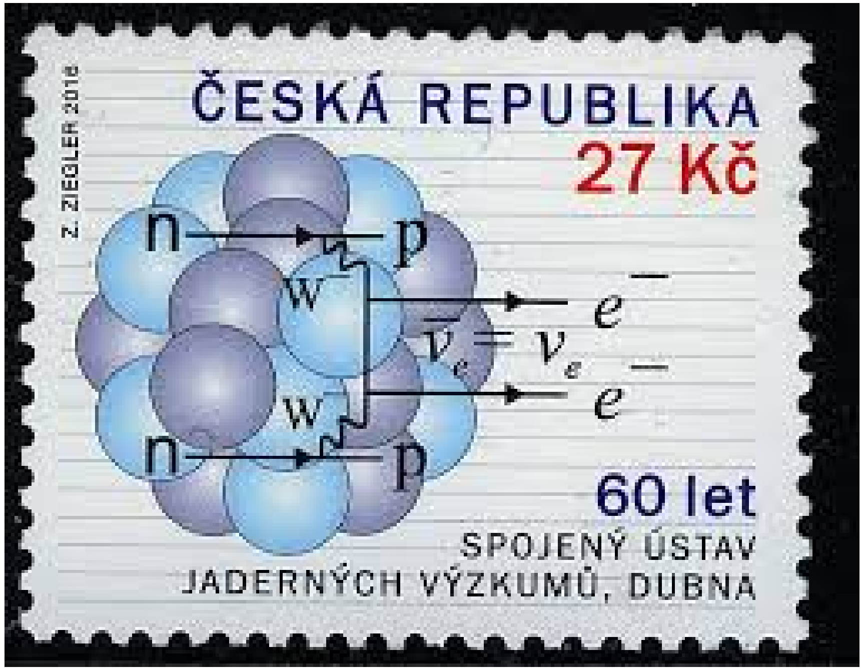
Majorana versus Dirac, Beyond $0\nu\beta\beta$



André de Gouvêa – Northwestern University

Hitoshi & Lawrence Fest, Berkeley

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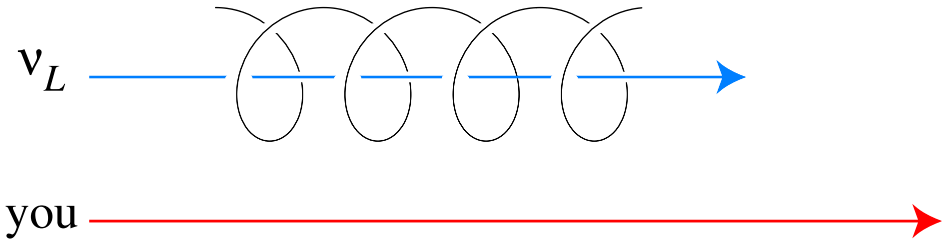
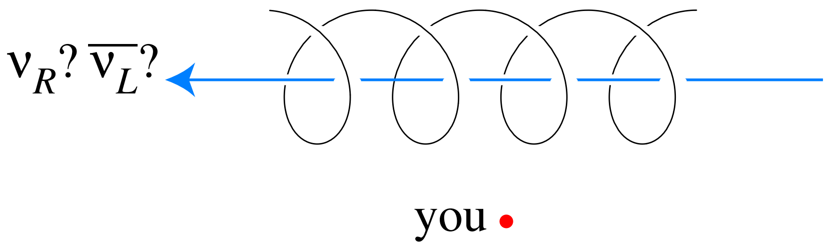
Are Neutrinos Majorana or Dirac Fermions?





How many degrees of freedom are required to describe massive neutrinos?

A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e^-_L \leftarrow \text{CPT} \rightarrow e^+_R)$$
$$\updownarrow \text{“Lorentz”}$$
$$(e^-_R \leftarrow \text{CPT} \rightarrow e^+_L)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{“Lorentz”} \qquad \text{‘DIRAC’}$$
$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\text{‘MAJORANA’} \qquad \updownarrow \text{“Lorentz”}$$
$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

Why Don't We Know the Answer?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

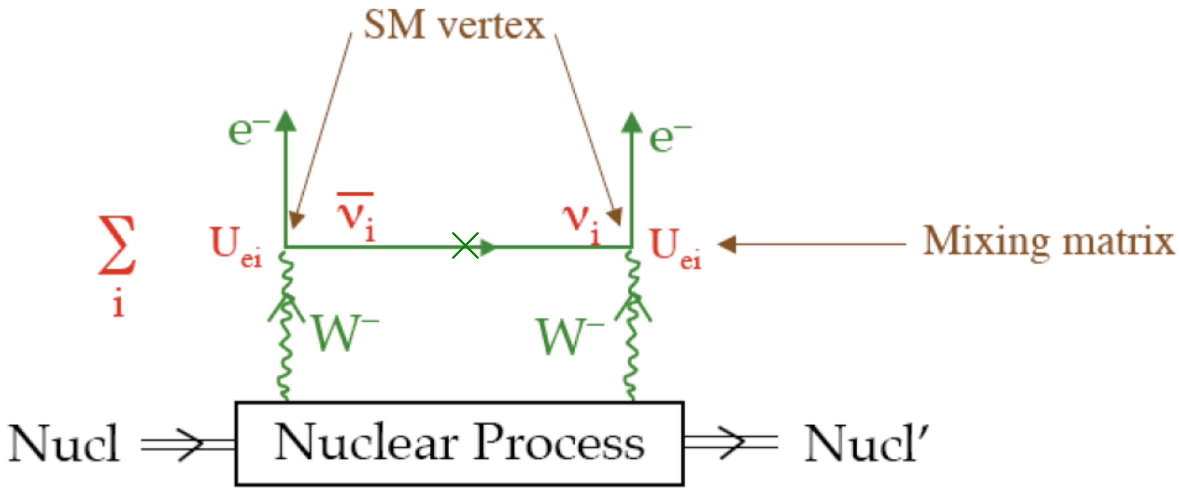
The "smoking gun" signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry **any** quantum number — including lepton number.

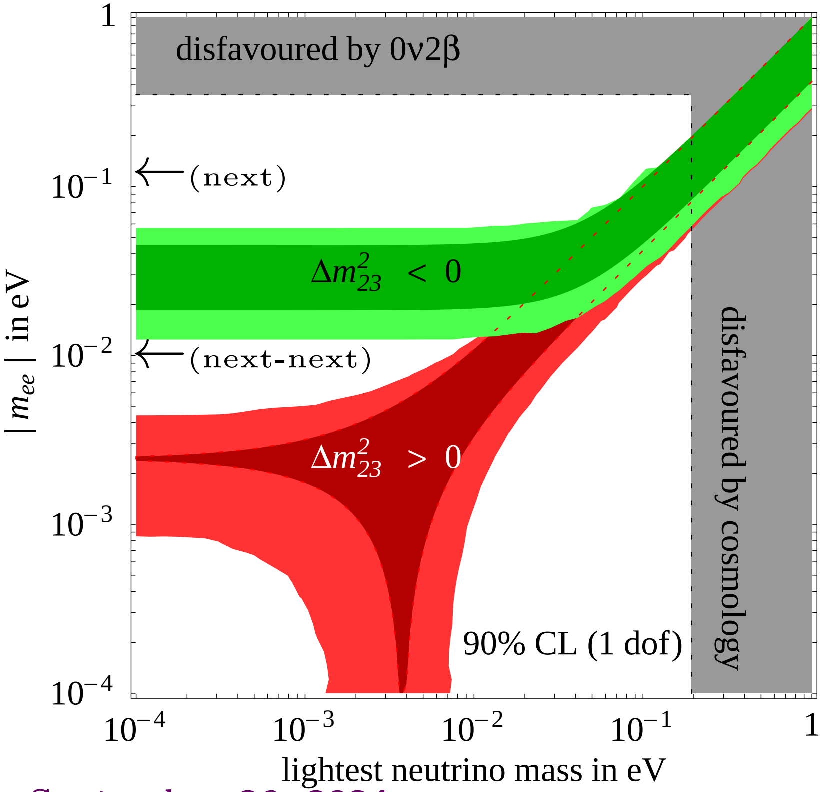
Search for the Violation of Lepton Number (or $B-L$)

Best Bet: search for

Neutrinoless Double-Beta

Decay: $Z \to (Z+2)e^-e^-$





Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

$\Leftarrow$ **information even if the answer is no**

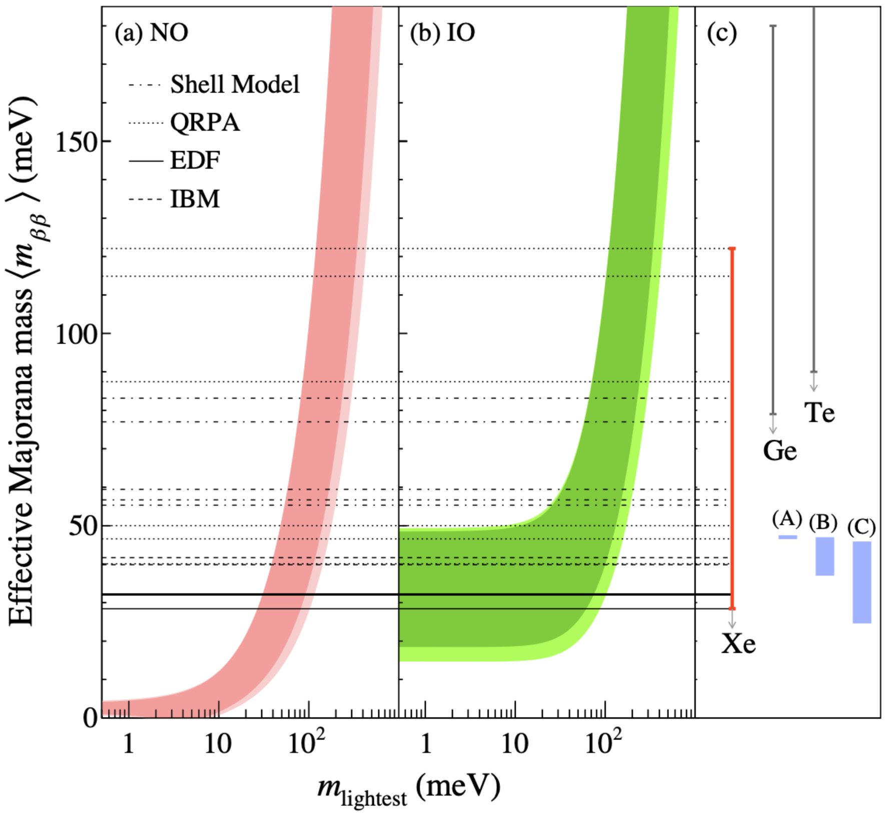


FIG. 4: Effective Majorana neutrino mass $\langle m_{\beta\beta}\rangle$ as a function of the lightest neutrino mass $m_{\rm lightest}$. The dark shaded regions are predictions based on best-fit values of neutrino oscillation parameters for (a) the normal ordering (NO) and (b) the inverted ordering (IO), and the light shaded regions indicate the 3σ ranges calculated from oscillation parameter uncertainties [30, 31]. The horizontal lines indicate 90% C.L. upper limits on $\langle m_{\beta\beta}\rangle$ with ^{136}Xe from KamLAND-Zen (this work), considering an improved phase space factor calculation [32, 33] and phenomenological NME calculations: shell model [34–37] (dot-dashed lines), quasiparticle randomphase approximation (QRPA) [38–42](dotted lines), energy-density functional (EDF) theory [43–45] (solid lines), interacting bo-

Lots of Experimental Activity!

Moving Towards Ton-Scale Expts.

(LEGEND, CUPID, nEXO, etc)

[KamLAND-Zen Coll. (Abe *et al*), 2406.11438 [hep-ex]]

What Else is There?

1. How about other searches for lepton number violation? Can they ever be competitive? How?

2. Are there other ways to tell whether the neutrinos are Majorana or Dirac fermions?

How about other searches for lepton number violation? Can they ever be competitive? How?

There are two major challenges one must face before embracing other searches for lepton number violation (LNV).

