Exploring QCD-like Dynamics with AMSB

Csaba Csáki with Raffaele d'Agnolo, Rick Gupta, Eric Kuflik, Tuhin Roy, Max Ruhdorfer, Taewook Youn + Andrew Gomes, Hitoshi Murayama, Ofri Telem

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Discrete Anomaly Matching[∗]

Csaba Csáki[†] and Hitoshi Murayama[‡]

Theoretical Physics Group Ernest Orlando Lawrence Berkeley National Laboratory University of California, Berkeley, California 94720

and

Department of Physics University of California, Berkeley, California 94720

csaki@thwk5.lbl.gov, murayama@thsrv.lbl.gov

Abstract

We extend the well-known 't Hooft anomaly matching conditions for continuous global symmetries to discrete groups. We state the matching conditions for all possible anomalies which involve discrete symmetries explicitly in Table 1. There are two types of discrete anomalies. For Type I anomalies, the matching conditions have to be always satisfied regardless of the details of the massive bound state spectrum. The Type II anomalies have to be also matched except if there are fractionally charged massive bound states in the theory. We check discrete anomaly matching in recent solutions of certain $N = 1$ supersymmetric gauge theories, most of which satisfy these constraints. The excluded examples include the chirally symmetric phase of $N = 1$ pure supersymmetric Yang-Mills theories described by the Veneziano–Yankielowicz Lagrangian and certain non-supersymmetric confining theories. The conjectured self-dual theories based on exceptional gauge groups do not satisfy discrete anomaly matching nor mapping of operators, and are viable only if the discrete symmetry in the electric theory appears as an accidental symmetry in the magnetic theory and vice versa.

Instantons in Partially Broken Gauge Groups[∗]

Csaba Csáki[†] and Hitoshi Murayama[‡]

Theoretical Physics Group Ernest Orlando Lawrence Berkeley National Laboratory University of California, Berkeley, California 94720

and

Department of Physics University of California, Berkeley, California 94720

csaki@thwk5.lbl.gov, murayama@lbl.gov

Abstract

We discuss the effects of instantons in partially broken gauge groups on the lowenergy effective gauge theory. Such effects arise when some of the instantons of the original gauge group G are no longer contained in (or can not be gauge rotated into) the unbroken group H . In cases of simple G and H , a good indicator for the existence of such instantons is the "index of embedding." However, in the general case one has to examine $\pi_3(G/H)$ to decide whether there are any instantons in the broken part of the gauge group. We give several examples of supersymmetric theories where such instantons exist and leave their effects on the low-energy effective theory.

``The Footnote paper"

Gauge Theories on an Interval: Unitarity without a Higgs

Csaba Csáki^a, Christophe Grojean^b, Hitoshi Murayama^c, Luigi Pilo^b and John Terning^d

^a Newman Laboratory of Elementary Particle Physics Cornell University, Ithaca, NY 14853, USA

 b Service de Physique Théorique, CEA Saclay, F91191 Gif–sur–Yvette, France

^c Department of Physics, University of California at Berkeley and Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

^d Theory Division T-8, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

csaki@mail.lns.cornell.edu, grojean@spht.saclay.cea.fr, murayama@lbl.gov, pilo@spht.saclay.cea.fr, terning@lanl.gov

Abstract

We consider extra dimensional gauge theories on an interval. We first review the derivation of the consistent boundary conditions (BC's) from the action principle. These BC's include choices that give rise to breaking of the gauge symmetries. The boundary conditions could be chosen to coincide with those commonly applied in orbifold theories, but there are many more possibilities. To investigate the nature of gauge symmetry breaking via BC's we calculate the elastic scattering amplitudes for longitudinal gauge bosons. We find that using a consistent set of BC's the terms in these amplitudes that explicitly grow with energy always cancel without having to introduce any additional scalar degree of freedom, but rather by the exchange of Kaluza–Klein (KK) gauge bosons. This suggests that perhaps the SM Higgs could be completely eliminated in favor of some KK towers of gauge fields. We show that from the low-energy effective theory perspective this seems to be indeed possible. We display an extra dimensional toy model, where BC's introduce a symmetry breaking pattern and mass spectrum that resembles that in the standard model.

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^a Newman Laboratory of Elementary Particle Physics Cornell University, Ithaca, NY 14853, USA b Service de Physique Théorique, CEA Saclay, F91191 Gif-sur-Yvette, France ^c Department of Physics, University of California at Berkeley and Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

^d Theory Division T-8, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

csaki@mail.lns.cornell.edu, grojean@spht.saclay.cea.fr, murayama@lbl.gov, pilo@spht.saclay.cea.fr, terning@lanl.gov

Abstract

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RS rules

Demonstration of Confinement and Chiral Symmetry Breaking in *SO*(*Nc*) Gauge Theories

Csaba Csáki,^{1,*} Andrew Gomes,^{1,†} Hitoshi Murayama,^{2, 3, 4, ‡} and Ofri Telem^{2, 4, §}

¹Department of Physics, LEPP, Cornell University, Ithaca, NY 14853, USA

²Department of Physics, University of California, Berkeley, CA 94720, USA

³Kavli Institute for the Physics and Mathematics of the Universe (WPI), University of Tokyo, Kashiwa 277-8583, Japan

⁴Ernest Orlando Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

We demonstrate that $SO(N_c)$ gauge theories with matter fields in the vector representation confine due to monopole condensation and break the $SU(N_F)$ chiral symmetry to $SO(N_F)$ via the quark bilinear. Our results are obtained by perturbing the $\mathcal{N}=1$ supersymmetric theory with anomalymediated supersymmetry breaking.

INTRODUCTION

Ever since quarks were proposed as fundamental constituents of the proton, neutron, and numerous hadrons by Gell-mann and Ne'eman [1, 2], it has been a mystery why they cannot be observed directly in experiments. At the same time, protons and neutrons bind in atomic nuclei due to the exchange of light pions predicted by Yukawa [3]. The binding of nuclei, and correspondingly the entire world of chemistry, hinges on pions being much lighter than protons, despite the fact that they are made of the same quarks. The first mystery was "explained" by postulating *confinement* of quarks by condensation of magnetic monopoles via the dual Meißner effect proturns out that we should focus on $N_F \leq N_c - 2$ where we can demonstrate monopole condensation.

In this Letter, we sketch the essence of the analysis, while details are presented in a forthcoming companion paper [28], that will also contain a discussion of the cases where $N_F > N_c - 2$.

ANOMALY MEDIATION

Anomaly mediation of supersymmetry breaking (AMSB) is parameterized by a single number *m* that explicitly breaks supersymmetry in two different ways. One is the tree-level contribution based on the superpotential

Outline

- Use of AMSB for studying QCD-like theories
- The η' Potential (and the Axion Mass)
- CP violation at $\theta = \pi$
- Chiral perturbation theory and heavy quark dynamics

The use of AMSB for studying QCD-like theories

- SUSY gives powerful constraints on strong dynamics
- Seiberg (+Intriligator, Hitoshi, …) was able to nail down phase structure of SUSY QCD in 1994 using
	- Holomorphy
	- 't Hooft anomaly matching
	- Instanton calculations
	- Integrating out/Higgsing
- Obtained many different phases depending on F vs N

The phases of SUSY QCD

• N=1 SUSY SU(N) gauge theory with F flavors \blacksquare in the function of \blacksquare in the function of the funct group.
C

• At low energies described in terms of mesons $M_{ij} = Q_i \bar{Q}_j \ \ \ \text{and baryons} \ \ \ B_{ij...k} = Q_i Q_j \ldots Q_k$ chiral symmetry of non-supersymmetric QCD with three flavors, while the *U*(1) **AI low energies described in terms of mesons** (formions $\overline{}$ representation of the gauge group) have opposite charges. There is an additional $M_{ij} = Q_i \bar{Q}_j$ and baryons $B_{ij...k} = Q_i Q_j \ldots Q_k$

The phases of SUSY QCD

F=0 - Pure SYM *V* (*R*) between two static test charges a distance *R* apart.² Up to an additive constant we expect the functional form of the potential will fall into one of the potential will fall into one o
The potential will fall into one of the potential will fall into one of the potential will fall into one of t

