

My paper with Lawrence and Hitoshi



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Sneutrino cold dark matter with lepton-number violation ¹

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Abstract

The tau sneutrino is proposed as a candidate for galactic halo dark matter, and as the cold dark matter (CDM) component of the universe. A lepton-number-violating sneutrino mass, $\tilde{\nu}\tilde{\nu}$, splits the tau sneutrino into two mass eigenstates: $\tilde{\nu} \rightarrow \tilde{\nu}_{\pm}$. The absence of a $Z\tilde{\nu}_-\tilde{\nu}_-$ coupling implies that the lighter mass eigenstate, $\tilde{\nu}_-$, does not annihilate via the s -channel Z -exchange to a low cosmological abundance, and furthermore, halo sneutrinos do not scatter excessively in Ge detectors. For the majority of the relevant parameter space, the event rate in Ge detectors is $\geq 10^{-2}$ events/kg/day. The lepton number violation required for sneutrino CDM implies that the tau neutrino mass is $m_{\nu_\tau} \geq 5$ MeV, large enough to be excluded by B factory experiments. Events of the form l^+l^-H or jjH , with low m_{ll} or m_{jj} , may be observed at LEP2. A seesaw mechanism is investigated as the origin for the lepton number violation, and several other cosmological and particle physics consequences of sneutrino CDM are discussed. © 1998 Elsevier Science B.V.

⇒ I learned a lot from Lawrence and Hitoshi about DM (and other) physics

DM Detection using Superconducting Qubits

Takeo Moroi (U. Tokyo)

Chen, Fukuda, Inada, TM, Nitta, Sichanugrist

arXiv 2212.03884 [PRL 131 (2023) 211001]

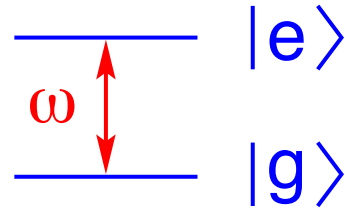
arXiv 2311.10413 [PRL 133 (2023) 021801]

arXiv 2407.19755

Workshop in Honor of Lawrence and Hitoshi, Berkeley, CA, '24.09.26 – 28

1. Introduction

What I discuss today: DM search with quantum bits (qubits)



Qubit: Two-level quantum system

- Qubit is an essential component for quantum computers
- Various types of qubits have been proposed and realized
- Qubits are excellent quantum sensors for DM detection

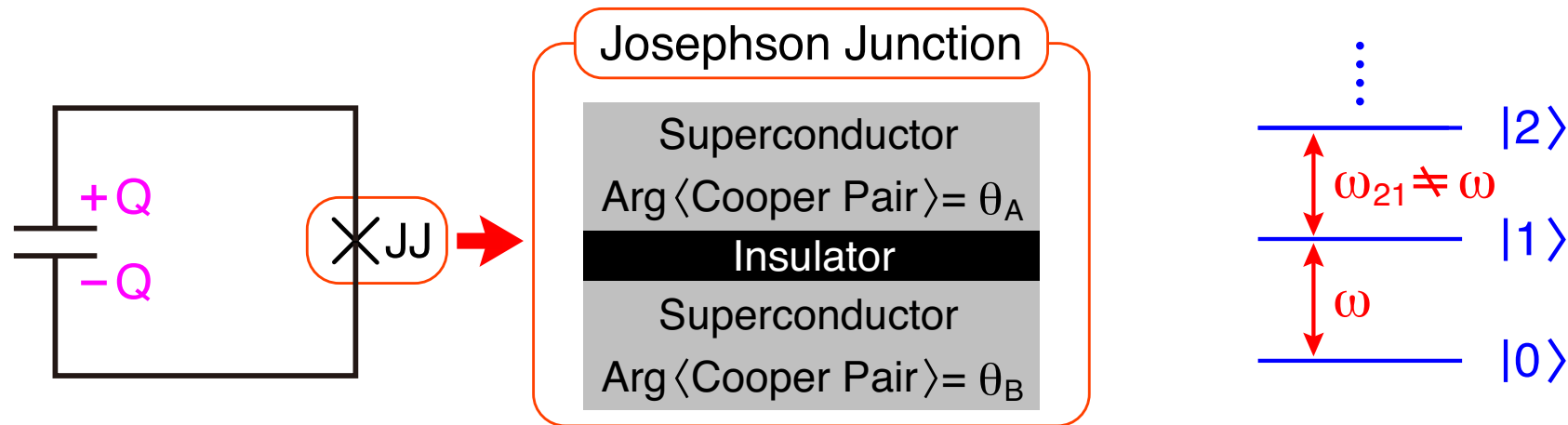
[Dixit et al. ('21); Chen, Fukuda, Inada, TM, Nitta, Sichanugrist ('22, '23, '24); Engelhardt, Bhoonah, Liu ('23); Chigusa, Hazumi, Herbschleb, Mizuochi, Nakayama ('23); Agrawal et al. ('23); Ito, Kitano, Nakano, Takai ('23); Braggio et al. ('24)]

Outline:

1. Introduction
2. Superconducting Qubit
3. DM Detection with Qubits
4. Cavity Effects / Quantum Enhancement
5. Experimental Status
6. Summary and Outlook

2. Superconducting Qubit

Superconducting qubit: Capacitor + Josephson junction (JJ)



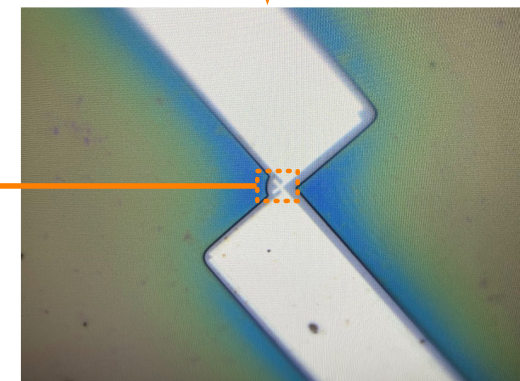
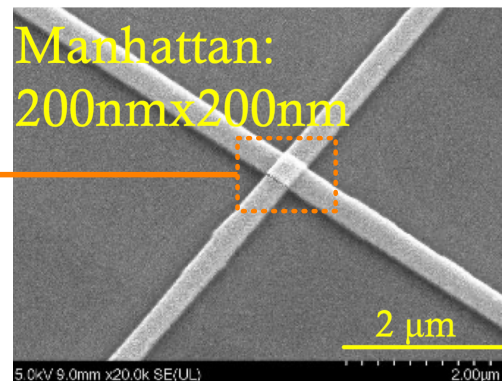
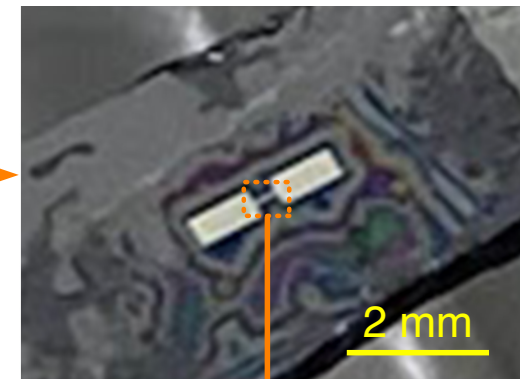
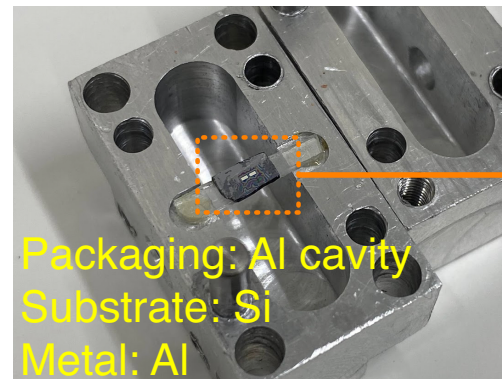
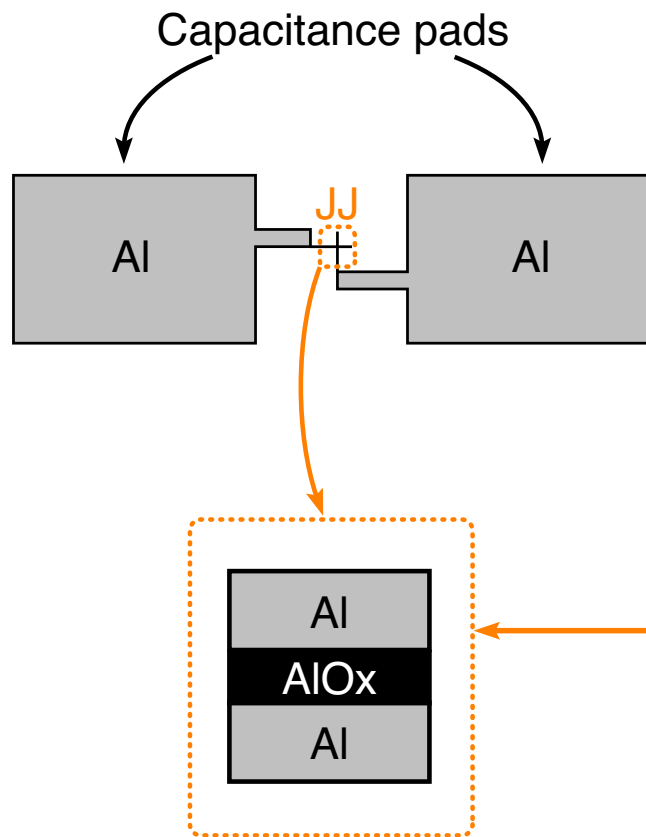
$\theta = \theta_B - \theta_A$: canonical variable of this system

$$\Rightarrow H_0 = \frac{1}{2C}Q^2 - J \cos \theta \simeq \frac{1}{2} \frac{C}{(2e)^2} \dot{\theta}^2 - J \cos \theta \Leftrightarrow Q = CV \simeq C \frac{\dot{\theta}}{2e}$$

Superconducting qubit has discrete energy levels

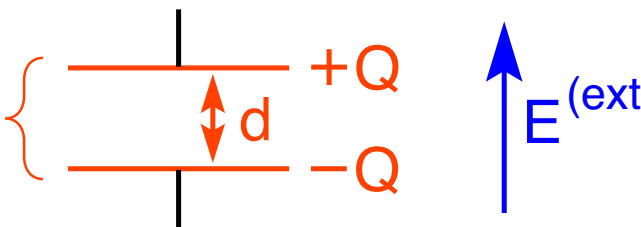
$\Rightarrow |0\rangle$ and $|1\rangle$ can be used as $|g\rangle$ and $|e\rangle$, respectively

Qubit developed by our colleagues (prototype)



- 2D object, fabricated on the surface of a substrate
- Operated with very low temperature $\sim O(10)$ mK

Superconducting qubit couples to external electric field

Capacitor  $\Leftrightarrow H_{\text{int}} = QdE^{(\text{ext})}$

Charge operator in the transmon limit: $CJ \gg (2e)^2$

$$Q \simeq \frac{C}{2e} \dot{\theta} \simeq \sqrt{\frac{C\omega}{2}} (|g\rangle\langle e| + |e\rangle\langle g|)$$

$|g\rangle \leftrightarrow |e\rangle$ transition occurs if DM field generates electric field

- Hidden photon
- Axion (if external magnetic field exists)
- ...