1. **Constraints from searches for $0\nu\beta\beta$ are too strong.** There is an "easy" way out: play with the flavor structure of the LNV physics.

2. **Neutrino masses are very small. Majorana neutrino masses are a consequence of LNV physics.** The relation between the LNV physics and the neutrino masses, however, is indirect so the real question is whether there are scenarios where LNV is accessible to laboratory experiments while, at the same time, the neutrino masses are tiny.

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to pe)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p\mu)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion		
$\sigma(\mu^- {}^{32}S \to e^+ {}^{32}Si^*)$ / $\sigma(\mu^- {}^{32}S \to \nu_\mu {}^{32}P^*)$		$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}I \to e^+ {}^{127}Sb^*)$ / $\sigma(\mu^- {}^{127}I \to$ anything)		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- Ti \to e^+ Ca)$ / $\sigma(\mu^- Ti \to$ capture)		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-\pi^-)/\Gamma_{total}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-\pi^-)/\Gamma_{total}$		$<3.9 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-K^-)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+K^-K^-)/\Gamma_{total}$		$<3.3 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-K^-)/\Gamma_{total}$		$<4.8 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+K^-K^-)/\Gamma_{total}$		$<4.7 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to p\mu^-\mu^-)/\Gamma_{total}$		$<4.4 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\mu^+\mu^-)/\Gamma_{total}$		$<3.3 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\gamma)/\Gamma_{total}$		$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0)/\Gamma_{total}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}2\pi^0)/\Gamma_{total}$		$<3.3 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\eta)/\Gamma_{total}$		$<8.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0\eta)/\Gamma_{total}$		$<2.7 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \Lambda\pi^-)/\Gamma_{total}$		$<7.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \overline{\Lambda}\pi^-)/\Gamma_{total}$		$<1.4 \times 10^{-7}$, CL = 90%
$t_{1/2}({}^{76}Ge \to {}^{76}Se + 2e^-)$		$>1.9 \times 10^{25}$ yr, CL = 90%
$\Gamma(\pi^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$	[q]	$<1.5 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^-\mu^+e^+)/\Gamma_{total}$		$<5.0 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^-e^+e^+)/\Gamma_{total}$		$<6.4 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$	[q]	$<1.1 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$	[q]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0e^+\overline{\nu}_e)/\Gamma_{total}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^-2e^+)/\Gamma_{total}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^-2\mu^+)/\Gamma_{total}$		$<2.2 \times 10^{-8}$, CL = 90%
$\Gamma(D^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \rho^-2\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^-2e^+)/\Gamma_{total}$		$<9 \times 10^{-7}$, CL = 90%
$\Gamma(D^+ \to K^-2\mu^+)/\Gamma_{total}$		$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<1.9 \times 10^{-6}$, CL = 90%

$\Gamma(D^+ \to K^*(892)^-2\mu^+)/\Gamma_{total}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^-2e^+ + c.c.)/\Gamma_{total}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^-2\mu^+ + c.c.)/\Gamma_{total}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-2e^+ + c.c.)/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-2\mu^+ + c.c.)/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2e^+ + c.c.)/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2\mu^+ + c.c.)/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^-\pi^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<5.7 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to pe^-)/\Gamma_{total}$	[r]	$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \overline{p}e^+)/\Gamma_{total}$	[s]	$<1.1 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-2e^+)/\Gamma_{total}$		$<4.1 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-2\mu^+)/\Gamma_{total}$		$<1.2 \times 10^{-7}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<8.4 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^-2e^+)/\Gamma_{total}$		$<5.2 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^-2\mu^+)/\Gamma_{total}$		$<1.3 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<6.1 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^*(892)^-2\mu^+)/\Gamma_{total}$		$<1.4 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^-e^+e^+)/\Gamma_{total}$		$<2.3 \times 10^{-8}$, CL = 90%
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$\Gamma(B^+ \to \rho^-e^+e^+)/\Gamma_{total}$		$<1.7 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^-\mu^+\mu^+)/\Gamma_{total}$		$<4.2 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^-e^+\mu^+)/\Gamma_{total}$		$<4.7 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^-e^+e^+)/\Gamma_{total}$		$<3.0 \times 10^{-8}$, CL = 90%
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$\Gamma(B^+ \to K^*(892)^-e^+e^+)/\Gamma_{total}$		$<4.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^-\mu^+\mu^+)/\Gamma_{total}$		$<5.9 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^-e^+\mu^+)/\Gamma_{total}$		$<3.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to D^-e^+e^+)/\Gamma_{total}$		$<2.6 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^-e^+\mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^-\mu^+\mu^+)/\Gamma_{total}$		$<6.9 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to D^{*-}\mu^+\mu^+)/\Gamma_{total}$		$<2.4 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to D_s^-\mu^+\mu^+)/\Gamma_{total}$		$<5.8 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to \overline{D}^0\pi^-\mu^+\mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to \Lambda^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \Lambda^0e^+)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0e^+)/\Gamma_{total}$		$<8 \times 10^{-8}$, CL = 90%

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$\sigma(\mu^- {}^{127}\mathrm{I} \to e^+ {}^{127}\mathrm{Sb}^*)$ / $\sigma(\mu^- {}^{127}\mathrm{I} \to$ anything)		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- \mathrm{Ti} \to e^+ \mathrm{Ca})$ / $\sigma(\mu^- \mathrm{Ti} \to$ capture)		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-\pi^-)/\Gamma_{total}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-\pi^-)/\Gamma_{total}$		$<3.9 \times 10^{-8}$, CL = 90%
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$\Gamma(\tau^- \to \overline{p}\mu^+\mu^-)/\Gamma_{total}$		$<3.3 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\gamma)/\Gamma_{total}$		$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0)/\Gamma_{total}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}2\pi^0)/\Gamma_{total}$		$<3.3 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\eta)/\Gamma_{total}$		$<8.9 \times 10^{-6}$, CL = 90%
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$\Gamma(D^+ \to \pi^-2e^+)/\Gamma_{total}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^-2\mu^+)/\Gamma_{total}$		$<2.2 \times 10^{-8}$, CL = 90%
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$\Gamma(D^+ \to \rho^-2\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^-2e^+)/\Gamma_{total}$		$<9 \times 10^{-7}$, CL = 90%
$\Gamma(D^+ \to K^-2\mu^+)/\Gamma_{total}$		$<1.0 \times 10^{-5}$, CL = 90%
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$\Gamma(D^0 \to K^-\pi^-2e^+ +$ c.c.$)/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-2\mu^+ +$ c.c.$)/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2e^+ +$ c.c.$)/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2\mu^+ +$ c.c.$)/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^-\pi^-e^+\mu^+ +$ c.c.$)/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-e^+\mu^+ +$ c.c.$)/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
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$\Gamma(D_s^+ \to \pi^-2\mu^+)/\Gamma_{total}$		$<1.2 \times 10^{-7}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<8.4 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^-2e^+)/\Gamma_{total}$		$<5.2 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^-2\mu^+)/\Gamma_{total}$		$<1.3 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<6.1 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^*(892)^-2\mu^+)/\Gamma_{total}$		$<1.4 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^-e^+e^+)/\Gamma_{total}$		$<2.3 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$		$<4.0 \times 10^{-9}$, CL = 95%
$\Gamma(B^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-7}$, CL = 90%
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$\Gamma(B^+ \to \rho^-e^+\mu^+)/\Gamma_{total}$		$<4.7 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^-e^+e^+)/\Gamma_{total}$		$<3.0 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to K^-\mu^+\mu^+)/\Gamma_{total}$		$<4.1 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<1.6 \times 10^{-7}$, CL = 90%
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$\Gamma(B^+ \to K^*(892)^-e^+\mu^+)/\Gamma_{total}$		$<3.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to D^-e^+e^+)/\Gamma_{total}$		$<2.6 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^-e^+\mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 90%
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$\Gamma(B^+ \to D^{*-}\mu^+\mu^+)/\Gamma_{total}$		$<2.4 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to D_s^-\mu^+\mu^+)/\Gamma_{total}$		$<5.8 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to \overline{D}^0\pi^-\mu^+\mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to \Lambda^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \Lambda^0e^+)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
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$\sigma(\mu^- {}^{32}S \to e^+ {}^{32}Si^*)$ / $\sigma(\mu^- {}^{32}S \to \nu_\mu {}^{32}P^*)$		$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}I \to e^+ {}^{127}Sb^*)$ / $\sigma(\mu^- {}^{127}I \to$ anything)		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- Ti \to e^+ Ca)$ / $\sigma(\mu^- Ti \to$ capture)		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-\pi^-)/\Gamma_{total}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-\pi^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to e^+\pi^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to e^+K^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \mu^+\pi^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \mu^+K^-K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to p\mu^-\mu^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\mu^+\mu^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\gamma)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\pi^0)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}2\pi^0)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\eta)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p}\pi^0\eta)/\Gamma_{total}$		
$\Gamma(\tau^- \to \Lambda\pi^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{\Lambda}\pi^-)/\Gamma_{total}$		
$t_{1/2}({}^{76}Ge \to {}^{76}Se + 2e^-)$		
$\Gamma(\pi^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^-\mu^+e^+)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^-e^+e^+)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^-\mu^+\mu^+)/\Gamma_{total}$	[q]	$<1.1 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+\overline{\nu}_e)/\Gamma_{total}$	[q]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0e^+\overline{\nu}_e)/\Gamma_{total}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^-2e^+)/\Gamma_{total}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^-2\mu^+)/\Gamma_{total}$		$<2.2 \times 10^{-8}$, CL = 90%
$\Gamma(D^+ \to \pi^-e^+\mu^+)/\Gamma_{total}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \rho^-2\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^-2e^+)/\Gamma_{total}$		$<9 \times 10^{-7}$, CL = 90%
$\Gamma(D^+ \to K^-2\mu^+)/\Gamma_{total}$		$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^-e^+\mu^+)/\Gamma_{total}$		$<1.9 \times 10^{-6}$, CL = 90%

$\Gamma(D^+ \to K^*(892)^-2\mu^+)/\Gamma_{total}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^-2e^+ + c.c.)/\Gamma_{total}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^-2\mu^+ + c.c.)/\Gamma_{total}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-2e^+ + c.c.)/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-2\mu^+ + c.c.)/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2e^+ + c.c.)/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-2\mu^+ + c.c.)/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^-\pi^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^-\pi^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^-e^+\mu^+ + c.c.)/\Gamma_{total}$		$<5.7 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to pe^-)/\Gamma_{total}$	[r]	$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \overline{p}e^+)/\Gamma_{total}$	[s]	$<1.1 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^-2e^+)/\Gamma_{total}$		$<4.1 \times 10^{-6}$, CL = 90%
		$<1.2 \times 10^{-7}$, CL = 90%
		$<8.4 \times 10^{-6}$, CL = 90%
		$<5.2 \times 10^{-6}$, CL = 90%
		$<1.3 \times 10^{-5}$, CL = 90%
		$<6.1 \times 10^{-6}$, CL = 90%
		$<1.4 \times 10^{-3}$, CL = 90%
		$<2.3 \times 10^{-8}$, CL = 90%
		$<4.0 \times 10^{-9}$, CL = 95%
		$<1.5 \times 10^{-7}$, CL = 90%
		$<1.7 \times 10^{-7}$, CL = 90%
		$<4.2 \times 10^{-7}$, CL = 90%
		$<4.7 \times 10^{-7}$, CL = 90%
		$<3.0 \times 10^{-8}$, CL = 90%
		$<4.1 \times 10^{-8}$, CL = 90%
		$<1.6 \times 10^{-7}$, CL = 90%
		$<4.0 \times 10^{-7}$, CL = 90%
		$<5.9 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^-e^+\mu^+)/\Gamma_{total}$		$<3.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to D^-e^+e^+)/\Gamma_{total}$		$<2.6 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^-e^+\mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^-\mu^+\mu^+)/\Gamma_{total}$		$<6.9 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to D^{*-}\mu^+\mu^+)/\Gamma_{total}$		$<2.4 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to D_s^-\mu^+\mu^+)/\Gamma_{total}$		$<5.8 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to \overline{D}^0\pi^-\mu^+\mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to \Lambda^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \Lambda^0e^+)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0\mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0e^+)/\Gamma_{total}$		$<8 \times 10^{-8}$, CL = 90%