- . No matter fields, no continuous flavor symmetry
	- Z_{2N} discrete R-symmetry rotating gauginos *R*
	- **Dynamics: gaugino condensation**

•
$$
W = N\Lambda^3
$$
 $\langle \lambda \lambda \rangle = -32\pi^2 \omega_k \Lambda^3$

• Should be truly confining $V(R) \sim \sigma R$

The phases of SUSY QCD

0<F<N: ADS superpotential field street strength term is the periodicity of the periodicity of the periodicity of the periodicity of the p
So the periodicity of the periodic e↵ective superpotential: *m* = 0 and *m* = 1. The *m* = 1 term is just the tree-level gauge theories. The supermaterial <u>IR behavior of many superpotential</u>

- First obtained by Affleck, Dine, Seiberg 1984 **Example 20 to be proportional to be proportional to be proportional to the gauge contribution of the gauge coupling receives the gauge coupling receives a contribution of the gauge coupling receives a coupling received an** nonperturbative renormalizations. The other term (*m* = 0) is the assembly (*n* = 0) is the A • First obtained by Affleck, Dine, Seiberg 1984
- Dynamics generates a non-perturbative superpotential of Dynamics generates a non-perturbative $/(N-F)$ • Dynamics generates a non-perturbative

$$
W_{\rm ADS} = (N - F) \left(\frac{\Lambda^{3N - F}}{\det M}\right)^{1/(N - F)}
$$

• For F=N-1 actually generated by instanton, calculable **2.2 Consistency of** *Calculable* where *CN,F* is in general renormalization scheme-dependent. remaining two gaugino legs can be converted to two quark legs by the insertion of two more two more verminon mass is \mathcal{L} **Calculable**

- $V(R) \sim \mathrm{constant}$ (at least for F=N-1) $\mathbf{E}^{\mathcal{F}}(\mathbf{D})$ is the colors of flavors by going out in the classical moduli modu $\mathcal{L}(\mu) \approx \text{constant}$ (at ieast for $\mathcal{L}(\mu)$) $V(P)$ a constant (at logat for $F-N$ 1) \bullet $V(R) \sim \text{constant}$ (at least for F=N-1) legs (wavy lines), connected by 2*N* squark VEVs (dashed lines with crosses). *R* From the instanton calculation we find the quark mass is given by \mathcal{L} • $V(R) \sim \mathrm{constant}$
- For F<N-1 gaugino condensation in unbroken group flavor is partially "eaten" by the Higgs mechanism (since there are 2*N* 1 broken Confining : *V* (*R*) ⇠ *R .* The explanation of the second forms is as follows. In a gauge theory is as follows. In a gauge theory is as follows.

The phases of SUSY QCD

The phases of SUSY QCD

• A beautiful picture, BUT very different from what we expect in non-SUSY QCD

- Lattice simulations suggest only 2 phases
	- Chiral symmetry breaking
	- For large number of flavors (perhaps as high as F>3N) conformal phase
- Would like to start making connection between SUSY and non-SUSY theories

Adding SUSY breaking

• A long history of perturbing with SUSY breaking terms, for example

- Aharony, Sonnenschein, Peskin, Yankielowicz '95
- Evans, Hsu, Schwetz '95
- Cheng & Shadmi 1998
- Arkani-Hamed & Rattazzi '98; Luty & Rattazzi '99
- Abel, Buican, Komargodsky '11
- Increasingly more systematic approach
- Lots of interesting results, but no clear pattern of what the actual phase structure is

The use of AMSB

• Proposal of Hitoshi in 2021: use anomaly mediated SUSY breaking for perturbing the Seiberg exact results

- AMSB: originally "designed" to provide a specific implementation for MSSM with predictive soft breaking patterns
- Here we will simply use it only to study phases of gauge theories, not as a BSM model
- Assumption of AMSB: SUSY breaking mediated purely by supergravity, no direct interaction between SUSY breaking sector and matter sector

Randall, Sundrum '98 Giudice, Luty, Murayama, Rattazzi '98 see also Arkani-Hamed, Rattazzi '98

• Assume matter sector sequestered - no direct interactions with SUSY breaking generated

• Only source of SUSY the auxiliary field of supergravity multiplet

AMSB

• Best way to describe effect of AMSB is via the introduction of the Weyl compensator Φ

- This conformal compensator is a spurion for super-Weyl transformations (SUSY rescaling + U(1) rotations) with weight 1 Ω with woight 1 (AMSB) can be formulated with the Weyl compensator know a priori whether the change in *m* is continuous.
- The effects of SUSY will show up through the coupling z
Z Z

$$
\mathcal{L} = \int d^4\theta \Phi^* \Phi K + \int d^2\theta \Phi^3 W + c.c.
$$

• With the spurion $\Phi = 1 + \theta^2 m$ of the theory, and *m* is the parameter of supersymme-

Pomarol, Rattazzi '99

- \cdot If the matter sector is conformal: can scale out Φ by ${\sf rescaling}$ the fields $\qquad \phi_i \; \rightarrow \;$ tter sector is conformal: can scale out Φ by σ ⁻ φ _{*i*} $\Phi^{-1}\phi_i$ of conformal invariance of Φ breaking parameters above depend on wave function ie lields $\varphi_i \to \Psi^- \varphi_i$
- For example if $K = \Phi^* \Phi \phi^+ \phi$ and $\overline{1}$ $\overline{$ α leads the leads to supersymmetry by $\alpha = \alpha + \beta$ $W = \Phi^3 \phi^3$ and the supersymmetry $\frac{1}{2}$ for all $\mathbf{v} = \mathbf{v} \boldsymbol{\varphi}$ $\mathsf{ple\ if}\ \ K=\Phi^*\Phi\phi^+\phi\ \ \mathsf{and}\ \ \ W=\Phi^3\phi^3$ of the theory, and *m* is the parameter of supersymmetry breaking. When the theory is confidentially in the theory
- $\phi_i \rightarrow \Phi^{-1} \phi_i$ rescaling will completely remove from the theory - no SUSY breaking from **s** interactions present at that energy scale. This point can ipletely remove $\boldsymbol \Phi$ very transparent in the DR scheme [17]. $\begin{array}{rcl} \bullet & \phi_i \rightarrow \Phi^{-1} \phi_i \; \textsf{rescaling will completely remove} \; \Phi \end{array}$ from the loops of heavy fields precisely give the necessary jump. removed from the theory by rescaling the fields *ⁱ* ! $\Phi^{-1}\phi_i$ rescaling will completely remove $\boldsymbol{\Phi}$ heory - no SUSY breaking
- SUSY breaking will be tied to violations of conformality! UV insensitive process! One way to intuitively understand the ultraviolet i s the analogy to i $\overline{\mathbf{S}}$ for auxiliary fields, the superpotential leads to the treefor auxiliary fields, the superpotential leads to the treeaking will be tied to violations of an experience terms of the supersymmetry of the supersymmetry of the supersymmetry \mathbf{r}_i

2000 induced AMSB effects
 A Symmetry breaking energy breaking e <u>value-level.</u> However, conformal in value-level. However, conformal in value-level. The conformal invariance in α $AMSB$ effects ²*g*² (*µ*)*m.* (6) stants, and there are loop-level supersymmetry breaking <u>luced AMSB effects</u>

• If scale invariance broken via RGE running: $\frac{1}{2}$ $\overline{2}$ *m*² *ⁱ* (*µ*) = ¹ ˙ *ⁱ*(*µ*)*m*² \bullet If scale invariance broken via RGF running \bullet ken via RGE running: loop-level supersymmetry breaking. $2F$ *runnin* @*ⁱ* Here, *ⁱ* = *µ ^d dµ* ln*Zi*(*µ*), ˙ ⁼ *^µ ^d ^Aijk*(*µ*) = ¹ *dµi*, and (*g*²) = *^µ ^d*

$$
\beta(g^2) = \frac{g^3}{16\pi^2} \left[S(R) - 3C(G) \right] + \dots \longrightarrow m_{\lambda}(\mu) = -\frac{\beta(g^2)}{2g^2}(\mu)m
$$

$$
\frac{d}{dt} Y^{ijk} = Y^{ijp} \left[\frac{1}{16\pi^2} \gamma_p^k + \dots \right] + (k \leftrightarrow i) + (k \leftrightarrow j)
$$

$$
m_i^2(\mu) = -\frac{1}{4} \gamma_i(\mu)m^2
$$

$$
A_{ijk}(\mu) = -\frac{1}{2} (\gamma_i + \gamma_j + \gamma_k)(\mu)m
$$

• For example in SUSY QCD

For example in SUSY QCD
$$
m_{\lambda} = \frac{g^2}{16\pi^2} (3N_c - N_f)m
$$

$$
m_Q^2 = m_{\tilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2} 2C_i (3N_c - N_f) m^2
$$

Loop induced AMSB effects

• Loop induced breaking terms provide positive squark masses and gaugino mass - massless spectrum that of ordinary QCD

• For AMSB version of MSSM slepton masses were problematic - right handed sleptons were tachyonic. Here only AF gauge group - AMSB gives perfect UV boundary condition

A surprise - tree-level AMSB effects removed from the theory by rescaling the fields *ⁱ* !

