



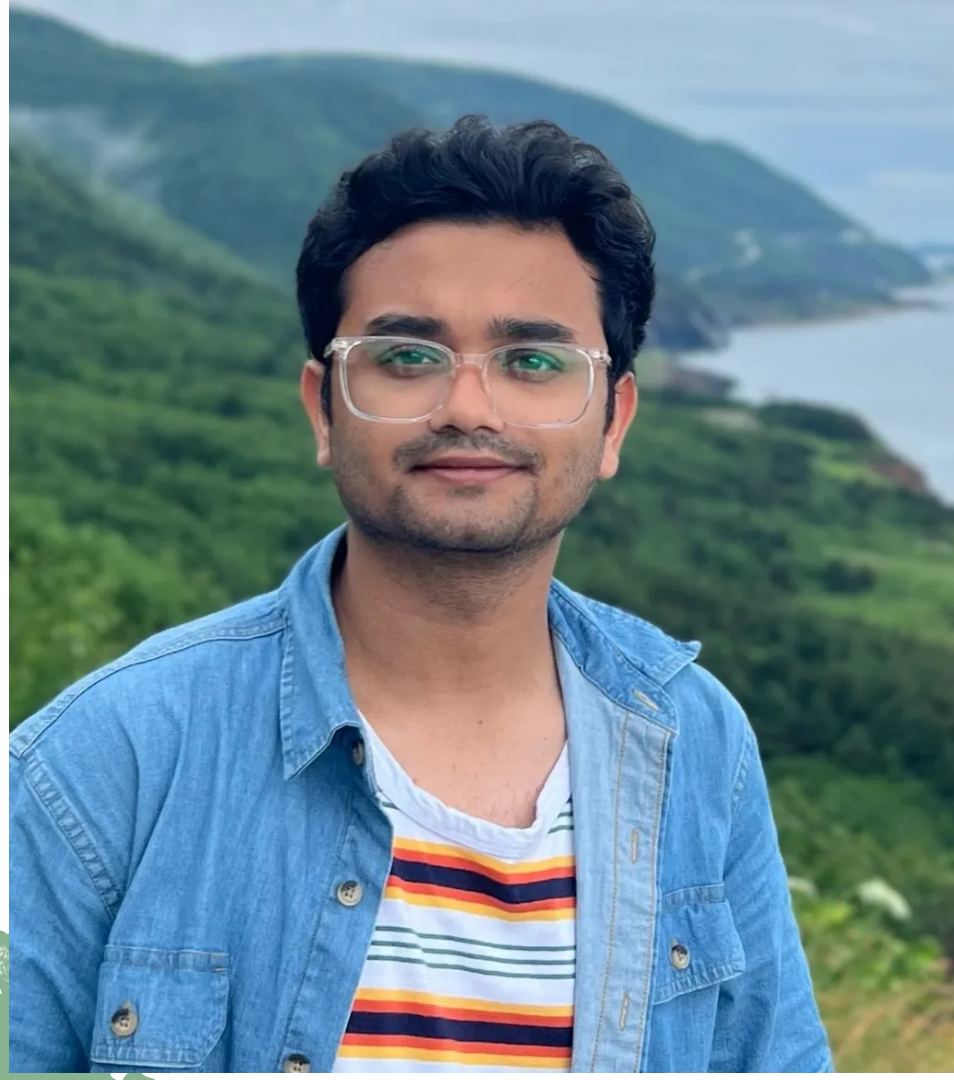
# LOOKING FOR NEW PHYSICS IN THE MUD

*Katelin Schutz, McGill University  
Lawrence-Hitoshi-fest, September 27 2024*

# The importance of saying “hi”



Based on Allan, B. (2008). Knowledge creation within a community of practice



Dr. Saniya Heeba



Nirmalya Brahma

Hugo Schérer





# LOOKING FOR NEW PHYSICS IN THE MUD

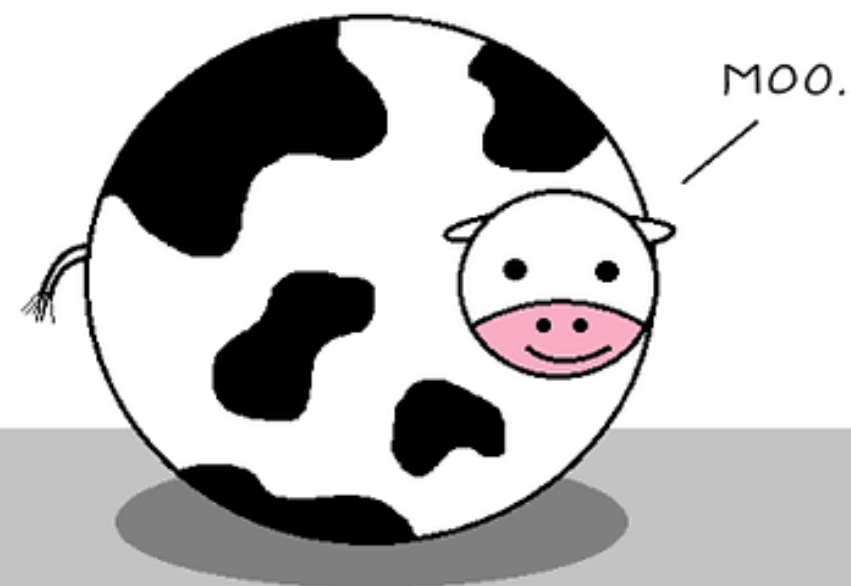
*Katelin Schutz, McGill University  
Lawrence-Hitoshi-fest, September 27 2024*

**NEWS FLASH: OUR UNIVERSE IS  
NOT A VACUUM!**

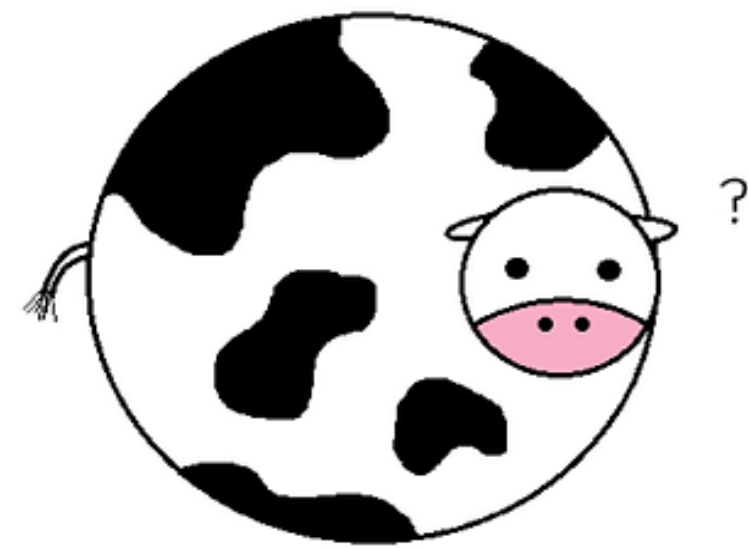


What happens when I go around at conferences telling people I've been thinking about finite-temperature field theory

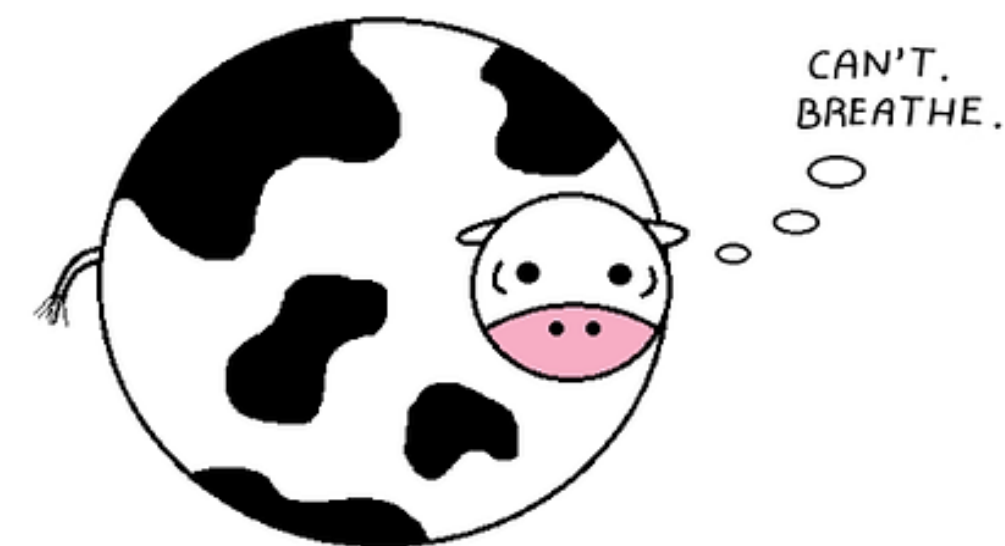
Assume a spherical cow of uniform density.



...while ignoring the effects of gravity.



...in a vacuum.



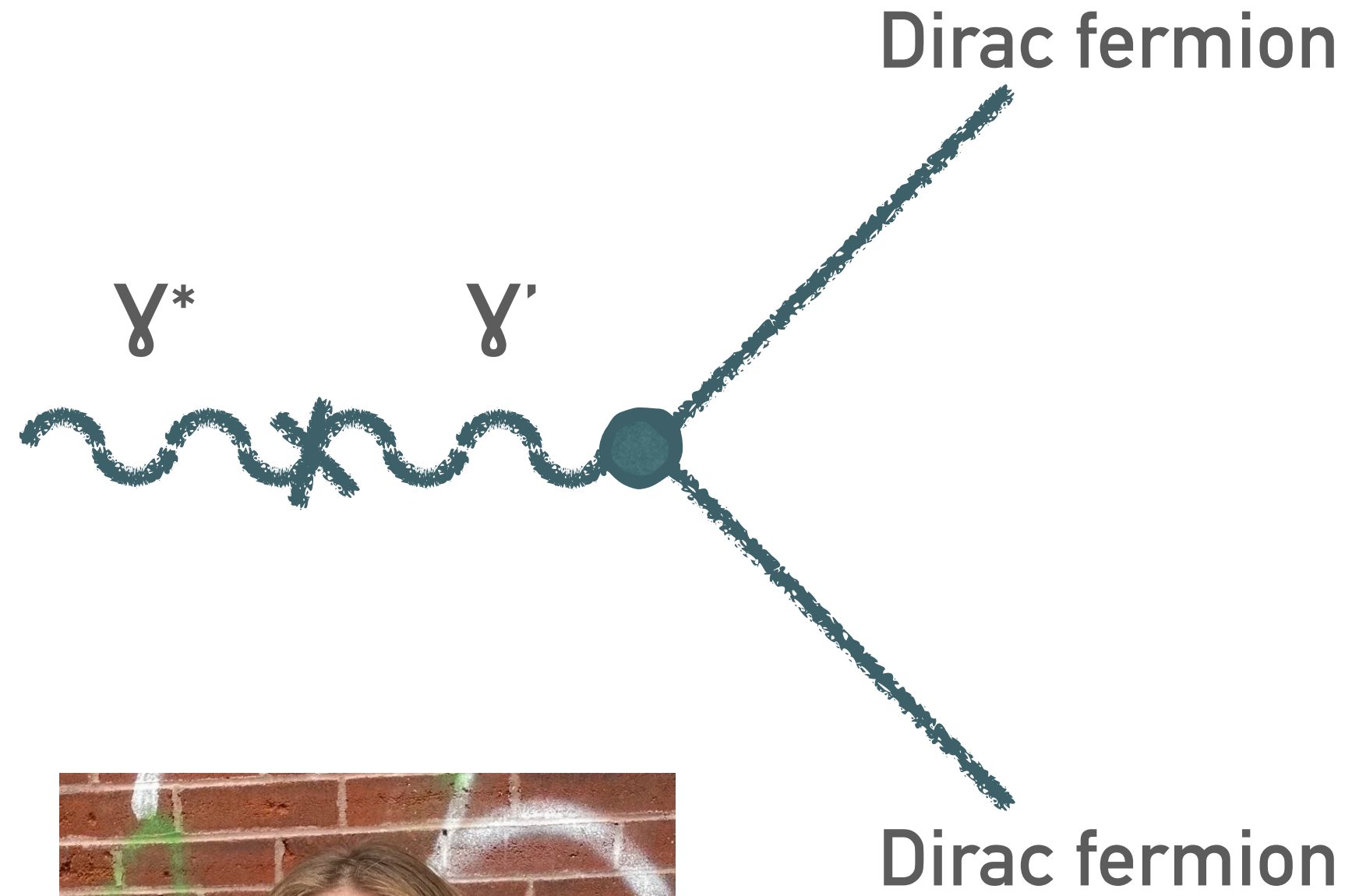
## CHALLENGE OR OPPORTUNITY?

