Quantum Computing for Transport and Energy Correlators

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InQubator for Quantum Simulation

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Real-Time Correlators

- **Definition**

$$\langle \mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\cdots\mathcal{O}_n(t_n)\rangle_T$$

$$T = 0, \text{ vacuum}$$

$$T \neq 0, \text{ thermal}$$

1. Time-ordered: e.g. in perturbation theory
2. Schwinger-Keldysh contour: out-of-equilibrium dynamics
3. Out-of-time-ordered (OTOC)

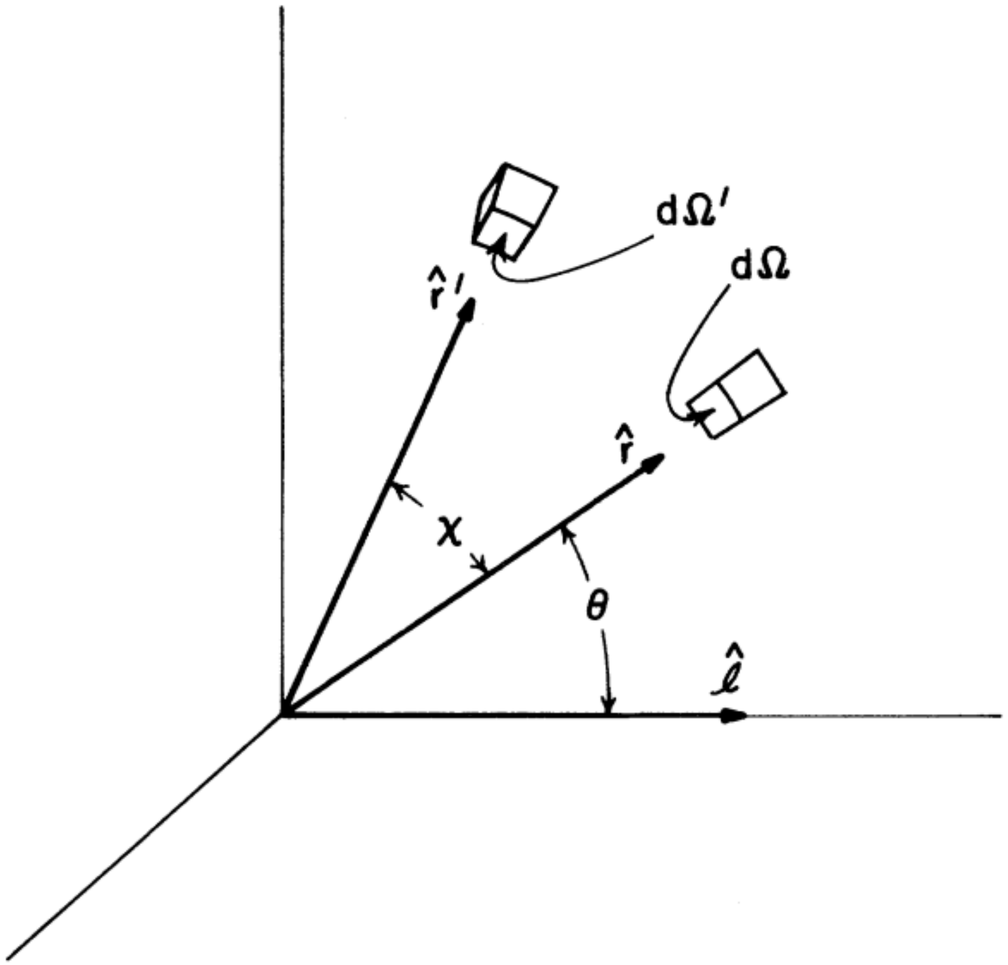
- **Contain useful physics information**

Parton distribution function	**Transport coefficients**
Jet/soft function in jet physics	Quantum chaos
Energy-energy correlators	And more

Energy Correlators

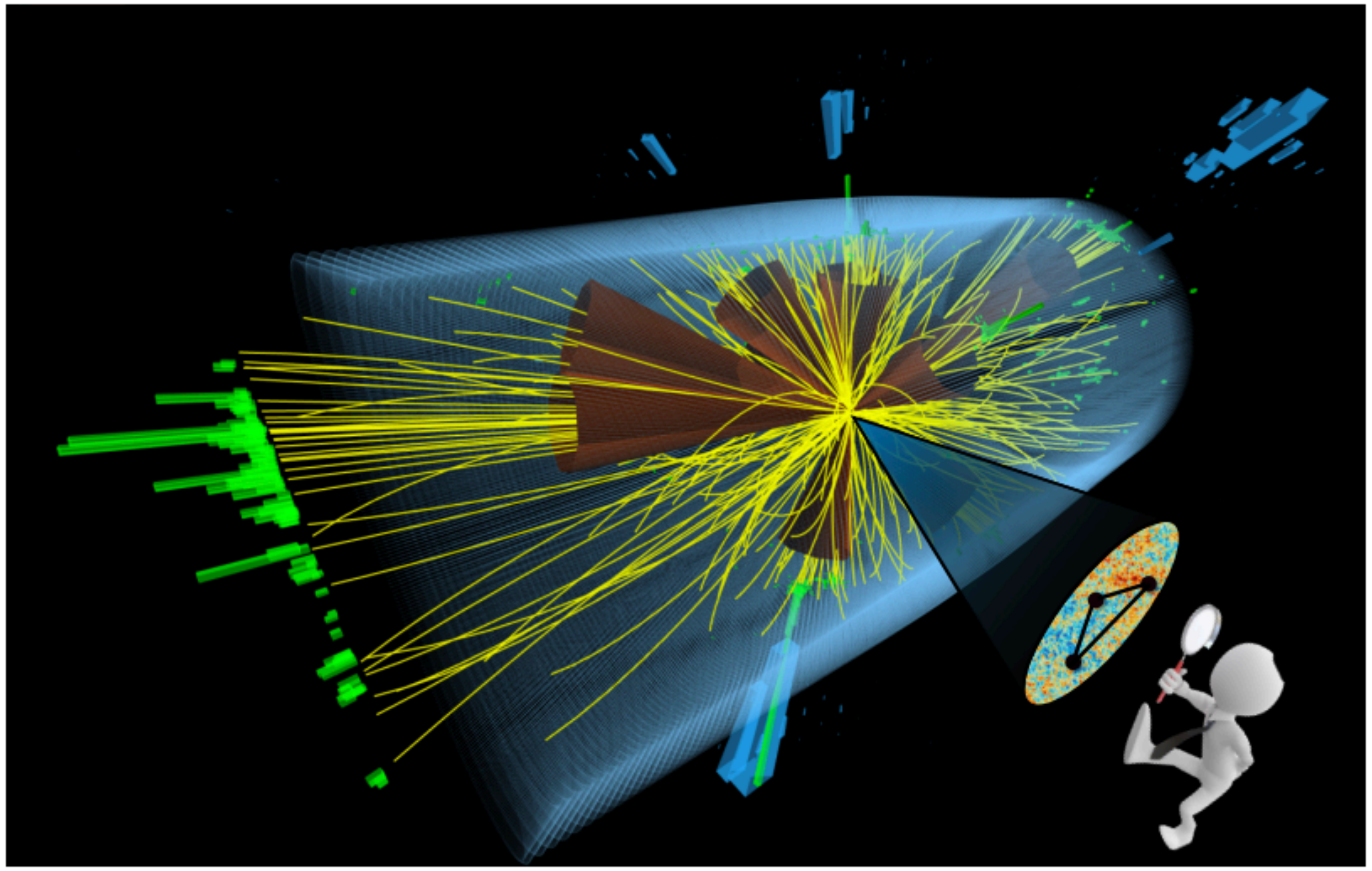
Introduction 1: Energy Correlators

- **Correlations of energy flow in collider events**



Sterman ILL-TH-75-32 (1975)

Basham, Brown, Ellis, and Love,
PRL 41, 1585 (1978)



Introduction 1: Energy Correlators

- **Field theory definition**

Energy flow operator from T_{0i}

$$\mathcal{E}(\theta) = \lim_{r\to\infty} r^2 \int_{-\infty}^{\infty} dt\, n^i T^0_{\ i}(t, r\vec{n}^i)$$

$$\langle \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\rangle \equiv \frac{\langle 0|\mathcal{O}^\dagger \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\mathcal{O}|0\rangle}{\langle 0|\mathcal{O}^\dagger\mathcal{O}|0\rangle}$$

- **Conformal field theory**

Strong coupling: flat in θ

$$\langle \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\rangle = \left(\frac{Q}{4\pi}\right)^n$$

Hofman, Maldacena, 0803.1467

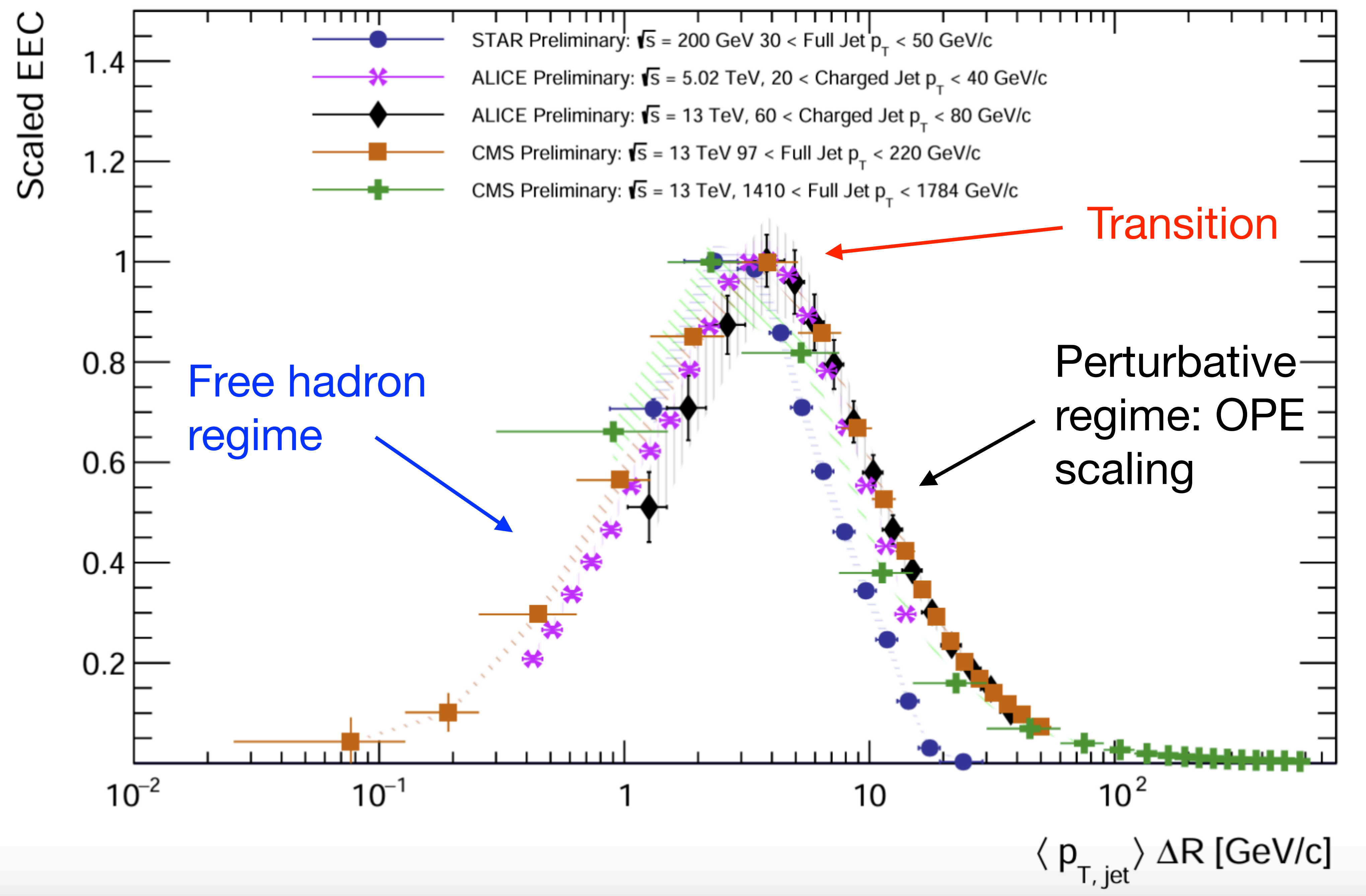
Weak coupling: OPE at small angle:, scaling controlled by anomalous dimension

$$\langle \mathcal{E}(\theta_1)\mathcal{E}(\theta_2)\cdots\rangle \sim \sum_n |\theta_{12}|^{\tau_n - 4}\langle \mathcal{U}_{3-1,n}(\theta_2)\cdots\rangle$$

