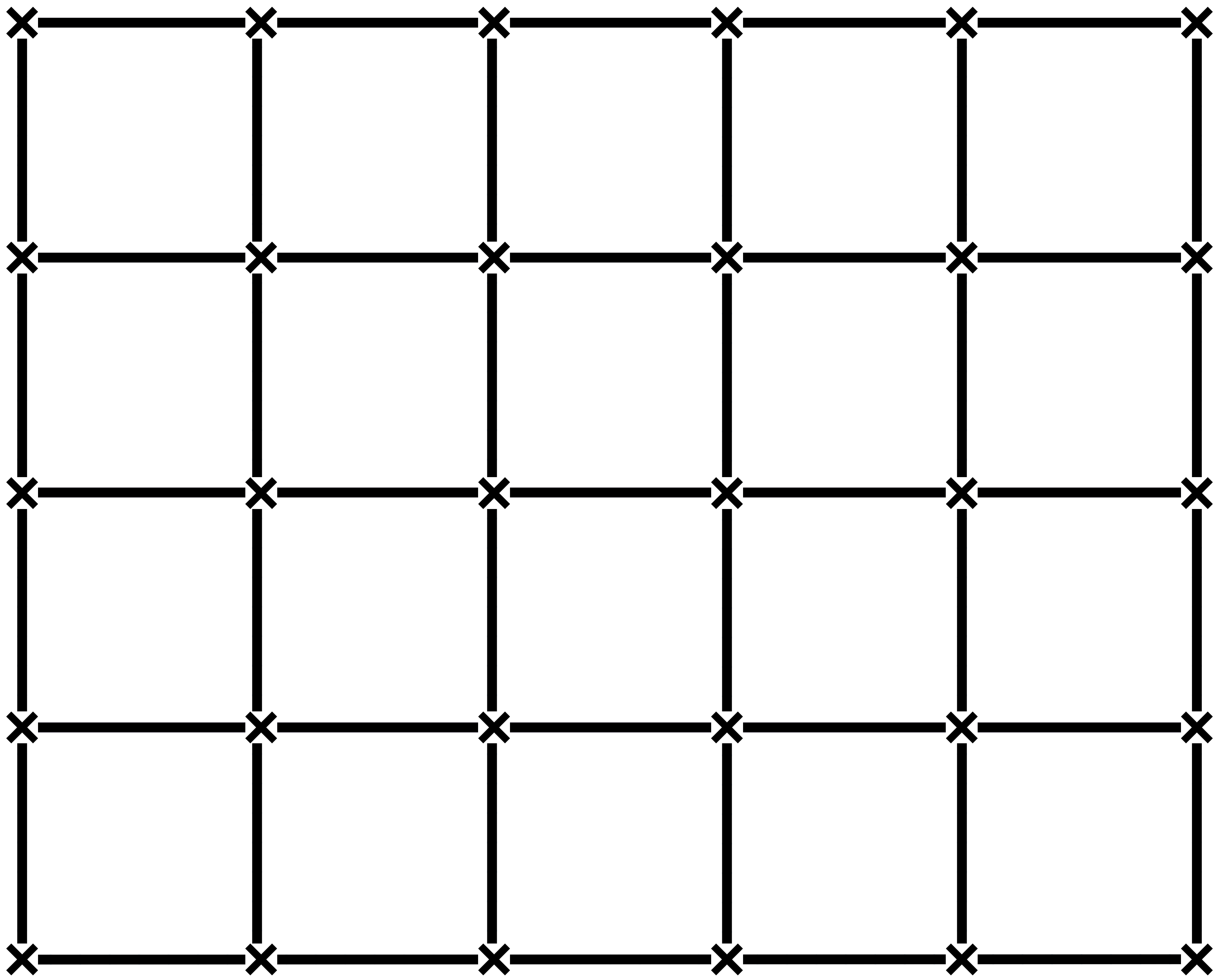
Phys. Rev. D 15, 1128 (1977)

Step One: Gauge-Fixing Procedure

Motivation: Gauge fixing allows for “importance sampling” when working in magnetic basis without worrying about breaking gauge-invariance

General Idea: Maximal-tree procedure provides a systematic method for determining which links can be eliminated

- Tree links: unphysical links that can be set to the identity
- Physical links: all other remaining links



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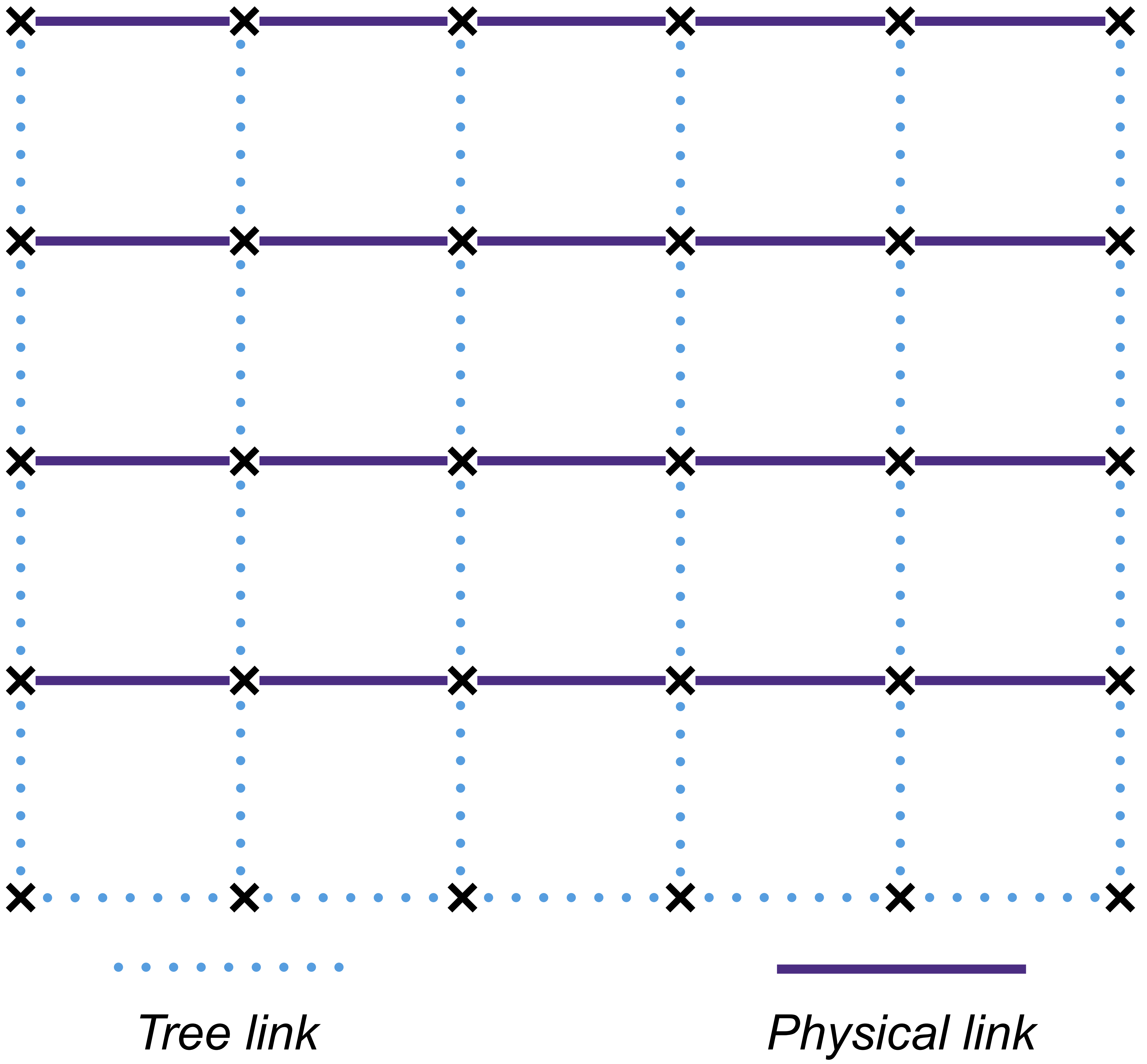

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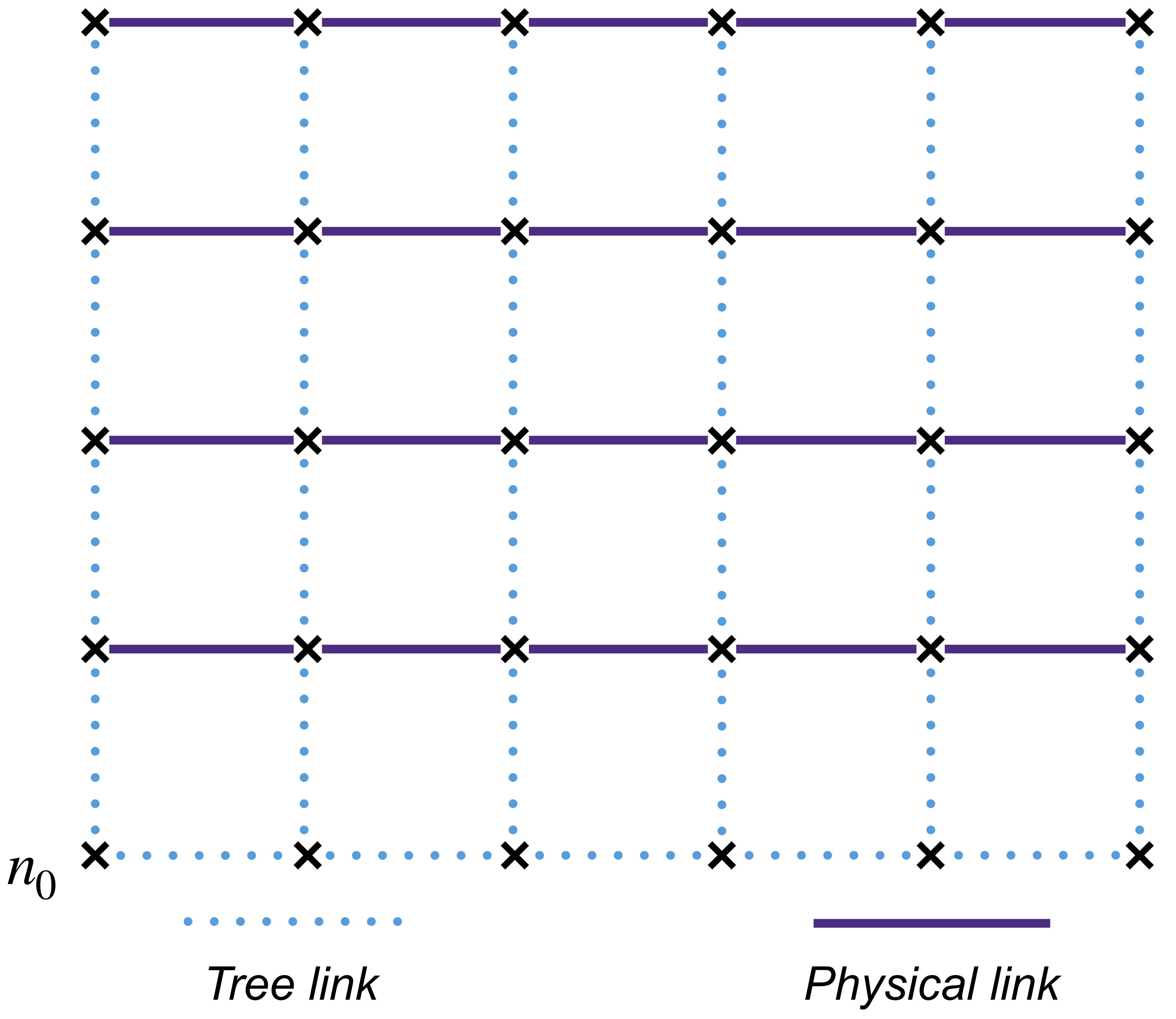

Still Incomplete: Procedure eliminates all local gauge transformations, but not global

- All gauge transformations are carried out relative to the origin

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SU(N)
2+1 and 3+1

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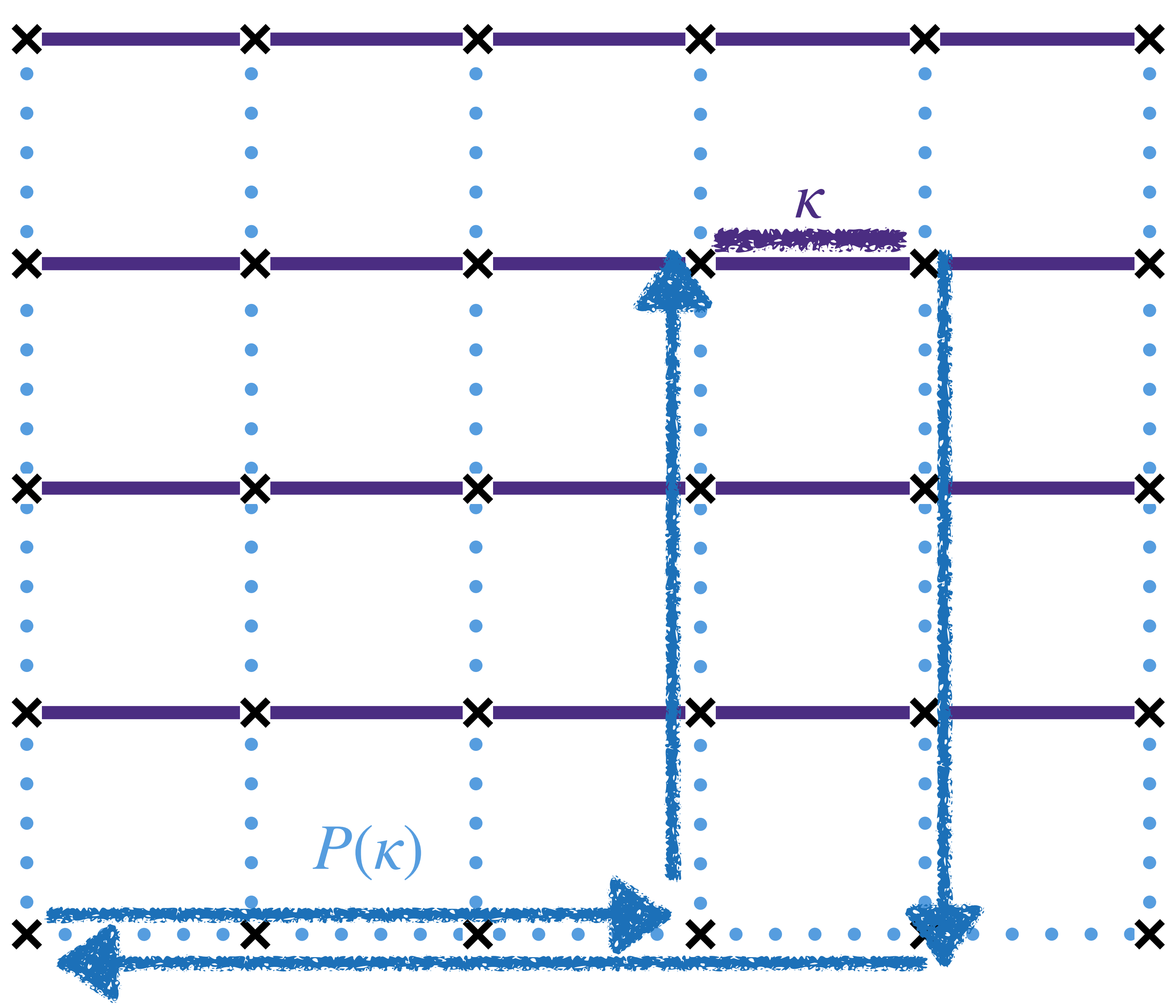

