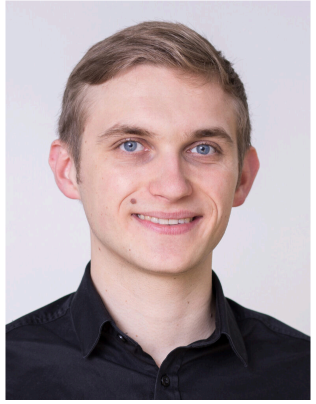
Towards a Quantum Information Theory of Hadronization

arXiv: 2503.22607

Zhiquan Sun
(UC Berkeley/LBNL)

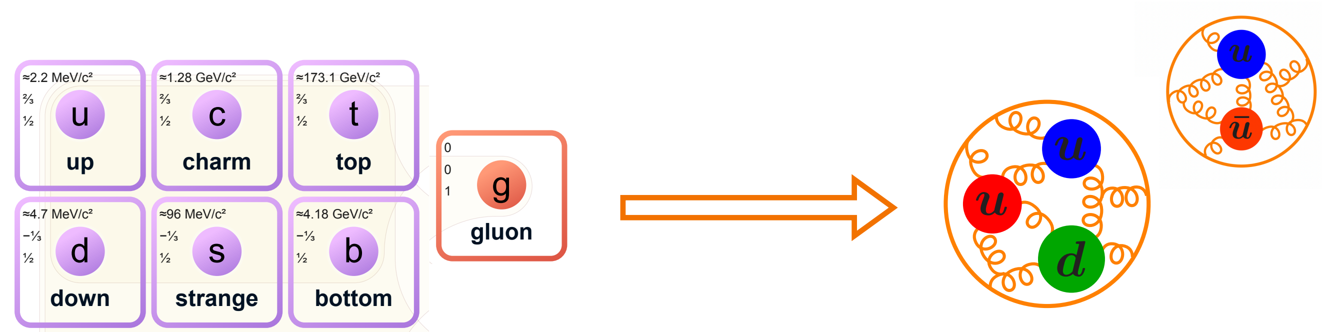
QuantHEP 2025, LBNL, October 2, 2025

Collaboration with



Rebecca von Kuk (Bern)
Johannes Michel (Nikhef)
Kyle Lee (Yale)

Is Hadronization Quantum?

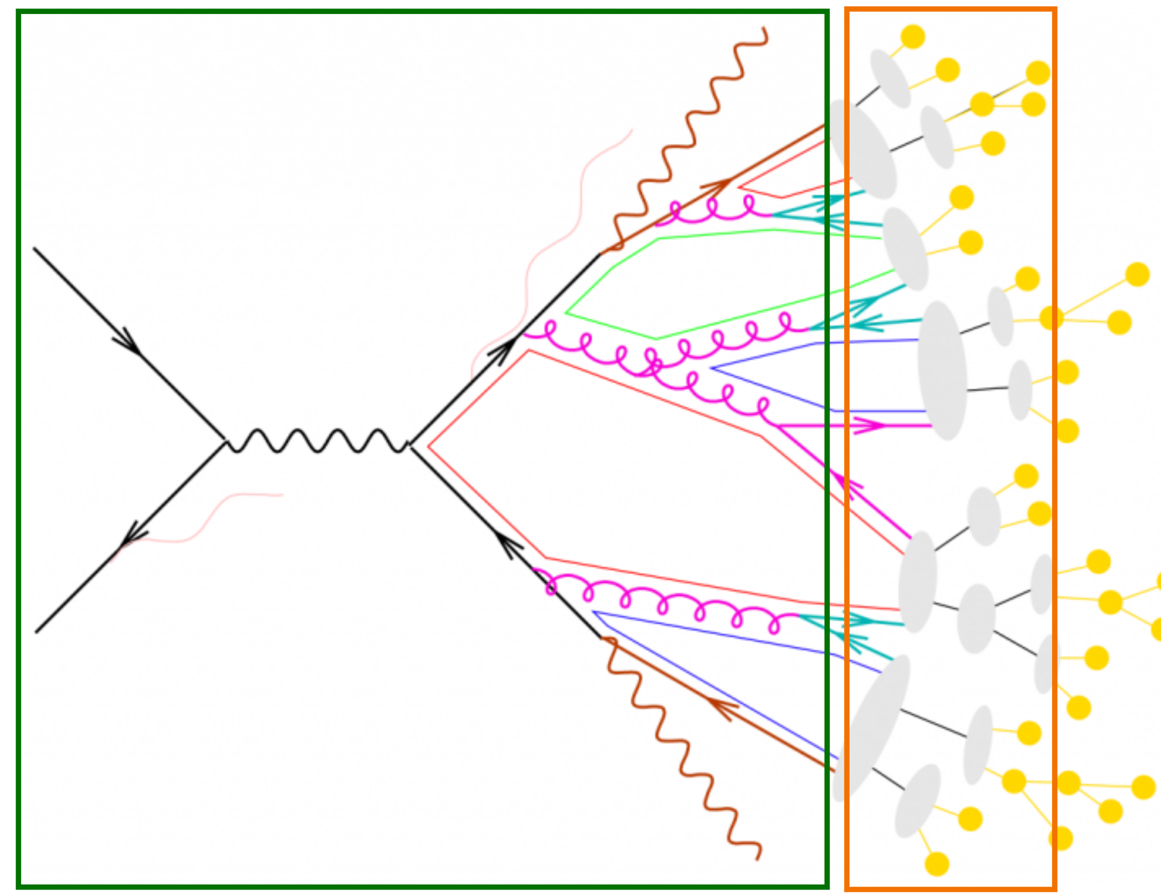


- Yes, of course!

- But: can we quantify this?

- Why are semi-classical Monte-Carlo generators so successful at describing the hadronization cascade?

  e.g. Lund string model: Pythia
  cluster model: Herwig, Sherpa



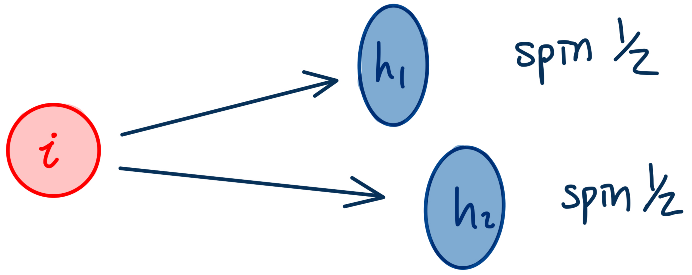
Perturbative interaction and parton shower

Nonperturbative hadronization

Outline

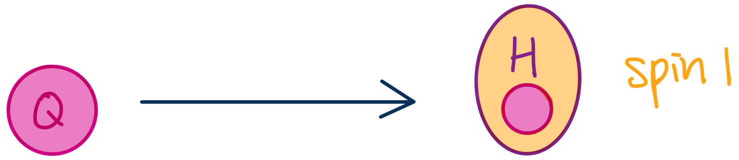
- We can use tools from **quantum information** to understand hadronization better!