Introduction 1: Experimental Measurement of EEC

- **Measurements in jets**

Tamis, Hard Probe 2024

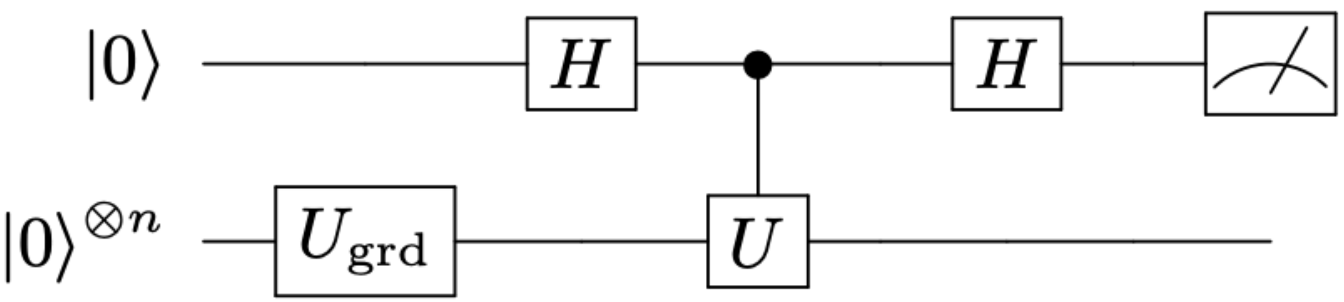


Understanding transition regime requires nonperturbative calculation

Quantum Algorithm for Real-Time Correlator

$$\sum_{i=0}^{N} \int_{t_i}^{\tilde{t}_i} \mathrm{d}t_1 \int_{t_i}^{\tilde{t}_i} \mathrm{d}t_2 \langle 0|\mathcal{O}^\dagger(0) n_1^i T_{0i}(t_1, r_i\vec{n}_1) n_2^j T_{0j}(t_2, r_i\vec{n}_2)\mathcal{O}(0)|0\rangle$$

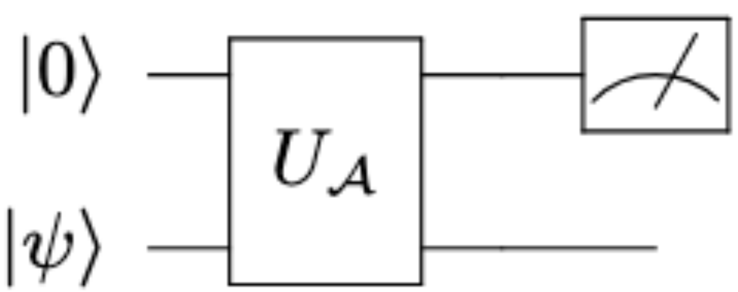
- **Hadamard test**



$$= \frac{1 + \mathrm{Re}\langle 0|U|0\rangle}{2}$$

$$U = \mathcal{O}^\dagger e^{iHt_1} T_{0i}(r\vec{n}_1) e^{iH(t_2 - t_1)} T_{0j}(r\vec{n}_2) e^{-iHt_2}\mathcal{O}$$

- **Diluted operator to realize nonunitary operator $\mathcal{A}$**



$$U_{\mathcal{A}} = e^{i\sigma^y \otimes \arccos\left(\frac{\mathcal{A}}{\|\mathcal{A}\|}\right)}$$

Success probability $$P_s = \langle\psi|\frac{\mathcal{A}^2}{\|\mathcal{A}\|^2}|\psi\rangle$$

Discretized Time Integration Path

- **Energy flux detectors spacelike separated**

$$[\mathcal{E}(\vec{n}_1), \mathcal{E}(\vec{n}_2)] = 0$$

Constant time integration not work

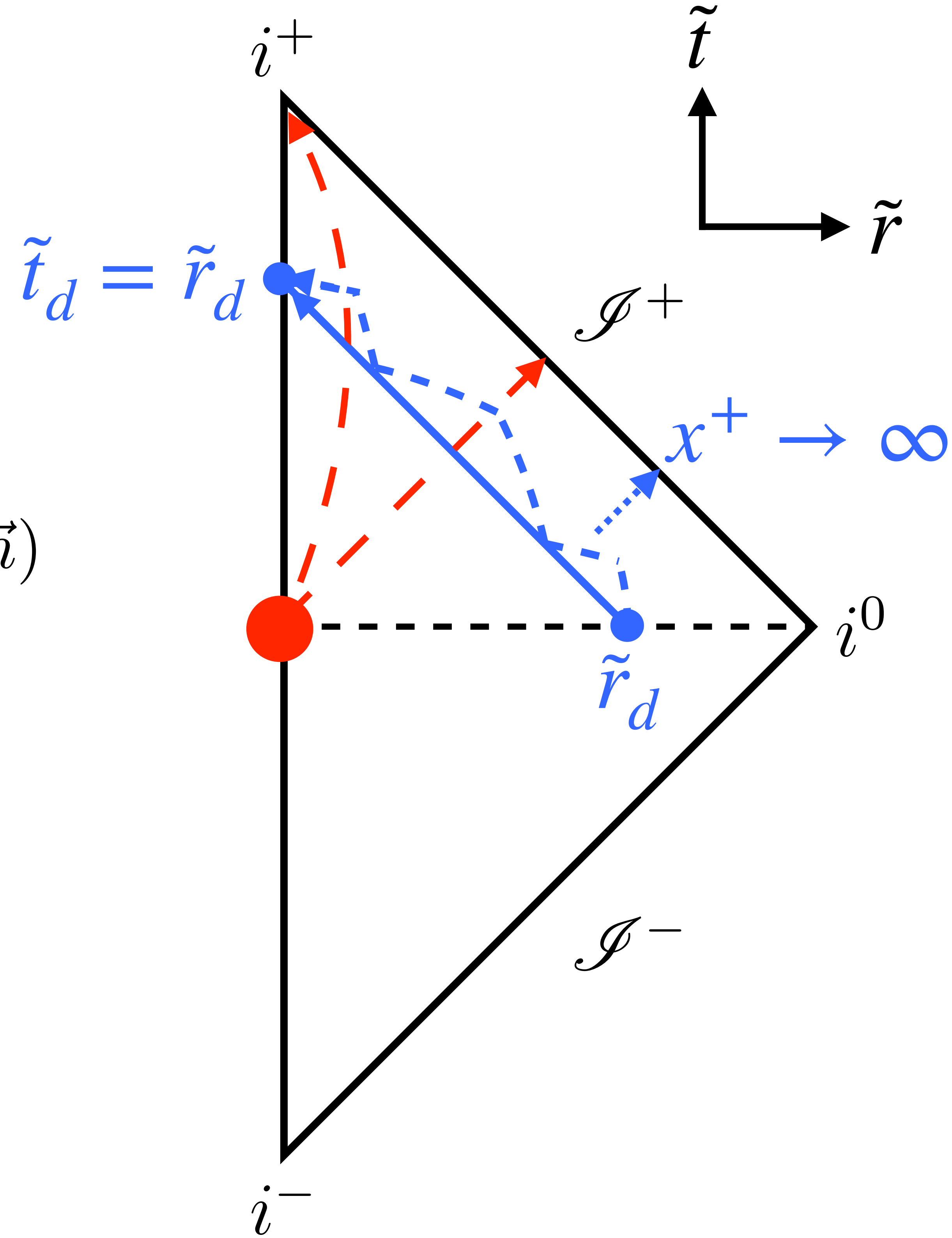
- **Integrate along x^- with x^+ fixed**

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty \mathrm{d}t\, r^{d-1}\, n^i T_{0i}(t, r\vec{n})$$

$$\int_0^\infty \mathrm{d}t\, n^i T_{0i} \to \int_{-x^+}^{x^+} \mathrm{d}x^-\, n^i T_{-i}$$