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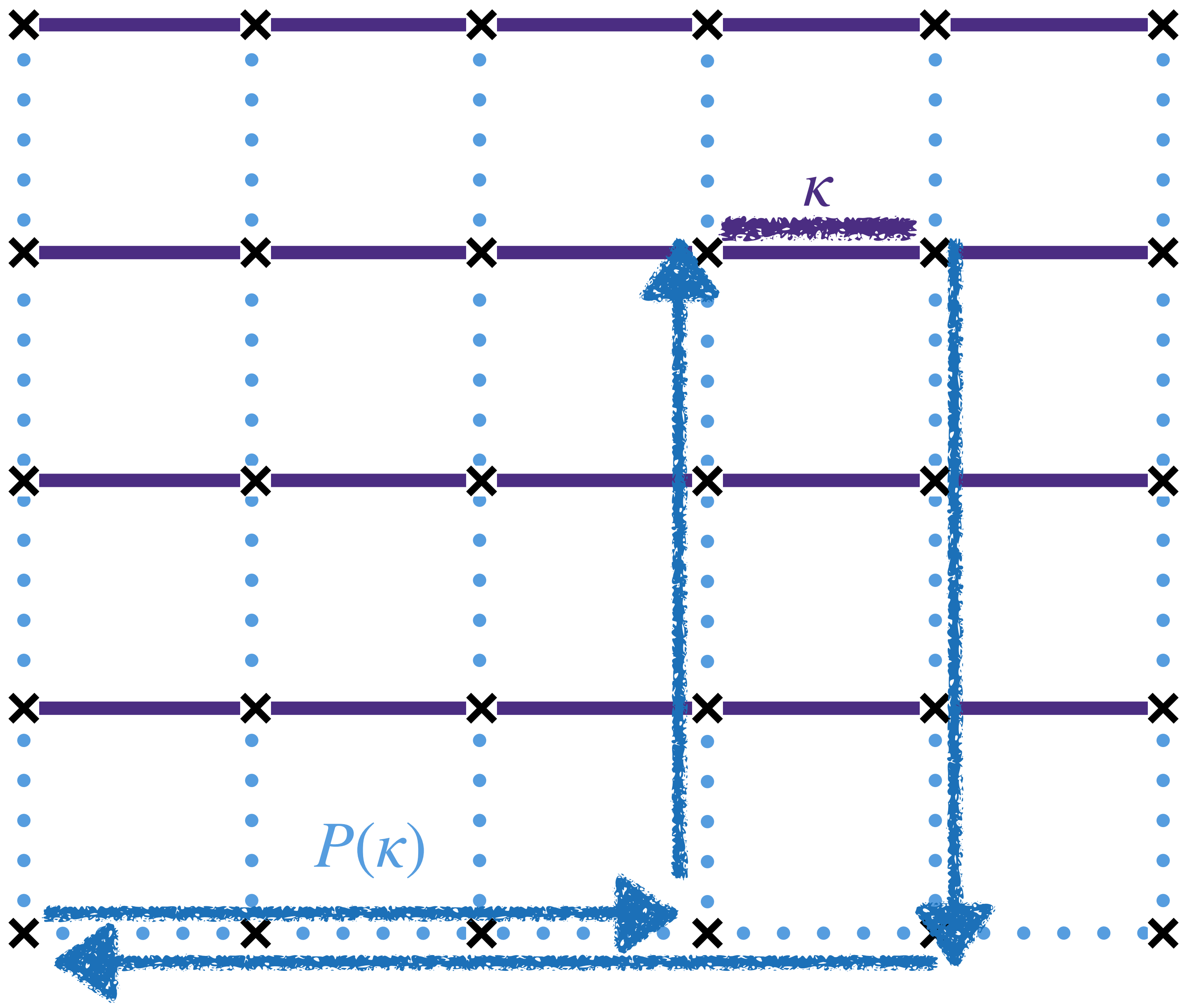

Commutation relations of new variables are canonical

$$[\hat{\mathscr{E}}^a_L(\kappa), \hat{X}(\kappa')] = T^a \hat{X}(\kappa)\delta_{\kappa,\kappa'} \qquad [\hat{\mathscr{E}}^a_R(\kappa), \hat{X}(\kappa')] = \hat{X}(\kappa) T^a \delta_{\kappa,\kappa'}$$

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2+1 and 3+1

Step One: Gauge-Fixing Procedure

Motivation: Gauge fixing allows for "importance sampling" when working in magnetic basis without worrying about breaking gauge-invariance

Non-Local Hamiltonian: Hamiltonian written in terms of new gauge-fixed variables is more complicated

$$H = \frac{1}{2a}\left[g^2 \sum_{\ell \in links} E_\ell E_\ell + \frac{1}{g^2}\sum_{p \in plaquettes} \text{Tr}\left(2I - P_p - P_p^\dagger\right)\right]$$

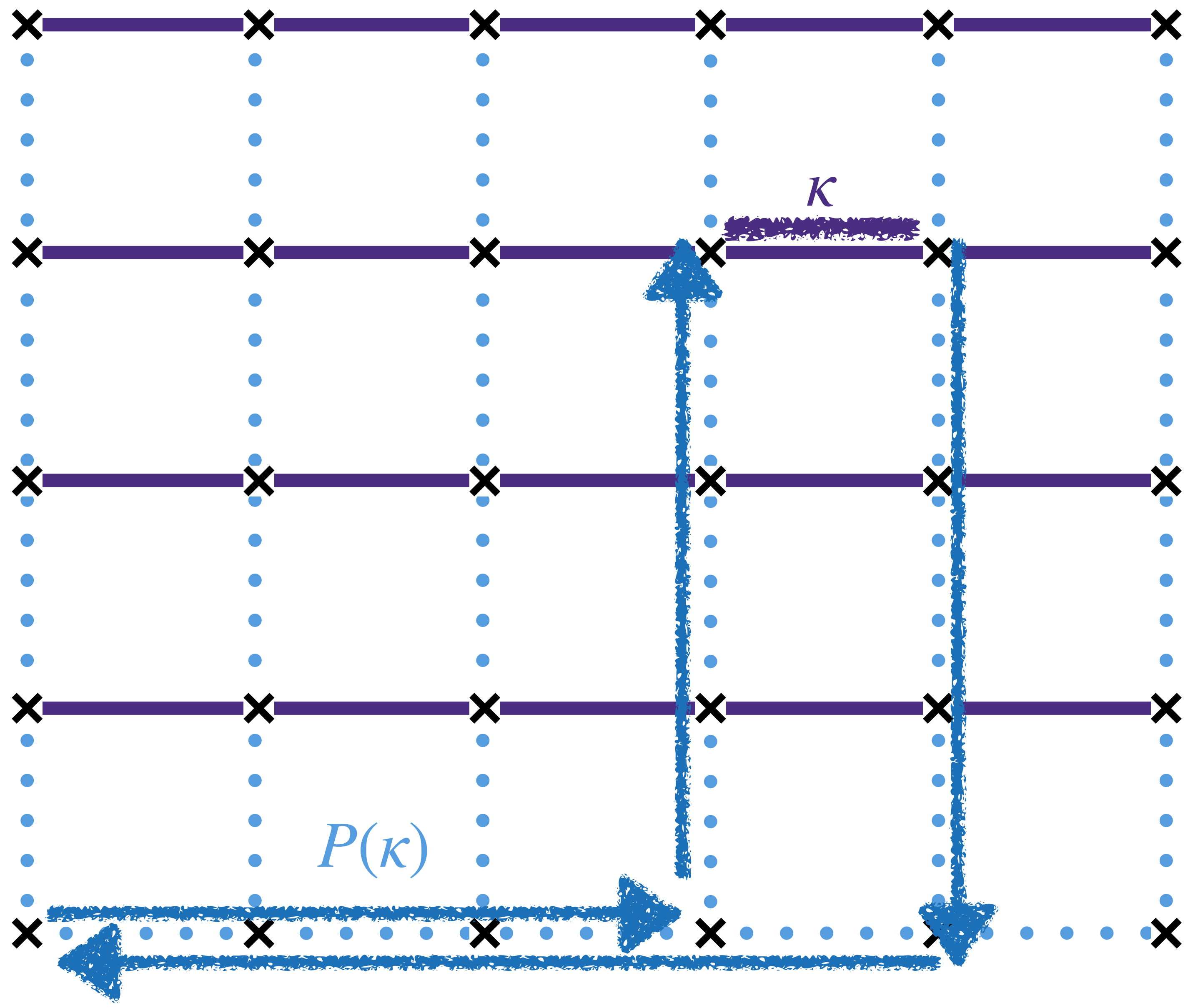
$$\Big\downarrow$$

Max. Tree Gauge Fixing

$$H = \frac{g^2}{2a}\sum_{\ell}\left(\sum_{\kappa \in t_+(\ell)} \hat{\mathscr{E}}^a_{L\kappa} - \sum_{\kappa \in t\_(\ell)} \hat{\mathscr{E}}^a_{R\kappa}\right)^2 + \left(\frac{1}{2g^2 a}\sum_{p} Tr\left(I - \prod_{\kappa \in p} \hat{X}(\kappa)^{\sigma(\kappa)}\right) + \text{h.c.}\right)$$

Commutation relations of new variables are canonical

$$[\hat{\mathscr{E}}^a_L(\kappa), \hat{X}(\kappa')] = T^a \hat{X}(\kappa)\delta_{\kappa,\kappa'} \qquad [\hat{\mathscr{E}}^a_R(\kappa), \hat{X}(\kappa')] = \hat{X}(\kappa) T^a \delta_{\kappa,\kappa'}$$



Big Question

How 'bad' is the non-locality?

Bauer, D'Andrea, Freytsis and **DMG**,
Phys.Rev.D 109 (2024) 7, 074501

Step Two: Parameterizing Operators

Motivation: Three quantum numbers of SU(2) Hamiltonian can be thought of as total angular momentum and projected angular momentums in lab frame and body frame: $\hat{L}^2, \hat{L}^z, \hat{L}'^z$

Bauer, D'Andrea, Freytsis and ***DMG****,*
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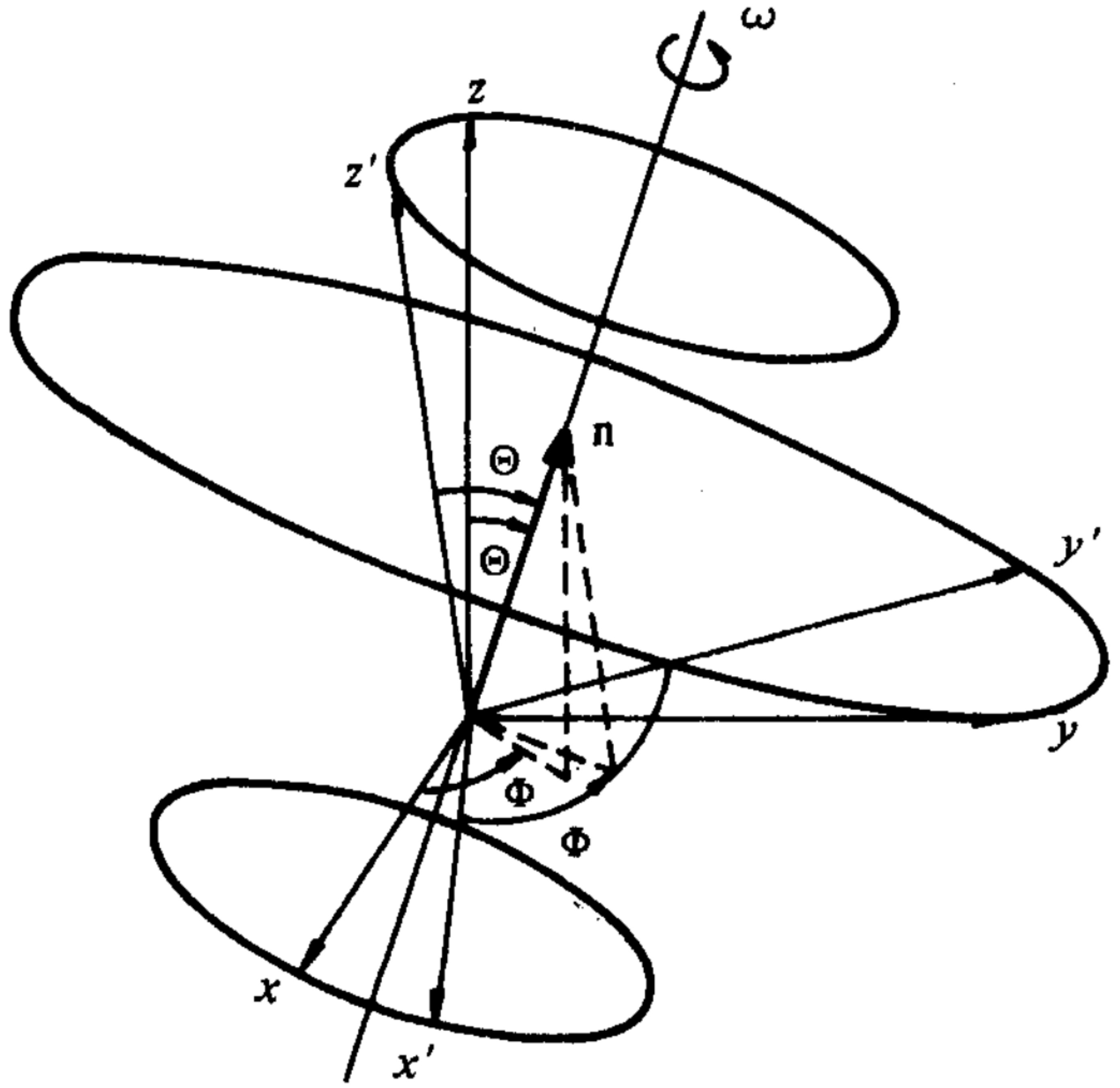
Eye towards Digitization: Axis-angle coordinates are particularly convenient parameterization of SU(2)

- Each loop variable is simply an SU(2) matrix

$$X = \begin{pmatrix} \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta & -i\sin\frac{\omega}{2}\sin\theta e^{-i\phi} \\ -i\sin\frac{\omega}{2}\sin\theta e^{i\phi} & \cos\frac{\omega}{2} + i\sin\frac{\omega}{2}\cos\theta \end{pmatrix}$$