$\Gamma(B^0 \to \Lambda_c^+\mu^-)/\Gamma_{total}$	$<1.4 \times 10^{-6}$, CL = 90%
$\Gamma(B^0 \to \Lambda_c^+e^-)/\Gamma_{total}$	$<4 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to \pi^+e^-)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^+\mu^-)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^-e^+)/\Gamma_{total}$	$<4 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^-\mu^+)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to K^+e^-)/\Gamma_{total}$	$<2 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^+\mu^-)/\Gamma_{total}$	$<3 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^-e^+)/\Gamma_{total}$	$<2 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^-\mu^+)/\Gamma_{total}$	$<3 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K_S^0\nu)/\Gamma_{total}$	$<2 \times 10^{-5}$, CL = 90%
$\Gamma(\Xi^- \to p\mu^-\mu^-)/\Gamma_{total}$	$<4 \times 10^{-8}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p}2e^+)/\Gamma_{total}$	$<2.7 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p}2\mu^+)/\Gamma_{total}$	$<9.4 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p}e^+\mu^+)/\Gamma_{total}$	$<1.6 \times 10^{-5}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \Sigma^-\mu^+\mu^+)/\Gamma_{total}$	$<7.0 \times 10^{-4}$, CL = 90%

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to pe)/\Gamma_{total}$	$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p\mu)/\Gamma_{total}$	$<1.8 \times 10^{-6}$, CL = 95%
$\Rightarrow$ limit on $\mu^- \to e^+$ conversion $\Leftarrow$	(Next Best Thing)
$\sigma(\mu^- {}^{32}S \to e^+ {}^{32}Si^*)$ / $\sigma(\mu^- {}^{32}S \to \nu_\mu {}^{32}P^*)$	$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}I \to e^+ {}^{127}Sb^*)$ / $\sigma(\mu^- {}^{127}I \to$ anything)	$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- Ti \to e^+ Ca)$ / $\sigma(\mu^- Ti \to$ capture)	$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-\pi^-)/\Gamma_{total}$	$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-\pi^-)/\Gamma_{total}$	$<3.9 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-K^-)/\Gamma_{total}$	$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+K^-K^-)/\Gamma_{total}$	$<3.3 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-K^-)/\Gamma_{total}$	$<4.8 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+K^-K^-)/\Gamma_{total}$	$<4.7 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to p\mu^-\mu^-)/\Gamma_{total}$	$<4.4 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\mu^+\mu^-)/\Gamma_{total}$	$<3.3 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\gamma)/\Gamma_{total}$	$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0)/\Gamma_{total}$	$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}2\pi^0)/\Gamma_{total}$	$<3.3 \times 10^{-5}$, CL = 90%

Are there other ways to tell whether the neutrinos are Majorana or Dirac fermions?

The answer is a qualified 'yes.'

The qualification is that we have to know the relevant physics – new physics may spoil everything! There are no "theorems" as far as I know…

Again: Why Don't We Know the Answer?

Neutrino Masses are Very Small!*

In fact, except for neutrino oscillation experiments, no consequence of a nonzero neutrino mass has ever been observed in any experiment. As far as all non-oscillation neutrino experiments are concerned, neutrinos are massless fermions.

*Very small compared to what? Compared to the typical energies and momentum transfers in your experiment. Another way to think about this: neutrinos are always **ultrarelativistic** in the lab frame.

There are two ways around it:

1. Find something that only Majorana fermions know how to do [e.g. violate lepton number] or
2. **find some non-ultrarelativistic neutrinos to work with!**

The Burden of Working with Non-Ultrarelativistic Neutrinos

In a nutshell: there aren't too many of them, and the weak interactions are weak. Remember, at low energies

$$\sigma \propto E \quad \text{(or worse)}$$

On the other hand, telling Majorana From Dirac neutrinos is "trivial." Indeed, it is an order one effect.

Examples, or

Where Can I Find Some Non-Relativistic Neutrinos?

- The Cosmic Neutrino Background;
- Low-Energy Reactions with (Not-To-Be-Detected) Neutrinos in the Final State;
- Decaying Neutrinos.

Example: The Cosmic Neutrino Background

[see, e.g., Long, Lunardini, Sabancilar, arXiv:1405.7654]

Assuming the Standard Model of Cosmology, at least two of the three neutrinos are mostly non-relativistic today:

$$T_\nu \sim 2\text{K} \sim 2 \times 10^{-4}\ \text{eV}.$$

Furthermore, it turns out that hitting a Majorana CνB with a charged-current process is easier than hitting a Dirac CνB, assuming the weak interactions. All of this is assuming one is measuring the CνB via neutrino-capture on nuclei, $\nu(Z,A) \to e^-(Z+1,A)$ (charged-current weak interaction on matter)

When you interact with a polarized (anti)neutrino at rest, it will either choose to behave like the left-chiral component or the right-chiral component, with the same probability.

In the Dirac case, the right-chiral component of the neutrino is sterile, i.e., it does not participate in the weak interactions and you can't interact with it. Furthermore, the antineutrinos have the opposite lepton number and can't be detected via $\nu(Z,A) \to e^-(Z+1,A)$.

In the Majorana case, the right-chiral component knows how to produce positrons (the "antineutrino") so both can interact via the weak interactions.

When it comes to the cosmic neutrino background detected via $\nu(Z,A) \to e^-(Z+1,A)$, we get a hit from the neutrinos – just like in the Dirac case – but we also get a hit from the "antineutrino," with the same rate, since the left-chiral component of the other polarization-state knows how to produce electrons!

This means that if we ever observe the cosmic neutrino background, we can determine the nature of the neutrino. If all neutrinos were at rest, for the same neutrino (+ "antineutrino") flux, we expect twice as many events in the experiment if the neutrinos are Majorana fermions. One can easily include finite temperature effects, effects related to the neutrino mass ordering, a potential primordial lepton asymmetry, etc.

Some challenges:

- We have never detected the cosmic neutrino background! (see, however, PTOLEMY [arXiv:1808.01892 and many uddates] for an idea that may work one day?);
- We measure flux times cross-section. While we know the average neutrino number density of the universe very well from the Standard Model of Cosmology, we don't know the number density of neutrinos *here* very well [Uncertainty around 10% so likely ok?].

Neutrinos Near Threshold

We looked at

$$e\gamma \to e\nu\bar{\nu}$$

at sub-eV energies, because it can be done, in principle (electron at rest, infrared photon). Best to do it in the mass basis! Using the Fermi theory...

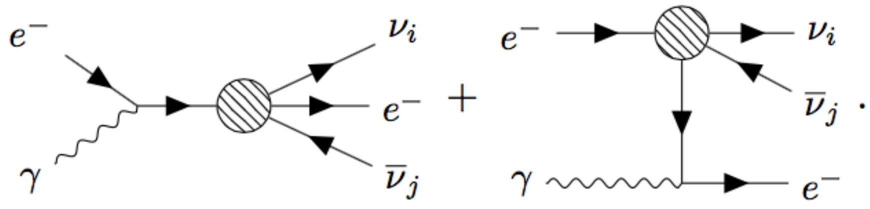
$$\mathcal{L}_{\rm CC} + \mathcal{L}_{\rm NC} = -\sqrt{2}G_F\left(\overline{\nu}_j\gamma^\mu P_L\nu_i\right)\left[\overline{\ell}_\alpha\gamma_\mu\left(g_V^{\alpha\beta ij}\mathbb{1} - g_A^{\alpha\beta ij}\gamma_5\right)\ell_\beta\right], \quad \text{(II.4)}$$

where we introduce the vector and axial couplings

$$g_V^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\left(1-4\sin^2\theta_W\right)\delta_{ij}\delta_{\alpha\beta}, \qquad g_A^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\delta_{ij}\delta_{\alpha\beta}. \quad \text{(II.5)}$$

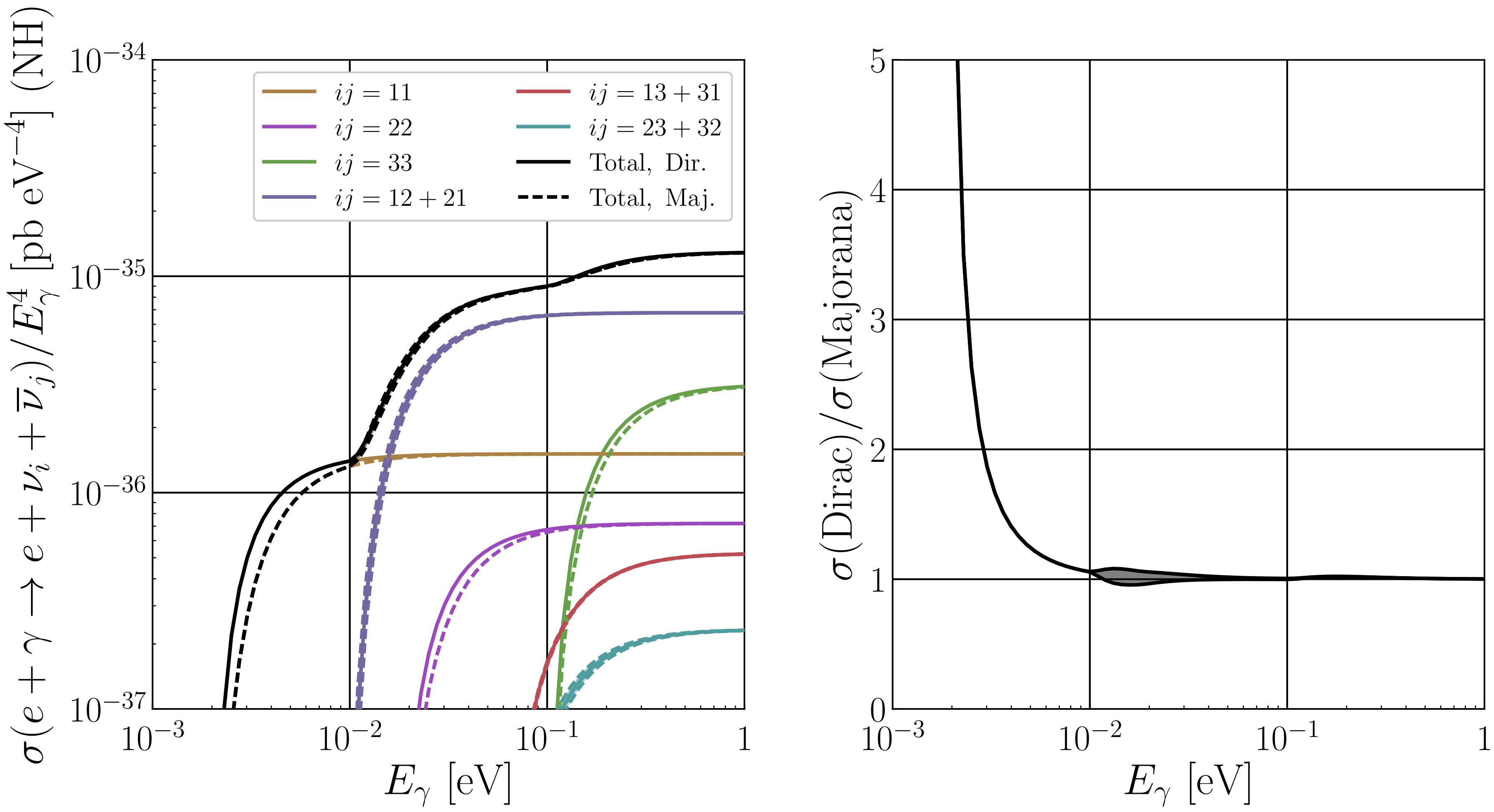
Since the only charged leptons considered in this work are electrons, we will make the simplification $g^{ij}_{V,A} \equiv g^{eeij}_{V,A}$.

The following diagrams are relevant to the evaluation of the amplitude:

$i\mathcal{M} \approx$  (II.6)

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]



Why Can't We Do This?