• If there is a non-scale invariant superpotential: will contribute to AMSB potential leads to the superpotential ¹*i*. On the other hand, violation of conformal invarilevel supersymmetry breaking terms ergy scales and depend only on the particle content and \blacksquare

$$
\mathcal{L}_{\text{tree}} = m \left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W \right) + c.c.
$$

- Vanishes for dim 3 superpotential, but not in general variance at the tree-level. However, conformal invariance at the tree-level. However, conformal invariance at t norol chiral Lagrangian, and I couple it to the same metric.
- Expression for general Kähler potential: is anomalously broken due to the running of coupling con-

C.C., Gomes, Murayama, Telem '21 $\begin{array}{c} \text{Tolom} \text{'} \Omega 1 \end{array}$ $\frac{1}{2}$ to the Weyl competition

$$
V_{\text{tree}} = \partial_i W g^{ij^*} \partial_j^* W^* + m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K \right) + m \left(\partial_i W g^{ij^*} \partial_j^* K - 3W \right) + c.c.
$$

A non-perturbative AMSB potential Manual Structure is a set of the meson of the meson fields *L* = *d*⁴✓⇤2*Nf*⇤

• Example: SU(N) for Nf < N_c. ADS Superpotential $SU(N)$ for N_f < ✓⇤³*NcN^f* ²*N^f*

$$
(N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)}
$$

• Will lead to induced term from z
Z ead to induced term from $\int d^2\theta \Phi^3 W_{ADS}$ \bullet \vee $\ddot{}$ t $\overline{}$ <u>**o** in</u> $\overline{}$ $\overline{}$ ✓⇤³*NcN^f* ²*N^f* $\overline{2}$ \mathbf{z} \overline{a}

 $Q =$

$$
-(3N_c - N_f)m\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{1/(N_c - N_f)} + c.c.
$$

• Along direction

$$
\begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix}
$$

$$
Q = \tilde{Q} = \begin{bmatrix} \vdots & \ddots & \vdots \\ \frac{0 & \cdots & 1}{0 & \cdots & 0} \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \phi, \qquad M = \phi^2.
$$

A non-perturbative AMSB potential l. E \overline{a} atantial which I work out to the first states which I work out to the first states which I work out to the fir
Alternatives which I would be a states which I would be a states which I would be a states which I would be a <u>olonisiai</u>

$$
\bullet \qquad - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}}\right)^{1/(N_c - N_f)} + c.c. \quad \text{term is key}
$$

- **Non-perturbative effect involving SUSY breaking** Note that there is now a well-defined minimum (see all α well-defined minimum (see all α
- AMSB allows us to pin down this term $MCD \neq$ $\overline{}$ $\overline{\phant$ *i*_b $\frac{1}{2}$
- · Formally tree-level but really must be a nonperturbative effect including SUSY breaking **FULLICAL SUPPRESSED IS 2018 IN 1998 IN** broken to *SU*(*N^f*)*^V* . The massless particle spectrum is The other is $\overline{ }$
- Will stabilize ADS superpotential! the corresponding Nambu–Goldstone bosons (pions) [34]. bosons (NGBs) have mass that grows with *m*. Naively
- Will give rise to proper symmetry breaking pattern! increasing material α increasing α on α the only remaining degrees of α increasing degrees of α pectations in QCD with small number of flavors. There *SU*(*N^f*)*^Q* ⇥ *SU*(*N^f*)*Q*˜ .

Phase for QCD* for Nf<Nc

- Symmetry breaking pattern $SU(N_f)_L \times SU(N_f)_R$ - \bullet S $\mathbf{S}U(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$
	- As in QCD, massless DOF's just pions
	- Could be continuously connected to actual QCD for $m>>\Lambda$ Id he septimuously

The phases of SUSY QCD/AMSB QCD

Chiral Lagrangian and n' potential - ``usual" $\overline{\mathbf{c}}$

- Naive assumption U(1)A anomalous, broken by instantons, so instanton effects will give mass to η '? *f*⌘⁰ and *f*⇡ are the ⌘⁰ and ⇡ decay constants. We will assume for simplicity that *f*⌘⁰ = *f*⇡ to ا
ا ا C 4 mption U(1)_A i • Naive assumption U(1)A anomalous, broken by where ⇤ ' 4⇡*f*⇡ is the dynamical scale of the gauge group, ↵ is an *O*(1) number, *m^Q* is the as Tr[*TaTb*]=2*ab*.
- Form of chiral Lagrangian would be • Form of chiral Lagrangian would be as Tr[*TaTb*]=2*ab*.

$$
\mathcal{L} = f_{\pi}^2 \text{Tr} \left[(\partial_{\mu} U)^{\dagger} \partial^{\mu} U \right] + a \Lambda f_{\pi}^2 \text{Tr} \, m_Q U + \text{h.c.}
$$

$$
\mathcal{L}_{\text{inst}} = b \Lambda^2 f_{\pi}^2 e^{-i\theta} \det U + \text{h.c.}
$$

 \cdot In terms of η' • In terms of η' $V_{\eta'} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta')$) *,* (2.10)

<u>Chiral Lagrangian and $\boldsymbol{\mathfrak{q}}'$ potential - ``usual"</u> potential term can only depend on the *U*(1)^A invariant ^and *A*³ **<u>Chiral Lagrangian and n' potential - ``usual"</u>** ³²⇡² Tr*GG ,* ^e (2.2) the *U*(1)*^A* symmetry. Thus it can be used as a building block in the chiral Lagrangian to to a spurion. Under a chiral rotation of the quarks \mathcal{L} <u>cunai Fagrangian and it potential - Insual</u>

- Would correspond to instanton effect because ~ $e^{i\theta}$ *^V*⌘⁰ ⁼ 2*b*⇤2*^f* ² \sim Wauld correspond to instanton offect hesewes \sim oih the case of the extreme integer integer. The condition of the condition σ *^j* ! *^ei*' *^j , ^c* • Would correspond to instanton effect because $\sim e^{i\theta}$
- Would give η' mass $\sim \Lambda$ a function which is explicitly 2^{α} periodic in α periodic in α without branch cuts or singularities. Furthera function which is explicitly 2^{α} periodic in α with branch cuts or singularities. Further-*^U* ! *^e*2*i*↵*ULUU†*
- Consistent with spurion analysis for axial U(1): potential term can only depend on the *U*(1)*^A* invariant combination ✓ *F*⌘⁰ *erit* with spurion analysis for $\frac{1}{2}$ • Consistent with spurion analysis for axial U(1):

$$
\theta \to \theta + 2 F \varphi
$$

$$
\eta' \to \eta' + 2 \varphi
$$

• After integrating out $\eta' = (\theta + 2k\pi)/F$ $V_{\pi} = -\alpha \Lambda f_{\pi}^2 e^{i(\bar{\theta} + 2\pi k)/F} \text{Tr}(m_q e^{i\pi^a T^a}) + \text{h.c.}$ $= -2\alpha\Lambda f_\pi^2\sum^F m_i\cos\left(\frac{\bar{\theta}+2\pi k}{F}-\sum^F t_i^j\pi^j\right).$ $\widetilde{i=1}$ and $\widetilde{j=1}$ and $\widetilde{j=1}$ and is given by $\widetilde{j=1}$ and is given by $\widetilde{j=1}$ and is given by $\widetilde{j=1}$ and \widetilde $= -2\alpha\Lambda f_{\pi}^2\sum m_i\cos\left(\frac{\theta+2\pi k}{\sigma}\right)+\sum t_i^j\pi^j\right]$ $\overline{i=1}$ $\overline{i=1}$ $\overline{j=1}$ $\overline{j=1}$ \overline{I} diagonalized. The resulting potential is \overline{f} is \over $=-2\alpha\Lambda f_\pi^2$ \sum *F i*=1 *mⁱ* cos $\sqrt{2}$ $\left(\frac{\bar{\theta}+2\pi k}{F}+\right.$ *F* X1 *j*=1 $t^j_i\pi^j$ $\left(\bar{\theta} + 2\pi k \right)$ $\left(\bar{B} + 2\pi k \right)$ $\left(\bar{C} + 2\pi k \right)$ \mathcal{A} $U = (U + 2NI)/I$ *ei*⇡*a^T ^a* $^{1/F}$