"Quantum Theory of Angular Momentum"
Varshalovich, Moskalev, Khersonskii

Bauer, D'Andrea, Freytsis and ***DMG****,*
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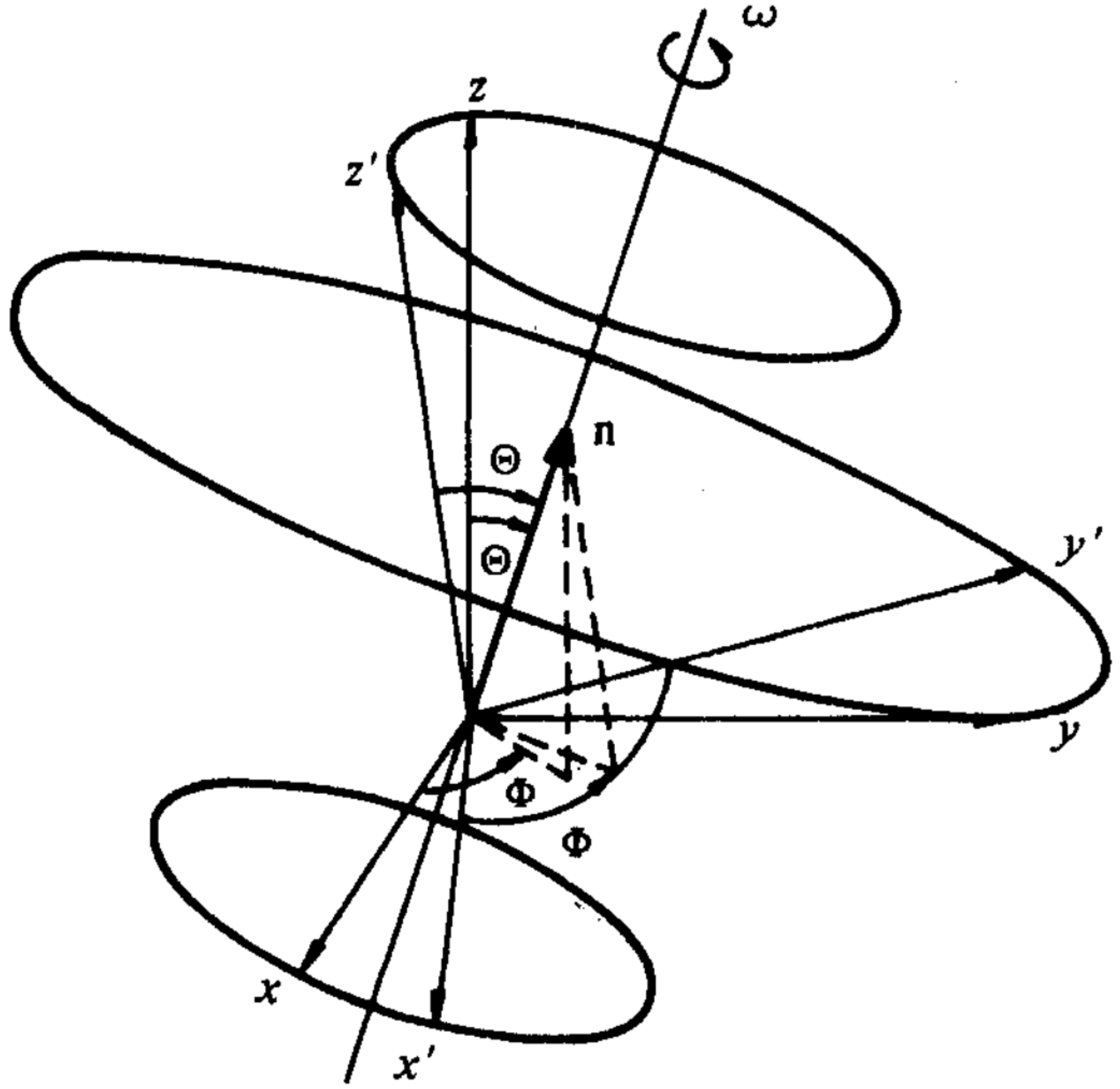
- Electric operators are differential operators

$$\mathcal{E}_{L/R} = \frac{\boldsymbol{\Sigma} \mp \boldsymbol{L}}{2} \qquad \boldsymbol{\Sigma} = 2i\mathbf{n}\partial_\omega + \cot\left(\frac{\omega}{2}\right)(\mathbf{n} \times \boldsymbol{L})$$



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Motivation: As currently written, $\left(\omega_\kappa, \theta_\kappa, \phi_\kappa\right)$ are all continuous variables and so cannot yet be implemented onto digital quantum computers

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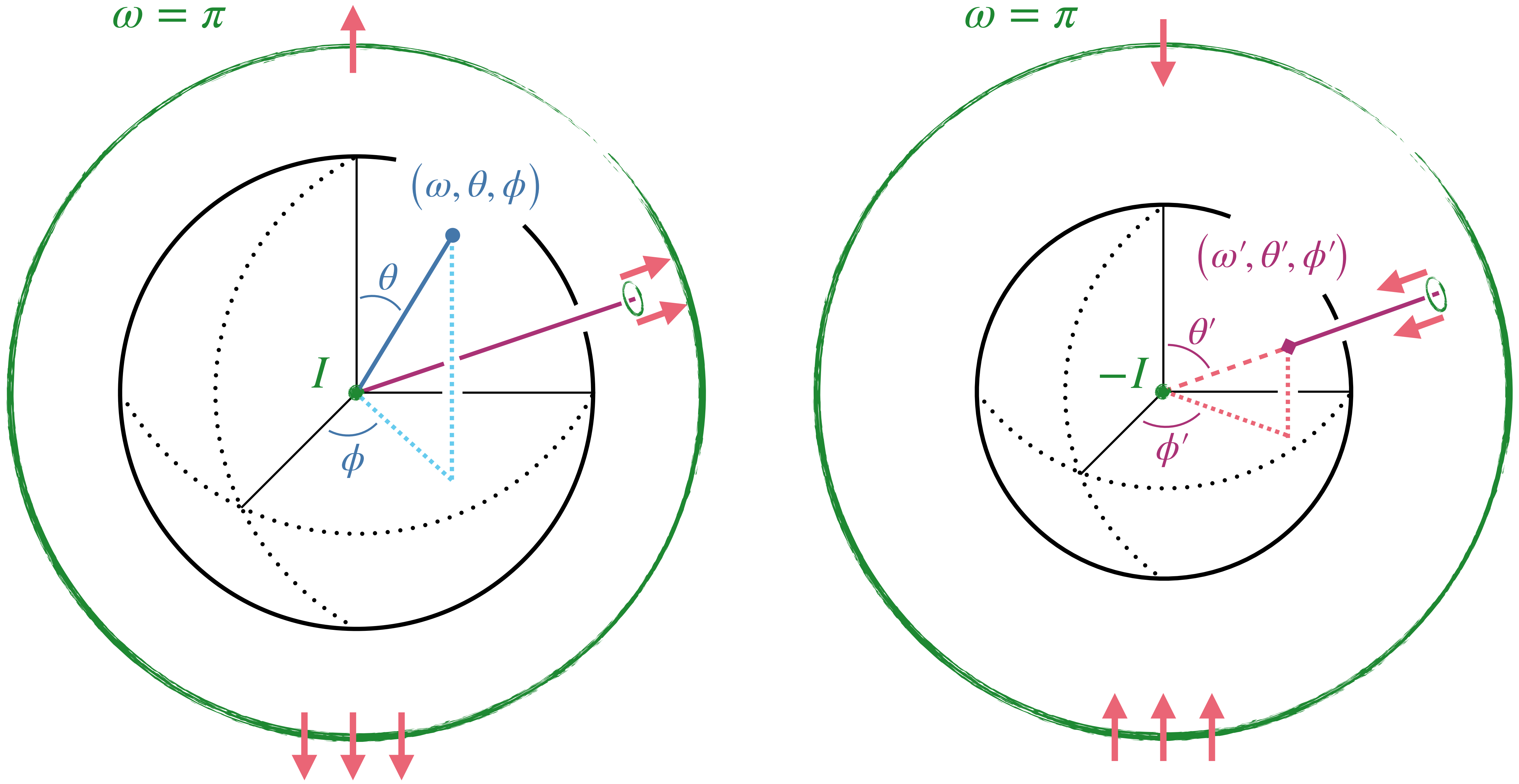
Shift of Intuition: Axis-angle coordinates are also hyperspherical coordinates of S^3

- Angular coordinates $\left(\theta_\kappa, \phi_\kappa\right)$ can be recast as spherical harmonic quantum numbers $\left(\ell_\kappa, m_\kappa\right)$
- Quantum numbers $\left(\ell_\kappa, m_\kappa\right)$ are discrete, with a natural truncation



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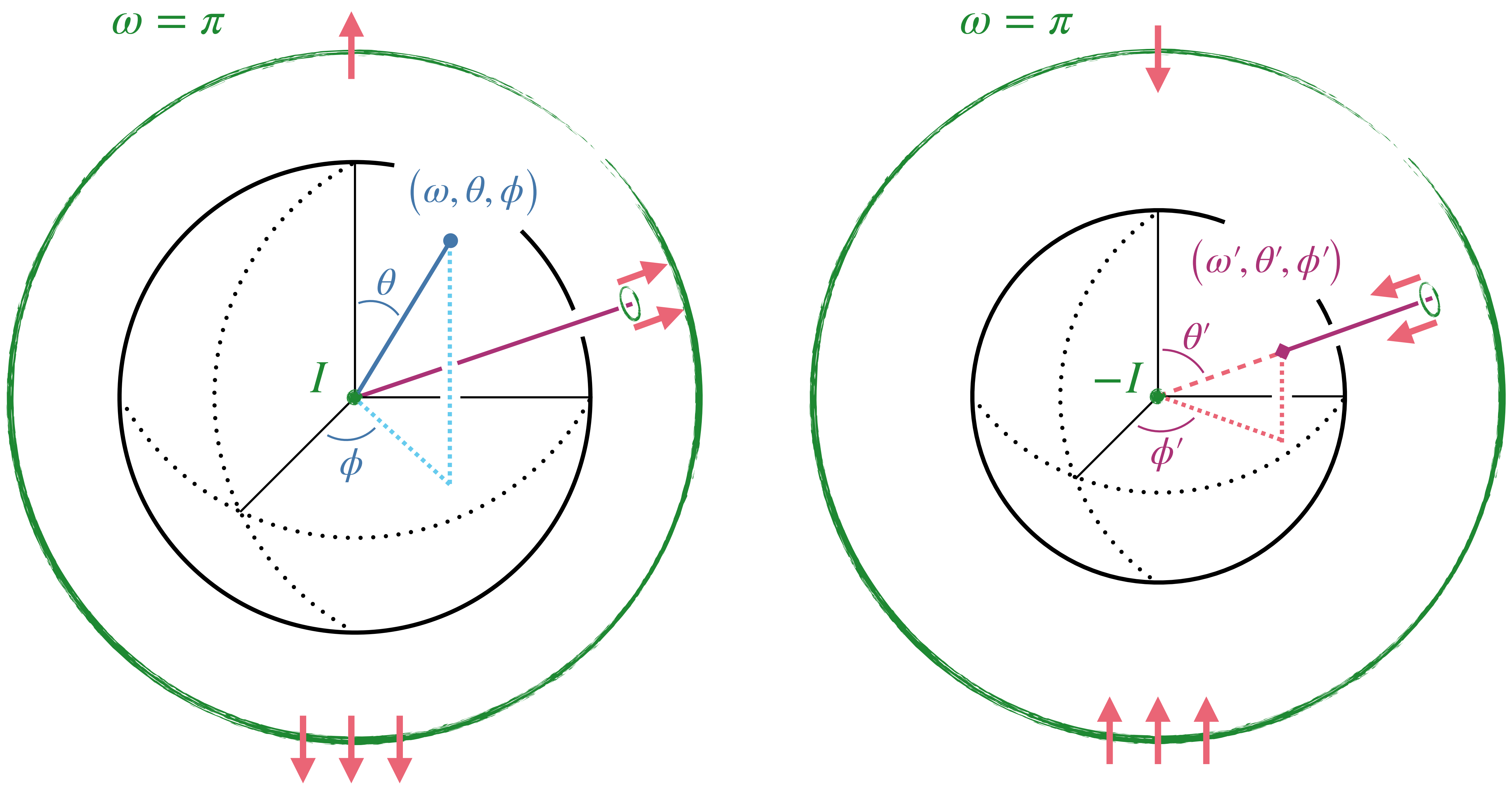
- Angular coordinates $(\theta_\kappa, \phi_\kappa)$ can be recast as spherical harmonic quantum numbers (ℓ_κ, m_κ)
- Quantum numbers (ℓ_κ, m_κ) are discrete, with a natural truncation
- Variable ω_κ is radial coordinate and can be digitized using previously developed methods*



"Mixed Basis": ω is magnetic basis variable and (ℓ, m) are electric basis

* *Bauer, C.W. and* ***DMG****, Phys. Rev.D 107 (2023) 3, L031503*

Finish Step One: Gauge Fix Fully

Observation: SU(2) Hamiltonian can be thought of as a system of rigid rods fixed together at the origin (axis-angle are hyperspherical coordinates)

Motivation: The quantum numbers $\left(\ell_\kappa, m_\kappa\right)$ are related to the total color charge of the system

$$\hat{G}^a(n_0) = \sum_\kappa \left[\hat{E}^a_L(\kappa) - \hat{E}^a_R(\kappa)\right] = -\sum_\kappa L^a_\kappa$$

("difference between lab and body frame")

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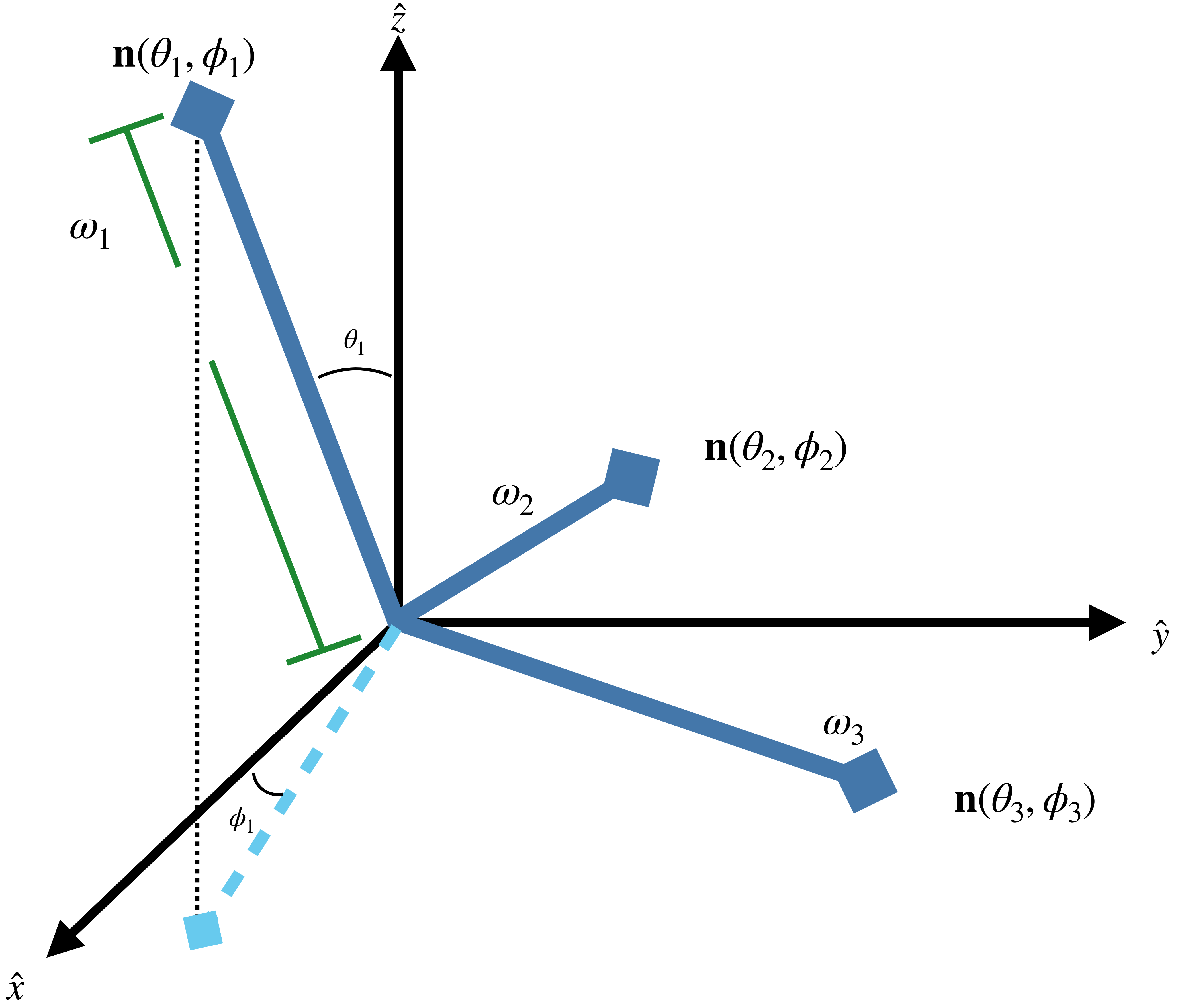