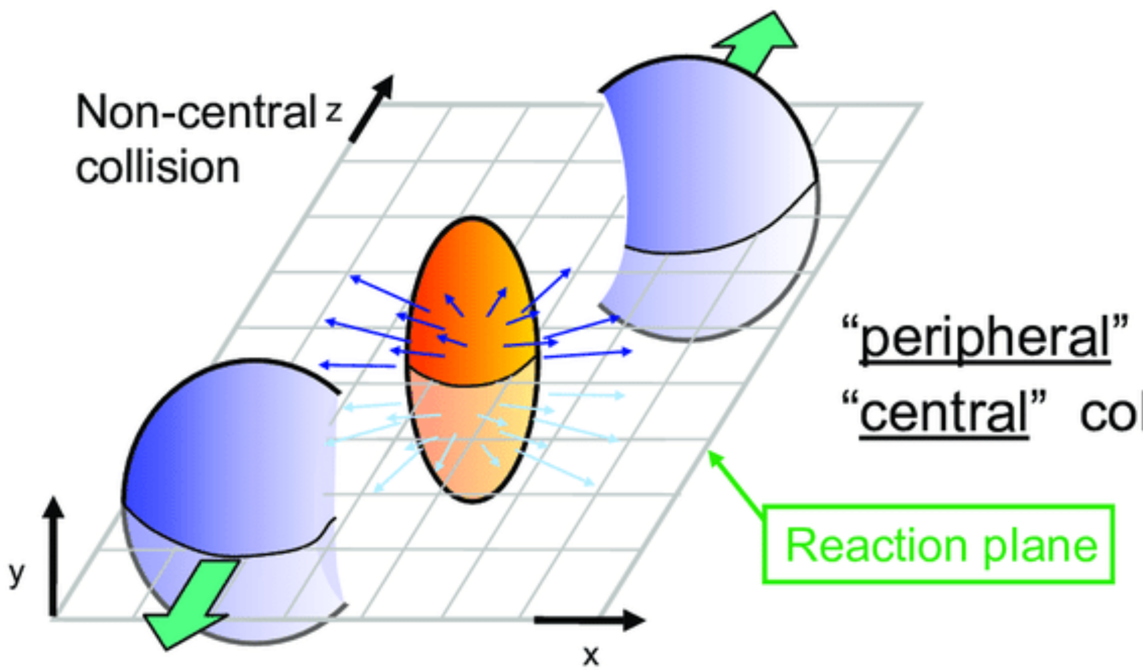
Then take $x^+ \to \infty$

- **On lattice: zigzag contour**

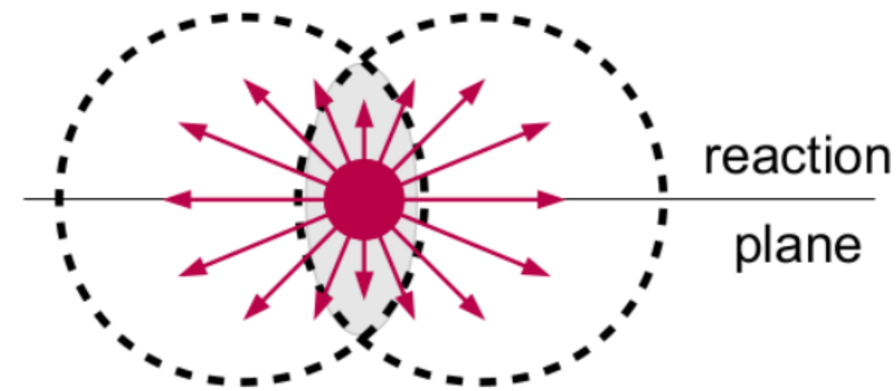


Transport Coefficients

Introduction 2: Flow in Heavy Ion Collisions



"peripheral" collision (b ~ b_{max})
"central" collision (b ~ 0)

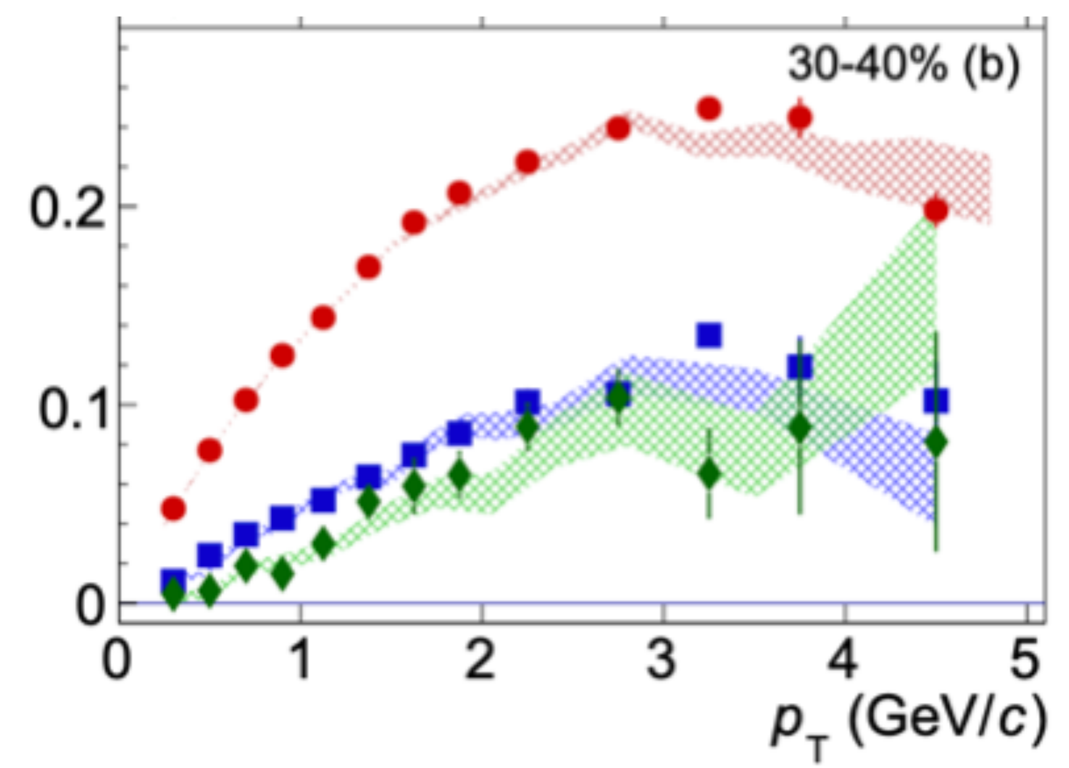


- **Anisotropic distribution —> collective behavior**

$$\rho(\phi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi)\right]$$

Flow coefficients

v_2: elliptic flow,
v_3: triangular flow



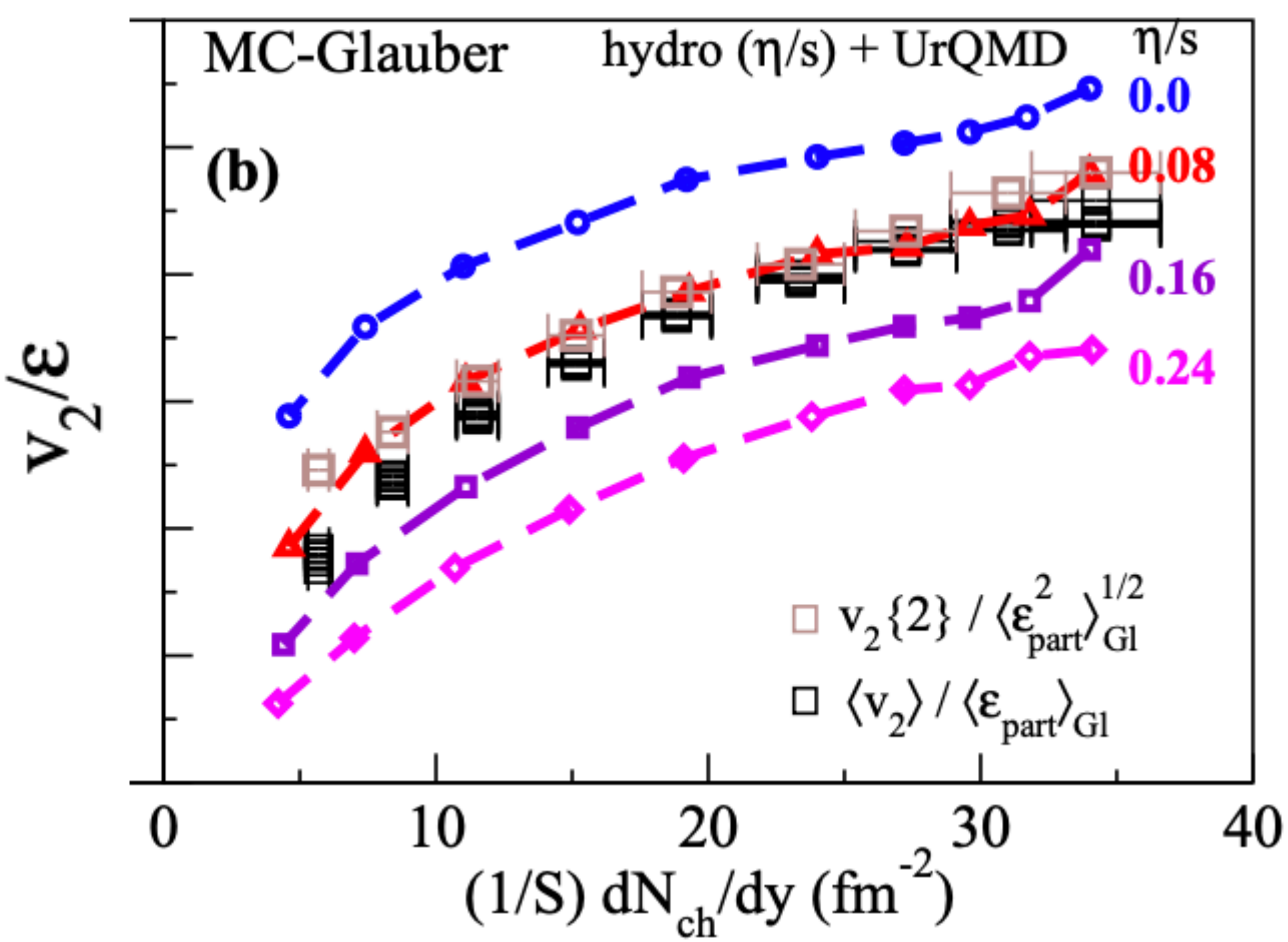
ALICE, 1602.01119

Introduction 2: Flow and Shear Viscosity

- **Relativistic hydrodynamic calculations indicate a small shear viscosity works**

$$\nabla_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} + 2\eta \nabla^{\langle\mu} u^{\nu\rangle}$$



Song, Bass, Heinz, Hirano, Shen, 1011.2783

$\eta/s = 0.08$ best describes data

$\eta/s \sim 1000$ for air

$\eta/s \sim 10$ for water

Calculating shear viscosity in QCD is hard

Moore, 2010.15704

Shear Viscosity from Linear Response

- **Kubo formula: transport determined by real-time correlation function**

$$\eta = \lim_{\omega \to 0} \frac{\partial}{\partial \omega} G_r^{xy}(\omega)$$

Baier, Romatschke, Son, Starinets, Stephanov, 0712.2451

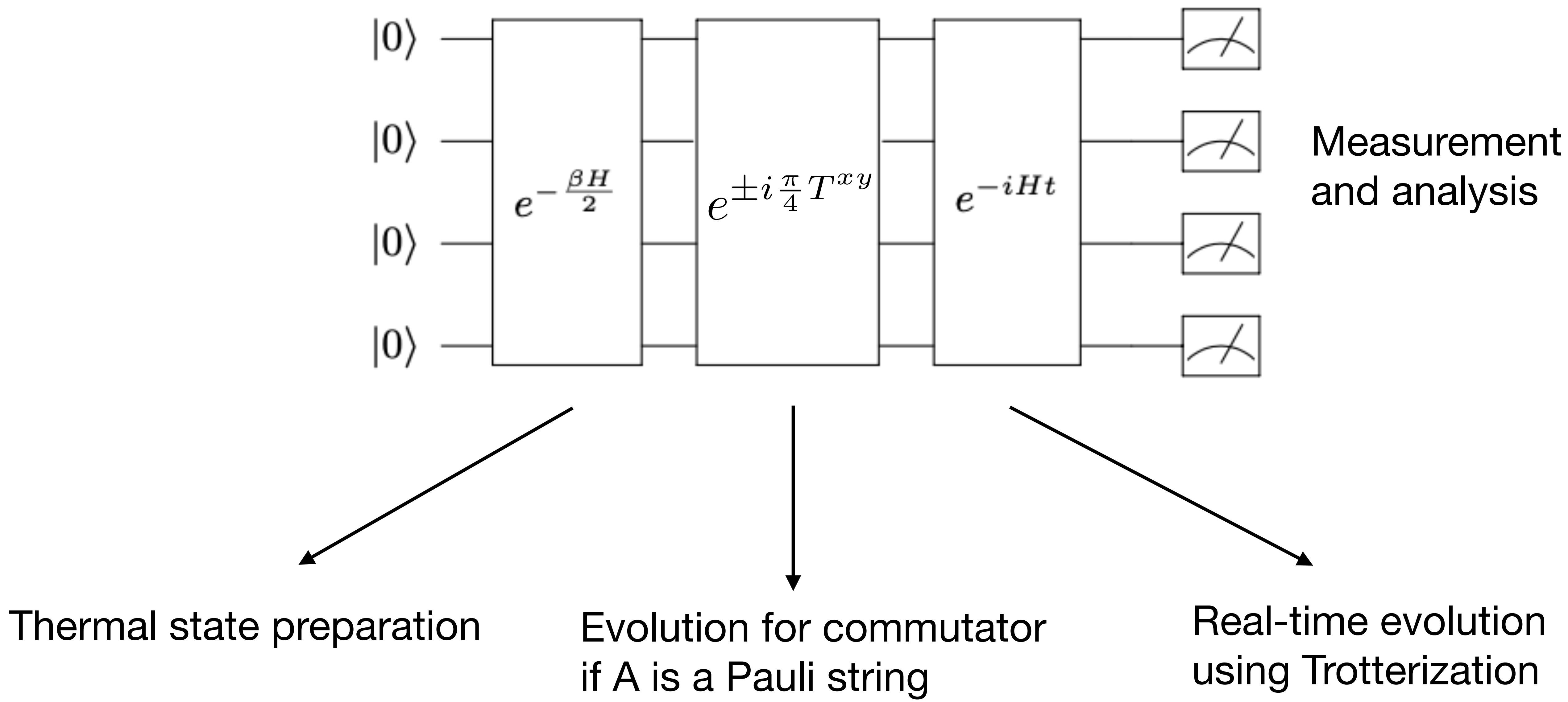
- **Retarded Green's function of T^{xy}**