- Doing QFT in a dense medium is quite technical (apologies for the number of equations in this talk!), do we really have to deal with this?
- Yes, if we care about “long”-wavelength physics (probing system beyond individual constituents)
- Not only technical but qualitatively new way of viewing a system, new dispersion relations, new propagating modes (e.g. longitudinal photon)
- Perhaps this implies qualitatively new signatures of physics beyond the Standard Model?
- Low energies imply we could look for low-dimensional portals/operators (kinetic mixing, axions, etc.) by exploiting the medium

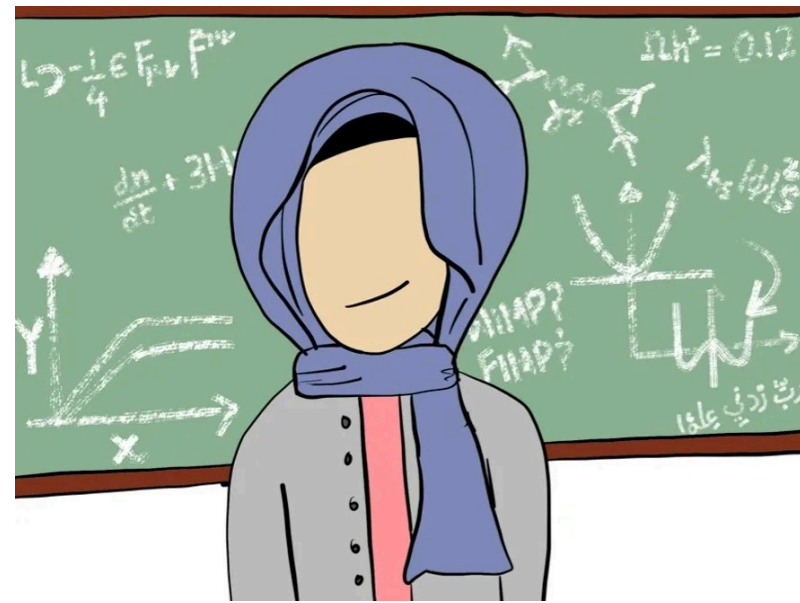
**HOW DID I GET INTERESTED IN FTFT AS  
AN ASTRO-CURIOUS BSM PERSON?**



# WHAT'S AT STAKE HERE FOR BSM PHYSICS?

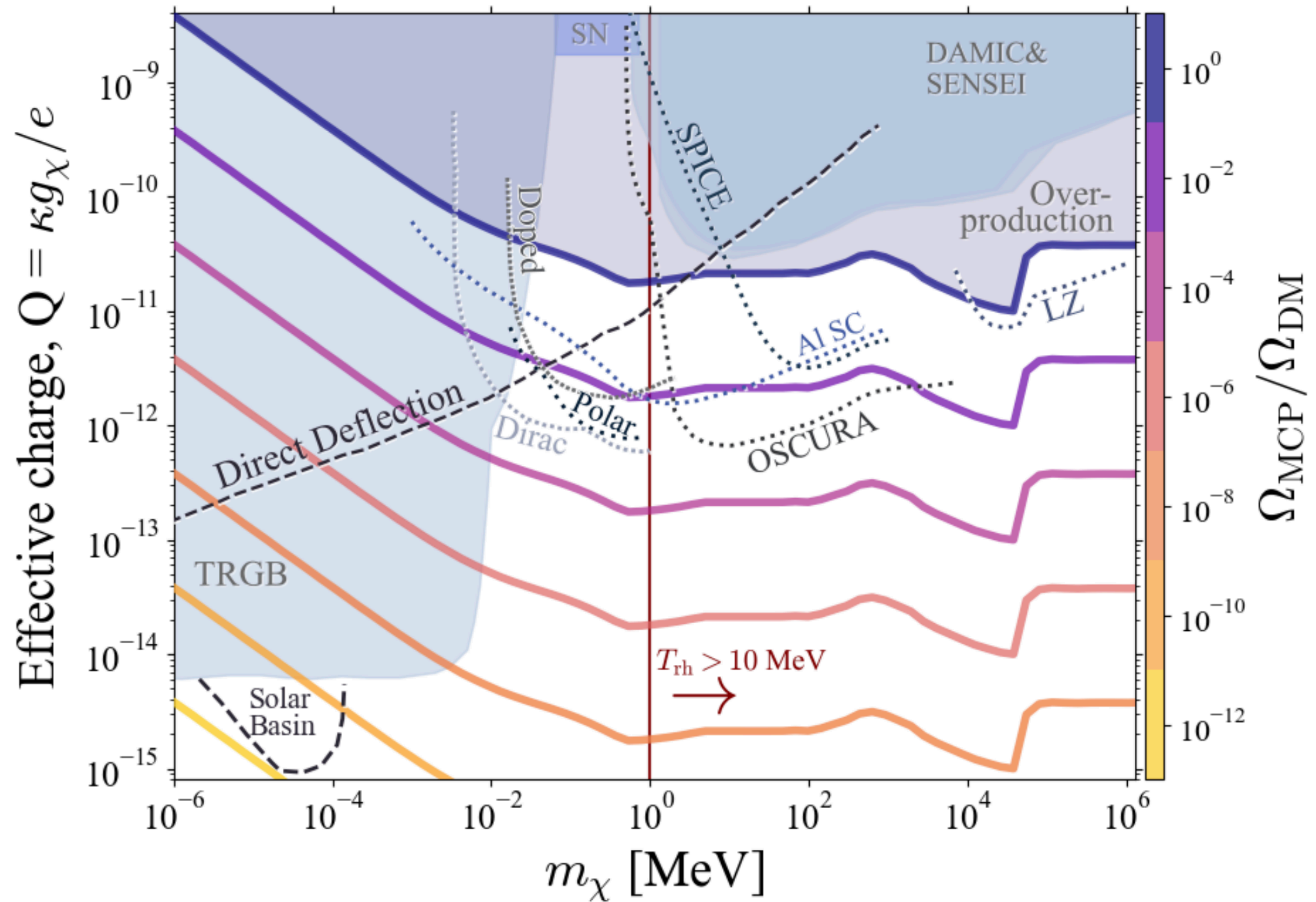


Ella Iles



Dr. Saniya Heeba

Iles, Heeba, KS 2407.21096

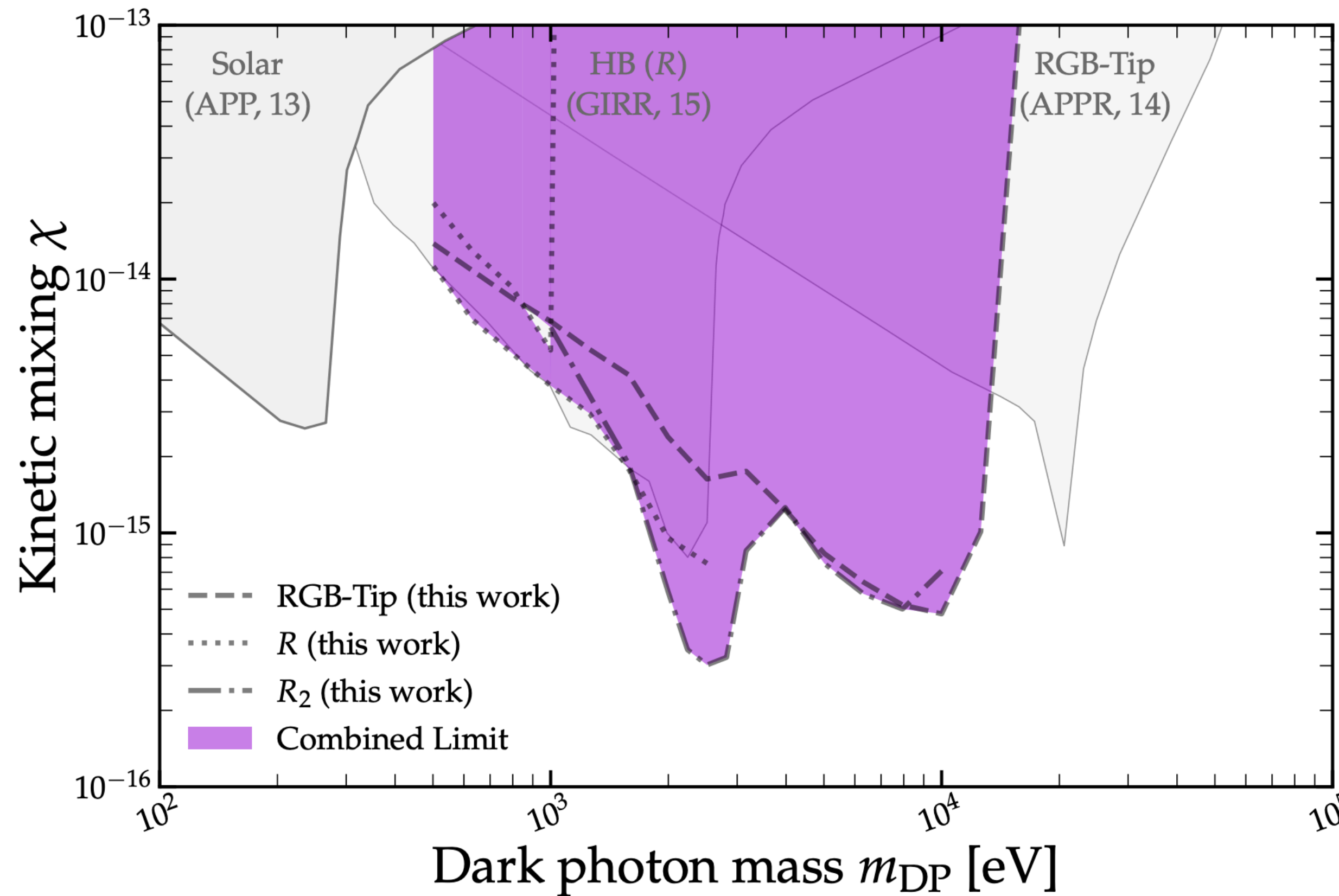
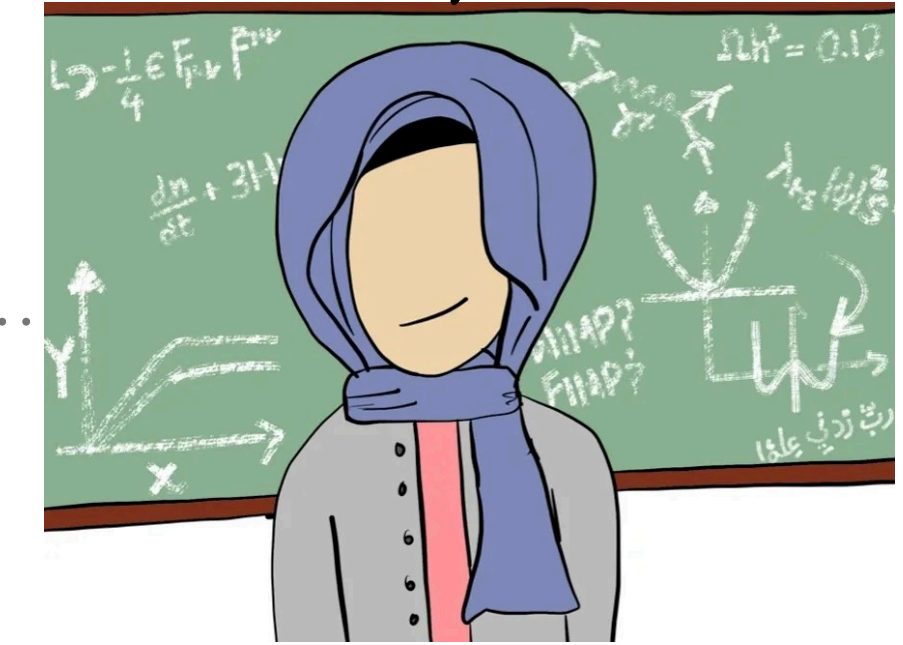