- Two physical examples:
  - Dihadron fragmentation



CHSH inequality

  - Heavy quark fragmentation to heavy baryon



KCBS inequality

❌ quantum simulation of hadronization

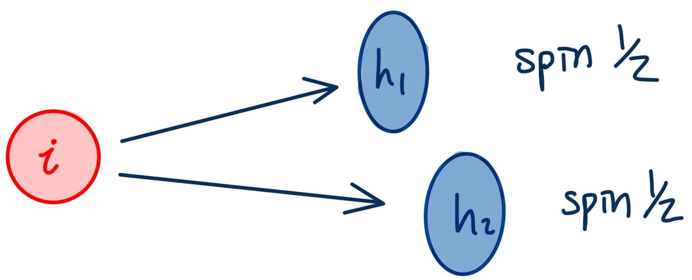
c.f. 2301.11991, 2404.06298, 2404.00087, 2406.05683, 2409.13830, 2503.16602, …

❌ testing nonlocality at colliders

c.f. 2108.12319, 2406.04402, 2501.03321, … but also see 2507.15949

Outline

- Dihadron fragmentation

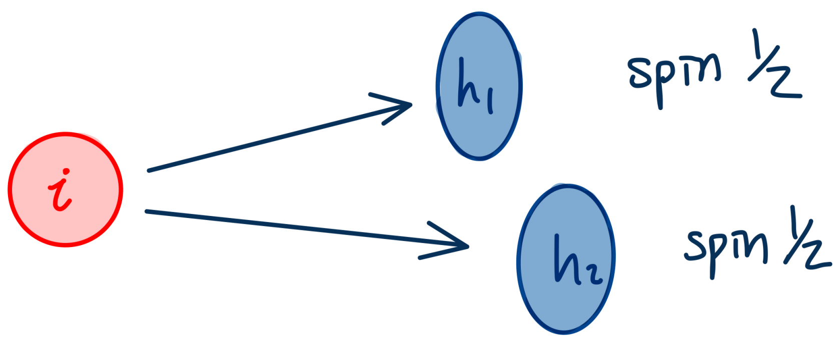


CHSH inequality

- Heavy quark fragmentation to heavy baryon

KCBS inequality

Dihadron fragmentation functions



- Hadronization process $i \to h_1 h_2 X$
- Spin density matrix at long distance $t_{\text{decay}} \sim 1/\Gamma_h$

$$\rho = \frac{\mathbb{1}}{4} + \tilde{D}_{LL} \hat{S}_{1,L} \otimes \hat{S}_{2,L} + \tilde{D}_{TT} \hat{\mathbf{S}}_{1,T} \otimes \hat{\mathbf{S}}_{2,T}$$

Fragmentation axis: along the direction of a hadron

- Dihadron fragmentation functions (**DiFFs**) $\tilde{D}_{LL}$ and $\tilde{D}_{TT}$ describe nonperturbative hadronization at scale $t_{\text{confine}} \sim 1/\Lambda_{\text{QCD}} \ll t_{\text{decay}}$
- Scale also separated from high energy perturbative scattering,

$$t_{\text{confine}} \gg t_{\text{hard}}$$

Dihadron fragmentation functions

- Constraints from QI are expressed in terms of $\tilde{D}_{LL}$ and $\tilde{D}_{TT}$
  - Positivity of eigenvalues

$$\tilde{D}_{LL} \geq -1 \qquad \tilde{D}_{LL} + 2|\tilde{D}_{TT}| \leq 1$$

  - Entanglement ⇔ positive-partial-transpose (PPT) criterion

M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A 223, 1 (1996)

$$|\tilde{D}_{TT}| > (\tilde{D}_{LL} - 1)/2$$

  - CHSH inequality

J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969)

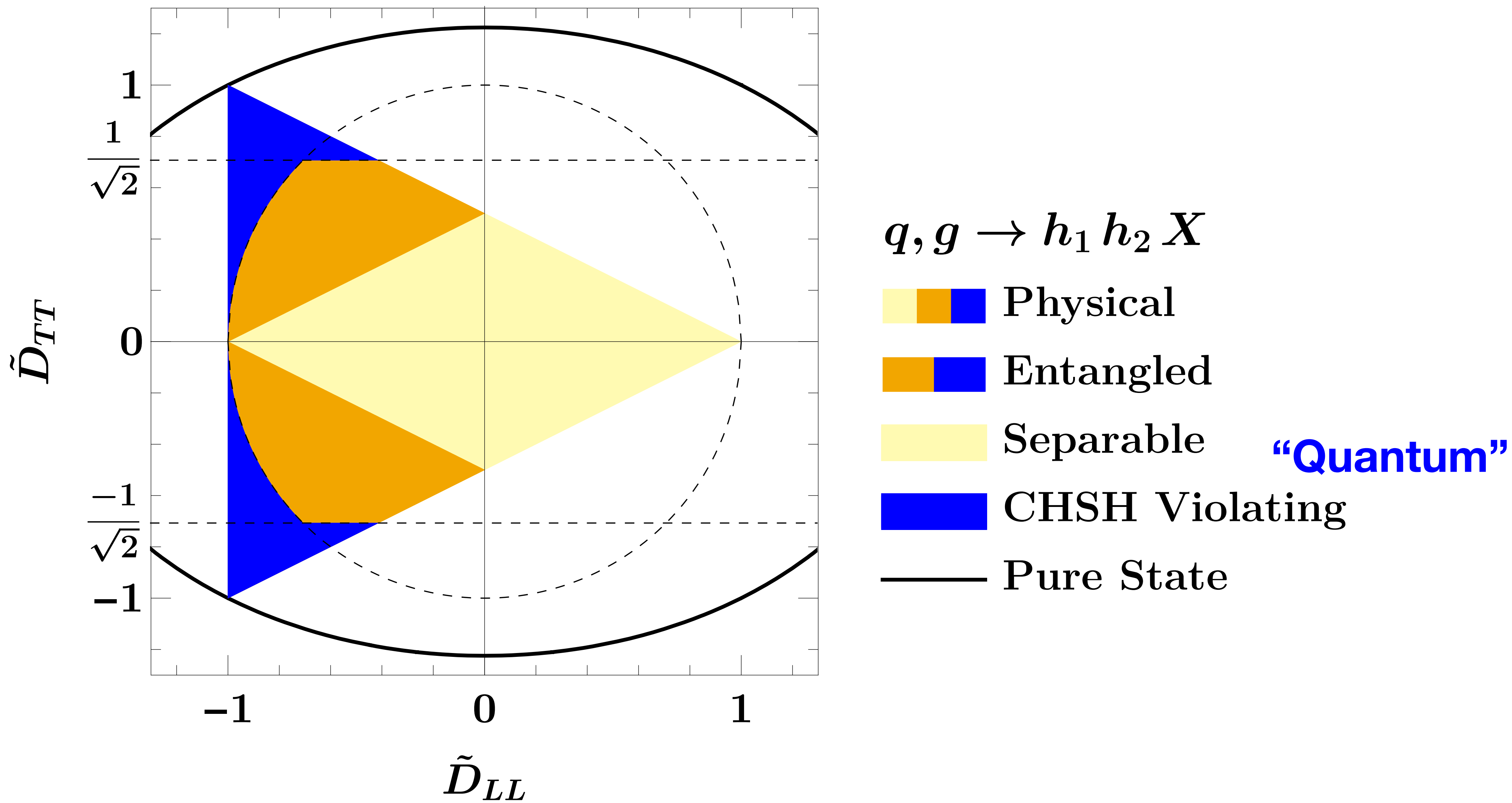
$p(\lambda)$ can correspond to an intermediate state in a MC event generator

