Measuring the strong coupling constant

Henoch Wong
UC Berkeley 290e seminar
6 April 2016

Demystifying this plot!

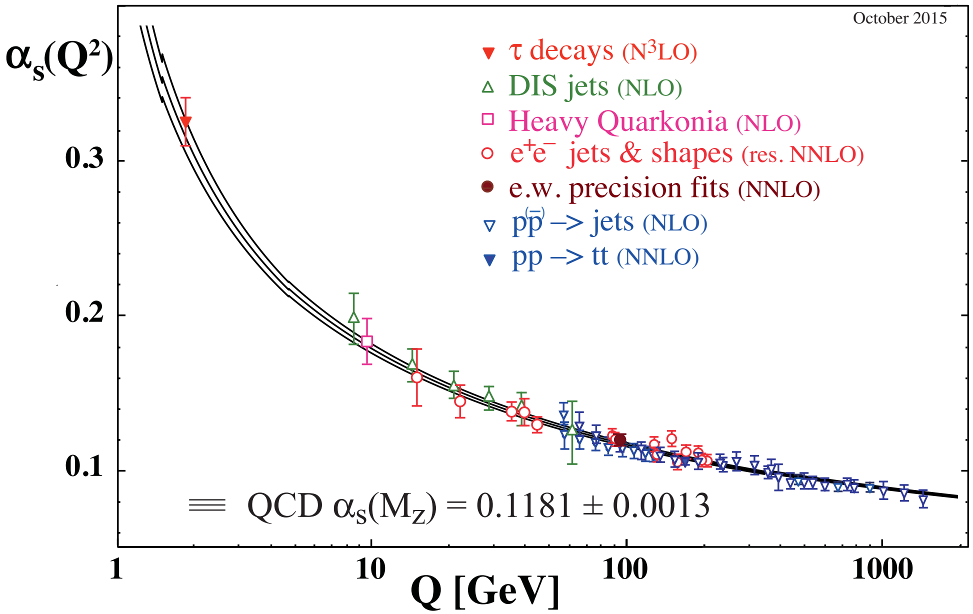
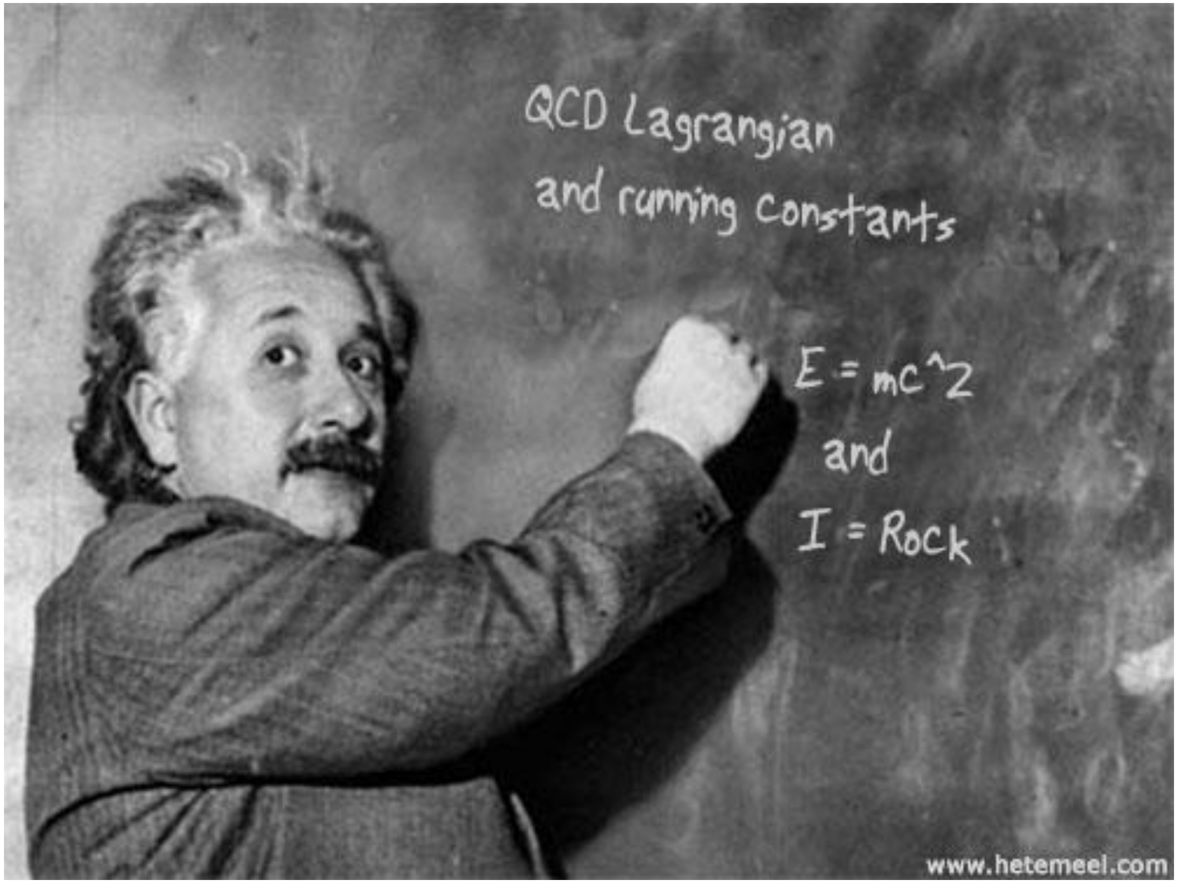


Figure 9.3: Summary of measurements of α_s as a function of the energy scale Q. The respective degree of QCD perturbation theory used in the extraction of α_s is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to leading order; res. NNLO: NNLO matched with resummed next-to-leading logs; N^3LO: next-to-NNLO).



A BIT OF THEORY

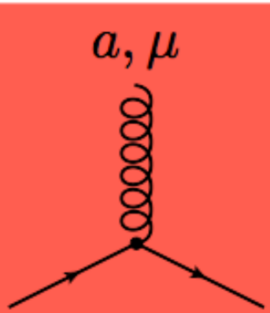
QCD Lagrangian

- QCD has one coupling parameter (g_s)

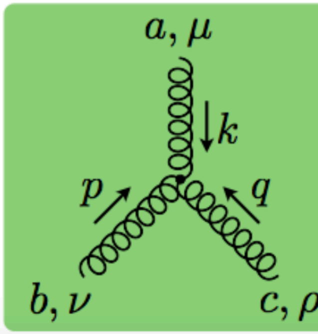
$$\mathcal{L} = \sum_q \bar{\psi}_{q,a}(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab})\psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu}$$

$$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu$$

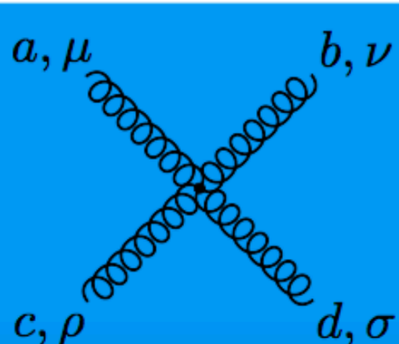
$$\alpha_s = \frac{g_s^2}{4\pi}$$



$$= i g_s \gamma^\mu t^a$$

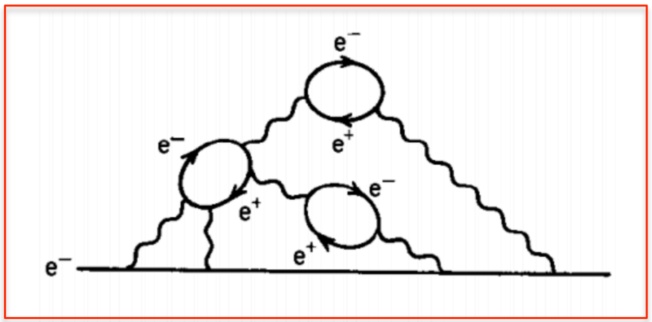


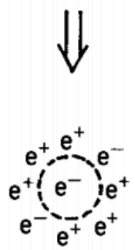
$$= g_s f^{abc} \left[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu \right]$$



$$= -i g_s^2 \left[f^{abe} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}\right) + f^{ace} f^{bde} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}\right) + f^{ade} f^{bce} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}\right) \right]$$

