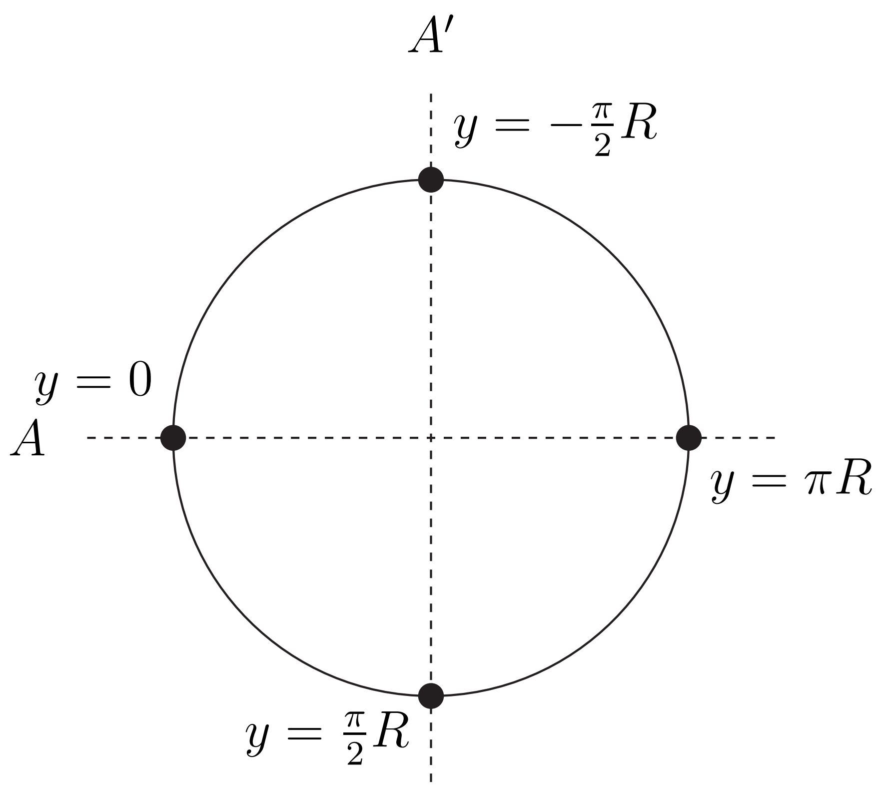
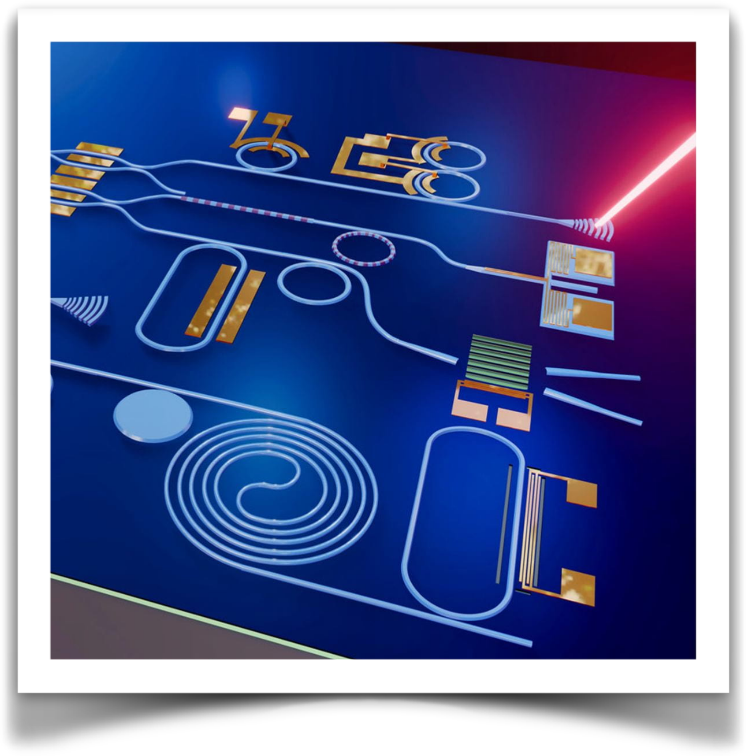
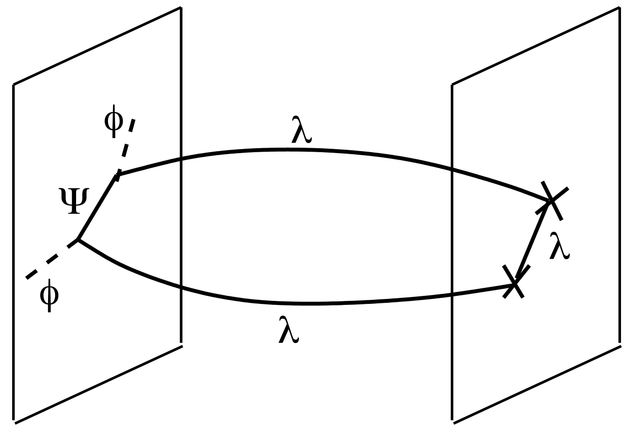
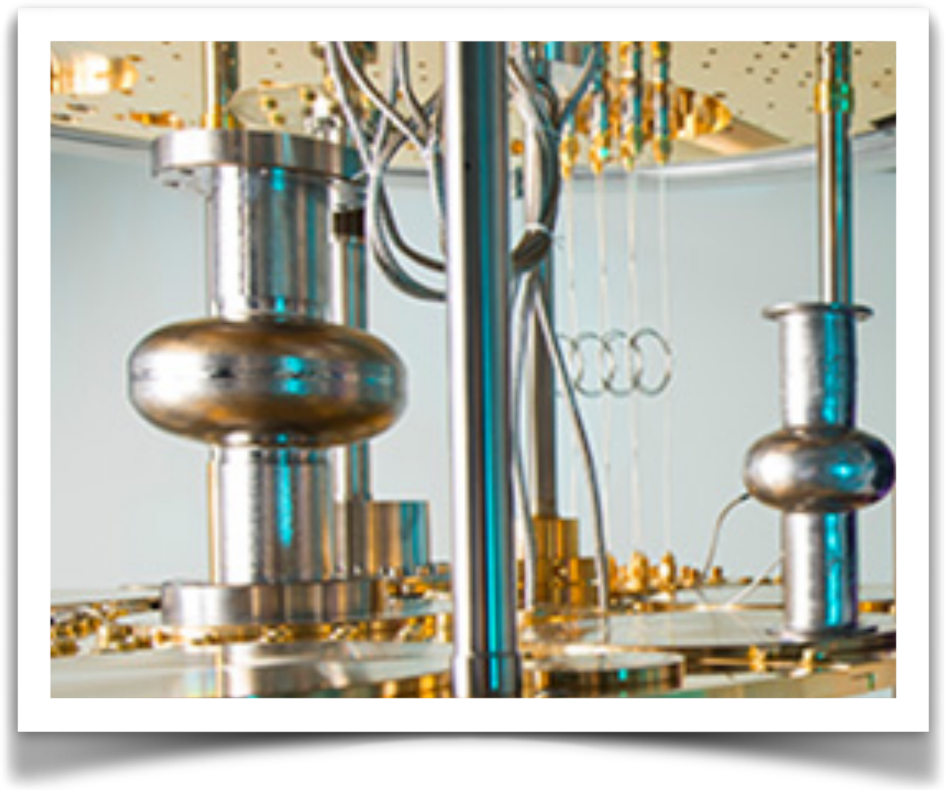
Quantum Devices for Model Builders

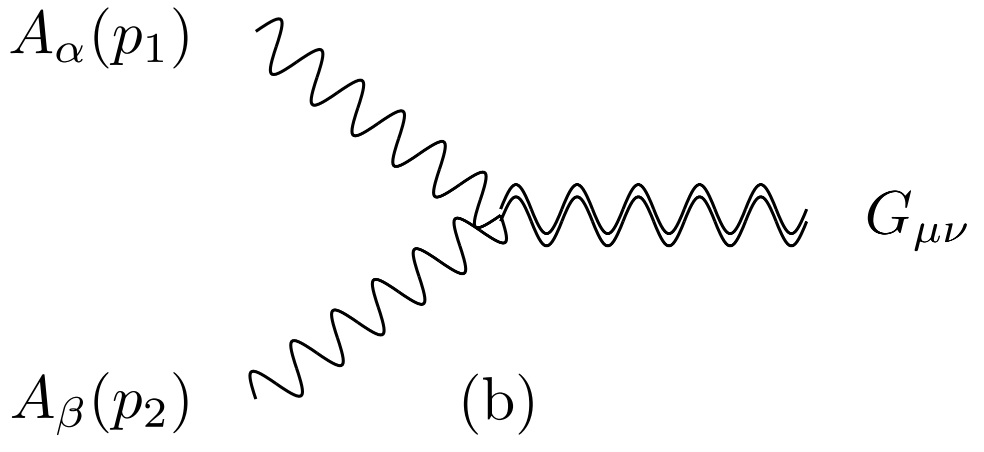
Roni Harnik, Fermilab











Fermilab

SQMS
SUPERCONDUCTING QUANTUM
MATERIALS & SYSTEMS CENTER

3061

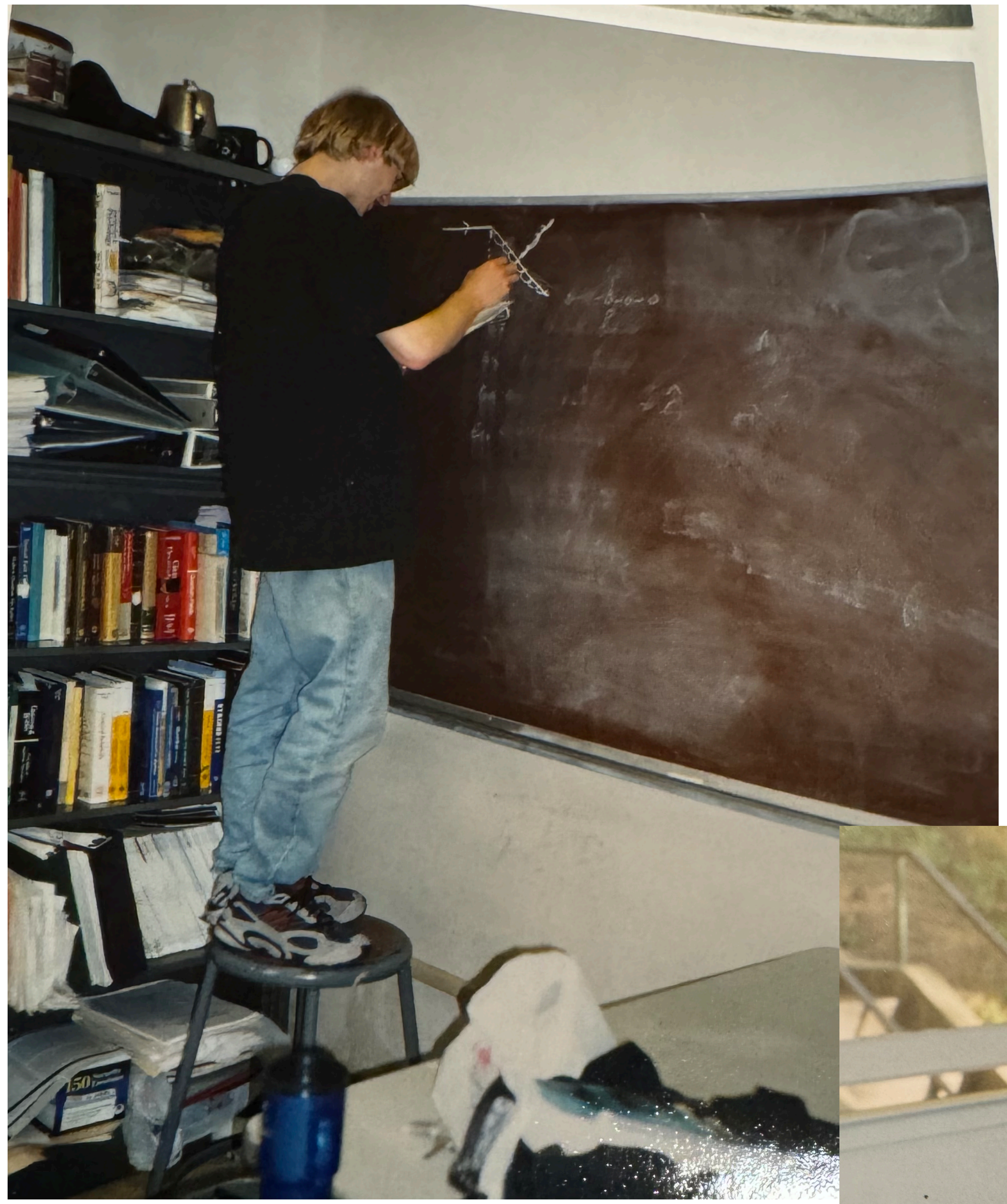
Time Capsule: HEP @ Berkeley in the Early 2000's