If hadronization model can be written as $\langle S_{1,k} \otimes S_{2,\ell} \rangle = \int \mathrm{d}\lambda \, p(\lambda) \, S_{1,k}(\lambda) \, S_{2,\ell}(\lambda)$

$$\tilde{D}_{TT}^2 + \max\{\tilde{D}_{LL}^2, \tilde{D}_{TT}^2\} \leq 1$$

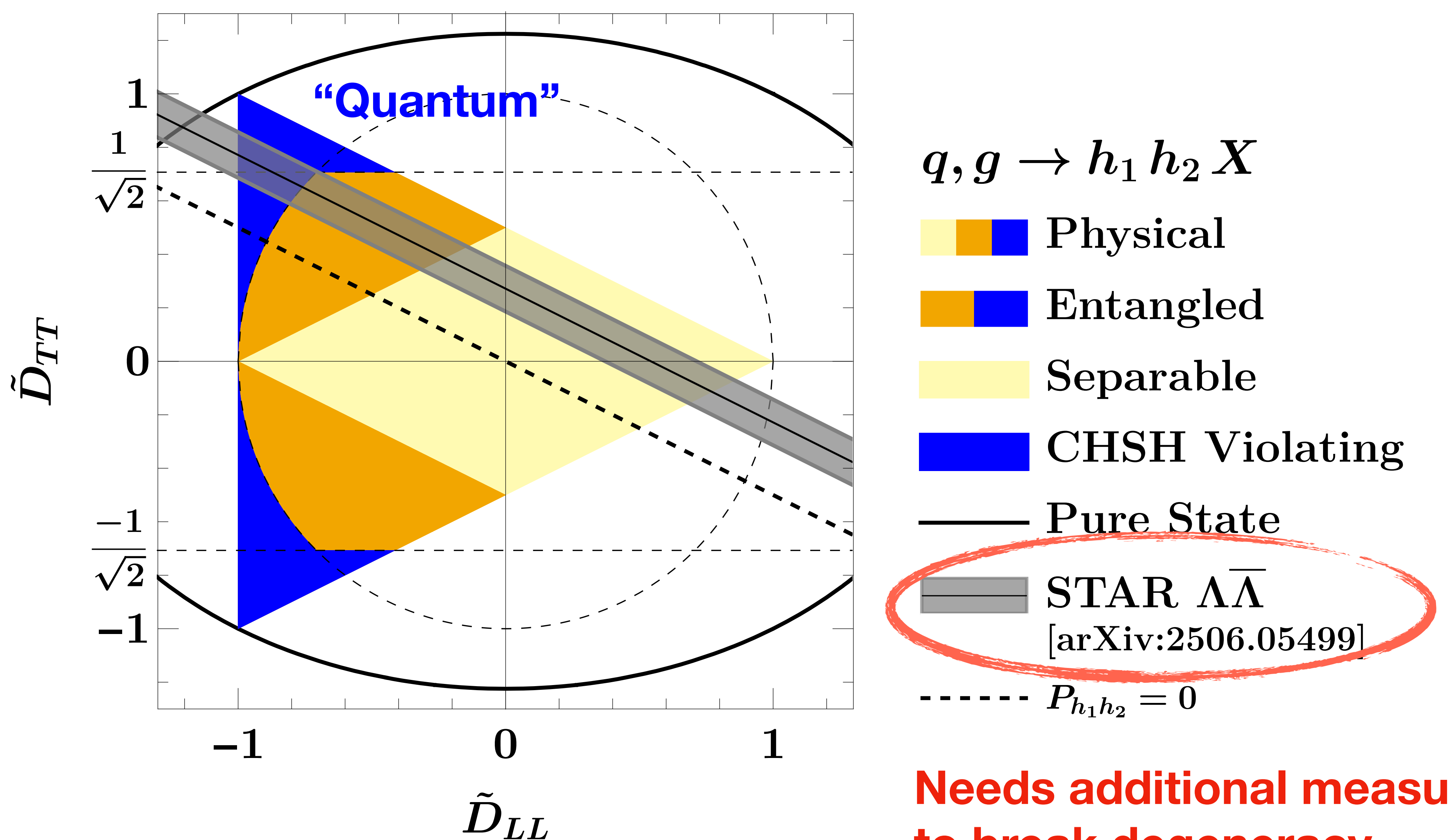
Dihadron fragmentation functions

- Experimental measurements can answer: whether the blackbox Hamiltonian of hadronization ***admits a classical description***



Dihadron fragmentation functions

- Experimental measurements can answer: whether the blackbox Hamiltonian of hadronization ***admits a classical description***



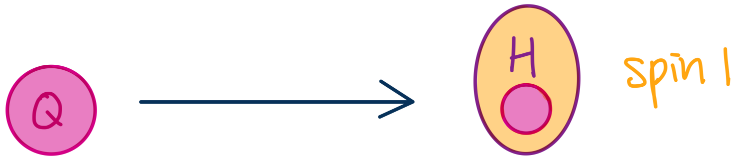
Needs additional measurement to break degeneracy

Outline

- Dihadron fragmentation

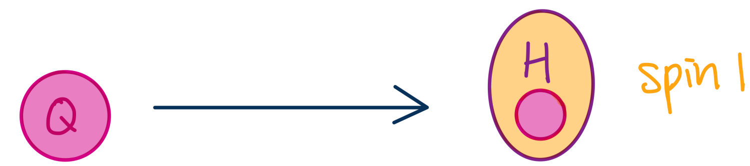
CHSH inequality

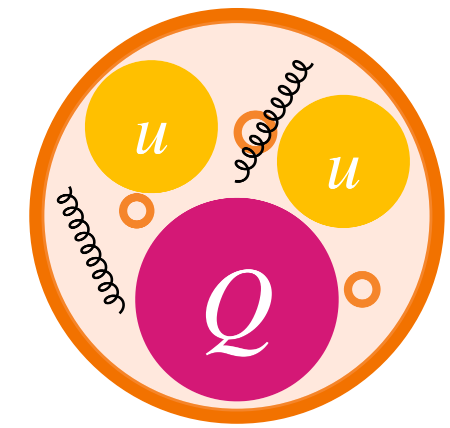
- Heavy quark fragmentation to heavy baryon



KCBS inequality

Excited heavy baryons: Qutrit





$\Sigma_Q^* = Quu, Qdd$

- Hadronization process $Q \to \Sigma_Q^* X$
- Light d.o.f has spin $s_\ell = 1$, and $s_{\Sigma_Q^*} = 3/2$
- Inclusive fragmentation: full azimuthal symmetry

$$\rho = \frac{w_1}{2}\big(|1_z\rangle\langle 1_z| + |-1_z\rangle\langle -1_z|\big) + \big(1 - w_1\big)|0_z\rangle\langle 0_z|$$

$$w_1 = \langle \hat{J}_z^2 \rangle$$

Falk-Peskin parameter

Falk, Peskin, Phys.Rev.D49, 3320-3332,1994, hep-ph/9308241

$\vec{z}$ is the direction of the fragmentation axis

KCBS inequality and contextuality

- **Contextuality** of a system: a measurement's outcome may depend on other quantities being measured
- Formulate with joint probability

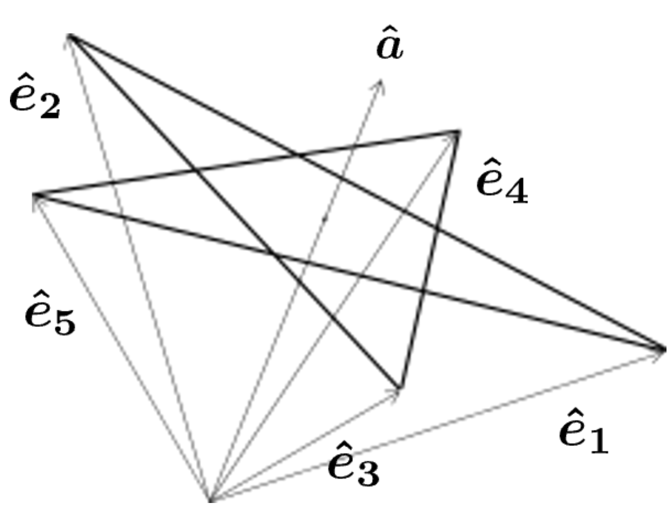
$$P(x_1, \dots, x_N | \boldsymbol{e}_1, \dots, \boldsymbol{e}_N)$$