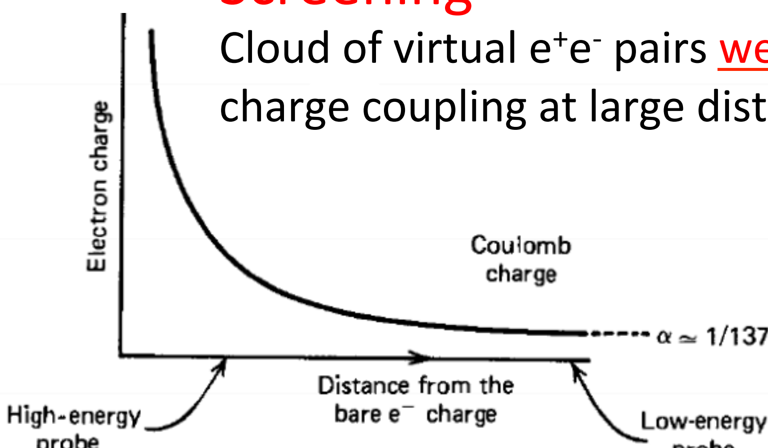
QED



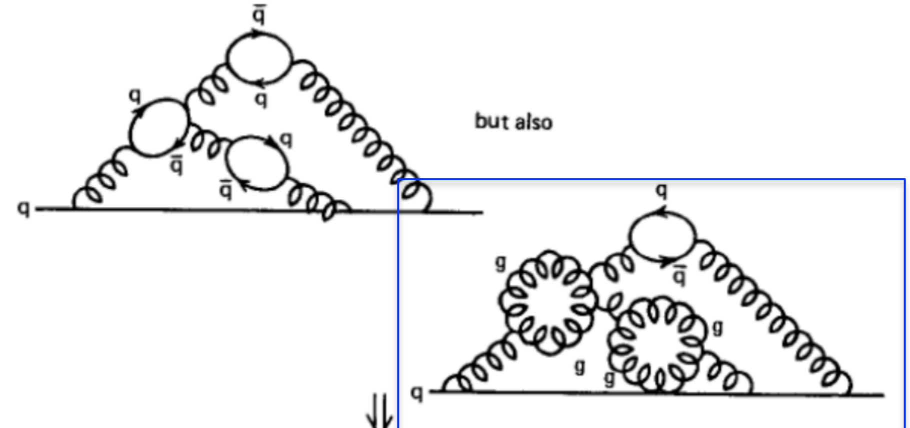


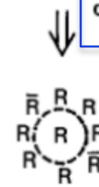
Screening

Cloud of virtual e^+e^- pairs weaken charge coupling at large distance



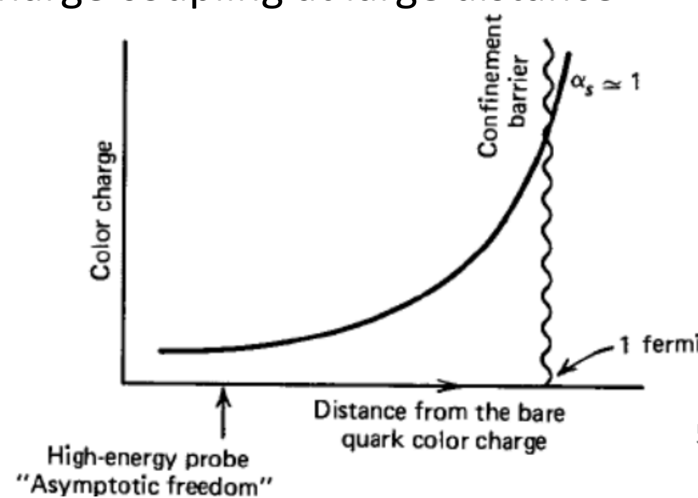
QCD





Anti-Screening

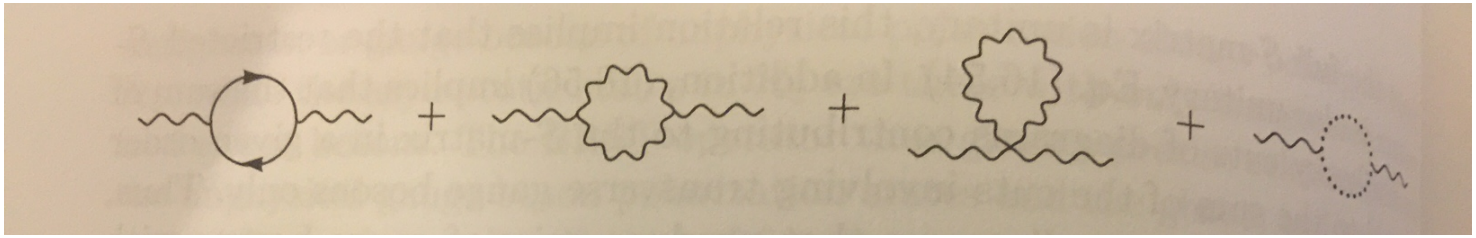
Gluon self-interaction strengthens charge coupling at large distance

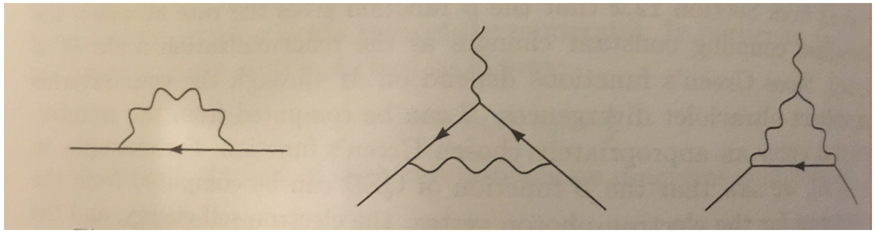


Renormalization of QCD

- Consider the one-loop correction to QCD:

An Introduction to QFT: Chapter 16 - Peskin & Schroeder





- Loops are divergent and requires the introduction of an arbitrary UV cut-off (Λ) to regulate

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + b_0 \alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$$

Renormalization Group Equation

Note: The renormalization scale are denoted $\Lambda_{\text{cut-off}}$, μ_R, M_{UV} interchangeably in the literature

- The renormalization scale (μ) is an unphysical scale to regulate the theory
- If "A" is an observable quantity, it should not depend on the arbitrary choice of μ
- Observable "A" will satisfy the renormalization group equation:

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s}\right] A\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

$$\alpha_s = \alpha_s(\mu^2) \qquad \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2}$$

β function

- A quick recap of the 2 main points:
  1. Physical observables should not depend on the unphysical scale μ
  2. Dependence on the energy scale μ is absorbed into the coupling constant

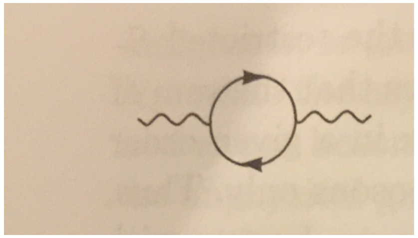
- The dependence of the coupling constant with the energy scale is called "running" and described by the β function:

$$\beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2}$$

SU(N) gauge theory

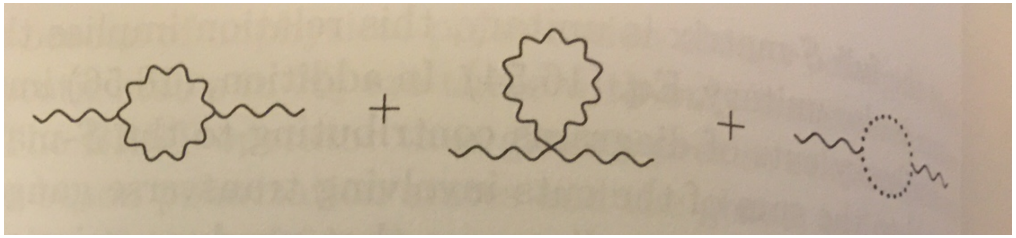
The 1-loop contributions to β function

- Quark loop vacuum polarization diagram:



$$+\frac{2n_f}{12\pi}$$

- Gluon loop diagrams:



$$-\frac{11N}{12\pi}$$

An Introduction to QFT: Chapter 16 - Peskin & Schroeder

β function in QCD

The Nobel Prize in Physics 2004

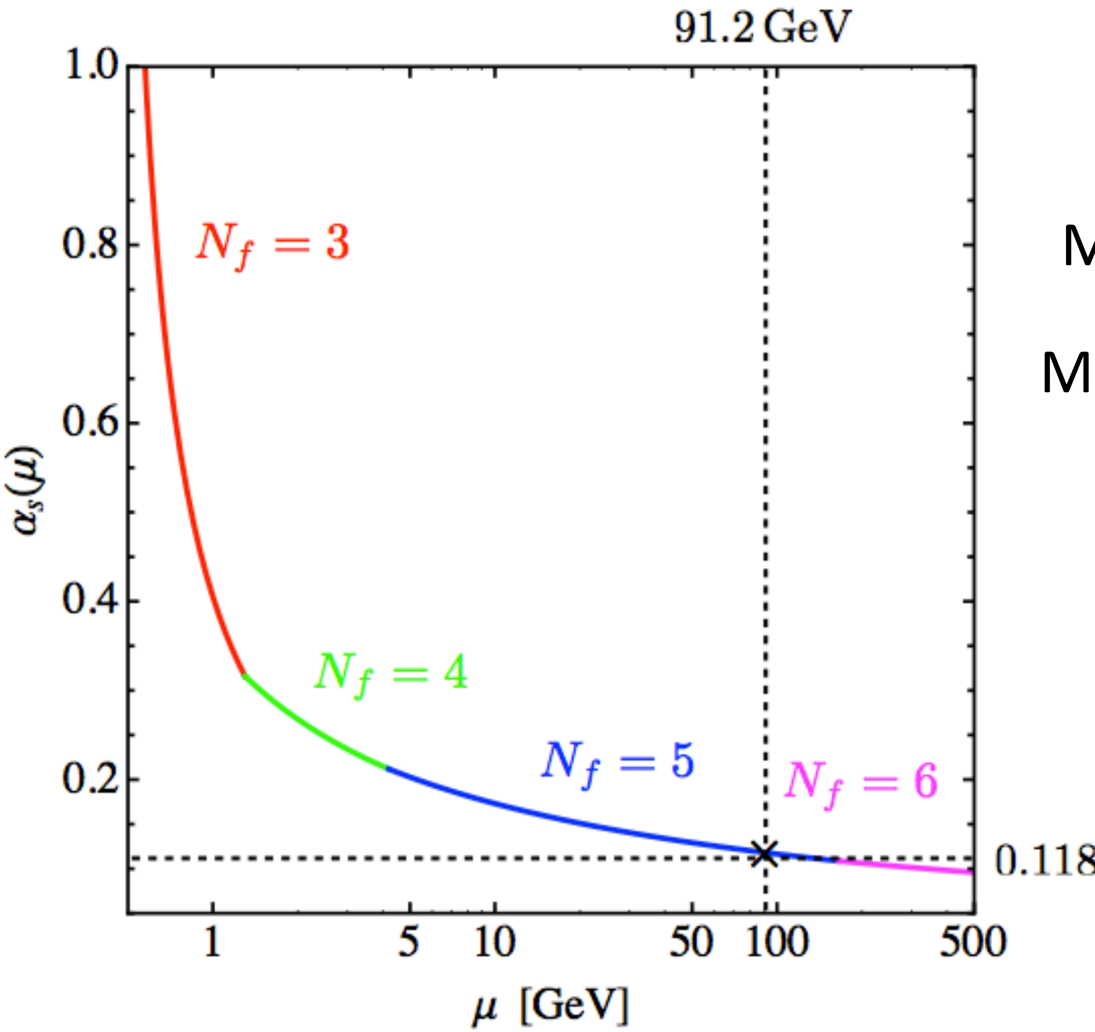
David J. Gross   H. David Politzer   Frank Wilczek

