Exploring QCD-like Dynamics with AMSB

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with Raffaele d'Agnolo, Rick Gupta, Eric Kuflik, Tuhin Roy, Max Ruhdorfer, Taewook Youn
+
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Discrete Anomaly Matching*

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Abstract

We extend the well-known 't Hooft anomaly matching conditions for continuous global symmetries to discrete groups. We state the matching conditions for all possible anomalies which involve discrete symmetries explicitly in Table 1. There are two types of discrete anomalies. For Type I anomalies, the matching conditions have to be always satisfied regardless of the details of the massive bound state spectrum. The Type II anomalies have to be also matched except if there are fractionally charged massive bound states in the theory. We check discrete anomaly matching in recent solutions of certain $N = 1$ supersymmetric gauge theories, most of which satisfy these constraints. The excluded examples include the chirally symmetric phase of $N = 1$ pure supersymmetric Yang-Mills theories described by the Veneziano–Yankielowicz Lagrangian and certain non-supersymmetric confining theories. The conjectured self-dual theories based on exceptional gauge groups do not satisfy discrete anomaly matching nor mapping of operators, and are viable only if the discrete symmetry in the electric theory appears as an accidental symmetry in the magnetic theory and vice versa.

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Instantons in Partially Broken Gauge Groups*

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Abstract

We discuss the effects of instantons in partially broken gauge groups on the low-energy effective gauge theory. Such effects arise when some of the instantons of the original gauge group G are no longer contained in (or can not be gauge rotated into) the unbroken group H. In cases of simple G and H, a good indicator for the existence of such instantons is the "index of embedding." However, in the general case one has to examine $\pi_3(G/H)$ to decide whether there are any instantons in the broken part of the gauge group. We give several examples of supersymmetric theories where such instantons exist and leave their effects on the low-energy effective theory.

``The Footnote paper”

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Saclay t03/061

Gauge Theories on an Interval: Unitarity without a Higgs

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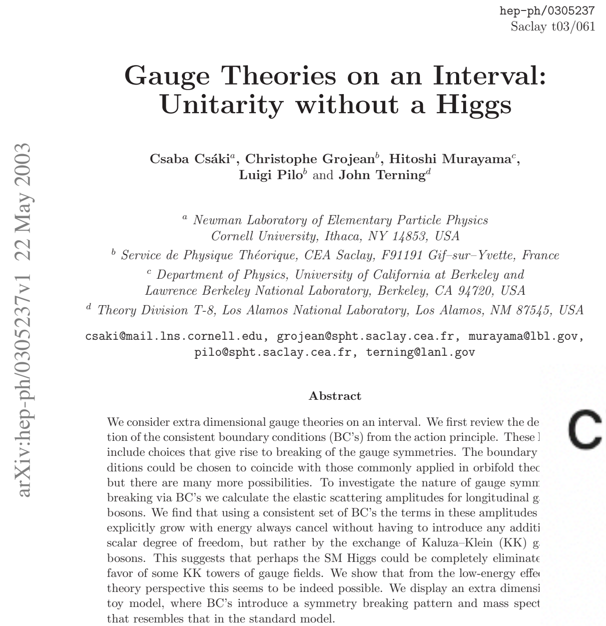
Abstract

We consider extra dimensional gauge theories on an interval. We first review the derivation of the consistent boundary conditions (BC's) from the action principle. These BC's include choices that give rise to breaking of the gauge symmetries. The boundary conditions could be chosen to coincide with those commonly applied in orbifold theories, but there are many more possibilities. To investigate the nature of gauge symmetry breaking via BC's we calculate the elastic scattering amplitudes for longitudinal gauge bosons. We find that using a consistent set of BC's the terms in these amplitudes that explicitly grow with energy always cancel without having to introduce any additional scalar degree of freedom, but rather by the exchange of Kaluza–Klein (KK) gauge bosons. This suggests that perhaps the SM Higgs could be completely eliminated in favor of some KK towers of gauge fields. We show that from the low-energy effective theory perspective this seems to be indeed possible. We display an extra dimensional toy model, where BC's introduce a symmetry breaking pattern and mass spectrum that resembles that in the standard model.



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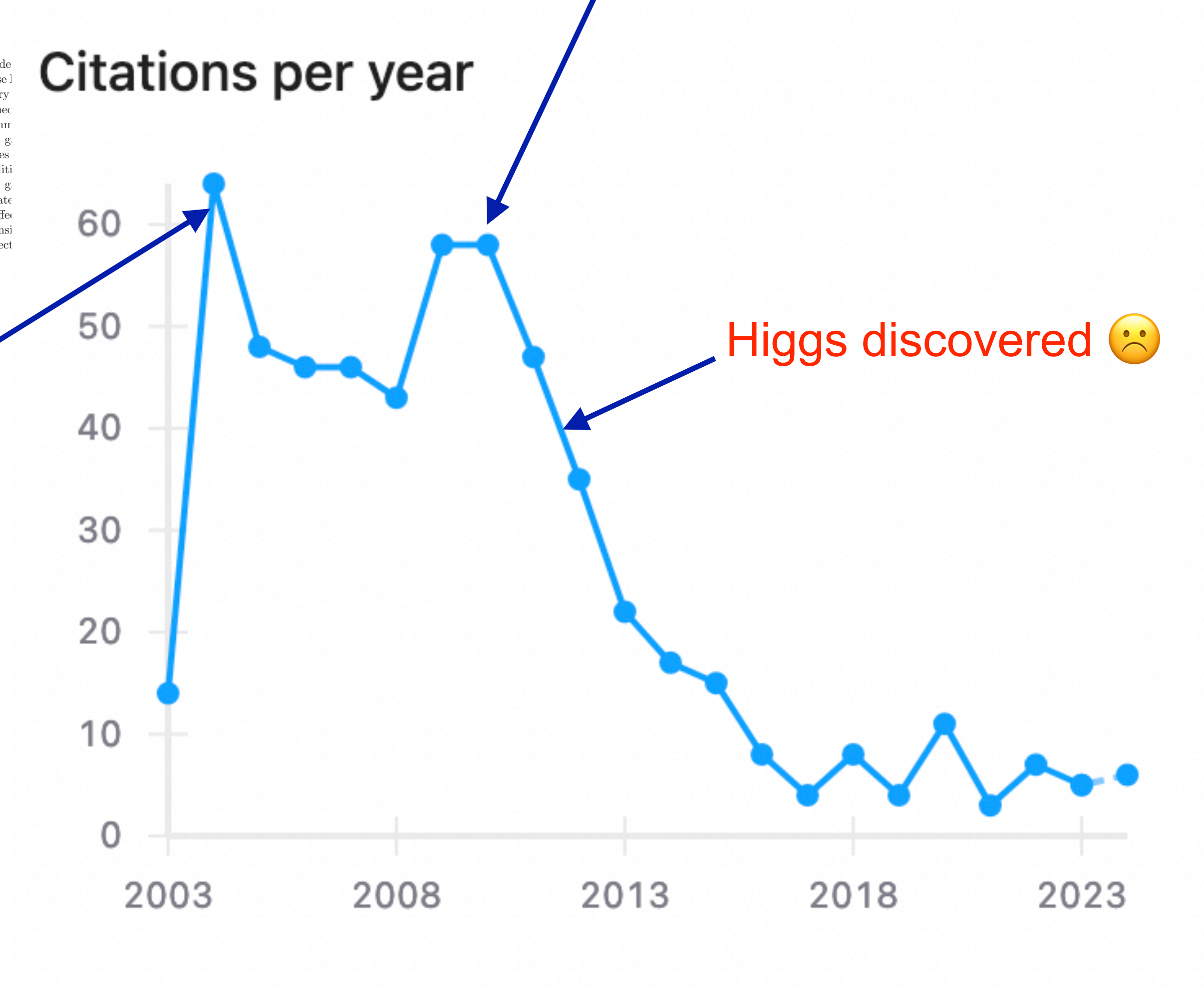
Maybe there really is no Higgs?