set of $\langle J_e^2 \rangle$ measurement outcomes, $x_i = 0,1$

set of unit directions

- Quantified by **KCBS inequality**

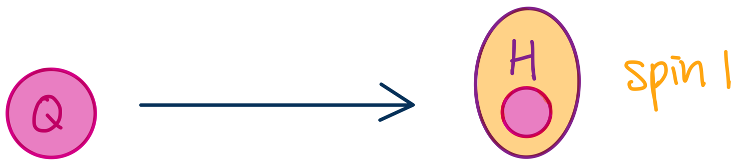
A. A. Klyachko, M. A. Can, S. Binicioglu, and A. S. Shumovsky, Phys. Rev. Lett. 101, 020403 (2008)



$$\sum_{e \in \star} \langle J_e^2 \rangle \geq 3$$

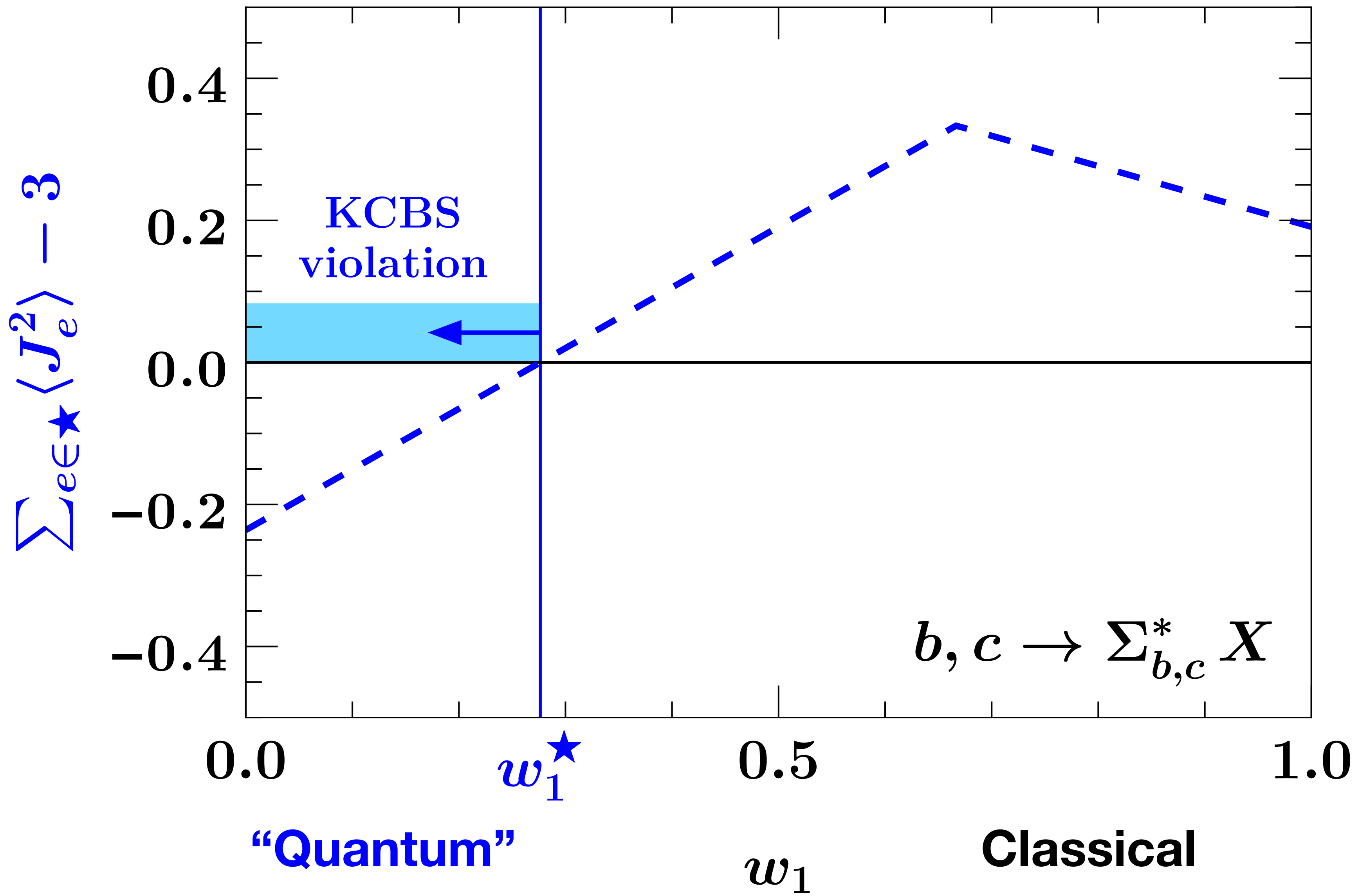
$$J_e^2 = (\boldsymbol{e} \cdot \boldsymbol{J})^2$$

Heavy quark to heavy excited baryon



- KCBS inequality applied to the hadronization $Q \to \Sigma_Q^* X$

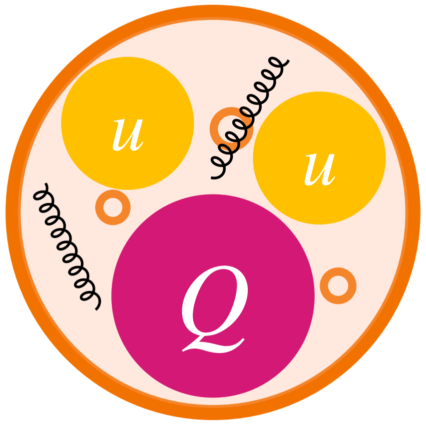
$$\sum_{e \in \star} J_e^2 \geq 3 \quad \Leftrightarrow \quad w_1 > w_1^\star \equiv \frac{1}{2} - \frac{1}{2\sqrt{5}} \approx 0.276$$