- For SU(N=3) gauge theory:

$$\mu_R^2 \frac{\partial \alpha_s}{\partial \mu_R^2} = \beta(\alpha_s) = -\left[\frac{33 - 2n_f}{12\pi}\right]\alpha_s^2$$

- There are 2 main features to the β function in QCD:
  1. Depends on n_f, the number of active fermion fields
  2. Since $n_f \leq 6$, $\beta < 0$ and the running coupling constant tends to zero at large momenta (asymptotically free)

Running of α_s



50

M_{charm} = 1.3 GeV

M_{bottom} = 4.2 GeV

M_{top} = 173 GeV

EXPERIMENTS

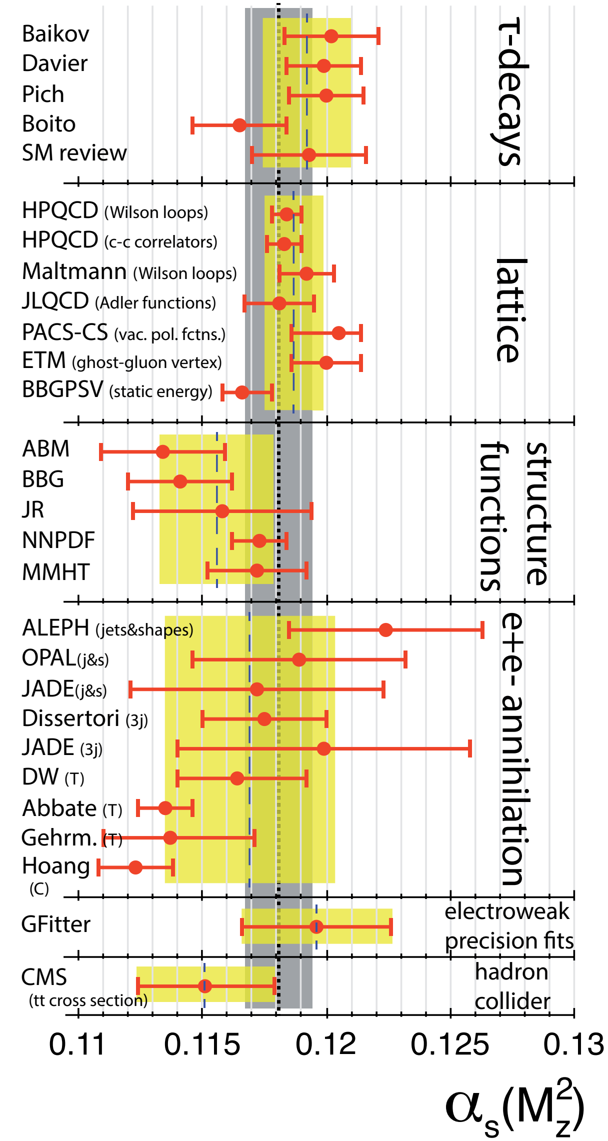
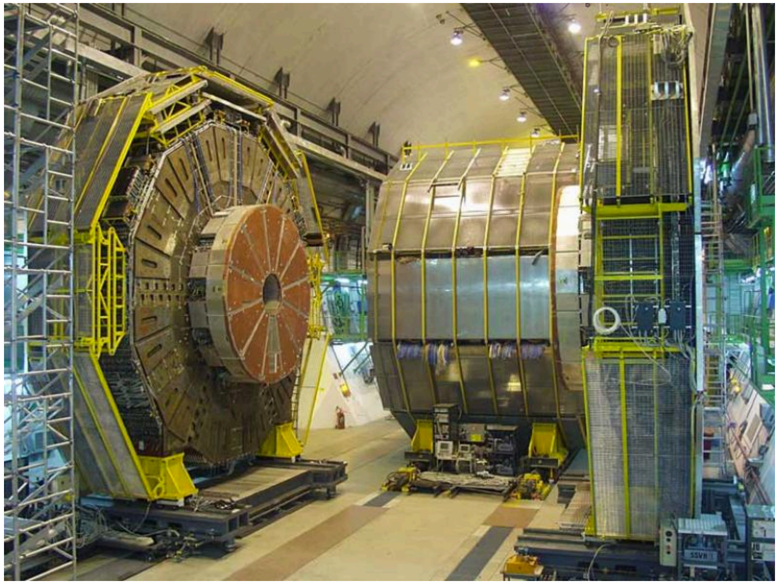
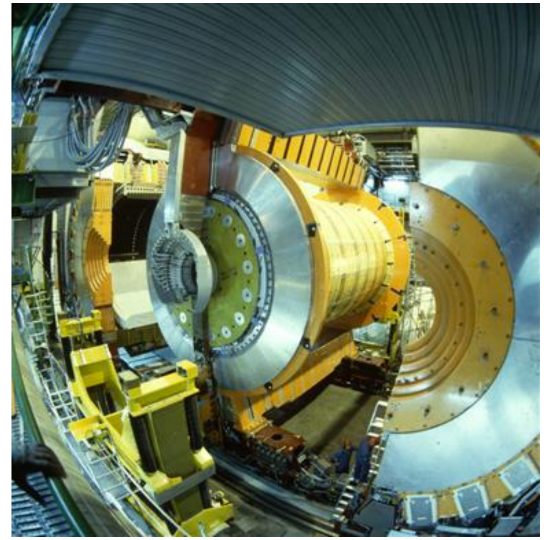


Figure 9.2: Summary of determinations of $\alpha_s(M_Z^2)$ from the six sub-fields discussed in the text. The yellow (light shaded) bands and dashed lines indicate the pre-average values of each sub-field. The dotted line and grey (dark shaded) band represent the final world average value of $\alpha_s(M_Z^2)$.

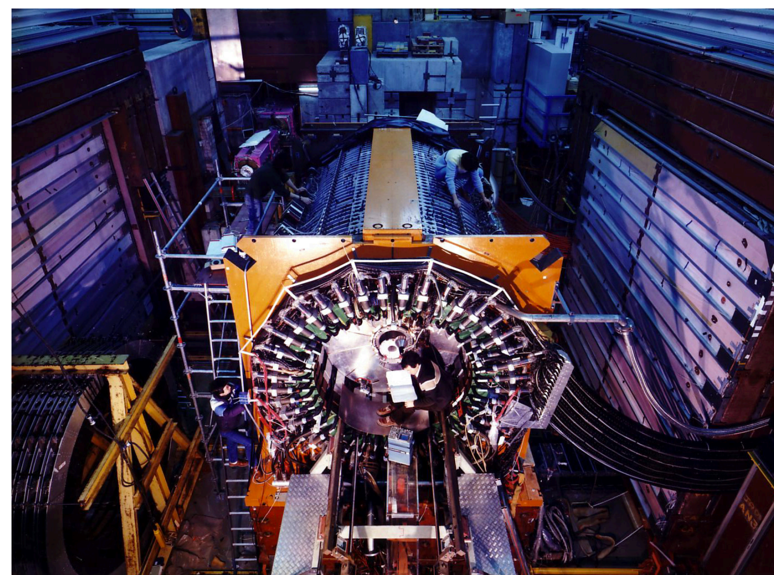


ALEPH detector, CERN



OPAL detector, CERN

e^+e^- annhilation

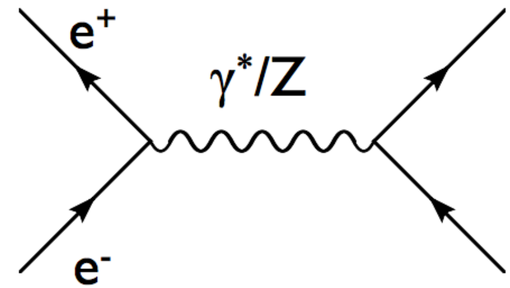


JADE detector, DESY

R-ratio

The R-ratio is defined as

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Since common factors cancel in numerator/denominator, to the lowest order:

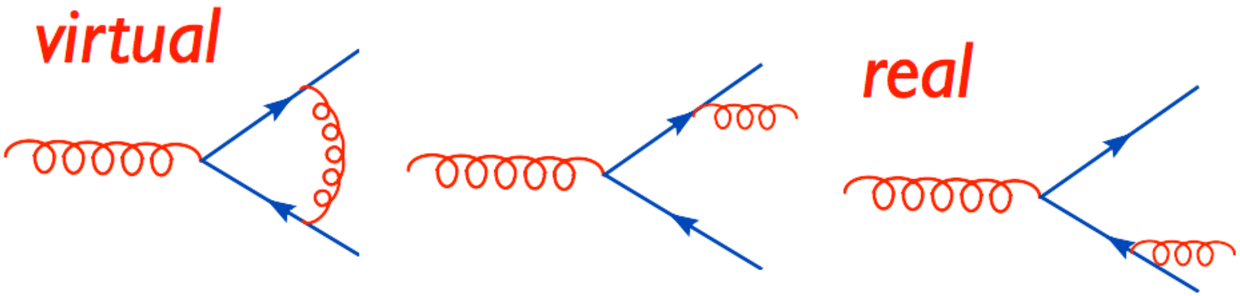
$$R(Q^2) = N_C \sum_f q_f^2$$

N_C= 3 for SU(3)

q_f is quark electric charge

Q is center-of-mass energy

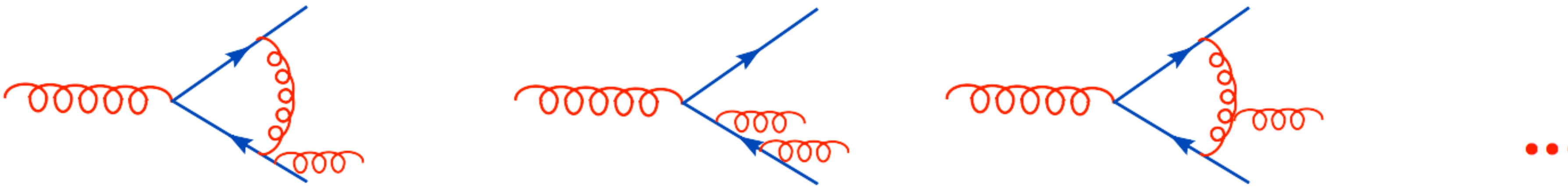
R-ratio (1st order correction)