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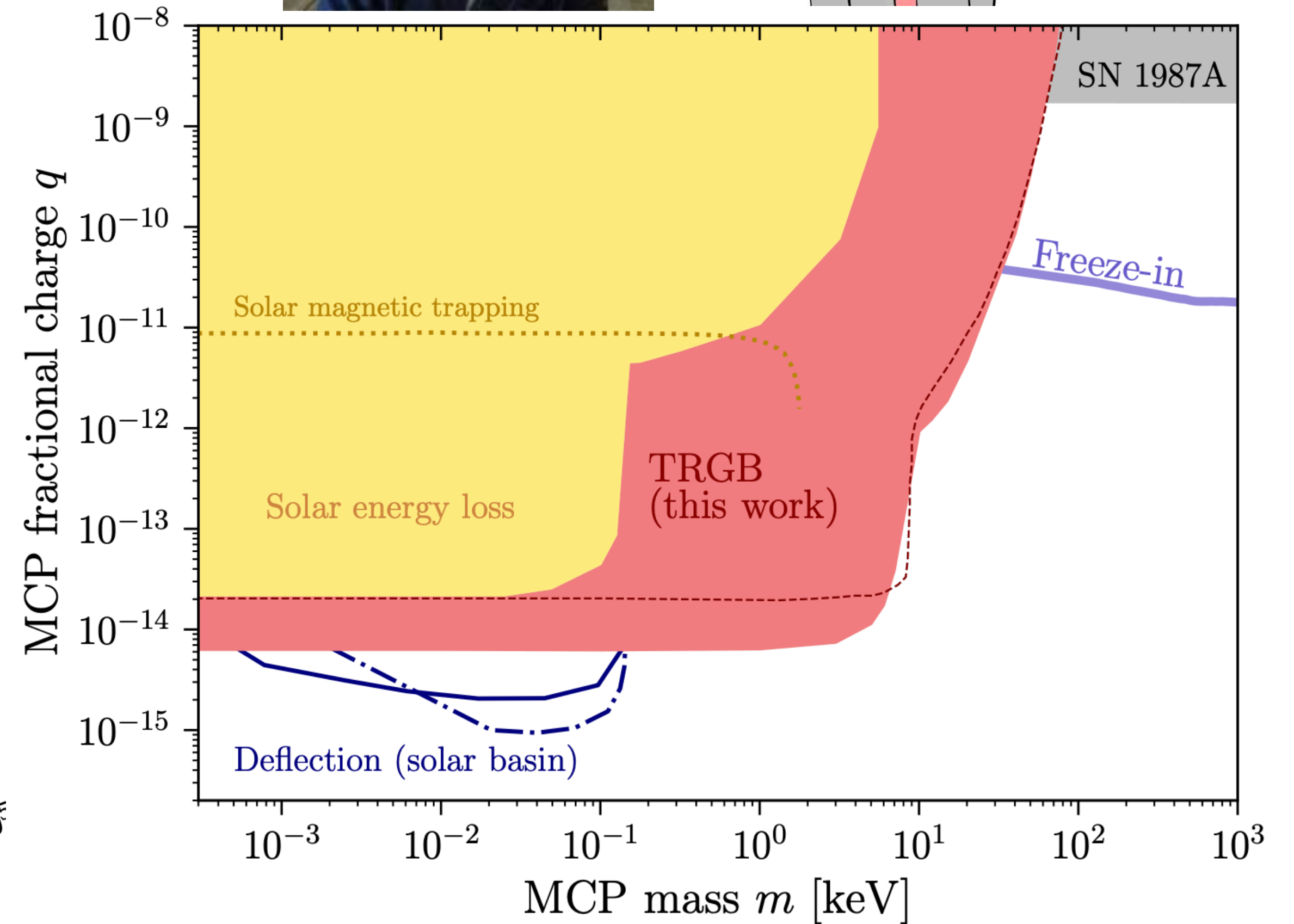
Audrey Fung



Dr. Saniya Heeba

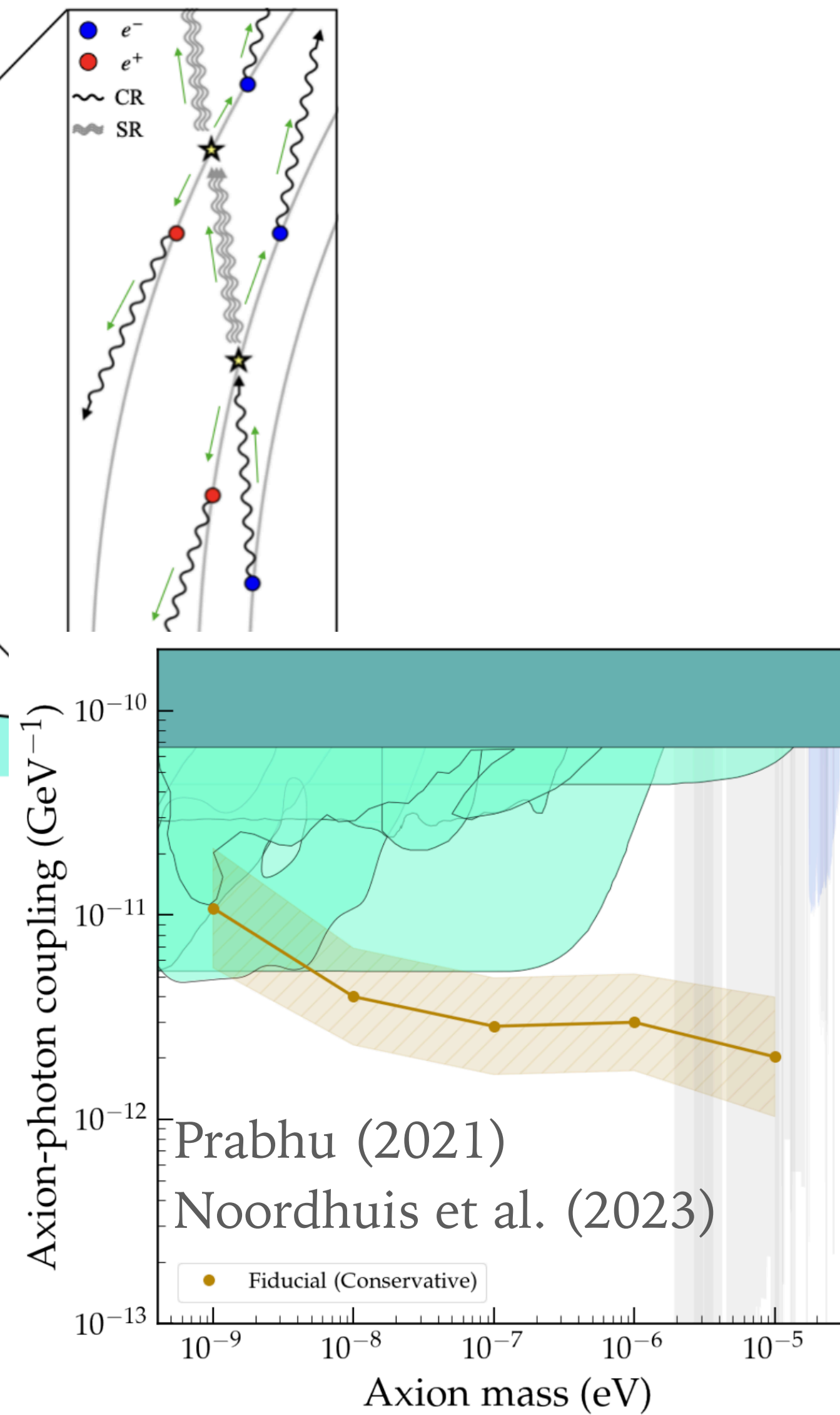
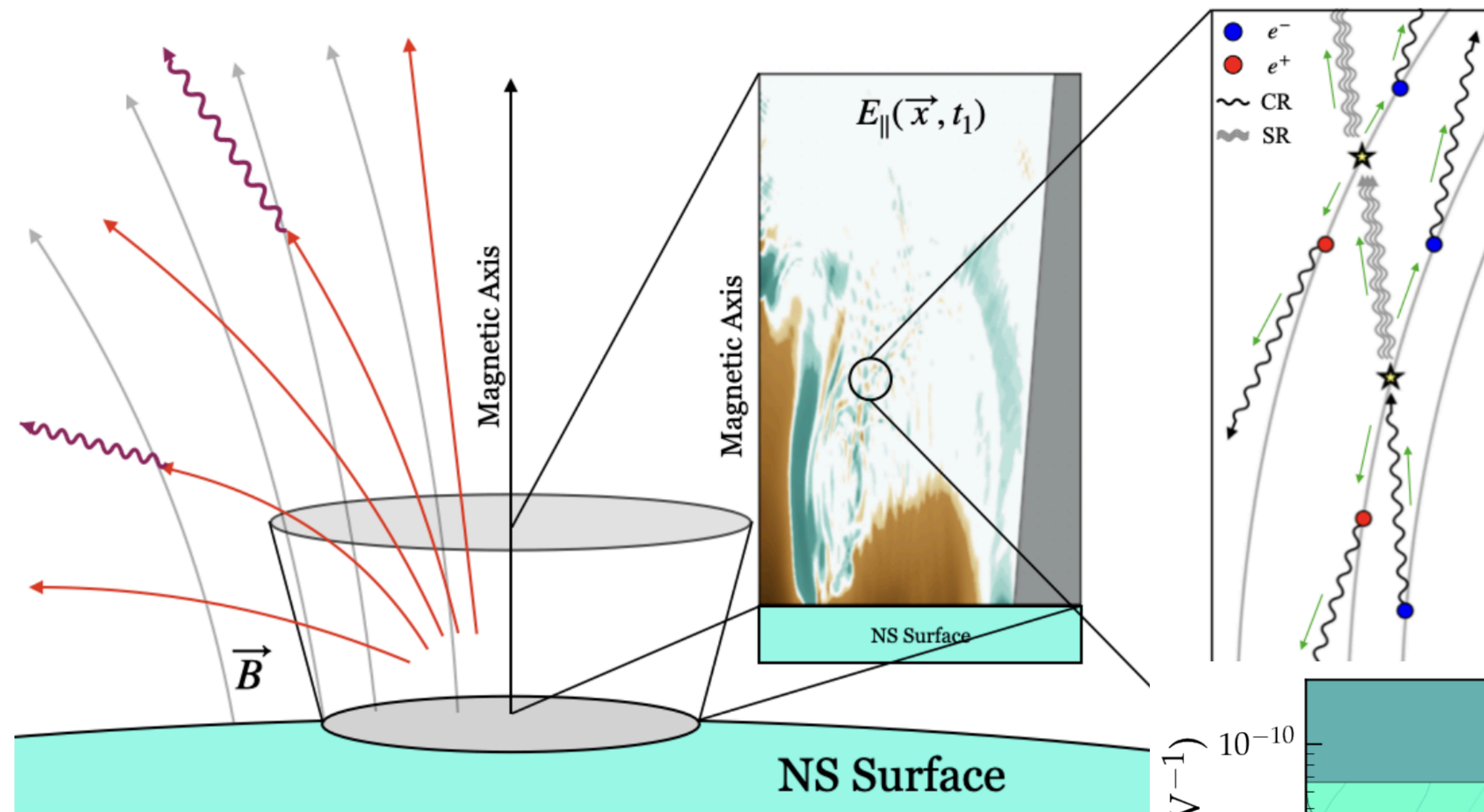
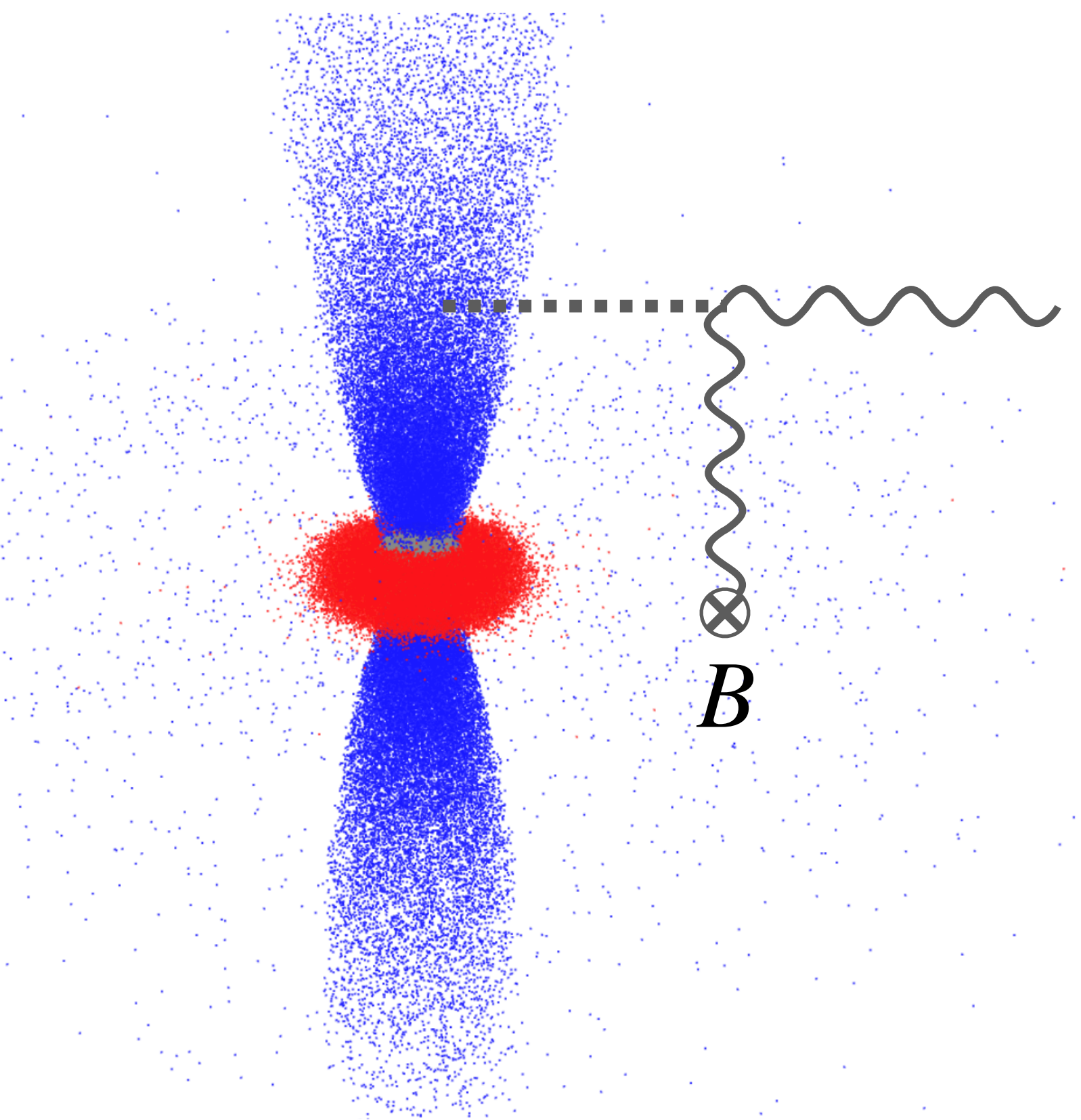