- Cross sections are small. Very, very small. For a 1 eV laser (1240 nm, near-infrared) with a power of 2 W (it exists) and a large target of electrons at rest with density 10^{23} cm^{-3} and a length of 1 m we expect one signal event every 10^{20} years.

- Backgrounds are overwhelming. The signal is a recoil electron and nothing else. This can be mimicked by $e + \gamma \to e + \gamma + \cdots + \gamma$ (n photons) when the photon(s) are very soft or fall within a dead-zone within the detector. Very naively,

$$\sigma_n \sim \alpha^{(n-1)} \sigma_{\rm Thomson} \left(\frac{E_\gamma^{\rm threshold}}{m_e} \right)^{2n}$$

where $\sigma_{\rm Thomson} \sim 0.7$ barn.

Neutrino Decay (Hint – Only Massive Particles Decay)

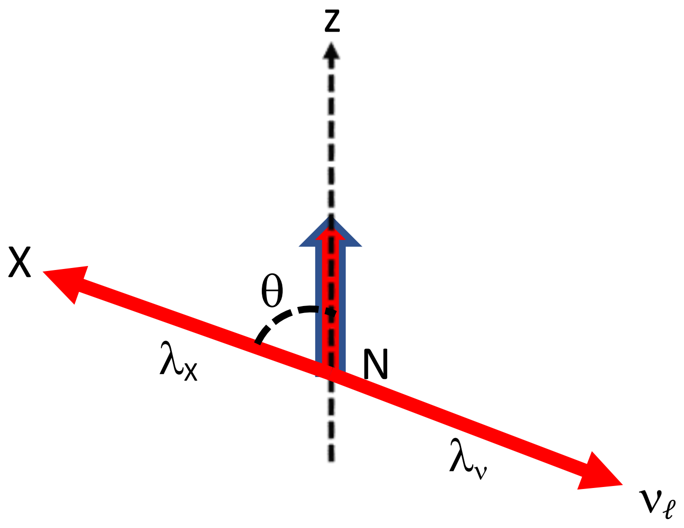
[Balantekin, AdG, Kayser, arXiv:1808.10518]

The two heavy neutrinos are expected to decay. E.g., if the neutrino mass ordering is normal, decay modes like $\nu_3 \to \nu_1\gamma$ and $\nu_3 \to \nu_1\nu_2\bar{\nu}_1$ are not only kinematically allowed, they are mediated by the weak interactions once mixing is taken into account.

Dirac and Majorana neutrinos "decay differently." In particular, the number of accessible final states, and the way in which they can potentially interfere, is such that the partial widths and the lifetimes are different – assuming the same mixing and mass parameters – if the neutrinos are Majorana or Dirac.

Obvious challenges. $\Gamma \propto (m_\nu)^n$ [n is some positive power] so the neutrino lifetimes are expected to be cosmological. Insult to injury, the $\nu \to \nu$'s decay mode is significant, which renders studying the final products of the decay a rather daunting task. Nonetheless, we proceed ...

CPT invariance [and at leading order]



We showed

$$\frac{d\Gamma(N \to \nu_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 + \alpha\cos\theta)$$

$$\frac{d\Gamma(\bar{N} \to \bar{\nu}_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 - \alpha\cos\theta)$$

Since $\alpha = -\bar{\alpha}$, for Majorana neutrinos we get $\alpha = 0$. This result holds for any self-conjugate boson X.

The two-body decay of a Majorana fermion into a self-conjugate final state is isotropic

A.B. Balantekin, B. Kayser, Ann. Rev. Nucl. Part. Sci. **68** (2018) 313-338 (arXiv:1805.00922)
A.B. Balantekin, A. de Gouvêa, B. Kayser, arXiv:1808.10518

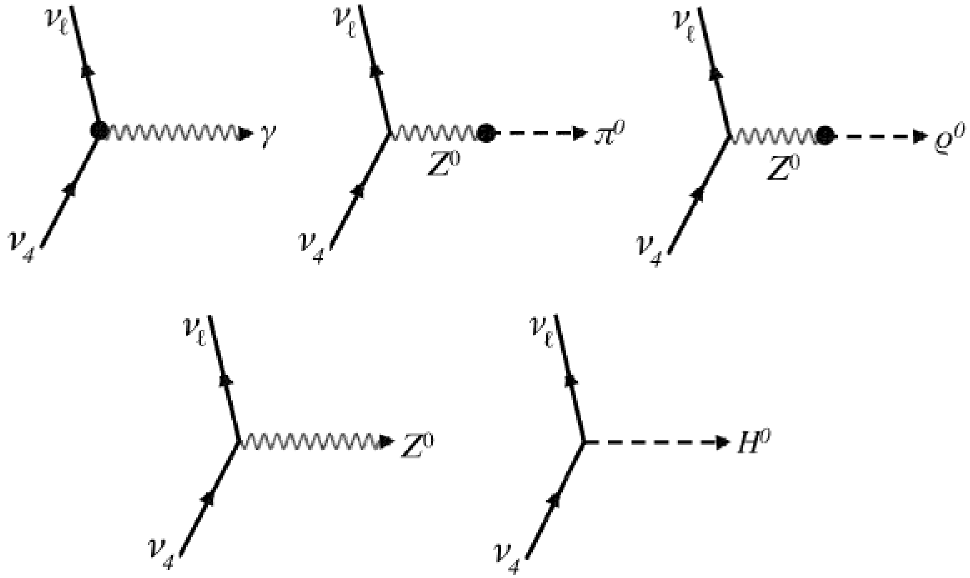
A More Realistic (?) Application – Neutral Heavy Leptons

If a neutral heavy lepton ν_4 is discovered somewhere – LHC, MicroBooNE, ICARUS, DUNE, SuperB Factory, SHiP, etc – in the future, after much rejoicing, we will want to establish whether this fermion is a *Majorana* or *Dirac* fermion.

How do we do it?

- Check for lepton-number violation. What does it take?
  - A lepton-number asymmetric initial state (easy). Or an even-by-event lepton number "tag" of the neutral heavy lepton (e.g. LHC environment).
  - Charge identification capability in the detector (sometimes absent or partially absent).
- **Kinematics**. Not only are the decay widths different (not super useful, since it requires we know unknown parameters) but the kinematics are also qualitatively different, as I showed in the last slide.

Heavy Neutral Leptons – More Realistic (?) Application



All of these decays are isotropic for a Majorana parent. Dirac case ⇓ (weak interactions)

Boson	γ	π^0	ρ^0	Z^0	H^0
α	$\frac{2\Im(\mu d^*)}{\lvert\mu\rvert^2+\lvert d\rvert^2}$	1	$\frac{m_4^2-2m_\rho^2}{m_4^2+2m_\rho^2}$	$\frac{m_4^2-2m_Z^2}{m_4^2+2m_Z^2}$	1

Can this be done, some day, in practice?

[AdG, Kayser, et al, 1912.07622, 2104.05719, 2105.06576, 2109.10358]

- Three-body decays (concentrating on $\nu\ell^+\ell^-$);
- Heavy neutrino production mechanism (choice is meson decay at rest);
- Consider different models for heavy neutrino decay, including the weak interactions (four-fermion interactions that preserve lepton number).

ν_4 at the Z-pole

[Blondel, AdG, Kayser, arXiv: 2105.06576]

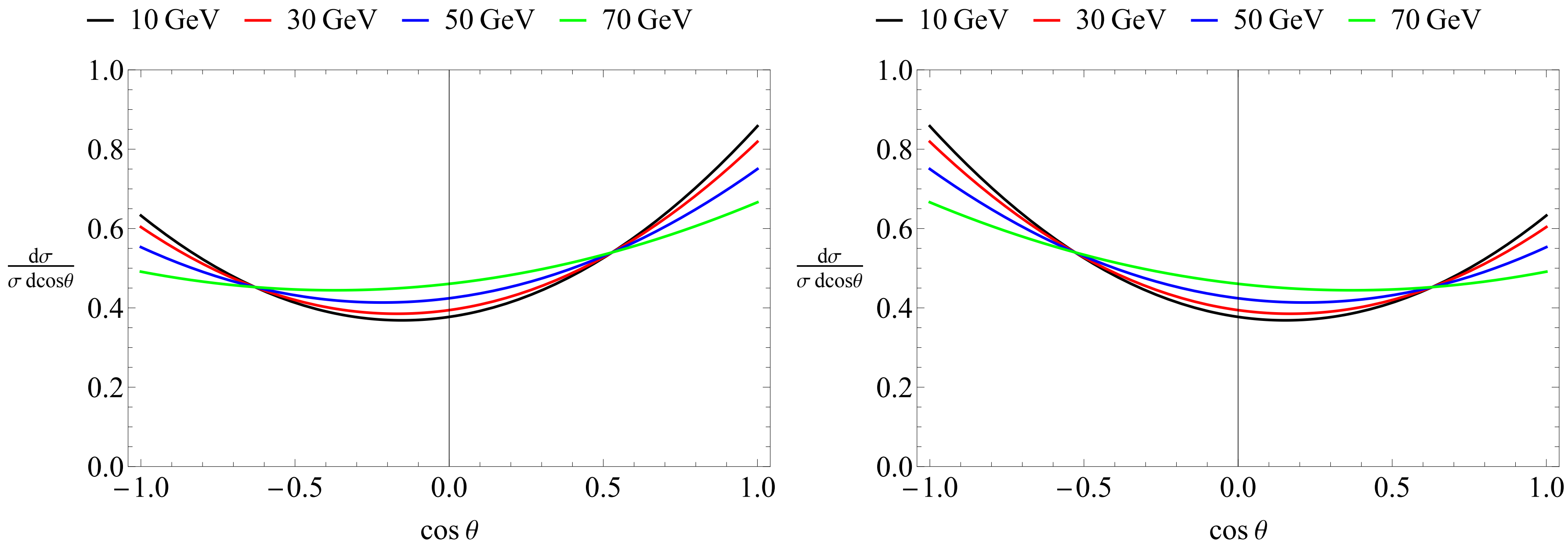
For $m_4 < M_Z$, heavy neutrinos can be produced in the decay of Z-bosons. For example,

$$e^+e^- \to Z \to \bar{\nu}_{\rm light}\nu_4 \text{ (or } \nu_{\rm light}\bar{\nu}_4)$$

followed by ν_4 decay.