$$G_r^{xy}(t, \boldsymbol{x}) \equiv \theta(t) \mathrm{Tr}\Big([T^{xy}(t, \boldsymbol{x}), T^{xy}(0, \mathbf{0})] \rho_T \Big)$$

$$\rho_T = \frac{1}{Z} e^{-\beta H}$$

A Quantum Algorithm for Retarded Correlator

- **An overview**



$$[A, B] = -i\left(e^{-i\frac{\pi}{4}A}Be^{i\frac{\pi}{4}A} - e^{i\frac{\pi}{4}A}Be^{-i\frac{\pi}{4}A}\right)$$

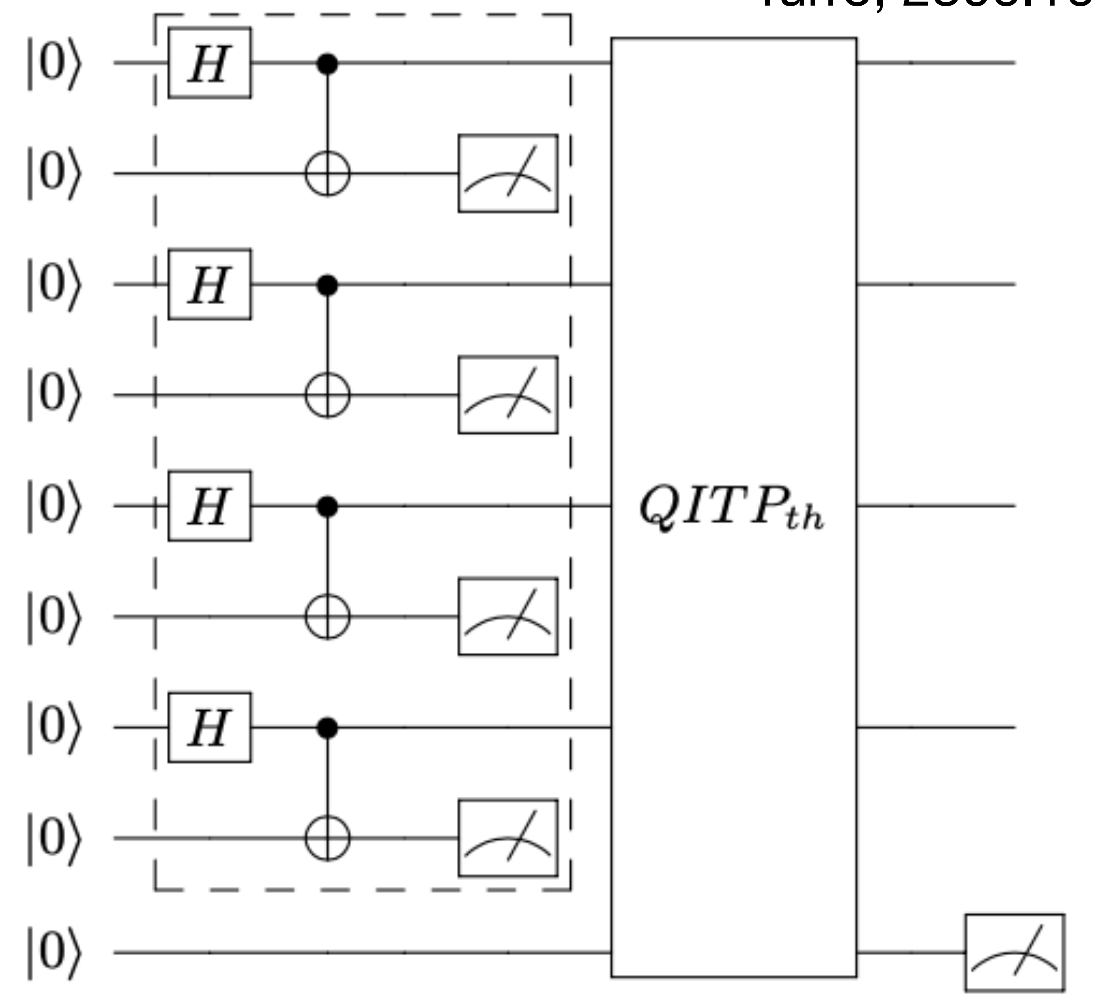
Quantum Imaginary Time Propagation

Turro, 2306.16580

- **Initialization: n_s system qubits + $(n_s + 1)$ ancillas**

Hadamard + CNOT + measurements
give maximally mixed state

$$\rho_s = \frac{1}{2^{n_s}} 1_{2^{n_s} \times 2^{n_s}}$$



- **Quantum imaginary time propagation**

$$QITP_{th} = \begin{pmatrix} \sqrt{p}\, e^{-\tau(H-E_T)} & \sqrt{1-p\, e^{-2\tau(H-E_T)}} \\ -\sqrt{1-p\, e^{-2\tau(H-E_T)}} & \sqrt{p}\, e^{-\tau(H-E_T)} \end{pmatrix}$$

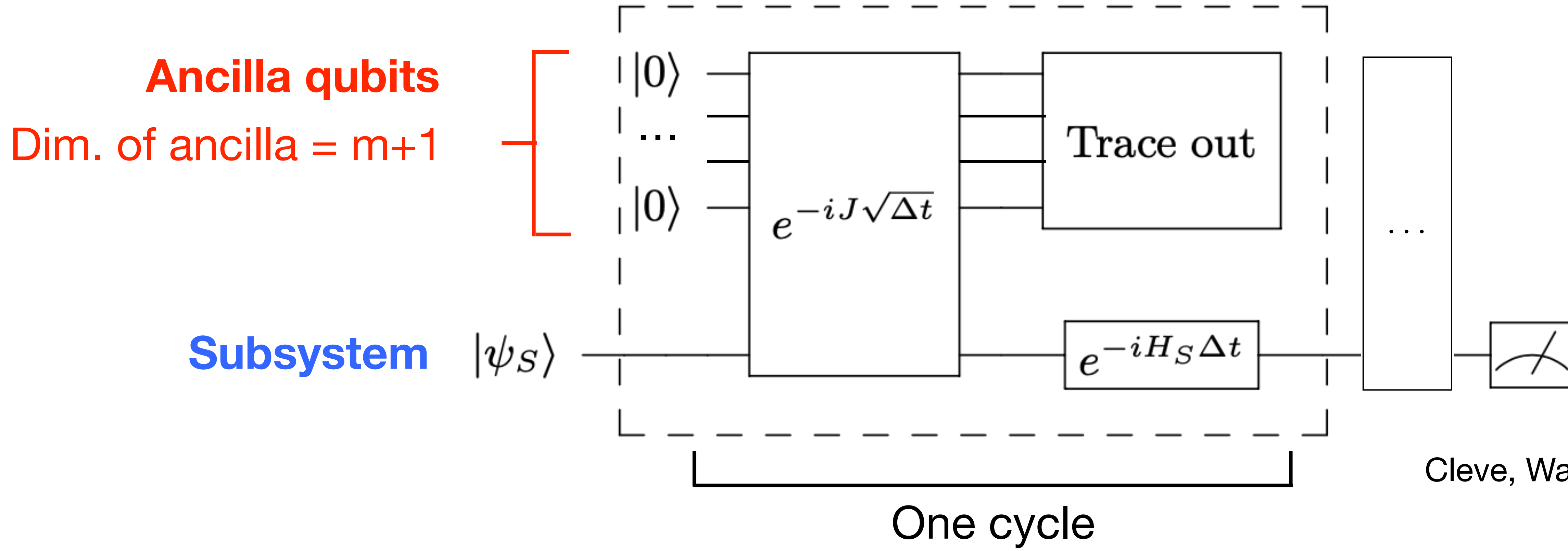
- **Measure the ancilla, success if $|0\rangle$ returned**

Success probability decays **exponentially** with system size

Thermal State Preparation: Open Quantum System

- **Lindblad equation in quantum Brownian motion limit, steady state ≈ thermal**

$$\frac{d\rho}{dt} = -i\left[H_S, \rho\right] + \sum_{j=1}^{m}\left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\}\right) \equiv \mathcal{L}\rho$$



Cleve, Wang, 1612.09512

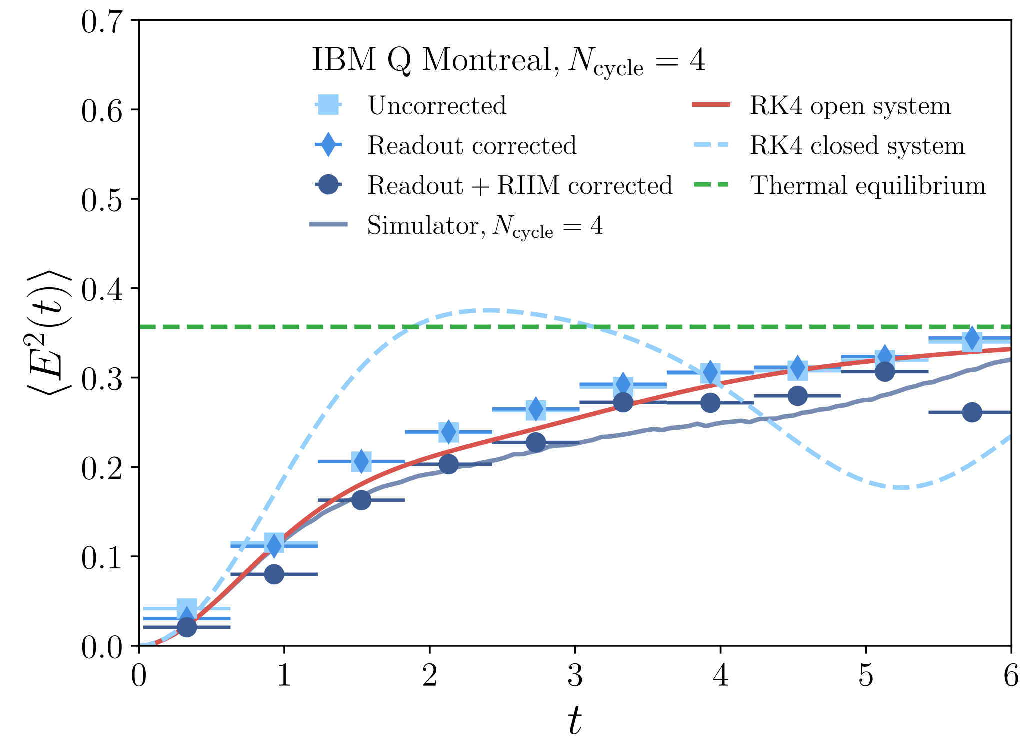
$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