When including 1st order correction:

$$R(Q^2) = N_C \sum_f q_f^2 \left(1 + \boxed{\frac{\alpha_s\left(Q^2\right)}{\pi}} \right)$$

R-ratio (2nd order correction)



When including 2nd order correction:

$$R(Q^2) = N_C \sum_f q_f^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi} + c_2 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 \right)$$

$C_2 = 1.9857 - 0.1152 n_f$ (from PDG)

Extracting α_s from R-ratio

1. Collide e^+e^-
2. Measure rate of events with hadron and lepton as final states
3. Take ratio of rates to obtain R
4. Extract α_s!

Mission Accomplished?

Extracting α_s from R-ratio

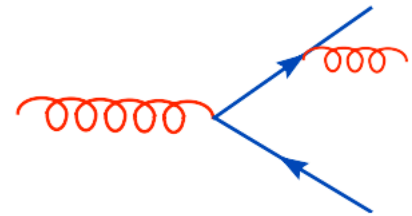
1. Collide e^+e^-
2. Measure rate of events with hadron and lepton as final states
3. Take ratio of rates to obtain R
4. Extract α_s!

Leading order term is independent of α_s

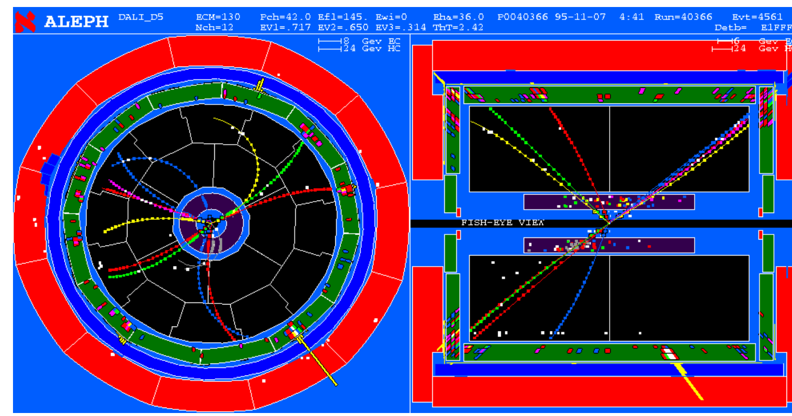
Radiative QCD correction is small

$$R(Q^2) \propto \left(1 + \left[\frac{\alpha_s(Q^2)}{\pi} + c_2 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 + \ldots \right] \right)$$

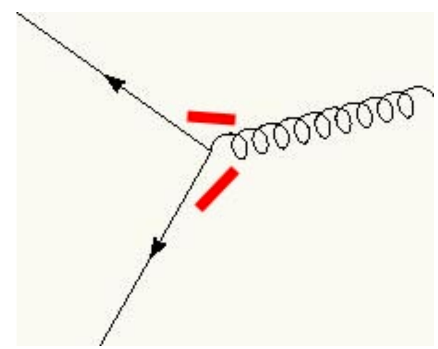
Jet Rates & Event Shapes



- Instead focus analysis with #jet ≥ 3, leading contribution is sensitive to α_s



- Jet Rates:
  - $e^+e^- \rightarrow$ multi (3,4,5,6) jets
- Event Shape:
  - Jet topology (momenta flow, angular distributions) affected by gluon emission



Eur. Phys. J. C 35, 457–486 (2004)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-01891-4

THE EUROPEAN PHYSICAL JOURNAL C

Studies of QCD at e^+e^- centre-of-mass energies between 91 and 209 GeV

The ALEPH Collaboration

- Jet Rates
  - Durham clustering algorithm

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{E_{\text{vis}}^2}$$

  - Jet multiplicity dependence on jet-algorithm clustering threshold y_{cut}

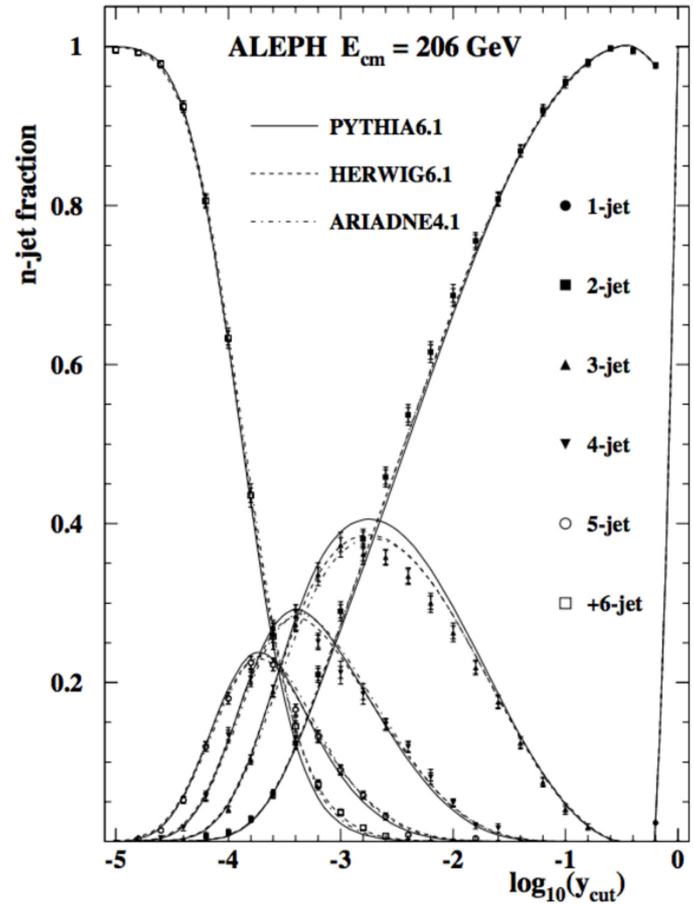


Fig. 7. Measured n-jet fractions for $n = 1, 2, 3, 4, 5$ and $n \geq 6$ and the predictions of Monte Carlo models, at a centre-of-mass energy of 206 GeV

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THE EUROPEAN PHYSICAL JOURNAL C

Studies of QCD at e^+e^- centre-of-mass energies between 91 and 209 GeV

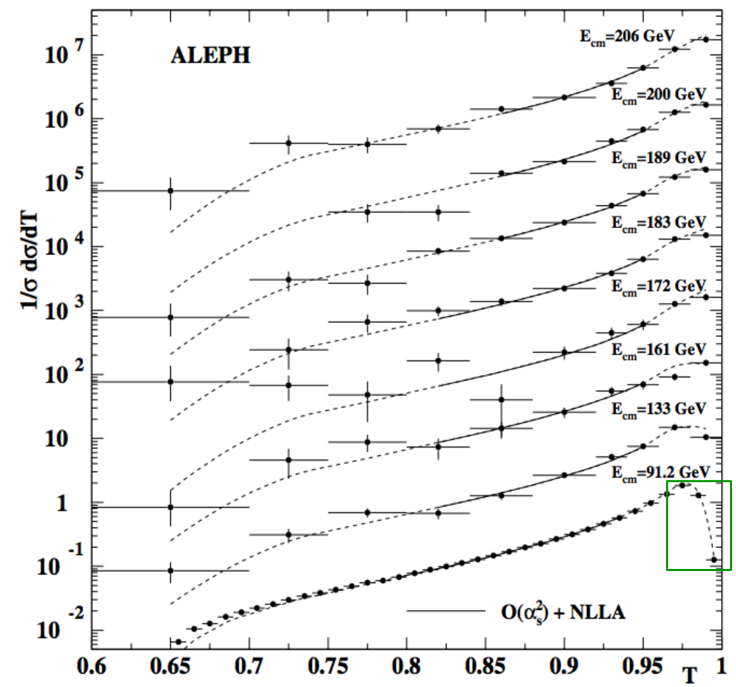
The ALEPH Collaboration

- # Event Shape
  - Define Thrust:

$$T = \max_{\mathbf{n}_T} \left(\frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}_T|}{\sum_i |\mathbf{p}_i|} \right)$$