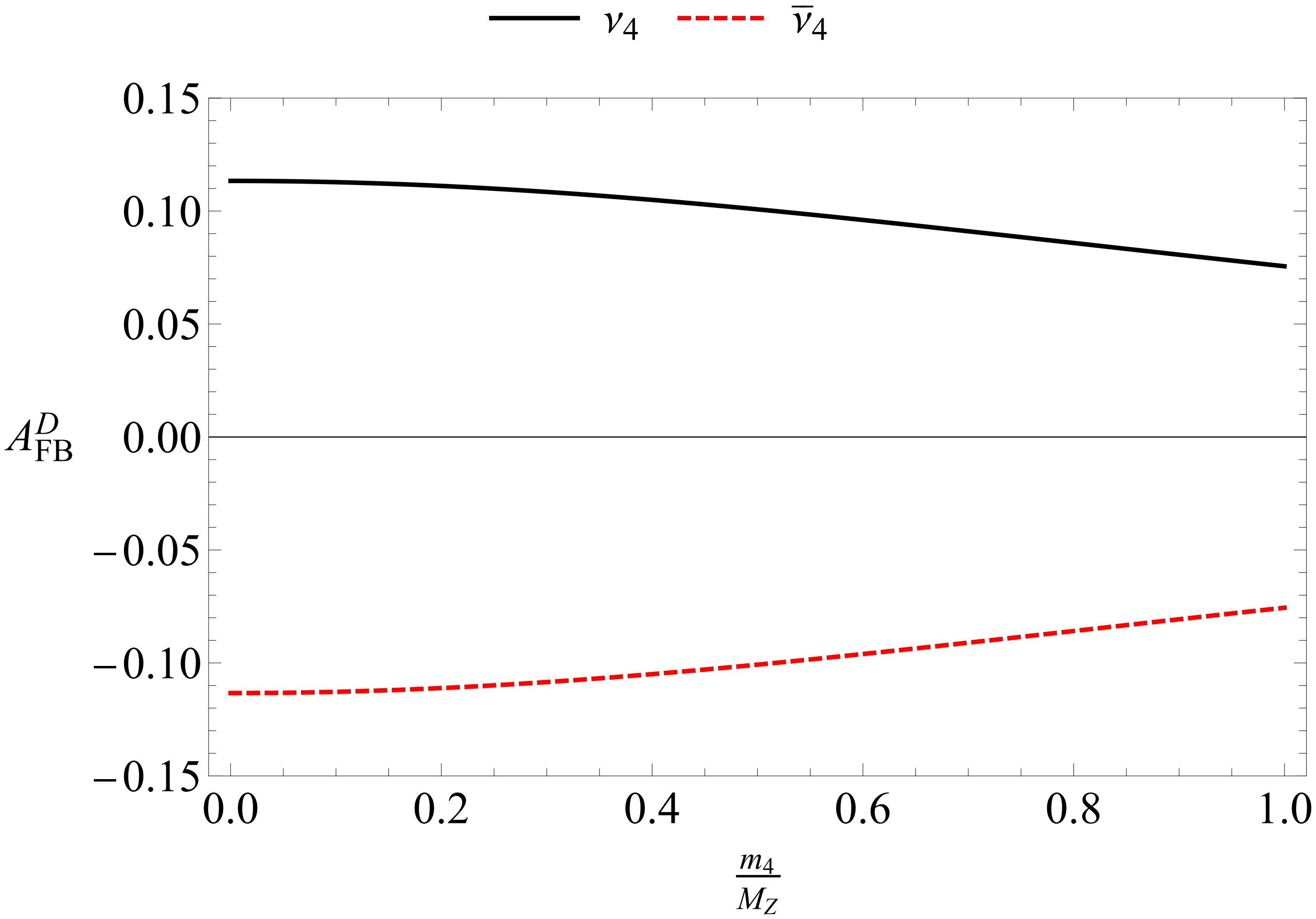
- Why do we care about this? Best constraints on m_4 around 10s of GeV from LEP! A Tera-Z-like experimental setup would be sensitive to seesaw-related heavy neutrinos. Unique opportunity.
- Challenge: Heavy neutrinos produced with light ones. We can't tell whether lepton-number is violated (at least in an event-by-event basis) since we don't get to the detect the light neutrinos.
- Main Message: If m_4 is large enough, Majorana and Dirac ν_4 are produced differently and decay differently!

[There are many related studies. E.g., P. Hernández, et al, arXiv:1810.07210 (ILC)]

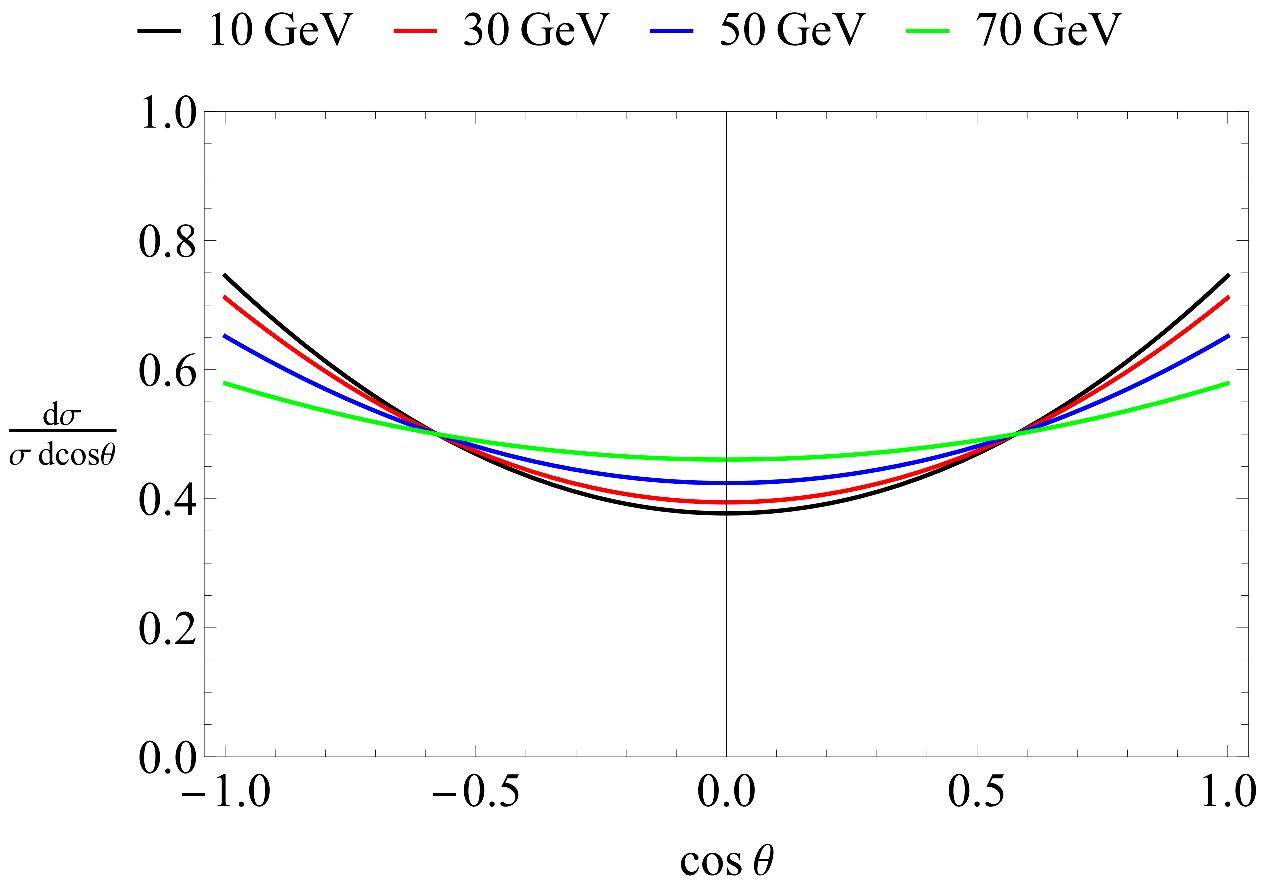


Normalized differential cross-section for $e^+e^- \to Z \to \nu_4\bar{\nu}_{\rm light}$ (left) and $e^+e^- \to Z \to \bar{\nu}_4\nu_{\rm light}$ (right) as a function of the direction of the heavy (anti)neutrino $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Dirac fermions.

[θ is defined relative to the direction of the e^--beam.]



The Forward-Backward Asymmetry A^D_{FB} of heavy neutrino or antineutrino production in $e^+e^- \to Z \to \nu_4\bar{\nu}_{\rm light}$ or $e^+e^- \to Z \to \bar{\nu}_4\nu_{\rm light}$ as a function of the heavy neutrino mass. The neutrinos are assumed to be Dirac fermions.



Normalized differential cross-section for $e^+e^- \to Z \to \nu_4\nu_{\rm light}$ as a function of the direction of the heavy neutrino, $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Majorana fermions.

The Forward-Backward Asymmetry A_{FB}^M vanishes exactly if ν_4 are Majorana fermions.

When the ν_4 decays via charged-current interactions, Dirac ν_4's decay like this:

$$\nu_4 \to \ell^- + X \quad \text{and} \quad \bar{\nu}_4 \to \ell^+ + X^*$$

so the ℓ^- inherits the angular distribution of the ν_4 while ℓ^+ inherits the angular distribution of the $\bar{\nu}_4$.

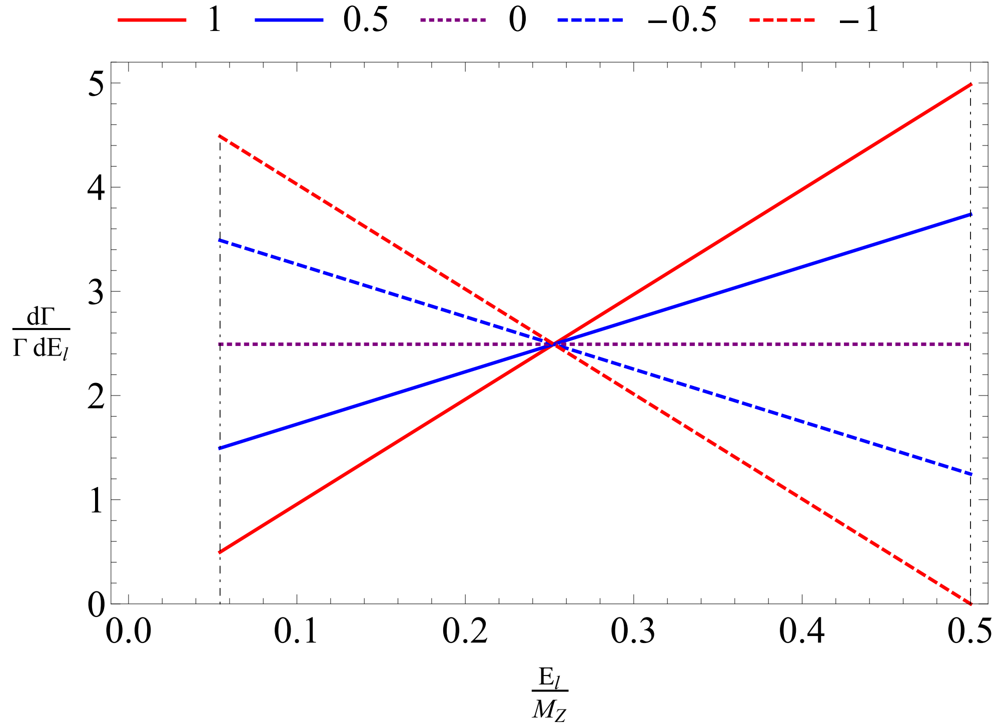
Meanwhile, Majorana ν_4's decay like this:

$$\nu_4 \to \ell^- + X \quad \text{or} \quad \nu_4 \to \ell^+ + X^*$$

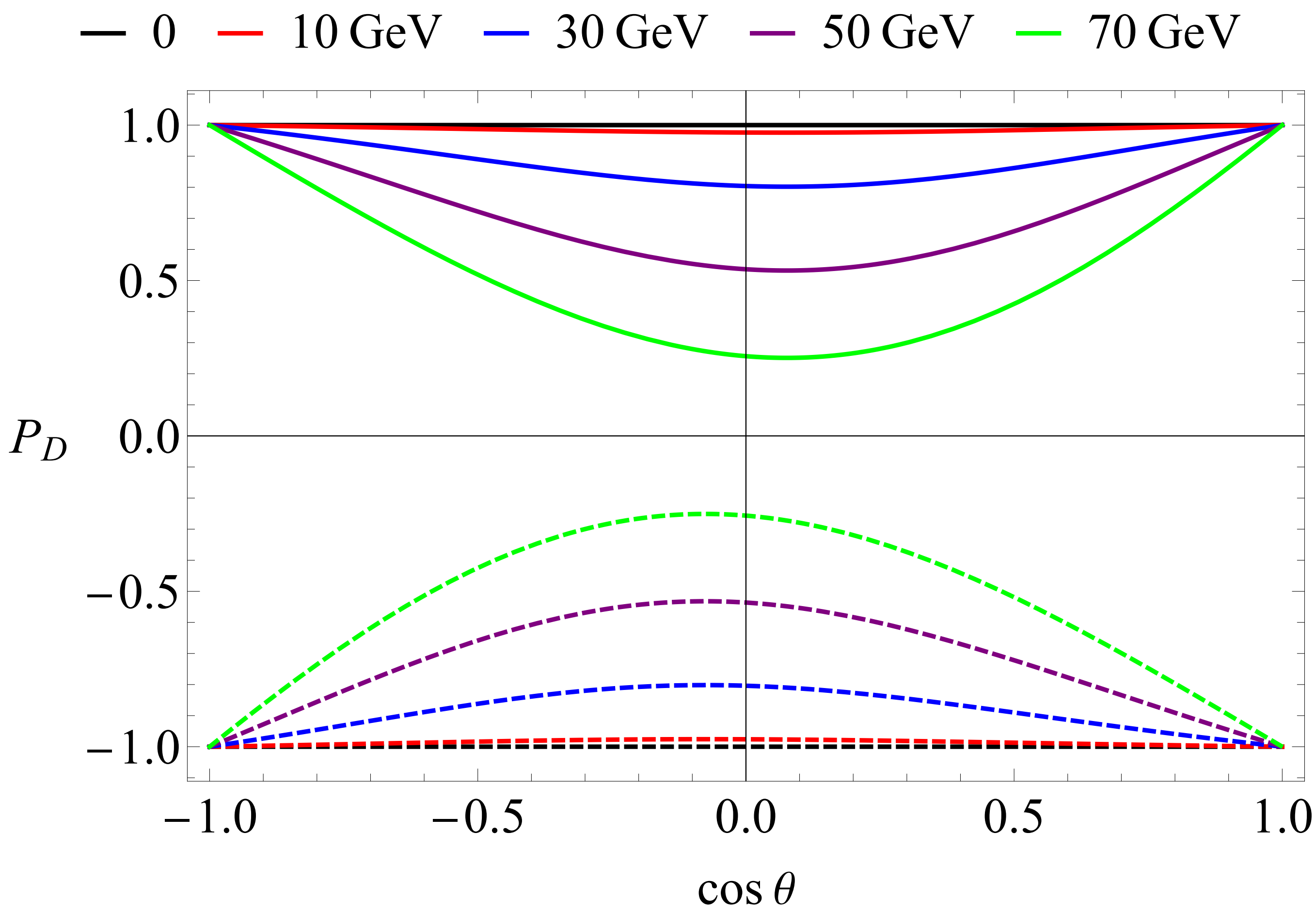
with equal probability. Both the ℓ^+ and the ℓ^- have the same angular distribution and no forward-backward asymmetry.