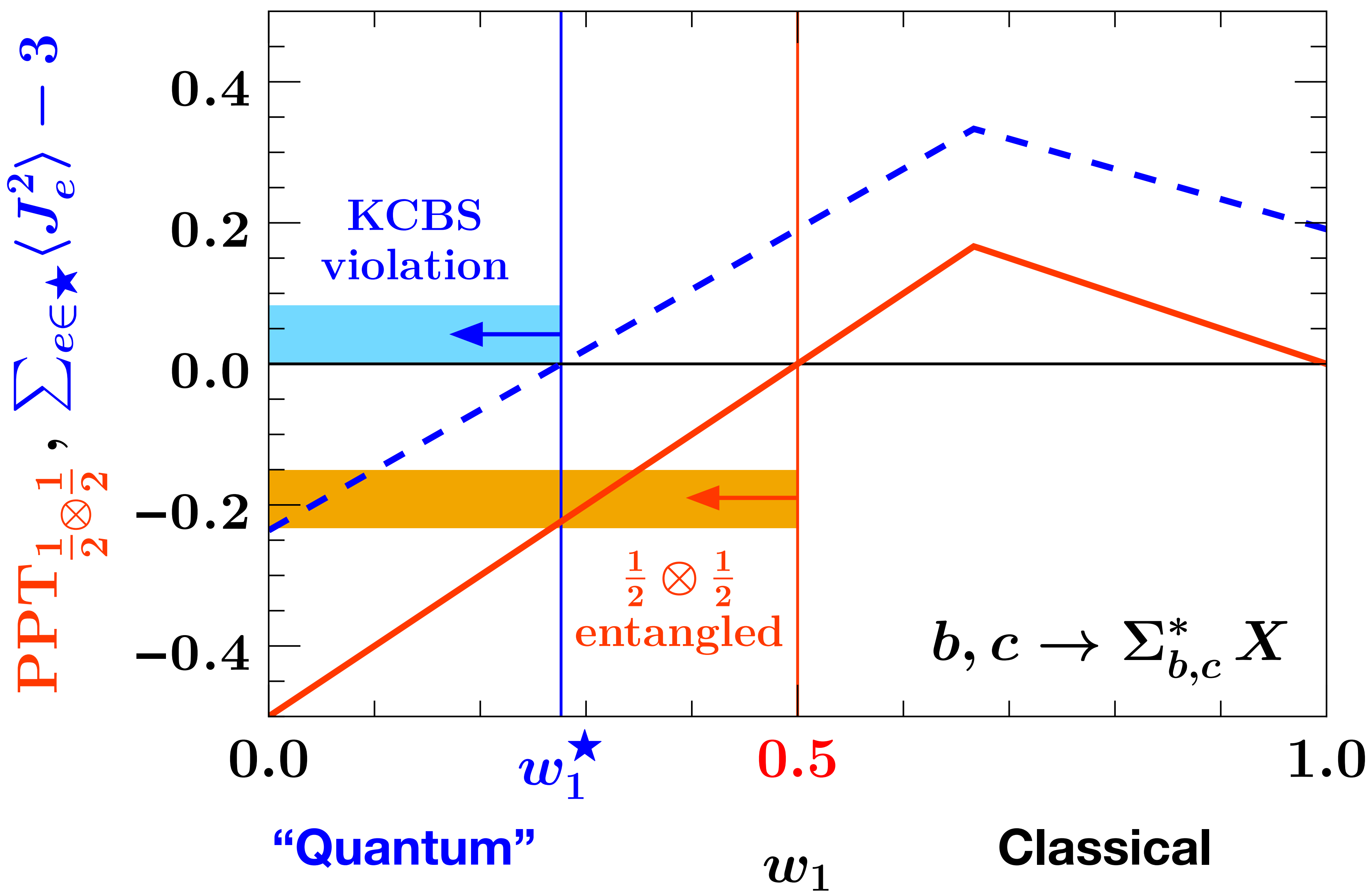


Bipartite model

- Consider a quantum bipartite **model** $\frac{1}{2} \otimes \frac{1}{2}$

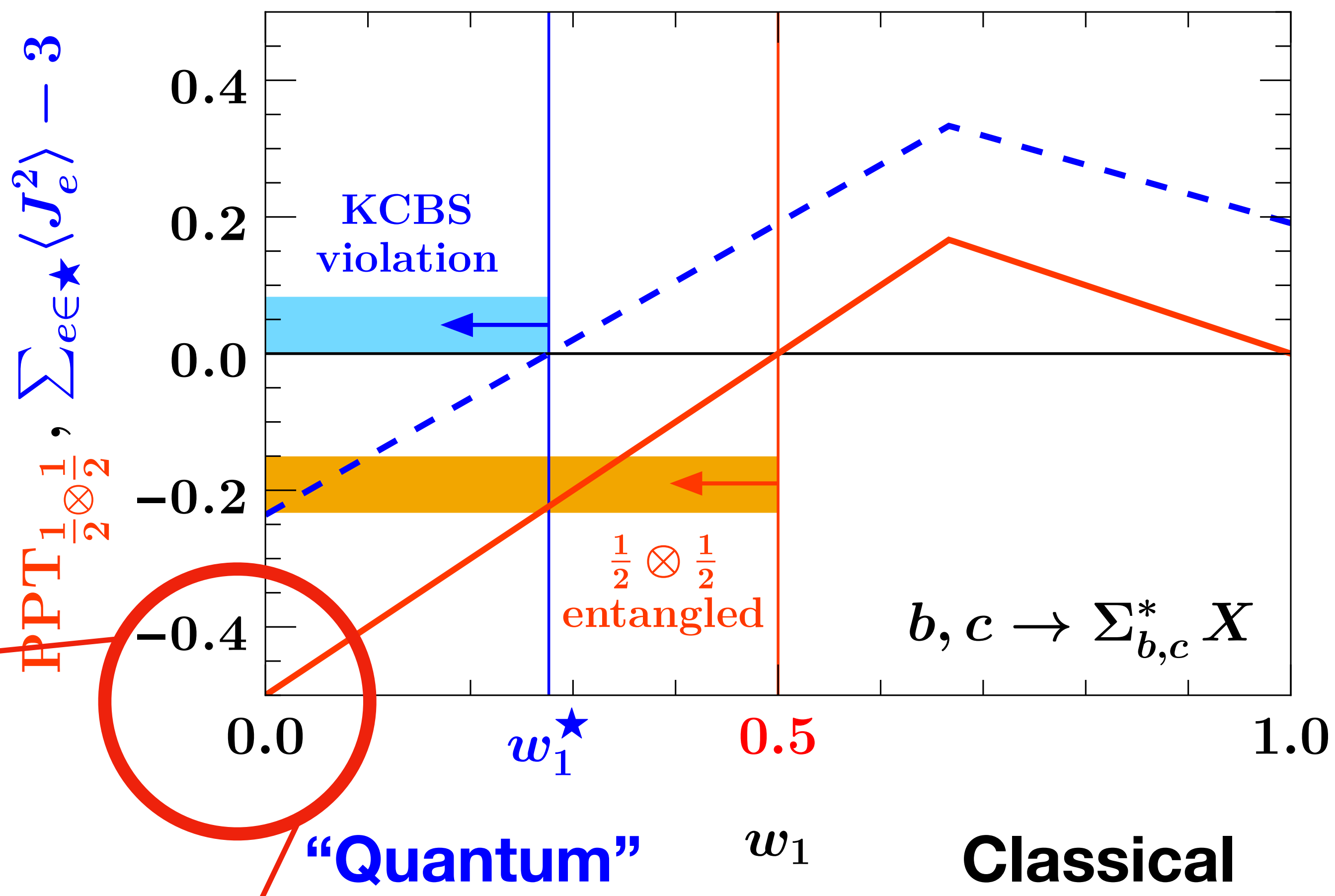
  Evaluate PPT criterion to map out entangled region with w_1





Bipartite model

- Evaluate PPT criterion in terms of w_1 for QM bipartite model



Example:

$$\rho = |0\rangle\langle 0|$$

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_q \downarrow_{\bar{q}}\rangle + |\downarrow_q \uparrow_{\bar{q}}\rangle\right)$$

PPT criterion evaluates to $-1/2$
$\Rightarrow$ maximally entangled
also maximally KCBS violating

Existing experimental data

- Two existing **data** in disagreement:

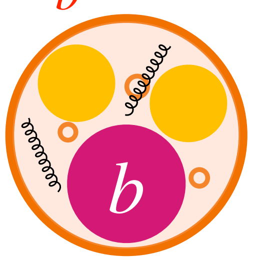
DELPHI '95, CERN-PPE/95-139

DELPHI $w_1 = -0.36 \pm 0.30 \pm 0.30$

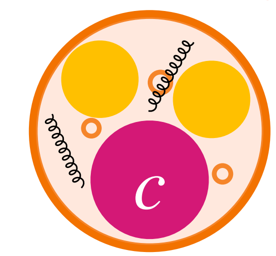
CLEO '95, Phys. Rev. Lett. 78 (1997) 2304-2308