Higgs discovered ☹️

RS rules

Demonstration of Confinement and Chiral Symmetry Breaking in $SO(N_c)$ Gauge Theories

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We demonstrate that $SO(N_c)$ gauge theories with matter fields in the vector representation confine due to monopole condensation and break the $SU(N_F)$ chiral symmetry to $SO(N_F)$ via the quark bilinear. Our results are obtained by perturbing the $\mathcal{N} = 1$ supersymmetric theory with anomaly-mediated supersymmetry breaking.

INTRODUCTION

Ever since quarks were proposed as fundamental constituents of the proton, neutron, and numerous hadrons by Gell-mann and Ne'eman [1, 2], it has been a mystery why they cannot be observed directly in experiments. At the same time, protons and neutrons bind in atomic nuclei due to the exchange of light pions predicted by Yukawa [3]. The binding of nuclei, and correspondingly the entire world of chemistry, hinges on pions being much lighter than protons, despite the fact that they are made of the same quarks. The first mystery was "explained" by postulating *confinement* of quarks by condensation of magnetic monopoles via the dual Meißner effect pro-

turns out that we should focus on $N_F \leq N_c - 2$ where we can demonstrate monopole condensation.

In this Letter, we sketch the essence of the analysis, while details are presented in a forthcoming companion paper [28], that will also contain a discussion of the cases where $N_F > N_c - 2$.

ANOMALY MEDIATION

Anomaly mediation of supersymmetry breaking (AMSB) is parameterized by a single number m that explicitly breaks supersymmetry in two different ways. One is the tree-level contribution based on the superpotential

Outline

- Use of AMSB for studying QCD-like theories
- The η′ Potential (and the Axion Mass)
- CP violation at $\theta=\pi$
- Chiral perturbation theory and heavy quark dynamics

The use of AMSB for studying QCD-like theories

- SUSY gives powerful constraints on strong dynamics

- Seiberg (+Intriligator, Hitoshi, …) was able to nail down phase structure of SUSY QCD in 1994 using
    - Holomorphy
    - ’t Hooft anomaly matching
    - Instanton calculations
    - Integrating out/Higgsing

- Obtained many different phases depending on F vs N

The phases of SUSY QCD

$F > 3N$	Theory IR free
$F = 3N$ to $F = \frac{3}{2}N$	Conformal - non-abelian Coulomb phase
$F = \frac{3}{2}N$ to $F = N+1$	Free magnetic phase
$F = N+1$	s-confinement
$F = N$	Quantum modified constraint
$F = N-1$ to $F = 0$	ADS superpotential, runaway vacuum
$F = 0$	Pure SYM - gaugino condensation

SUSY QCD

- N=1 SUSY SU(N) gauge theory with F flavors

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
$\Phi,\ Q$	$\square$	$\square$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\Phi},\ \overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{F-N}{F}$

- At low energies described in terms of mesons $M_{ij} = Q_i \bar{Q}_j$ and baryons $B_{ij\ldots k} = Q_i Q_j \ldots Q_k$

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F=0 - Pure SYM

- No matter fields, no continuous flavor symmetry
- Z_{2N} discrete R-symmetry rotating gauginos
- Dynamics: gaugino condensation
- $W = N\Lambda^3$ $\qquad\qquad$ $\langle\lambda\lambda\rangle = -32\pi^2\omega_k\Lambda^3$
- Should be truly confining $\qquad$ $V(R) \sim \sigma R$

The phases of SUSY QCD

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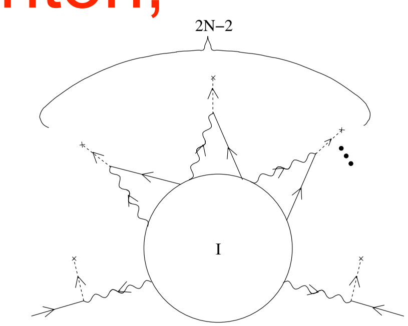
0<F<N: ADS superpotential

- First obtained by Affleck, Dine, Seiberg 1984

- Dynamics generates a non-perturbative superpotential

$$W_{\rm ADS} = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)}$$

- For F=N-1 actually generated by instanton, calculable



- Gauge group (partially) Higgsed

- $V(R) \sim \mathrm{constant}$ (at least for F=N-1)

- For F<N-1 gaugino condensation in unbroken group

The phases of SUSY QCD

$F > 3N$ Theory IR free

$F = 3N$

Conformal - non-abelian Coulomb phase

$F = \frac{3}{2}N$

Free magnetic phase

$F = N+1$ s-confinement

$F = N$ Quantum modified constraint

$F = N-1$

ADS superpotential, runaway vacuum

$F = 0$ Pure SYM - gaugino condensation

The phases of SUSY QCD

- A beautiful picture, BUT very different from what we expect in non-SUSY QCD

- Lattice simulations suggest only 2 phases
    - Chiral symmetry breaking
    - For large number of flavors (perhaps as high as F>3N) conformal phase

- Would like to start making connection between SUSY and non-SUSY theories

Adding SUSY breaking

- A long history of perturbing with SUSY breaking terms, for example

- Aharony, Sonnenschein, Peskin, Yankielowicz ’95
- Evans, Hsu, Schwetz ‘95
- Cheng & Shadmi 1998
- Arkani-Hamed & Rattazzi ‘98; Luty & Rattazzi ’99
- Abel, Buican, Komargodsky ’11

- Increasingly more systematic approach
- Lots of interesting results, but no clear pattern of what the actual phase structure is

The use of AMSB

- Proposal of Hitoshi in 2021: use anomaly mediated SUSY breaking for perturbing the Seiberg exact results

- AMSB: originally ``designed" to provide a specific implementation for MSSM with predictive soft breaking patterns

- Here we will simply use it only to study phases of gauge theories, not as a BSM model

- Assumption of AMSB: SUSY breaking mediated purely by supergravity, no direct interaction between SUSY breaking sector and matter sector

AMSB

Randall, Sundrum '98
Giudice, Luty, Murayama, Rattazzi '98
see also Arkani-Hamed, Rattazzi '98

SUGRA

~~SUSY~~ Matter sector/SQCD

- Assume matter sector sequestered - no direct interactions with SUSY breaking generated

- Only source of ~~SUSY~~ the auxiliary field of supergravity multiplet

AMSB

- Best way to describe effect of AMSB is via the introduction of the Weyl compensator Φ

Pomarol, Rattazzi '99

- This conformal compensator is a spurion for super-Weyl transformations (SUSY rescaling + U(1) rotations) with weight 1

- The effects of ~~SUSY~~ will show up through the coupling

$$\mathcal{L} = \int d^4\theta \Phi^* \Phi K + \int d^2\theta \Phi^3 W + c.c.$$

- With the spurion $\Phi = 1 + \theta^2 m$

AMSB

- If the matter sector is conformal: can scale out $\mathbf{\Phi}$ by rescaling the fields $\phi_i \;\rightarrow\; \Phi^{-1}\phi_i$
- For example if $K = \Phi^*\Phi\phi^+\phi$ and $W = \Phi^3\phi^3$
- $\phi_i \;\rightarrow\; \Phi^{-1}\phi_i$ rescaling will completely remove $\mathbf{\Phi}$ from the theory - no SUSY breaking
- SUSY breaking will be tied to violations of conformality! UV insensitive process!