And there is more…

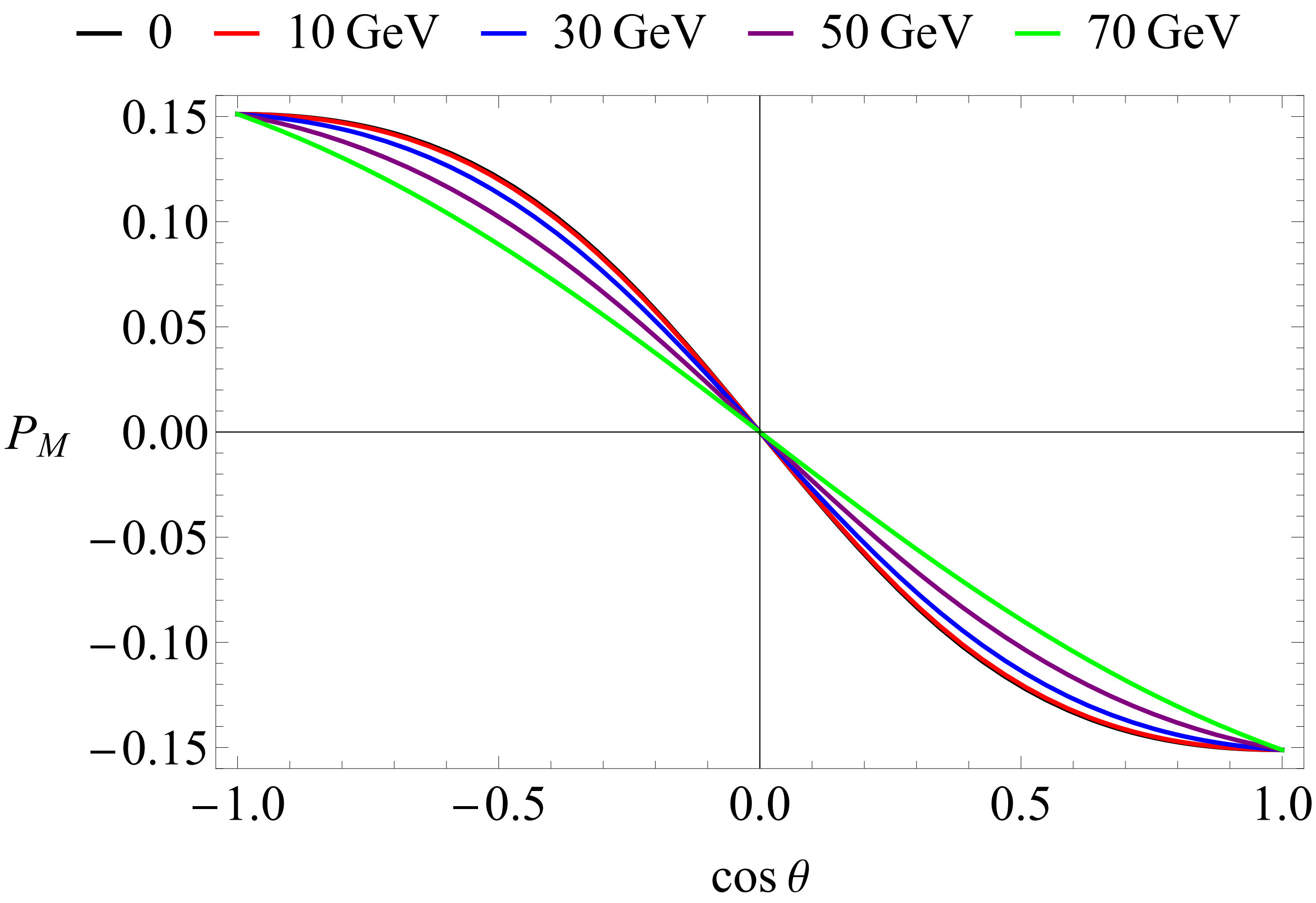
... the energy distribution of the daugther-charged-leptons depends on the polarization of the parent ν_4 (and $\bar{\nu}_4$ if ν_4 is a Dirac fermion). For example,



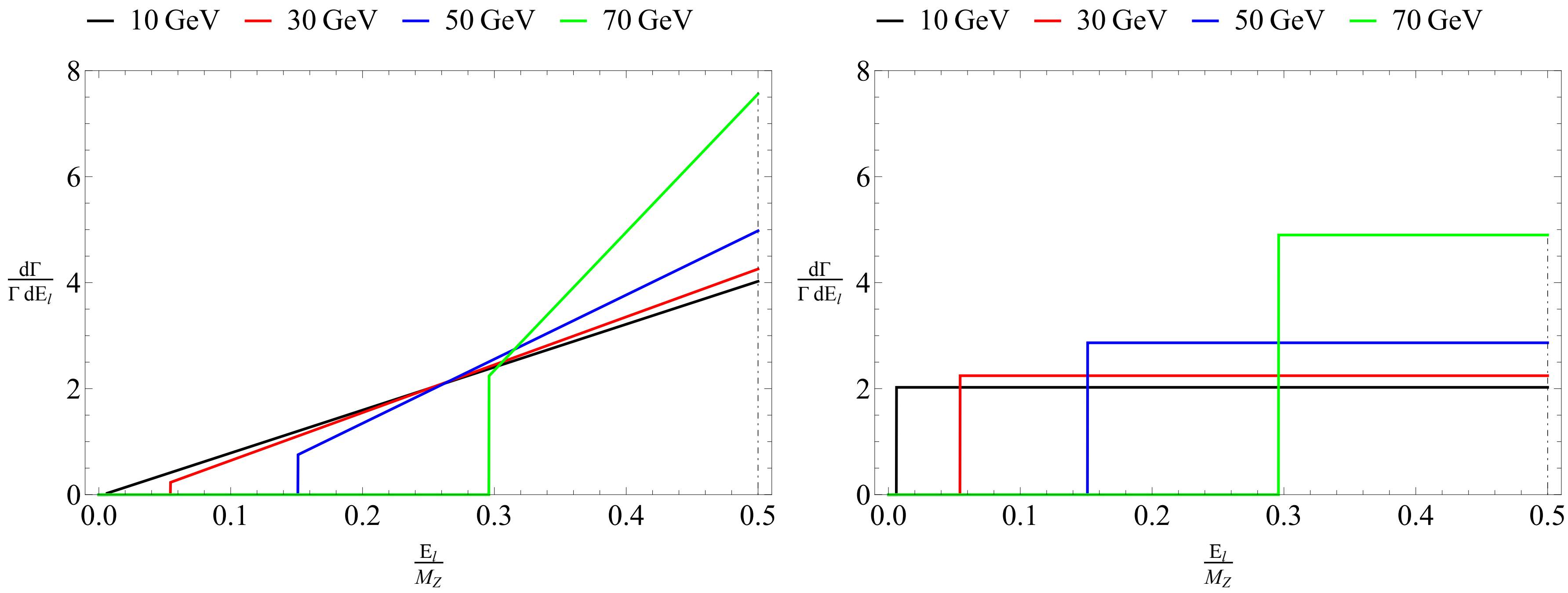
Normalized differential decay widths of $\nu_4 \to \ell^\pm \pi^\mp$ as a function of the energy of the charged-lepton, for ν_4 produced in Z-decay-at-rest. The different curves correspond to different values of $\alpha_\pm P \in [-1, 1]$ and $m_4 = 30$ GeV. $\alpha_\pm$ is the decay-asymmetry parameter and is a property of the physics responsible for the decay. For the SM, $\alpha_+ = +1 = -\alpha_-$



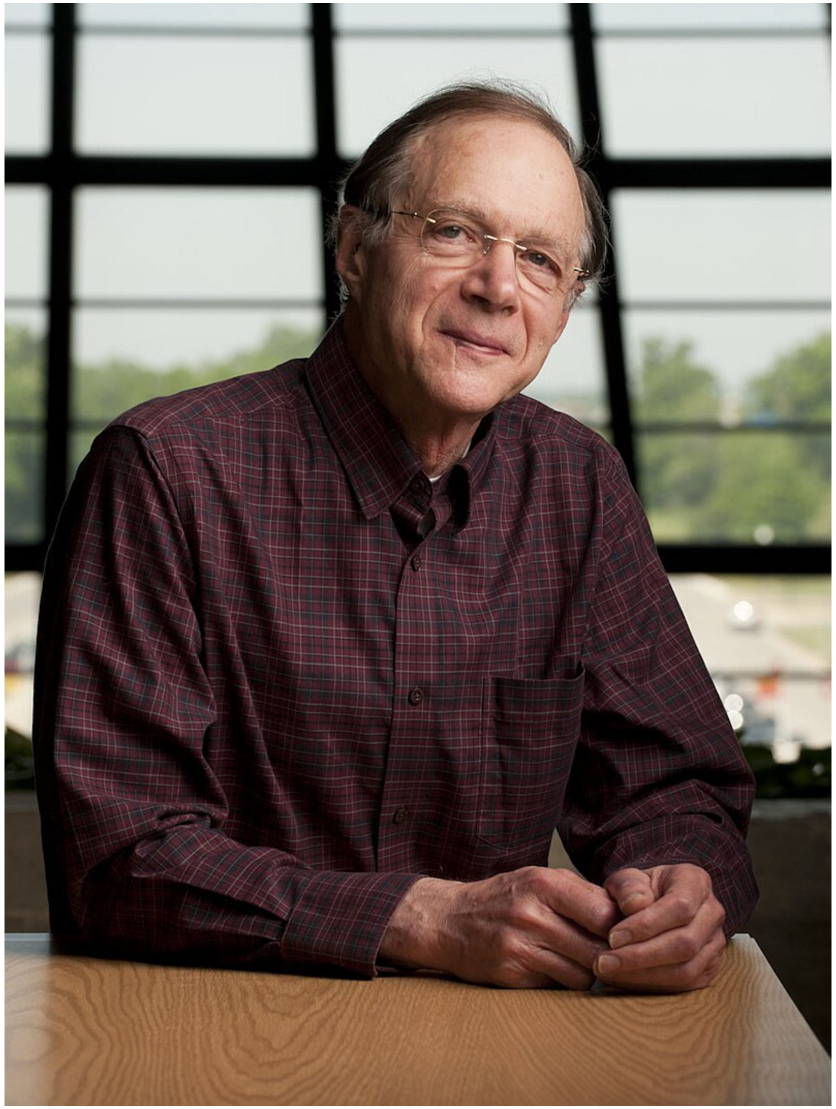
The polarizarion P_D of heavy neutrinos (dashed lines) or antineutrinos (solid lines) produced in $e^+e^- \to Z \to \nu_4\bar{\nu}_{\rm light}$ or $e^+e^- \to Z \to \bar{\nu}_4\nu_{\rm light}$ as a function of the direction of the heavy (anti)neutrino $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Dirac fermions.