Dolan et al. 2022



Fung, Heeba, Liu, Muralidharan, KS, Vincent (2024)

# WHAT'S AT STAKE HERE FOR BSM PHYSICS?



Hook, Kahn, Safdi, Sun (2018),  
Safdi, Sun, Chen (2018)

# HOW MIGHT THESE NEW SIGNATURES ARISE?

---

Dark photons in vacuum:  $\mathcal{L} = -\frac{1}{4}F^2 - \frac{1}{4}F'^2 + \frac{1}{2}\kappa FF' + \frac{1}{2}m_{A'}^2 A'^2 + ej_\mu A^\mu, \quad \kappa \ll 1$

$$A_1 = A - \kappa A', \quad A_2 = A'$$



Rotate away kinetic mixing term  
("mass basis")

$$\mathcal{L} \supset \frac{1}{2}m_{A'}^2 A_2^2 + ej_\mu (A_1^\mu + \kappa A_2^\mu)$$



$$\mathcal{A} = A, \quad \mathcal{S} = A' - \kappa A$$

Rotate away kinetic mixing term  
("active/sterile basis" analog of neutrinos)

$$\mathcal{L} \supset \frac{1}{2}m_{A'}^2 \mathcal{S}_\mu (\mathcal{S}^\mu + \mathcal{A}^\mu) + ej_\mu \mathcal{A}^\mu$$

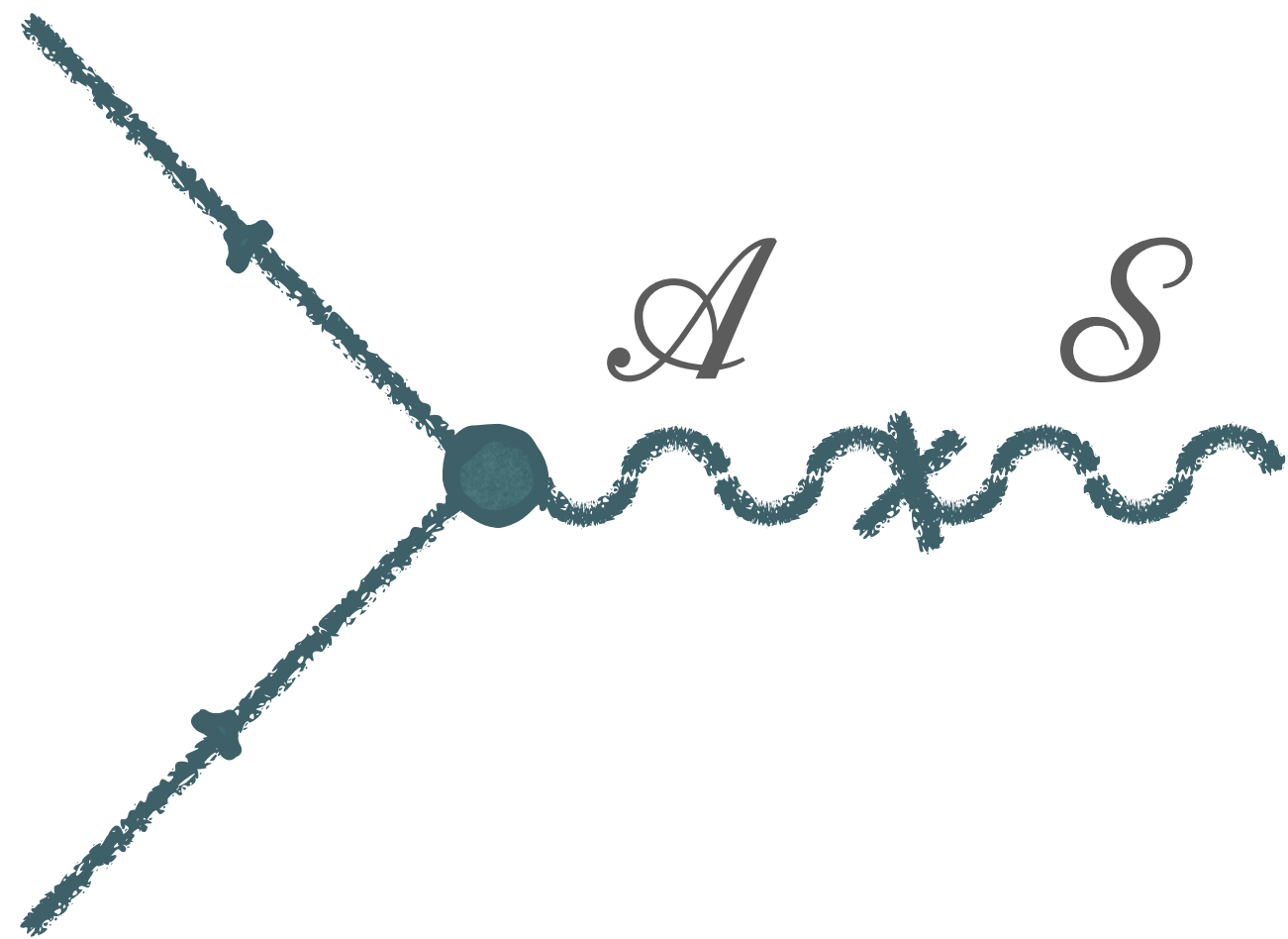
# HOW IS THIS AFFECTED BY AN AMBIENT MEDIUM?

---

$$j_\mu = j_\mu^{\text{ext}} + j_\mu^{\text{ind}}, \quad j_\mu^{\text{ind}} \equiv \Pi^{\mu\nu}(\omega, k)A_\nu$$

Polarization tensor of  
linear response theory

$$\mathcal{L} \supset \frac{1}{2}m_{A'}^2 \mathcal{S}_\mu (\mathcal{S}^\mu + \mathcal{A}^\mu) + \mathcal{A}_\mu \Pi^{\mu\nu} \mathcal{A}_\nu + e j_\mu^{\text{ext}} \mathcal{A}^\mu$$



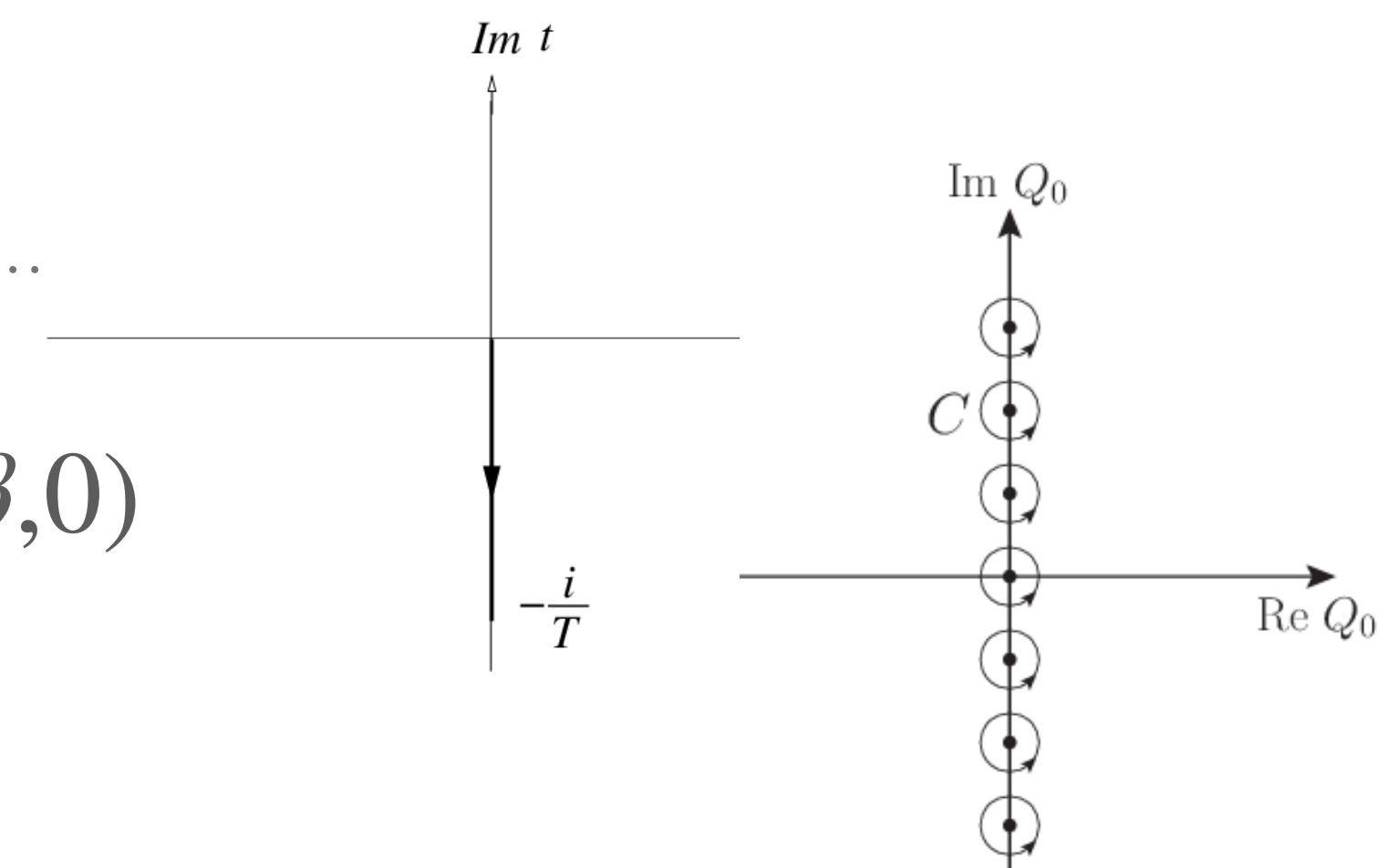
Active state is constantly getting  
bombarded (“dressed”) by background  
particles before oscillating to sterile state