3. DM Detection with Qubits

AC electric field due to oscillating DM field:

$$E^{(\text{DM})} = \bar{E} \cos(m_X t + \alpha) \quad \text{with } m_X = \text{DM mass}$$

Hamiltonian for qubit + DM system

$$H = \omega |e\rangle\langle e| - 2\eta \cos(m_X t + \alpha) (|e\rangle\langle g| + |g\rangle\langle e|)$$

$$\eta \simeq \frac{1}{2\sqrt{2}} d\sqrt{C\omega}\bar{E}$$

Schrödinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle \quad \Rightarrow \quad |\psi(t)\rangle = U_{\text{DM}}(t) |\psi(0)\rangle$$

$$|\psi(t)\rangle \equiv \psi_g(t) |g\rangle + e^{-i\omega t} \psi_e(t) |e\rangle$$

Resonance limit $\omega = m_X$ (for $\eta t \ll 1$)

$$\begin{pmatrix} \psi_g(t) \\ \psi_e(t) \end{pmatrix} = U_{\text{DM}}(t) \begin{pmatrix} \psi_g(0) \\ \psi_e(0) \end{pmatrix} \simeq \begin{pmatrix} 1 & ie^{-i\alpha}\eta t \\ ie^{i\alpha}\eta t & 1 \end{pmatrix} \begin{pmatrix} \psi_g(0) \\ \psi_e(0) \end{pmatrix}$$

$|g\rangle \rightarrow |e\rangle$ transition probability (assuming $|\psi(0)\rangle = |g\rangle$)

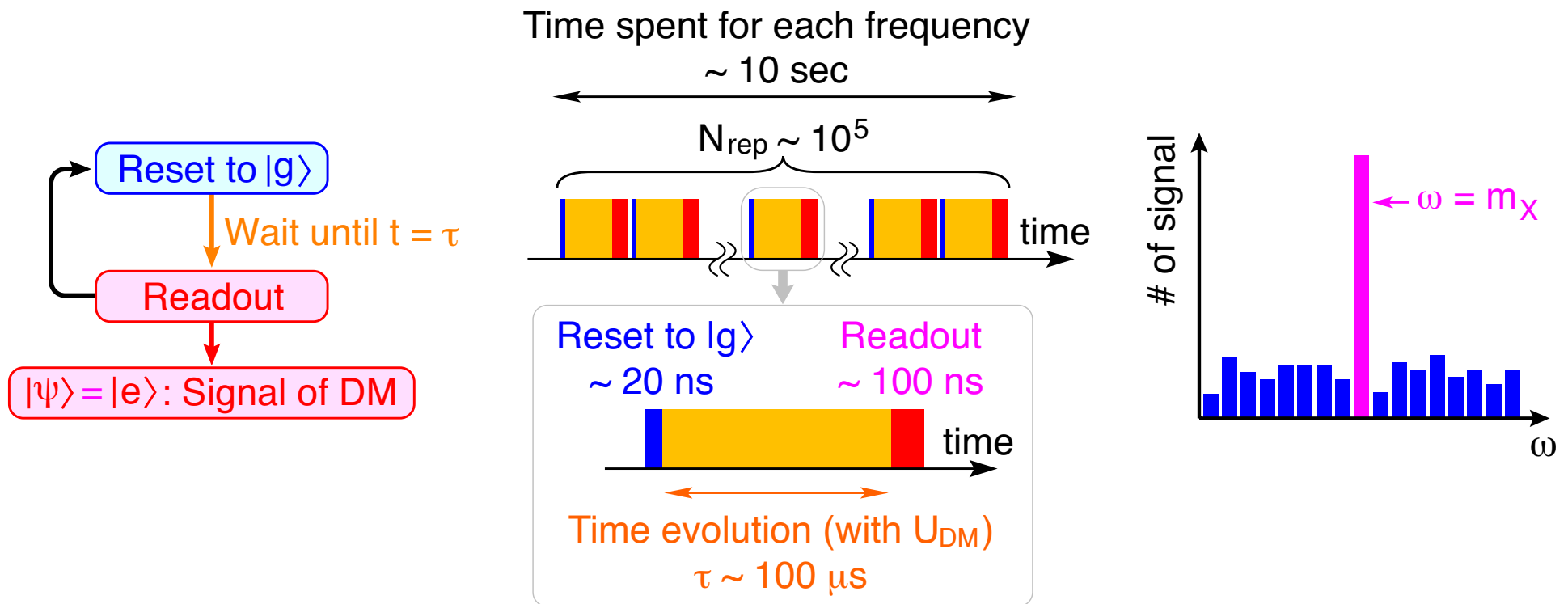
$$|\psi_e(t)|^2 \simeq \begin{cases} \eta^2 t^2 & : \omega = m_X \text{ (on-resonance)} \\ \sim \eta^2 (\omega - m_X)^{-2} & : \omega \neq m_X \text{ (off-resonance)} \end{cases}$$

Excitation can be the signal of wave-like DM

- When $\omega \simeq m_X$, the transition rate is proportional to t^2
 \Rightarrow We should take t as long as the coherence time τ
- DM mass is unknown, so we should scan the frequency

Search strategy (with frequency-tunable SQUID qubits)

- For fixed ω , repeat the measurement cycle (reset, wait, and readout) as many time as possible
- Scan the qubit frequency ω



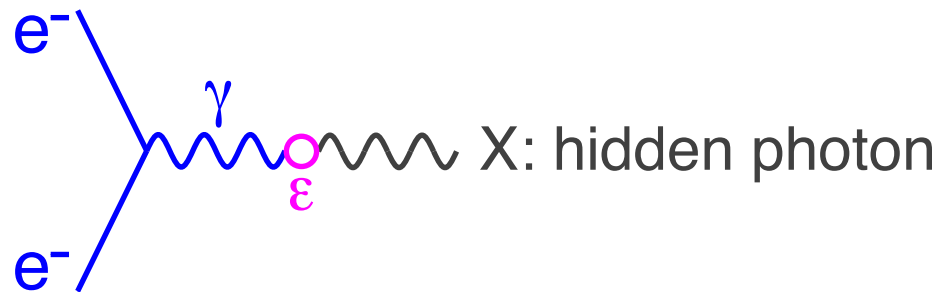
One of possible targets: hidden photon X_μ

$$\mathcal{L} \ni -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{2}\epsilon F_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu$$

$F_{\mu\nu}$: EM field

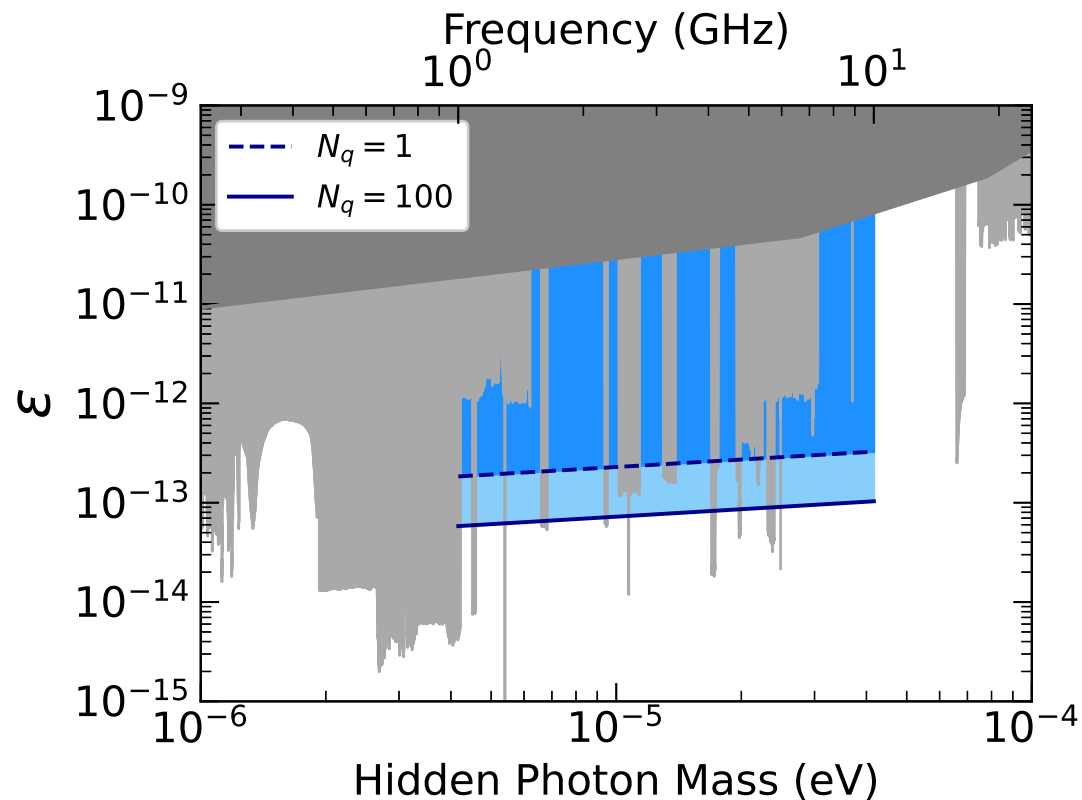
Hidden photon DM induces effective electric field

$$\vec{X} \simeq \bar{X} \vec{n} \sin(m_X t + \alpha) \quad \text{with} \quad \rho_{\text{DM}} = \frac{1}{2}m_X^2 \bar{X}^2$$



$$\vec{E}^{(\text{DM})} = -\epsilon \dot{\vec{X}} = -\epsilon m_X \bar{X} \vec{n} \cos(m_X t + \alpha) \Leftrightarrow |\vec{E}^{(\text{DM})}| = \epsilon \sqrt{2\rho_{\text{DM}}}$$

Hidden photon DM: 1 year frequency scan ($1 \leq f \leq 10$ GHz)



- $d = 100 \mu\text{m}$

- $C = 0.1 \text{ pF}$

- $Q = 10^6$

- Error rate / qubit = 0.1 %

⇔ For $C = 0.1 \text{ pF}$ and $d = 100 \mu\text{m}$:

$$p_{g \rightarrow e} \simeq 0.1 \times \left(\frac{\epsilon}{10^{-11}} \right)^2 \left(\frac{f}{1 \text{ GHz}} \right) \left(\frac{\tau}{100 \mu\text{s}} \right)^2$$