Loop induced AMSB effects

- If scale invariance broken via RGE running:

$$\beta(g^2) = \frac{g^3}{16\pi^2}\left[S(R) - 3C(G)\right] + \dots \longrightarrow m_\lambda(\mu) = -\frac{\beta(g^2)}{2g^2}(\mu) m$$

$$\frac{d}{dt}Y^{ijk} = Y^{ijp}\left[\frac{1}{16\pi^2}\gamma_p^k + \dots\right] + (k \leftrightarrow i) + (k \leftrightarrow j)$$

$$m_i^2(\mu) = -\frac{1}{4}\dot{\gamma}_i(\mu) m^2$$

$$A_{ijk}(\mu) = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)(\mu) m$$

- For example in SUSY QCD

$$m_\lambda = \frac{g^2}{16\pi^2}(3N_c - N_f)m$$

$$m_Q^2 = m_{\tilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2} 2C_i(3N_c - N_f)m^2$$

Loop induced AMSB effects

• Loop induced breaking terms provide positive squark masses and gaugino mass - massless spectrum that of ordinary QCD

• For AMSB version of MSSM slepton masses were problematic - right handed sleptons were tachyonic. Here only AF gauge group - AMSB gives perfect UV boundary condition

A surprise - tree-level AMSB effects

- If there is a non-scale invariant superpotential: will contribute to AMSB potential

$$\mathcal{L}_{\rm tree} = m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c.$$

- Vanishes for dim 3 superpotential, but not in general

- Expression for general Kähler potential:

C.C., Gomes, Murayama, Telem '21

$$\begin{aligned} V_{\rm tree} =& \partial_i W g^{ij^*} \partial_j^* W^* + m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K\right) \\ &+ m\left(\partial_i W g^{ij^*} \partial_j^* K - 3W\right) + c.c. \end{aligned}$$

A non-perturbative AMSB potential

- Example: SU(N) for $N_f < N_c$. ADS Superpotential

$$(N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)}$$

- Will lead to induced term from $\int d^2\theta \Phi^3 W_{ADS}$

$$-(3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$$

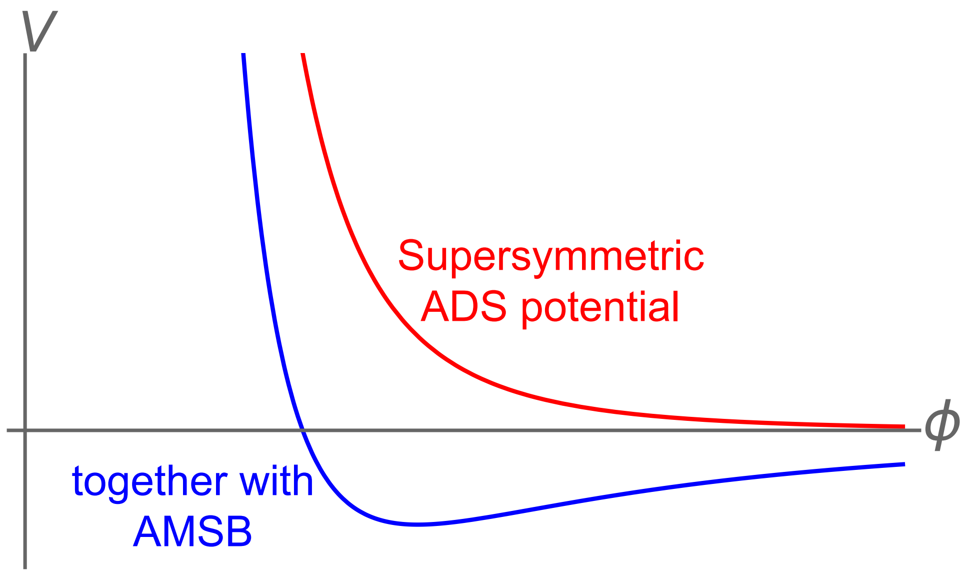
- Along direction

$$Q = \tilde{Q} = \left(\begin{array}{ccc} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \hline 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{array}\right)\phi, \qquad M = \phi^2.$$

A non-perturbative AMSB potential

- $$-(3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$$ term is key
- Non-perturbative effect involving SUSY breaking
- AMSB allows us to pin down this term
- Formally tree-level but really must be a non-perturbative effect including SUSY breaking
- Will stabilize ADS superpotential!
- Will give rise to proper symmetry breaking pattern!

Phase for QCD* for $N_f<N_c$



- Symmetry breaking pattern $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

- As in QCD, massless DOF's just pions

- Could be continuously connected to actual QCD for m>>Λ

The phases of SUSY QCD/AMSB QCD

PURE SUSY QCD

$F > 3N$ Theory IR free

$F = 3N$

Conformal

$F = \frac{3}{2}N$

Free magnetic phase

$F = N + 1$ s-confinement

$F = N$ Quantum modified constraint

$F = N - 1$

ADS

$F = 0$ Pure SYM - gaugino condensation

AMSB QCD

QCD-like

Runaway?
+QCD-like
local

QCD-like

Non-calc +QCD

QCD-like

Chiral Lagrangian and η' potential - ``usual"

- Naive assumption $U(1)_A$ anomalous, broken by instantons, so instanton effects will give mass to **η**'?

- Form of chiral Lagrangian would be

$$\mathcal{L} = f_\pi^2 \mathrm{Tr}\left[(\partial_\mu U)^\dagger \partial^\mu U\right] + a\Lambda f_\pi^2 \mathrm{Tr}\, m_Q U + \mathrm{h.c.}$$

$$\mathcal{L}_{\mathrm{inst}} = b\Lambda^2 f_\pi^2 e^{-i\theta} \det U + \mathrm{h.c.}$$

- In terms of **η**' $\quad V_{\eta'} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta')$

Chiral Lagrangian and η' potential - ``usual"

- Would correspond to instanton effect because ~ $e^{i\theta}$

- Would give η' mass ~ Λ

- Consistent with spurion analysis for axial U(1):

$$\theta \to \theta + 2\,F\varphi$$

$$\eta' \to \eta' + 2\varphi$$

- After integrating out η' $\quad \eta' = (\theta + 2k\pi)/F$

$$V_\pi = -\alpha\Lambda f_\pi^2 e^{i(\bar{\theta}+2\pi k)/F}\mathrm{Tr}(m_q e^{i\pi^a T^a}) + \mathrm{h.c.}$$
$$= -2\alpha\Lambda f_\pi^2 \sum_{i=1}^{F} m_i \cos\left(\frac{\bar{\theta}+2\pi k}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

The axion mass

R. d'Agnolo, R. Gupta, E. Kuflik, T. Roy, M. Ruhdorfer and C.C.

- To get θ dependence of potential also integrate out pions:

$$V_2 = -2\alpha\Lambda f_\pi^2 \left[m_u \cos\left(\frac{\bar\theta}{2} + k\pi + \pi^0\right) + m_d \cos\left(\frac{\bar\theta}{2} + k\pi - \pi^0\right)\right]$$

$$V_{\min} = -2|\alpha|\Lambda f_\pi^2 \sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos\bar\theta}$$

- If we also had a physical QCD axion: separate PQ symmetry

$$a \to a + \varphi, \quad \theta \to \theta - n\varphi$$

$$V_a = -2\alpha\Lambda^2 f_\pi^2 \sum_{i=1}^{F} \frac{m_i}{\Lambda} \cos\left(\frac{\bar\theta + an}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

- To get axion mass: integrate out pions! With axion easy to solve EOM's, quadratic term around minimum

$$V_a = \alpha\Lambda^2 f_\pi^2 \sum_{i=1}^{F} \frac{m_i}{\Lambda} \left(\frac{an}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)^2$$

Condensates and large N limit

- Issue: large N limit anomaly vanishes

$$\partial_\mu j^\mu_A \sim F\frac{g^2}{16\pi^2}\mathrm{Tr}G\tilde{G} \sim \frac{\lambda}{16\pi^2}\frac{F}{N}\mathrm{Tr}G\tilde{G} \to 0$$

- **η'** mass should vanish in this limit

- But from $V_{\eta'} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta')$ does not vanish for large N