$$\begin{aligned}
& \text{Diagram: A circle with two wavy lines labeled } k \text{ and two straight lines labeled } p \text{ and } p+k. \\
& = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} \left\{ f(E_p) \left[ \begin{array}{c} \text{Diagram 1: } k \text{ wavy line up, } p \text{ straight line right, } p+k \text{ straight line right, } p \text{ straight line down, } k \text{ wavy line right.} \\ \text{Diagram 2: } k \text{ wavy line up, } p \text{ straight line right, } p-k \text{ straight line up, } p \text{ straight line down, } k \text{ wavy line right.} \end{array} \right] \right. \\
& \quad \left. + \bar{f}(E_p) \left[ \begin{array}{c} \text{Diagram 3: } k \text{ wavy line up, } p \text{ straight line left, } p+k \text{ straight line left, } p \text{ straight line down, } k \text{ wavy line right.} \\ \text{Diagram 4: } k \text{ wavy line up, } p \text{ straight line left, } p-k \text{ straight line down, } p \text{ straight line up, } k \text{ wavy line right.} \end{array} \right] \right\}
\end{aligned}$$

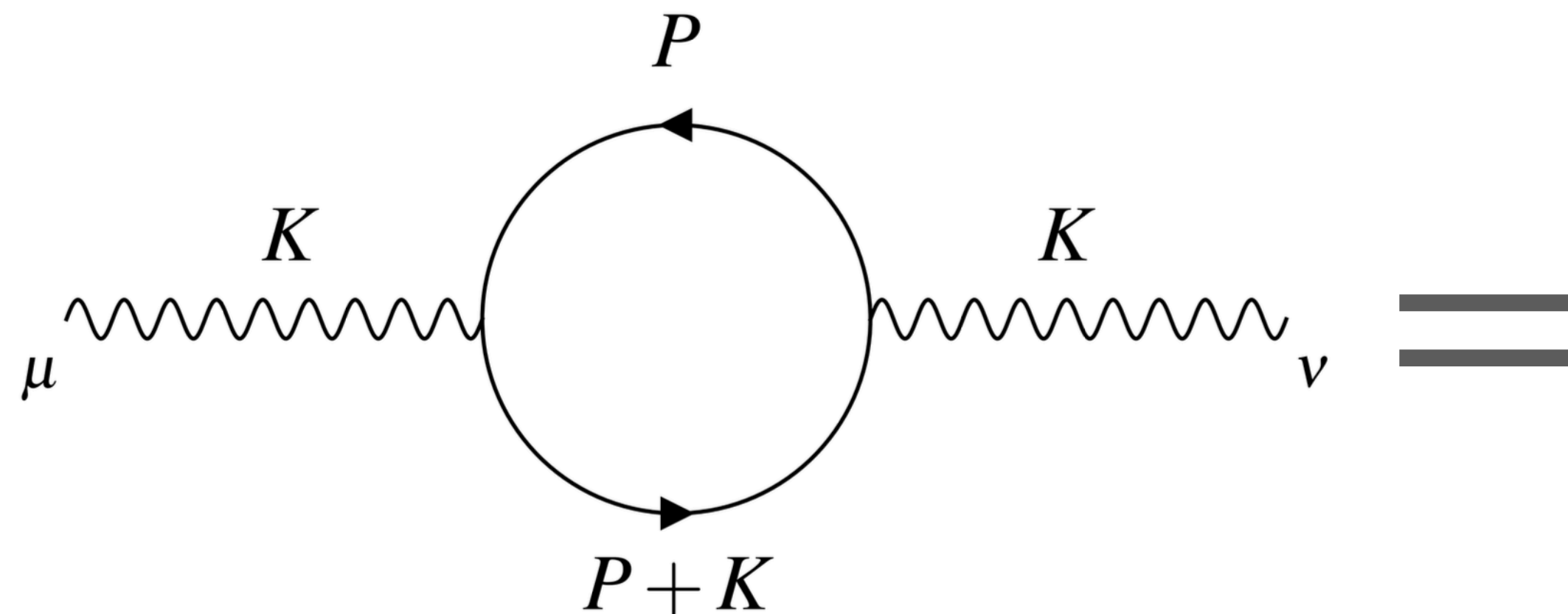
# HOW DO WE COMPUTE POLARIZATION TENSORS?

If system is in thermal equilibrium,  $\rho = e^{-H\beta} = e^{-iH\Delta t} = U(-i\beta, 0)$

finite imaginary time interval, bosons have periodic boundary conditions, so Fourier transforming we get discrete spectrum of imaginary “Matsubara frequencies”



$$\int \frac{d^4 p}{(2\pi)^4} M(p_0) \rightarrow \frac{i}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} M(p_0 = i\omega_n)$$

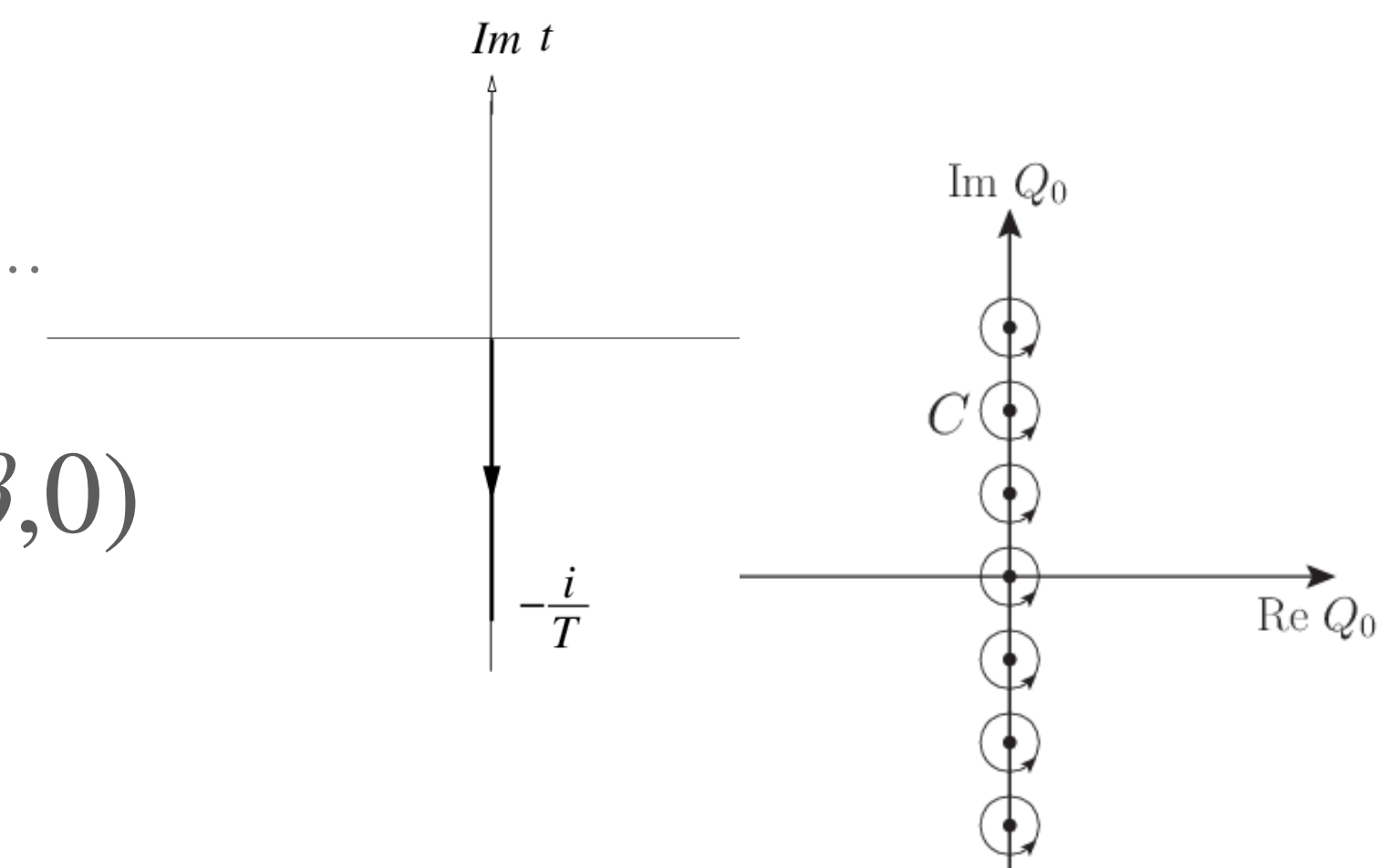


$$= 16\pi\alpha \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} [f(E_p) + \bar{f}(E_p)] \times \frac{(p \cdot k)(k^\mu p^\nu + k^\nu p^\mu) - (k^2)p^\mu p^\nu - (p \cdot k)^2 \eta^{\mu\nu}}{(p \cdot k)^2 - \frac{1}{4}(k^2)^2}$$

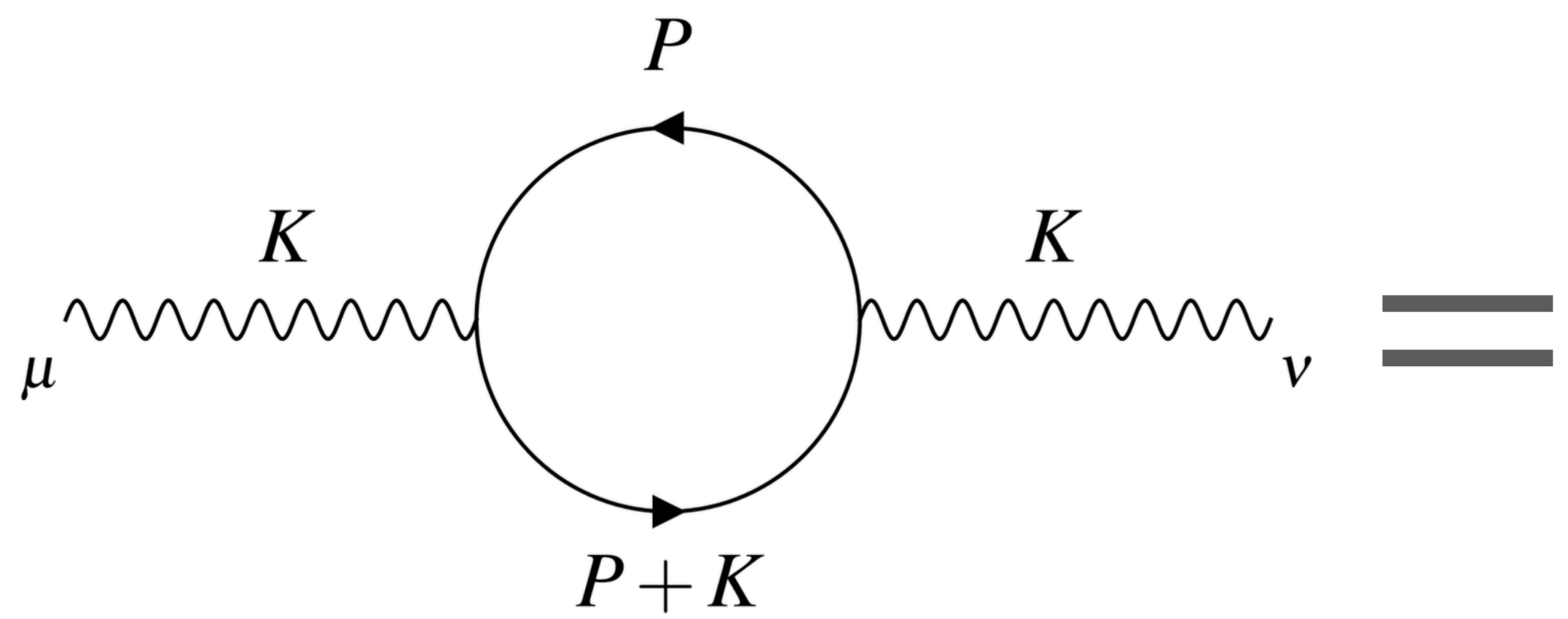
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soft photon approximation used in Braaten & Segel (1993)