FR 6 ANION MESS Of the Curis E Kuflik **The axion mass** . The sum of the sum of contact the sum of contact to one contact to o **THE GAIVIT HIGSS**
R. d'Agnolo, R. Gupta, E. Kuflik, T. Roy, combination ✓ + *na F*⌘⁰

✓ *F*⌘⁰ + *na* + 2⇡*k*

• To get θ dependence of potential also integrate out pions: H. all *m*_i and *n*ⁱ and *n*ⁱ and *n*ⁱ and *n*ⁱ and *n*ⁱ and *n*ⁱ cosines W. RUNDOTTER and \sim . **Et a** dependence of potential also integrate out $V_2 = -2\alpha\Lambda f_\pi^2$ $\left\lceil m_u \cos \left(\frac{\bar{\theta}}{2} \right) \right\rceil$ $\frac{0}{2} + k\pi + \pi^0$ $\Big\}+m_d\cos\Big(\frac{\bar{\theta}}{2}\Big)$ $\frac{0}{2} + k\pi - \pi^0$ \bigcap *^V*² ⁼ 2↵⇤*^f* ² ⇡ \mathbf{R} [:] \mathbf{R} + \mathbf{R} + dimensionless μ ^{*a*} *f*_{**a**} which dimensionless μ ^{*a*} $To not a denominator of the interval $0$$ is the axion.

$$
V_{\rm min}=-2|\alpha|\Lambda f_\pi^2\sqrt{m_u^2+m_d^2+2m_u m_d \cos\bar{\theta}}
$$

in which case the inequivalent choices for *k* are *k* = 0*,* 1. with the minimum of the potential

F

• If we also had a physical QCD axion: separate PQ $symmetry$ $a \mapsto a + \varphi, \quad \theta \mapsto \theta - n\varphi$ $a \rightarrow a + \varphi, \theta \rightarrow \theta -$ ³ ⇡⁰ ⁺ ⌘ ⁺*m^s* cos ✓ ¯✓ + 2*k*⇡ ⁺*m^s* cos ✓ ¯✓ + 2*k*⇡ \cdot If we also had a physical OCD axion; separate **Symmetry** $a \to a + \varphi, \; \theta \to \theta - n\varphi$ \cdot If we also had a physical QCD axion: separate PQ

$$
V_a = -2\alpha\Lambda^2 f_\pi^2 \sum_{i=1}^F \frac{m_i}{\Lambda} \cos\left(\frac{\bar{\theta} + an}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)
$$

• To get axion mass: integrate out pions! With axion easy to solve EOM's, quadratic term around minimum [MR: Comment on the branch structure and possibility to eliminate 2*k*⇡ by shifting ⇡⁰ • To get axion mass: integrate out pions! With axion 3 Instanton vs. condensates: large *N* limit and branched potential easy to solve EOM's, quadratic term around minimure massive field will be strongly dominated by \mathcal{L} , and the mass one to a good approximation \mathbf{r} approximation \mathbf{r} where ¯✓ = ✓ + 2⇡*k* + *F*✓*^q* is the physical ¯✓. 1 got amon maco. Integrato cat piono. That and $\frac{1}{\sqrt{2}}$ to the axion potential. Thus at trying to draw instanton diagrams representing $\frac{1}{\sqrt{2}}$ T_{α} and ovien messes integrate of the mel M^{th} evients , to get axion mass, integrate out pions: with axion

$$
V_a = \alpha \Lambda^2 f_\pi^2 \sum_{i=1}^F \frac{m_i}{\Lambda} \left(\frac{an}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j \right)^2
$$

Condensates and large N limit Witten and Veneziano pointed out however that the situation regarding the situation regarding the situation reg <u>condensates and large in thing</u> $\mathbf C$ ondensates and large N limit **Condensates and large N limit** type of instanton-inspired term, Eq. (2.9), that we have used in the previous Section does not

• Issue: large N limit anomaly vanishes is by considering the large *N* limit of the theory, keeping the 't Hooft coupling *g*2*N* = fixed. Tobas. Targo IV in the anomaly vanishes as Tr[*TaTb*]=2*ab*. • Issue: large N limit anomaly vanishes

$$
\partial_{\mu} j_A^{\mu} \sim F \frac{g^2}{16\pi^2} {\rm Tr} G \tilde{G} \sim \frac{\lambda}{16\pi^2} \frac{F}{N} {\rm Tr} G \tilde{G} \rightarrow 0
$$

- η' mass should vanish in this limit h_n meass also uld vanish in this limit. **in** thas should valush in this limit plicit is the space restored in the promote \mathbf{r}_i to a spurion. Eq. (2.9) corresponds to a spurion to a spurion to a spurion. Eq. (2.9) corresponds to a spurion to a spurion to a spurion of \mathbf{r}_i • η´ mass should vanish in this limit From Eq. (3.4) it is approved that \mathbf{r} and \mathbf{r} is a problem that \mathbf{r}
- But from $V_{\eta'} = -2b\Lambda^2 f_\pi^2 \cos(\theta F\eta')$ does not vanish for large N **EURNIC SHOULD V** $V_{\eta\prime} = -20\Lambda$ J_{π} $\cos(\theta - F\eta)$ and the instanton in the image this may not be correct due to infrared divergences and the growth of the growth of the growth of the number of z $V_{\eta\prime} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta')$ does not vanish for a function which is explicitly $2^{\frac{1}{2}}$ periodic in $\frac{1}{2^{\frac{1}{2}}}$ periodic in $\frac{1}{2^{\frac{1}{2}}}$ • But from $V_{\eta'} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta')$ *n* does not vanish for or large limit. This further justifies the ansatz for the ansatz for the vacuum energy in Eq. (3.4). T **• But from** $V_{\eta\prime} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta')$ **does flot vanish for large** (without fermions), the vacuum energy is proportional to *N*² —scaling with the number of
- Witten: η' needs to cancel θ dependence of pure QCD vacuum energy Another convincing argument of Witten is to consider the e $\mathbf 1$ vacuum energy $E(\theta) = N^2 f(\theta/N)$ is $m_{\omega}^2 \propto \frac{1}{2\pi}$ away by a chiral rotation on the massless fermion. However the vacuum energy of \mathcal{N} To analyze the value of the theory (and the axion mass) of the theory (and the axion mass) to its vev determined from Eq. (2.10), ⌘0 = (✓ + 2*k*⇡)*/F*, where *k* is an arbitrary integer. We $m_{\eta'}^2 \propto$ 1 *N* Ω Lagrangian we have the concel adependence of pure Ω $\frac{1}{2}$ mass. Instead of the property $\frac{1}{2}$ the form showled rather be of the form showledge of $E(\theta) = N^2 f(\theta/N)$ (3.4) $m_{\eta'}^2 \propto \frac{1}{N^2}$

• Form of potential π ^{*F*}(*x*) π ²(*x*) (3.2) (3.2) $\mathbf{r} = \begin{bmatrix} \mathbf{r} & \mathbf{r} & \mathbf{r} & \mathbf{r} \\ \mathbf{r} & \mathbf{r} & \mathbf{r} & \mathbf{r} \end{bmatrix}$ • Form of potential $\mathcal{L}_{\eta'} = N \Lambda^2 f_\pi^2 (e^{-i\theta} \mathrm{det} \, U)^{1/N} + h.c.$