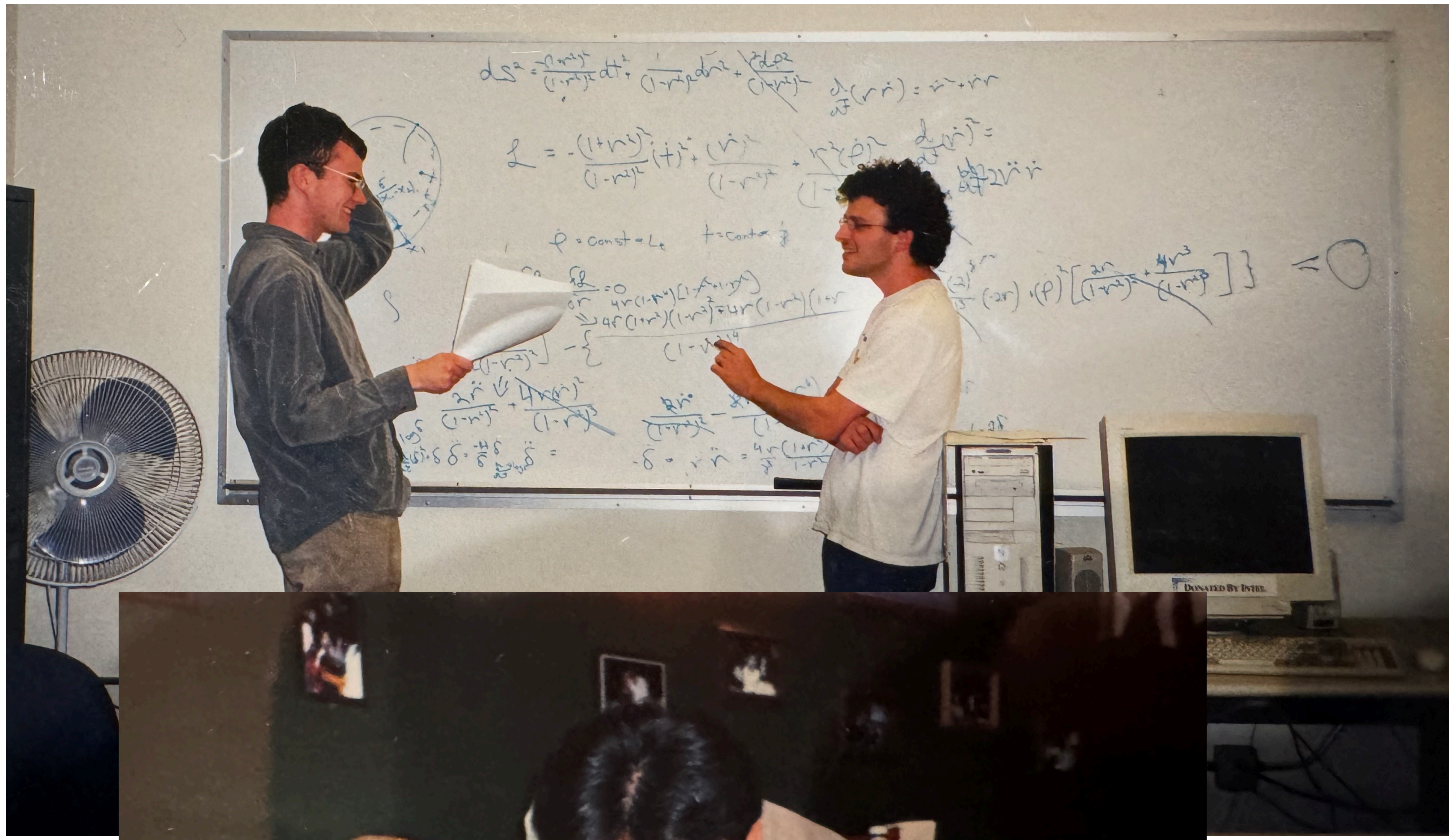
- LEP concluding. LHC construction beginning.
- Tevatron was on, but already in late years.
- Flavor physics had data w/ Babar+Belle
- Neutrino masses recently established
- DM direct detection was CDMS vs DAMA and that's it.

A lot of theorists, are actively **model building.**

Grad students joined the fun!

Exploring a lot on our own ···





Emotional roller coaster.

Model alive, model dead, alive, dead ···

Model Building Themes

- Hierarchy problem
- SUSY mediation mechanisms
- Problems with SUSY mediation mechanisms
- Flavor in SUSY models
- Extra dimensions of various kinds
- Little Higgs & Little Hierarchy Problem

A bag of tricks!

super potentials, symmetries, soft symmetry breaking, collective symmetry breaking, compositness, warped dimensions, branes, brane localized terms, NDA, Seiberg duality,

Phenomenology was confined to "we predict particles within LHC's reach.

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- How the hell can Hitoshi be so productive ???

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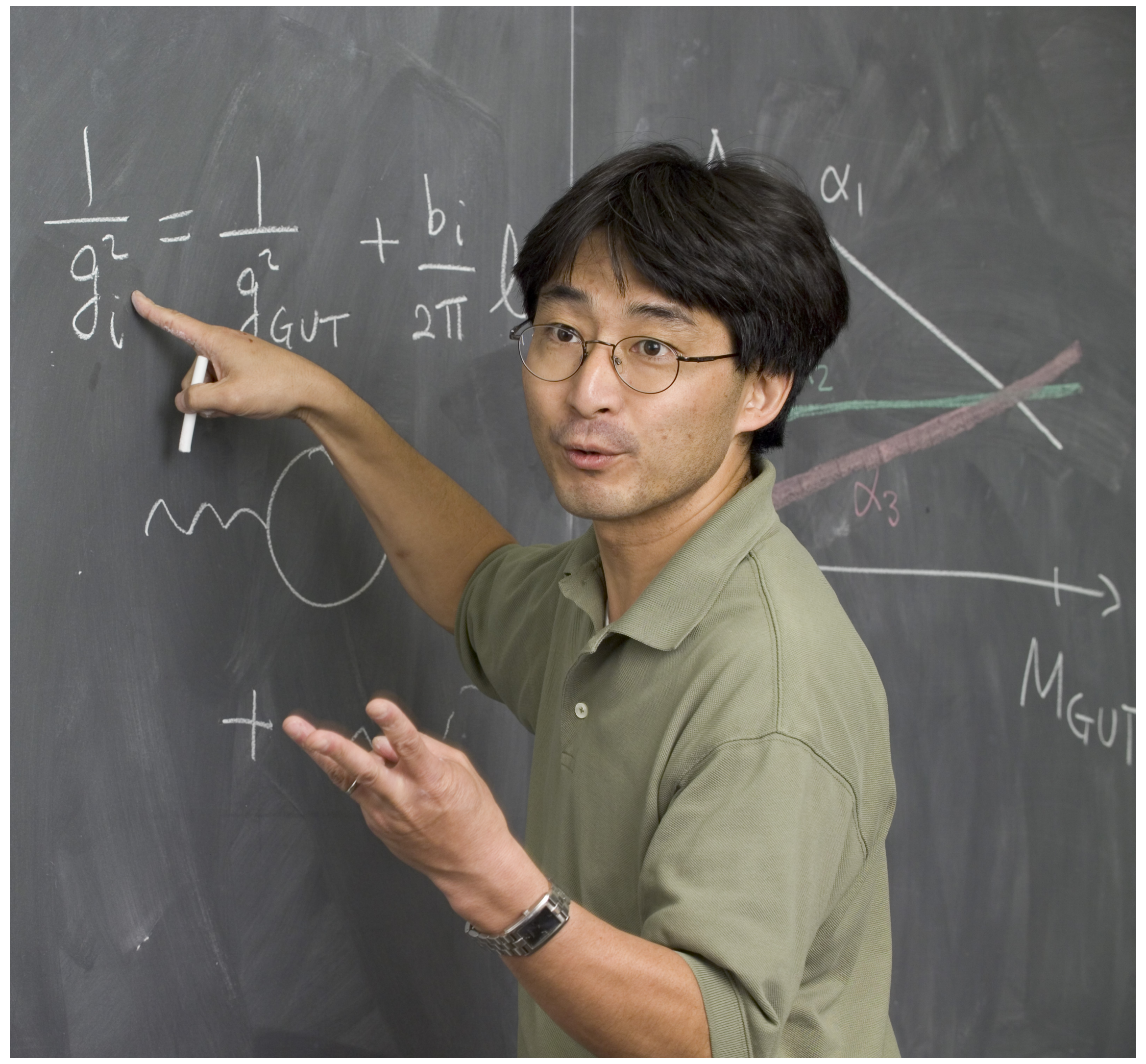
Twin Hitoshi Theory

Abstract: We present a model that naturally explains how Hitoshi can get so much done by invoking a (softly broken) Z_2 symmetry.

Take Hitoshi. Add a Z_2 symmetry.



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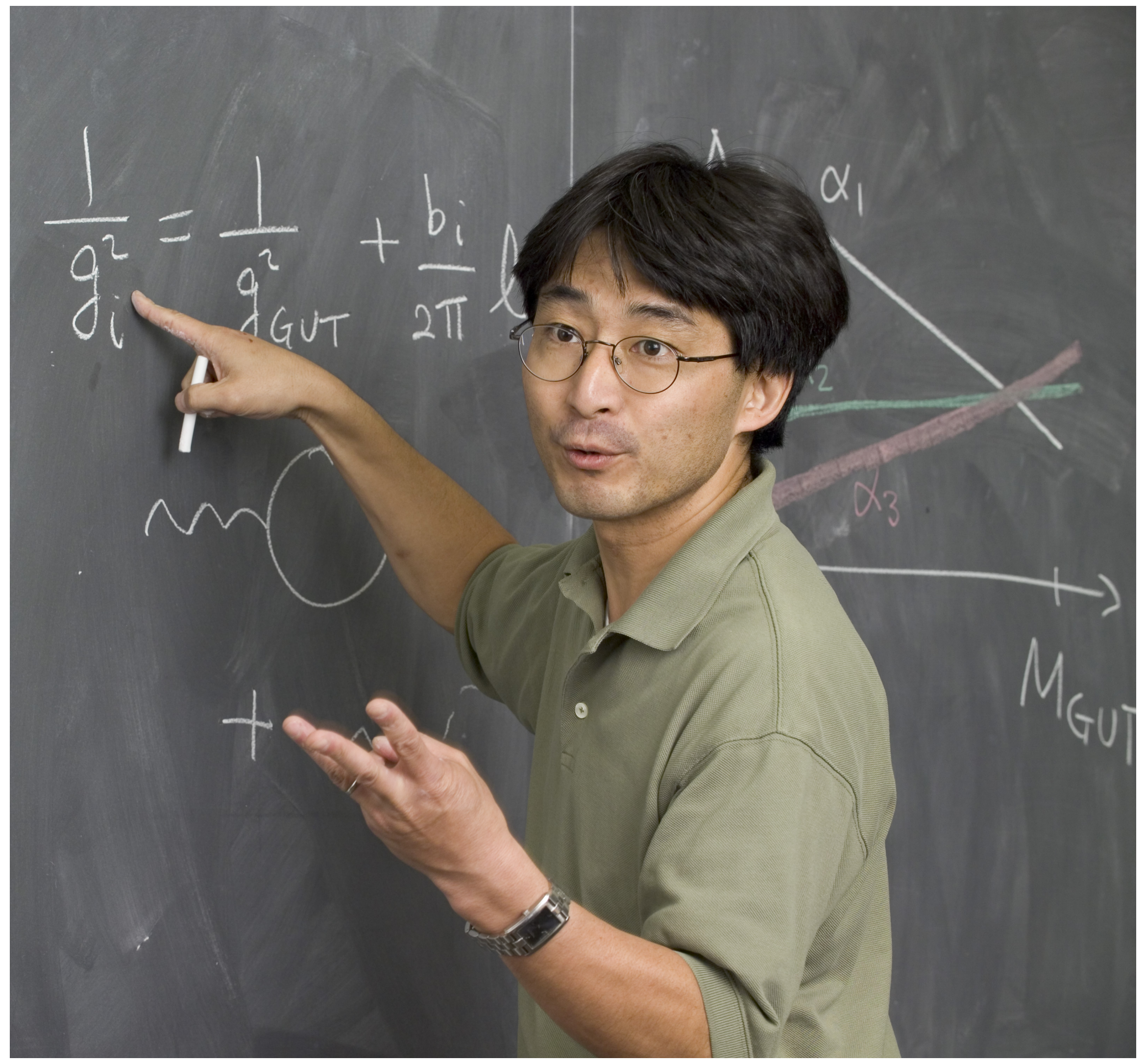

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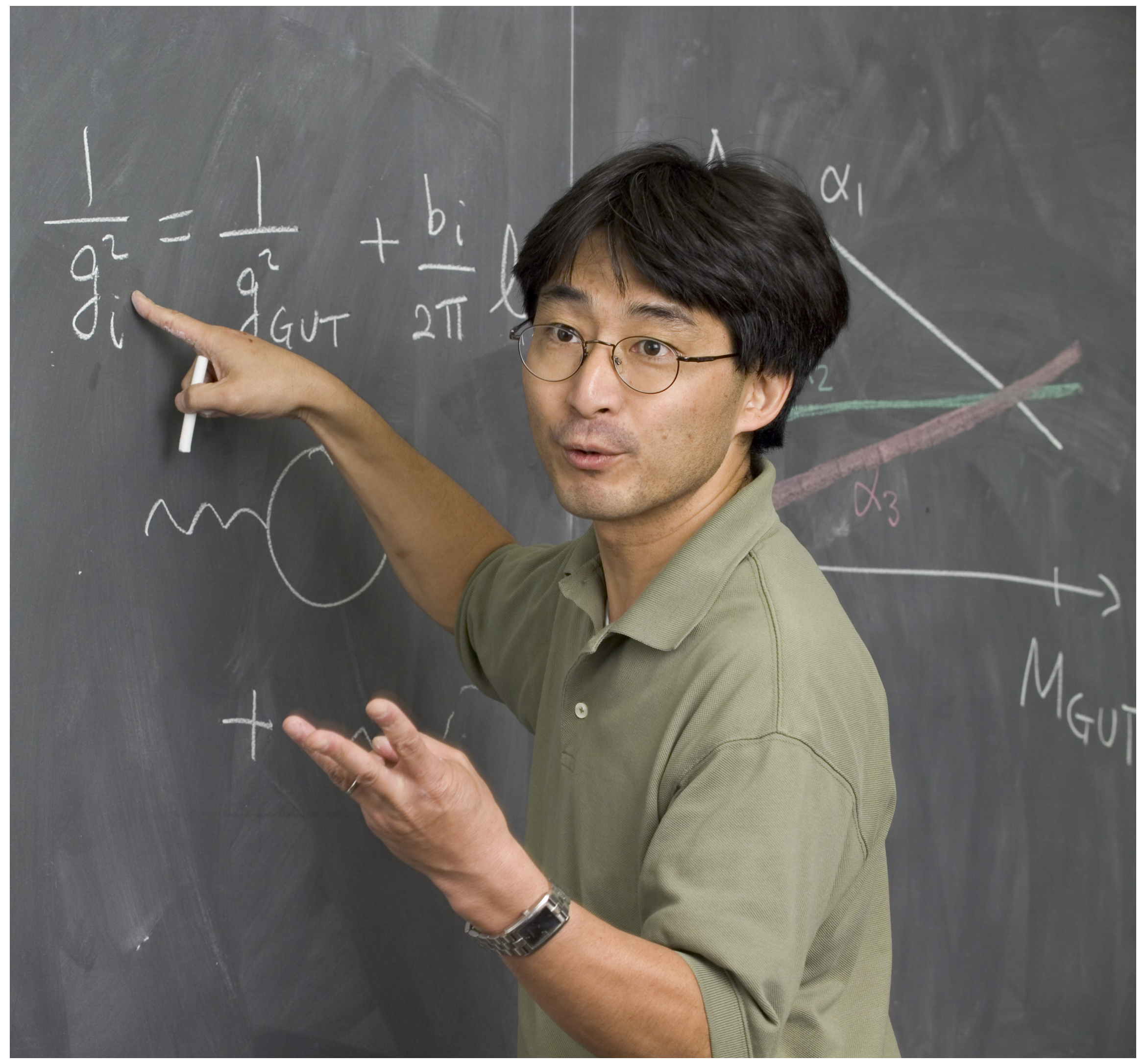

Our model explains:

- How they get so much done.
- How he calculated overnight and looked refreshed in the morning.
- Other rare phenomena:

Teaching at Berkeley while being in Japan on Kamland shift[1].