# USAGE OF BRAATEN & SEGEL APPROXIMATION

Decompose polarization tensor by projection

$$P_L^{\mu\nu} = \epsilon_L^\mu \epsilon_L^\nu, \quad P_T = \epsilon_{T1}^\mu \epsilon_{T1}^\nu + \epsilon_{T2}^\mu \epsilon_{T2}^\nu$$

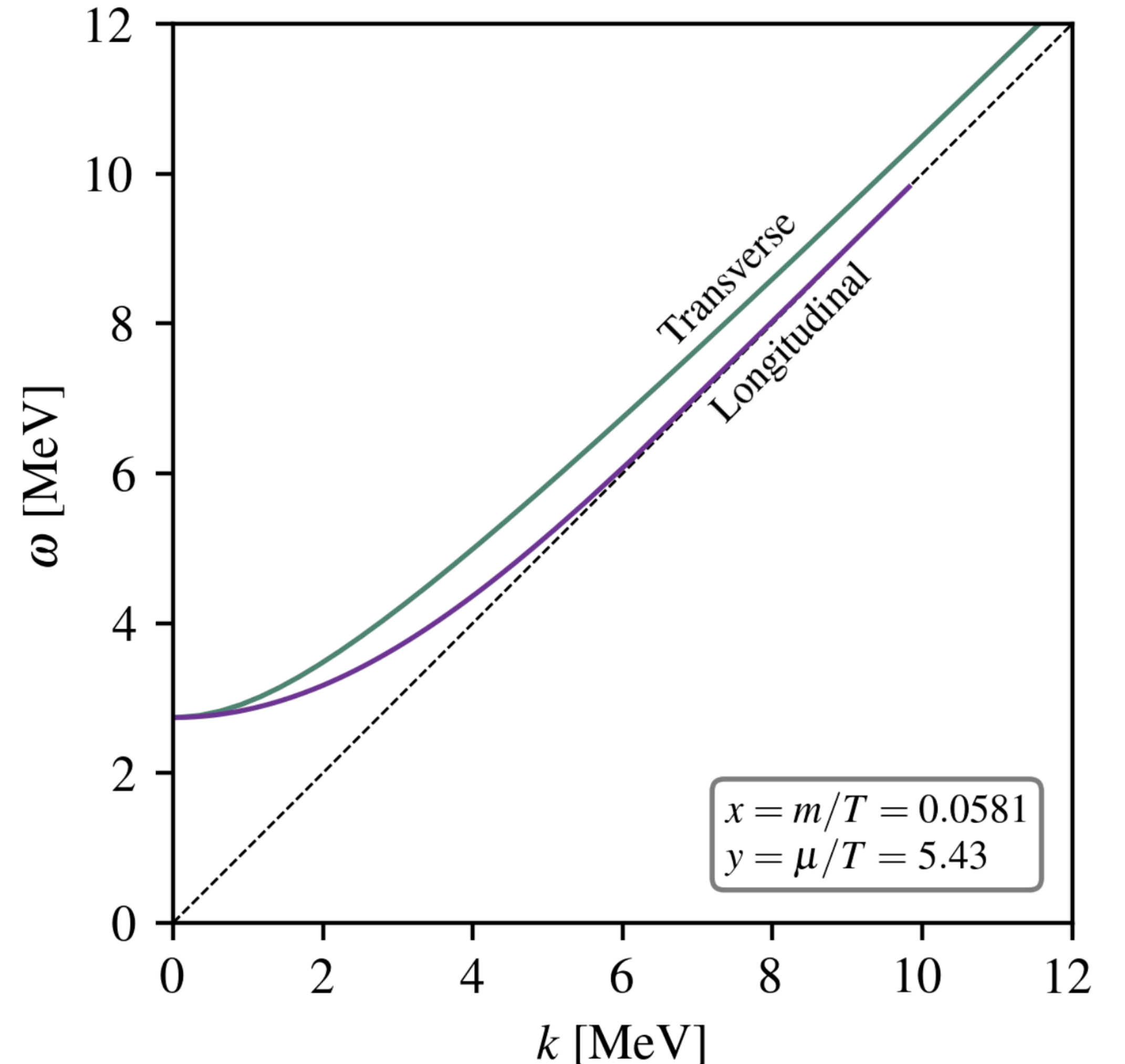
$$\Pi^{\mu\nu} = \Pi_L P_L^{\mu\nu} + \Pi_T P_T^{\mu\nu}$$

Read off dispersion relations from poles in propagator

$$\omega_T^2 = k^2 + \Pi_T, \quad \omega_L = \frac{\omega_L^2}{k^2} \Pi_L$$

$$\Pi_L^{\text{On}} = \frac{3\omega_p^2}{v_*^2} \left( \frac{1-n^2}{n^2} \right) \left[ \frac{1}{2nv_*} \log \left( \frac{1+nv_*}{1-nv_*} \right) - 1 \right]$$

$$\Pi_T^{\text{On}} = \frac{3\omega_p^2}{2v_*^2} \left[ \frac{1}{n^2} - \left( \frac{1-n^2v_*^2}{n^2} \right) \frac{1}{2nv_*} \log \left( \frac{1+nv_*}{1-nv_*} \right) \right]$$



# USAGE OF BRAATEN & SEGEL APPROXIMATION

Hugo Schérer

## Neutrino energy loss from the plasma process at all temperatures and densities

Eric Braaten (Northwestern U.), Daniel Segel (Northwestern U.)

Jan, 1993

38 pages

Published in: *Phys.Rev.D* 48 (1993) 1478-1491

e-Print: [hep-ph/9302213](https://arxiv.org/abs/hep-ph/9302213) [hep-ph]

DOI: [10.1103/PhysRevD.48.1478](https://doi.org/10.1103/PhysRevD.48.1478)

Report number: NUHEP-TH-93-1

View in: [OSTI Information Bridge Server](#), [ADS Abstract Service](#)

[pdf](#) [cite](#) [claim](#)

[reference search](#) [222 citations](#)

### Citations per year

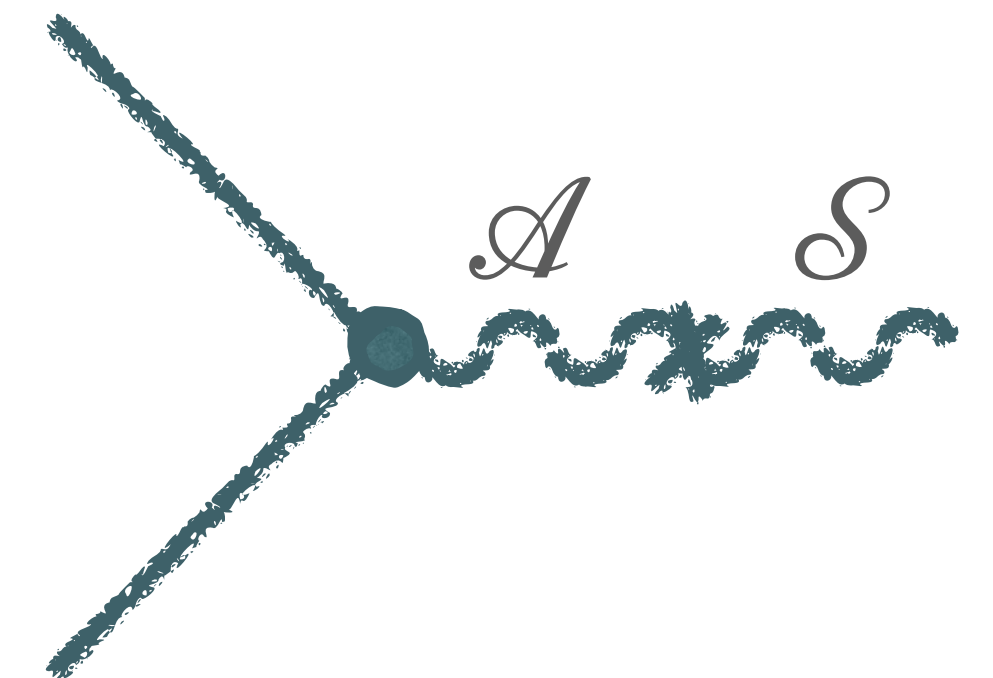


Raffelt's book, "Stars as Laboratories for fundamental physics"

This version (08 July 2023) with some errata fixed  
See also new Appendix E

## 6.3.4 Lowest-Order QED Calculation of the Polarization Tensor

This section was aiming at the dispersion relations of transverse and longitudinal plasmons, following Braaten and Segel (1993) [4] who provided beautiful analytic approximations. The expressions for the polarization tensor that obtain after dropping  $(K^2)^2/4$  in Eq. (6.36) are accurate to lowest order in  $\alpha$  only in the neighborhood of  $\omega \sim k$  and thus are only useful to find the dispersion relations. *They should not be used in the off-shell regime.* After dropping this term, Braaten and Segel arrive at their Eqs. (A16) and (A17), corresponding to Eqs. (6.37)



# RESULT WITHOUT ASSUMING ON-SHELL



$$\begin{aligned} \Pi_L &= \omega_p^2 \left[ -\frac{2K^2}{k^2 v_*^2} + \frac{K^2}{4E_*^2 v_*^3} \log\left(\frac{1+v_*}{1-v_*}\right) + \frac{\omega K^2 (3 + (\omega^2 - 3k^2)/4E_*^2)}{4k^3 v_*^3} \log\left|\frac{(\omega + kv_*)^2 - (K^2)^2/4E_*^2}{(\omega - kv_*)^2 - (K^2)^2/4E_*^2}\right| \right. \\ &\quad \left. - \frac{E_* K^2 (1 + 3K^2/4E_*^2)}{2k^3 v_*^3} \log\left|\frac{\omega^2 - (kv_* - K^2/2E_*)^2}{\omega^2 - (kv_* + K^2/2E_*)^2}\right| - \frac{(1 - v_*^2 + K^2/2E_*^2)}{2v_*^3} \sqrt{\left|\frac{4m^2}{K^2} - 1\right|} C \right] \\ \Pi_T &= \omega_p^2 \left[ \frac{k^2 + 2\omega^2}{2k^2 v_*^2} + \frac{K^2}{4E_*^2 v_*^3} \log\left(\frac{1+v_*}{1-v_*}\right) - \frac{\omega (3(\omega^2 - k^2 v_*^2) + (\omega^2 + 3k^2)K^2/4E_*^2)}{8k^3 v_*^3} \log\left|\frac{(\omega + kv_*)^2 - (K^2)^2/4E_*^2}{(\omega - kv_*)^2 - (K^2)^2/4E_*^2}\right| \right. \\ &\quad \left. + \frac{E_* (3(\omega^2 - v_*^2 k^2) - 2K^2 + 3(\omega^2 + k^2)K^2/4E_*^2)}{4k^3 v_*^3} \log\left|\frac{\omega^2 - (kv_* - K^2/2E_*)^2}{\omega^2 - (kv_* + K^2/2E_*)^2}\right| - \frac{(1 - v_*^2 + K^2/2E_*^2)}{2v_*^3} \sqrt{\left|\frac{4m^2}{K^2} - 1\right|} C \right] \end{aligned}$$