The Chiral Lagrangian and n' potential <u>v († 789 ka</u>ponso_v
Jean-Amirie III
Jean-Amirie III **grangian and it potential
W is it 2s periodic in 42**

- Non-analytic how is it 2π periodic in θ ?
- Need to have several branches, potential of the form Lagrangian responsible for the form the form of the fo

$$
V(\theta, \eta') = \text{Min}_k - 2N\Lambda^2 f_\pi^2 \cos(\frac{\theta - F\eta' + 2\pi k}{N}), \quad k = 0, \dots, N - 1
$$

The Chiral Lagrangian and n' potential the Okinal Leonarchan and Justantial so as to cancel the \mathbf{r} <u>heavy axion.</u>

• η' acts like a (heavy) axion and relaxes to minimum of potential to cancel θ dependence (and wash out branch structure) of potential to cancel θ der $\langle \eta' \rangle =$ $\theta + 2\pi k$ $\frac{2\pi}{F}$

- Check this picture in AMSB QCD 1. Chack this picture in AMSR OCD **2. Expanding the Expanding order will give example the contract of the quartic service expanding order, which contract the quartic service of the quartic service order order order order, which contract the quartic service**
- Similar work & results by Dine, Draper, Stephenson-Haskins & Xu (2016). They were using soft squark and gluino masses - can reliably do only for F<N be suppressed by *N*⁴ as expected.

' potential for F<N in AMSB QCD *U*(*F*) ⇥ *U*(*F*) flavor symmetry, just like the quark masses in regular QCD. **n' potential for F<N in AMSB QCD**
B d'Agnele B Cunte E Kuflik T Bev

F F 2 F \cdot **F** \cdot **F** \cdot d'Agnolo, R. Gupta, E. Kuflik, T. Roy, and determine the scalar potential for R , d^2 2 ◆(*NF*)*/*(2*N*) M. Ruhdorfer and C.C.*O*(*m*) and *O*(*O*).

• Consider first F<N - with quark mass $\frac{1}{2}$ *d* nsider fir \mathbf{r} J. 2*F* \mathbf{r} ✓⇤3*N^F N - with quark m* ◆1*/*(*NF*) i
L $\overline{\mathbf{a}}$ ľ $=$ $\frac{1}{2}$ *N* - with

$$
W = (N - F) \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} + \text{Tr}(M_QM)
$$

• The meson VEV as usual where the first term is the first term is the non-perturbative ADS superpotential α τ be we seem λ/Γ λ seems after putting the derivatives after putting the λ/Γ if included in the second was the second reference a typo in Eq. (2) or the second reference of the second reference

$$
\phi=\Lambda\left(\frac{N+F}{3N-F}\frac{\Lambda}{m}\right)^{(N-F)/(2N)}+{\cal O}(m_Q/m)
$$

• The meson matrix: ŕ r Ļ \overline{P} ľ ✓⇤3*N^F* 2*^F* • The meson matrix: **Looking associated with the spontaneous breaking at anomalous symmetry of the spontaneous symmetry** \overline{L}

$$
Q_f^a=|\phi|\delta_f^a\,,\quad \bar Q_f^a=Q_{f'}^aU_{f'f}\,,\quad M=|\phi|^2U
$$

• η' part of U matrix, need to make sure we keep the whole phase everywhere **the result in the result in the result in the parenthesis.** ✓ *N* + *F* 3*N F* ⇤ *m* ◆(*NF*)*/*(2*N*) + *O*(*mQ/m*)*.* (6.3) [RTD: I see that you're using *gij* = 2*Fij* , but the Kaler to me looks canonical. Looking at the Goldstone bosons (GBs) in a unitary matrix *in a unitary matrix in a una be identified* with the interest of the interest the pseudo-GB associated with the spontaneous breaking of the anomalous *U*(1)*^A* symmetry whole phase everywhere *^f* ⁼ *[|]|^a ^f , ^Q*¯*^a* The scalar potential for *U* can be obtained from the potential for *Q* and *Q*¯ after the \mathbb{F} part of \mathbb{F}

FILL AMSB QCD WHERE WE USE THE PASS OF THE PHASE OF THE PHASE OF THE PHASE OF CAN BE ABSORBED IN THE DEFINITION OF THE USE OF THE US *^f* ⁼ *[|]|^a ^f*0*Uf*0*^f , M* = *||*

• Chiral Lagrangian: The scalar potential for *U* can be obtained from the potential for *Q* and *Q*¯ after the

$$
V = -m \left[(3N - F) \left(\frac{\Lambda^{3N-F}}{|\phi|^{2F}} \right)^{1/(N-F)} \det(U)^{-1/(N-F)} + |\phi|^{2} \text{Tr}(m_Q U) \right] + c.c.
$$

- 2 $\left(\frac{\Lambda^{3N-F}}{|\phi|^{2F}} \right)^{1/(N-F)} \det(U)^{-1/(N-F)} \text{Tr}(m_Q^{\dagger} U^{\dagger}) + c.c.$

• Has the branch structure like Witten predicted, but 1/ $(N-F)$ power. η' potential: *U* ∴ is the station of station of the scalar production, but it *<i>A*¹ **b** $\frac{1}{2}$ **n**₁*n***₁** *n***₁^{***n***} ***n***₁^{***n***}** *n***₁^{***n***} ***n*_{*n***¹ ***n*_{*n***¹ ***n*_{*n*} *n*}} to the contract of the contrac

$$
V = -2(3N - F)\left(\frac{N + F}{3N - F}\right)^{-F/N} \left(\frac{m}{|\Lambda|}\right)^{F/N} m|\Lambda|^3 \cos\left(\frac{F}{N - F}\frac{\eta'}{f_{\eta'}} - \frac{\theta + 2\pi k}{N - F}\right)
$$

- 2F\left(\frac{N + F}{3N - F}\right)^{1 - F/N} \left(\frac{m}{|\Lambda|}\right)^{F/N} |m_Q||\Lambda|^3 \cos\left(\frac{\eta'}{f_{\eta'}} + \theta_Q\right)
- 4F\left(\frac{N + F}{3N - F}\right)^{-F/N} \left(\frac{m}{|\Lambda|}\right)^{F/N} |m_Q||\Lambda|^3 \cos\left(\frac{N}{N - F}\frac{\eta'}{f_{\eta'}} + \theta_Q - \frac{\theta + 2\pi k}{N - F}\right),

FERM IN AMSB QCD simplest case is pure SYM theory with *F* = 0. In this scenario the contribution to the scalar \blacksquare It is instructive to consider a few special cases for the scalar potential in Eq. (7.6). The scalar potential simplest case is pure SYM the *F* ϵ *N* in this school to the scalar contribution to th

• **Pure QCD:**
$$
V_k \xrightarrow{F=0} -6N^2m|\Lambda_{\text{phys}}|^3 \cos\left(\frac{\theta + 2\pi k}{N}\right)
$$

- Just like Witten predicted (also Dine et al) • Just like Witten predicted (also Dine et al)
	- For small number of flavors: *N*2*f*(✓*/N*) with *N* branches. a small number of flavors; i.e. taking the potential simplifies \mathcal{L} is possible potential simplifies \mathcal{L}

$$
V_k \stackrel{N \gg F}{\rightarrow} -6N^2m|\Lambda_{\rm phys}|^3 \cos\left(\frac{F}{N}\eta' - \frac{\theta + 2\pi k}{N}\right) - \frac{14}{3}N|\Lambda_{\rm phys}|^3 \sum_{i=1}^F m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)
$$

• Again just as Witten predicted, and as in Dine et al. also straightforward to see that for *m^Q* = 0, i.e. when the axial symmetry at the classical part of the group, whereas quark contributions are suppressed by one power of *N*. It is •Again just as Witten predicted, and as in Dine et al.