Having enough frequent flyer mile for two people[2]

On few occasions I told Hitoshi something, and the next day he did not remember...[3]

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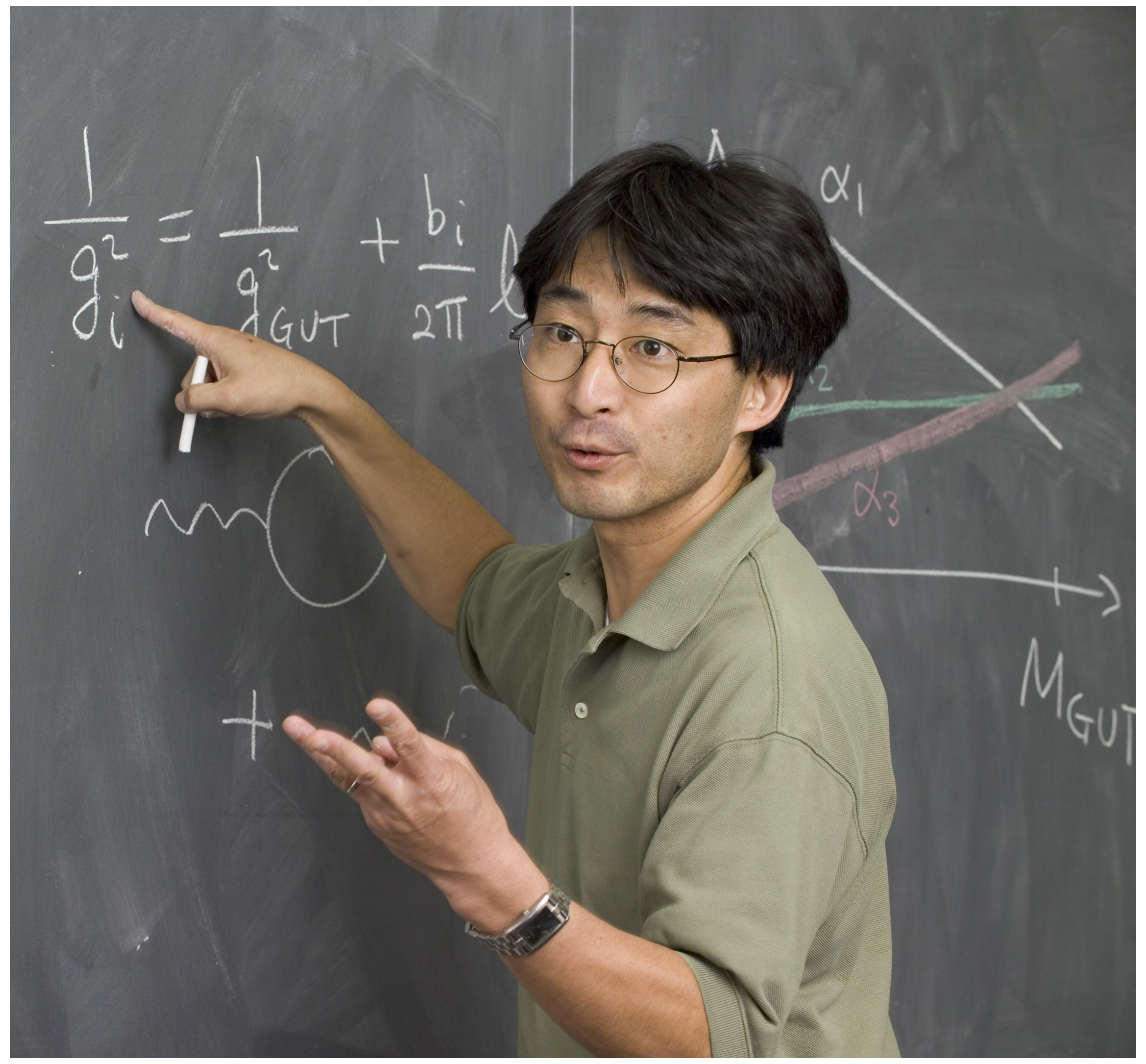

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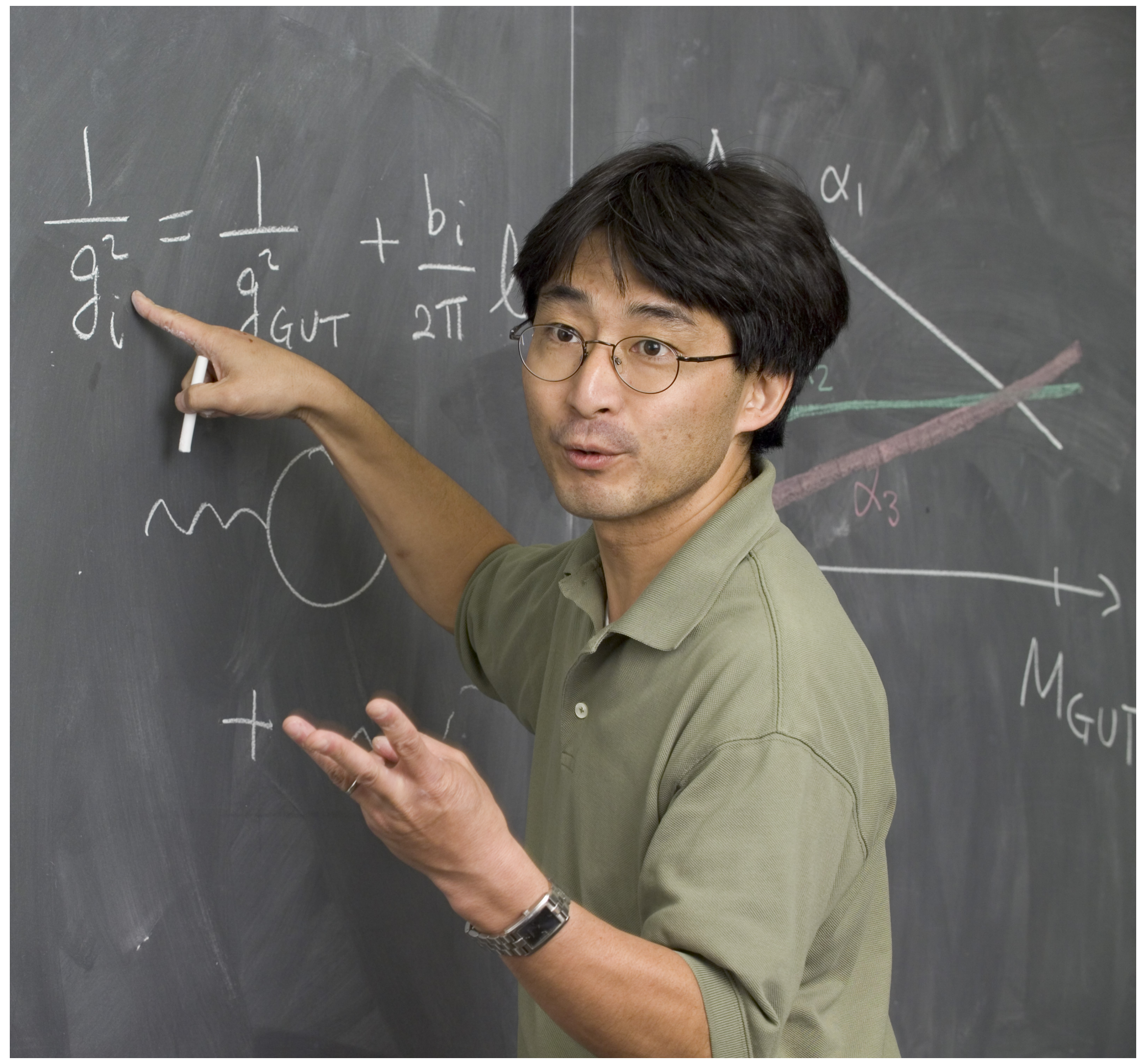

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[3] However, after interacting with Lawrence, I concluded that this effect was not statistically significant.

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Model Building Themes

- Hierarchy problem
- SUSY mediation mechanisms
- Problems with SUSY mediation mechanisms
- Flavor in SUSY models
- Extra dimensions of various kinds
- Little Higgs & Little Hierarchy Problem
- Cosmology sometimes played a more minor role…

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hep-ph 0011311

In 2001 Lawrence gave a talk about this paper:

A Constrained Standard Model from a Compact Extra Dimension

Riccardo Barbieri[a], Lawrence J. Hall[b,c], Yasunori Nomura[b,c]

A $SU(3) \times SU(2) \times U(1)$ supersymmetric theory is constructed with a TeV sized extra dimension compactified on the orbifold $S^1/(Z_2 \times Z_2')$. The compactification breaks supersymmetry leaving a set of zero modes which correspond precisely to the states of the 1 Higgs doublet standard model. …

… yielding a Higgs mass prediction of 127 ± 8 GeV. The masses of the all superpartners, and the Kaluza-Klein excitations are also predicted. The lightest supersymmetric particle is a top squark of mass 197 ± 20 GeV

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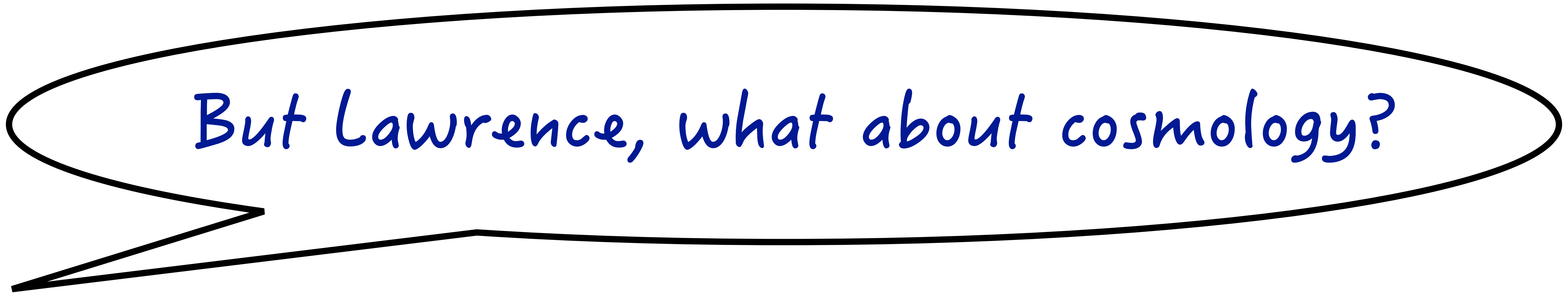

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But Lawrence, what about cosmology?

Cosmology Schmosmology.

(+ chuckle for effet)

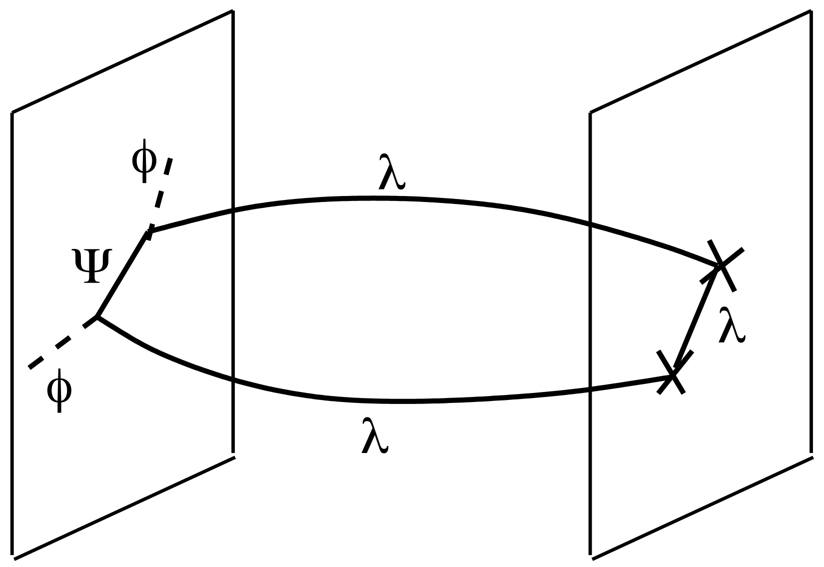
Our bag of tricks

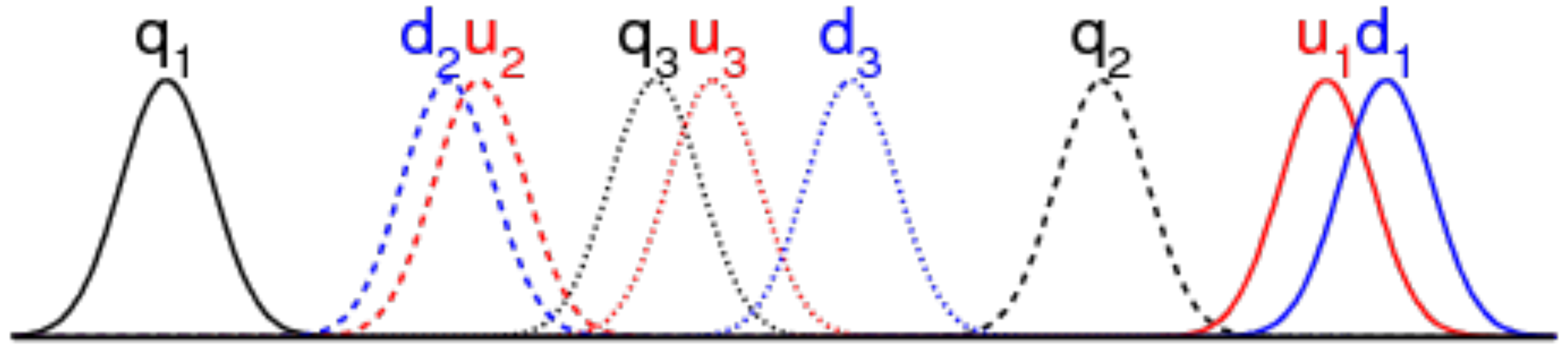
- In recent years I'm working quite a bit at the interface of HEP and QIS.

- QIS folks are playing with a lot of our tricks!

  But:
  - Calling them by different names.
  - Different goals.
  - They are not only building models. They are building stuff!

compositness,
EFT
compact dimensions
branes
brane localized terms
NDA
neutrino oscillations
Matter effects
...