Reproduce Lindblad equation if expanded to linear order in Δt

Efficient Thermal State Preparation: OQS

- **Implementation on IBM hardware for Schwinger model**

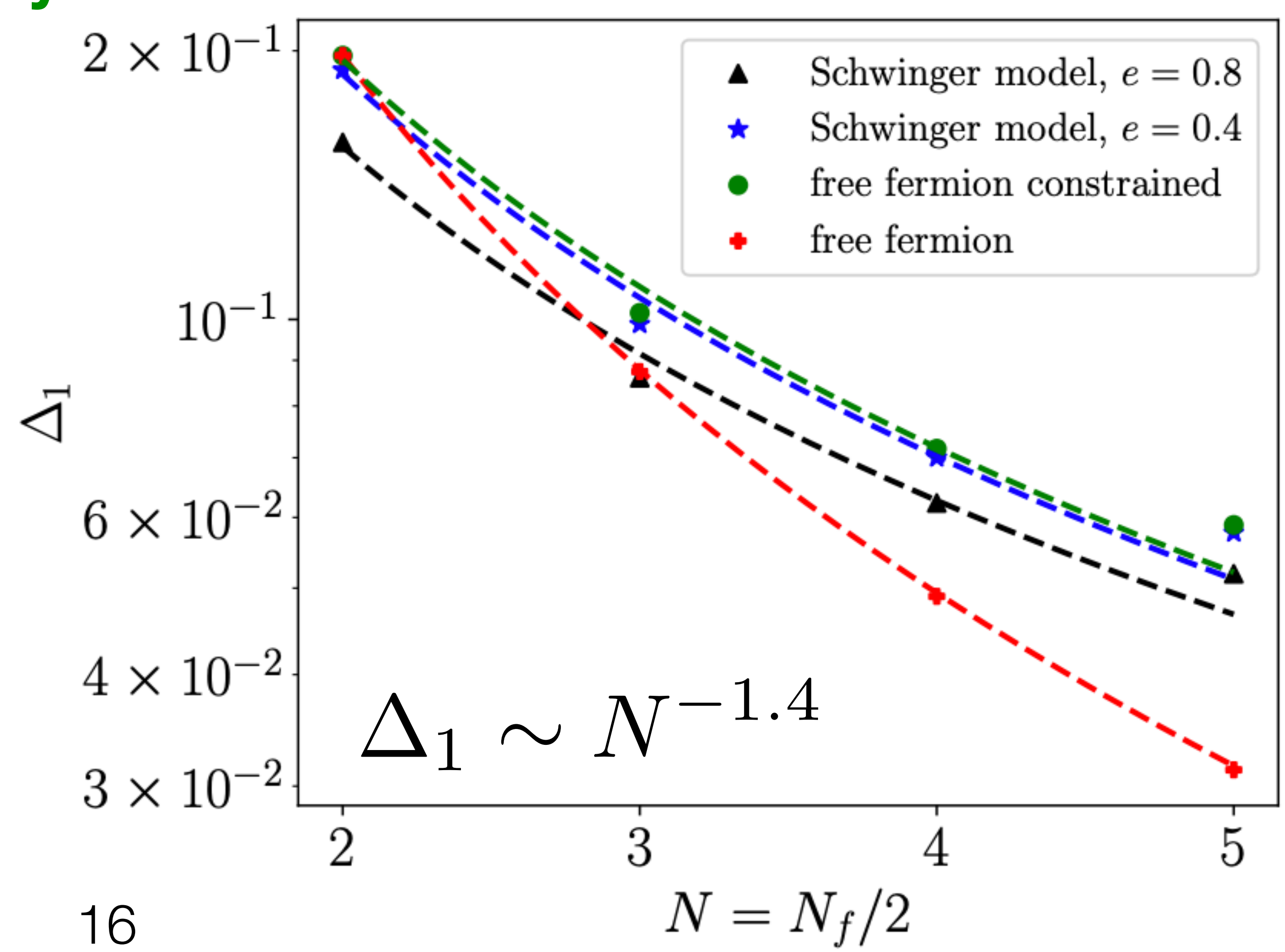
de Jong, Lee, Mulligan, Ploskon, Ringer and XY, 2106.08394



- **Equilibration time scales polynomially**

$$\rho(t) = \rho_0 + \sum_j c_j e^{\lambda_j t} \rho_j^R$$

Liouvillian gap $\Delta_j = -\,\mathrm{Re}\,\lambda_j$

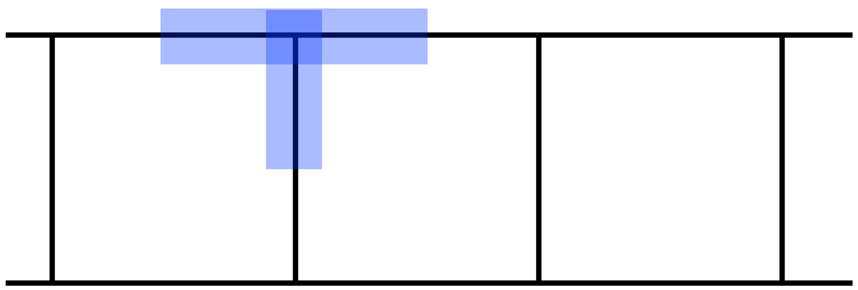


Lee, Mulligan, Ringer and XY, 2308.03878

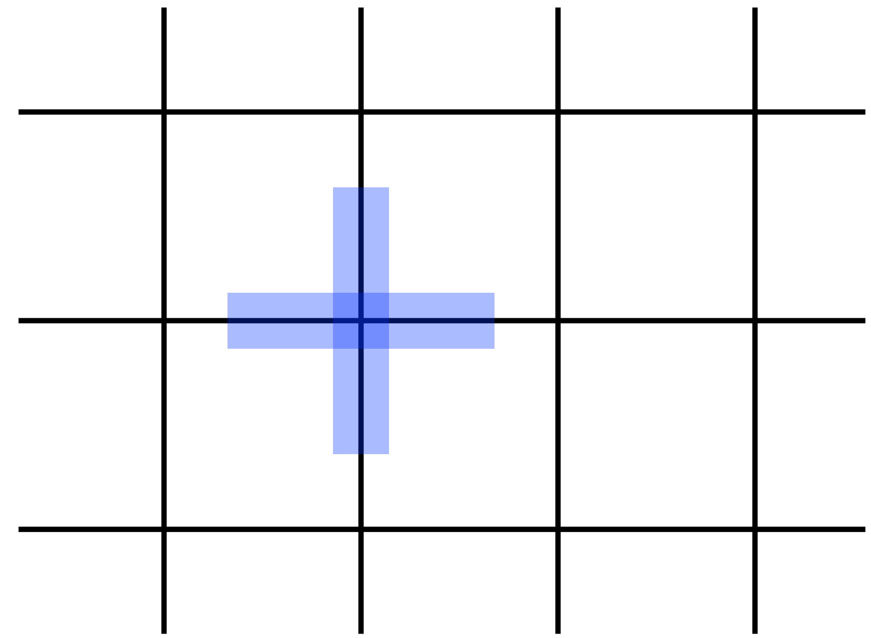
Application to 2+1D SU(2)

Honeycomb Lattice for SU(2)

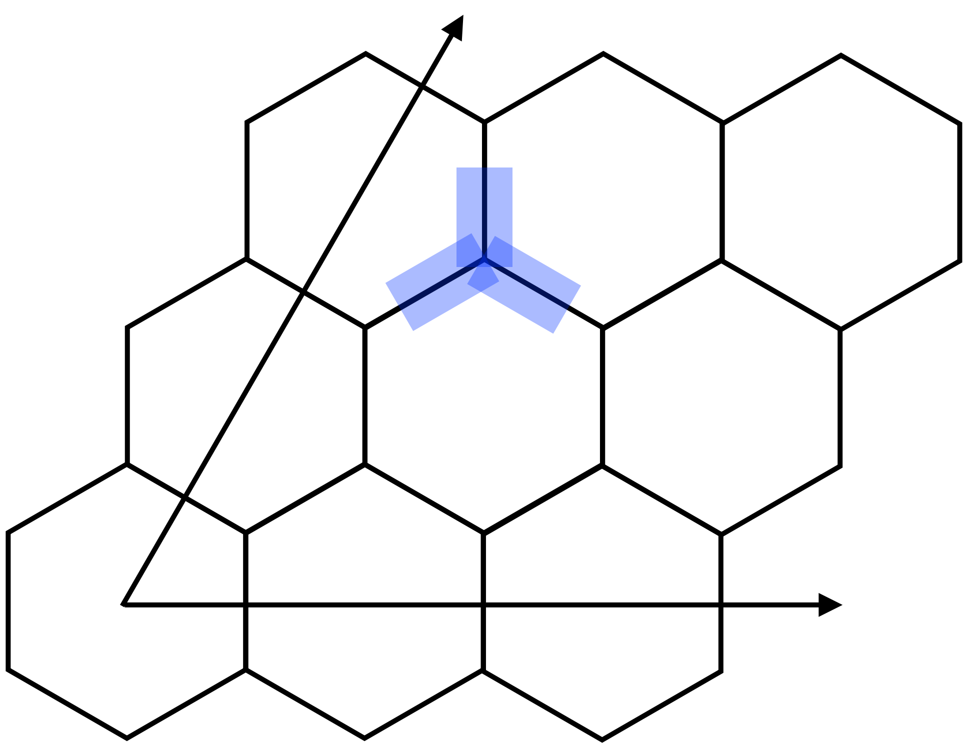
- **Problem on square lattice:** each vertex has four links —> singlet is **not uniquely** defined by four j values