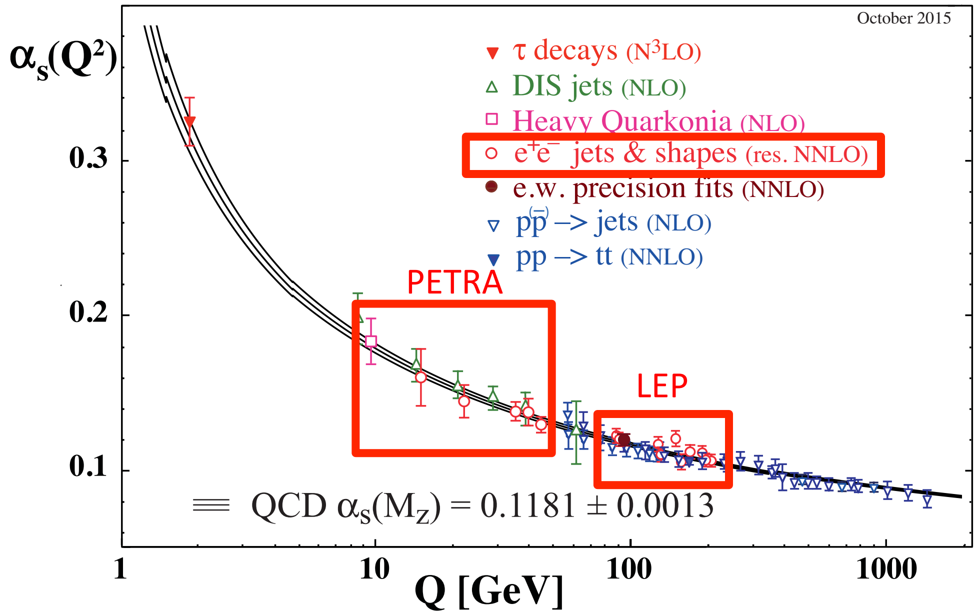
p_i: Momentum of particle

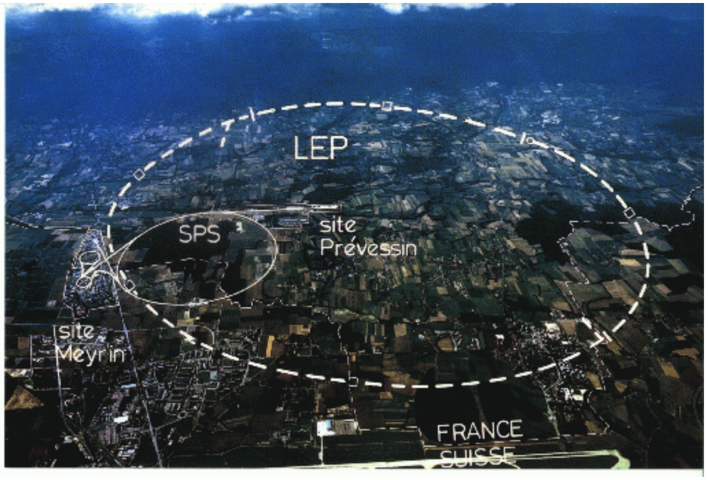
n_T: Thrust axis



T=1
Collinear jets

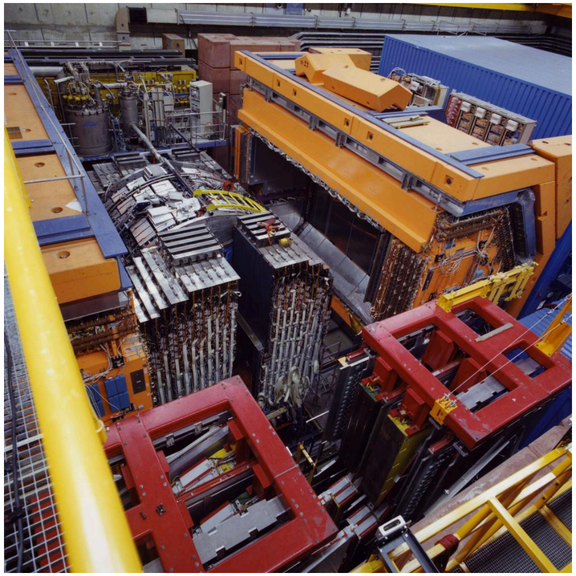
  - By definition, n_T points in direction that is collinear with the maximum flow of momenta of the event
  - T=1 for back-to-back jets
  - T=0.5 for perfectly spherical distribution of momenta



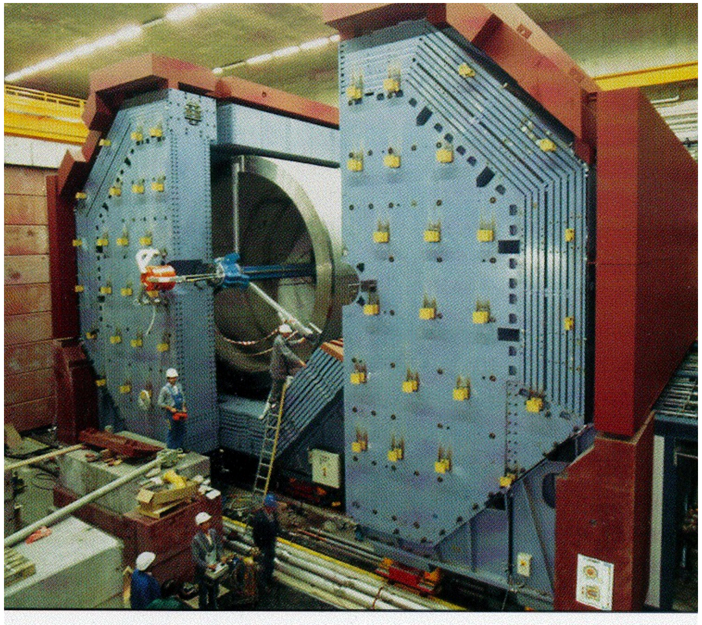


PETRA collider, DESY
e^+e^- collider @ 14-44 GeV

LEP collider, CERN
e^+e^- collider @ 91-206 GeV



ZEUS detector, DESY

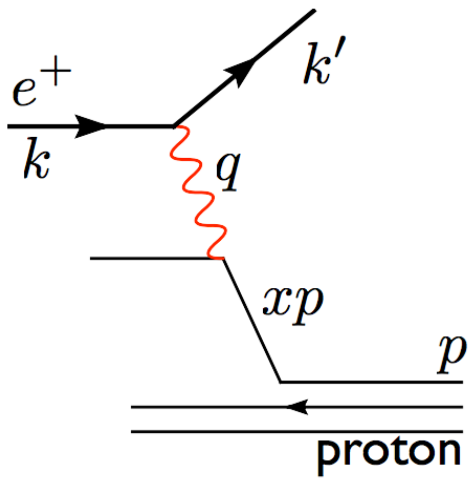


H1 detector, DESY

Deep Inelastic Scattering

Deep Inelastic Scattering (0^{th} order)

- Lepton-Proton collider



Kinematics:

$$Q^2 = -q^2 \quad s = (k+p)^2 \quad x_{Bj} = \frac{Q^2}{2p\cdot q} \quad y = \frac{p\cdot q}{k\cdot p}$$

Cross Section:

$$\frac{d\sigma}{dy\,dx} = \frac{2\pi\alpha_{em}^2 s}{Q^4}\left(1+(1-y^2)\right)F_2(x) \qquad F_2(x) = \sum_l x q_l^2 f_l^{(p)}(x)$$

Structure function

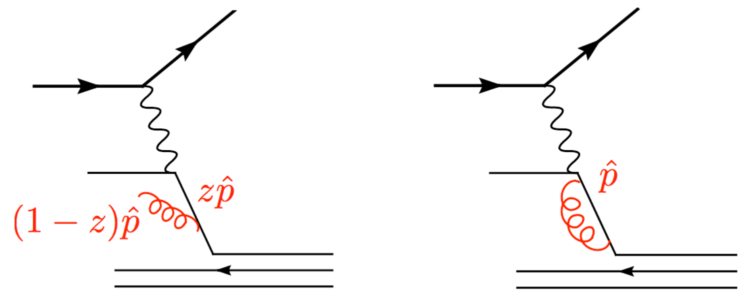
Parton distribution function (PDF)

- $f_l(x)$ independent of Q^2, invariant to scaling
- Partons are point-like (Bjorken scaling)

Deep Inelastic Scattering (1st order)

Radiative corrections

To first order in the coupling:
need to consider the emission of one real gluon and a virtual one



- PDF is no longer scale invariant (not point-like free quarks)
- Evolution of PDF given by DGLAP equation:

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

F_2 with pertubative QCD

- Including higher orders in pQCD, the structure function becomes:

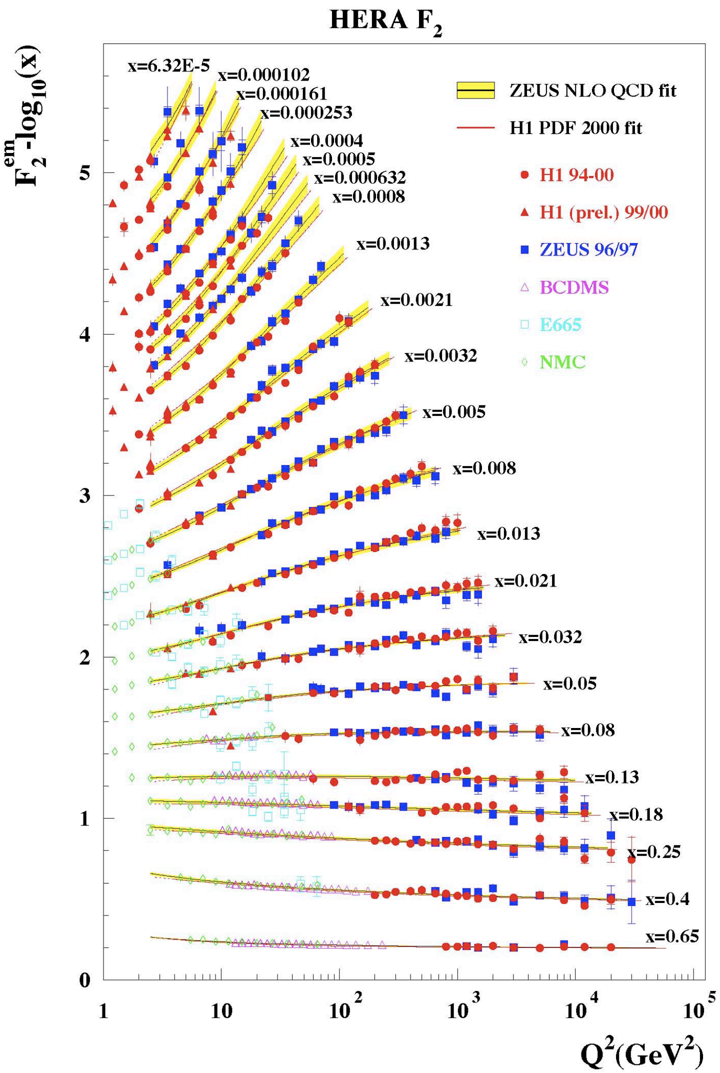
$$F_2(x,Q^2) = x\sum_{n=0}^{\infty}\frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n}\sum_{i=q,g}\int_x^1\frac{dz}{z}C_{2,i}^{(n)}(z,Q^2,\mu_R^2,\mu_F^2)\, f_{i/p}\left(\frac{x}{z},\mu_F^2\right)$$