$$
m_{\eta'}^2 \propto F^2 m |\Lambda_{\rm phys}|^3/f_\pi^2 \sim 1/N
$$

F<N in AMSB QCD

- F~N both large the situation is very different! structure due to the non-analyticity induced by gaugino condensation. In condensation. In condensation. In con \bullet **The number of anumerical property is not property.**
- For example F=N-1 and both large **• For example F=N-1 and both large** the limit *N* 1, with *F* = *N* 1 fixed, the potential takes the form *For example F=N-1 and bot* <u>0</u> • For example F=N-1 and both large

$$
V_k \stackrel{N = F + 1 \gg 1}{\rightarrow} - 4N^{3/2}m^2 |\Lambda_{\text{phys}}|^2 \cos ((N - 1)\eta' - \theta) - 2N^{1/2}m|\Lambda_{\text{phys}}|^2 \sum_{i=1}^F m_i \cos \left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)
$$

$$
- 4N^{1/2}m|\Lambda_{\text{phys}}|^2 \sum_{i=1}^F m_i \cos \left(N\eta' + \theta_Q - \theta + \sum_{j=1}^{F-1} t_i^j \pi^j\right)
$$

• No branches, η' mass does not go to zero @*N*⌘ The magnitude of this potential is set by *m*2⇤2, and is not vanishing in the large *N* limit. The magnitude of this potential is set by *m*2⇤2, and is not vanishing in the large *N* limit. \cdot No branches, η mass does hot go to zero • No branches **n**' mass does not go to zero

$$
m_{\eta'}^2 = \frac{(x-3)^2 x}{(x+1)(x-1)^2} m^2, \text{ with } x = \frac{F}{N}
$$

• Large F,N qualitatively different from large N, fixed F limits! limit where *N F* = *p* is held fixed (rather than the *x* = *F/N* ratio). becomes strongly coupled in the large *N* limit, just like the more general Eq. (8.6) for the limit where *N F* = *p* is held fixed (rather than the *x* = *F/N* ratio). becomes strongly coupled in the large *N* limit, just like the more general Eq. (8.6) for the • Large F,N qualitatively different from large N, fixed F 8.1 Vacuum structure and phase transition and phase limited

F<N in AMSB QCD

- For N-F>1 NOT an instanton effect
- We know it is actually gaugino condensation
- For F=N-1 it actually IS an instanton effect, and no branches in QCD
- \cdot In that case the η' mass does not vanish for large N
- But also anomaly does not vanish, since both F,N $\rightarrow \infty$
- Which one is QCD? Does QCD with F=N have branches or not?

F=N in AMSB QCD Note that we chose to implement the constraint on det(*M*) and *BB*¯ such that *X* does not

• The F=N,N+1 special cases • The F=N,N+T special cases *X†X* $\overline{}$ *B*¯*†B*¯

• Only consider mesonic VEV, assume other branches OK OK implemented in the superpotential with the superpotential with the help of a Lagrange multiplier multiplier multiplier CN ⇤2*^N* ¹ $\overline{\mathbf{a}}$ *m*_m and the *mail* of α phy consider mesonic V EV assume other brang f uly turisiuci thesume v∟v, assume unici biahl *M, X, B, ^B*¯ ⌧ ⇤, which will turn out not to be the case. A more solid approach is to start

•
$$
\mathbf{F} = \mathbf{N}
$$
 $W = X \left(\frac{\det(M) - \bar{B}B}{\Lambda^{2N}} - 1 \right) + m_Q \text{Tr}(M)$

$$
K = \frac{\text{Tr}(M^{\dagger}M)}{\alpha |\Lambda|^2} + \frac{X^{\dagger}X}{\beta |\Lambda|^4} + \frac{\bar{B}^{\dagger} \bar{B}}{\gamma |\Lambda|^{2N-2}} + \frac{B^{\dagger}B}{\delta |\Lambda|^{2N-2}}
$$

- Resulting η' potential 2 ($|\Lambda|^{2}$ - $\sqrt{(M-9)^2}$ ↵*|*⇤*|* n^2) cos (*X†X |*⇤*|* θ) – : $\sum_{k=2}^N$ *|*⇤*|* $\sum_{i=1}^{n} m_i \cos$ \overline{f} \overline{f} *|*⇤*|* $\left(2-1\right)\eta'-\theta_{Q}-\theta-\sum_{j=1}t_{i}^{j}\pi^{j}\left(\theta_{j}-\theta_{j}\right)$ $-\frac{4m|\Lambda|^2}{\lambda-1}m_i\cos\left(\eta'+\theta_Q+\sum_{i=1}t_i^2\pi^j\right).$ following only the $\frac{1}{2}$ only the $\frac{1}{2}$ is only in the K¨ahler potential is only justified if $\frac{1}{2}$ where **Repulting n'** potential following potential N and $N-1$ $V=-2|\Lambda|^2(|\Lambda|^2+(N-2)m^2)\cos\left(N\eta'-\theta\right)-2m|\Lambda|^2\sum_{i=1}m_i\cos\left((N-1)\eta'-\theta_Q-\theta-\sum_{i=1}t_i^j\pi^j\right)$ start from *F* = *N* + 1 and then give one flavor a heavy mass *µ* with ⇤ *µ m m^Q* $a_1 = 4m|\Lambda|^2 \sum m_i \cos \left(\eta' + \theta_Q + \sum t_i^j \pi^j\right)$. $i=1$ and $j=1$ *N i*=1 *mⁱ* cos $\sqrt{ }$ $\int (N-1)\eta' - \theta_Q - \theta -$ *N* X1 *j*=1 $t_i^j \pi^j$ \setminus A $-4m|\Lambda|^2\sum$ *N i*=1 *mⁱ* cos $\sqrt{ }$ $\eta' + \theta_Q +$ *N* X1 *j*=1 $t_i^j \pi^j$ \setminus \vert \vert \vert
- No branches looks like an instanton effect! **• No branches - looks like an instanton effect!** ²*ij , X* ⁼ *m|*⇤*[|]* **2** *a***_n** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **c** $\frac{1}{2}$ **c** $\frac{1}{2}$ **c** $\frac{1}{2}$ **c**

n' potential for F≥N with our simplified and *m* an

- The F=N,N+1 special cases *Mf f*⁰ = *|*⇤*|* ²*f f*⁰ *, X* ⁼ *m|*⇤*[|]* ² *, B* = *B*¯ = 0 *.* (8.4)
- No branches looks like an instanton effect! E.g. F=N $N-1$

$$
V = -2|\Lambda|^2(|\Lambda|^2 + (N-2)m^2)\cos(N\eta' - \theta) - 2m|\Lambda|^2 \sum_{i=1}^N m_i \cos\left((N-1)\eta' - \theta_Q - \theta - \sum_{j=1}^{N-1} t_i^j \pi^j\right) - 4m|\Lambda|^2 \sum_{i=1}^N m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{N-1} t_i^j \pi^j\right).
$$

- F>N+1 Seiberg duality. Dual quarks will be massive, get again gaugino condensation **instant of the evening of the first terms in the first terms is the first terms in the first terms**
- We will get very similar results as for F<N, with $N-F \leftrightarrow F-N$ $V_k = -4(3N-2F)\left(\frac{2F-3N}{N}\right)$ **WITH STILL MUCH IS STILL MULLER IN A 1999 WITH THE NEUTRAL FOR THE NEUTRAL FORM** $-2F\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-1)}$ *N m* $|\Lambda|$ \sum *F/*(2*N-F*) $m|\Lambda|^3 \cos\left(\frac{F}{F-N}\eta' - \frac{\theta+2\pi k}{F-N}\right)$ ◆ *N m* $|\Lambda|$ $\bigwedge^{N/(2N-F)}$ $|\Lambda|^3 \sum^F$ *i*=1 *mⁱ* cos $\sqrt{ }$ $\overline{1}$ $\frac{N}{F-N}\eta' - \theta_Q - \frac{\theta+2\pi k}{F-N}$ *F*⁻¹ *j*=1 $t_i^j\pi_j$ 1 A $-\frac{4FN}{2F-3N}$ $\sqrt{\frac{2F-3N}{2}}$ *N m* $|\Lambda|$ $\bigwedge N/(2N-F)$ $|\Lambda|^3 \sum^F$ *i*=1 *mⁱ* cos $\sqrt{ }$ $\eta' + \theta_Q +$ *F*⁻¹ *j*=1 $t_i^j\pi_j$ 1 \vert ,