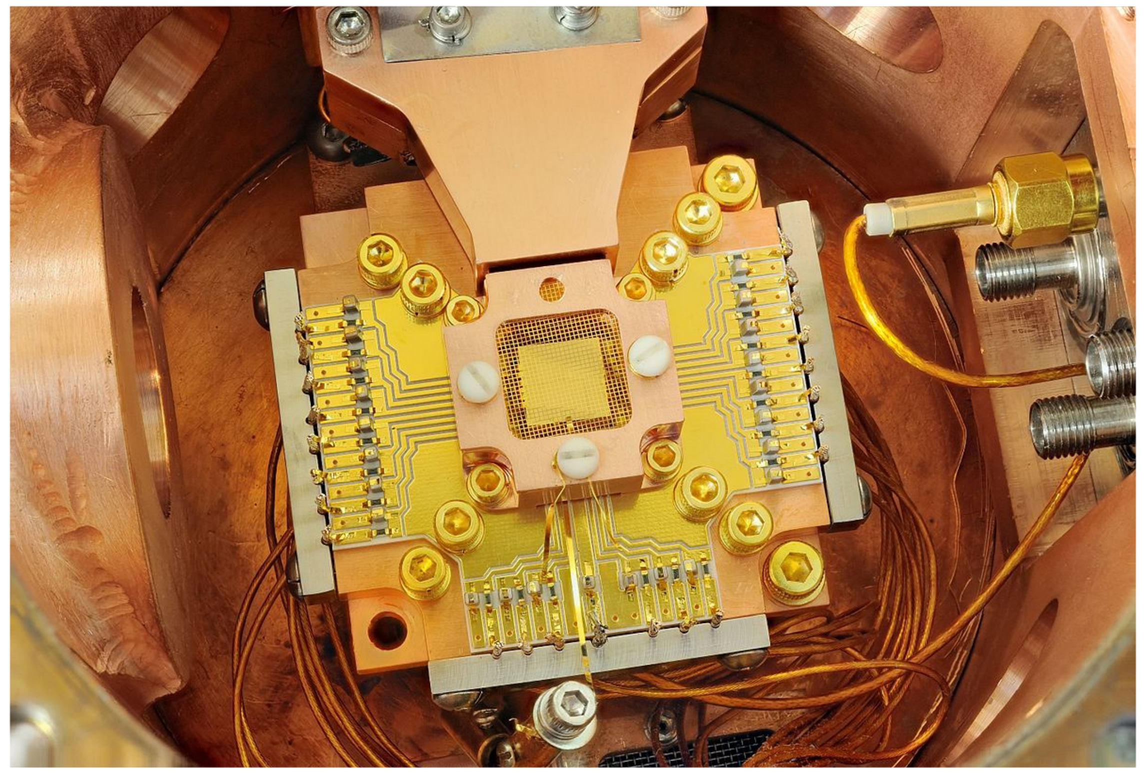




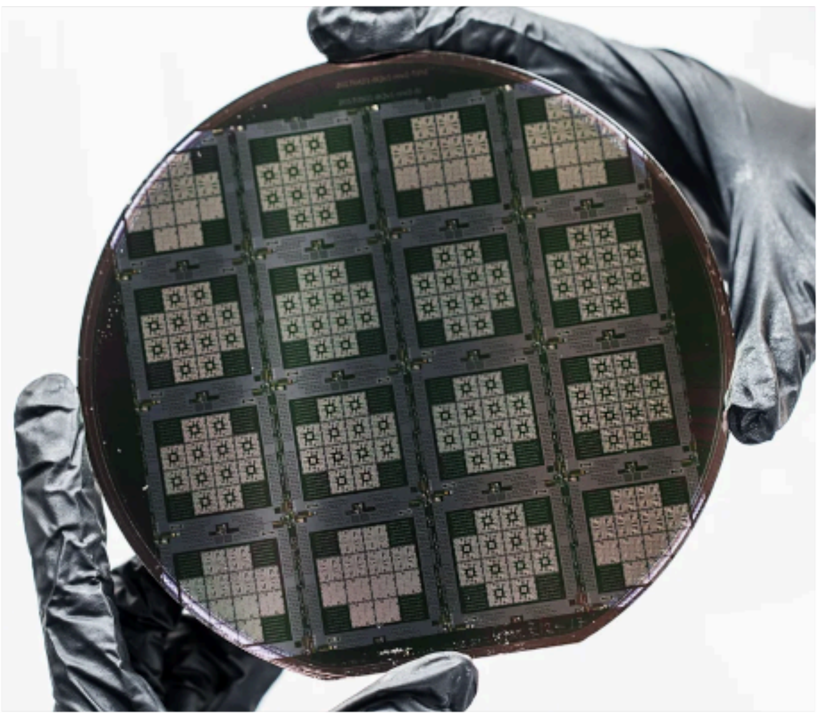
Quantum Devices

(But in model builder's language)

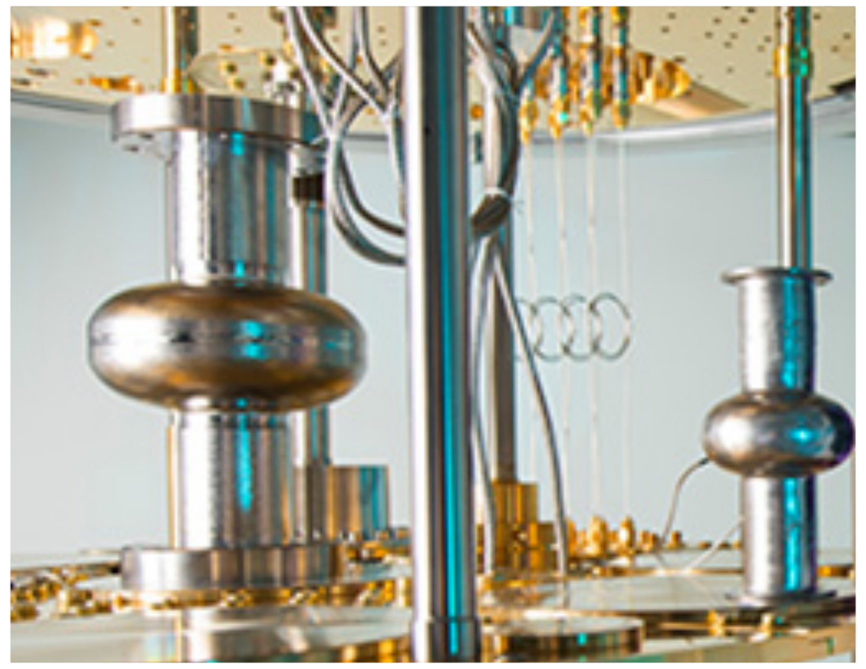
Ion traps



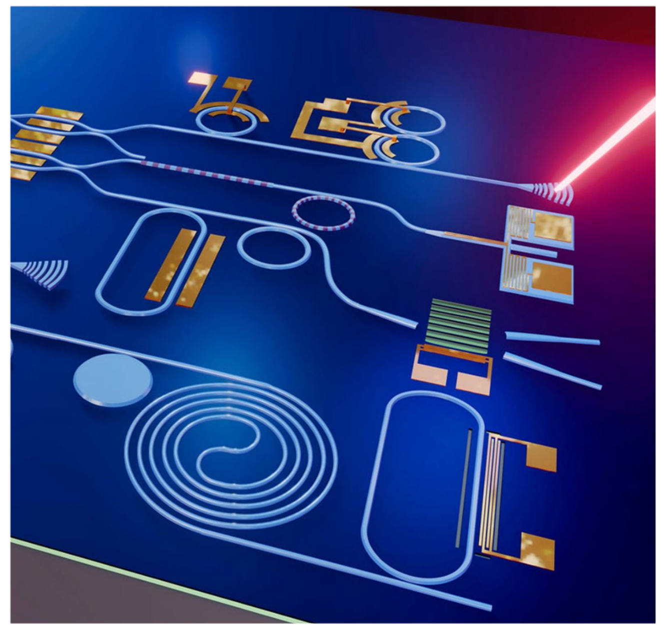
SC qubits



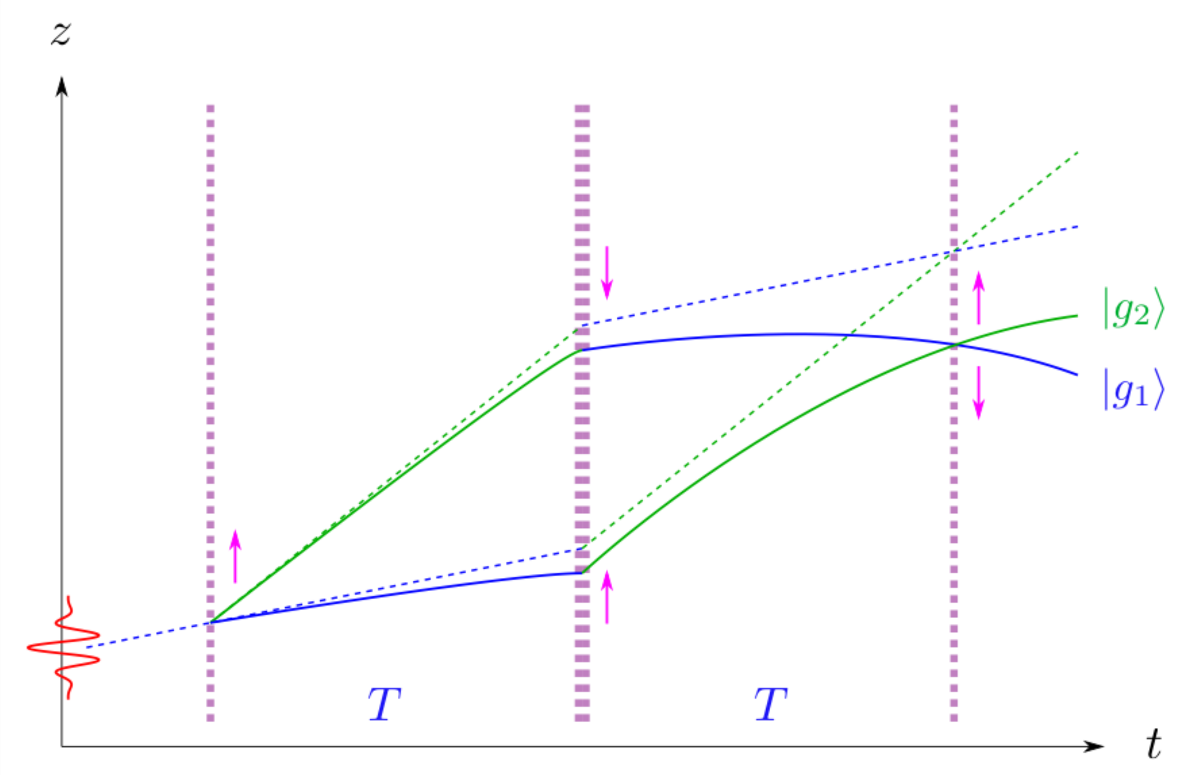
SC cavities



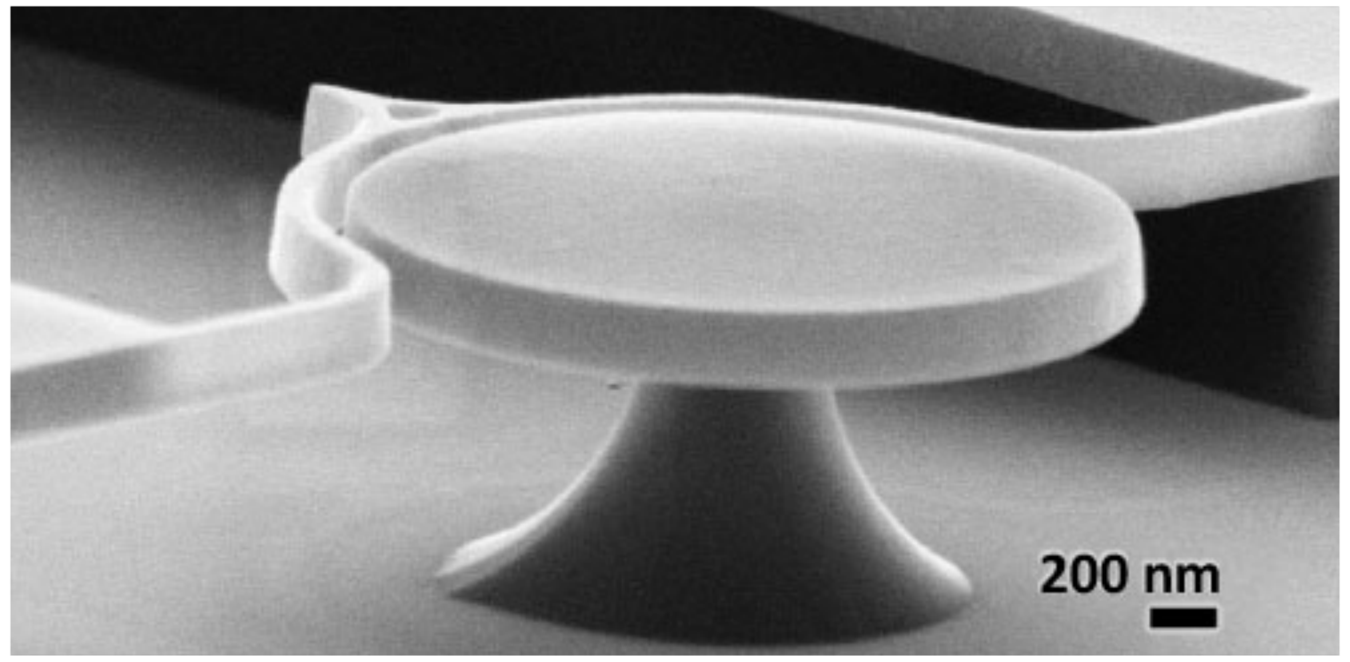
Integrated photonics

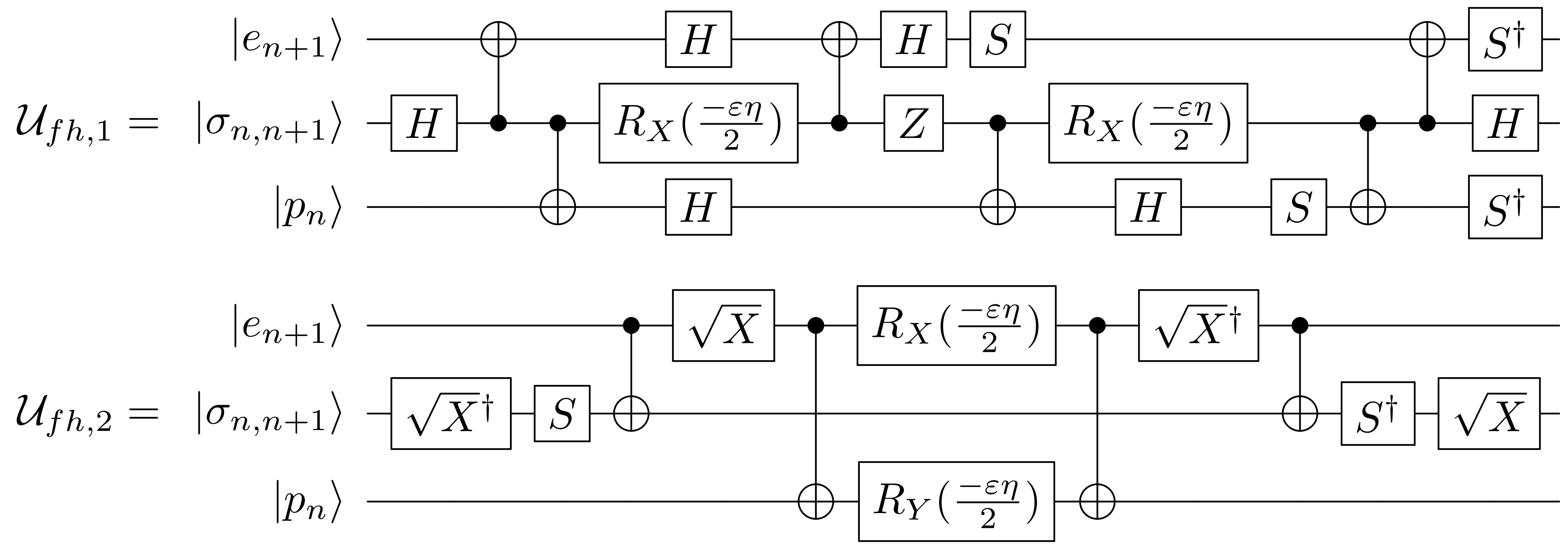


Atom interferometers



Optomechanical sensors





Quantum algorithms

Quantum Field Theory

- At the heart of QFT is a mode expansion.
  We get to pick the modes. Something like -

$$\phi(x_\mu) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega}}\left(a_{\vec{k}}u_{\vec{k}}(\vec{x})e^{i\omega t} + a^\dagger_{\vec{k}}u_{\vec{k}}{}^*(\vec{x})e^{-i\omega t}\right)$$

Quantize: a, $a^\dagger$ are operators.