Klco, Stryker, Savage, 1908.06935



- **Use honeycomb lattice**



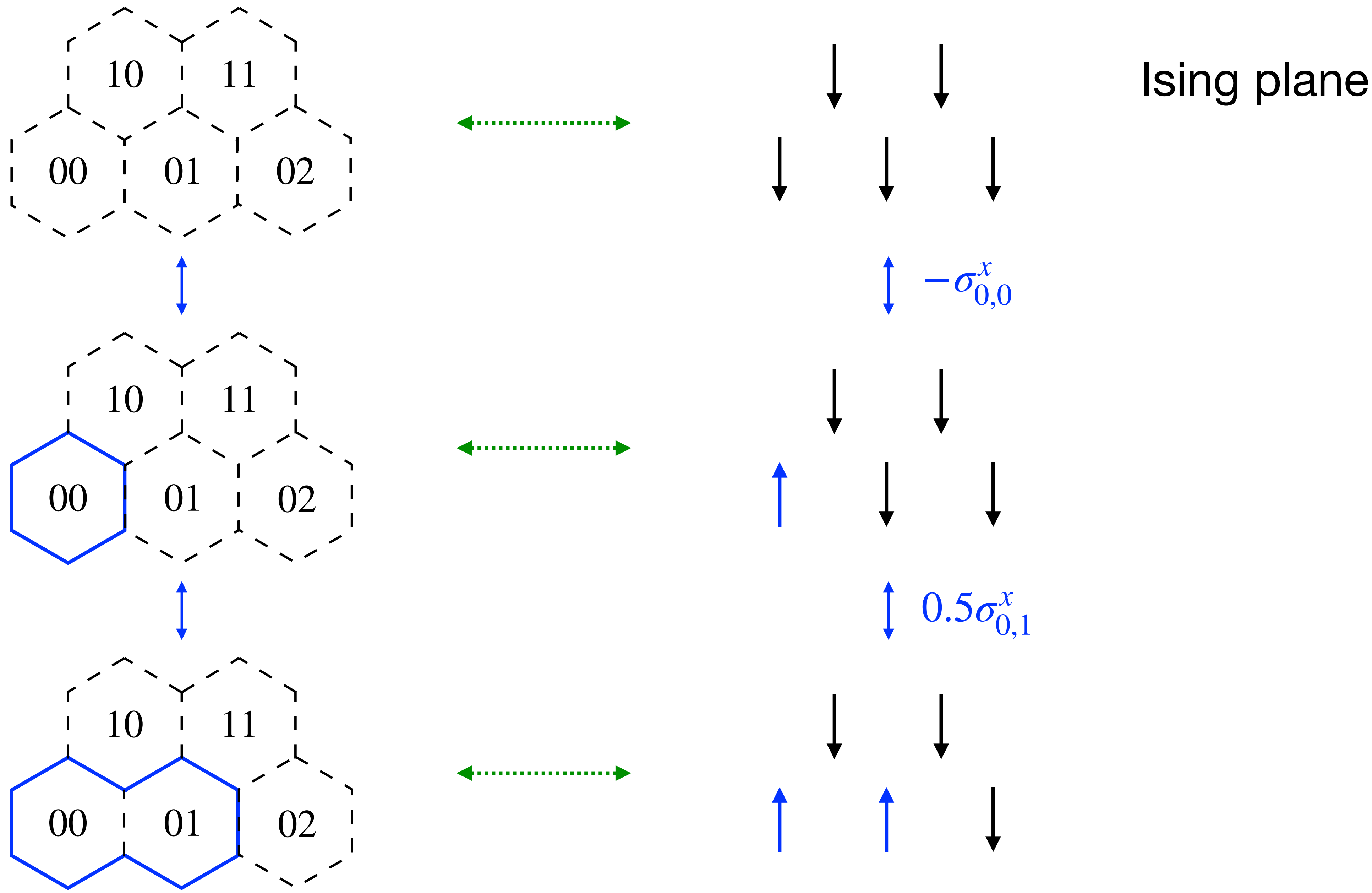
Müller, XY, 2307.00045
Illa, Savage, XY, 2503.09688

$$H_{\text{el}} \propto g^2 \sum_{\text{links}} E_i^a E_i^a$$

$$H_{\text{mag}} \propto -\frac{1}{a^2 g^2} \sum_{\text{plaqs}} \hexagon$$

Simplify Hamiltonian with $j_{\max} = 1/2$

SU(2) w/ $j_{\max} = \frac{1}{2}$



$$aH = h_+ \sum_{(i,j)} \Pi^+_{i,j} - h_{++} \sum_{(i,j)} \Pi^+_{i,j} \left(\Pi^+_{i+1,j} + \Pi^+_{i,j+1} + \Pi^+_{i+1,j-1} \right) + h_x \sum_{(i,j)} \sigma^x_{ij} D_{ij}$$

$$\Pi^+_{i,j} = (1 + \sigma^z_{i,j})/2$$

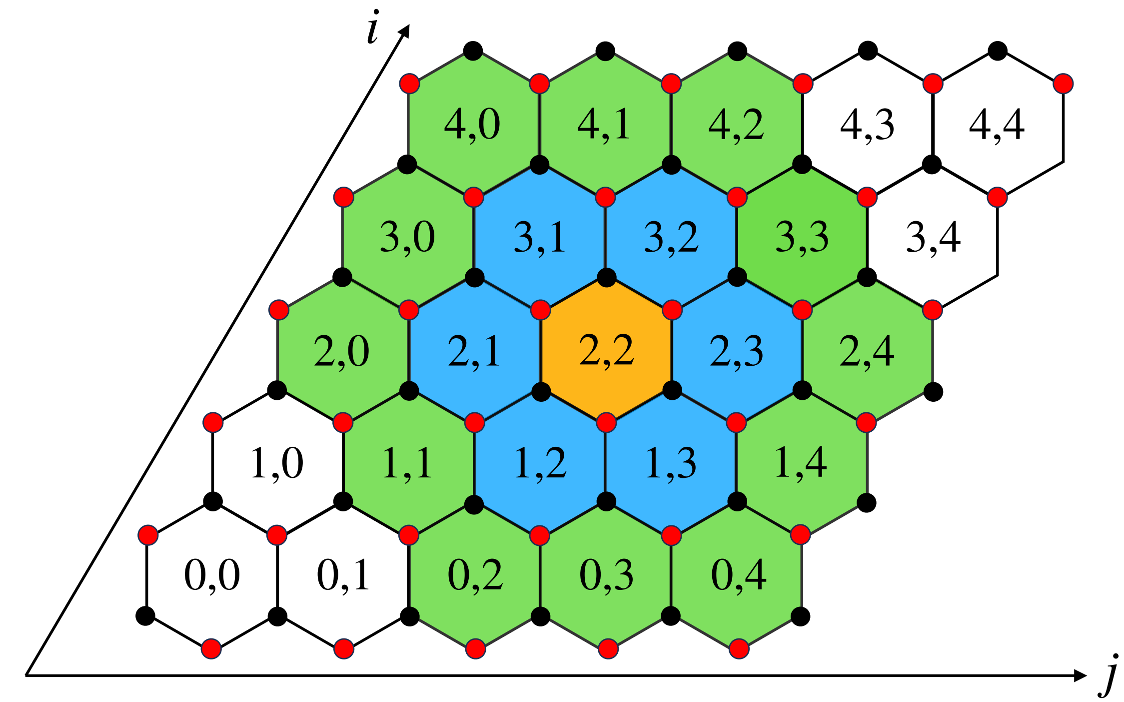
Müller, XY, 2307.00045

Results

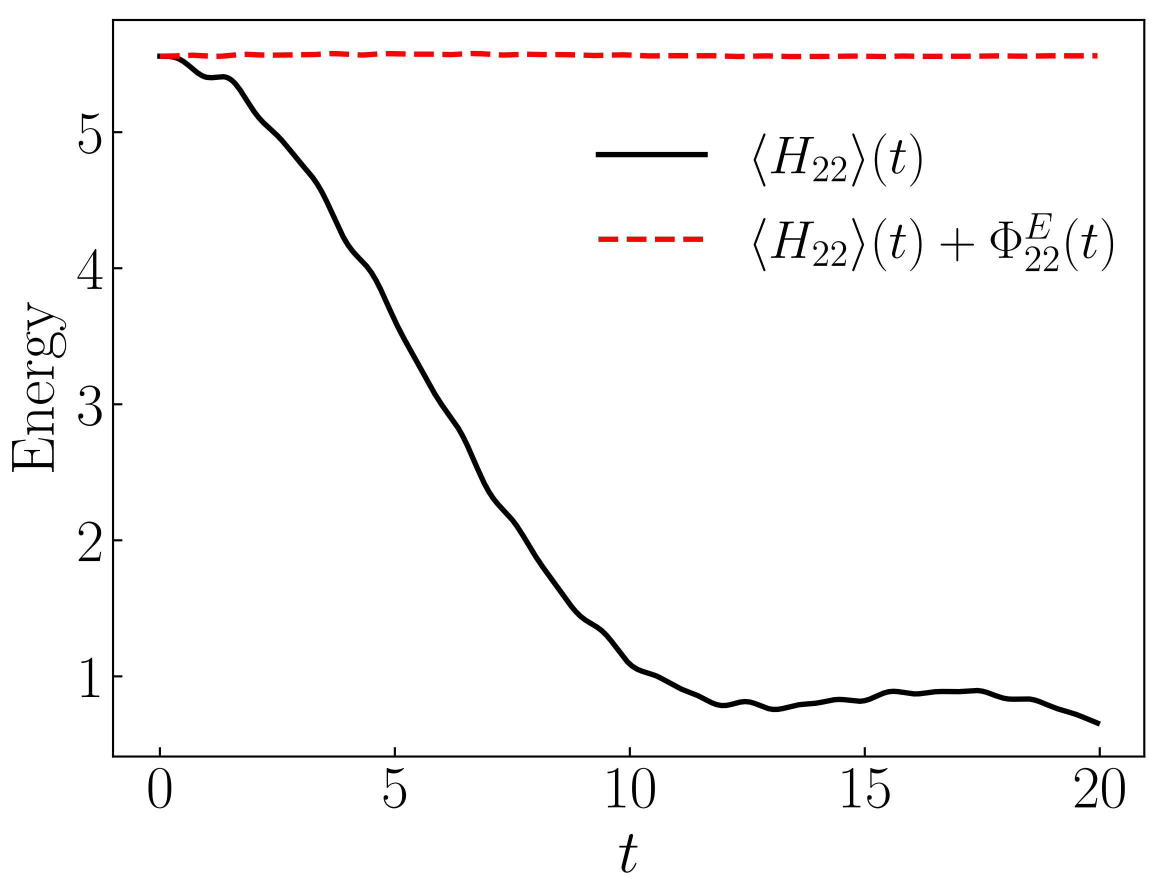
Energy Correlator for 2+1D SU(2) on Small Lattice