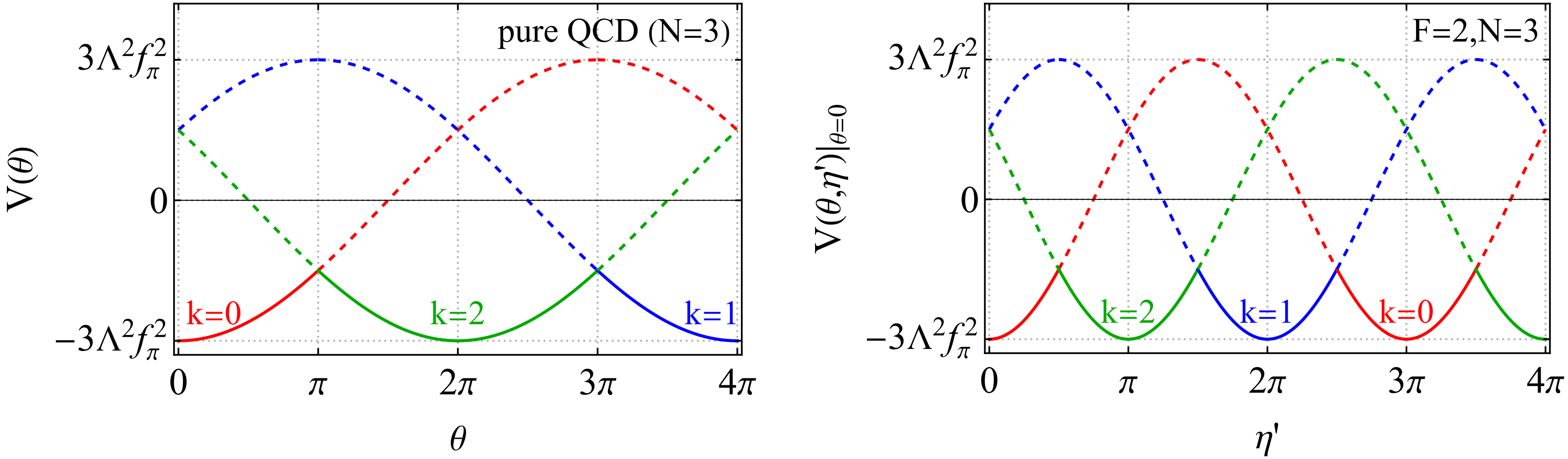
- Witten: **η'** needs to cancel θ dependence of pure QCD vacuum energy $E(\theta) = N^2 f(\theta/N)$ $m^2_{\eta'} \propto \dfrac{1}{N}$
- Form of potential $\mathcal{L}_{\eta'} = N\Lambda^2 f_\pi^2 (e^{-i\theta}\det U)^{1/N} + h.c.$

The Chiral Lagrangian and η' potential

- Non-analytic - how is it 2π periodic in θ?

- Need to have several branches, potential of the form

$$V(\theta,\eta') = \mathrm{Min}_k \; - 2N\Lambda^2 f_\pi^2 \cos(\frac{\theta - F\eta' + 2\pi k}{N}), \quad k = 0,\ldots,N-1$$



The Chiral Lagrangian and η' potential

- η' acts like a (heavy) axion and relaxes to minimum of potential to cancel θ dependence (and wash out branch structure)

$$\langle \eta' \rangle = \frac{\theta + 2\pi k}{F}$$

- Check this picture in AMSB QCD

- Similar work & results by Dine, Draper, Stephenson-Haskins & Xu (2016). They were using soft squark and gluino masses - can reliably do only for F<N

η' potential for F<N in AMSB QCD

R. d'Agnolo, R. Gupta, E. Kuflik, T. Roy,
M. Ruhdorfer and C.C.

- Consider first F<N - with quark mass

$$W = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} + \mathrm{Tr}(M_Q M)$$

- The meson VEV as usual

$$\phi = \Lambda\left(\frac{N+F}{3N-F}\frac{\Lambda}{m}\right)^{(N-F)/(2N)} + \mathcal{O}(m_Q/m)$$

- The meson matrix:

$$Q_f^a = |\phi|\delta_f^a\,, \quad \bar{Q}_f^a = Q_{f'}^a U_{f'f}\,, \quad M = |\phi|^2 U$$

- η' part of U matrix, need to make sure we keep the whole phase everywhere

F<N in AMSB QCD

- Chiral Lagrangian:

$$V = -m\left[(3N-F)\left(\frac{\Lambda^{3N-F}}{|\phi|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)} + |\phi|^2\mathrm{Tr}(m_Q U)\right] + c.c.$$
$$-2\left(\frac{\Lambda^{3N-F}}{|\phi|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)}\mathrm{Tr}(m_Q^\dagger U^\dagger) + c.c\,.$$

- Has the branch structure like Witten predicted, but 1/(N-F) power. η' potential:

$$V = -2(3N-F)\left(\frac{N+F}{3N-F}\right)^{-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N} m|\Lambda|^3\cos\left(\frac{F}{N-F}\frac{\eta'}{f_{\eta'}} - \frac{\theta+2\pi k}{N-F}\right)$$
$$-2F\left(\frac{N+F}{3N-F}\right)^{1-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N}|m_Q||\Lambda|^3\cos\left(\frac{\eta'}{f_{\eta'}} + \theta_Q\right)$$
$$-4F\left(\frac{N+F}{3N-F}\right)^{-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N}|m_Q||\Lambda|^3\cos\left(\frac{N}{N-F}\frac{\eta'}{f_{\eta'}} + \theta_Q - \frac{\theta+2\pi k}{N-F}\right),$$

F<N in AMSB QCD

- Pure QCD: $$V_k \xrightarrow{F=0} -6N^2 m|\Lambda_{\rm phys}|^3 \cos\left(\frac{\theta+2\pi k}{N}\right)$$

- Just like Witten predicted (also Dine et al)

- For small number of flavors:

$$V_k \stackrel{N\gg F}{\to} -6N^2 m|\Lambda_{\rm phys}|^3 \cos\left(\frac{F}{N}\eta' - \frac{\theta+2\pi k}{N}\right) - \frac{14}{3}N|\Lambda_{\rm phys}|^3 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

- Again just as Witten predicted, and as in Dine et al.

$$m_{\eta'}^2 \propto F^2 m|\Lambda_{\rm phys}|^3/f_\pi^2 \sim 1/N$$

F<N in AMSB QCD

- F~N both large the situation is very different!
- For example F=N-1 and both large

$$V_k \overset{N=F+1\gg 1}{\rightarrow} -4N^{3/2}m^2|\Lambda_{\rm phys}|^2\cos\left((N-1)\eta'-\theta\right) - 2N^{1/2}m|\Lambda_{\rm phys}|^2\sum_{i=1}^{F} m_i\cos\left(\eta'+\theta_Q+\sum_{j=1}^{F-1}t_i^j\pi^j\right)$$
$$-4N^{1/2}m|\Lambda_{\rm phys}|^2\sum_{i=1}^{F}m_i\cos\left(N\eta'+\theta_Q-\theta+\sum_{j=1}^{F-1}t_i^j\pi^j\right)$$

- No branches, η' mass does not go to zero

$$m_{\eta'}^2 = \frac{(x-3)^2 x}{(x+1)(x-1)^2}m^2\,, \quad \text{with} \quad x=\frac{F}{N}$$

- Large F,N qualitatively different from large N, fixed F limits!

F<N in AMSB QCD

- For N-F>1 NOT an instanton effect

- We know it is actually gaugino condensation

- For F=N-1 it actually IS an instanton effect, and no branches in QCD

- In that case the η' mass does not vanish for large N

- But also anomaly does not vanish, since both F,N $\to\infty$

- Which one is QCD? Does QCD with F=N have branches or not?

F=N in AMSB QCD

- The F=N,N+1 special cases

- Only consider mesonic VEV, assume other branches OK

- F=N

$$W = X\left(\frac{\det(M) - \bar{B}B}{\Lambda^{2N}} - 1\right) + m_Q \mathrm{Tr}(M)$$

$$K = \frac{\mathrm{Tr}(M^\dagger M)}{\alpha|\Lambda|^2} + \frac{X^\dagger X}{\beta|\Lambda|^4} + \frac{\bar{B}^\dagger \bar{B}}{\gamma|\Lambda|^{2N-2}} + \frac{B^\dagger B}{\delta|\Lambda|^{2N-2}}$$

- Resulting η' potential

$$V = -2|\Lambda|^2(|\Lambda|^2 + (N-2)m^2)\cos\left(N\eta' - \theta\right) - 2m|\Lambda|^2 \sum_{i=1}^{N} m_i \cos\left((N-1)\eta' - \theta_Q - \theta - \sum_{j=1}^{N-1} t_i^j \pi^j\right)$$
$$-4m|\Lambda|^2 \sum_{i=1}^{N} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{N-1} t_i^j \pi^j\right).$$

- No branches - looks like an instanton effect!