QCD review PDG

1. $F_2(x,Q^2)$ has Q^2 dependence because of QCD radiative correction
2. Strong coupling constant α_s gives size of correction
3. $C_{2,i}^{(n)}$ coefficient is calculable from Feynman diagrams

F_2 dependence on Q^2

DIS analysis done to NLO



Extraction of α_s from low x curves

http://www.mit.edu/~hasell/DKH_zeus.html

Jets cross-section in DIS

- Production of jets is a more direct measurement of α_s

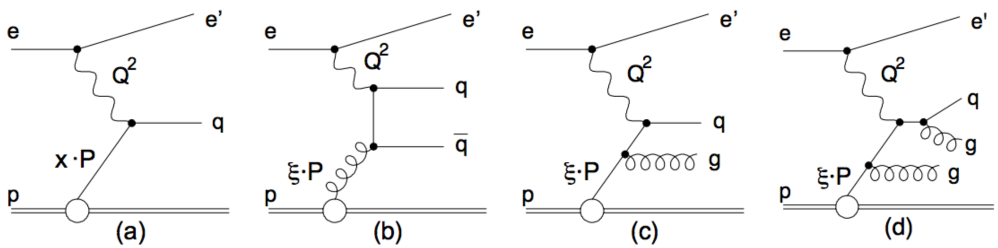
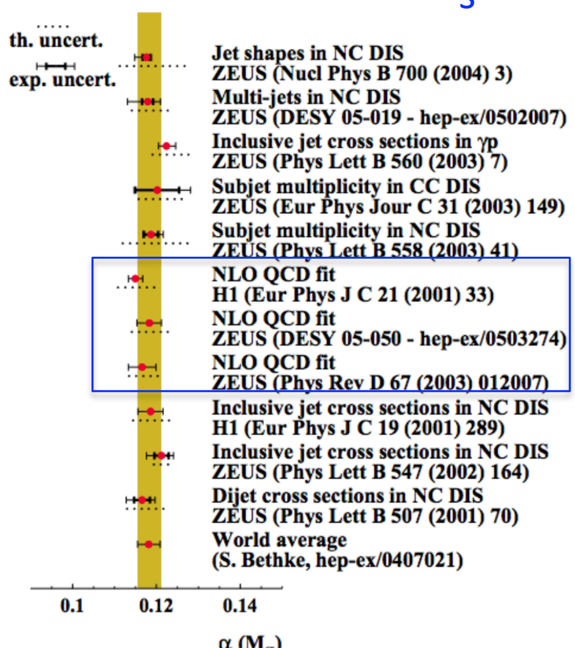
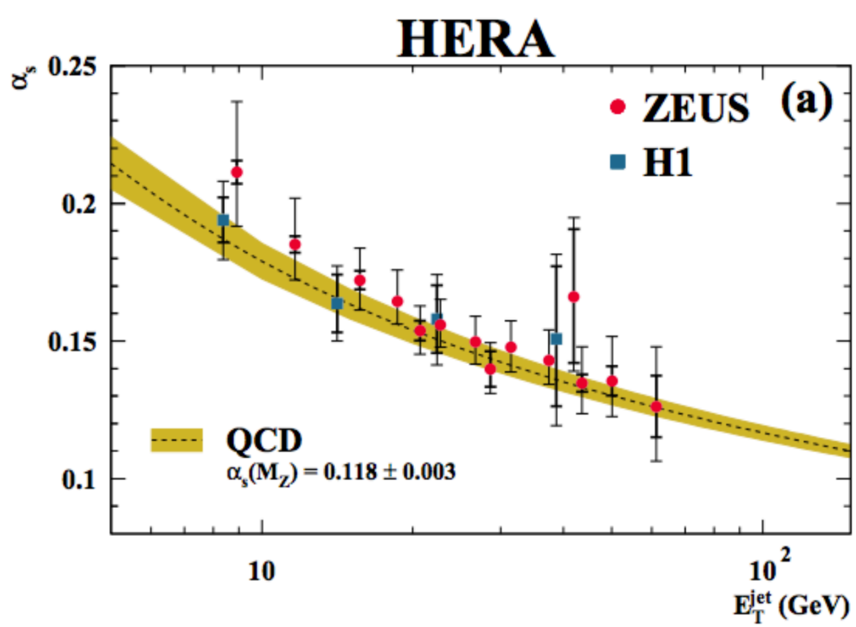


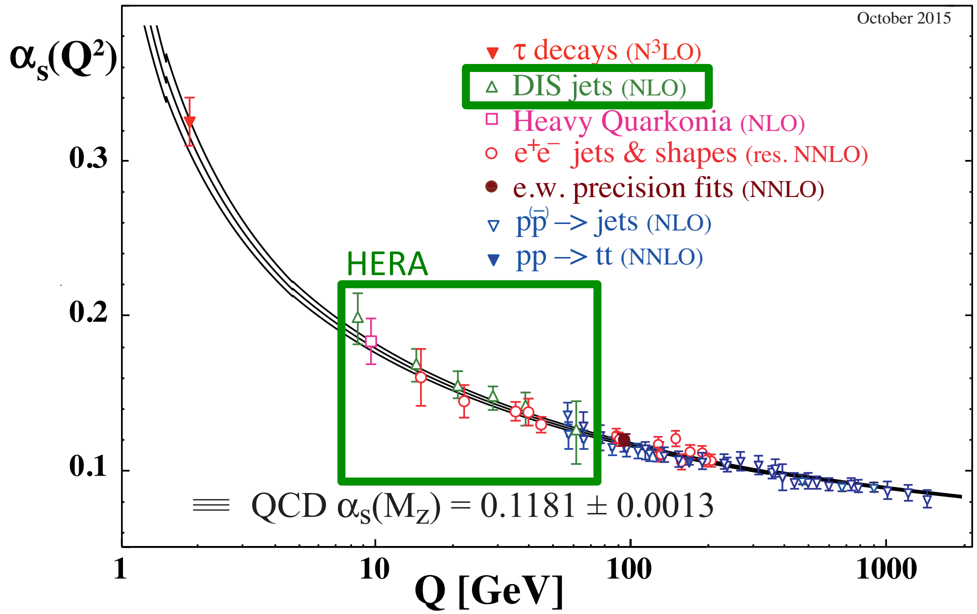
Figure 1: Deep-inelastic *ep* scattering at different orders in α_s: (a) Born contribution $O(\alpha_{em}^2)$, (b) example of boson-gluon fusion $O(\alpha_{em}^2\alpha_s)$, (c) example of QCD Compton scattering $O(\alpha_{em}^2\alpha_s)$ and (d) example of a trijet process $O(\alpha_{em}^2\alpha_s^2)$.