Spontaneous CP breaking

M. Ruhdorfer, T. Youn and C.C.

- Tr $G\tilde{G}$ odd under CP a θ term generically breaks CP explicitly (strong CP problem)
- However for $\theta = \pi$ the effect of CP is $\theta = \pi \leftrightarrow \theta = -\pi$
- But these two are related by 2π shift in θ , so CP is not explicitly broken here. But is it spontaneously broken?
- Claim: in ordinary QCD it may.
- Results of Gaiotto, Kapustin, Komargodski, Seiberg (2017-18) + di Vecchia, Rossi, Veneziano, Yankielowicz (2017)

Spontaneous CP breaking

- Also for large N Witten 1979-80, di Vecchia, Veneziano 1980
- For F=0 CP spontaneoulsy broken. Also holds at finite N (Zohar et al - anomaly matching arguments)
- For F>1 and all masses equal CP also spontaneously broken
- For F=1 there is a critical mass at which second order phase transition
- Similarly for F>1 unequal masses critical surface

F=1 in AMSB QCD theory <u>F = 1 in Amob gob giveny</u> $\frac{1}{2}$ and the limiting case by the limiting case $\frac{1}{2}$ as suggested by the limiting case when $\frac{1}{2}$ <u>contine and a continent masses is much had the other.</u>

• F=1 (implying F<N) the potential is • F=1 (implying F<N) the potential is

$$
V = \frac{1}{2} \left| \frac{2}{\phi} \left(\frac{\Lambda^{3N-1}}{\phi^2} \right)^{\frac{1}{N-1}} - 2\phi \, m_Q \right|^2 - m \left[(3N-1) \left(\frac{\Lambda^{3N-1}}{\phi^2} \right)^{\frac{1}{N-1}} + \phi^2 m_Q \right] + \text{h.c.}.
$$

- where $M = \phi^2$ and $K = 2 \, \phi^\dagger \phi$. Separating the magnitude and phase of the complex scalar , i.e. = *f* exp(*i^f*), • where $M = \phi^2$ and $K = 2 \phi^{\dagger} \phi$. • where $M = \phi^2$ and $K = 2 \phi^{\dagger} \phi$. $\mathbf{z} = \mathbf{z} - \mathbf{z} + \mathbf{z}$ \bullet where $M = \phi^2$ a \overline{D} $M = \phi^2$ and $K = 2 \phi^{\dagger} \phi$ Separating the magnitude and phase of the complex scalar , i.e. = *f* exp(*i^f*),
- Since under $\textbf{CP} \quad M \stackrel{\text{CH}}{=}$ want to separate out phase $\phi = f \exp(i \delta_f)$ ✓*|*⇤*[|] f* $\sum_{i=1}^n$ chiral superfield $\sum_{i=1}^n$ $\sum_{i=1}^n$ $\sum_{i=1}^n$ $\sum_{i=1}^n$ $\sum_{i=1}^n$ $\sum_{i=1}^n$ \bullet Since under CP $M \stackrel{\text{CP}}{\longrightarrow} M^{\dagger}$ want to separate out phase $\phi = f \exp(i \delta_x)$ anomaly-free combination of *R* and global *U*(1) symmetries is broken. $\longrightarrow M^{\dagger}$ want to separate out phase $\phi = f \exp(i \delta_f)$ Cinese weden CD *x* = *|*⇤*|* $\,$ se ϕ : *n*₁
	- Potential to minimize: • Potential to minimize: *^Q* ⁺ *^x*² *mm^Q* cos(2*^f*) *^m*(3*^N* 1)*^x* cos broken along the *D*-flat direction, hence it does not produce additional Goldstone bosons, while an anomaly-free combination of *R* and global *U*(1) symmetries is broken.

$$
V_{\ell} = 2 x^{\frac{1-N}{N}} |\Lambda|^{\frac{3N-1}{N}} \left(m_Q^2 + x^2 - m m_Q \cos(2\delta_f) - m(3N-1)x \cos\left[\frac{2\delta_f - \bar{\theta} - 2\pi\ell}{N-1}\right] \right)
$$

$$
-2m_Q x \cos\left[\frac{2N\delta_f - \bar{\theta} - 2\pi\ell}{N-1}\right], \quad \ell = 0, 1, \dots, N-2,
$$

F=1 in AMSB QCD theory <u>**For an Annon Qob theory</u>**</u> branch with minimal energy at the lower en For *N* = 2*,* 3 the minimum of the potential can be found analytically for ¯✓ = ⇡.

• Here $x = |\Lambda| \left(\frac{|\Lambda|}{f}\right)^{N-1}$. For N=2,3 can analytically find minimum. From this find critical masses $x = |\Lambda|$ $\sqrt{|$ ^{Λ} *f* $\sqrt{\frac{2N}{N-}}$ $N-1$ $\overline{\mathsf{a}}$ the potential can be written as *V* = min` *V*` with $\left(\begin{array}{c} 1 & 1 \end{array} \right)$ $\frac{2N}{N-1}$ • Here $x = |\Lambda| \left(\frac{|\Lambda|}{c} \right)$ For N=2.3 can analytically f vaining up a pear, this final suitisel messesse values of the critical point and control independent are given by α $2N$ Here $x-|\Lambda|\left(\frac{|\Lambda|}{2}\right)^{N-1}$ For NI-2.3 can analytically fine $\sum_{i=1}^{N}$ spontaneously. For $\sum_{i=1}^{N}$ spontaneously. For $\sum_{i=1}^{N}$ minimum. From this lind critical masses $F(5) = \frac{F}{\sqrt{f}}$ and $F(6) = \frac{F}{\sqrt{f}}$ and $F(7) = \frac{F}{\sqrt{f}}$ and $F(8) = \frac{F}{\sqrt{f}}$ and $F(9) = \frac{F}{\sqrt{f}}$ for $F(9) = \frac{F}{\sqrt{f}}$ and $F(1) = \frac{F}{\sqrt{f}}$ and $F(1) = \frac{F}{\sqrt{f}}$ for $F(1) = \frac{F}{\sqrt{f}}$ and $F(1) = \frac{F}{\sqrt{f}}$ and *SUMMIQIIII* **F** FOIT GIO MIG ORGO *FROSS*

$$
\frac{m_{Q,0}}{m}\Big|_{N=2} = \frac{3}{2}\cos\left(\frac{1}{3}\arccos\left(-\frac{25}{27}\right) - \frac{2\pi}{3}\right) \approx 0.576511
$$

$$
\frac{m_{Q,0}}{m}\Big|_{N=3} = \frac{14}{9}\cos\left(\frac{1}{3}\arccos\left(-\frac{289}{343}\right) - \frac{2\pi}{3}\right) - \frac{1}{9} \approx 0.398853
$$

• Can also find for large N limit \sim Can algo find for lerge N limit \bullet Can also into tor targe to infinitionally ⁵Note that for *^m^Q < m* the potential is unbounded in the *^f* = 0 and *^x* ! 0 direction, i.e. \bullet Gan arso into for large in infiit \bullet ⁵Note that for *^m^Q < m* the potential is unbounded in the *^f* = 0 and *^x* ! 0 direction, i.e. \bullet Can also find for large N limit

$$
\left. \frac{m_{Q,0}}{m}\right|_{N\gg 1} = \frac{9}{7}\frac{1}{N} + \mathcal{O}\left(\frac{1}{N}\right)^2
$$

• However for N>3 only numerical solution For *N* = 2*,* 3 the minimum of the potential can be found analytically for ¯✓ = ⇡. 5 We observe the existence of a critical value for the SUSY quark mass *mQ,*⁰ below For *N >* 3 the minimum of the potential and the critical value *mQ,*⁰ can only be found numerical setting in the right panel of Figure 2, the right panel of Figure 2, the critical setting 2, the c

F=1 in AMSB QCD theory

The phase structure for F=1 *SU*(2) and *SU*(3) with *F* = 1. The critical point occurs at *mQ/m* ⇡ 0*.*576511 and

F=1 in AMSB QCD theory

- The η' potential as we pass through the critical mass respectively. We show the potential for several values of *m^Q* close to the critical
- At the critical mass η' itself becomes massless

F=1 phase structure

Phase structure for F>1

- $\frac{1}{2}$ The shaded region shows where CP is spontaneously broken. For *m^Q*¹ */m* 1 and • For equal masses CP always broken
- Unequal masses have phase boundary critical mass reappears in decoupling limit ratio and down quark mass ratio in the up and down $\frac{1}{\sqrt{2}}$
- QCD-like mass ratios?