η’ potential for F≥N

- The F=N,N+1 special cases

- No branches - looks like an instanton effect! E.g. F=N

$$V = -2|\Lambda|^2(|\Lambda|^2 + (N-2)m^2)\cos\left(N\eta' - \theta\right) - 2m|\Lambda|^2 \sum_{i=1}^{N} m_i \cos\left((N-1)\eta' - \theta_Q - \theta - \sum_{j=1}^{N-1} t_i^j \pi^j\right)$$
$$- 4m|\Lambda|^2 \sum_{i=1}^{N} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{N-1} t_i^j \pi^j\right) .$$

- F>N+1 - Seiberg duality. Dual quarks will be massive, get again gaugino condensation

- We will get very similar results as for F<N, with N-F↔F-N

$$V_k = -4(3N-2F)\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{F/(2N-F)} m|\Lambda|^3 \cos\left(\frac{F}{F-N}\eta' - \frac{\theta+2\pi k}{F-N}\right)$$
$$- 2F\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\frac{N}{F-N}\eta' - \theta_Q - \frac{\theta+2\pi k}{F-N} - \sum_{j=1}^{F-1} t_i^j \pi_j\right)$$
$$- \frac{4FN}{2F-3N}\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi_j\right) ,$$

Spontaneous CP breaking

M. Ruhdorfer, T. Youn and C.C.

- $\mathrm{Tr} G\tilde{G}$ odd under CP - a θ term generically breaks CP explicitly (strong CP problem)

- However for $\theta = \pi$ the effect of CP is $\theta = \pi \leftrightarrow \theta = -\pi$

- But these two are related by 2π shift in θ, so CP is not explicitly broken here. But is it spontaneously broken?

- Claim: in ordinary QCD it may.

- Results of Gaiotto, Kapustin, Komargodski, Seiberg (2017-18) + di Vecchia, Rossi, Veneziano, Yankielowicz (2017)

Spontaneous CP breaking

- Also for large N Witten 1979-80, di Vecchia, Veneziano 1980
- For F=0 CP spontaneoulsy broken. Also holds at finite N (Zohar et al - anomaly matching arguments)
- For F>1 and all masses equal CP also spontaneously broken
- For F=1 there is a critical mass at which second order phase transition
- Similarly for F>1 unequal masses critical surface

F=1 in AMSB QCD theory

- F=1 (implying F<N) the potential is

$$V = \frac{1}{2}\left|\frac{2}{\phi}\left(\frac{\Lambda^{3N-1}}{\phi^2}\right)^{\frac{1}{N-1}} - 2\phi\, m_Q\right|^2 - m\left[(3N-1)\left(\frac{\Lambda^{3N-1}}{\phi^2}\right)^{\frac{1}{N-1}} + \phi^2 m_Q\right] + \text{h.c.}\,.$$

- where $M = \phi^2$ and $K = 2\,\phi^\dagger\phi$.
- Since under CP $M \xrightarrow{\text{CP}} M^\dagger$
  want to separate out phase $\phi = f\exp(i\delta_f)$
- Potential to minimize:

$$V_\ell = 2\,x^{\frac{1-N}{N}}|\Lambda|^{\frac{3N-1}{N}}\left(m_Q^2 + x^2 - m m_Q\cos(2\delta_f) - m(3N-1)x\cos\left[\frac{2\delta_f - \bar{\theta} - 2\pi\ell}{N-1}\right]\right.$$
$$\left. -2m_Q x\cos\left[\frac{2N\delta_f - \bar{\theta} - 2\pi\ell}{N-1}\right]\right)\,, \quad \ell = 0, 1, \ldots, N-2\,,$$

F=1 in AMSB QCD theory

- Here $x = |\Lambda| \left(\frac{|\Lambda|}{f}\right)^{\frac{2N}{N-1}}$. For N=2,3 can analytically find minimum. From this find critical masses

$$\left.\frac{m_{Q,0}}{m}\right|_{N=2} = \frac{3}{2}\cos\left(\frac{1}{3}\arccos\left(-\frac{25}{27}\right) - \frac{2\pi}{3}\right) \approx 0.576511$$

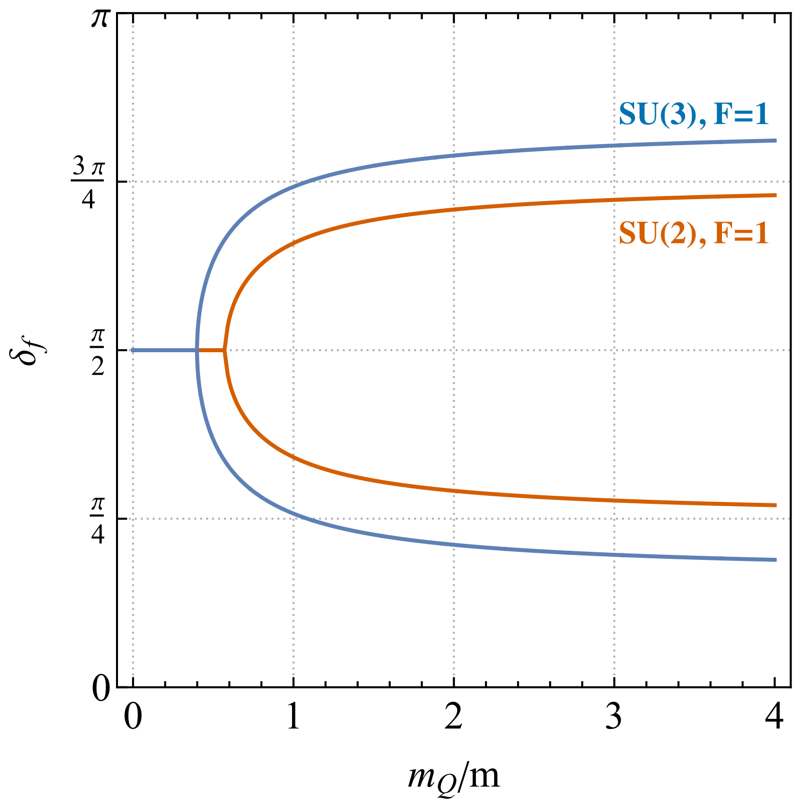
$$\left.\frac{m_{Q,0}}{m}\right|_{N=3} = \frac{14}{9}\cos\left(\frac{1}{3}\arccos\left(-\frac{289}{343}\right) - \frac{2\pi}{3}\right) - \frac{1}{9} \approx 0.398853$$