Satisfy: $[a_k, a_{k'}^\dagger] = \delta_{kk'}$

- This is sometimes referred to as "second quantization". For us its first!

Quantum Fields in Small Devices

- In this big Universe, fields sometimes get localized to a finite regions. Either "naturally" or in a lab.

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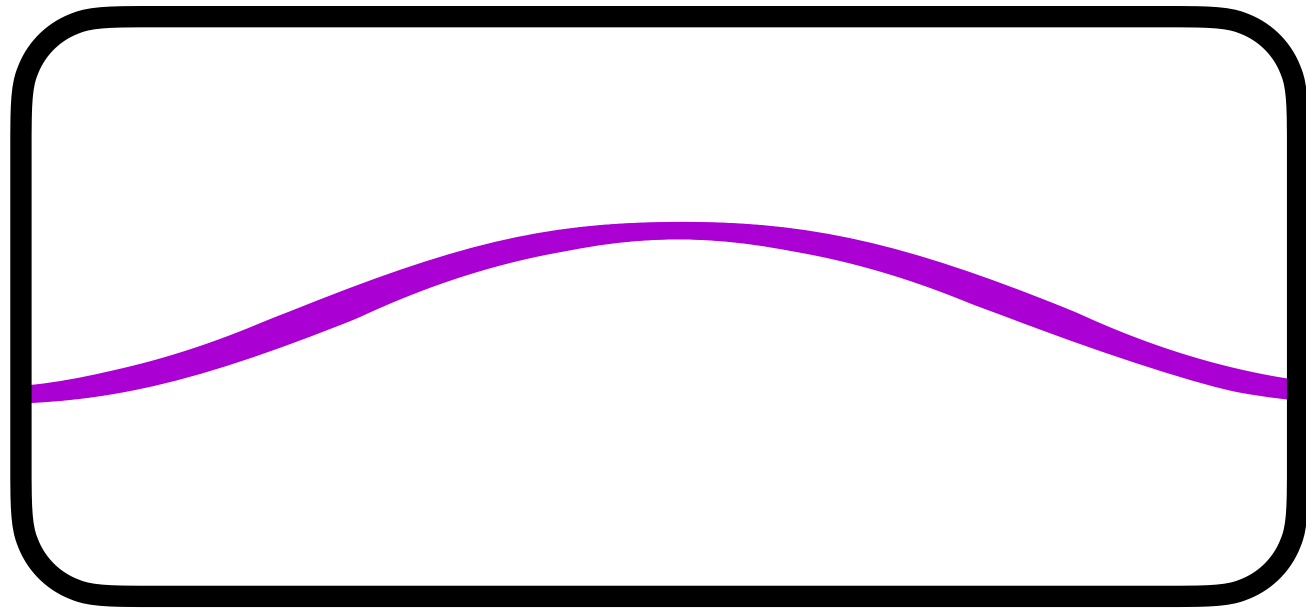

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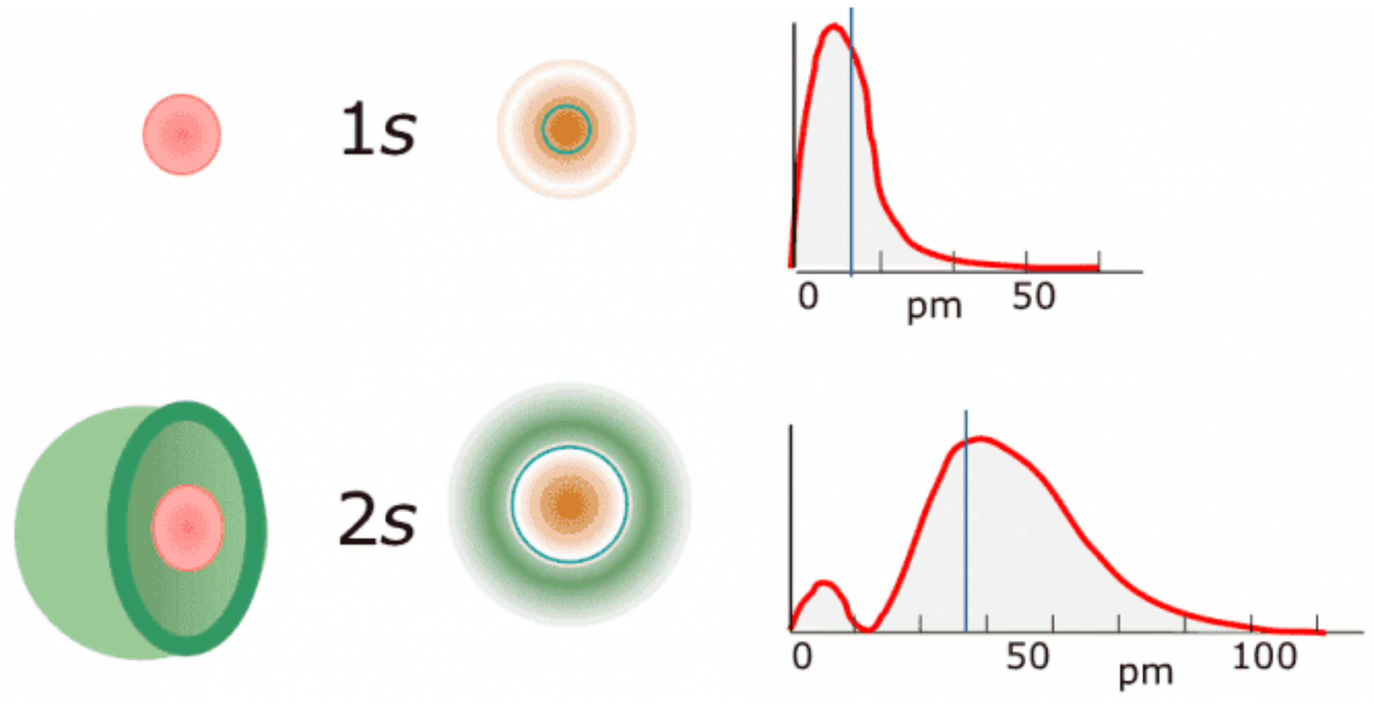

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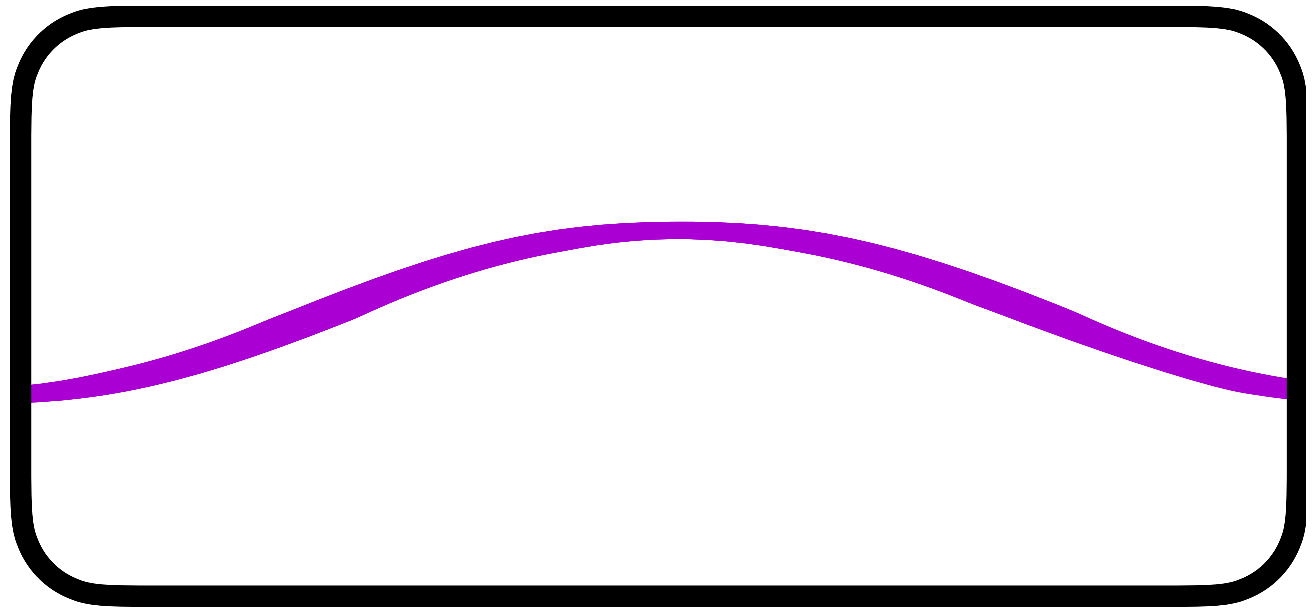
$$+ \sum_j \frac{1}{\sqrt{2\omega}} \left(a_j u_j(\vec{x}) e^{i\omega t} + a^\dagger_j u_j{}^*(\vec{x})(e^{-i\omega t} \right)$$

Only a discretum satisfies boundary conditions.



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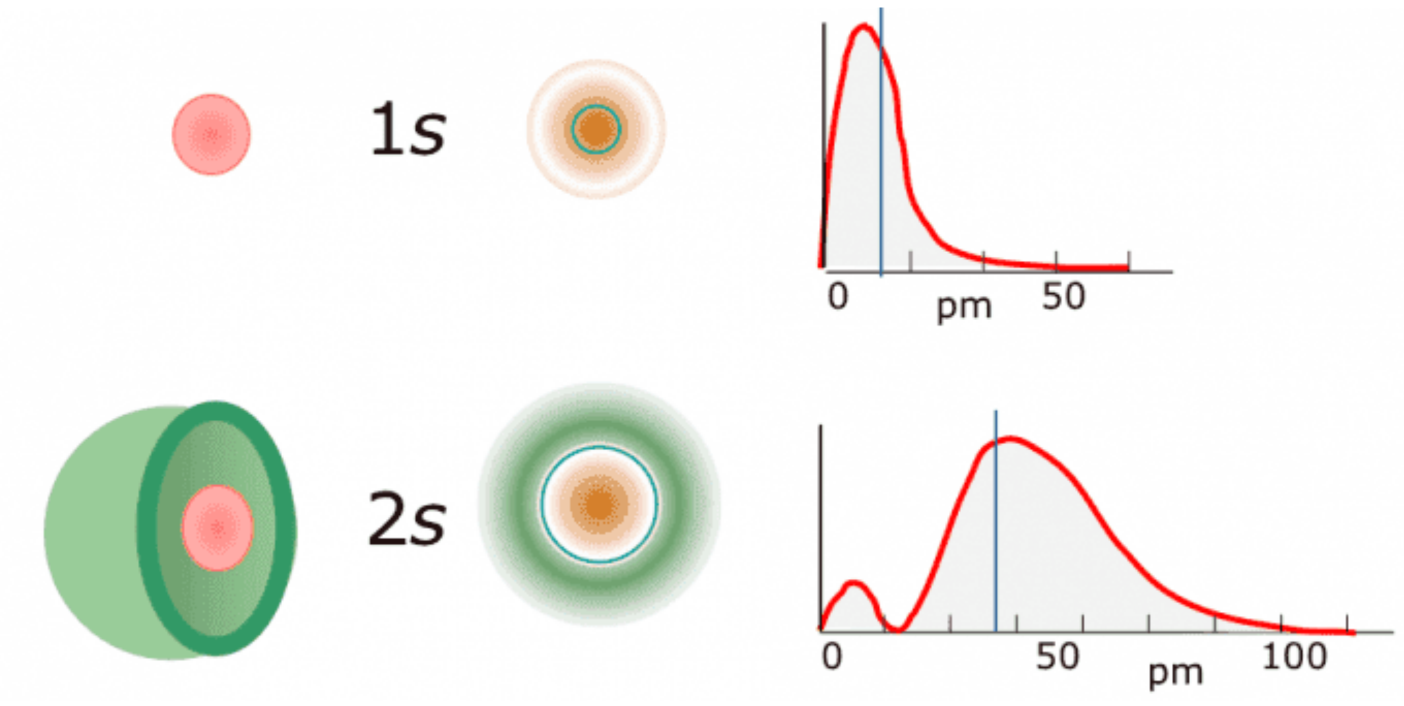
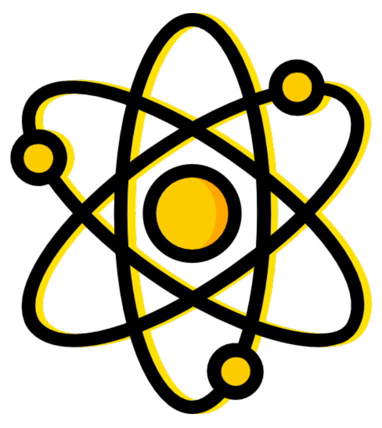