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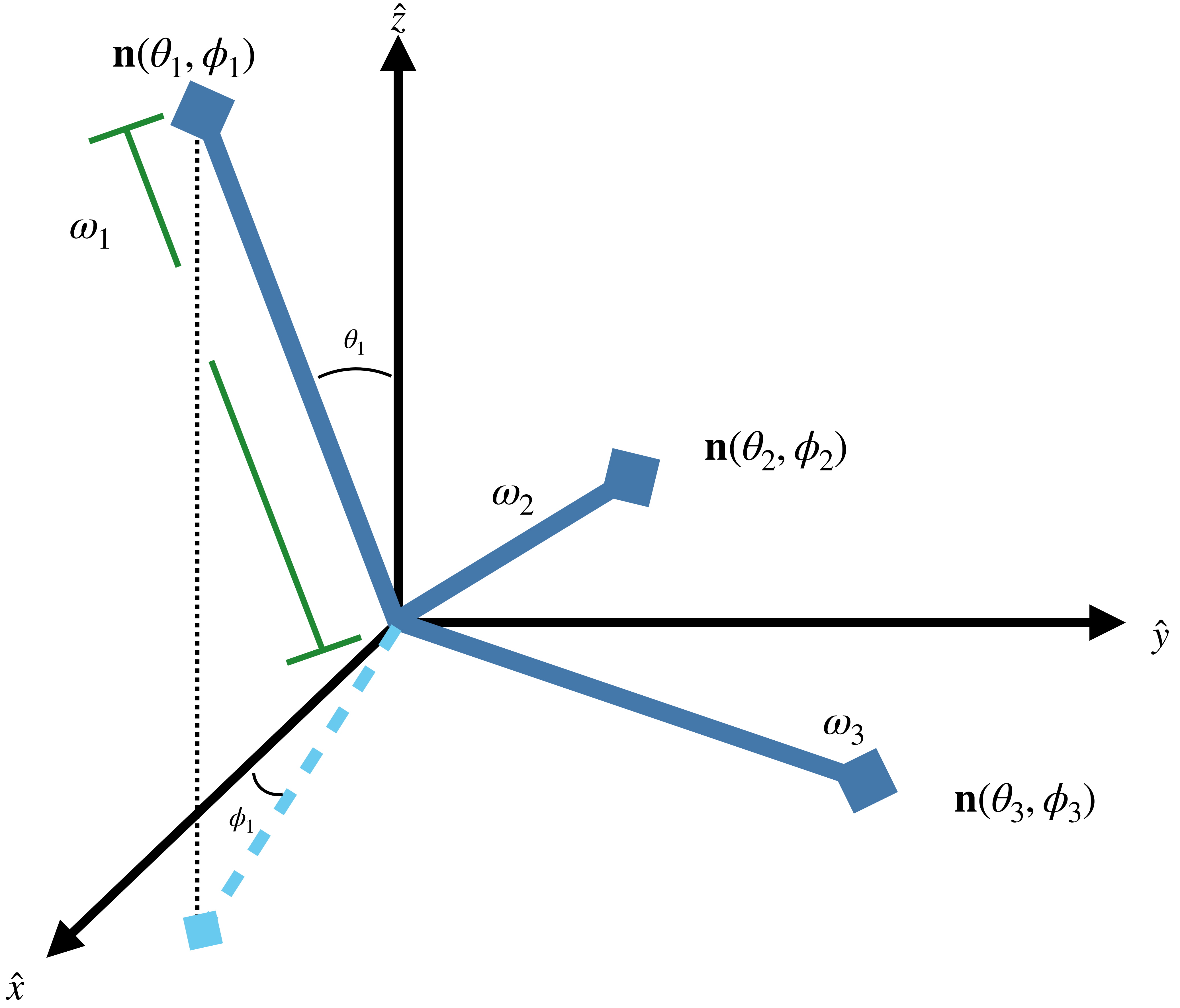
("difference between lab and body frame")

Thought: Is the remaining gauge redundancy related to the rotation between the lab and body frame?



DMG, *Kane and Bauer, Phys.Rev.D 111 (2025) 11, 114516*

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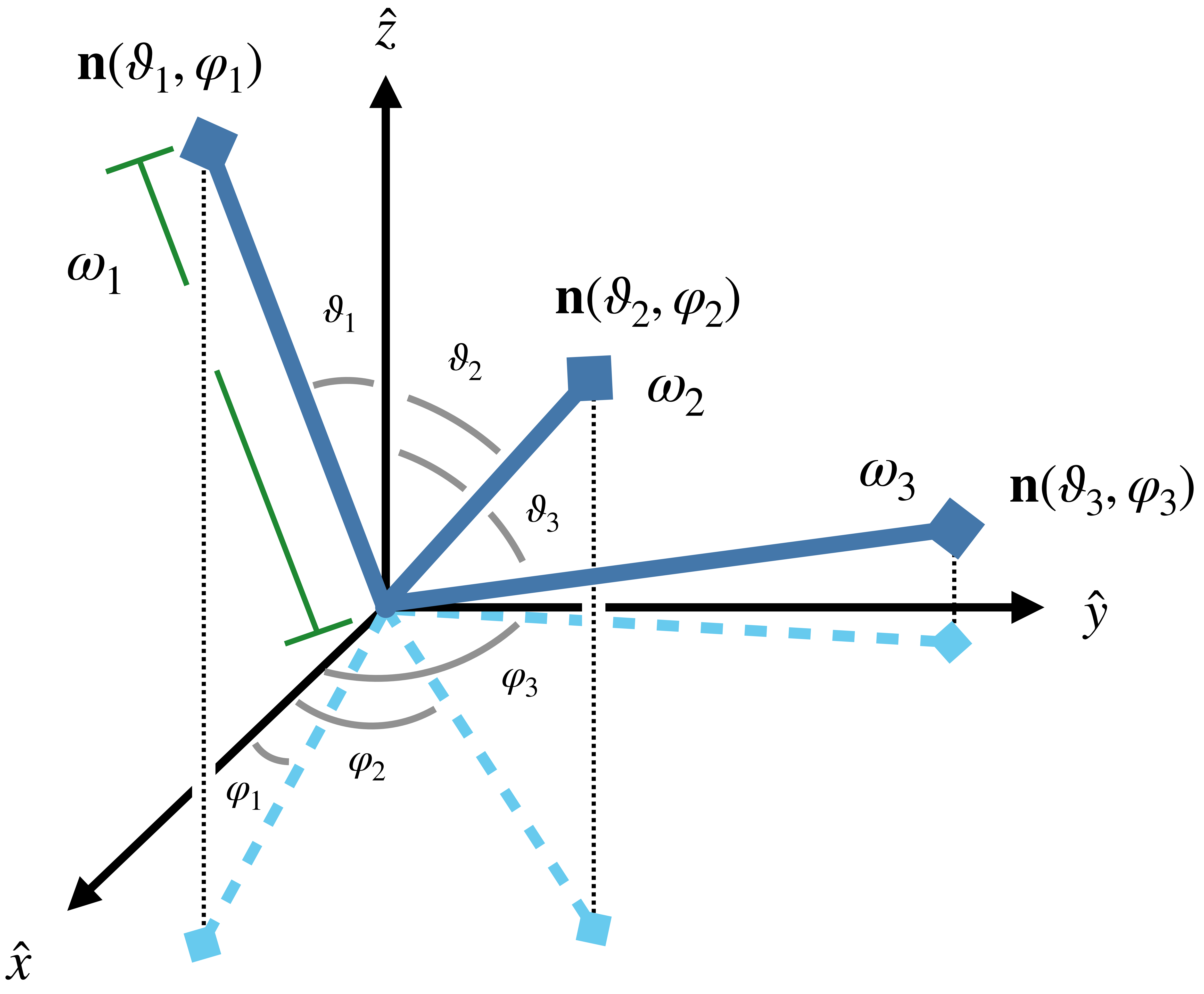

Lab Frame

DMG, *Kane and Bauer, Phys.Rev.D 111 (2025) 11, 114516*

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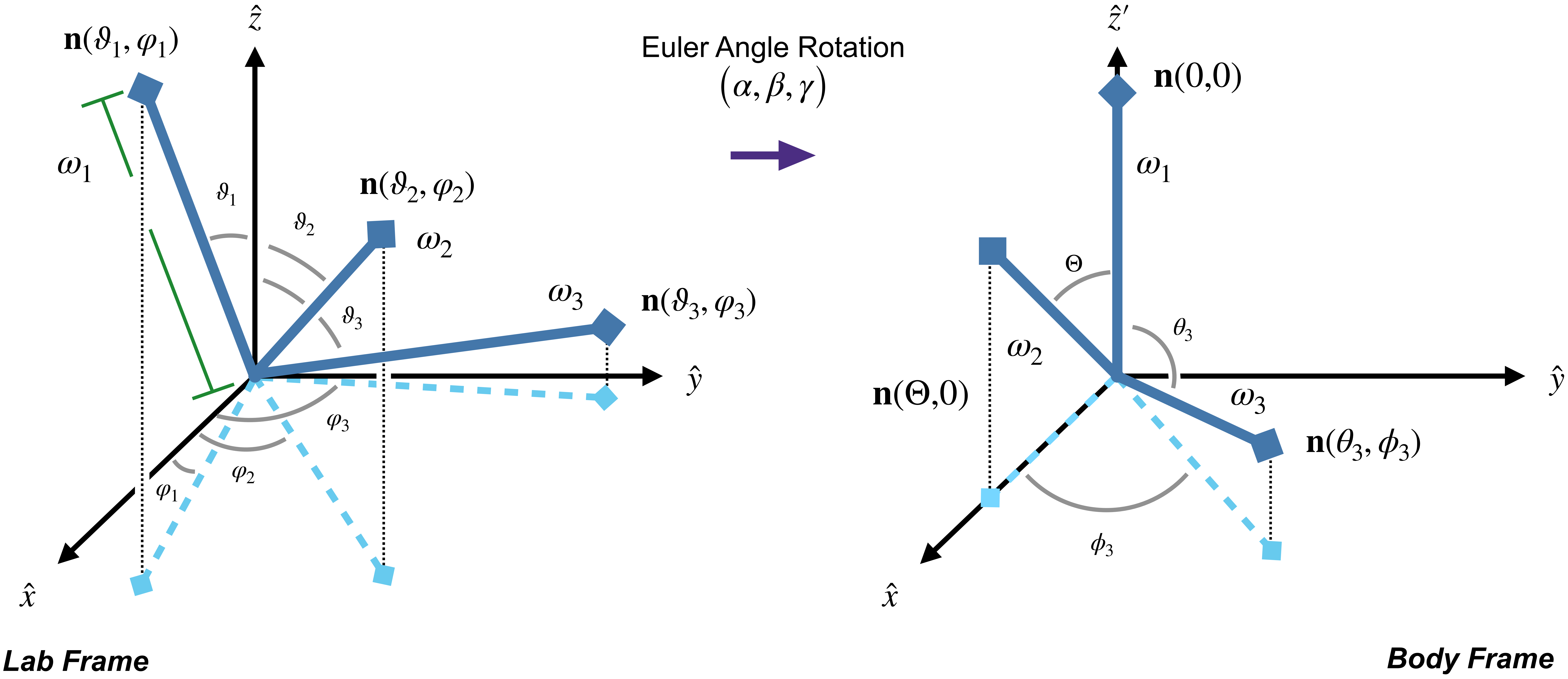

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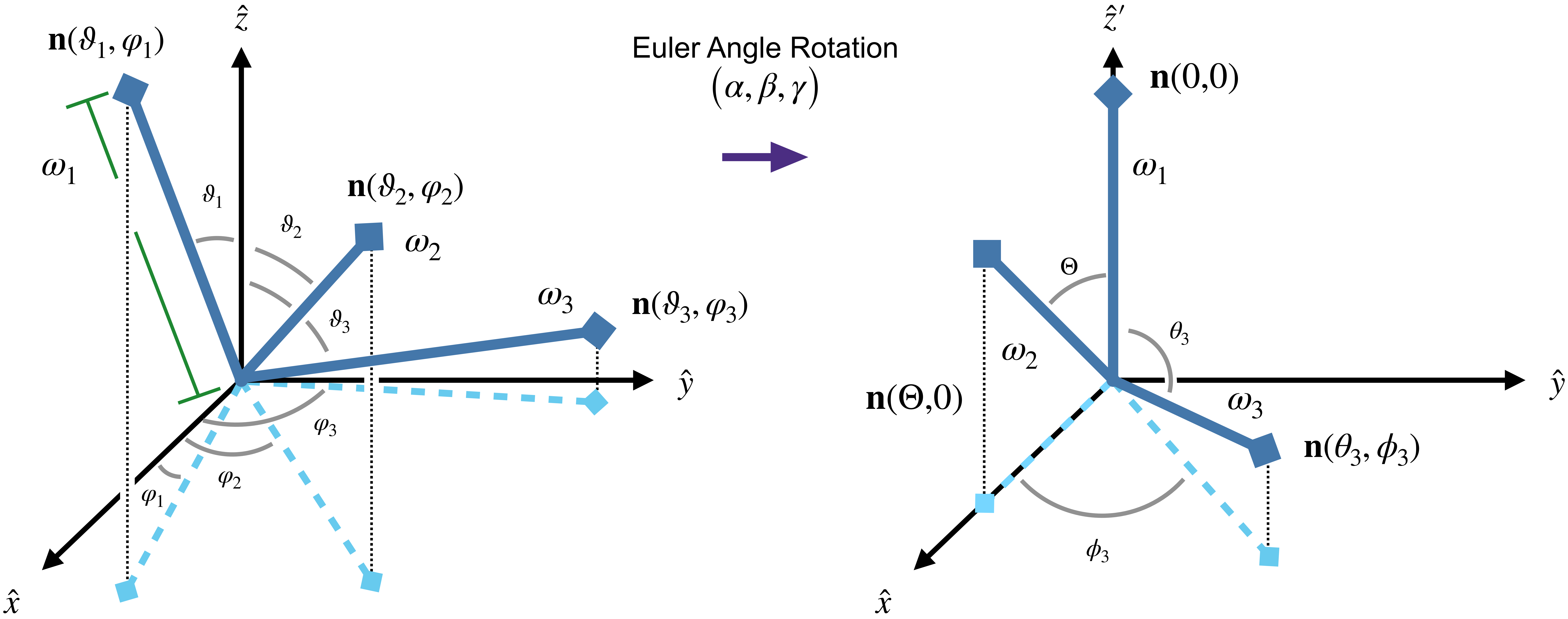

Lab Frame

$$|\,\omega_\kappa, \vartheta_\kappa, \varphi_\kappa\rangle \rightarrow |\,\omega_\kappa, \Theta, \theta_\mu, \theta_\mu; \alpha, \beta, \gamma\rangle$$

Body Frame

Finish Step One: Gauge Fix Fully

SU(2)
2+1 and 3+1

General Idea: Appropriate basis change will lead us to a fully gauge-fixed theory for arbitrary volumes

(Magnetic) Basis Change:

$$|\,\omega_\kappa, \vartheta_\kappa, \varphi_\kappa\rangle \rightarrow |\,\omega_\kappa, \Theta, \theta_\mu, \theta_\mu; \alpha, \beta, \gamma\rangle$$

(Mixed) Basis Change:

$$|\,\omega_\kappa, \ell_\kappa, m_\kappa\rangle \rightarrow |\,\omega_\kappa, n_{12}, \ell_\mu, m_\mu; \Lambda, M, N\rangle$$

Finish Step One: Gauge Fix Fully

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(Magnetic) Basis Change: $|\omega_\kappa, \vartheta_\kappa, \varphi_\kappa\rangle \rightarrow |\omega_\kappa, \Theta, \theta_\mu, \theta_\mu; \alpha, \beta, \gamma\rangle$

Total Charge!