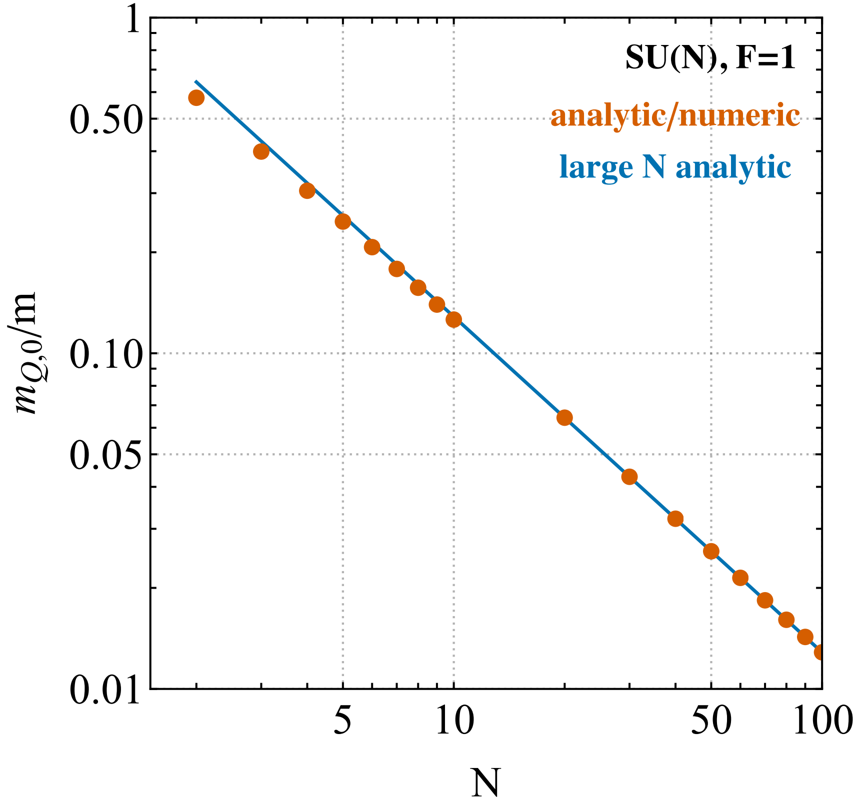
- Can also find for large N limit

$$\left.\frac{m_{Q,0}}{m}\right|_{N\gg 1} = \frac{9}{7}\frac{1}{N} + \mathcal{O}\left(\frac{1}{N}\right)^2$$