Axion DM detection with qubits

Magnetic field is necessary to convert axion to electric field

$$\mathcal{L}_{\text{int}} = g_{a\gamma\gamma} a \vec{E} \vec{B} \Rightarrow \vec{E} \simeq g_{a\gamma\gamma} a \langle \vec{B}^{(\text{ext})} \rangle$$

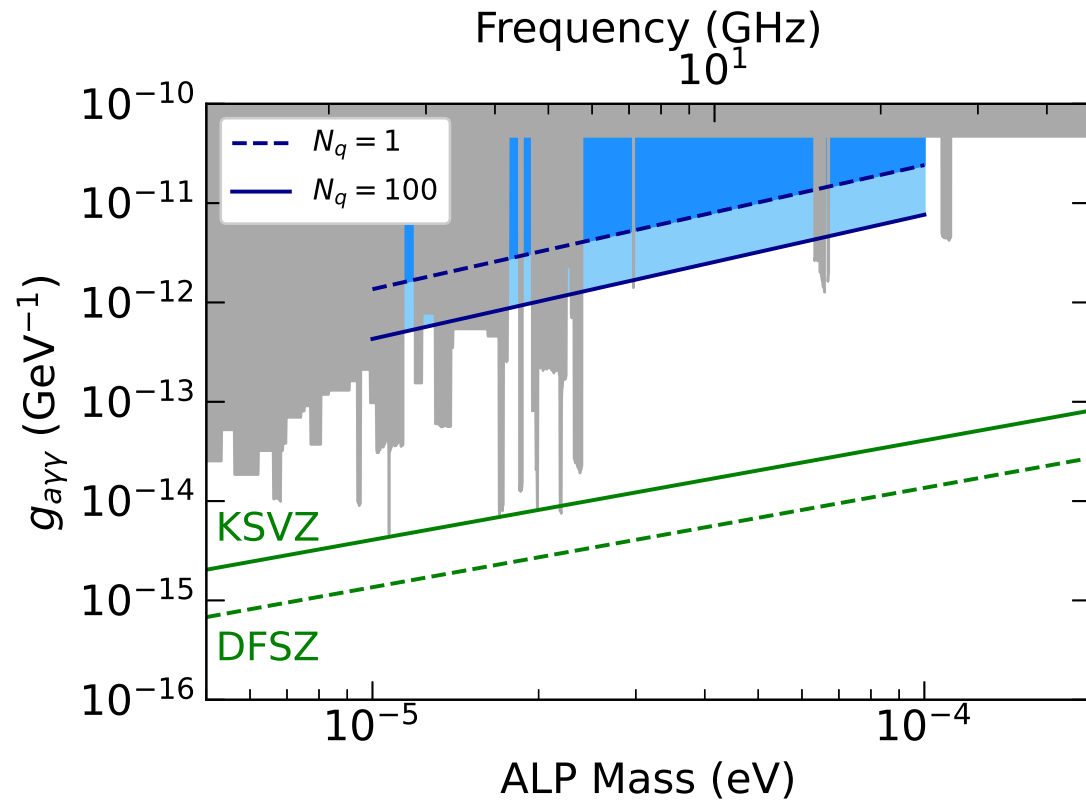
Magnetic field onto the superconductor may be a concern

- Transmon qubit is fabricated on the surface of Si substrate (2D object)
- Transmon qubit works with magnetic field of ~ 1 T, if the magnetic field is parallel to the surface

[Krause et al., 2111.01115]

\Leftrightarrow More detailed study is underway

Axion DM search: 1-year scan



• $B = 5 \text{ T}$

\Leftrightarrow For $C = 0.1 \text{ pF}$ and $d = 100 \text{ }\mu\text{m}$:

$$p_{g \rightarrow e} \simeq 0.1 \times \left(\frac{g_{a\gamma\gamma}}{10^{-10} \text{ GeV}^{-1}} \right)^2 \left(\frac{m_a}{1 \text{ }\mu\text{eV}} \right)^{-1} \left(\frac{B}{1 \text{ T}} \right)^2 \left(\frac{\tau}{100 \text{ }\mu\text{s}} \right)^2$$

4. Cavity Effect / Quantum Enhancement

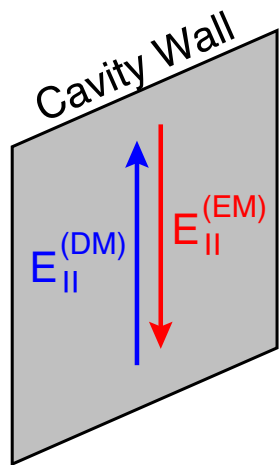
Signal rate can be enhanced with cavity effect

⇔ Qubits are usually set in a “cavity”

Effective electric field: $\vec{E}^{(\text{eff})} \equiv \vec{E}^{(\text{DM})} + \vec{E}^{(\text{EM})} \equiv \kappa \vec{E}^{(\text{DM})}$

$$\square \vec{E}^{(\text{EM})} = 0 \quad \text{and} \quad \vec{\nabla} \cdot \vec{E}^{(\text{EM})} = 0$$

$$[\vec{E}_{\parallel}^{(\text{EM})} + \vec{E}_{\parallel}^{(\text{DM})}]_{\text{wall}} = 0 \quad \Leftrightarrow \quad \vec{E}_{\parallel}^{(\text{eff})} = 0 \text{ at the cavity wall}$$

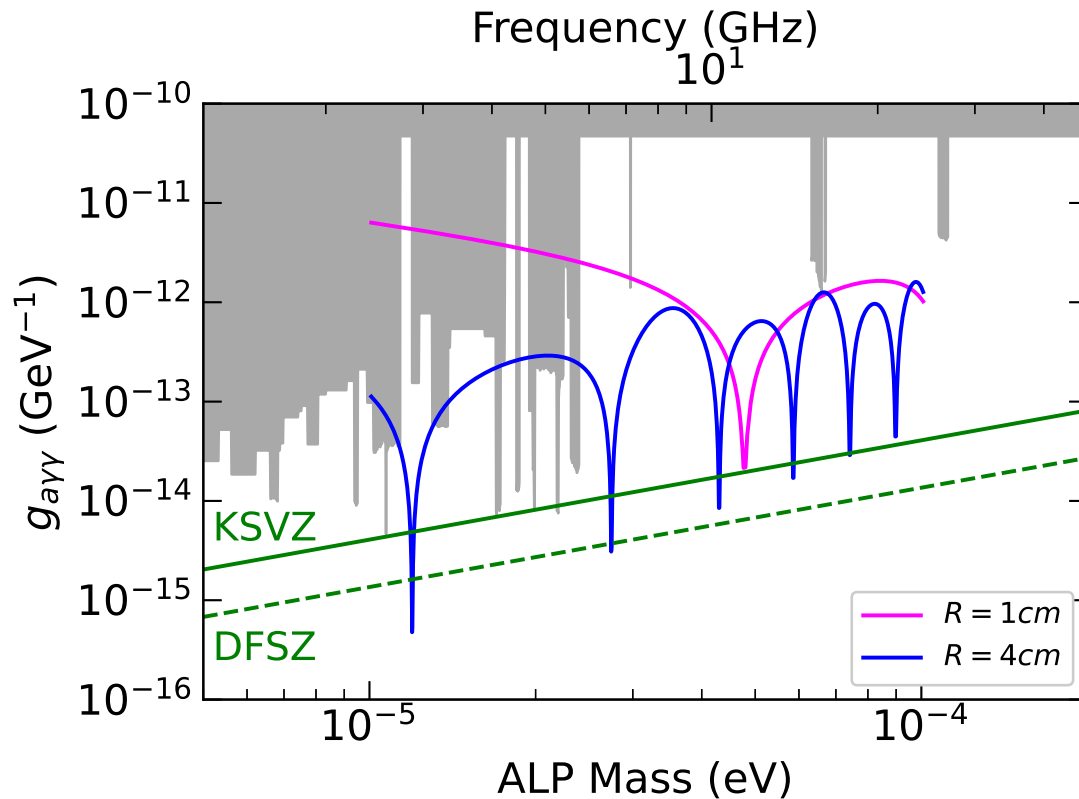


$$\Rightarrow H_{\text{int}} = QdE^{(\text{eff})}$$

$$\Rightarrow p_{g \rightarrow e} \propto \kappa^2$$

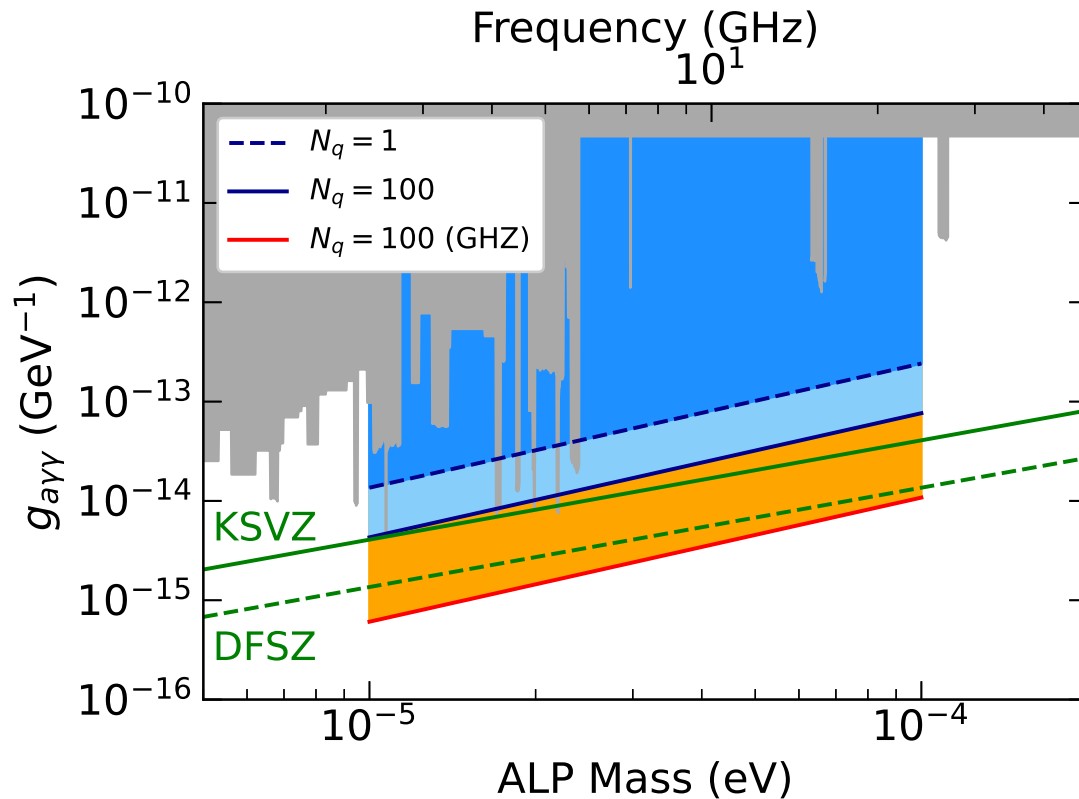
⇒ $\kappa \gg 1$, if m_{DM} is close to one of cavity frequencies