The polarizarion P_M of heavy neutrinos produced in $e^+e^- \to Z \to \nu_4\nu_{\rm light}$ as a function of the direction of the heavy neutrino $\cos\theta$, for different values of the heavy neutrino mass m_4. The neutrinos are assumed to be Majorana fermions. [Range of P_M values much smaller than range of P_D values (previous slide).]



Normalized differential decay widths of $\nu_4 \to \ell^- \pi^+$ as a function of the energy of the charged-lepton, averaged over the heavy-neutrino production angle, for ν_4 produced in Z-decay-at-rest assuming the heavy neutrinos are Dirac (left) and Majorana (right) fermions. The different curves correspond to different values of m_4. The same curves apply, both in the left-hand and in the right-hand panels, to the $\ell^+\pi^-$ final-states.



IS HINCHLIFFE'S RULE TRUE?*

Boris Peon

Abstract

Hinchliffe has asserted that whenever the title of a paper is a question with a yes/no answer, the answer is always no. This paper demonstrates that Hinchliffe's assertion is false, but only if it is true.

*Accepted for publication in *Annals of Gnosis*.

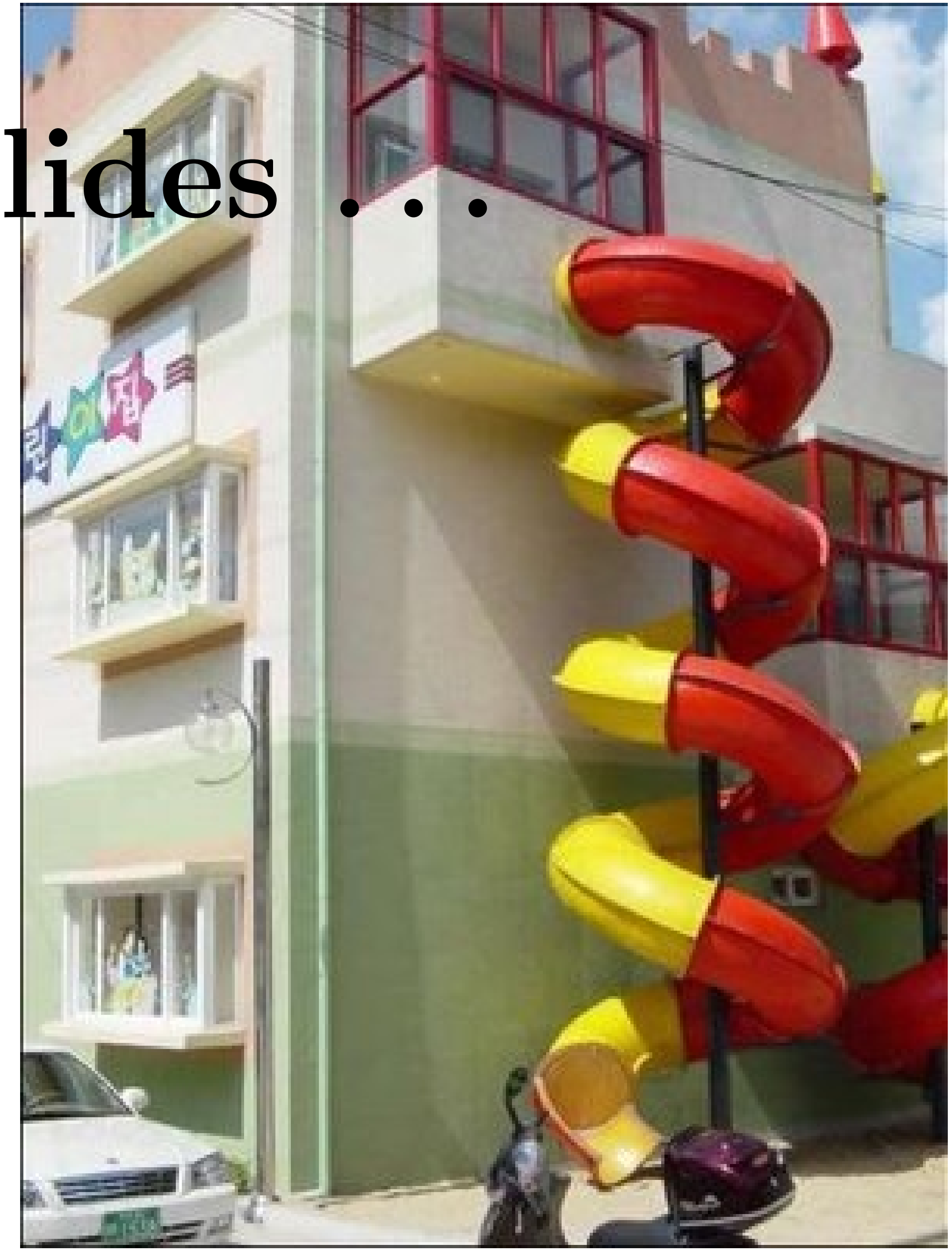
Quick Summary

- Majorana and Dirac Fermions are Qualitatively Different. However, massless Majorana and Dirac fermions are “the same” – Majorana-versus-Dirac is a nonquestion! Since neutrinos are always ultra-relativistic, it is very difficult to address whether they are Majorana or Dirac. Neutrinos are massless as far as most experiments are concerned.

- One solution is to look for phenomena that can only occur if the neutrino is a Majorana fermion (e.g., LNV). Even for very rare phenomena, any positive result establishes that neutrinos are Majorana fermions.

- The other way is to find circumstances where the neutrinos are not ultra-relativistic. In this case, the Majorana versus Dirac differences are large. The rates, on the other hand...

Backup Slides . . .



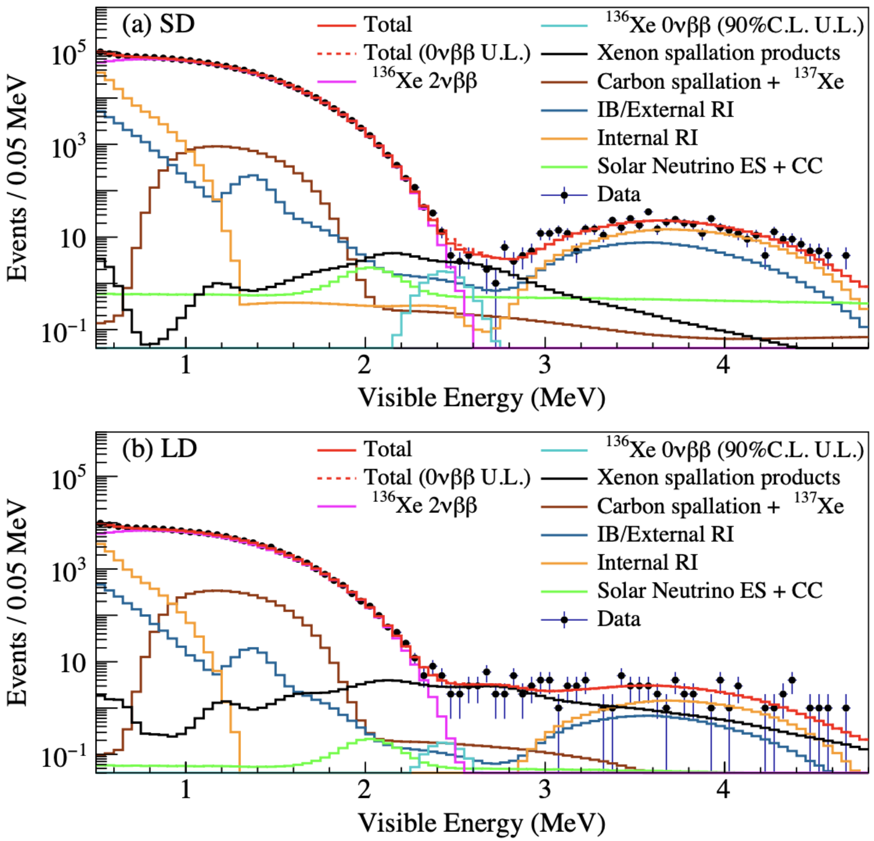


FIG. 2: Energy spectra of selected $\beta\beta$ candidates within a 1.57-m-radius spherical volume drawn together with best-fit backgrounds, the $2\nu\beta\beta$ decay spectrum, and the 90% C.L. upper limit for $0\nu\beta\beta$ decay of (a) singles data (SD), and (b) long-lived data (LD). The LD exposure is about 10% of the SD exposure.

Lots of Experimental Activity!

Moving Towards Ton-Scale Expts.

(LEGEND, CUPID, nEXO, etc)

[KamLAND-Zen Coll. (Abe *et al*), 2406.11438 [hep-ex]]

Another Example of Neutrinos Near Threshold (Brief)

Atomic process: $A^* \to A\gamma$, where A (A^*) is a neutral atom (in some excited state). Now replace the γ with an off-shell Z, which manifests itself as two neutrinos:

$$A^* \to A\nu\bar{\nu}.$$

It is easy to imagine sub-eV energies and hence the neutrinos are not ultra-relativistic.

For all the details including rates – tiny – and difference between Majorana and Dirac neutrinos – large – see, for example, Yoshimura, hep-ph/0611362, Dinh *et al.*, arXiv:1209.4808, and Song *et al.* arXiv:1510.00421, and references therein.

ν_4 and Lepton-Number Violation at Hadron Colliders

Heavy neutrinos, when produced at a collider experiment, may also mediate lepton-number violation if they are Majorana fermions. For example,

$$XW^{+(*)} \to X\ell^+\nu_4(\to \ell^- + q\bar{q}').$$

versus

$$XW^{+(*)} \to X\ell^+\nu_4(\to \ell^+ + q\bar{q}').$$

Smoking-gun if the lepton-number of X is known (e.g., zero).

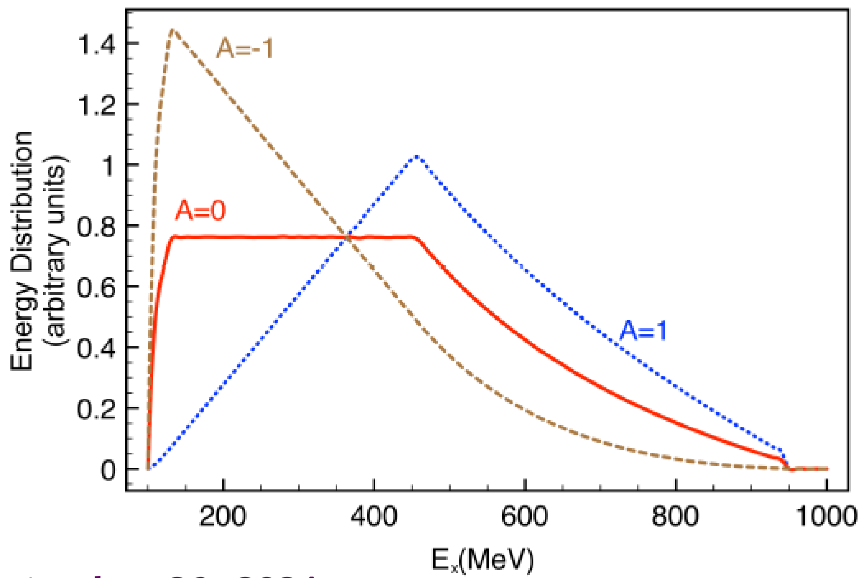
Energy distribution in the Laboratory ("same" as angular distribution)

Parent's rest frame for $N \to \nu_\ell + X$

$$\frac{dn_X}{d\cos\theta_X} \propto (1 + A\cos\theta_X), \quad A = \alpha \times \text{polarization}$$

Lab frame with $r = m_X^2/m_N^2 < 1$

$$\frac{dn_X(E_N, E_X)}{dE_X} \propto \frac{2}{p_N(1-r)}\left[1 + A\left(\frac{2}{(1-r)}\frac{E_X}{p_N} - \left(\frac{1+r}{1-r}\right)\frac{E_N}{p_N}\right)\right]$$



$m_X = 100\,\text{MeV}$

$m_N = 300\,\text{MeV}$

$500\,\text{MeV} < E_N < 100\,\text{MeV}$

Aside: We Can Use Charged Final States Too!