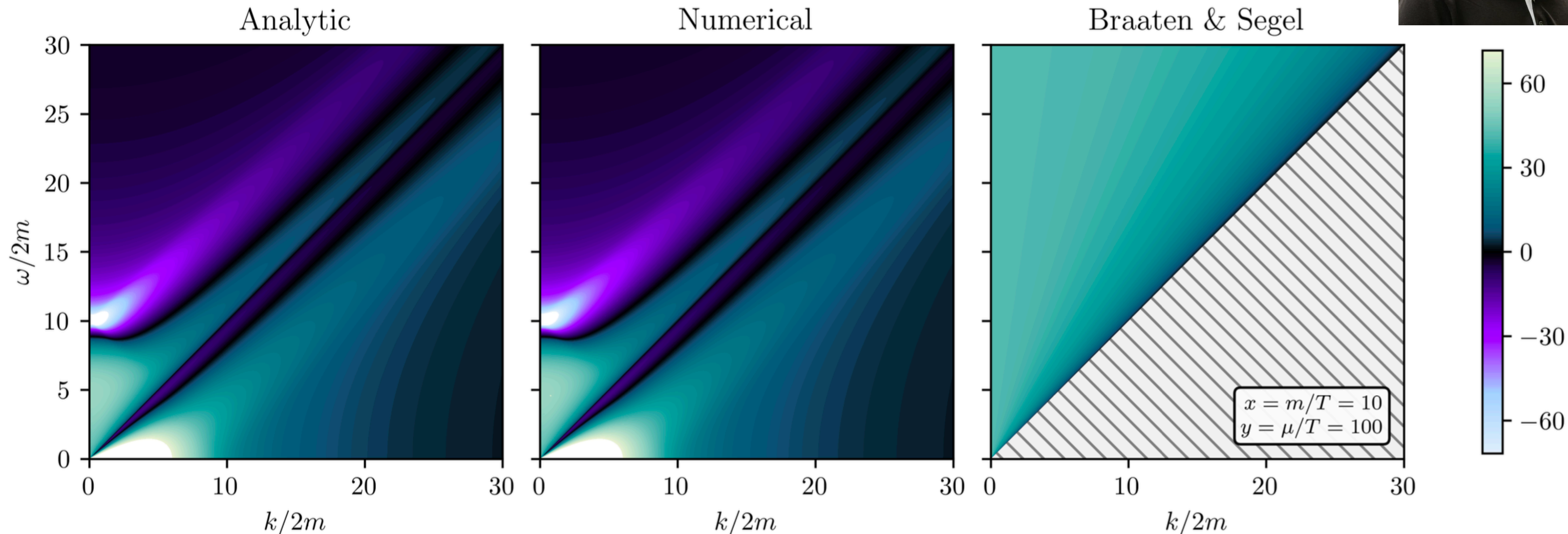
where

$$C = \begin{cases} \tan^{-1} \left( \frac{((K^2)^2/4m^2 + k^2)v_* - \omega k}{((K^2)^2/4m^2) \sqrt{4m^2/K^2 - 1}} \right) + \tan^{-1} \left( \frac{((K^2)^2/4m^2 + k^2)v_* + \omega k}{((K^2)^2/4m^2) \sqrt{4m^2/K^2 - 1}} \right) & n < 1 \text{ and } \xi < 1 \\ \frac{1}{2} \log \left| \frac{\left( v_* ((K^2)^2/4m^2 + k^2) + ((K^2)^2/4m^2) \sqrt{1 - 4m^2/K^2} \right)^2 - \omega^2 k^2}{\left( v_* ((K^2)^2/4m^2 + k^2) - ((K^2)^2/4m^2) \sqrt{1 - 4m^2/K^2} \right)^2 - \omega^2 k^2} \right| & n > 1 \text{ or } \xi > 1 \end{cases}$$

# HOW WELL DOES THE APPROXIMATION DO?

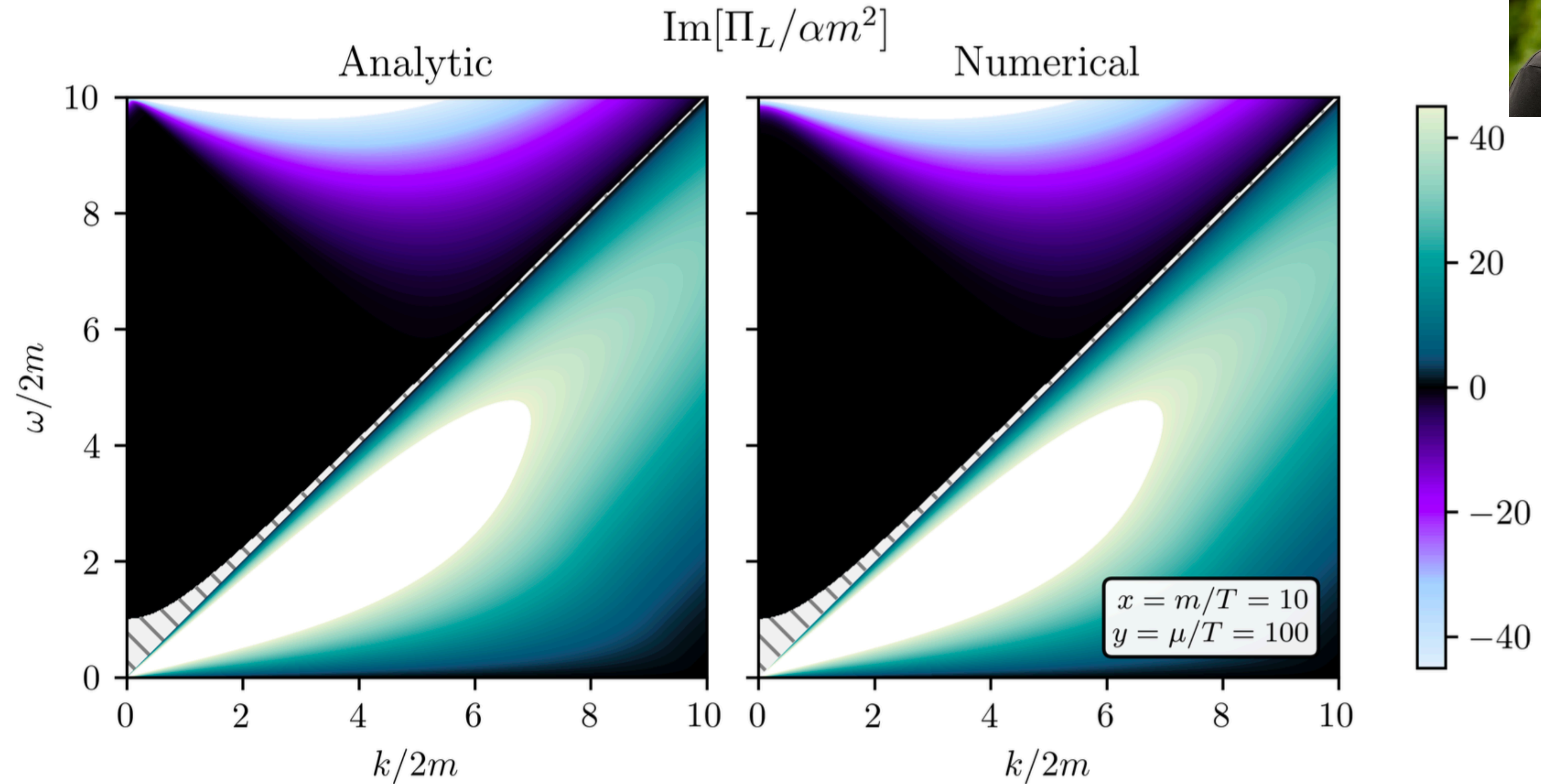


$$\text{Re}[\Pi_L/\alpha m^2]$$

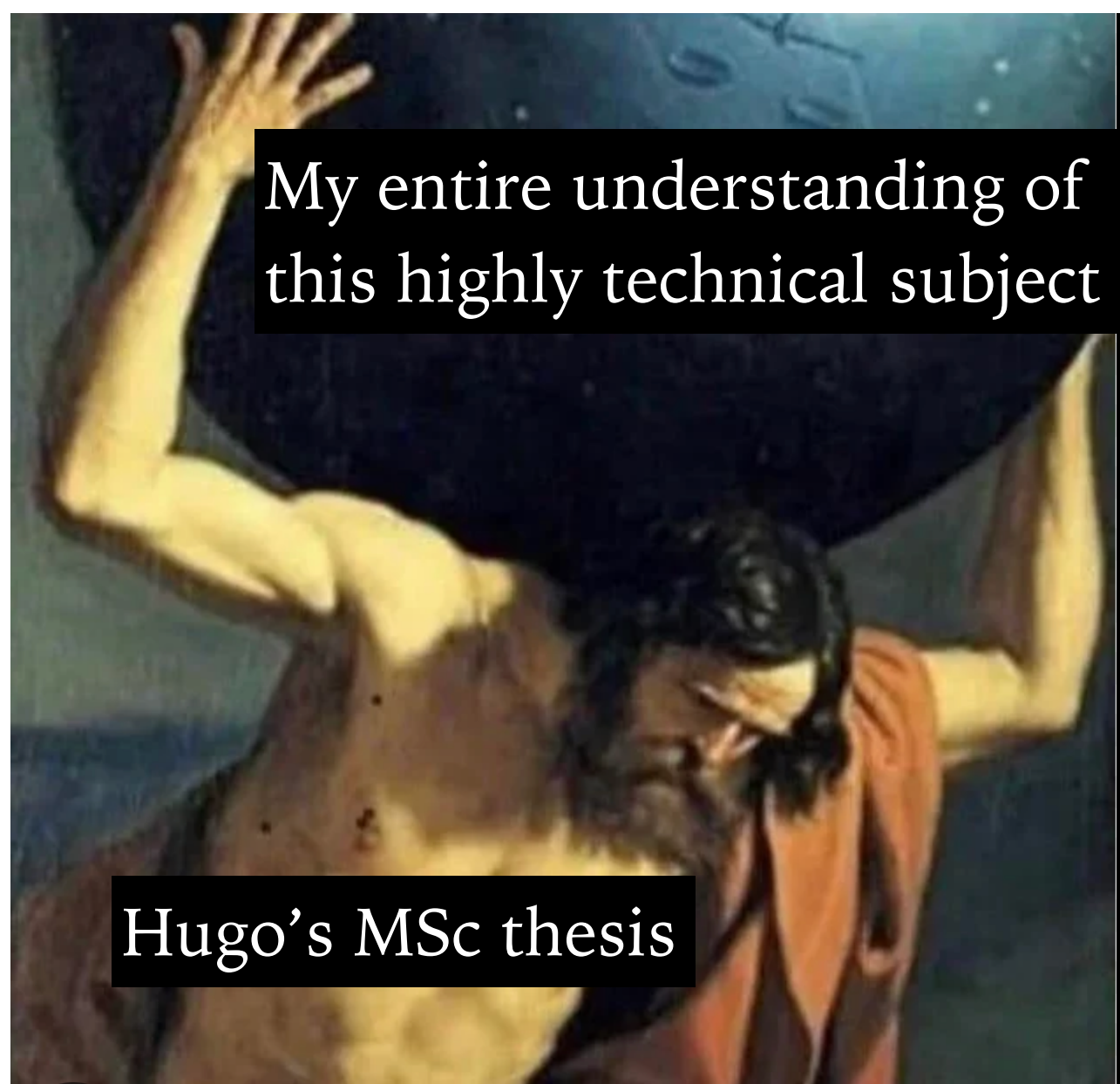


This is secretly the finite-T Lindhard formula for the longitudinal dielectric!!!!