Axion DM search: with fixed cavity geometry (cylinder)



- $B = 5\text{ T}$
- $N_q = 1$
- Error rate / qubit = 0.1 %

Axion DM search: with frequency-tunable cavity



• $B = 5 \text{ T}$

• $\kappa = 100$

⇒ In order to always realize $|\kappa| \gg 1$, cavity frequency should be tuned during the frequency scan

Signal rate can be $O(N_q^2)$ with quantum operations

We may perform quantum operations onto qubits

\Rightarrow “DM detection with quantum computers”

U_{DM} induces pure phase rotation of its eigenstates

E.g. for $\alpha = 0$: $U_{\text{DM}} \simeq \begin{pmatrix} \cos \delta & i \sin \delta \\ i \sin \delta & \cos \delta \end{pmatrix}$ with $\delta \equiv \eta\tau$

$\Rightarrow U_{\text{DM}}|\pm\rangle = e^{\pm i\delta}|\pm\rangle$ with $|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$

$\Rightarrow U_{\text{DM}}^{\otimes N_q}|\pm\rangle^{\otimes N_q} = e^{\pm iN_q\delta}|\pm\rangle^{\otimes N_q}$

We can design the following quantum (unitary) operation

$$U_{\text{GHZ}} : \begin{pmatrix} |g\rangle^{\otimes N_q} \\ |e\rangle^{\otimes N_q} \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} |+\rangle^{\otimes N_q} + |-\rangle^{\otimes N_q} \\ |+\rangle^{\otimes N_q} - |-\rangle^{\otimes N_q} \end{pmatrix}$$

Starting with $|g\rangle^{\otimes N_q}$:

1. Apply U_{GHZ} : $\frac{1}{\sqrt{2}} (|+\rangle^{\otimes N_q} + |-\rangle^{\otimes N_q})$

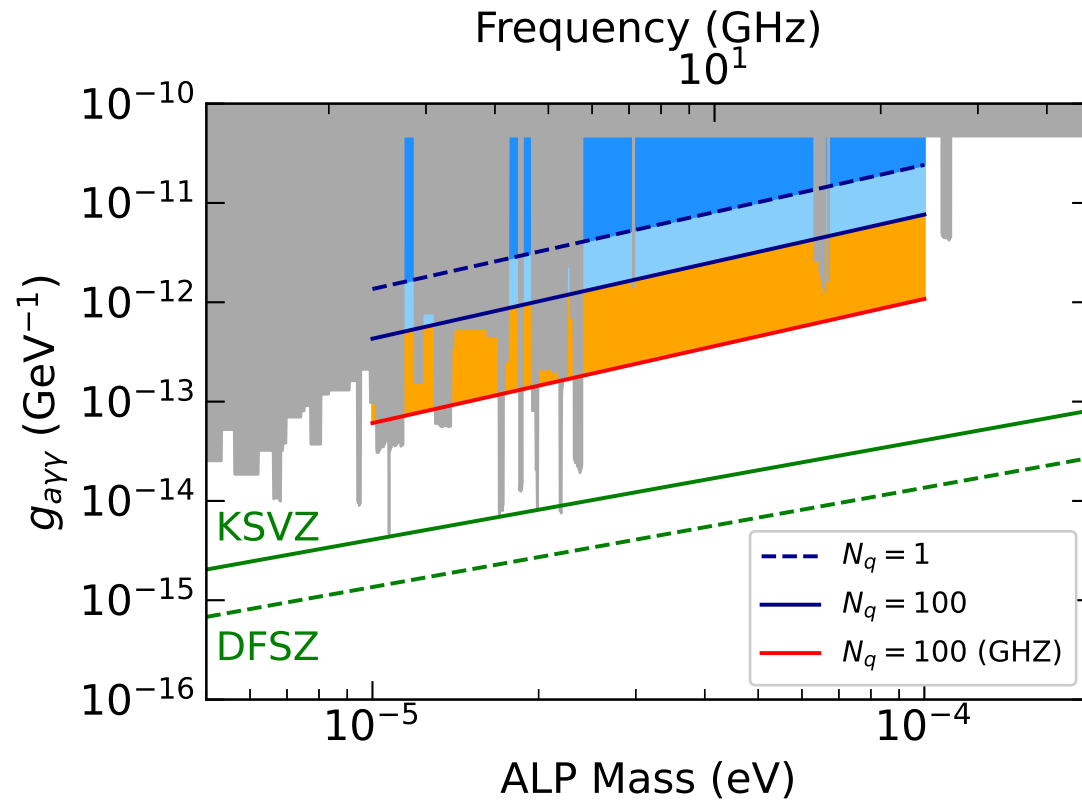
2. Evolution with DM: $\frac{1}{\sqrt{2}} (e^{iN_q\delta} |+\rangle^{\otimes N_q} + e^{-iN_q\delta} |-\rangle^{\otimes N_q})$

3. Apply U_{GHZ}^{-1} : $\cos N_q\delta |g\rangle^{\otimes N_q} + i \sin N_q\delta |e\rangle^{\otimes N_q}$

Transition probability:

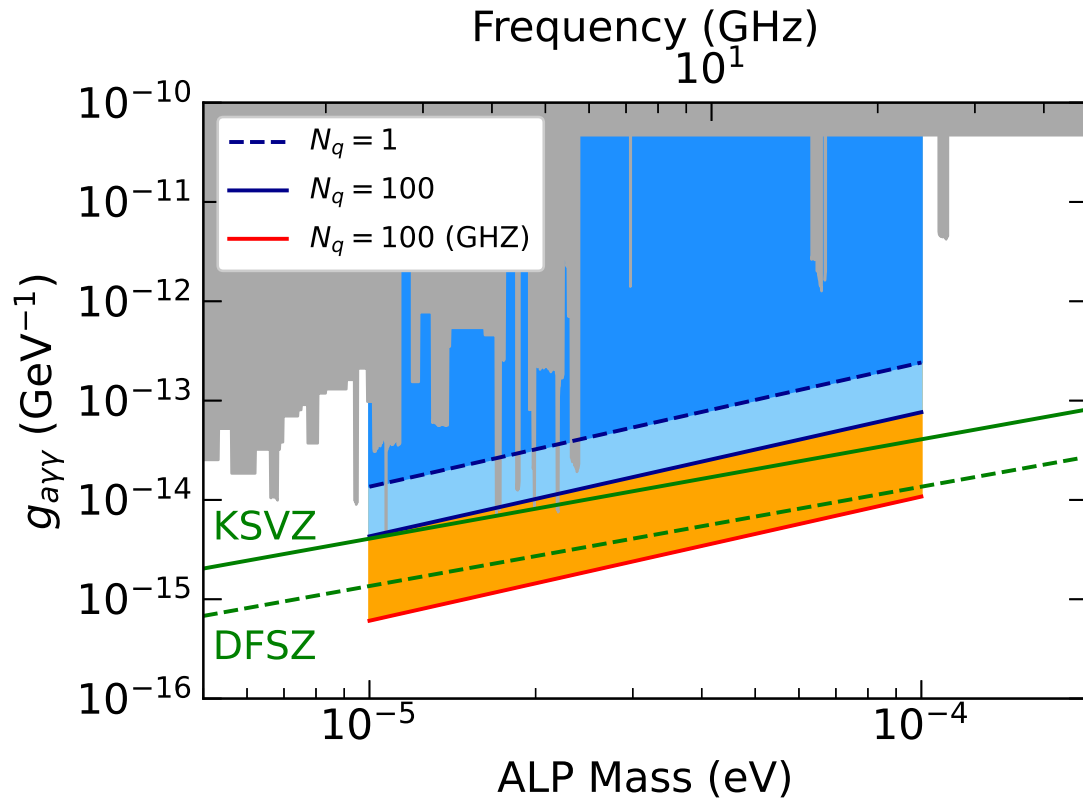
$$p_{g \rightarrow e} = \sin^2 N_q\delta \simeq N_q^2\delta^2 \quad \Rightarrow \quad \frac{S}{\sqrt{B}} \propto N_q^{3/2}\delta^2$$