(Mixed) Basis Change: $|\omega_\kappa, \ell_\kappa, m_\kappa\rangle \rightarrow |\omega_\kappa, n_{12}, \ell_\mu, m_\mu; \Lambda, M, N\rangle$

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Key Points: After calculating all matrix possible matrix elements in Hamiltonian, we make three important observations

Implication: trivial to construct Hamiltonian that spans only one total charge sector

1. No operator can change Λ, the total global charge

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Key Points: After calculating all matrix possible matrix elements in Hamiltonian, we make three important observations

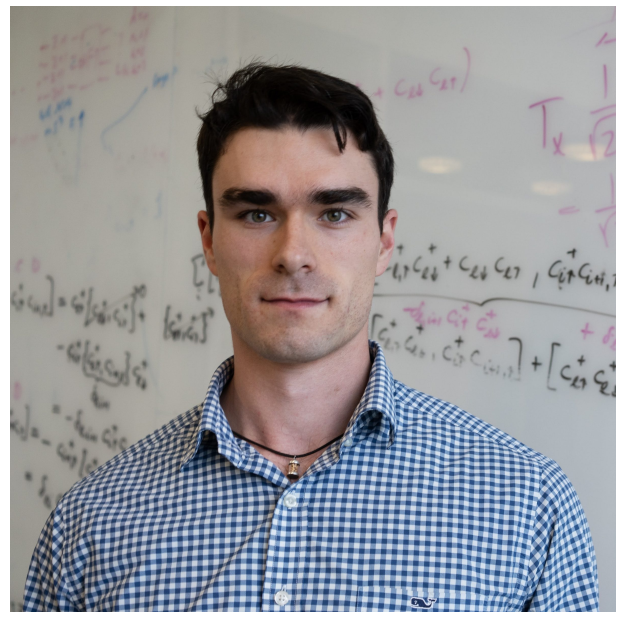
1. No operator can change Λ, the total global charge
2. No one operator can change more than four (discrete) quantum numbers at a time
3. (Discrete) quantum numbers can only change by $\{-1, 0, 1\}$

Implication: trivial to construct Hamiltonian that spans only one total charge sector

Implication: Hamiltonian is sparse

Two Plaquette System

Work in Progress



Henry Froland

Zhiyao Li

Explicit Hamiltonian in Differential Form

General Idea: Fully gauge-fixing reduces the number of degrees of freedom

$$
\begin{aligned}
H =& \frac{1}{g^2}\left(4-2\cos\frac{\omega_1}{2}-2\cos\frac{\omega_2}{2}\right)-\frac{g^2}{2}\left[4\left(\frac{\partial^2}{\partial\omega_1^2}+\cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_1}\right)+4\left(\frac{\partial^2}{\partial\omega_2^2}+\cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_2}\right)\right.\\
& -2\cos\theta\frac{\partial}{\partial\omega_1}\frac{\partial}{\partial\omega_2}+\sin\theta\left(\cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_2}+\cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_1}+\frac{1}{2}\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2}\right)\frac{\partial}{\partial\theta}\\
& \left.-\left(2\csc^2\frac{\omega_1}{2}+2\csc^2\frac{\omega_2}{2}+\frac{1}{2}\cos\theta\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2}-\frac{1}{2}\right)\hat{\mathcal{N}}\right]
\end{aligned}
$$

Explicit Hamiltonian in Differential Form

General Idea: Fully gauge-fixing reduces the number of degrees of freedom

$$
\begin{aligned}
H = &\frac{1}{g^2}\left(4 - 2\cos\frac{\omega_1}{2} - 2\cos\frac{\omega_2}{2}\right) - \frac{g^2}{2}\left[4\left(\frac{\partial^2}{\partial\omega_1^2} + \cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_1}\right) + 4\left(\frac{\partial^2}{\partial\omega_2^2} + \cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_2}\right)\right. \\
&- 2\cos\theta\frac{\partial}{\partial\omega_1}\frac{\partial}{\partial\omega_2} + \sin\theta\left(\cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_2} + \cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_1} + \frac{1}{2}\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2}\right)\frac{\partial}{\partial\theta} \\
&\left. - \left(2\csc^2\frac{\omega_1}{2} + 2\csc^2\frac{\omega_2}{2} + \frac{1}{2}\cos\theta\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2} - \frac{1}{2}\right)\hat{\mathcal{N}}\right]
\end{aligned}
$$

Legendre Differential Operator

$$\hat{\mathcal{N}} = -\frac{\partial^2}{\partial\theta^2} - \cot\theta\frac{\partial}{\partial\theta}$$

$$\hat{\mathcal{N}}P_\nu(\theta) = \nu(\nu+1)P_\nu(\theta)$$

Explicit Hamiltonian in Differential Form

General Idea: Fully gauge-fixing reduces the number of degrees of freedom

Magnetic

$$
\begin{aligned}
H = {} & \frac{1}{g^2}\left(4 - 2\cos\frac{\omega_1}{2} - 2\cos\frac{\omega_2}{2}\right) - \frac{g^2}{2}\Bigg[4\left(\frac{\partial^2}{\partial\omega_1^2} + \cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_1}\right) + 4\left(\frac{\partial^2}{\partial\omega_2^2} + \cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_2}\right) \\
& - 2\cos\theta\frac{\partial}{\partial\omega_1}\frac{\partial}{\partial\omega_2} + \sin\theta\left(\cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_2} + \cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_1} + \frac{1}{2}\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2}\right)\frac{\partial}{\partial\theta} \\
& - \left(2\csc^2\frac{\omega_1}{2} + 2\csc^2\frac{\omega_2}{2} + \frac{1}{2}\cos\theta\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2} - \frac{1}{2}\right)\hat{\mathcal{N}}\Bigg]
\end{aligned}
$$

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Explicit Hamiltonian in Differential Form

General Idea: Fully gauge-fixing reduces the number of degrees of freedom

Magnetic

Electric

$$H = \frac{1}{g^2}\left(4 - 2\cos\frac{\omega_1}{2} - 2\cos\frac{\omega_2}{2}\right) - \frac{g^2}{2}\left[4\left(\frac{\partial^2}{\partial\omega_1^2} + \cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_1}\right) + 4\left(\frac{\partial^2}{\partial\omega_2^2} + \cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_2}\right)\right.$$

$$- 2\cos\theta\frac{\partial}{\partial\omega_1}\frac{\partial}{\partial\omega_2} + \sin\theta\left(\cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_2} + \cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_1} + \frac{1}{2}\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2}\right)\frac{\partial}{\partial\theta}$$

$$\left. - \left(2\csc^2\frac{\omega_1}{2} + 2\csc^2\frac{\omega_2}{2} + \frac{1}{2}\cos\theta\cot\frac{\omega_1}{2}\cot\frac{\omega_2}{2} - \frac{1}{2}\right)\hat{\mathcal{N}}\right]$$

Legendre Differential Operator

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Explicit Hamiltonian in Differential Form

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Electric

$$H = \frac{1}{g^2}\left(4 - 2\cos\frac{\omega_1}{2} - 2\cos\frac{\omega_2}{2}\right) - \frac{g^2}{2}\left[4\left(\frac{\partial^2}{\partial\omega_1^2} + \cot\frac{\omega_1}{2}\frac{\partial}{\partial\omega_1}\right) + 4\left(\frac{\partial^2}{\partial\omega_2^2} + \cot\frac{\omega_2}{2}\frac{\partial}{\partial\omega_2}\right)\right.$$

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Two Important Questions:

- Are there efficient ways to implement this on digital quantum devices?
- Can these methods easily generalize to larger number of plaquettes?

Different Discrete Bases for Two Plaquette System

General Idea: Different bases work well for different values of the gauge coupling

Gauge Coupling

Weak ⟷ *Strong*

Different Discrete Bases for Two Plaquette System

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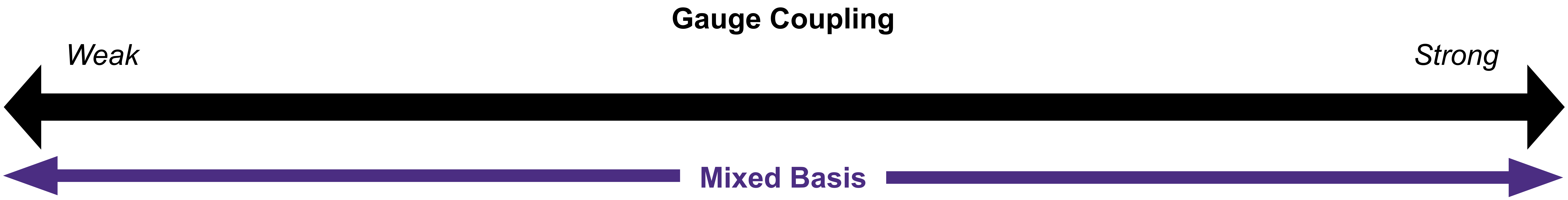

Gauge Coupling

Weak

Strong

Mixed Basis

$$|\,\omega_1, \omega_2, \nu\rangle$$

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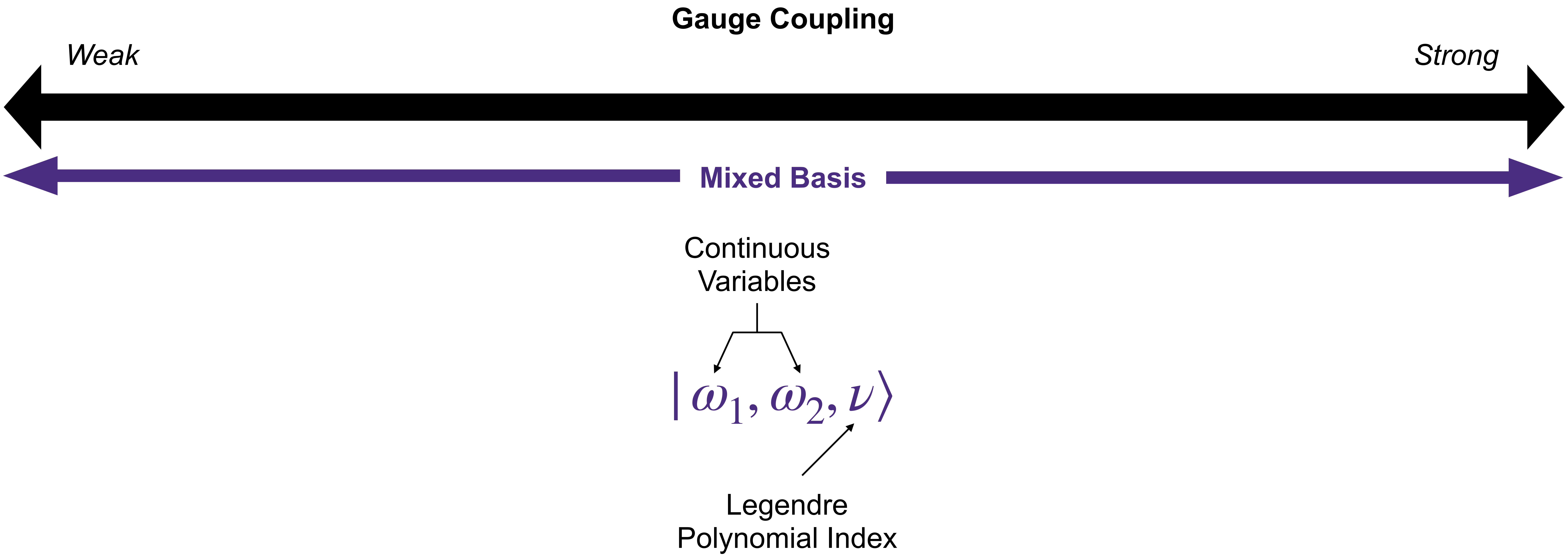

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Weak

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Mixed Basis

Continuous Variables

$$|\,\omega_1, \omega_2, \nu\rangle$$

Legendre Polynomial Index

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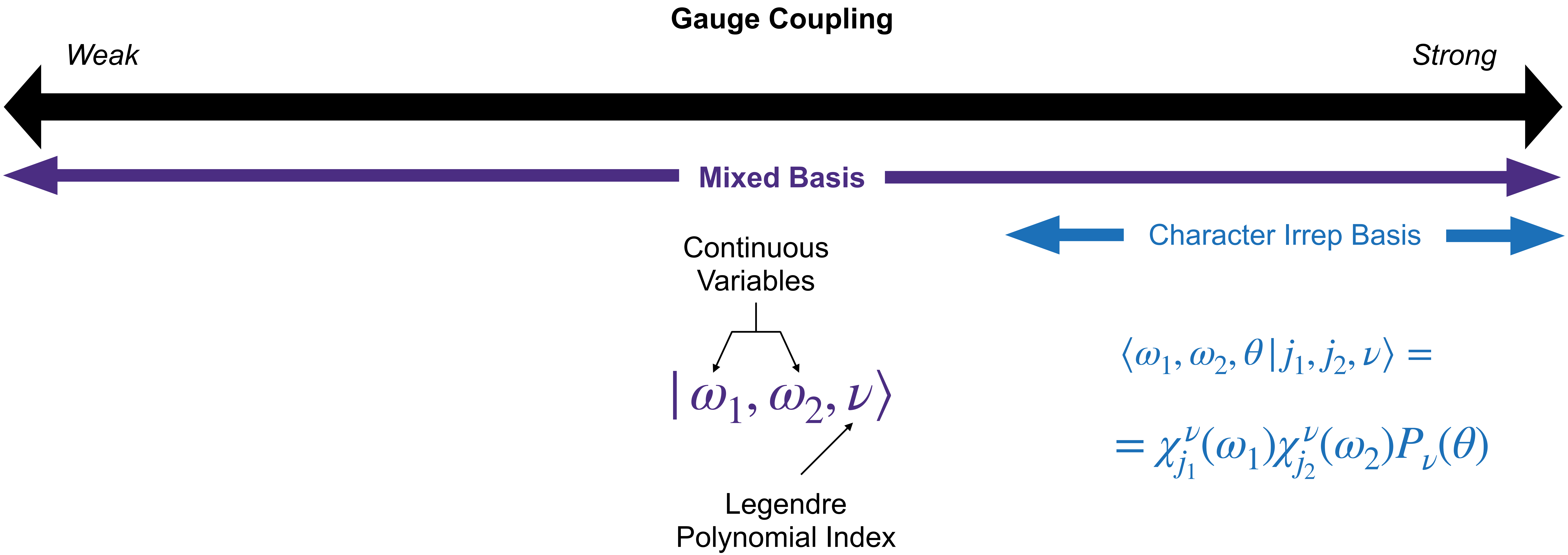

Gauge Coupling

Weak

Strong

Mixed Basis

Character Irrep Basis

Continuous Variables

$$|\omega_1, \omega_2, \nu\rangle$$

Legendre Polynomial Index

$$\langle \omega_1, \omega_2, \theta \,|\, j_1, j_2, \nu \rangle =$$

$$= \chi^{\nu}_{j_1}(\omega_1)\chi^{\nu}_{j_2}(\omega_2)P_{\nu}(\theta)$$

Different Discrete Bases for Two Plaquette System

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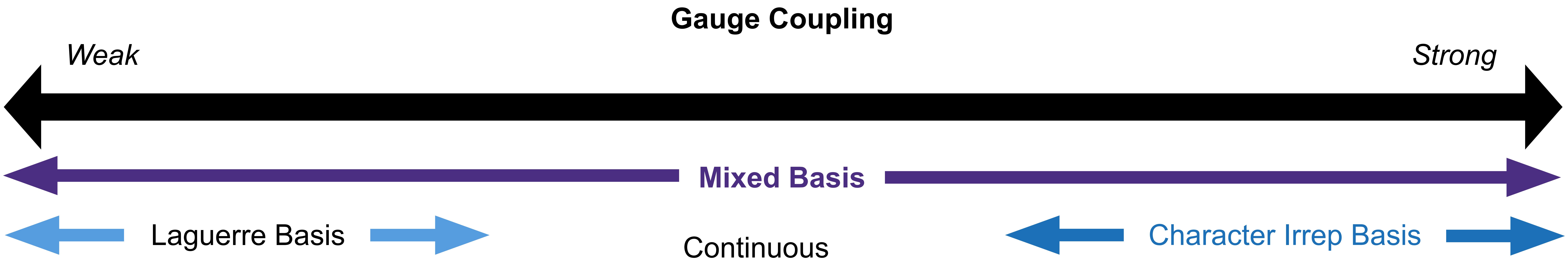


Gauge Coupling

Weak

Strong

Mixed Basis

Laguerre Basis

Character Irrep Basis

$$\langle \omega_1, \omega_2, \theta \,|\, n_1, n_2, \nu \rangle =$$

$$= e^{-\frac{2\sqrt{2}r^2}{g^2}} r^\nu L_n^{\nu+\frac{1}{2}}\left(\frac{4\sqrt{2}}{g^2}r^2\right) P_\nu(\theta)$$