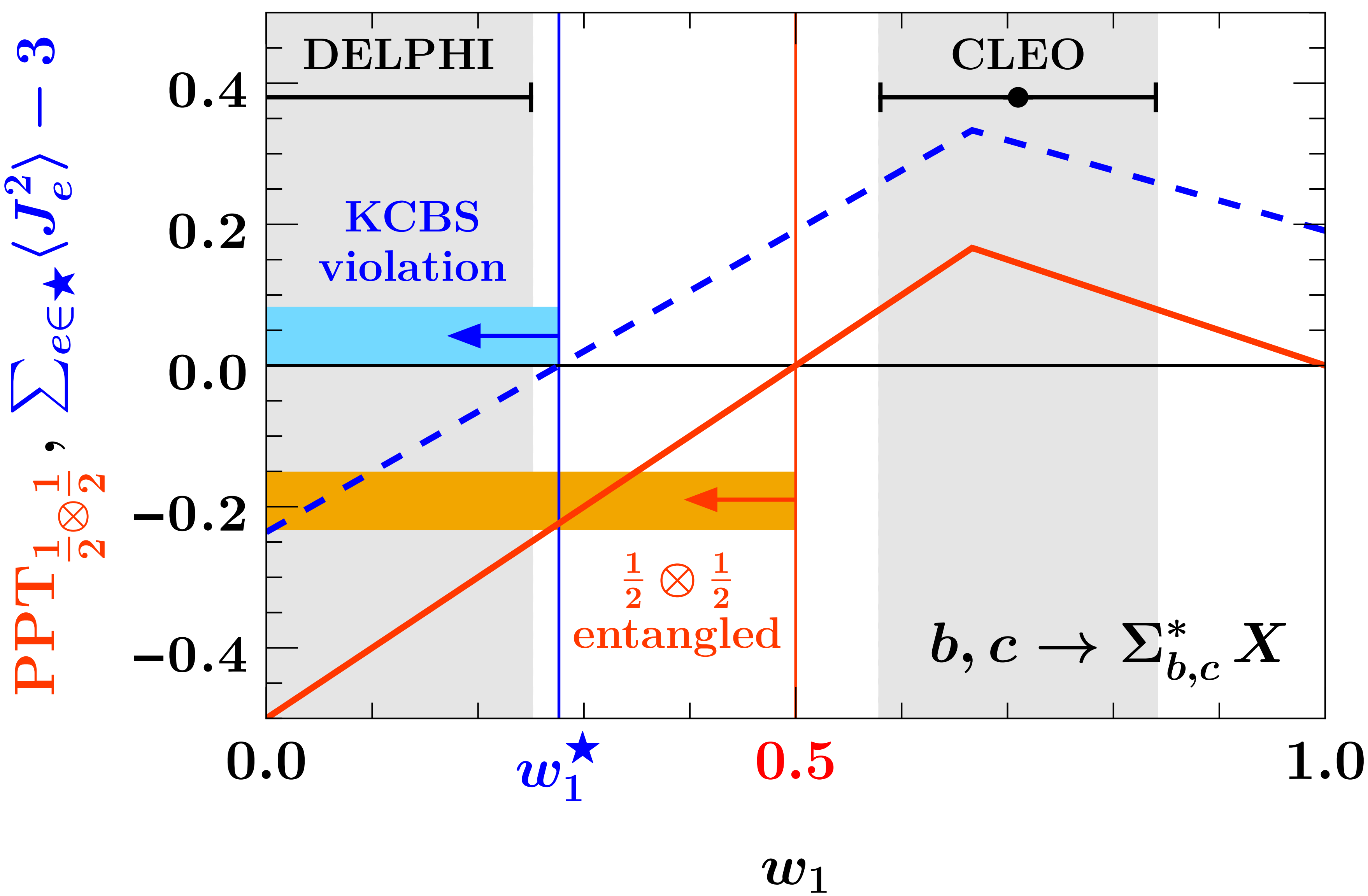
CLEO $w_1 = 0.71 \pm 0.13$

Σ_b^*



Σ_c^*





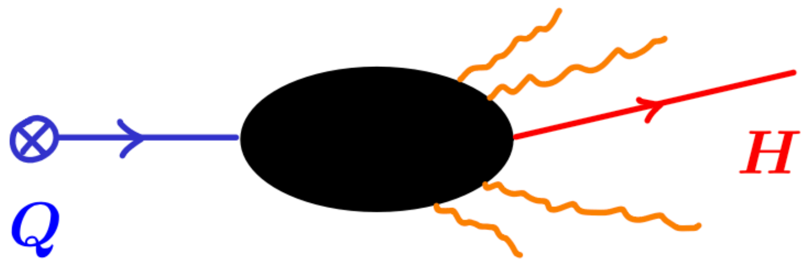
Breaking azimuthal symmetry: TMD

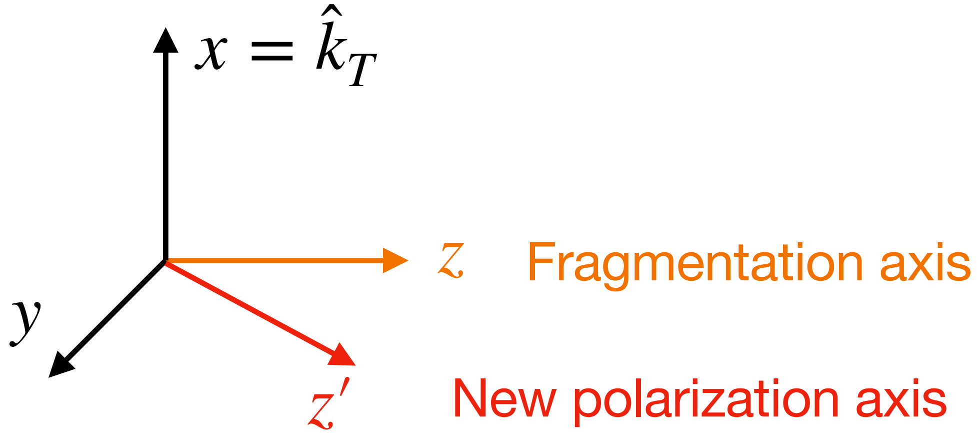
- Recall light spin-density matrix with **transverse momentum** k_T

$$\rho = \frac{\mathbf{1}}{3} - \tfrac{1}{2}\hat{J}_y\, w^{(1,1)}(k_T) + \left(\tfrac{3}{2}\hat{J}_z^2 - \mathbf{1}\right) w^{(2,0)}(k_T) + \left(\hat{J}_x\hat{J}_z + \hat{J}_z\hat{J}_x\right) w^{(2,1)}(k_T) + \left(\hat{J}_x^2 - \hat{J}_y^2\right) w^{(2,2)}(k_T)$$

- Define new polarization axis z' by minimizing $w_1' \equiv \min_{\boldsymbol{e}} \langle J_e^2 \rangle$