Axion DM search: 1-year scan with the entangled state



- $\kappa = 1$
- $B = 5 \text{ T}$
- Error rate / qubit = 0.1 %

Axion DM search: 1-year scan with the entangled state



- $\kappa = 100$
- $B = 5 \text{ T}$
- Error rate / qubit = 0.1 %

- We need reliable quantum gates
- Frequencies of all the qubits should be equal

5. Experimental Status

Now, the real search experiment is in progress

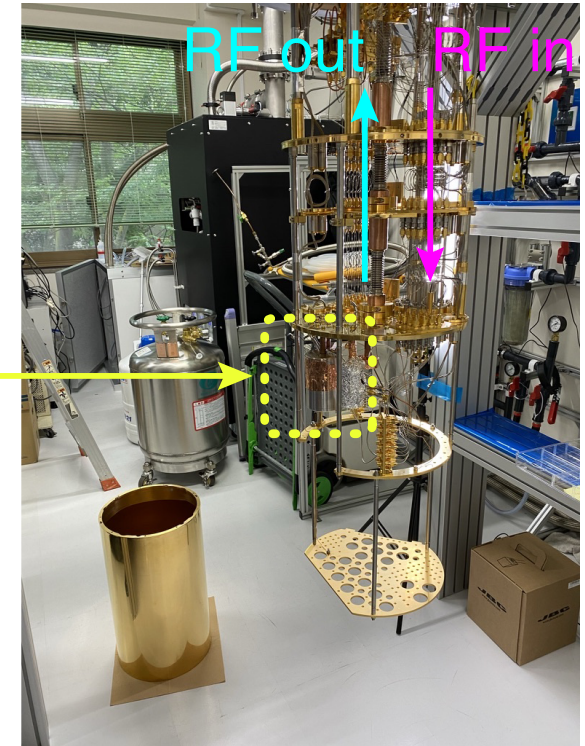
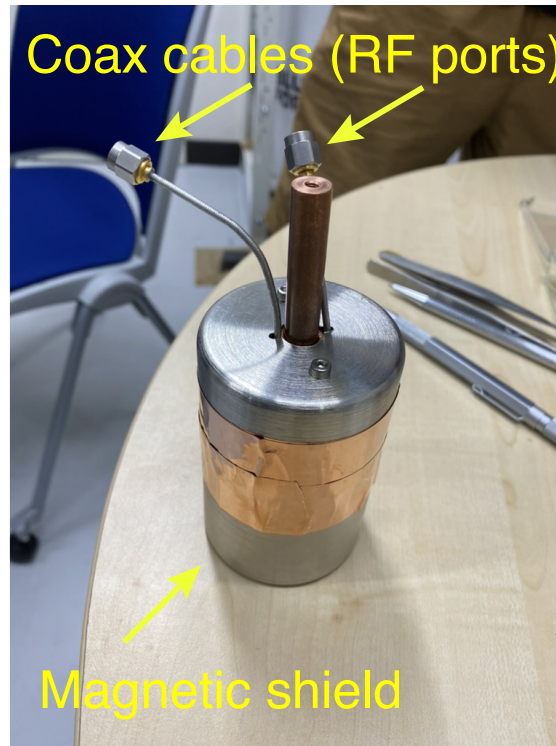
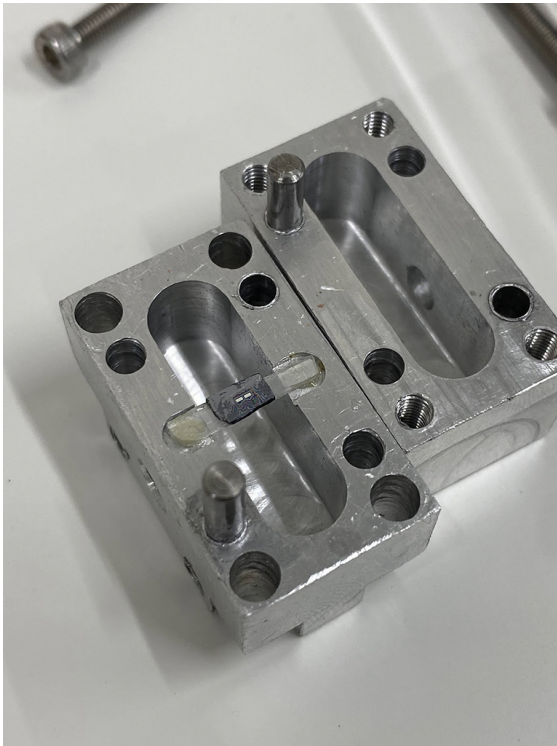
Ex.: Watanabe, Nakazono, Nitta, Chen, Inada, Terashi, Sawada, Shirai,
Noguchi

Th.: Fukuda, TM, Sichanugrist

- Qubit development is underway
 - ⇒ Currently, $\tau_q \sim O(1 - 10) \mu\text{sec}$
- Dilution refrigerator is available
- The first detection run was performed this summer
 - ⇒ The result will come out soon

Setup into the dilution refrigerator

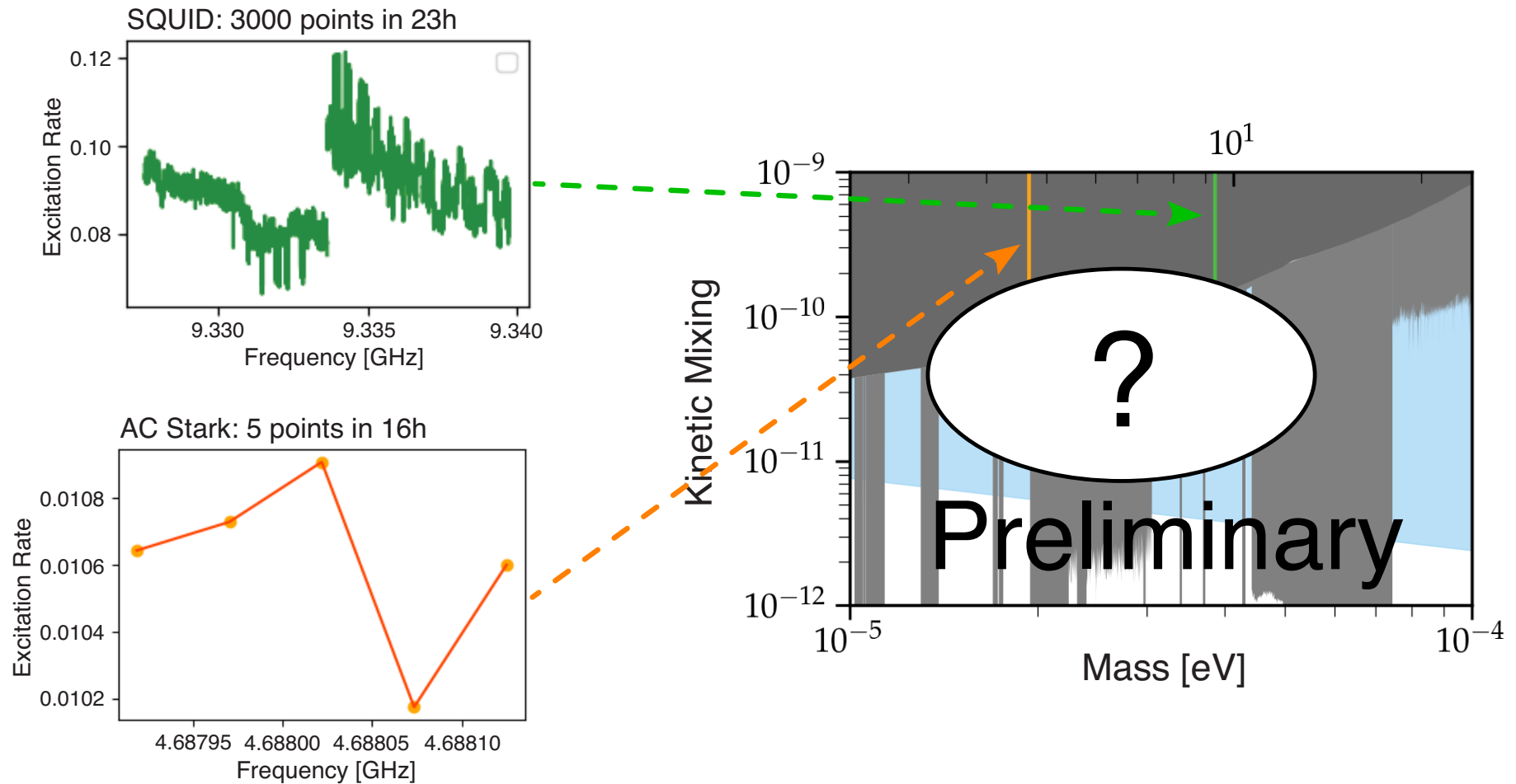
[Cryogenic Research Center, U.Tokyo]



Encapsulate

Attach to the coldest part
of the fridge (~ 10 mK)

Preliminary data from the 2024 summer run



⇒ We hope to announce the first result soon

6. Summary and Outlook

Superconducting qubit is an excellent DM detector

- We may reach parameter region which is unexplored
- We may use its quantum properties to enhance the signal

The real experiment has been started

- We need to fabricate high-quality qubit
- First run was performed this summer
- Effects of the magnetic field is under study (for axion DM detection)
- We hope to announce the first result soon, so stay tuned!

Thank you very much for your listening and

I wish to celebrate great achievements of
Lawrence and Hitoshi

Backup: Hidden Photon DM

Case of hidden photon X_μ

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} \epsilon F'_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu$$

$F'_{\mu\nu}$: EM field (in gauge eigenstate)

Vector bosons in the mass eigenstates

$$A_\mu \simeq A'_\mu - \epsilon X_\mu \text{ and } X_\mu$$

Interaction with electron

$$\mathcal{L}_{\text{int}} = e \bar{\psi}_e \gamma^\mu A'_\mu \psi_e = e \bar{\psi} \gamma^\mu \psi (A_\mu + \epsilon X_\mu)$$

Hidden photon as dark matter

$$\vec{X} \simeq \bar{X} \vec{n}_X \cos m_X t$$

Energy density of hidden photon DM

$$\rho_{\text{DM}} = \frac{1}{2} \dot{\vec{X}}^2 + \frac{1}{2} m_X^2 \vec{X}^2 \simeq \frac{1}{2} m_X^2 \bar{X}^2$$

$$\Leftrightarrow \rho_{\text{DM}} \sim 0.45 \text{ GeV/cm}^3$$

Effective electric field induced by the hidden photon

$$\vec{E}^{(X)} = -\epsilon \dot{\vec{X}} = \bar{E}^{(X)} \vec{n}_X \sin m_X t$$

$$\bar{E}^{(X)} = \epsilon m_X \bar{X} = \epsilon \sqrt{2\rho_{\text{DM}}}$$

Backup: Transmon Qubit

Hamiltonian

$$H_0 = \frac{1}{2C} Q^2 - J \cos \theta = \frac{1}{2Z} n^2 - J \cos \theta$$

$$Z \equiv (2e)^{-2} C$$

Transmon limit: $CJ \gg (2e)^2 \Rightarrow \langle \theta^2 \rangle \ll 1$

[Koch et al. ('07); see also Roth, Ma & Chew (2106.11352)]

$$\Rightarrow H_0 = \frac{1}{2Z} n^2 + \frac{1}{2} J \theta^2 + O(\theta^4)$$

$$\hat{a} \equiv \frac{1}{\sqrt{2\omega Z}} (n - i\omega Z \theta), \quad \hat{a}^\dagger \equiv \frac{1}{\sqrt{2\omega Z}} (n + i\omega Z \theta)$$

$$\Rightarrow [\hat{a}, \hat{a}^\dagger] = 1$$

In the transmon limit, anharmonicity is small:

$$\Rightarrow |e\rangle \simeq \hat{a}^\dagger |g\rangle$$

$$\Rightarrow \omega_{21} \simeq \left(1 - \frac{1}{8} \frac{2e}{\sqrt{CJ}}\right) \omega$$

Charge operator in the transmon limit

$$Q = 2en = \sqrt{\frac{C\omega}{2}} (\hat{a} + \hat{a}^\dagger) \simeq \sqrt{\frac{C\omega}{2}} (|g\rangle\langle e| + |e\rangle\langle g|)$$

Interaction Hamiltonian

$$H_{\text{int}} = QdE^{(\text{ext})} \simeq \sqrt{\frac{C\omega}{2}} dE^{(\text{ext})} (|g\rangle\langle e| + |e\rangle\langle g|)$$

Backup: Schrödinger Equation

Effective Hamiltonian

$$H = \omega|e\rangle\langle e| + 2\eta \sin m_X t (|e\rangle\langle g| + |g\rangle\langle e|)$$

η : Small parameter

Schrödinger equation:

$$i \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

$$|\Psi(t)\rangle \equiv \psi_g(t)|g\rangle + e^{-i\omega t} \psi_e(t)|e\rangle$$

$$\Rightarrow i \frac{d}{dt} \begin{pmatrix} \psi_g \\ \psi_e \end{pmatrix} = 2\eta \sin m_X t \begin{pmatrix} 0 & e^{-i\omega t} \\ e^{i\omega t} & 0 \end{pmatrix} \begin{pmatrix} \psi_g \\ \psi_e \end{pmatrix}$$