Continuous Variables

$$|\,\omega_1, \omega_2, \nu\rangle$$

Legendre Polynomial Index

$$\langle \omega_1, \omega_2, \theta \,|\, j_1, j_2, \nu \rangle =$$

$$= \chi^\nu_{j_1}(\omega_1)\chi^\nu_{j_2}(\omega_2)P_\nu(\theta)$$

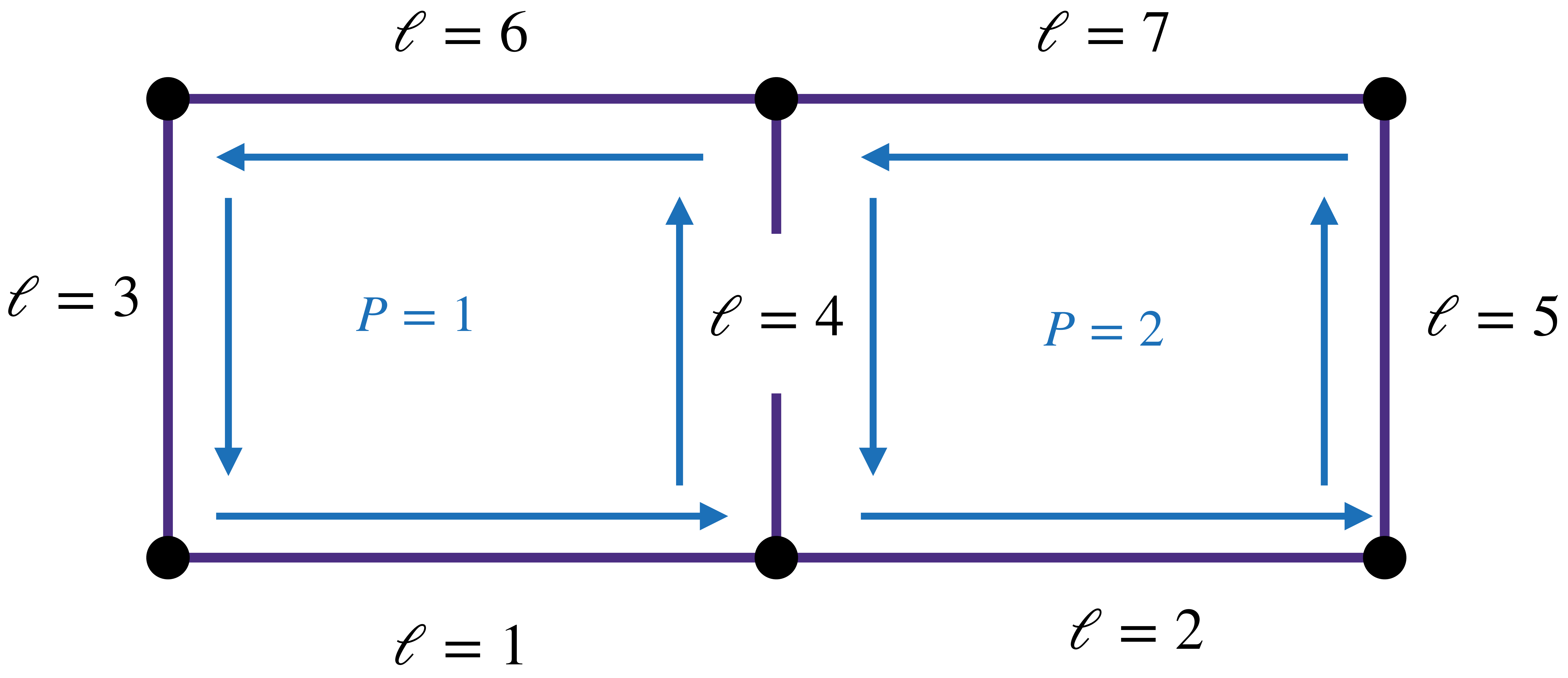
Example: Two Plaquette Universe

General Idea: Full gauge-fixing can result in resource savings

Kogut - Susskind

Irrep Basis

$$|j_\ell, m_{L\ell}, m_{R\ell}\rangle$$



Most naive implementation!

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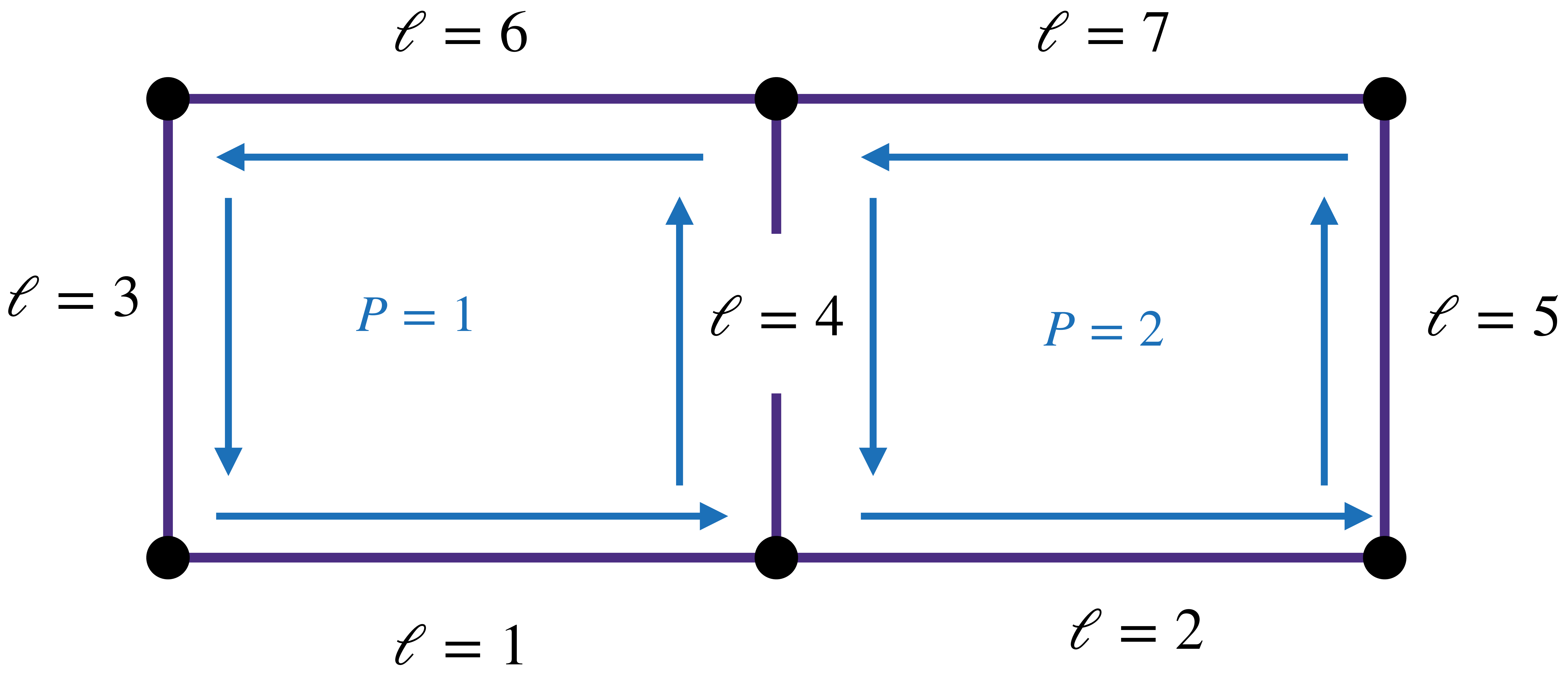

$$Dim(\mathscr{H}(j_{max})) : \left(\frac{1}{3}\right)^7 \left(8j_{max}^3 + 18j_{max}^2 + 13j_{max} + 3\right)^7$$

$$Dim(\mathscr{H}(j_{max} = 1)) \sim 10^8$$

Most naive implementation!

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General Idea: Full gauge-fixing can result in resource savings

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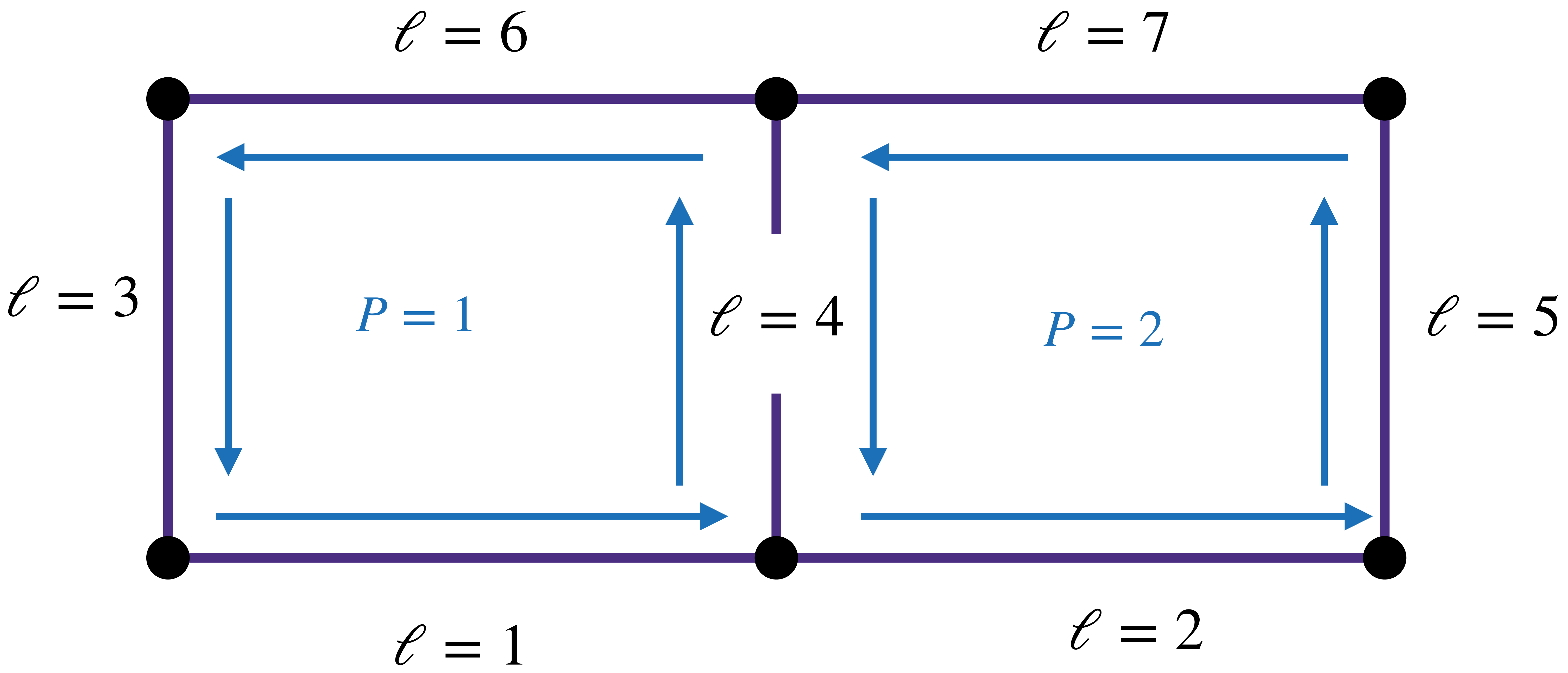
Irrep Basis

$$|j_\ell, m_{L\ell}, m_{R\ell}\rangle$$



$$Dim(\mathscr{H}(j_{max})) : \left(\frac{1}{3}\right)^7 \left(8j_{max}^3 + 18j_{max}^2 + 13j_{max} + 3\right)^7$$

$$Dim(\mathscr{H}(j_{max} = 1)) \sim 10^8$$

Additional concerns: state prep and gauge violation due to truncation/Trotter

Example: Two Plaquette Universe

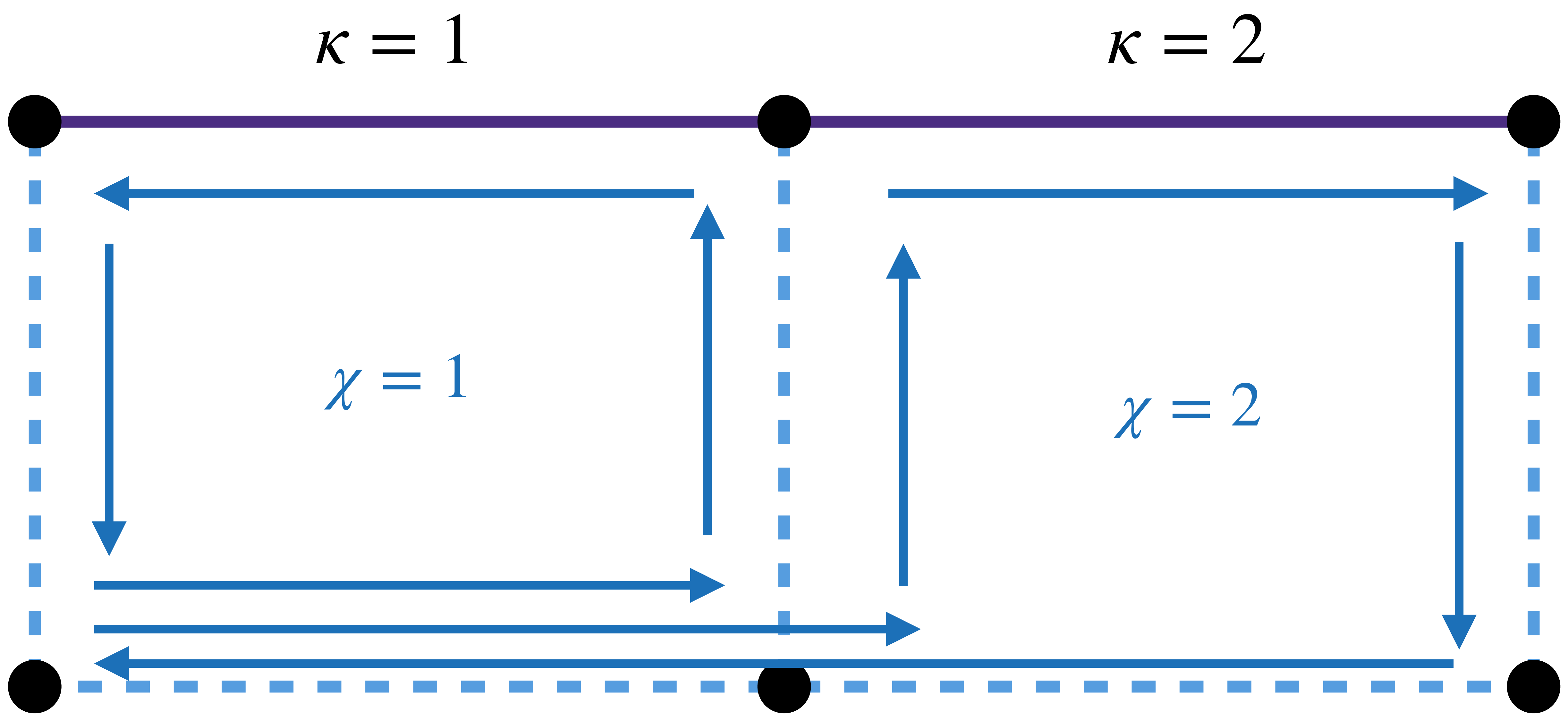
General Idea: Full gauge-fixing can result in resource savings