Device EFT

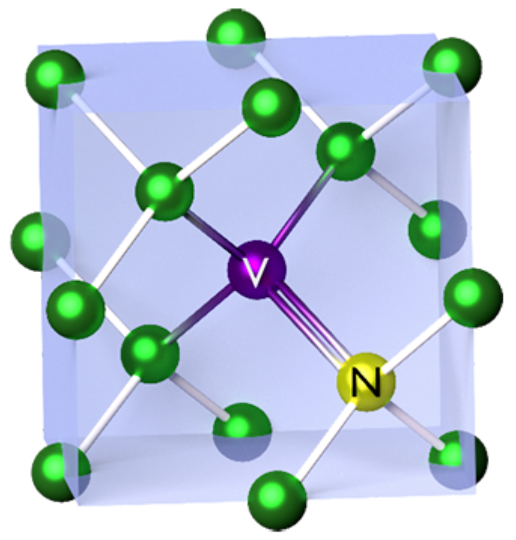
- Consider the low energy EFT of the discretum. Often in terms of a, $a^\dagger$

$$\phi_j(x_\mu) = \frac{1}{\sqrt{2\omega}}\left(a_j u_j(\vec{x})e^{i\omega t} + a_j^\dagger u_j{}^*(\vec{x})(e^{-i\omega t}\right)$$

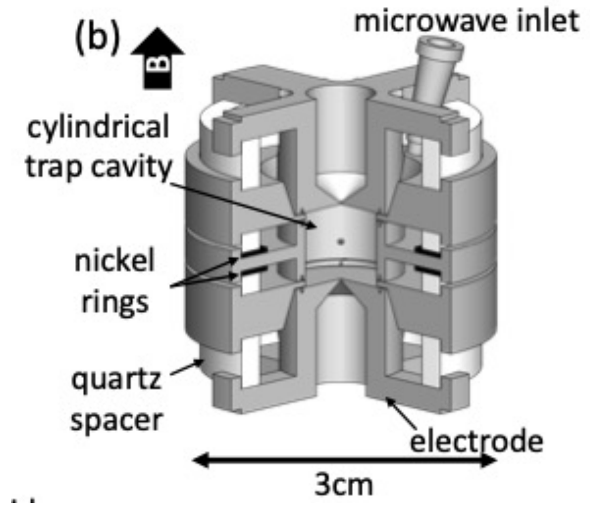
- In these EFTs, modes separate from the continuum, Quantum Mechanics shines:



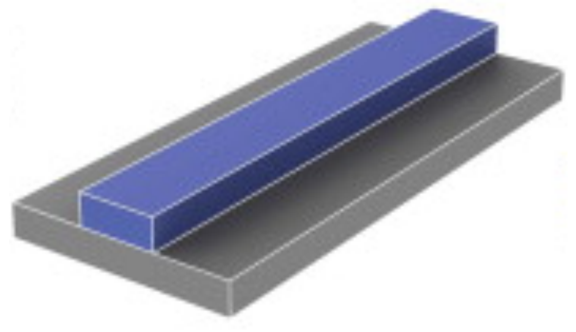
Atoms



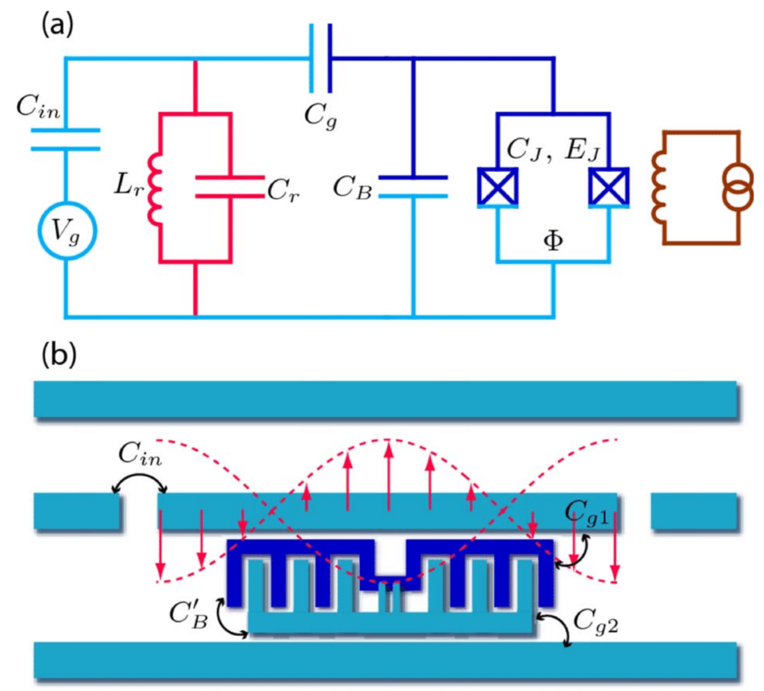
Defects



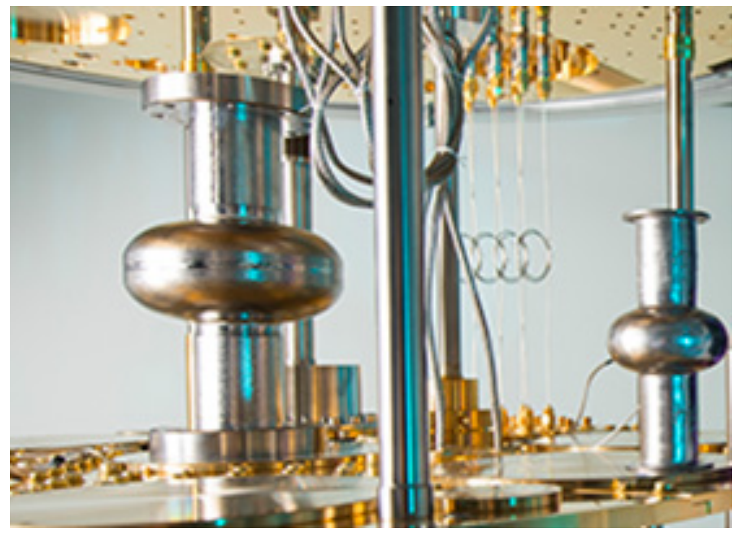
Artificial Atoms (particle in trap)



Optical waveguide



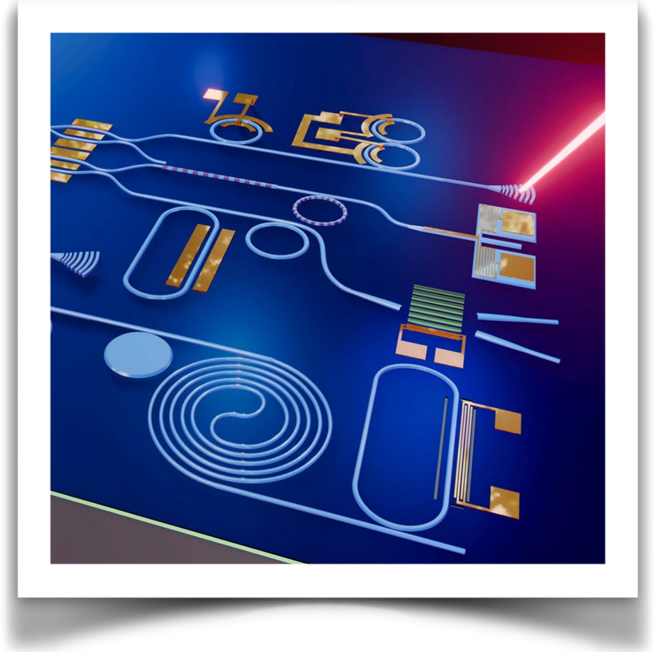
Superconducting circuits



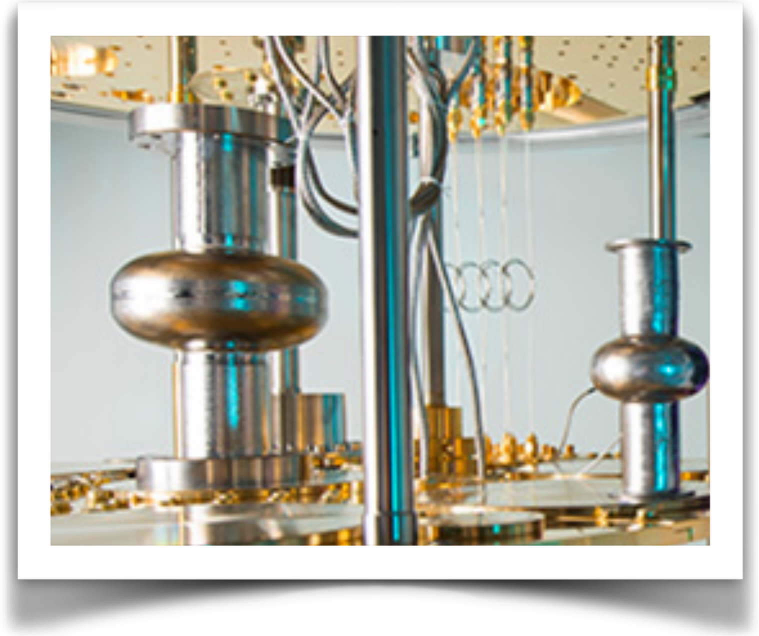
Electromagnetic Cavities

Examples

Optical Devices (e.g. "photonics")



Superconducting qubits and cavities



Estrada, RH, Senger, Rodriguez
***PRX Quantum* 2 (2021) 3, 030340**

RH
In preparation

Blinov, Gao, RH, Janish, Sinclair
2401.17260

[Chapter 6 of Jackson]

Optics

- The EFT of light traveling through a medium, made of atoms: $N_{atoms} \sim 10^{23}$!!

$\omega_{atoms} \sim \alpha^2 m_e$

$\delta x_{atoms} \sim a_{Bohr} \sim (\alpha\, m_e)^{-1}$

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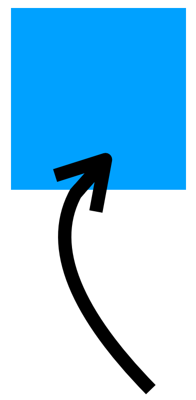
$\delta x_{atoms} \ll \lambda_{light} \rightarrow$ atoms react collectively!

At the renormalizable level: $\partial_t^2 E - \partial_x^2 E = 0 \longrightarrow \partial_t^2 E - \frac{1}{n^2}\partial_x^2 E = 0$

Optical Device

- Consider a small 1D optical device in large dimensions. Lets think of it as a fat brane.
- On the brane we can write a brane localized kinetic term

$$\partial_t^2 E - \partial_x^2 E$$



$$\partial_t^2 E - \frac{1}{n^2}\partial_x^2 E$$

* The field can be brane-localized.

* On the brane the field obeys: $k = n\,\omega$

* Index of refraction n can depend on other UV effects

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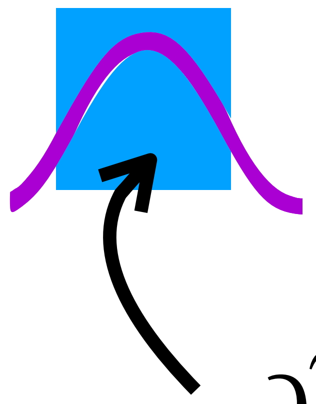

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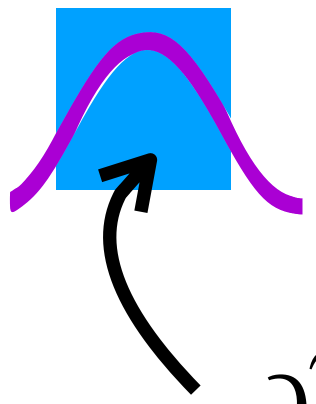

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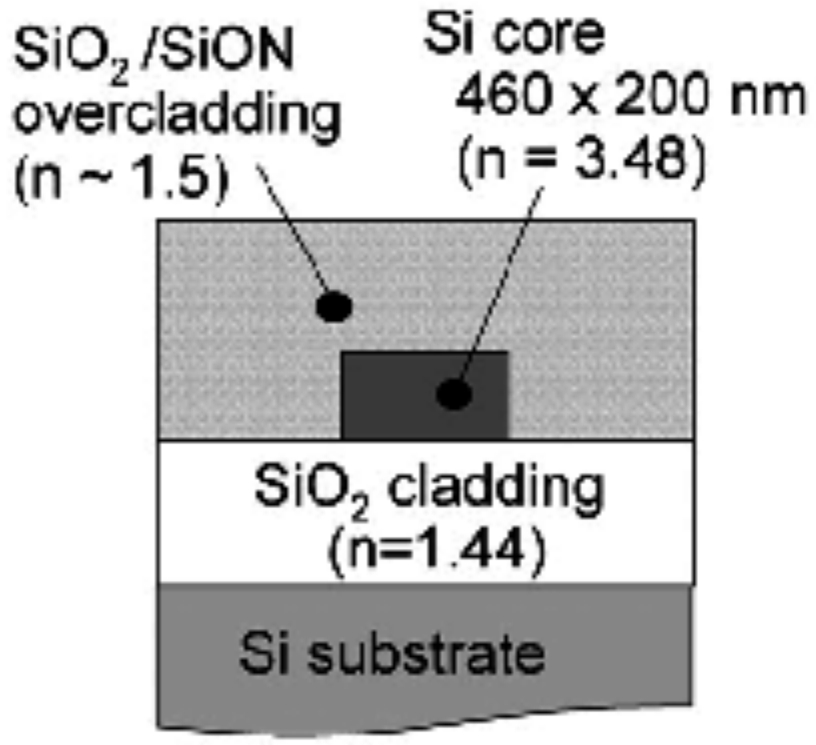
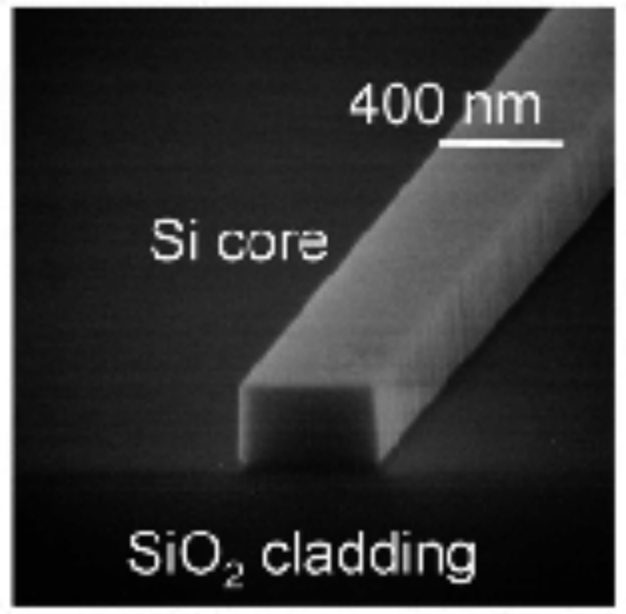
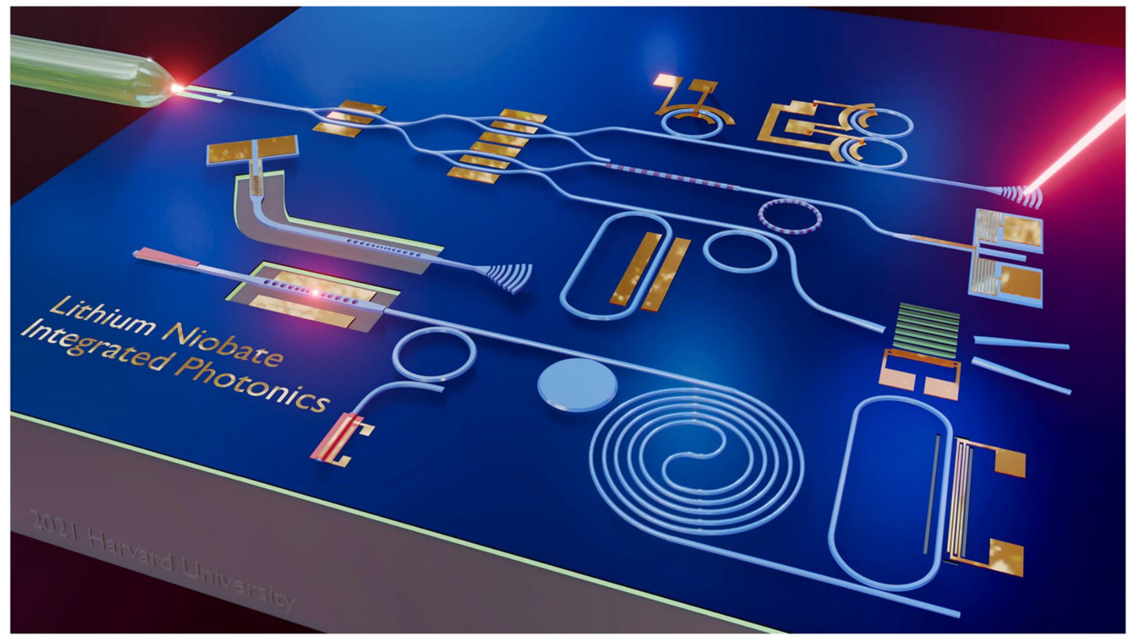