z' is generated by transverse dynamics

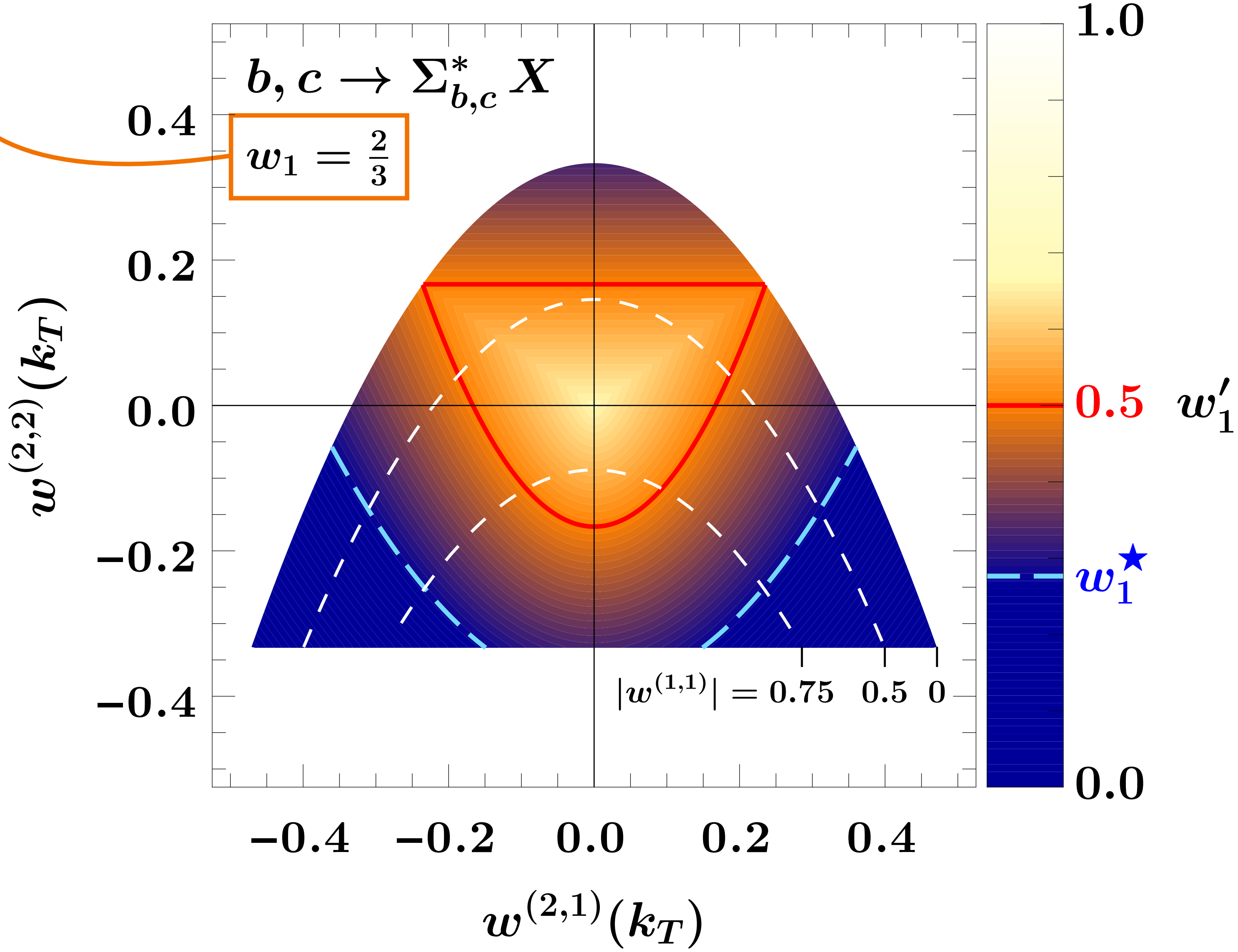


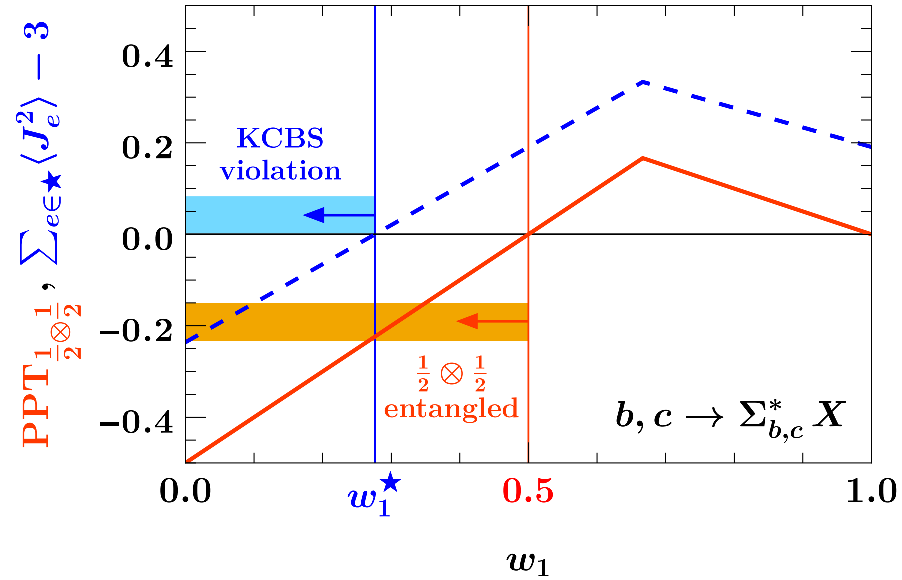


TMD as QI probe!

- TMD structure functions can give us quantum interpretation **even with classical longitudinal polarization**!

Isotropic longitudinal polarization



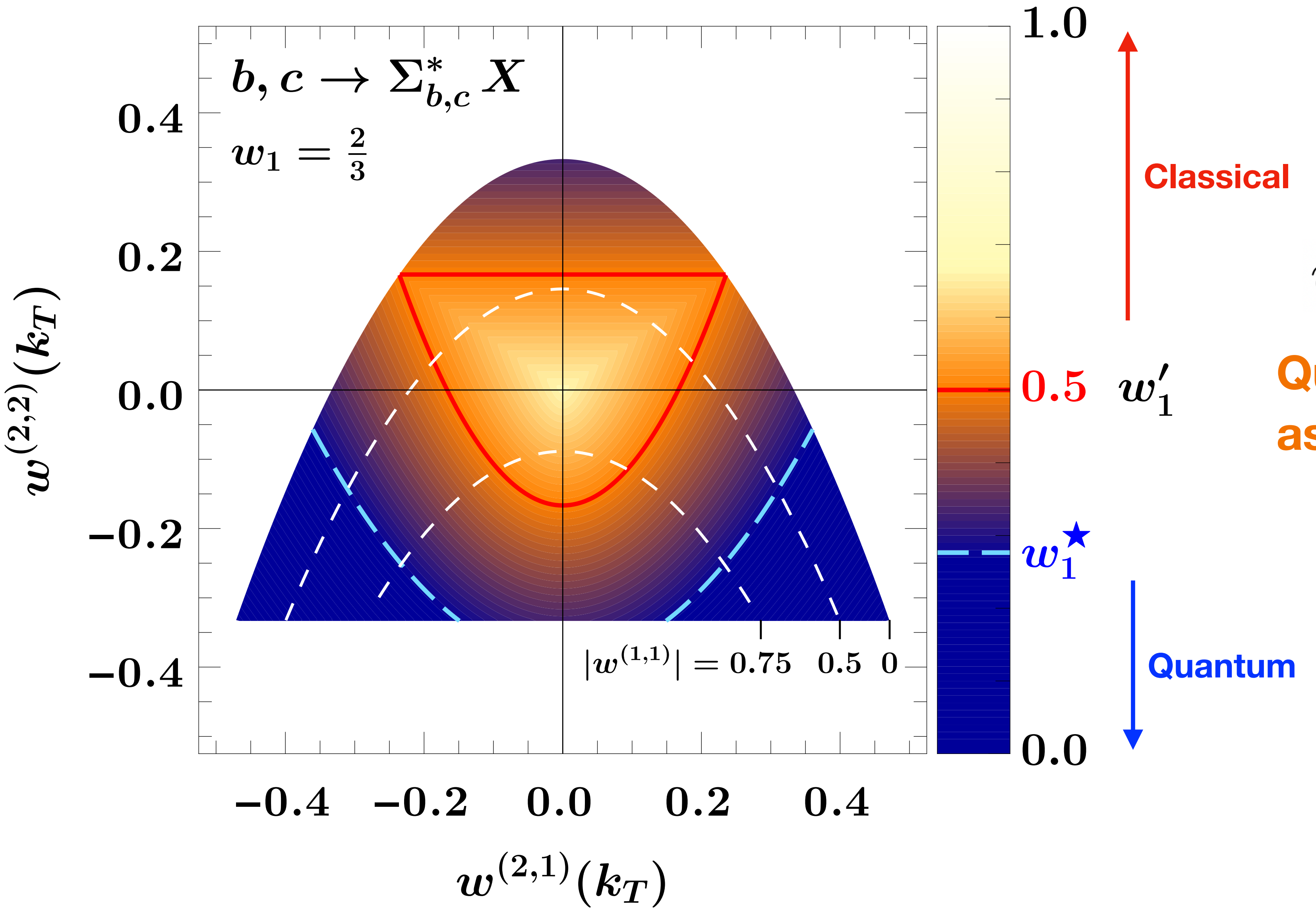


Entangled in $\frac{1}{2} \otimes \frac{1}{2}$

KCBS violation

TMD as QI probe!