Zero Charge Sector

Fully-Gauge Fixed

Irrep Basis

$$|j_1, j_2, \nu\rangle$$



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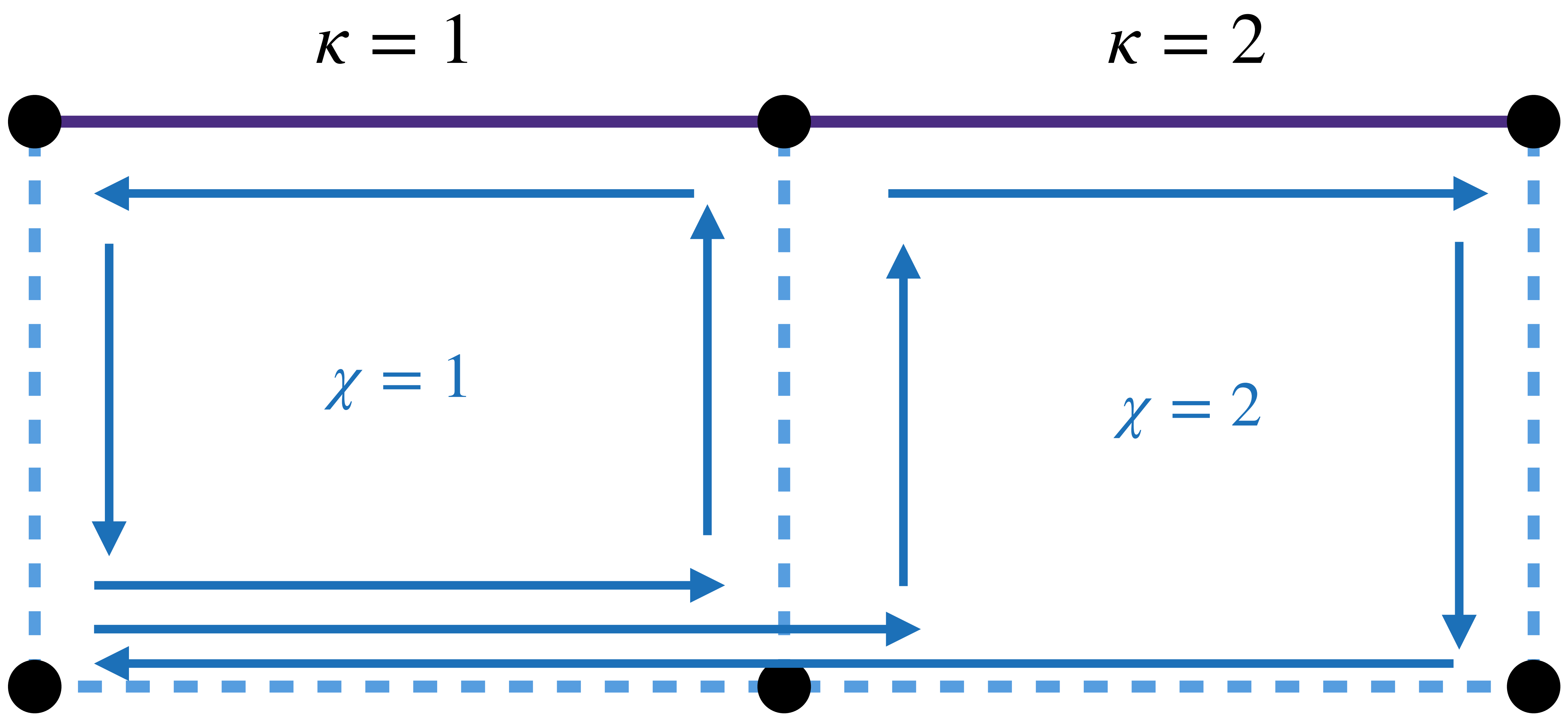

$Dim(\mathscr{H}(j_{max}, \nu_{max}) : (2j_{max} + 1)^2 (\nu_{max} + 1)$

$Dim(\mathscr{H}(j_{max} = 2, \nu_{max} = 2)) = 75$

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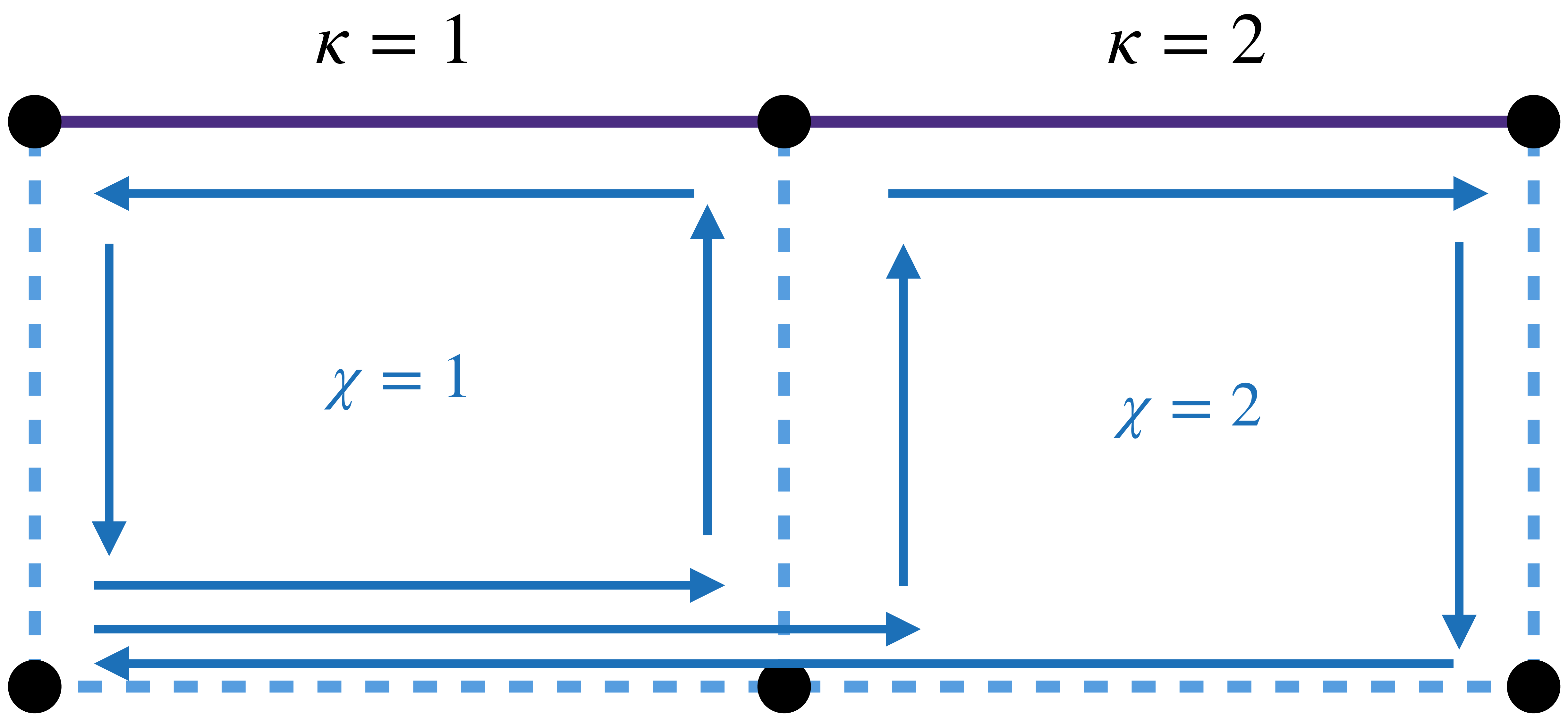

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$Dim(\mathscr{H}(j_{max} = 2, \nu_{max} = 2)) = 75$

Additional concerns: effects of non-locality as lattice volume grows

Mixed Basis Circuit Construction

General Idea: Construct circuit for each type of term independently and stitch them together

Seven types of terms

$\partial_{\omega_1}^2$ $\quad$ $f_0(\omega_1)$ $\quad$ $f_1(\omega_1)\partial_{\omega_1}$ $\quad$ $f_2(\omega_2, \nu)\partial_{\omega_1}$ $\quad$ $f_4(\omega_1, \nu)$

$f_3(\nu)\partial_{\omega_1}\partial_{\omega_2}$ $\quad$ $f_5(\omega_1, \omega_2, \nu)$

$\partial_{\omega_2}^2$ $\quad$ $f_0(\omega_2)$ $\quad$ $f_1(\omega_2)\partial_{\omega_2}$ $\quad$ $f_2(\omega_1, \nu)\partial_{\omega_2}$ $\quad$ $f_4(\omega_2, \nu)$

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$$\begin{array}{ccccccc} \partial^2_{\omega_1} & f_0(\omega_1) & f_1(\omega_1)\partial_{\omega_1} & f_2(\omega_2,\nu)\partial_{\omega_1} & & f_4(\omega_1,\nu) & \\ & & & & f_3(\nu)\partial_{\omega_1}\partial_{\omega_2} & & f_5(\omega_1,\omega_2,\nu) \\ \partial^2_{\omega_2} & f_0(\omega_2) & f_1(\omega_2)\partial_{\omega_2} & f_2(\omega_1,\nu)\partial_{\omega_2} & & f_4(\omega_2,\nu) & \end{array}$$

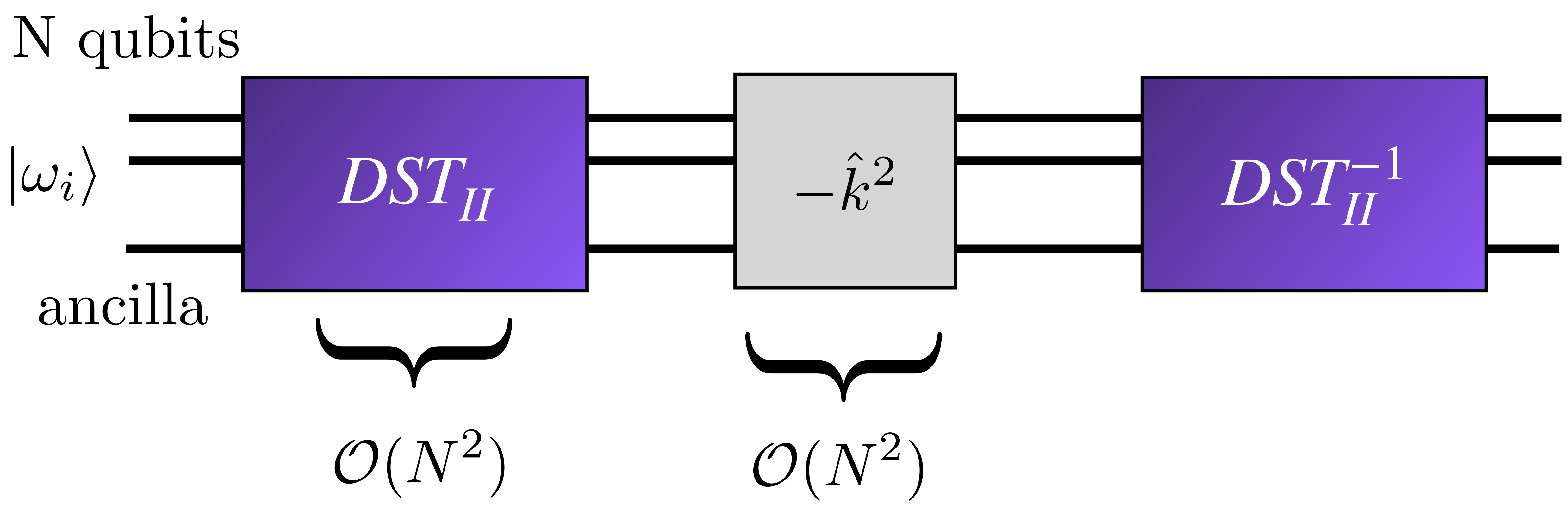
Two Possible Approaches: Implementing these terms can be done in two (related) ways

- (Asymptotic Approach): Determine circuits for each term individually
- NISQ Approach: Decompose terms into Pauli strings and use truncation and clever orderings to cancel as many CNOTs as possible

Mixed Basis Circuit Construction, Asymptotic Approach

Mixed Basis Circuit Construction, Asymptotic Approach

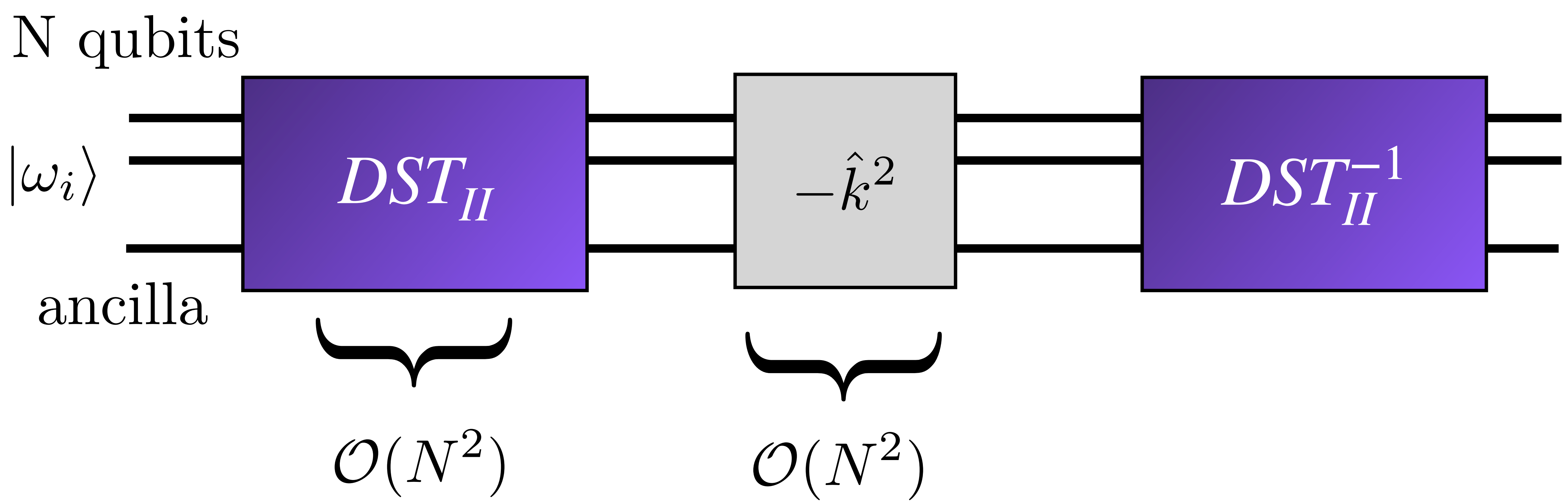
$\partial^2_{\omega_i}$



** N. Klco and M.J. Savage, Phys. Rev. A 99, 052335*

Mixed Basis Circuit Construction, Asymptotic Approach

$\partial^2_{\omega_i}$



N qubits

$|\omega_i\rangle$

ancilla

DST_{II}

$-\hat{k}^2$

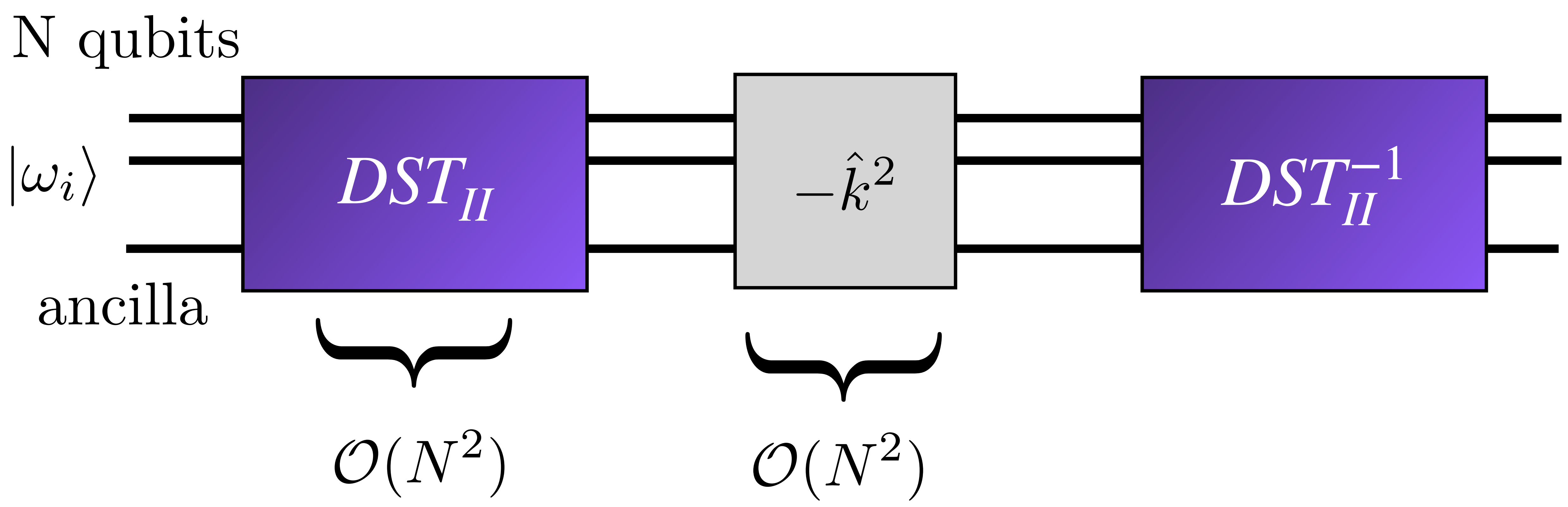
DST_{II}^{-1}

$\underbrace{\phantom{DST_{II}}}_{\mathcal{O}(N^2)}$ $\underbrace{\phantom{-\hat{k}^2}}_{\mathcal{O}(N^2)}$