Measurement of α_s at HERA



Structure Function

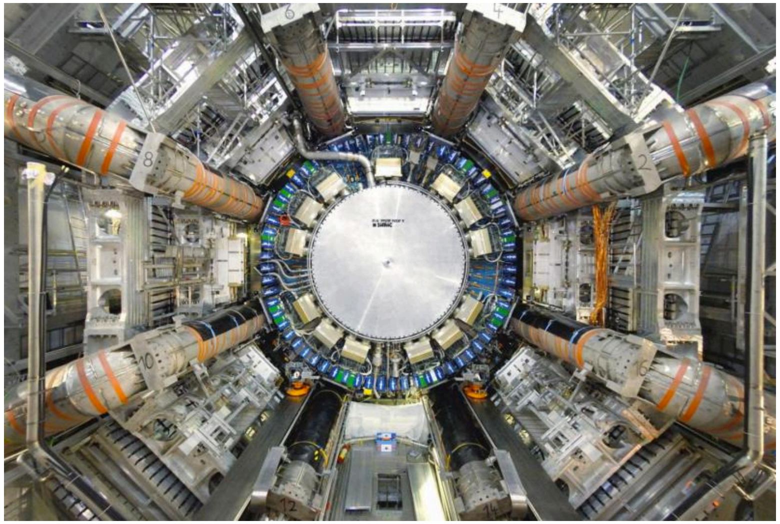




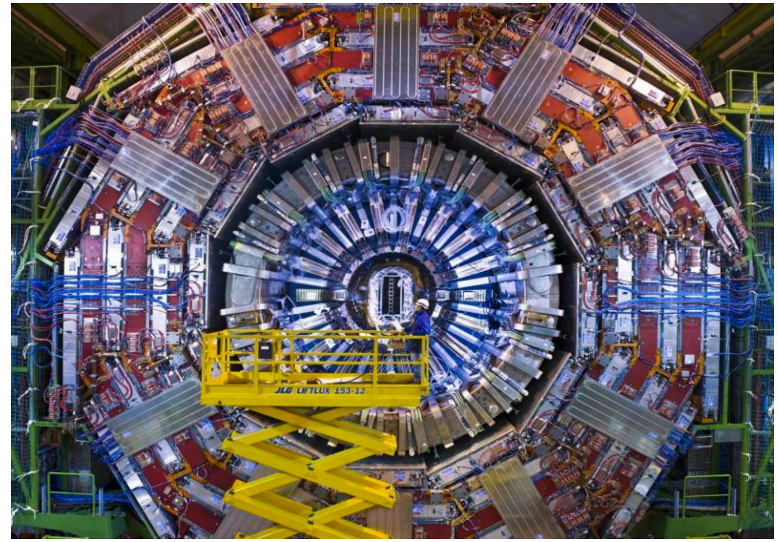
HERA collider, DESY

e^-/e^+ @ 27.5 GeV

Proton @ 920 GeV



ATLAS detector, CERN



CMS detector, CERN

Hadron-Hadron Collisions

Probing QCD with Hadron Collisions

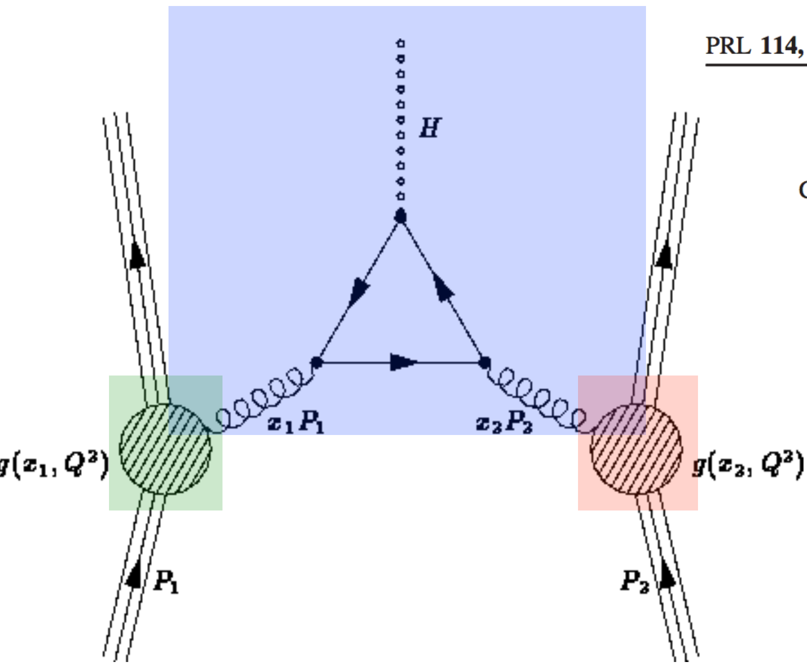
Cross Section

Parton Distribution Functions

Partonic Cross Section

$$\sigma(h_1 h_2 \to W + X) = \sum_{n=0}^{\infty} \alpha_s^n(\mu_R^2) \sum_{i,j} \int dx_1 dx_2\, f_{i/h_1}\left(x_1, \mu_F^2\right) f_{j/h_2}\left(x_2, \mu_F^2\right) \times \hat{\sigma}^{(n)}_{ij \to W+X}\left(x_1 x_2 s, \mu_R^2, \mu_F^2\right)$$

- Higgs production from gluon fusion



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Higgs Boson Gluon-Fusion Production in QCD at Three Loops

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We present the cross section for the production of a Higgs boson at hadron colliders at next-to-next-to-next-to-leading order (N^3LO) in perturbative QCD. The calculation is based on a method to perform a series expansion of the partonic cross section around the threshold limit to an arbitrary order. We perform this expansion to sufficiently high order to obtain the value of the hadronic cross at N^3LO in the large top-mass limit. For renormalization and factorization scales equal to half the Higgs boson mass, the N^3LO corrections are of the order of $+2.2\%$. The total scale variation at N^3LO is 3%, reducing the uncertainty due to missing higher order QCD corrections by a factor of 3.

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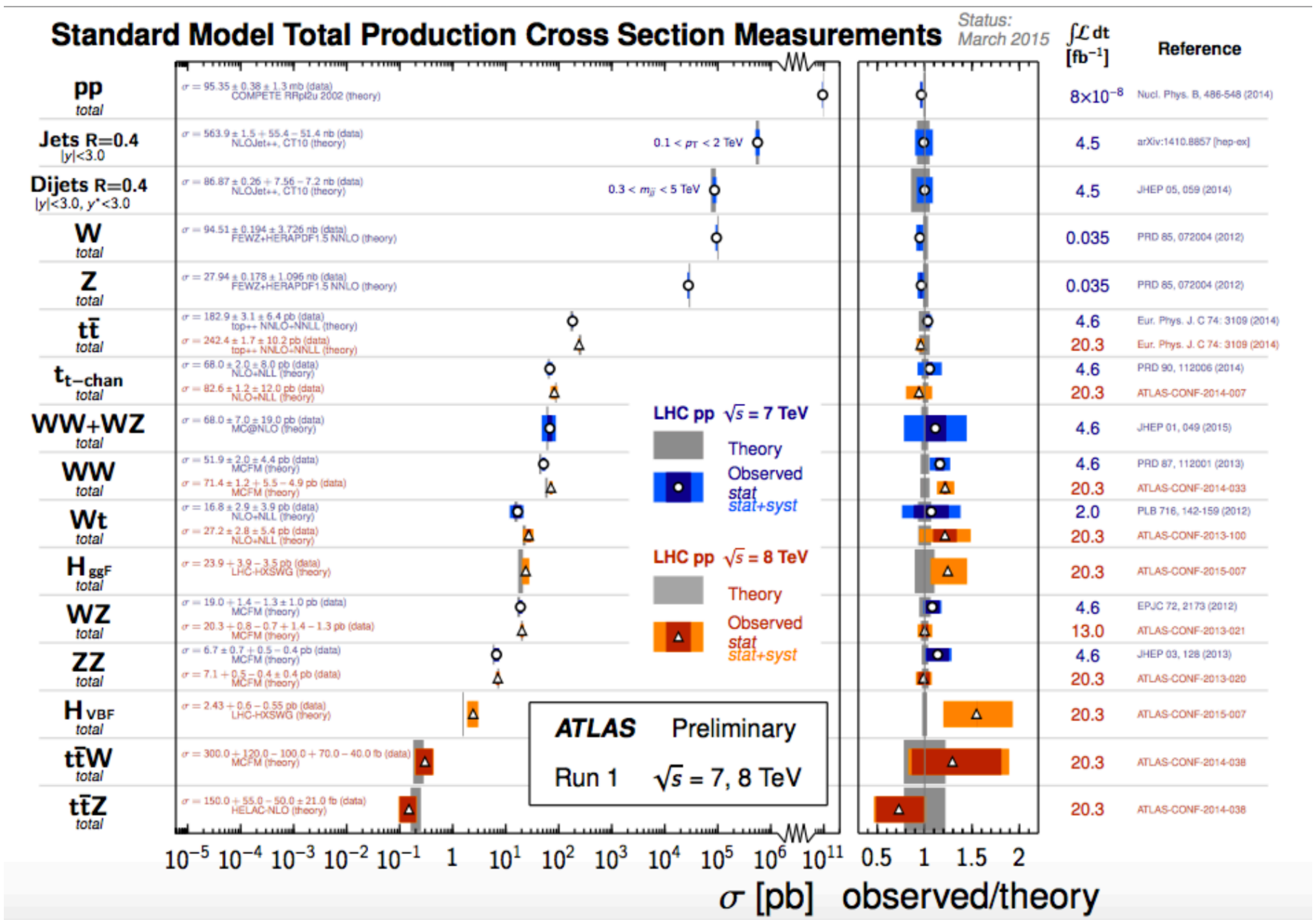
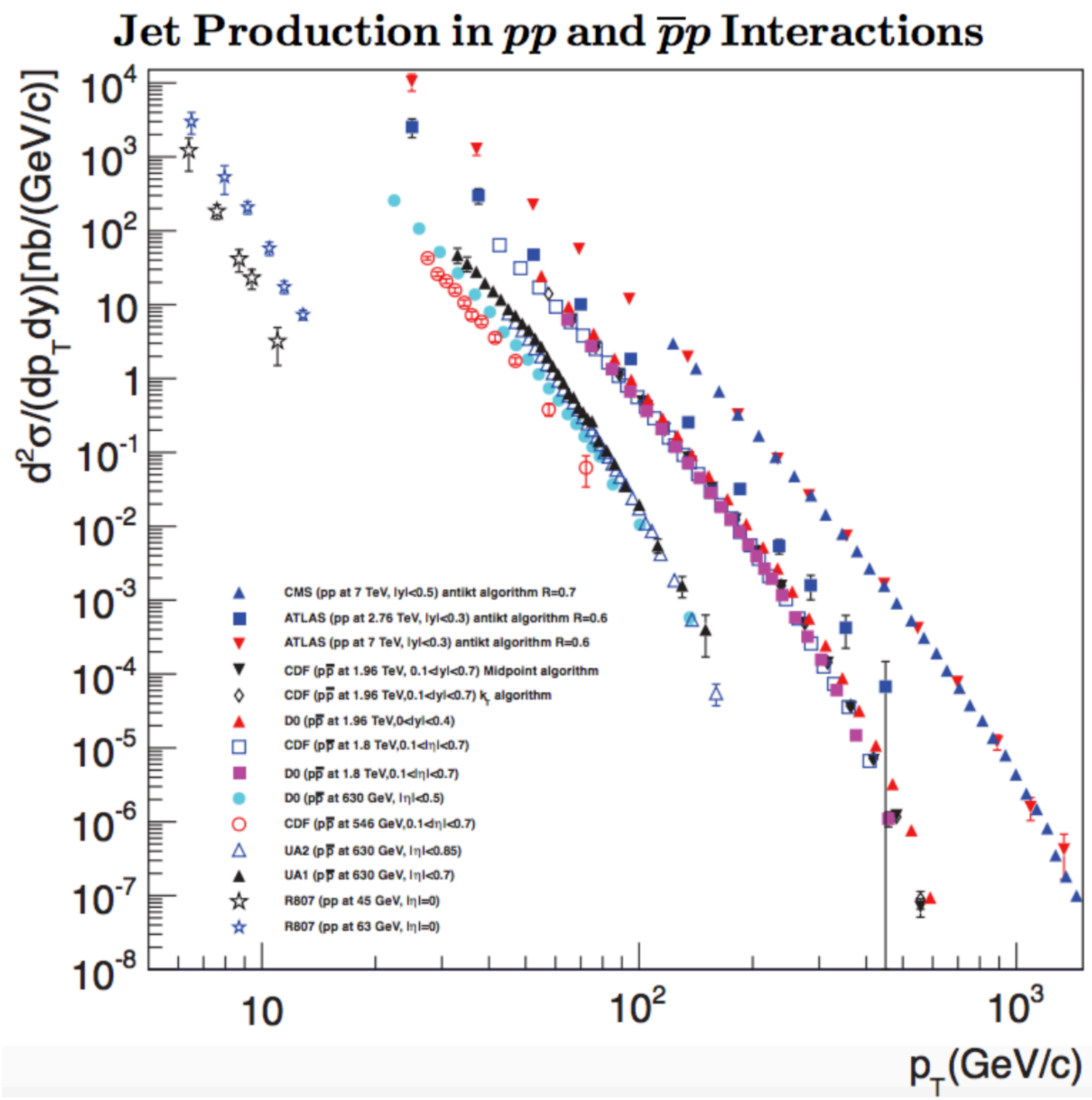
Cross Section Measurements