- However for N>3 only numerical solution

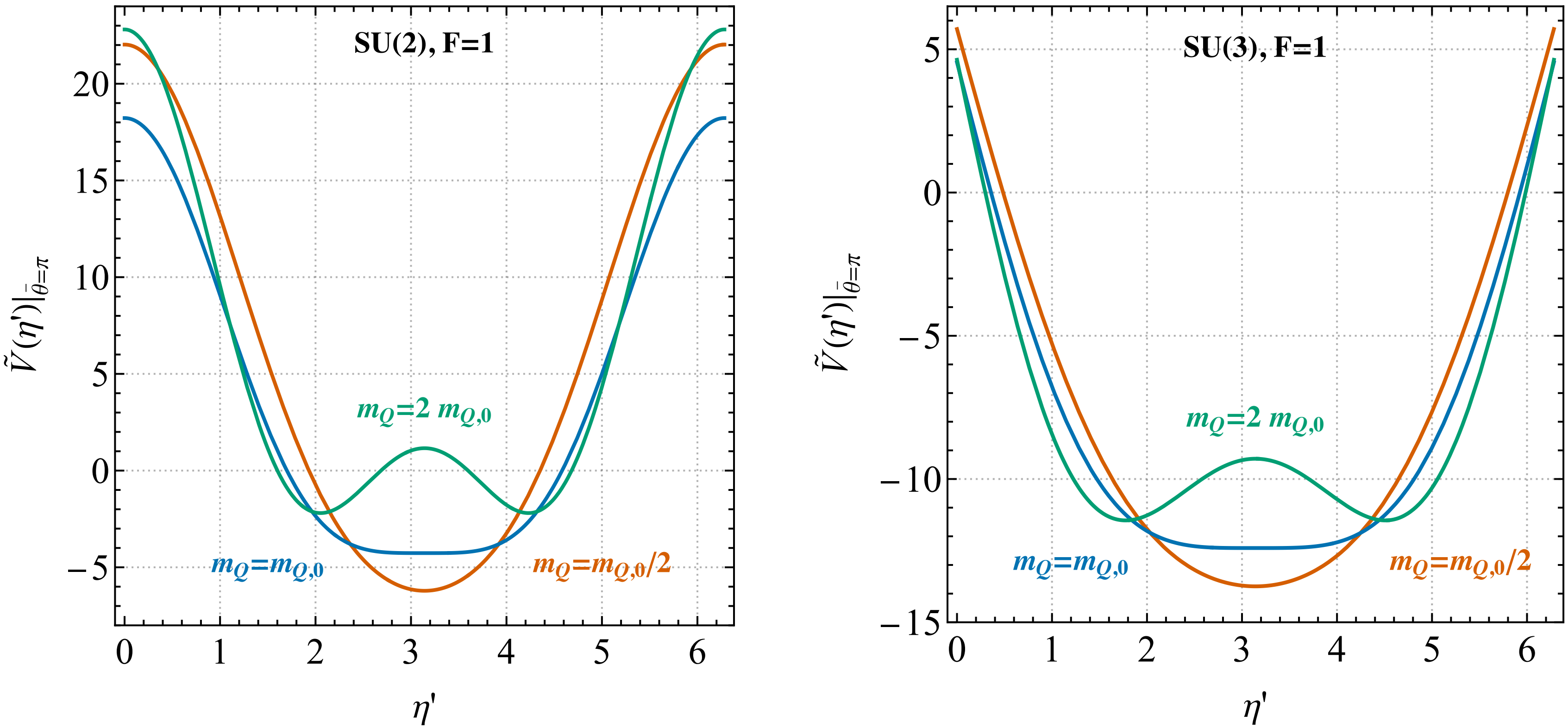
F=1 in AMSB QCD theory





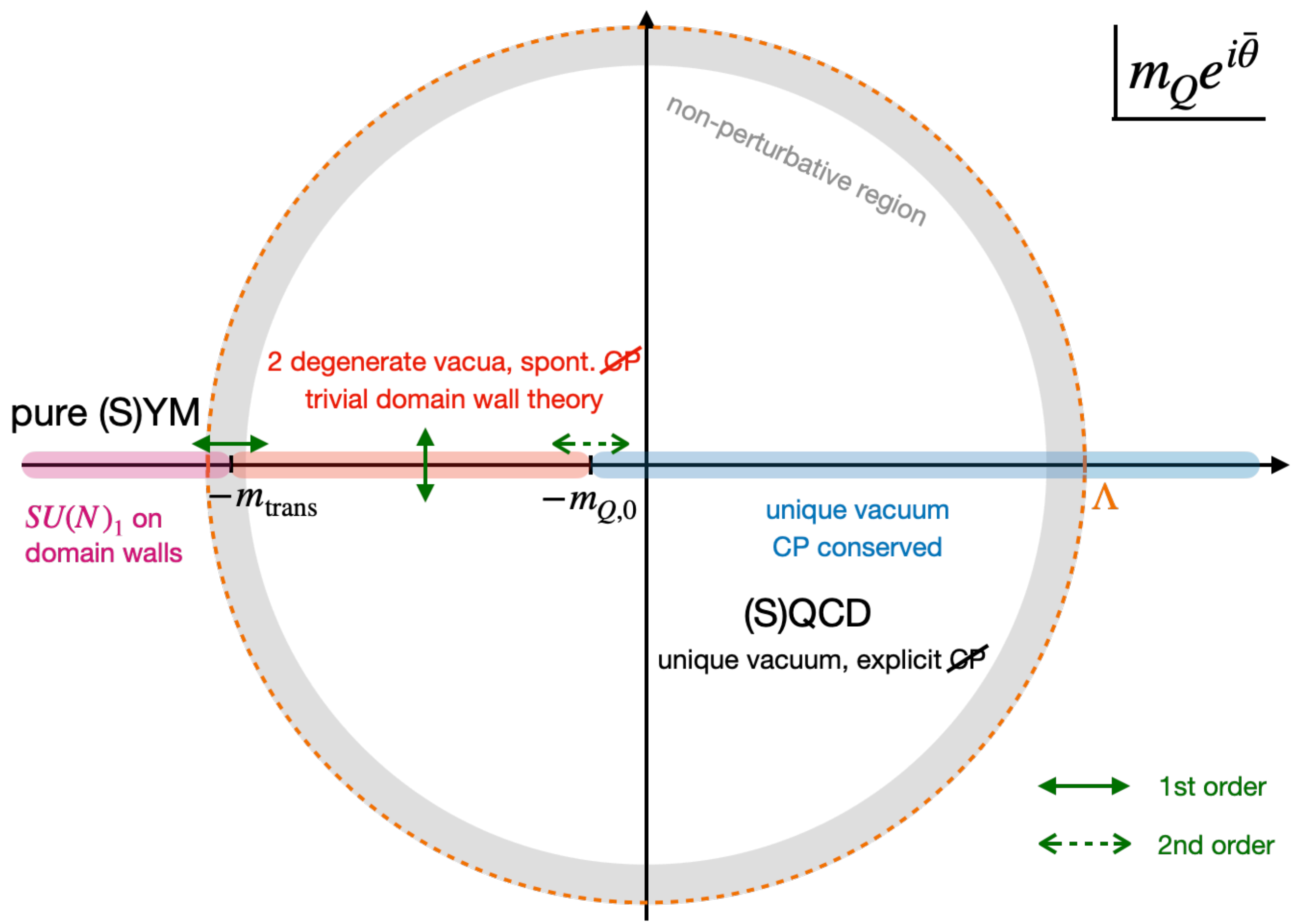
The phase structure for F=1

F=1 in AMSB QCD theory

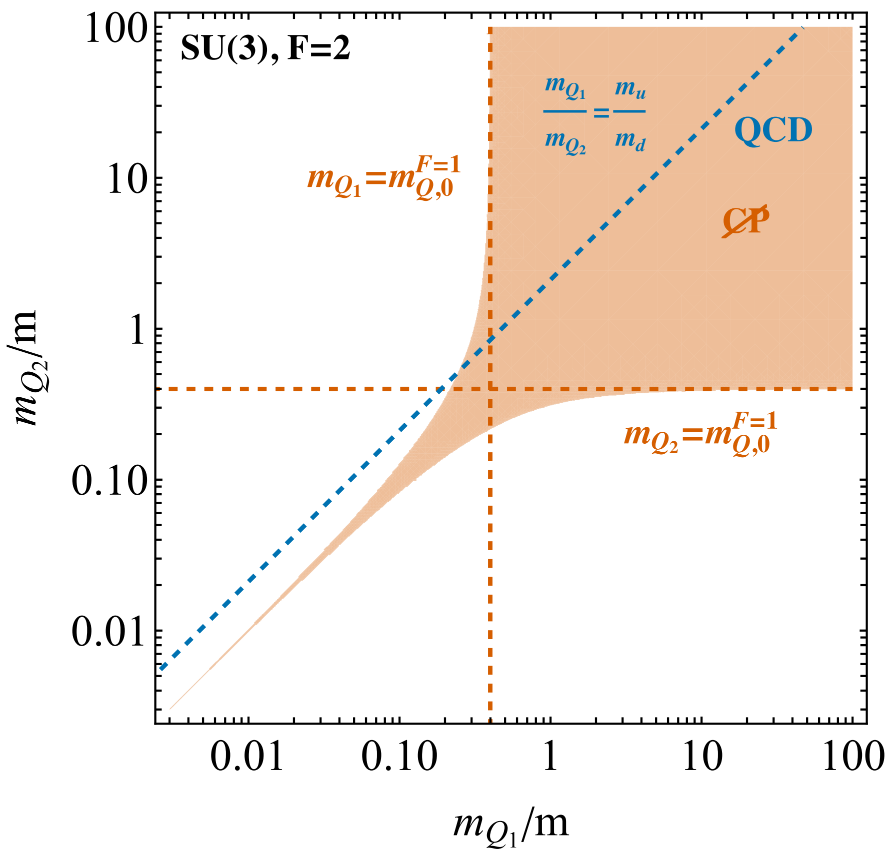


- The **η'** potential as we pass through the critical mass
- At the critical mass **η'** itself becomes massless

F=1 phase structure

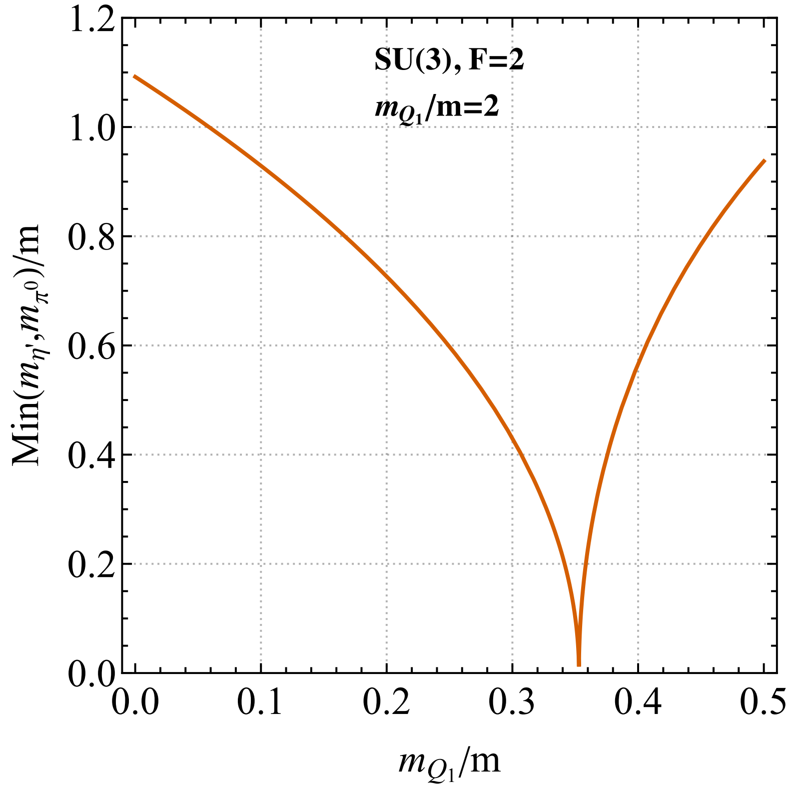


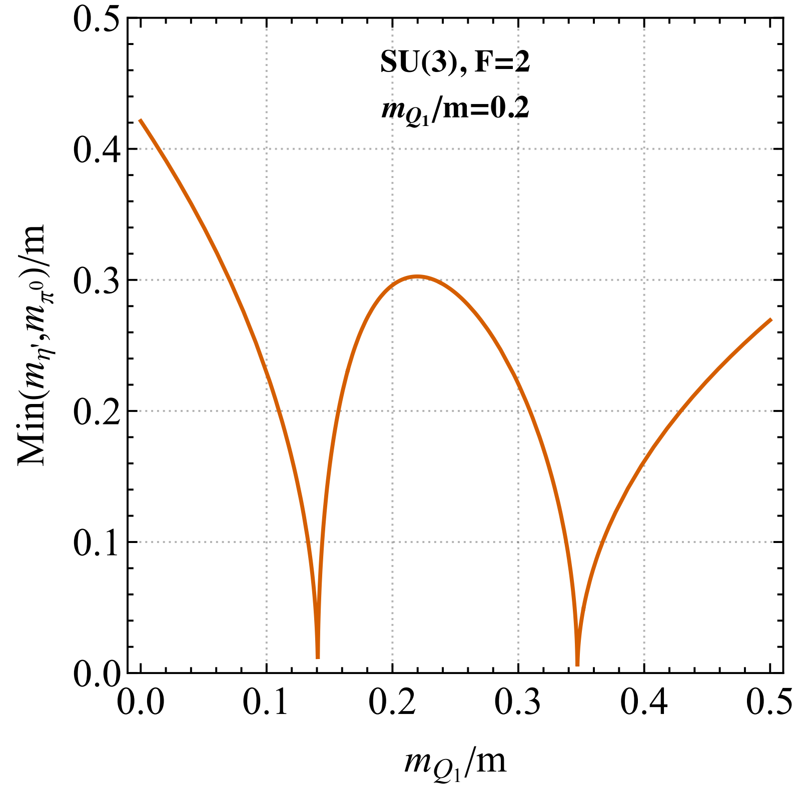
Phase structure for F>1



- For equal masses CP always broken
- Unequal masses have phase boundary - critical mass reappears in decoupling limit
- QCD-like mass ratios?