Standard way of implementing second derivatives with exponential convergence*

Mixed Basis Circuit Construction, Asymptotic Approach

$$\partial^2_{\omega_i}$$



Standard way of implementing second derivatives with exponential convergence*

$$f_2(\omega_j)\partial_{\omega_i}$$

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Mixed Basis Circuit Construction, Asymptotic Approach

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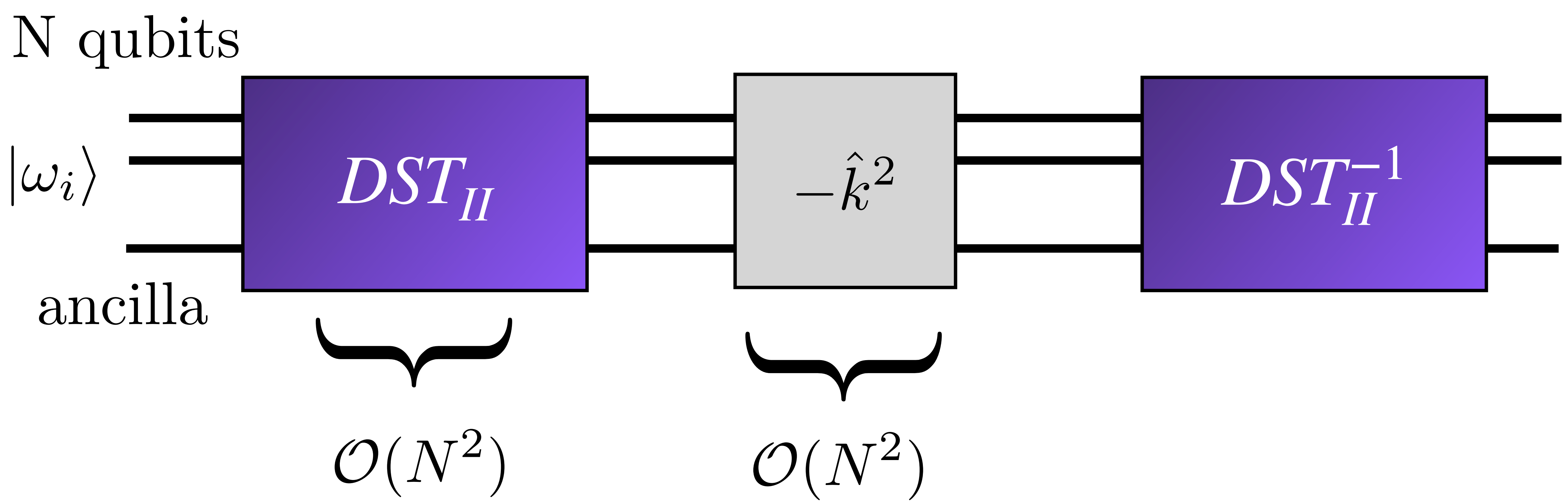

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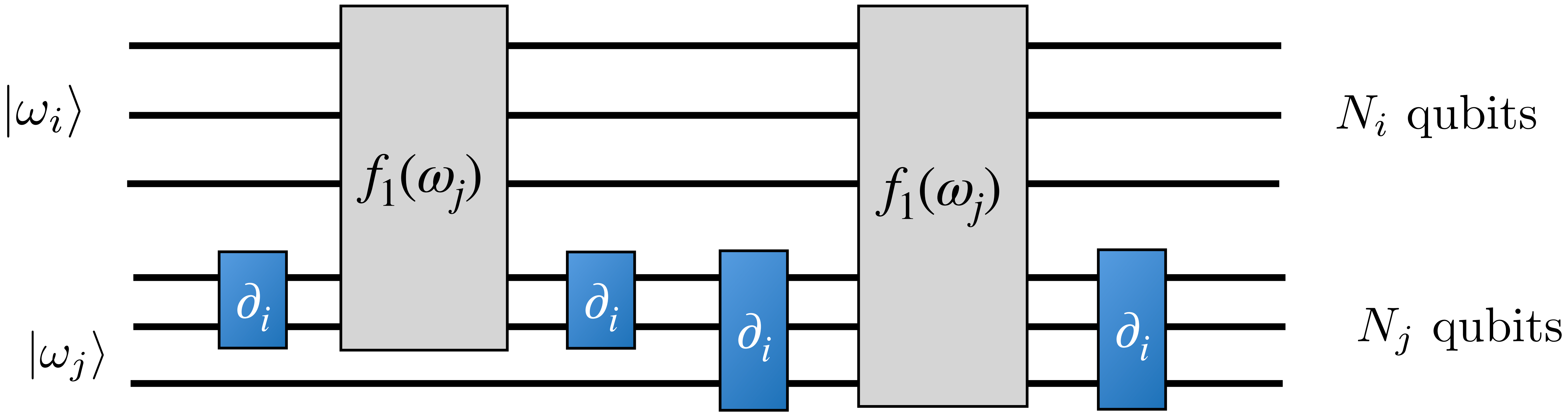

Mixed Basis Circuit Construction, Asymptotic Approach

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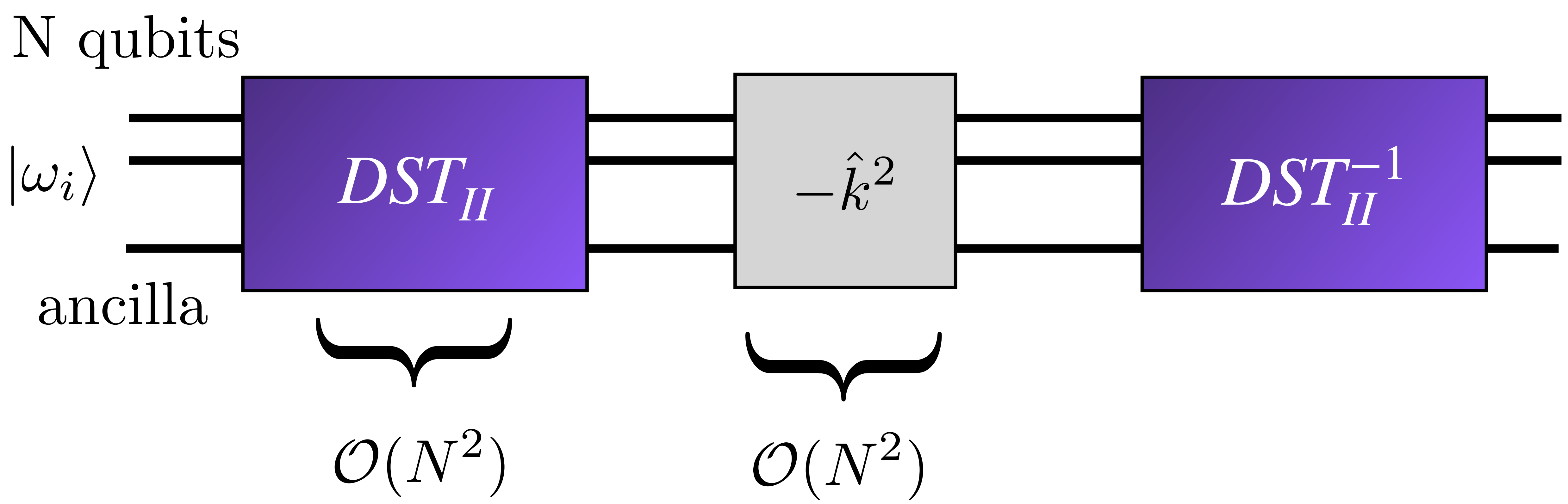

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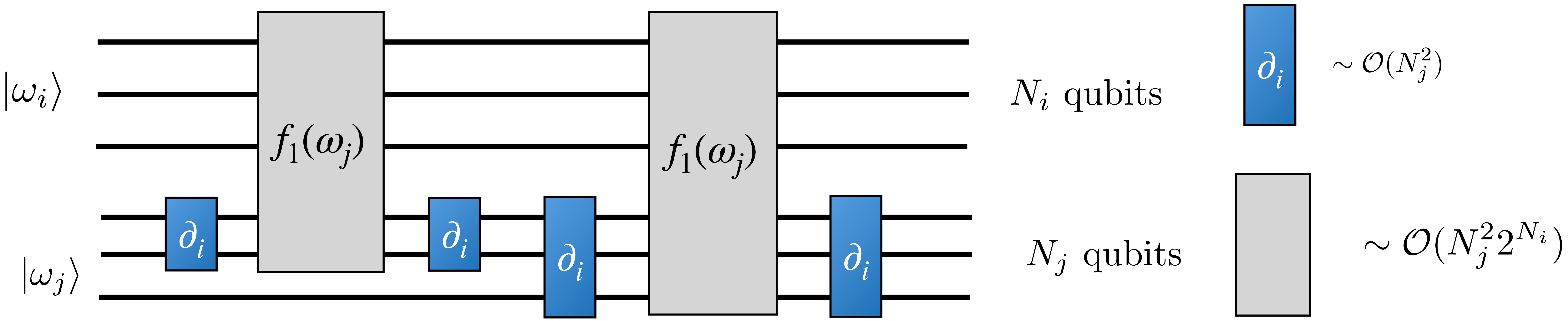

Exponential CNOT gate is $f_1(\omega_j)$ is due to this being a trigonometric function - approximation dramatically reduces overhead cost

* N. Klco and M.J. Savage, Phys. Rev. A 99, 052335

Mixed Basis Circuit Construction, NISQ Approach

General Idea: Decompose each term in Hamiltonian into Pauli strings and optimize

$$H = \sum_k \beta_k H_k = \sum_k \sum_{i \in S(P_k)} c_i \mathscr{P}_i \qquad \mathscr{P}_i = \otimes_{\ell \in i} \sigma_\ell^{\alpha_{i\ell}}$$

S_k= support of H_k and P_k is power set

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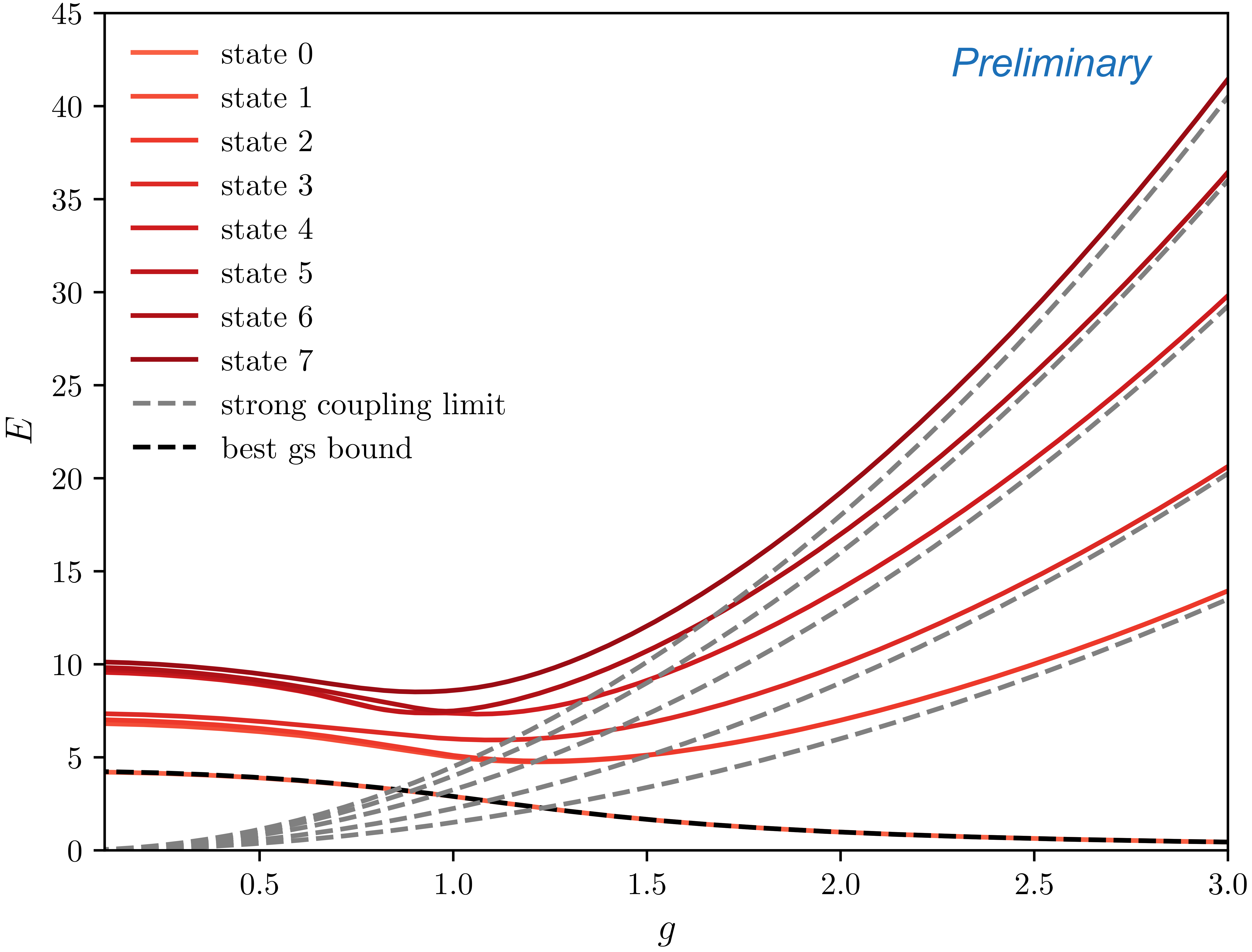
General Comments:

- Generically there are $4^{\#(S_k)}$ terms
- Choose some truncation scale ϵ and ignore all smaller rotations
- Hardest terms to implement (most non-local, highest weight) have smaller coefficients - even mild truncations gives biggest savings
- Usefulness depends on truncation

Energy Spectrum Results

Energy Spectrum for Two Plaquette System using mixed basis formulation

- Three qubits per ω and one for ν
- Strong coupling limit is result for character irrep formulation
- Best ground state bound is PDE (FEM) solver result
- Laguerre results will be added



Conclusions

Simulating non-Abelian gauge theories on digital quantum devices necessitates balancing the requirements of gauge invariance, efficiency for fine lattices and systematic improvability

Main Take-Away Point 1: Gauge fixing allows for constructing Hamiltonians in the group element basis, allowing for efficient simulations at weak coupling*

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* Do not worry: we are thinking about how to extend this to go to SU(3) and include fermions

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Simulating non-Abelian gauge theories on digital quantum devices necessitates balancing the requirements of gauge invariance, efficiency for fine lattices and systematic improvability

Main Take-Away Point 1: Gauge fixing allows for constructing Hamiltonians in the group element basis, allowing for efficient simulations at weak coupling*

Main Take-Away Point 2: Non-local interactions do not always result in highly connected systems and exponential resource scaling

* Do not worry: we are thinking about how to extend this to go to SU(3) and include fermions