# HOW WELL DOES THE APPROXIMATION DO?



IF YOU WANT A  
PEDAGOGICAL REFERENCE  
TO LEARN MORE IN DETAIL



Finite temperature field theory  
for the masses

Hugo Schérer

Department of Physics  
McGill University, Montréal

April, 2024

**CRUCIAL CAVEAT: ALL OF THIS  
ASSUMES AN ISOTROPIC  
PLASMA!**



Nirmalya Brahma

**MOST ASTROPHYSICAL SYSTEMS  
HAVE MAGNETIC FIELDS— NOT  
ISOTROPIC!**



# HOW DOES ANISOTROPY PLAY A ROLE?

Brahma, KS 2409.monday

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$$\text{EOM} \quad (K^2(g^{\mu\nu} - K^\mu k^\nu / K^2) + \Pi^{\mu\nu})A_\nu = 0$$



Project onto e.g. transverse subspace

$$(\epsilon_\mu^T)^* (K^2(g^{\mu\nu} - K^\mu k^\nu / K^2) - \Pi^{\mu\nu}) \epsilon_\nu^T A_T = (\omega^2 - k^2 - (\epsilon_\mu^T)^* \Pi^{\mu\nu} \epsilon_\nu^T) A_T = 0$$

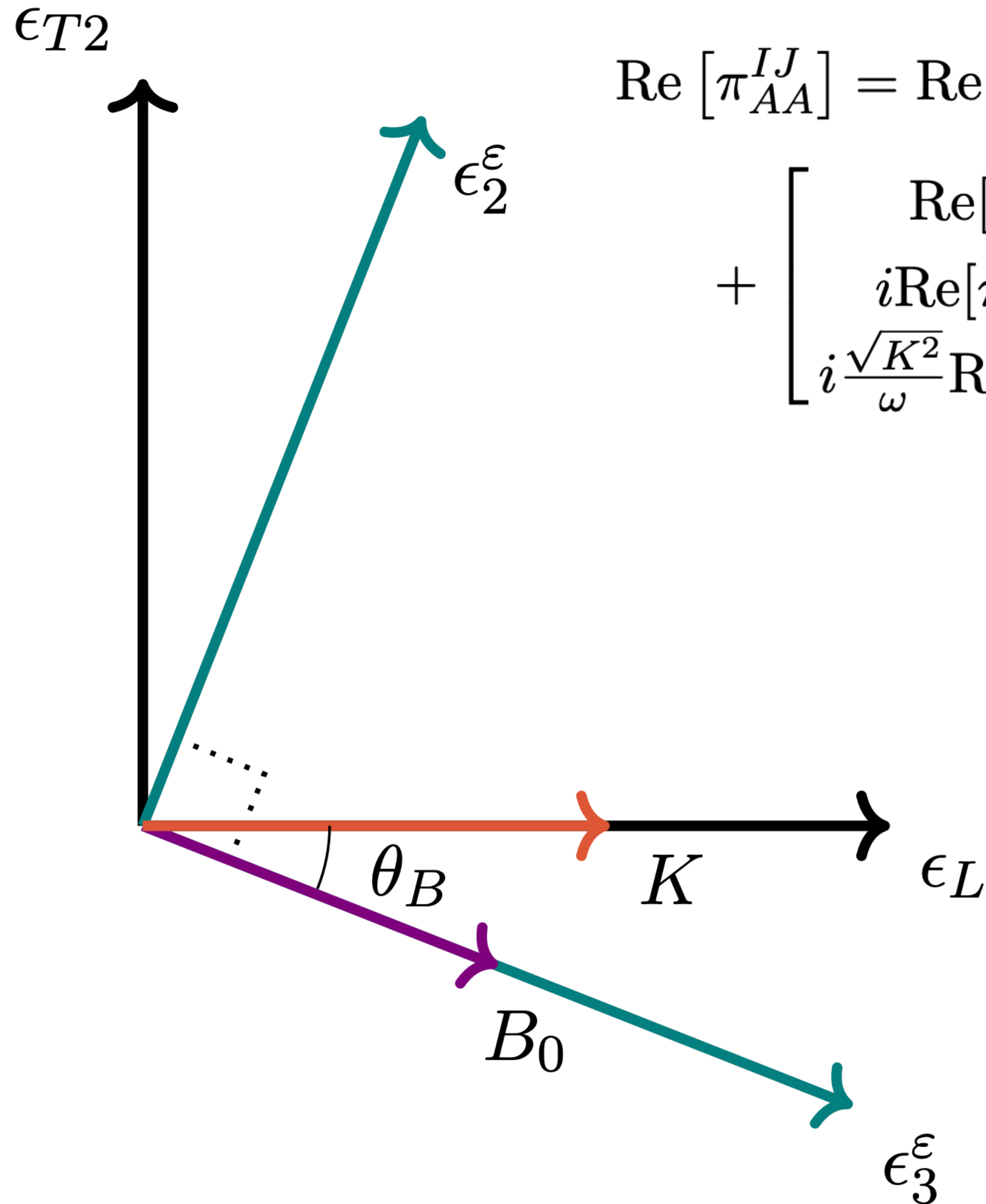
$\Pi_T$  if plasma is isotropic

in general, for modes I, J  $\pi^{IJ} = (\epsilon_\mu^I)^* \Pi^{\mu\nu} \epsilon_\nu^J$  is the mode mixing matrix

in isotropic plasmas,  $\pi^{IJ} = \text{diag}(\Pi_L, \Pi_T, \Pi_T)$  so transverse and longitudinal modes are the normal modes of the system!!

# PLASMA NORMAL MODES

Brahma, KS 2409.monday



$$\text{Re} [\pi_{AA}^{IJ}] = \text{Re} \left[ -(\epsilon^I)^* \cdot \hat{\Pi}^{BV} \cdot \epsilon^J \right]$$

$$+ \begin{bmatrix} \text{Re}[\pi_{\perp}] & -i\text{Re}[\pi_{\times}]c_{\theta} & -i\frac{\sqrt{K^2}}{\omega}\text{Re}[\pi_{\times}]s_{\theta} \\ i\text{Re}[\pi_{\times}]c_{\theta} & \text{Re}[\pi_{\perp}]c_{\theta}^2 + \text{Re}[\pi_{\parallel}]s_{\theta}^2 & \frac{\sqrt{K^2}}{\omega}(\text{Re}[\pi_{\perp}] - \text{Re}[\pi_{\parallel}])c_{\theta}s_{\theta} \\ i\frac{\sqrt{K^2}}{\omega}\text{Re}[\pi_{\times}]s_{\theta} & \frac{\sqrt{K^2}}{\omega}(\text{Re}[\pi_{\perp}] - \text{Re}[\pi_{\parallel}])c_{\theta}s_{\theta} & \frac{K^2}{\omega^2}(\text{Re}[\pi_{\parallel}]c_{\theta}^2 + \text{Re}[\pi_{\perp}]s_{\theta}^2) \end{bmatrix}$$

where  $\text{Re}[\pi_i(\omega, B)] = \frac{e^3 B}{4\pi} \sum_{n=0}^{\infty} \pi_i^{(n)}(\omega, B), \quad i = \perp, \parallel, \times$

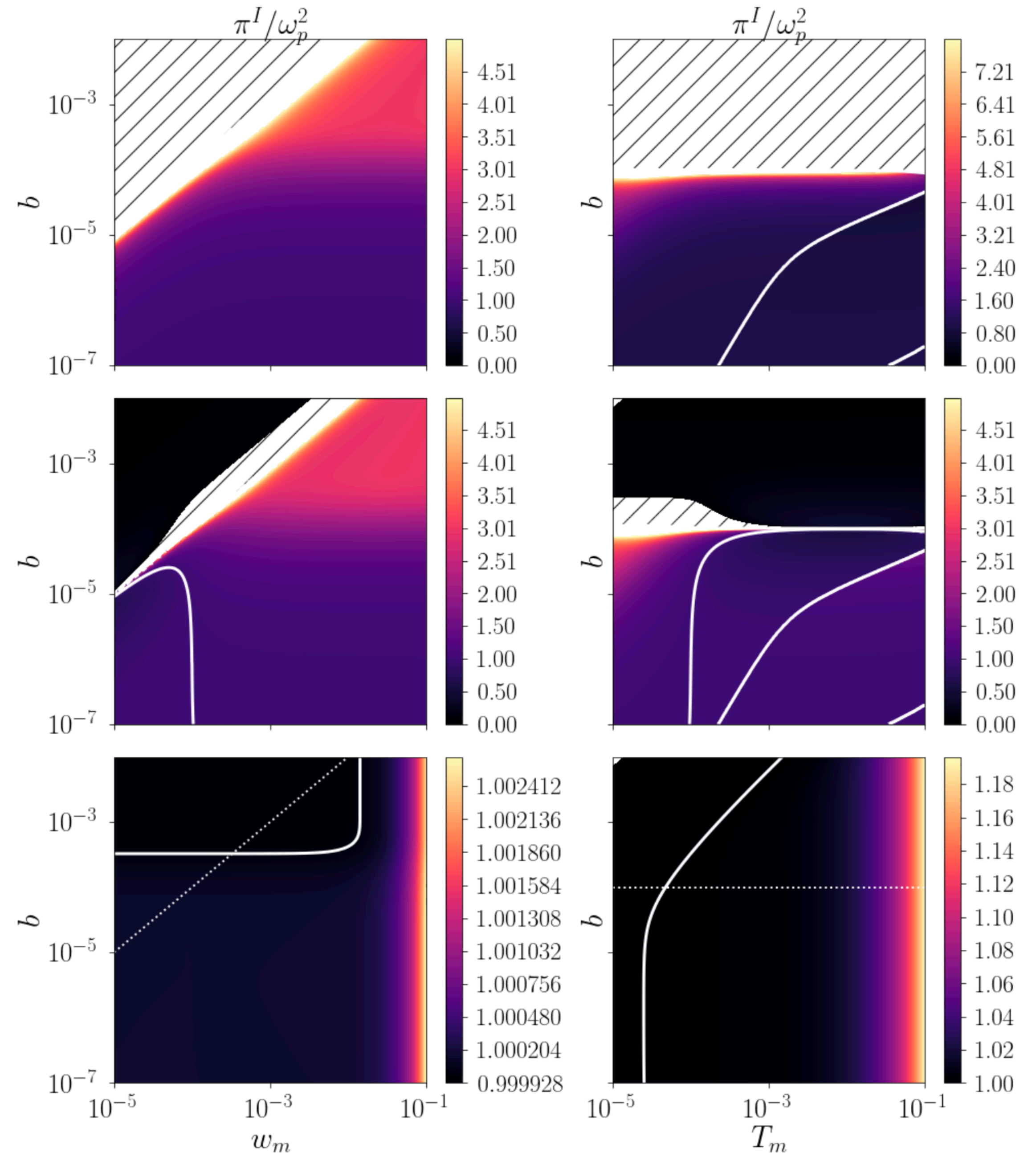
e.g.  $\pi_{\parallel}^{(n)} = \int \frac{dq_{\parallel,b}}{2\pi} \frac{f_e(E_q^n) + f_{\bar{e}}(E_q^n)}{2E_q^n} \frac{(2 - \delta_0^n) 4m_e^2 + (1 - \delta_0^n) 16neB}{(E_q^n)^2 - \frac{\omega^2}{4}}$

energy of nth Landau level  $E_q^n \equiv \sqrt{q_{\parallel,b}^2 + m_e^2 + 2neB}$

# PLASMA NORMAL MODES- CLASSICAL

- “mass” of normal modes is not just simply the plasma frequency!
- In some parts of phase space, the eigenvalue of the mixing matrix is negative — no mixing with BSM particles is possible!
- As photons propagate in astrophysical media, temperatures and plasma frequencies scan a wide range of values, normal modes will rotate — lots of opportunities to hit resonances!

Brahma, KS 2409.monday



# CONCLUSIONS

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- There's a lot of variety in the universe!  
Many temperatures and densities to probe different energy scales
- We've developed some hammers... what are the nails? Mix and match different astro systems, models, observables, etc.
- When you create a group atmosphere where people feel welcomed and supported, you can do hard things (finite-temperature QFT!)
- Thanks for the memories and inspiration!



me at Comal  
45 minutes  
after walking  
across stage  
in 2019



me at Comal  
last night