The phase boundaries





- The η' becomes massless at the phase boundaries
- Second order PT

Meson physics in AMSB QCD

T. Roy, M. Ruhdorfer, T. Youn and C.C.

- Once we add the quark masses, can also study
    - Detailed structure of VEVs
    - Meson masses - corrections to GMO
    - Meson decays

- Take F=3 and use either F<N (easier but less dynamics) or F=N (harder but more similar to actual QCD dynamics)

$$F = 3\text{: } F < N \text{ and } F = N \text{ \& } \Lambda \gg m > m_s > m_d > m_u$$

Meson physics in AMSB QCD

- Leading order in quark masses:
  - All VEVs equal
  - Gell-Mann Okubo satisfied

Mass / Theory	$F < N$	$F = N$
$m^2_{\pi^0}$	$(m_d + m_u)A$	$(m_d + m_u)B$
$m^2_{\pi^\pm}$	$(m_d + m_u)A$	$(m_d + m_u)B$
$m^2_{K^0}$	$(m_d + m_s)A$	$(m_d + m_s)B$
$m^2_{K^\pm}$	$(m_u + m_s)A$	$(m_u + m_s)B$
m^2_{η}	$\frac{1}{3}(m_d + m_u + 4m_s)A$	$\frac{1}{3}(m_d + m_u + 4m_s)B$
$m^2_{\eta'}$	$\frac{2}{3}(m_d + m_u + m_s)A$	$\frac{6\beta\Lambda^2}{m} - \frac{6m}{\alpha} - \frac{4}{3}(m_d + m_u + m_s)B$

$$A = \frac{\Lambda^3}{f^2}\left(\frac{\Lambda}{f}\right)^{\frac{6}{N-3}} + \frac{m}{2}, \quad B = \frac{\alpha^2 X}{2\Lambda^2} + \alpha m$$

Meson physics in AMSB QCD

- Corrections to quark condensates:

$$f_i = f^0 + m_s f_i^1 + m_q f_i^2 + \Delta m_q f_i^3 + \cdots$$

$$m_q \equiv m_d + m_u, \qquad \Delta m_q \equiv m_d - m_u$$

For $F < N$

$$f_d \simeq \left(1 + \frac{m_d}{18}\right)\frac{\Lambda^{3/2}}{\sqrt{3m}}, \quad f_u \simeq \left(1 + \frac{m_u}{18}\right)\frac{\Lambda^{3/2}}{\sqrt{3m}}, \quad f_s \simeq \left(1 + \frac{m_s}{18}\right)\frac{\Lambda^{3/2}}{\sqrt{3m}}$$

For $F = N$

$$f_d \simeq \left(1 - \frac{m^2}{6\alpha\beta\Lambda^2} + \frac{m(m_s + m_q)}{18\beta\Lambda^2} - \alpha\frac{m_s - m_q/2}{12m} + \alpha\frac{\Delta m_q}{8m}\right)\Lambda$$

$$f_u \simeq \left(1 - \frac{m^2}{6\alpha\beta\Lambda^2} + \frac{m(m_s + m_q)}{18\beta\Lambda^2} - \alpha\frac{m_s - m_q/2}{12m} - \alpha\frac{\Delta m_q}{8m}\right)\Lambda$$

$$f_s \simeq \left(1 - \frac{m^2}{6\alpha\beta\Lambda^2} + \frac{m(m_s + m_q)}{18\beta\Lambda^2} + \alpha\frac{m_s - m_q/2}{6m}\right)\Lambda$$

Meson physics in AMSB QCD

- For both cases

$$\frac{f_d}{f_u} = 1 - \frac{\Delta m_q}{m_s - m_q/2}\left(1 - \frac{f_s}{f_u}\right) + \cdots$$

- In agreement with ChPT Gasser/Leutwyler

$$\frac{\langle 0|\bar{d}d|0\rangle}{\langle 0|\bar{u}u|0\rangle} = 1 - \frac{m_d - m_u}{m_s - \hat{m}}\left\{1 - \frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle} + \frac{1}{16\pi^2 F_0^2}\left(M_K^2 - M_\pi^2 - M_\pi^2 \ln \frac{M_K^2}{M_\pi^2}\right)\right\}$$

- Corrections to Gell-Mann Okubo sum rule

For $F < N$

$$\Delta_{\mathrm{GMO}}^{\mathrm{EXP}} \sim -70000\ \mathrm{MeV}^2$$

$$\Delta_{\mathrm{GMO}} = -\left(\frac{7}{27} + \frac{1}{648}\frac{m_q}{m}\right) m_s^2 + \frac{7}{27} m_s m_q \sim -2000\ \mathrm{MeV}^2$$

For $F = N$

$$\Delta_{\mathrm{GMO}} = \left(\alpha^2 + \frac{\alpha^3}{4}\frac{m_q}{m} - \frac{7\alpha^2}{36\beta}\frac{m m_q}{\Lambda^2}\right) m_s^2 + \left(\frac{\alpha^2}{2} + \frac{7\alpha^2}{36\beta}\frac{m m_q}{\Lambda^2}\right) m_s m_q$$

Δ_{GMO} is negative for $F < N$, while not for $F = N$ unless $m \gg \Lambda$, which is ordinary QCD

Heavy quark physics?

- Can calculate meson decays

- Semi-leptonic decays: ChPT:

$$\langle \partial_\mu U^\dagger \partial^\mu U \rangle \to \langle \partial_\mu U^\dagger G_F [Q_W] \bar{\ell} \gamma^\mu \nu U \rangle \to G_F (\partial_\mu K \pi)(\bar{\ell} \gamma^\mu \nu)$$

$$\partial^\mu \to D^\mu = \partial^\mu + G_F Q_W J_W^\mu \qquad J_W^\mu = \bar{\ell} \gamma^\mu \nu$$

- In AMSB QCD:

$$\langle M^\dagger M \rangle \to \langle M^\dagger e^{Q_W J_W} M e^{-Q_W J_W} \rangle \to G_F (\partial_\mu K \pi)(\bar{\ell} \gamma^\mu \nu)$$

- In progress, also want to look at hadronic decays

Meson physics in AMSB QCD

- Some of the issues in heavy quark physics can be addressed - those the don't involve mixing with generic heavy QCD states

- Charm decay constant

$$f_q^2 = \left(\frac{m_c}{m_*^{N-F+1}} \right)^{\frac{1}{N}} \Lambda^{\frac{3N-F}{N}}, \qquad f_c^2 = \frac{m_q}{m_c} f_q^2$$

- η_c - η' mixing

$$\sin\theta_\eta = \sqrt{\frac{m_*}{m_c}} \text{ for } F < N$$

$$\sin\theta_\eta = \frac{m_*}{m_c} \text{ for } F = N \text{ and } N+1 < F < \frac{3}{2}N$$

$$\sin\theta_\eta = \frac{1}{\sqrt{2}} \text{ for } F = N+1$$

- η_c , η' masses

For $F < N$ and $N+1 < F < \frac{3}{2}N$

$$m_{\eta'}^2 \sim m_*^2 \, , \, m_{\eta_c}^2 \sim m_c^2$$

For $F = N, N+1$

Heavily depends on the form of Kähler potentials

$m_{\eta'}^2 \sim \Lambda^2 \, , \, m_{\eta_c}^2 \sim \Lambda^2$ or $m_{\eta'}^2 \sim m_c^2 \, , \, m_{\eta_c}^2 \sim m_c^2$

Need a better understanding

Summary

- Softly broken SUSY theories lab for studying confinement and QCD physics
- AMSB is UV insensitive, produces QCD-like phase structure
- η' potential: for most cases not instanton induced, except for special cases F=N-1,N,N+1
- Reproduce the structure of CP phases at $\theta=\pi$
- Can study some meson physics - vacuum structure, mass sum rules, semi-leptonic decays, heavy charm physics,….

Happy birthdays - and please more physics!!!!