The phase boundaries

- of ⌘⁰ and ⇡⁰ becomes massless at the phase boundary, signaling a second-order phase • The η' becomes massless at the phase boundaries
- Second order PT

Meson physics in AMSB QCD

T. Roy, M. Ruhdorfer, T. Youn and C.C.

- Once we add the quark masses, can also study
	- Detailed structure of VEVs
- Meson masses corrections to GMO
	- Meson decays

• Take F=3 and use either F<N (easier but less dynamics) or F=N (harder but more similar to actual QCD dynamics) perturbation theory (Chemptre Chemptre)

 $F = 3: F < N$ and $F = N & \Lambda \gg m > m_s > m_d > m_u$

Meson physics in AMSB QCD

- **Leading order in quark masses:**
- All VEVs equal **Meson Mass Spectrum**
	- **Gell-Mann Okubo satisfied**

$$
A = \frac{\Lambda^3}{f^2} \left(\frac{\Lambda}{f}\right)^{\frac{6}{N-3}} + \frac{m}{2}, \quad B = \frac{\alpha^2 X}{2\Lambda^2} + \alpha m
$$

Meson physics in AMSB QCD Phenomenological Application

 \cdot Corrections to quark condensates:

$$
f_i = f^0 + m_s f_i^1 + m_q f_i^2 + \Delta m_q f_i^3 + \cdots
$$

 $m_q \equiv m_d + m_u$, $\Delta m_q \equiv m_d - m_u$

For $F < N$

$$
f_d \simeq \left(1 + \frac{m_d}{18}\right) \frac{\Lambda^{3/2}}{\sqrt{3m}}, \quad f_u \simeq \left(1 + \frac{m_u}{18}\right) \frac{\Lambda^{3/2}}{\sqrt{3m}}, \quad f_s \simeq \left(1 + \frac{m_s}{18}\right) \frac{\Lambda^{3/2}}{\sqrt{3m}}
$$

For $F = N$

$$
f_d \simeq \left(1 - \frac{m^2}{6\alpha\beta\Lambda^2} + \frac{m(m_s + m_q)}{18\beta\Lambda^2} - \alpha \frac{m_s - m_q/2}{12m} + \alpha \frac{\Delta m_q}{8m}\right) \Lambda
$$

$$
f_u \simeq \left(1 - \frac{m^2}{6\alpha\beta\Lambda^2} + \frac{m(m_s + m_q)}{18\beta\Lambda^2} - \alpha \frac{m_s - m_q/2}{12m} - \alpha \frac{\Delta m_q}{8m}\right) \Lambda
$$

$$
f_s \simeq \left(1 - \frac{m^2}{6\alpha\beta\Lambda^2} + \frac{m(m_s + m_q)}{18\beta\Lambda^2} + \alpha \frac{m_s - m_q/2}{6m}\right) \Lambda
$$

Meson physics in AMSB QCD Phenomenological Application Correction to meson VEVs

• For both cases

$$
\frac{f_d}{f_u} = 1 - \frac{\Delta m_q}{m_s - m_q/2} \left(1 - \frac{f_s}{f_u} \right) + \dots
$$

• In agreement with ChPT Gasser/Leutwyler *ms* [−] *mq*/2 (¹ [−] *^f* **h** agree

$$
\frac{\langle 0|\bar{d}d|0\rangle}{\langle 0|\bar{u}u|0\rangle} = 1 - \frac{m_{\rm d} - m_{\rm u}}{m_{\rm s} - \hat{m}} \left\{ 1 - \frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle} + \frac{1}{16\pi^2 F_0^2} \left(M_{\rm K}^2 - M_{\pi}^2 - M_{\pi}^2 \ln \frac{M_{\rm K}^2}{M_{\pi}^2} \right) \right\}
$$

 \cdot Corrections to Gell-Mann Okubo sum rule

4

For
$$
F < N
$$

\n
$$
\Delta_{\text{GMO}} = -\left(\frac{7}{27} + \frac{1}{648} \frac{m_q}{m}\right) m_s^2 + \frac{7}{27} m_s m_q \sim -2000 \text{ MeV}^2
$$
\nFor $F = N$
\n
$$
\Delta_{\text{GMO}} = \left(\alpha^2 + \frac{\alpha^3}{4} \frac{m_q}{m} - \frac{7\alpha^2}{36\beta} \frac{m m_q}{\Lambda^2}\right) m_s^2 + \left(\frac{\alpha^2}{2} + \frac{7\alpha^2}{36\beta} \frac{m m_q}{\Lambda^2}\right) m_s m_q
$$

 Δ_{GMO} is negative for $F < N$, while not for $F = N$ unless $m \gg \Lambda$, which is ordinary QCD

2

36*β*

Heavy quark physics? Form Factors Phenomenological Application

- Can calculate meson decays Can calculate mes **Form Factors**
- Semi-leptonic decays: ChPT: In ChPT,

$$
\langle \partial_{\mu} U^{\dagger} \partial^{\mu} U \rangle \to \langle \partial_{\mu} U^{\dagger} G_F [Q_W] \bar{\ell} \gamma^{\mu} \nu U \rangle \to G_F (\partial_{\mu} K \pi) (\bar{\ell} \gamma^{\mu} \nu)
$$

$$
\partial^{\mu} \to D^{\mu} = \partial^{\mu} + G_F Q_W J_W^{\mu} \quad J_W^{\mu} = \bar{\ell} \gamma^{\mu} \nu
$$

W

• In AMSB QCD: MSB QCI

$$
\langle M^{\dagger}M\rangle\rightarrow\langle M^{\dagger}e^{{\mathcal{ Q}}_WJ_W}Me^{-{\mathcal{ Q}}_WJ_W}\rangle\rightarrow G_F(\partial_\mu K\pi)(\bar{\ell}\gamma^\mu\nu)
$$

. In progress, also want to look at hadronic decays

Meson physics in AMSB QCD <u>too mis mir</u>

- Some of the issues in heavy quark physics can be addressed 3 those the don't involve mixing with generic heavy QCD states $\overline{}$ $\sqrt{2}$ *neric* heavy QCD sta
- Charm decay constant

onstant
$$
f_q^2 = \left(\frac{m_c}{m_*^{N-F+1}}\right)^{\frac{1}{N}} \Lambda^{\frac{3N-F}{N}}, \qquad f_c^2 = \frac{m_q}{m_c} f_q^2
$$

• η_c - η' mixing

$$
\sin \theta_{\eta} = \sqrt{\frac{m_*}{m_c}} \text{ for } F < N
$$
\n
$$
\sin \theta_{\eta} = \frac{m_*}{m_c} \text{ for } F = N \text{ and } N + 1 < F < \frac{3}{2}N
$$
\n
$$
\sin \theta_{\eta} = \frac{1}{\sqrt{2}} \text{ for } F = N + 1
$$

For
$$
F < N
$$
 and $N + 1 < F < \frac{3}{2}N$

- η_c , η' masses
- $m_{\eta'}^2 \sim m_*^2$, $m_{\eta_c}^2 \sim m_c^2$

For $F = N, N + 1$

Heavily depends on the form of Kähler potentials

$$
m_{\eta'}^2 \sim \Lambda^2, m_{\eta_c}^2 \sim \Lambda^2 \text{ or } m_{\eta'}^2 \sim m_c^2, m_{\eta_c}^2 \sim m_c^2
$$

Need a better understanding

- Softly broken SUSY theories lab for studying confinement and QCD physics
- AMSB is UV insensitive, produces QCD-like phase structure
- η' potential: for most cases not instanton induced, except for special cases F=N-1,N,N+1
- Reproduce the structure of CP phases at $\theta = \pi$
- Can study some meson physics vacuum structure, mass sum rules, semi-leptonic decays, heavy charm physics,….

Happy birthdays - and please more physics!!!!