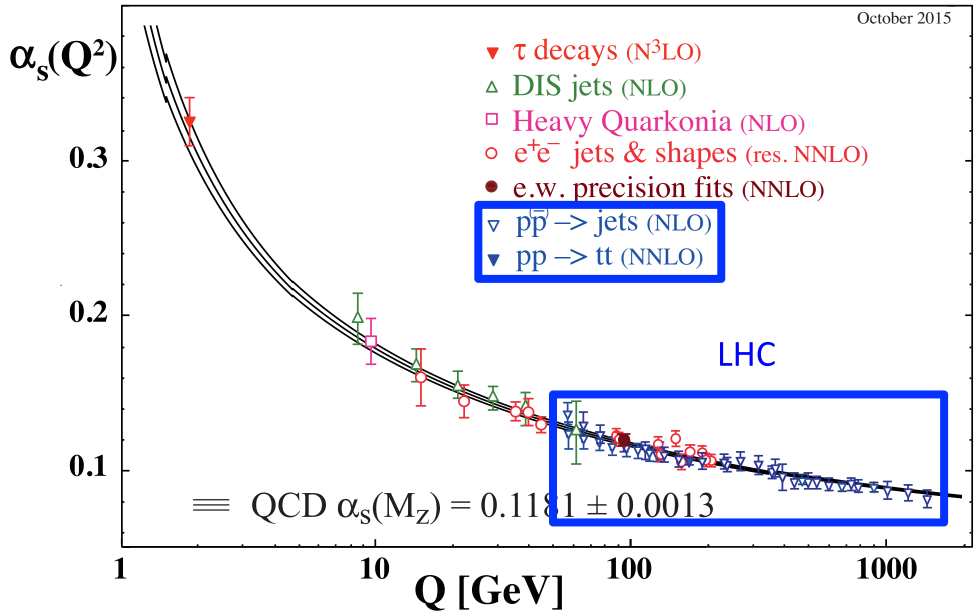
Good agreement with NLO QCD predictions!



Jet production in Hadron Collisions

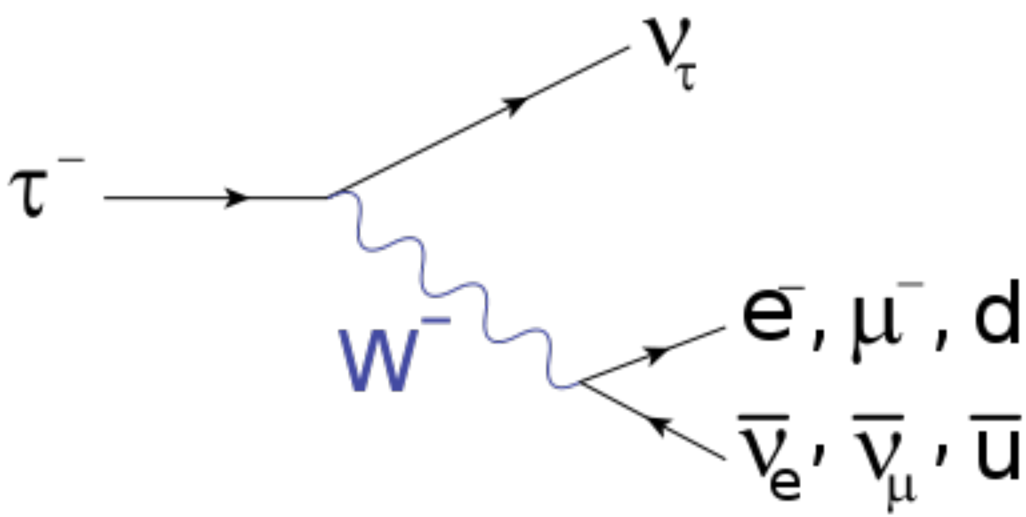


- σ(Jets) is sensitive to α_s at lowest order
- Dependence on energy measured by LHC from ~10 to 10^3 GeV



LHC, CERN

pp⁻ collision @ 7, 8 and now 13 TeV



τ DECAYS

Why τ?

- The only lepton that can decay hadronically
- Can probe QCD at energy scales of M_τ = 1.8 GeV

Naïve lowest order

$$R_\tau \equiv \frac{\Gamma[\tau^- \to \mathrm{hadrons}\,\nu_\tau]}{\Gamma[\tau^- \to e^- \overline{\nu}_e \nu_\tau]} = 3\left|V_{ud}\right|^2$$

- QCD perturbation would cause a correction to the hadronic decay width

Correction identical to the R-ratio in e^+e^-

$$R_\tau \propto \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)\left(1+\frac{2s}{M_\tau^2}\right)\left[1+\alpha_s(s)+\left(1.9857-0.1152n_f\right)\alpha_s(s)^2+\ldots\right]$$

Integrated over allowed range of invariant masses from decay of τ

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Order α_s^4 QCD Corrections to Z and τ Decays

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$$R = 1 + a_s + (1.9857 - 0.1152 n_f) a_s^2 + (-6.63694 - 1.20013 n_f - 0.00518 n_f^2) a_s^3 + (-156.61 + 18.77 n_f - 0.7974 n_f^2 + 0.0215 n_f^3) a_s^4.$$

- Using τ hadronic decay width from LEP:

$$\alpha_s(M_\tau) = 0.332 \pm 0.005_{\rm exp} \pm 0.015_{\rm th}.$$

M_τ=1.8 GeV

PDG world average

$$\alpha_S(M_Z) = 0.1181 \pm 0.0013$$

M_Z=91 GeV

Summary

Theory

- Running of coupling constants from introduction of scale to regulate theory
- QCD is asymptotically free because of gluon self-interaction
- Kinks in curve from active n_f

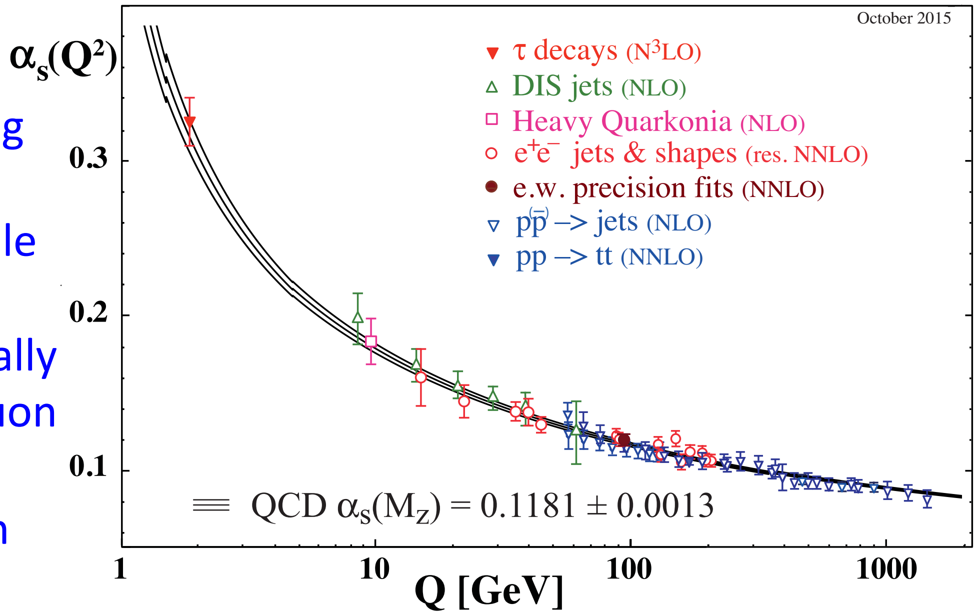

Figure 9.3: Summary of measurements of α_s as a function of the energy scale Q. The respective degree of QCD perturbation theory used in the extraction of α_s is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to leading order; res. NNLO: NNLO matched with resummed next-to-leading logs; N^3LO: next-to-NNLO).

Experiments

- e^+e^- (Multiple Jets & Event Shape)
- DIS
- Hadron Collision
- τ decay

Resources

- http://pdg.lbl.gov/2015/reviews/rpp2015-rev-standard-model.pdf (PDG 2015 QCD review)
- https://www2.physics.ox.ac.uk/sites/default/files/QCDLectures.pdf (Giulia Zanderighi lecture)
- http://www.nikhef.nl/~h24/qcdcourse/section-6.pdf