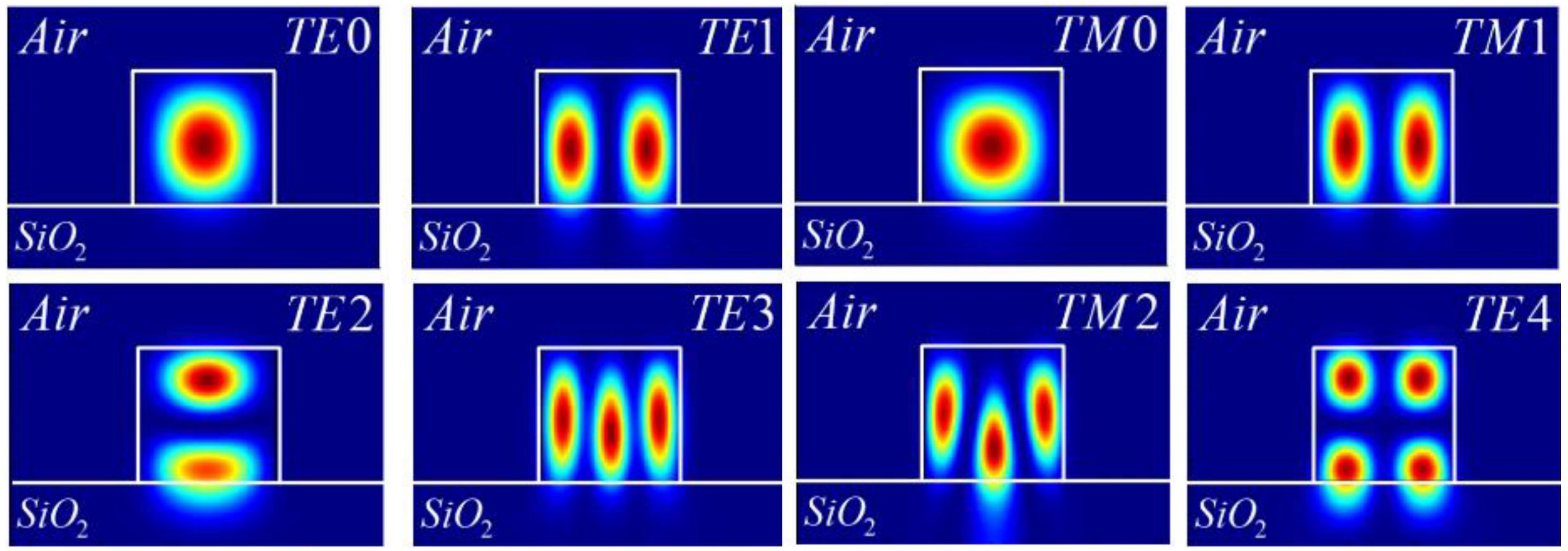




"Integrated photonics"

Degree of Freedom in Optical Devices

- The dispersion relation determines kinematic properties! Would be nice to control it.
  - Fields on the fat brane live in a compact dimension. The field can be expanded in KK modes
  - If the material is anisotropic, n can depend on polarization.



"Flavor!!"

But we have no interactions between flavors…

Linear Optics: $H = E^2 + B^2 = \Sigma\, \hbar\omega(a^\dagger a + 1/2)$

Nonlinear Devices

- Like any EFT, in a quantum device there is a UV cutoff.
- We can add higher dim operators. For example, in optics

Dim-6: $$H_{\mathrm{SPDC}} = \int_{\mathrm{crystal}} d^3\vec{x} \left(\chi^{(2)}_{jkl} E_j E_k E_l \right)$$

Dim-8: $$H_{\text{4-wave}} = \int_{\mathrm{crystal}} d^3\vec{x} \left(\chi^{(3)}_{jkl} E_j E_k E_l E_m \right)$$

Nonlinear Devices

- Like any EFT, in a quantum device there is a UV cutoff.
- We can add higher dim operators. For example, in optics

Dim-6: $$H_{\text{SPDC}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(2)}_{jkl} E_j E_k E_l\right)$$

Dim-8: $$H_{\text{4-wave}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(3)}_{jkl} E_j E_k E_l E_m\right)$$

We can estimate χ's in naive dimensional analysis:

When the field is set to that in an atom, we set (Dim-4 ~ Dim-6 ~ Dim-8):

$$E_{\text{atom}} \sim e/4\pi a_0^2$$

$$\chi^{(2)} \sim \frac{\sqrt{4\pi}}{\alpha^{5/2} m_e^2}$$
$$\chi^{(3)} \sim \frac{4\pi}{\alpha^5 m_e^4}$$

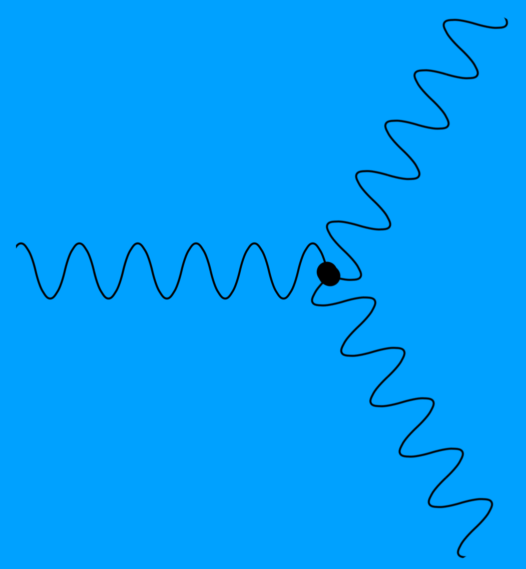
(by comparison, in vacuum $\chi^{(2)} = 0$, $\chi^{(3)} = \frac{2\alpha^2}{45 m_e^4}$)

SPDC

- Now we can have "flavor" changing decays: Photon -> two other photons

Pump ⟶ Signal + Idler

- Kinematics:

$H \supset$ 

and

$k_p = n_p\,\omega_p$

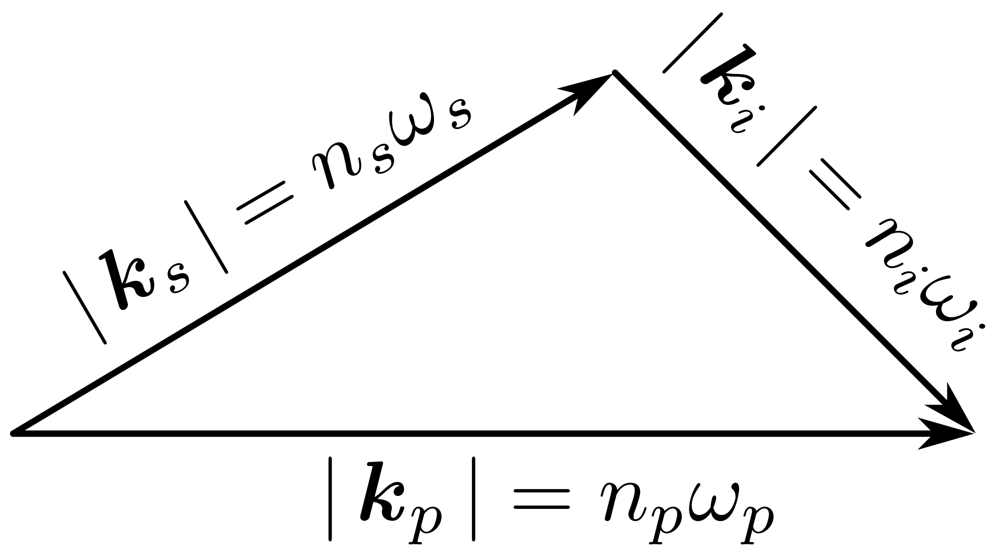
$k_s = n_s\,\omega_s$

$k_i = n_i\,\omega_i$

Indices can be different for p, s, i

"Phase matching":

$$\begin{cases} \omega_p = \omega_s + \omega_i \\ \boldsymbol{k}_p = \boldsymbol{k}_s + \boldsymbol{k}_i \end{cases}.$$



When n's are non trivial, phase matching happens in interesting ways

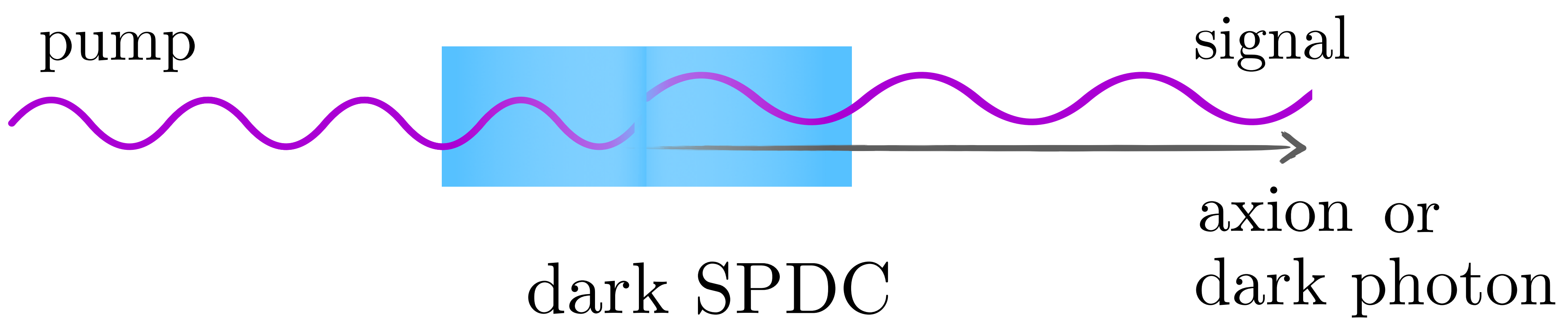
…. can we use this to search for BSM?

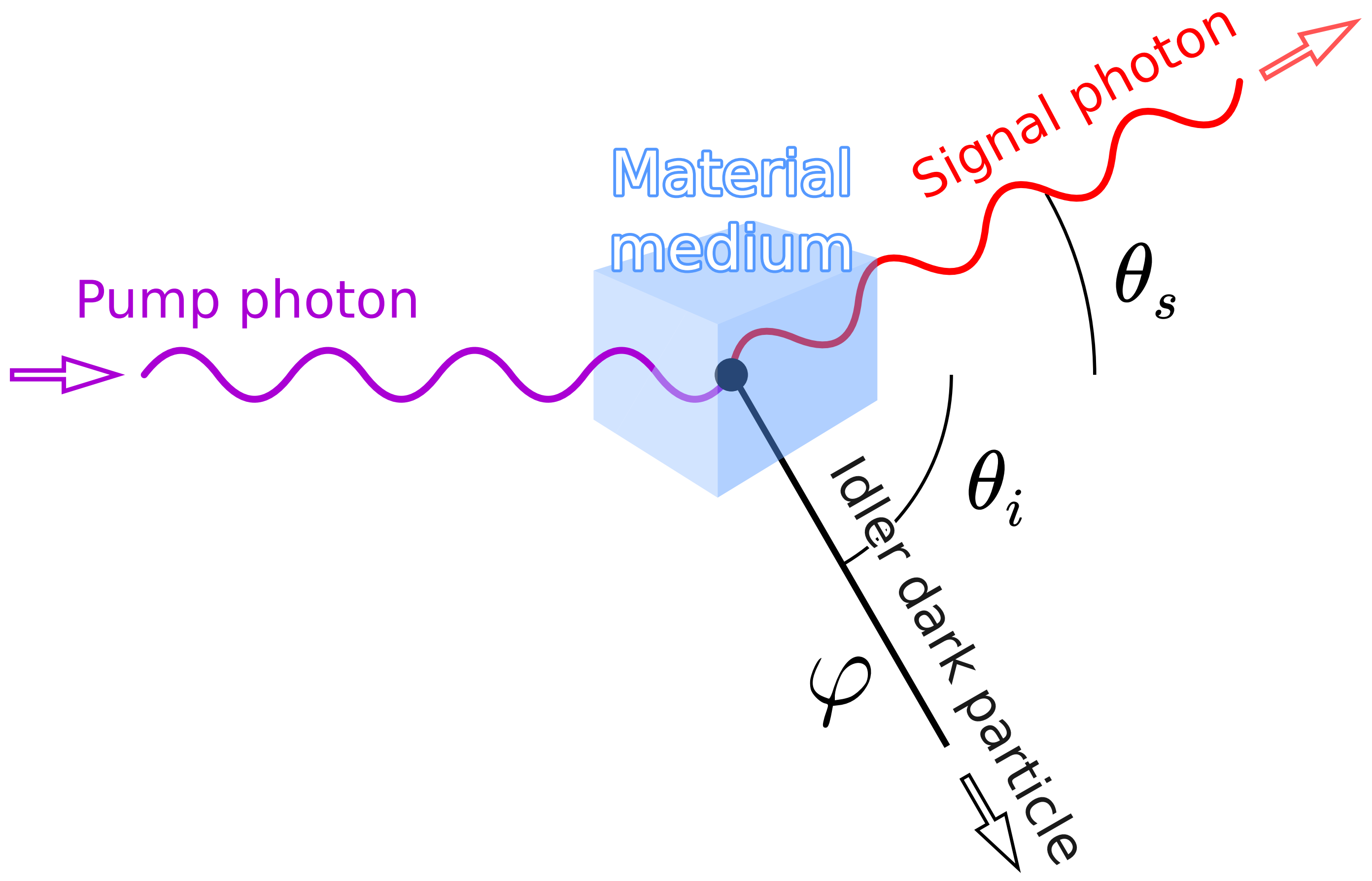
Dark SPDC

In SPDC we infer (herald) the presence of the idler.