- Even with classical w_1, transverse momentum k_T acts like a dial of quantumness!

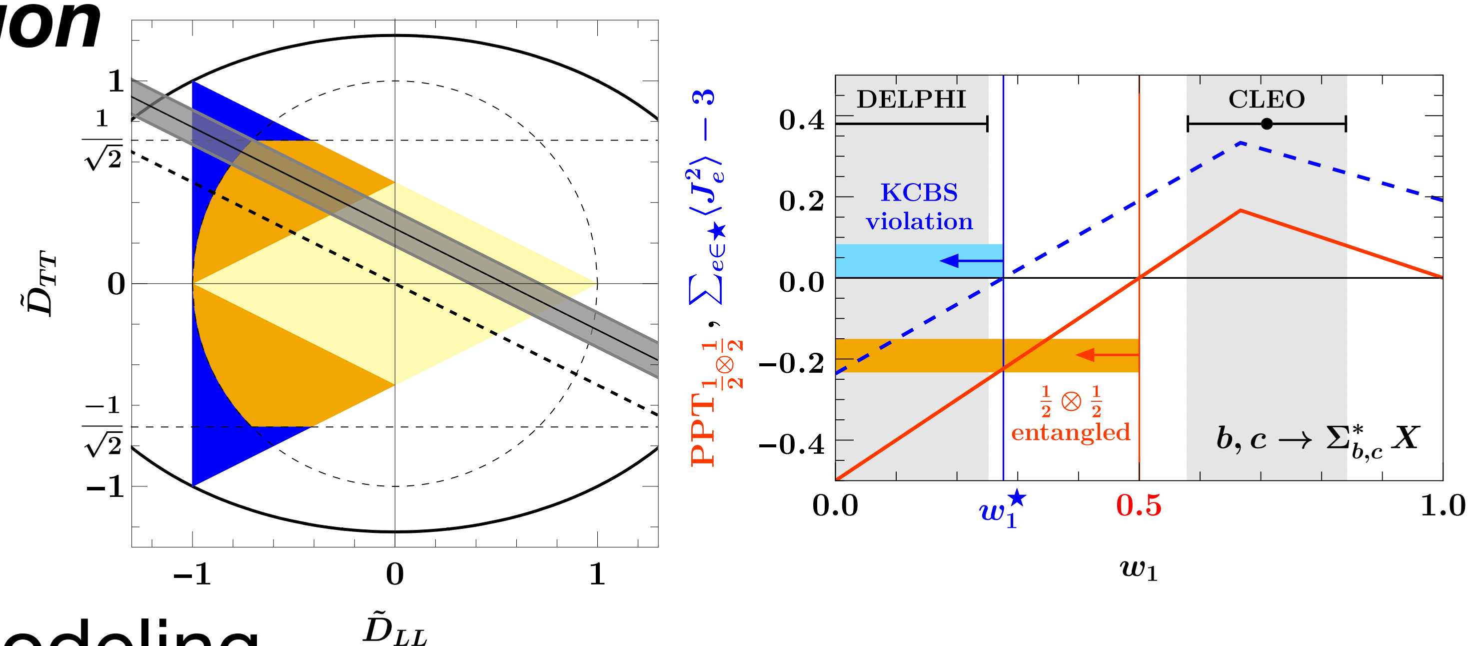


$$w^{(N,n)}(k_T) \sim (\Lambda_{\rm QCD}/k_T)^n$$

Quantum / classical transition as a function of k_T

Conclusion

- We can use tools from **quantum information** to quantitatively test for quantum properties of **hadronization!**
  - Can guide us in developing better event generators
- **Experimental measurements** can tell us whether hadronization ***admits a classical description***

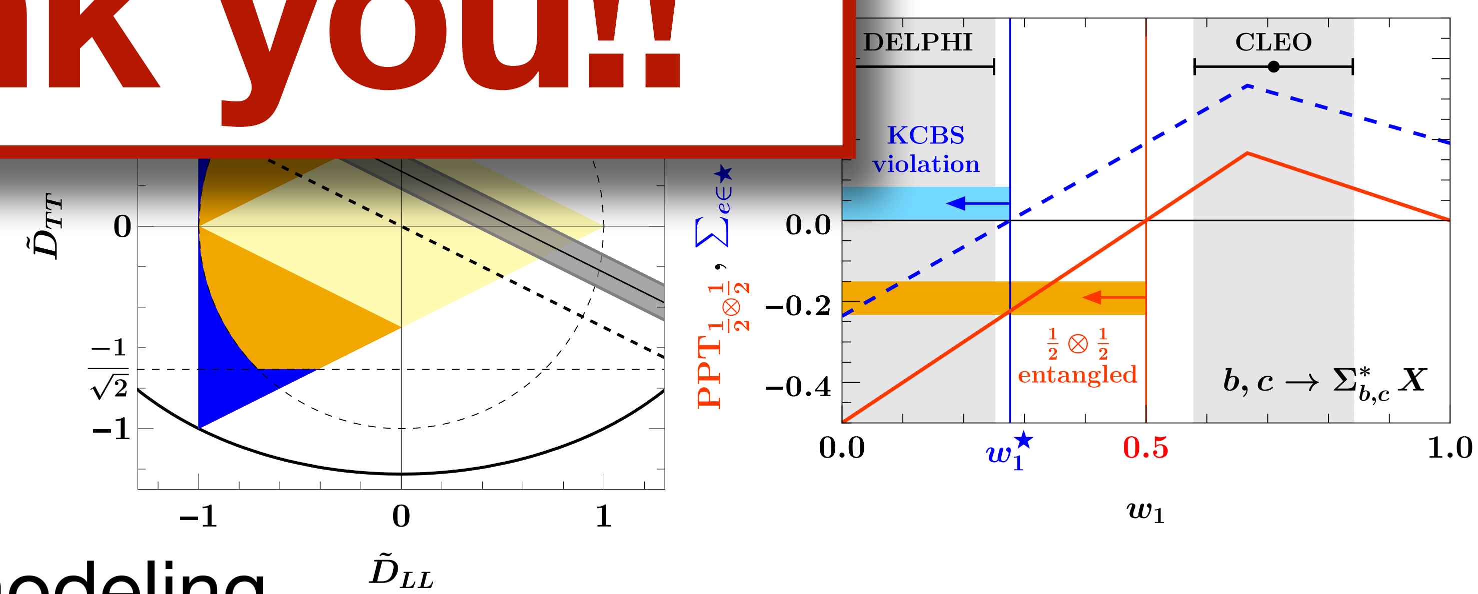


- Future outlook:
  - Constrain hadronization modeling,
  - Adapt more QI concepts/tools to other processes, ……

Conclusion

- We can use tools from **quantum information** to quantitatively test for quantum properties of **hadronization!**
  - Can guide us in developing better event generators
- **Experimen** ... adronization
  admits a c

Thank you!!



- Future outlook:
  - Constrain hadronization modeling,
  - Adapt more QI concepts/tools to other processes, ……