Solution with $|\Psi(0)\rangle = |g\rangle$ (for $|\omega \pm m_X|^{-1} \ll t \ll \eta^{-1}$)

$$\psi_g(t) \simeq 1 + O(\eta^2)$$

$$\psi_e(t) \simeq \eta \left(\frac{e^{i(\omega - m_X)t} - 1}{i(\omega - m_X)} - \frac{e^{i(\omega + m_X)t} - 1}{i(\omega + m_X)} \right)$$

Resonance limit: $\omega \rightarrow m_X$

$$\Rightarrow \psi_e(t) \rightarrow \eta t + (\text{non-growing})$$

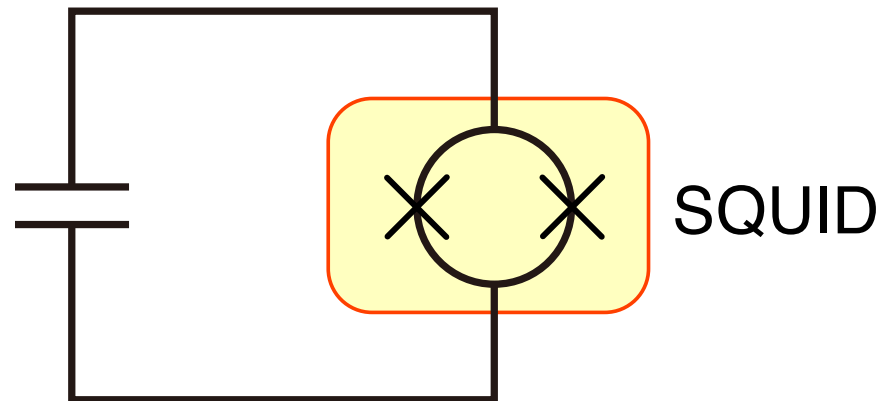
$|g\rangle \rightarrow |e\rangle$ transition rate (for $t \ll \eta^{-1}$)

$$P_{ge} = |\psi_e(t)|^2 \simeq \begin{cases} \sim \eta^2 (\omega - m_X)^{-2} & : \omega \neq m_X \\ \eta^2 t^2 & : \omega = m_X \end{cases}$$

Backup: Frequency Scan

Frequency scan

Frequency scan is possible with qubit consisting of SQUID and capacitor

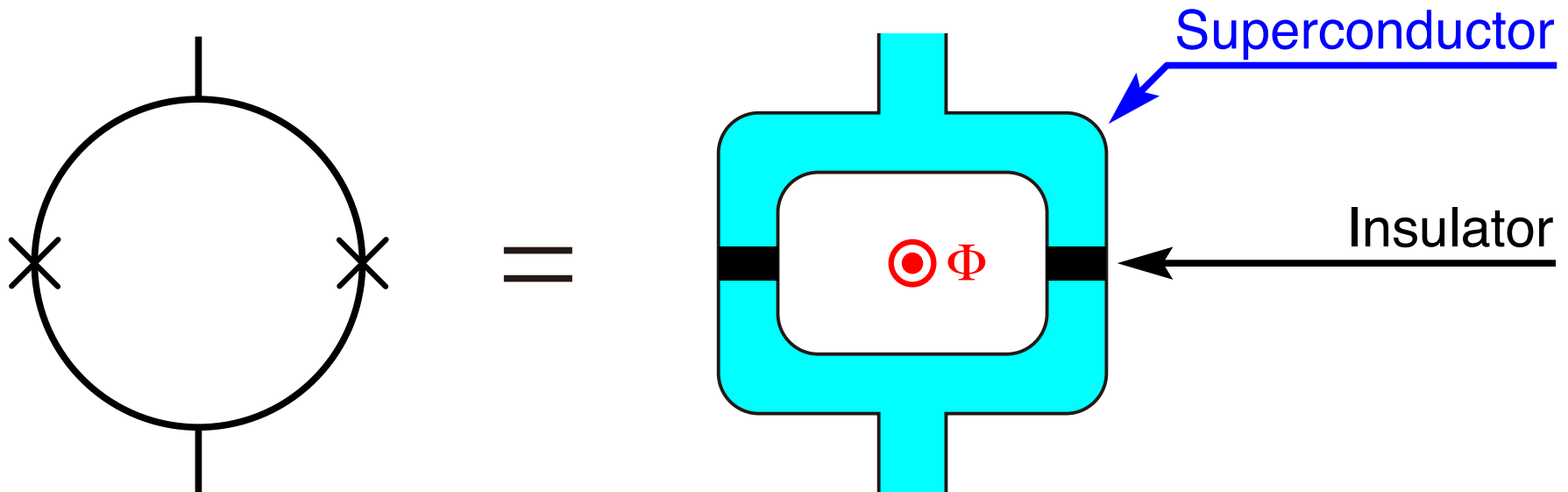


SQUID: superconducting quantum interference device

- Quantum device sensitive to magnetic flux
- With SQUID, the qubit frequency ω can be changed

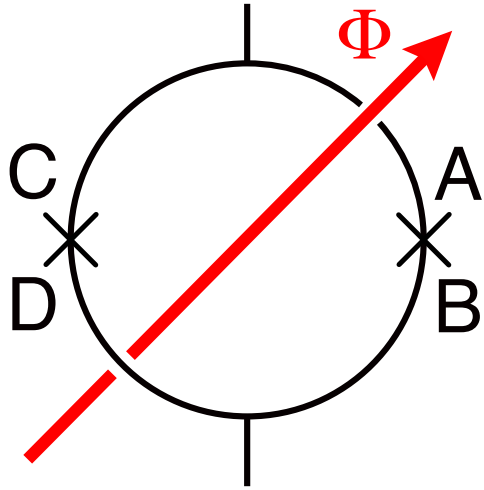
SQUID

- Loop-shaped superconductors separated by insulating layers



- We consider the case with external magnetic flux Φ going through the loop

Phases in the presence of magnetic flux



$$\theta_C - \theta_A = (2e) \int_{A \rightarrow C} \vec{A}(\vec{x}) d\vec{x}$$

$$\theta_B - \theta_D = (2e) \int_{D \rightarrow B} \vec{A}(\vec{x}) d\vec{x}$$

$$\theta_{BA} - \theta_{DC} = (2e) \oint \vec{A}(\vec{x}) d\vec{x} = (2e) \Phi = \frac{2\pi}{\Phi_0} \Phi$$

$$\theta_{YX} = \theta_Y - \theta_X$$

$$\Phi_0 = \frac{h}{2e}: \text{magnetic flux quantum}$$

Define: $\theta \equiv (\theta_{BA} + \theta_{DC})/2$

$$H_{\text{SQUID}} \simeq -J (\cos \theta_{BA} + \cos \theta_{DC}) = -2J \cos(e\Phi) \cos \theta$$

Based on the previous analysis with $J \rightarrow 2J \cos(e\Phi)$

$$\omega \simeq \sqrt{\frac{2J}{Z} \cos(e\Phi)}$$

$$Z = (2e)^{-2} C$$

The excitation energy depends on Φ

\Rightarrow Frequency scan is possible with varying the external magnetic field

Backup: Quantum Circuit

Basic unitary operations (quantum gates)

- Z gate

$$Z = |g\rangle\langle g| - |e\rangle\langle e| \Rightarrow |+\rangle \xrightarrow{Z} |-\rangle \quad \text{with} \quad |\pm\rangle \equiv \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$$

- Hadamard gate

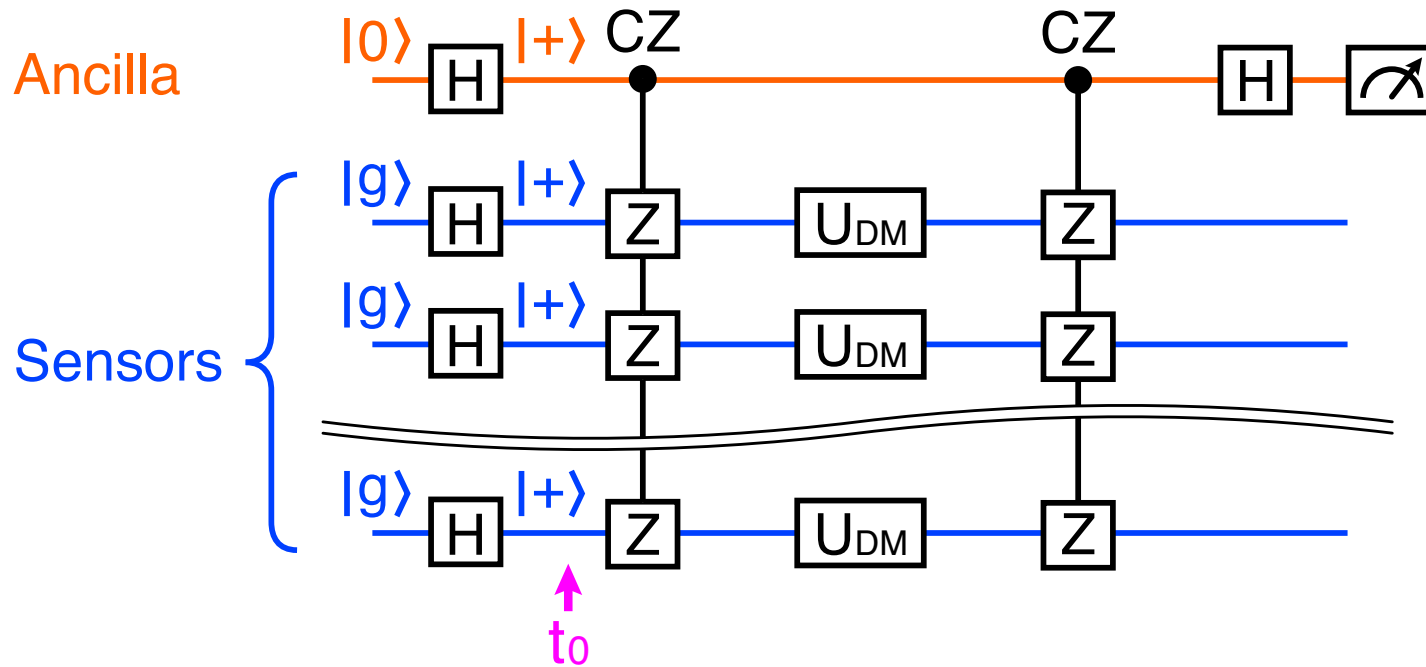
$$H = |+\rangle\langle g| + |-\rangle\langle e| \Rightarrow |g\rangle \xrightarrow{H} |+\rangle, \quad |e\rangle \xrightarrow{H} |-\rangle$$

- Controlled Z gate

$$CZ = |0\rangle\langle 0| \otimes \mathbf{1} + |1\rangle\langle 1| \otimes Z$$

$$\Rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |+\rangle \xrightarrow{CZ} \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle$$