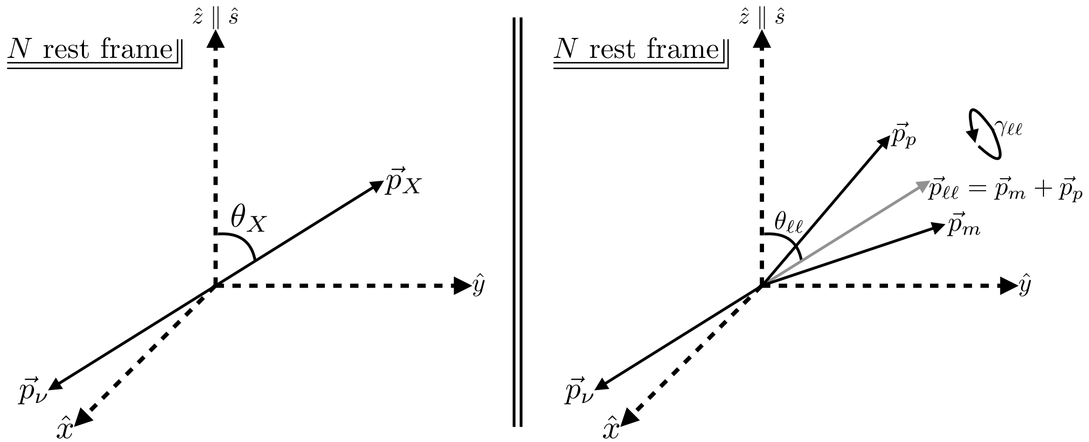
The two-body final states here all involve a neutrino and a neutral boson. Impossible to reconstruct the parent rest-frame and it requires measuring the properties of a neutral boson, which is sometimes challenging. Can we use the charged final states? E.g.,

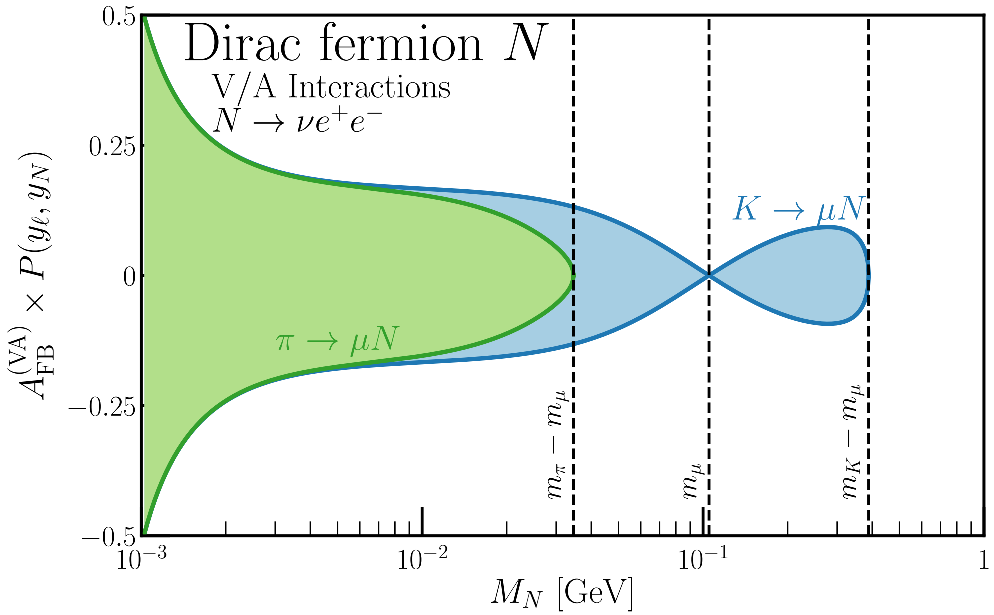
$$\nu_4 \to \mu^+\pi^-$$

Most of the time, 'yes'! The reason is as follows. CPT invariance (at leading order) implies, for 100% polarized Majorana fermions,

$$\frac{d\Gamma(\nu_4 \to \mu^+\pi^-)}{d\cos\theta} \propto (1+\alpha\cos\theta) \quad \text{while} \quad \frac{d\Gamma(\nu_4 \to \mu^-\pi^+)}{d\cos\theta} \propto (1-\alpha\cos\theta)$$

so the **charge-blind sum of the two is also isotropic**. This is not the case for Dirac neutrinos as long as the production of neutrinos and antineutrinos is asymmetric, which is usually the case.





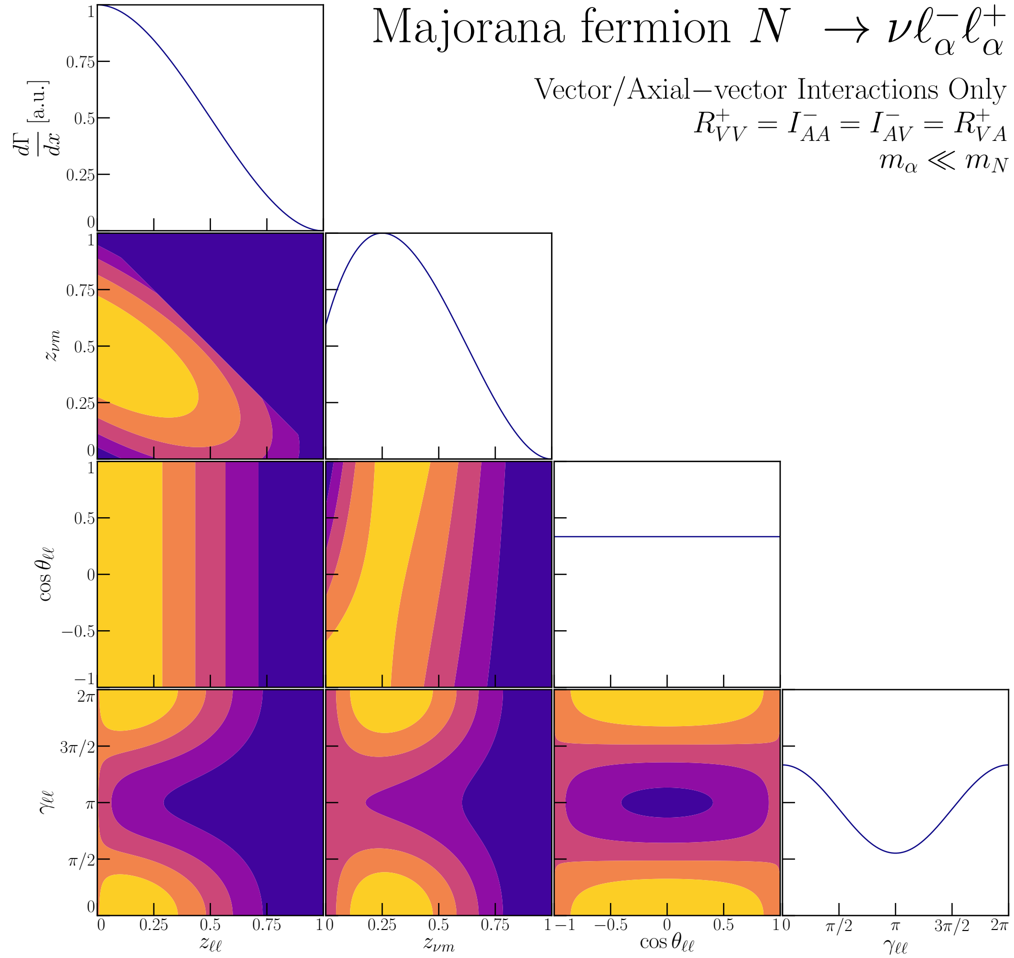
[AdG et al, arXiv: 2109.10358]

Majorana fermion $N \to \nu \ell_\alpha^- \ell_\alpha^+$

Vector/Axial−vector Interactions Only

$R_{VV}^+ = I_{AA}^- = I_{AV}^- = R_{VA}^+$

$m_\alpha \ll m_N$

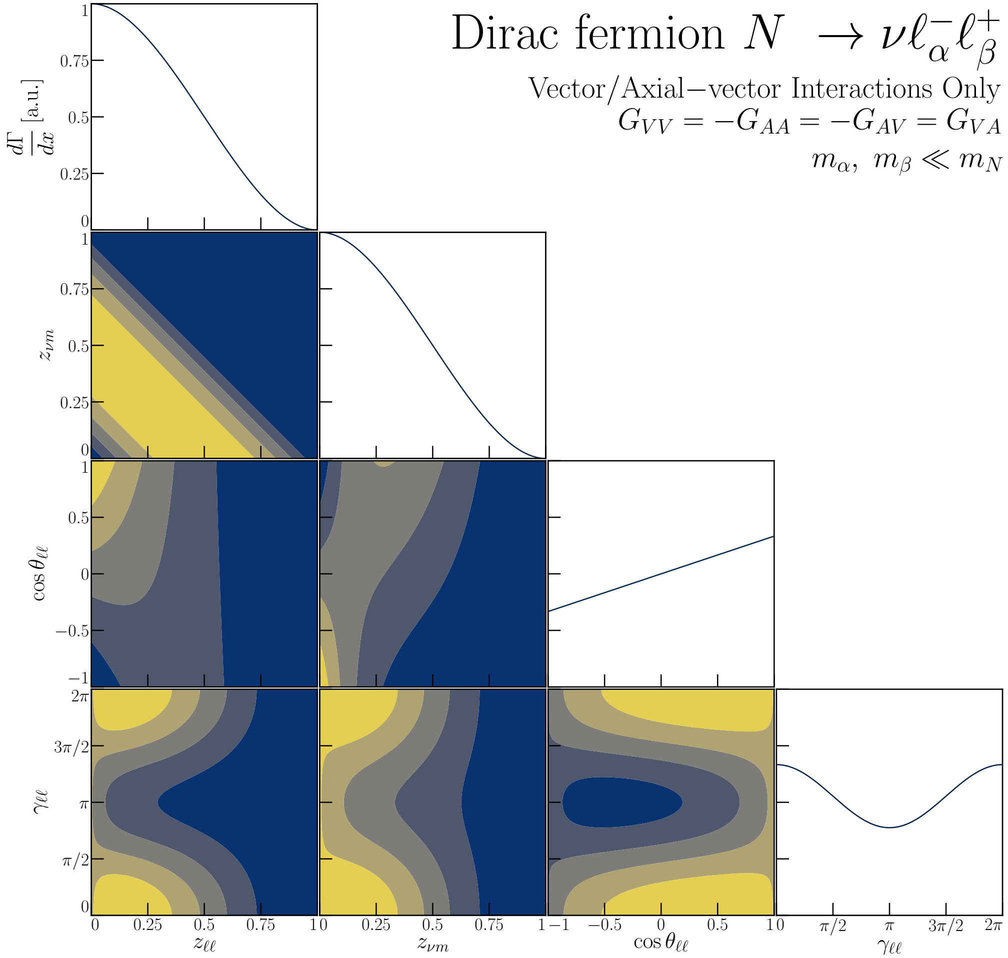


Dirac fermion $N \to \nu \ell_\alpha^- \ell_\beta^+$

Vector/Axial−vector Interactions Only

$G_{VV} = -G_{AA} = -G_{AV} = G_{VA}$

$m_\alpha,\ m_\beta \ll m_N$



[AdG et al, arXiv: 2104.05719]