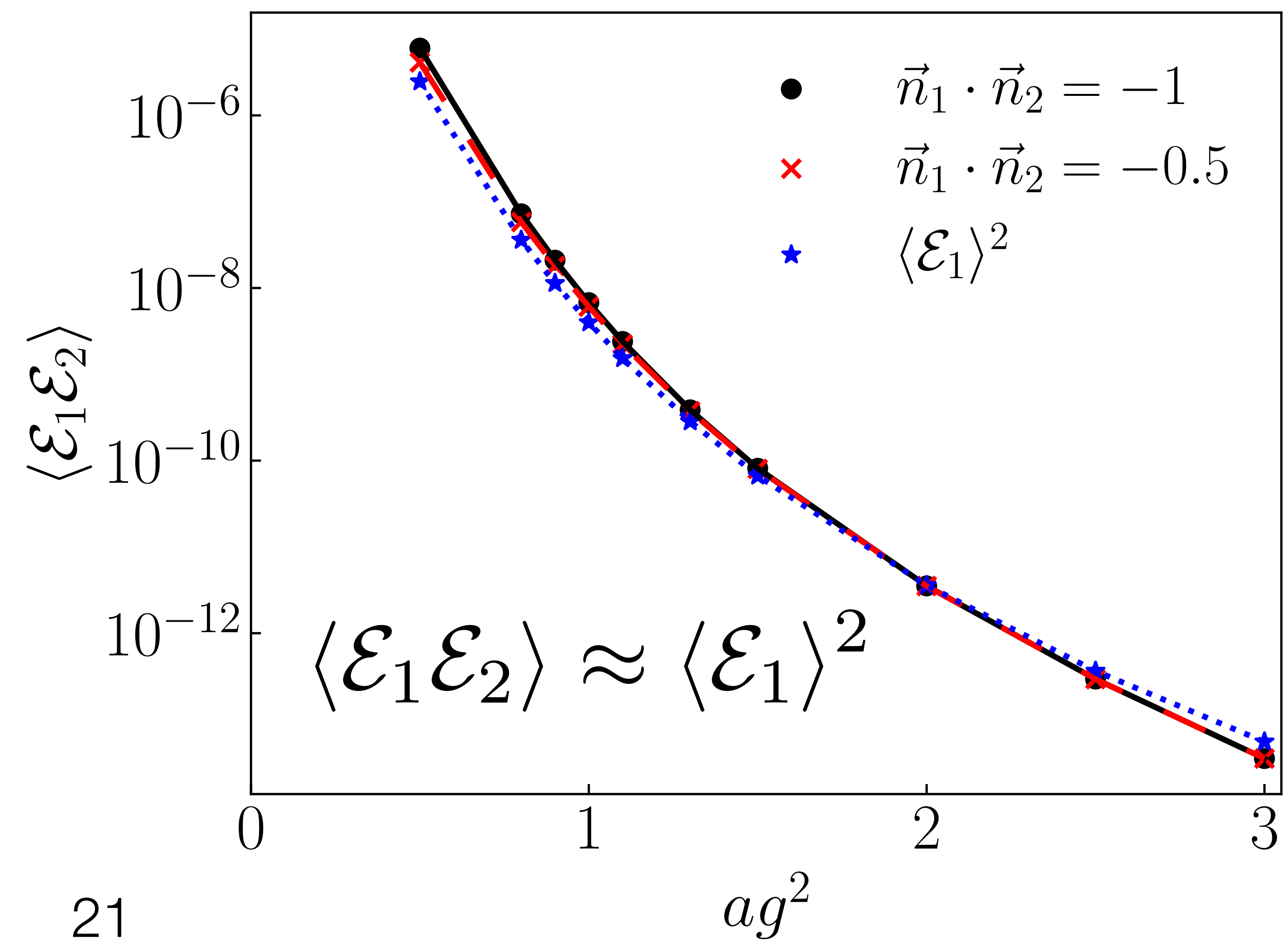
- 5×5 **lattice with** $ag^2 = 1$**,** $j_{\max} = 1/2$

Perturbation source at center (2,2)



Energy conservation works well

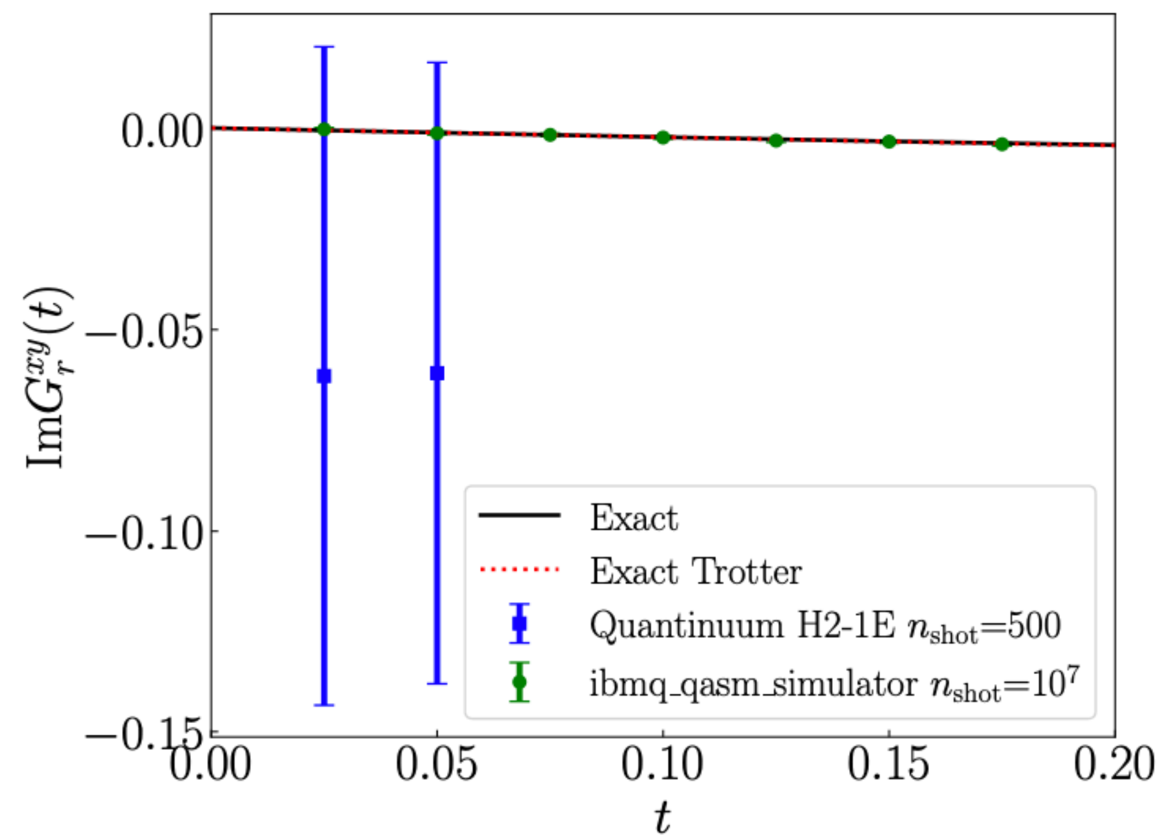




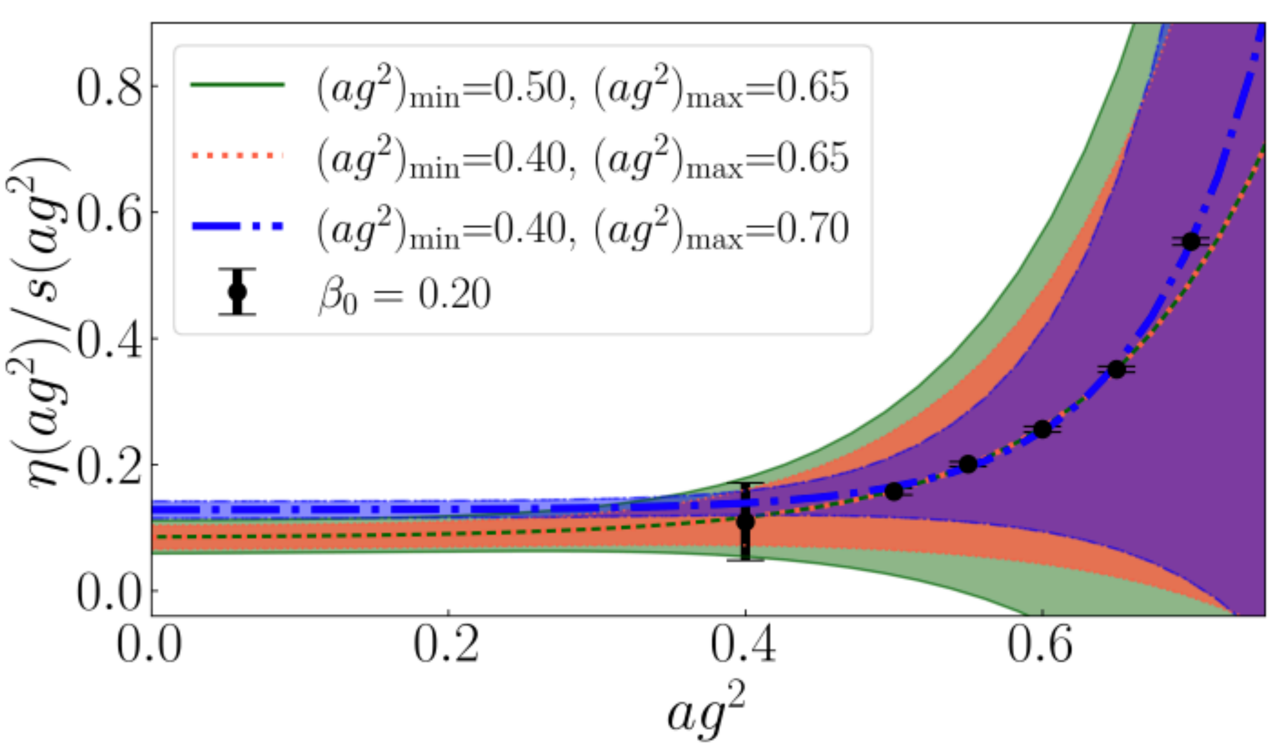
Shear Viscosity on Small Lattice

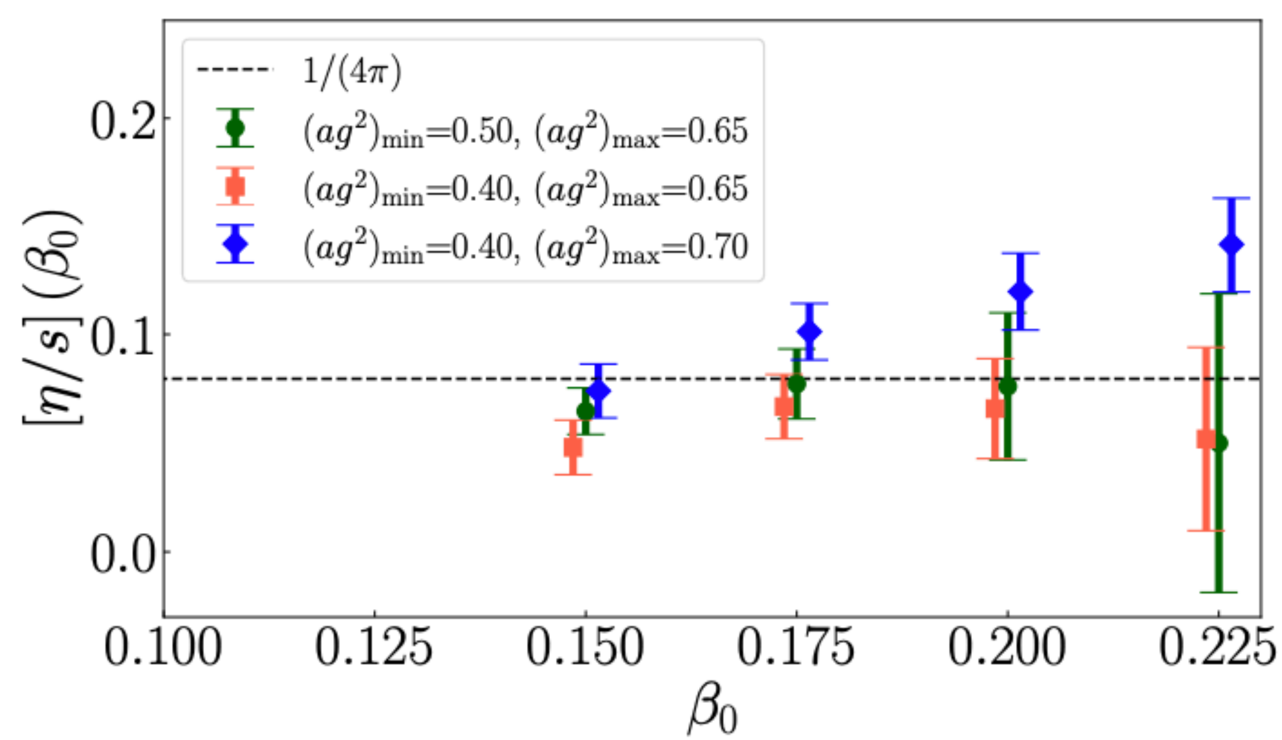
- **Quantum simulator results for retarded correlator on 2×2 lattice**