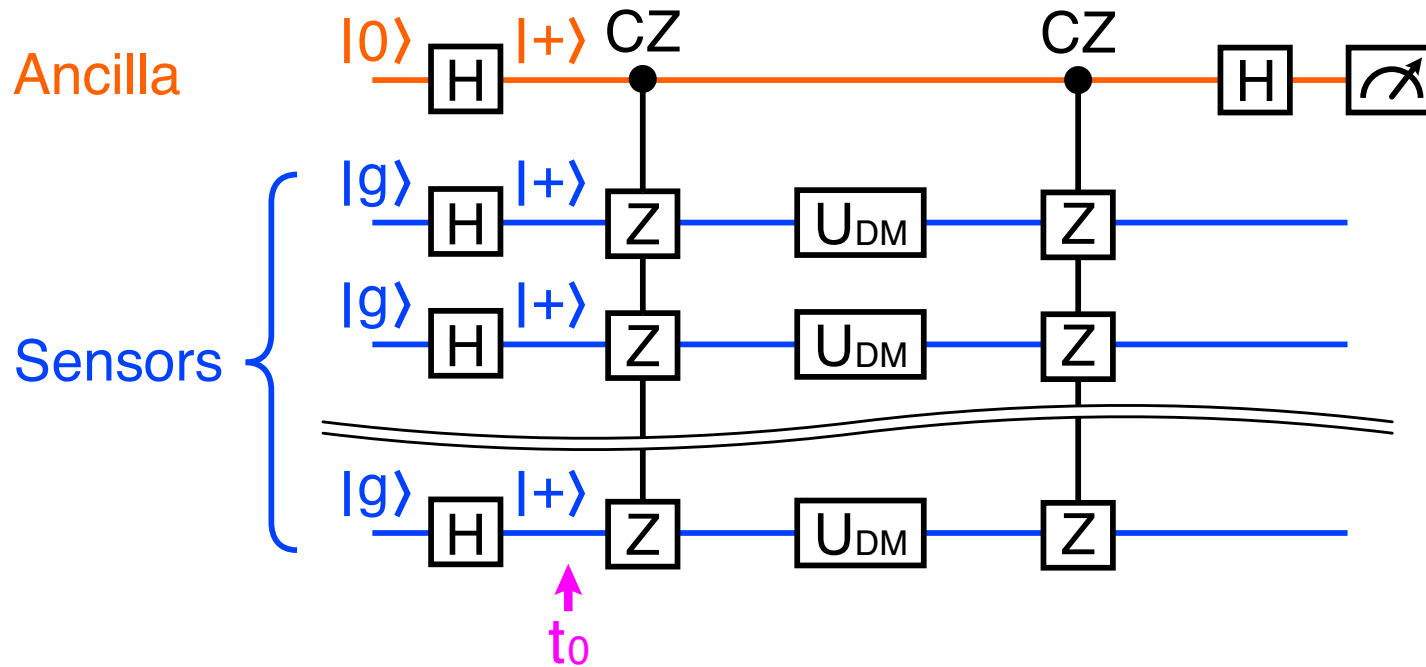
One measurement cycle for the signal enhancement



The above is an example of the quantum circuit

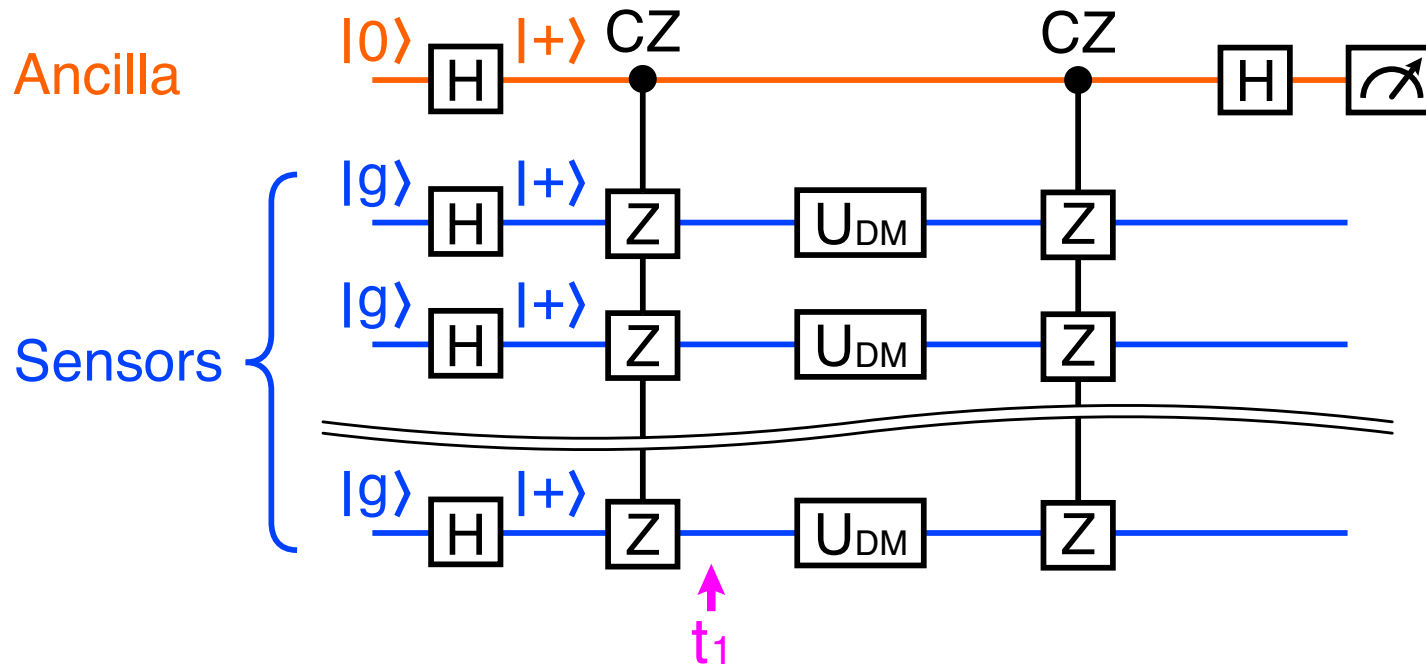
\Rightarrow Let us first see how it works when $\alpha = 0$

One measurement cycle for the signal enhancement



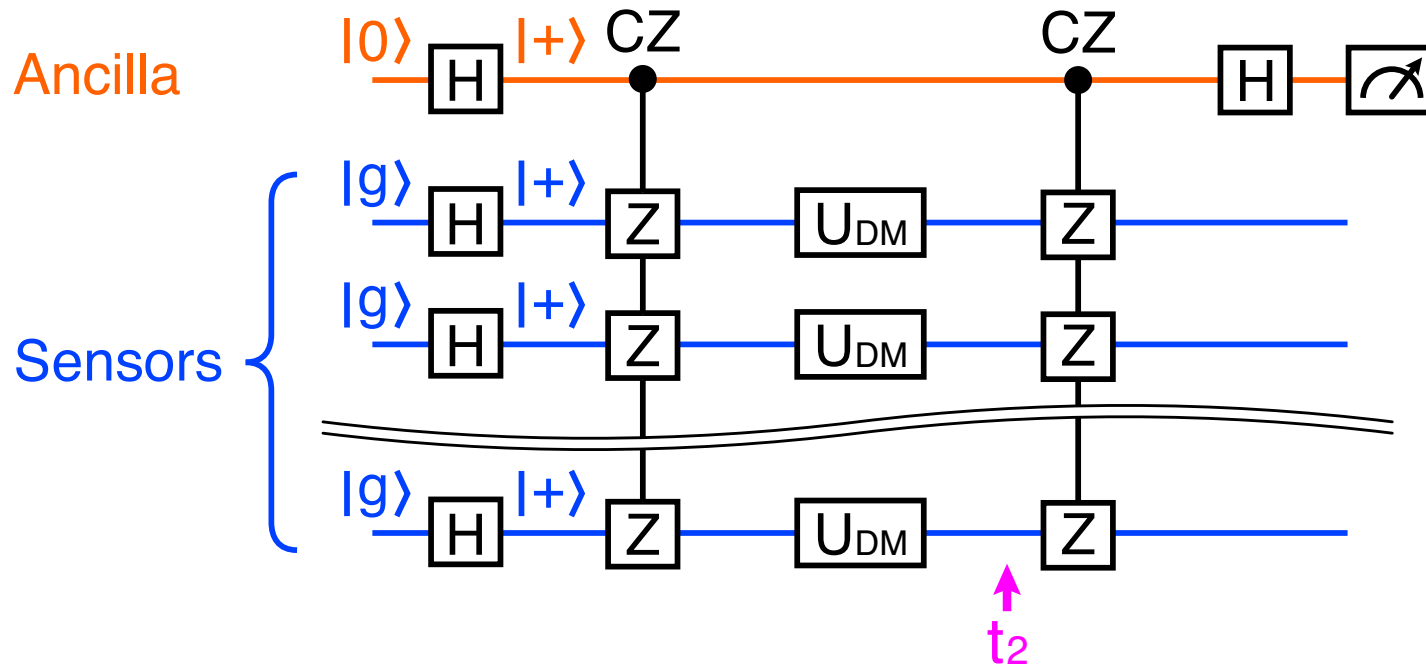
$$|\Psi(t_0)\rangle = |+\rangle \otimes |+\rangle^{\otimes N_q} = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}}|1\rangle \otimes |+\rangle^{\otimes N_q}$$