  $j_{\max} = 1/2,\ ag^2 = 1,\ \beta = 0.15,\ \Delta t = 0.025$



- **A naive "continuum" extrapolation on 4×4 lattice w/ $j_{\max} = 1/2$**

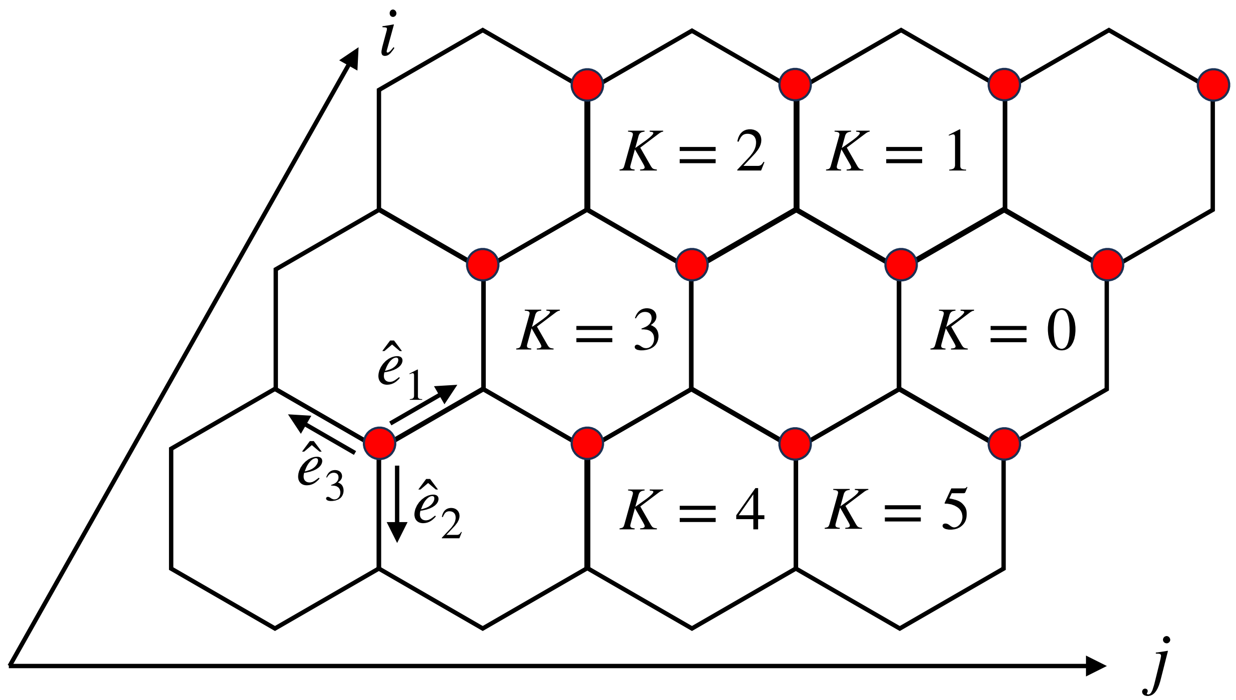




Conclusions

- Real-time correlators contain interesting physics information
  - Energy-energy correlators
  - Shear viscosity
- For 2+1D SU(2) with $j_{\max} = 1/2$ on honeycomb lattice, each Trotter step takes about 200 CNOT depth

Backup: Magnetic Interaction w/ $j_{\max} = 1/2$



Factors of $(-0.5)^n$ can appear, consequence of CG coefficients

$$H^{\text{mag}} = h_x \sum_{(i,j)} \sigma^x_{i,j} \prod_{K=0}^{5} \left[\left(\frac{1}{2} - \frac{i}{2\sqrt{2}} \right) \sigma^z_K \sigma^z_{K+1} + \frac{1}{2} + \frac{i}{2\sqrt{2}} \right]$$

Backup: Lattice Version of T_{0i}

- **Obtain from energy conservation**

Start with an energy density operator on lattice H_{ij} (per plaquette)

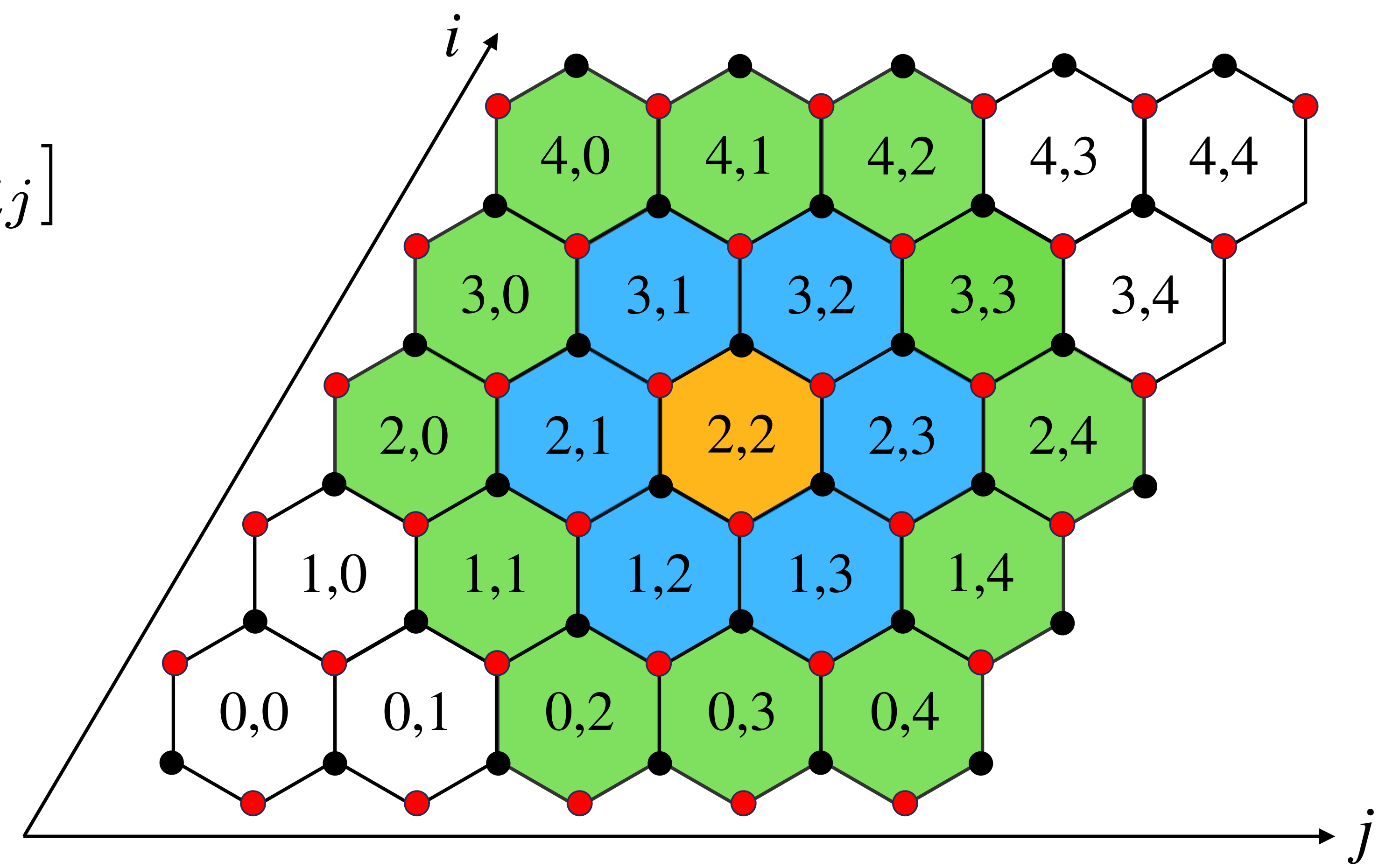
Evolution in Heisenberg picture v.s. energy conservation equation

$$\frac{\mathrm{d}H_{ij}(t)}{\mathrm{d}t} = i[H, H_{ij}(t)]$$

$$\partial_t e = -\nabla \cdot \vec{J}^e$$

$$T_{0,(i,j)\to(i',j')} = -i[H_{i'j'}, H_{ij}]$$

(i', j') denotes neighbors of (i, j)



Backup: Relation to Naively Discretized Operator

- **A particular discretization** $T_{0i} = -T^{0i} = \frac{1}{g^2} F^a_{0j} F^a_{ij}$

$$T^{\text{bare}}_{0i} \propto \epsilon_{ijz} \left(E^a_j \text{Tr}(T^a U_{\hexagon}) + \text{Tr}(T^a U_{\hexagon}) E^a_j - E^a_j \text{Tr}(T^a U^\dagger_{\hexagon}) - \text{Tr}(T^a U^\dagger_{\hexagon}) E^a_j \right)$$

↓ Poynting vector $\vec{E} \times \vec{B}$

- **With arbitrary cutoff of irrep dimensions** j_{max}

$$\langle \{J\} | \text{Tr}(T^a U_{\hexagon}) E^a_{R1} | \{j\} \rangle = - \frac{\langle \{J\} | \text{Tr}(U_{\hexagon}) | \{j\} \rangle}{\begin{Bmatrix} j^{ex}_6 & j_6 & j_1 \\ \frac{1}{2} & J_1 & J_6 \end{Bmatrix}}$$
$$\times (-1)^{s'_1 + j^{ex}_6 - M_6 - M_1 + J_1 - j_1} T^a_{s_1 s'_1} T^{(j_1)a}_{m_1 m'_1} \begin{pmatrix} j_6 & j^{ex}_6 & j_1 \\ m_6 & m^{ex}_6 & m_1 \end{pmatrix}$$
$$\times \begin{pmatrix} J_6 & j^{ex}_6 & J_1 \\ M_6 & m^{ex}_6 & M_1 \end{pmatrix} \begin{pmatrix} j_1 & \frac{1}{2} & J_1 \\ m'_1 & s_1 & -M_1 \end{pmatrix} \begin{pmatrix} j_6 & \frac{1}{2} & J_6 \\ m_6 & -s'_1 & -M_6 \end{pmatrix},$$

- **With cutoff** $j_{\text{max}} = 1/2$**, we find**

$$T_{0,(i,j)\to(i',j')} = T^{\text{bare}}_{0i} + \frac{\#}{a^2 g^4}$$