One measurement cycle for the signal enhancement



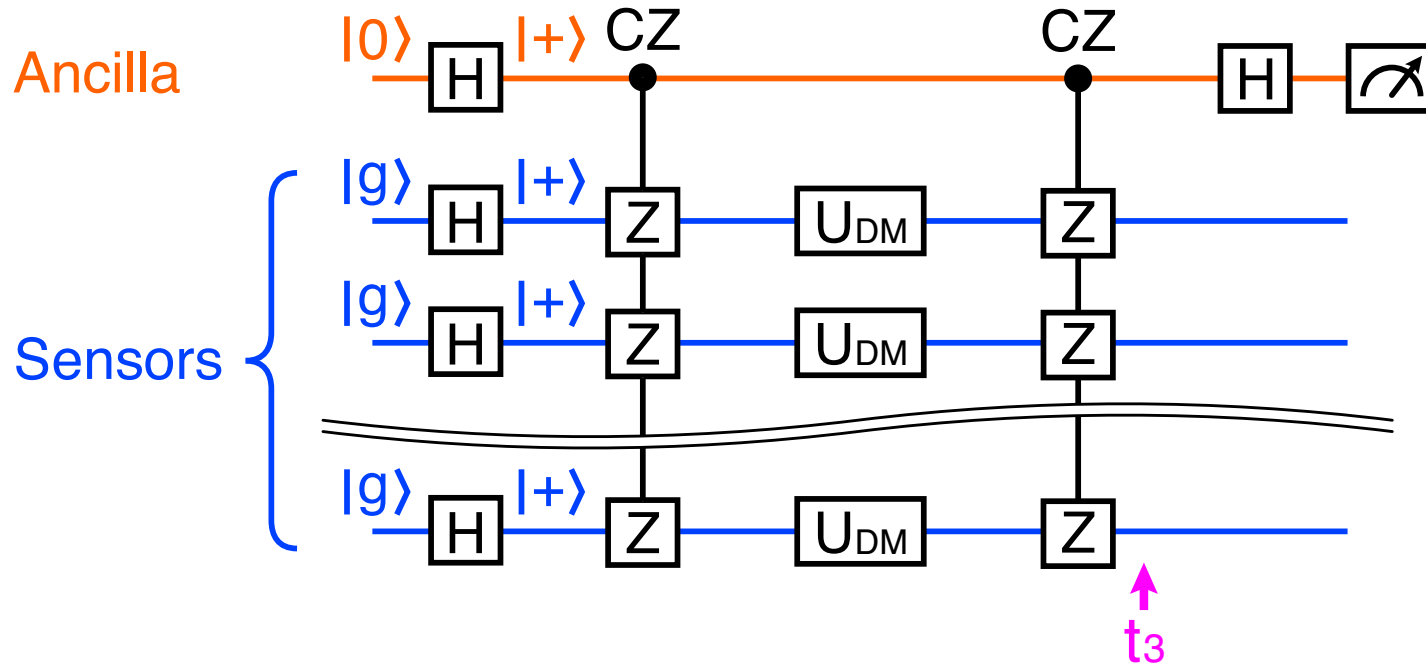
$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle^{\otimes N_q}$$

One measurement cycle for the signal enhancement



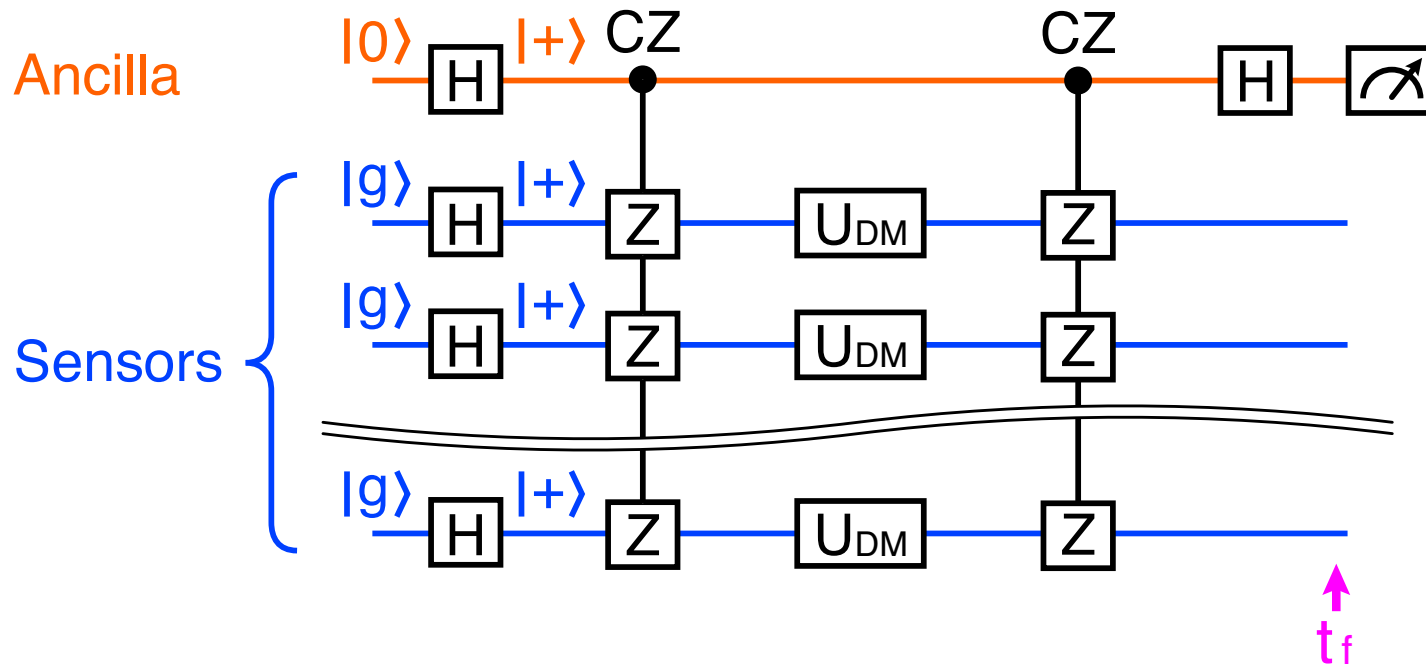
$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}} e^{iN_q\delta} |0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta} |1\rangle \otimes |-\rangle^{\otimes N_q}$$

One measurement cycle for the signal enhancement



$$\begin{aligned}
 |\Psi(t_3)\rangle &= \frac{1}{\sqrt{2}} e^{iN_q\delta} |0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta} |1\rangle \otimes |+\rangle^{\otimes N_q} \\
 &= (\cos N_q\delta |+\rangle + i \sin N_q\delta |-\rangle) \otimes |+\rangle^{\otimes N_q}
 \end{aligned}$$

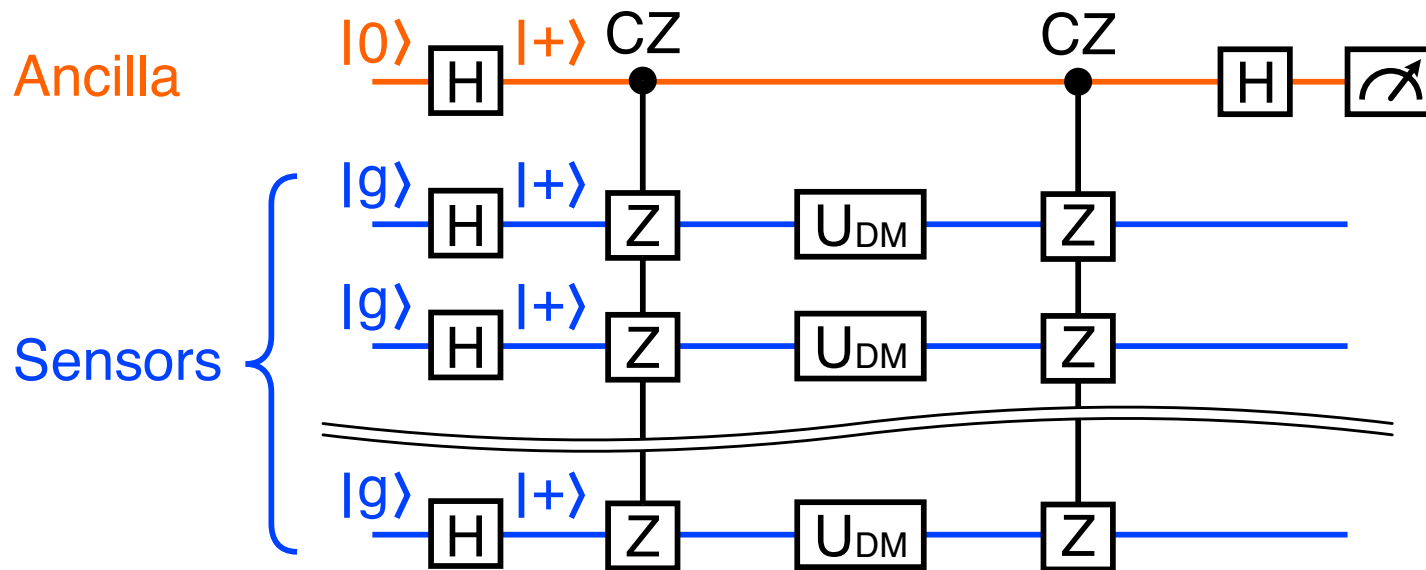
One measurement cycle for the signal enhancement



$$|\Psi(t_f)\rangle = (\cos N_q \delta |0\rangle + i \sin N_q \delta |1\rangle) \otimes |+\rangle^{\otimes N_q}$$

\Rightarrow Ancilla qubit can be excited: $P_{0 \rightarrow 1} \simeq \sin^2 N_q \delta \simeq N_q^2 \delta^2$

The phase α is unknown in the actual search, but...



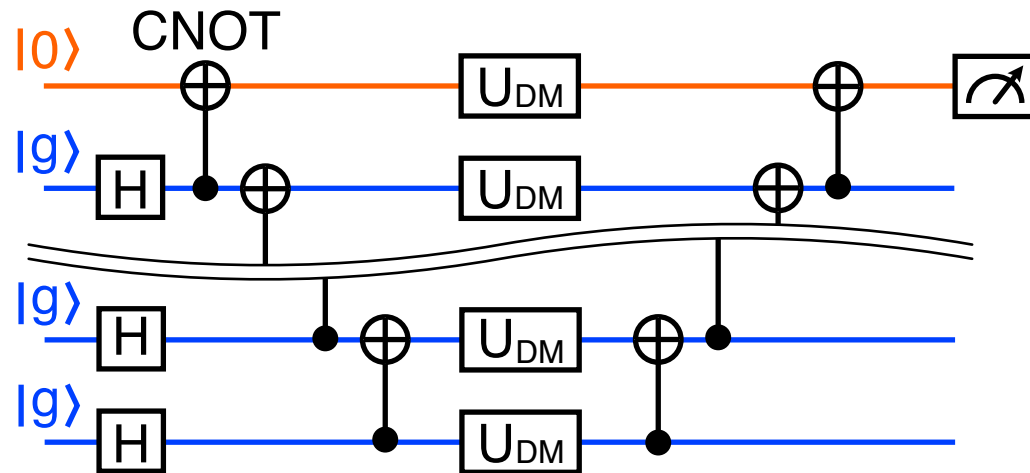
$$P_{0 \rightarrow 1} \simeq N_q^2 \delta^2 \cos^2 \alpha \rightarrow \frac{1}{2} N_q^2 \delta^2$$

\Rightarrow Signal rate can be of $O(N_q^2)$

\Rightarrow The number of gate operation can be $O(N_q)$

Circuit only with nearest neighbor interactions

\Rightarrow (# of gates) $\sim O(N_q)$



$$\Rightarrow P_{0 \rightarrow 1} \simeq \frac{1}{2} N_q^2 \delta^2$$

$$\text{CNOT (Controlled-NOT)} = |g\rangle\langle g| \otimes \mathbf{1} + |e\rangle\langle e| \otimes X$$

\Rightarrow (# of signals) $\sim O(N_q^2)$

\Rightarrow (# of errors & noises) $\sim O(N_q) \ll$ (# of signals), for $N_q \gg 1$