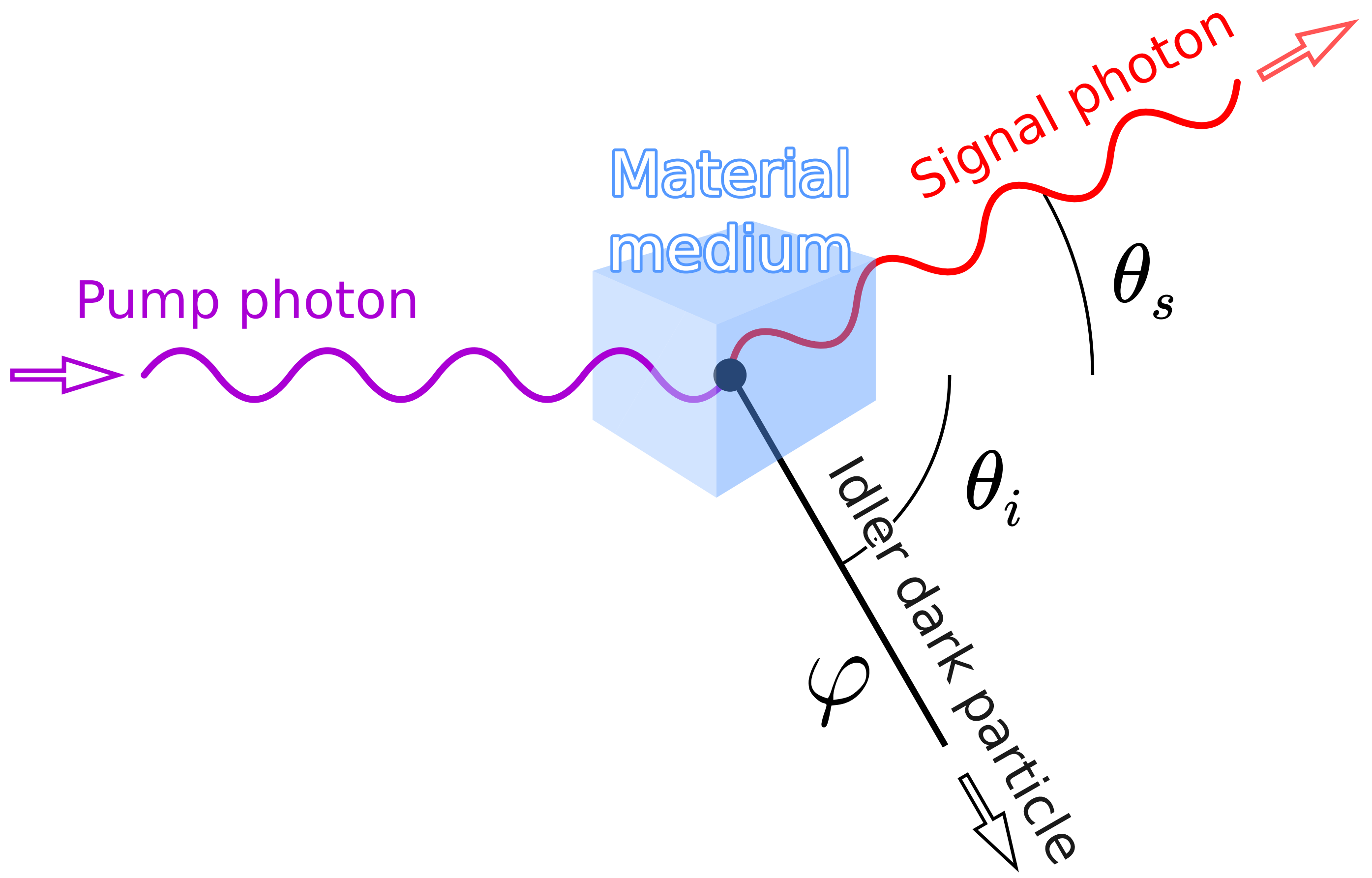
Particle physicists look for missing energy at colliders.

Can we search for mono-photon on the optics table?

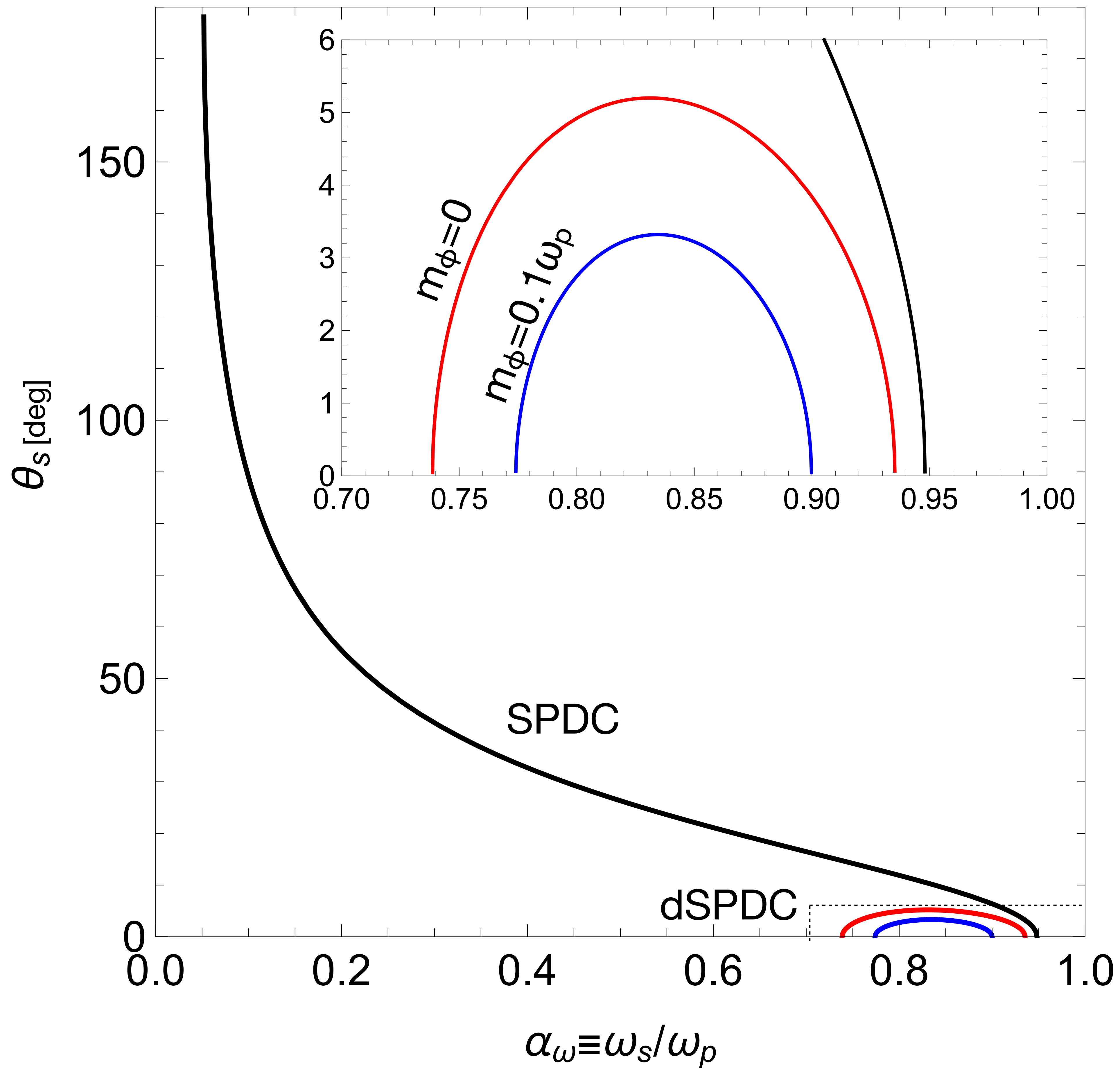


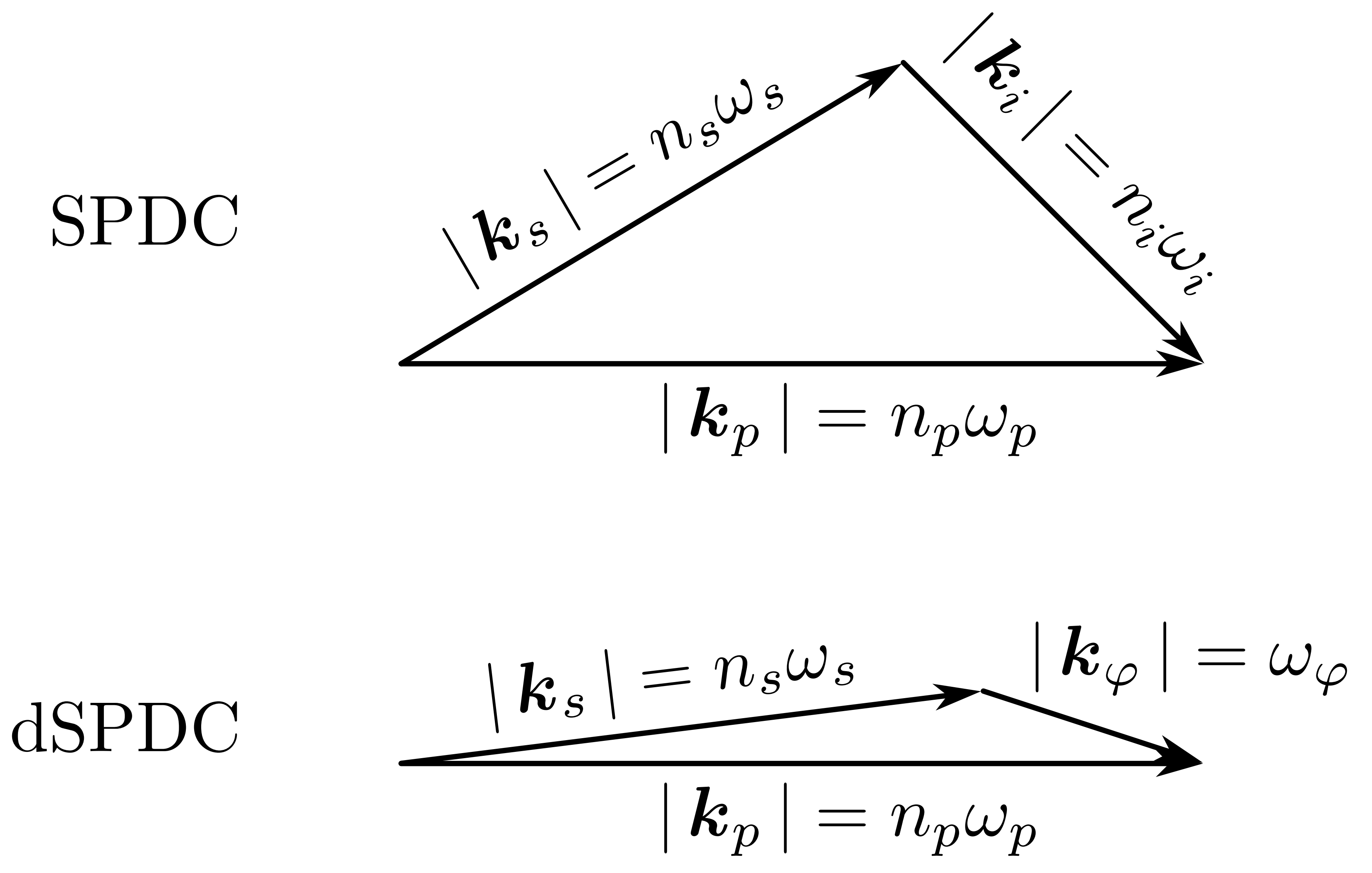


Phase Matching in dSPDC



- Two observables: signal frequency and angle ω_s and θ_s

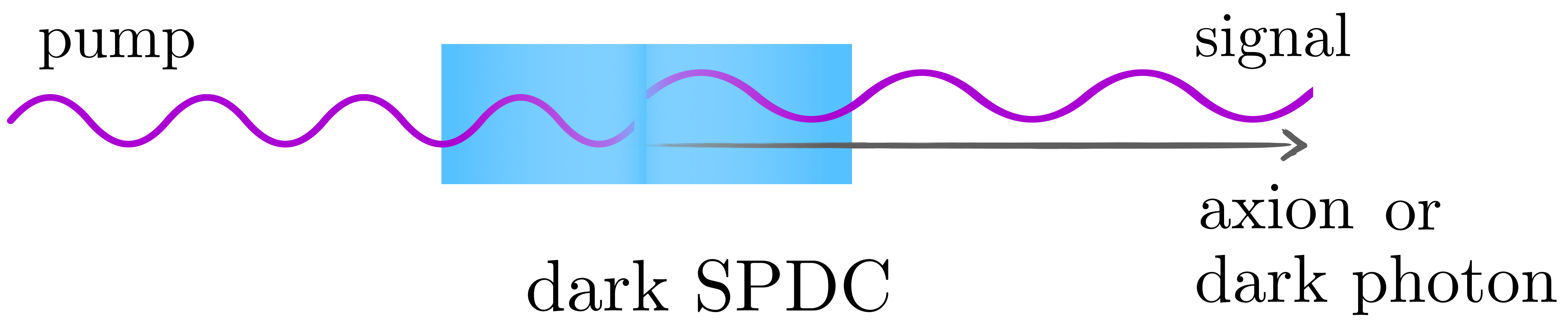


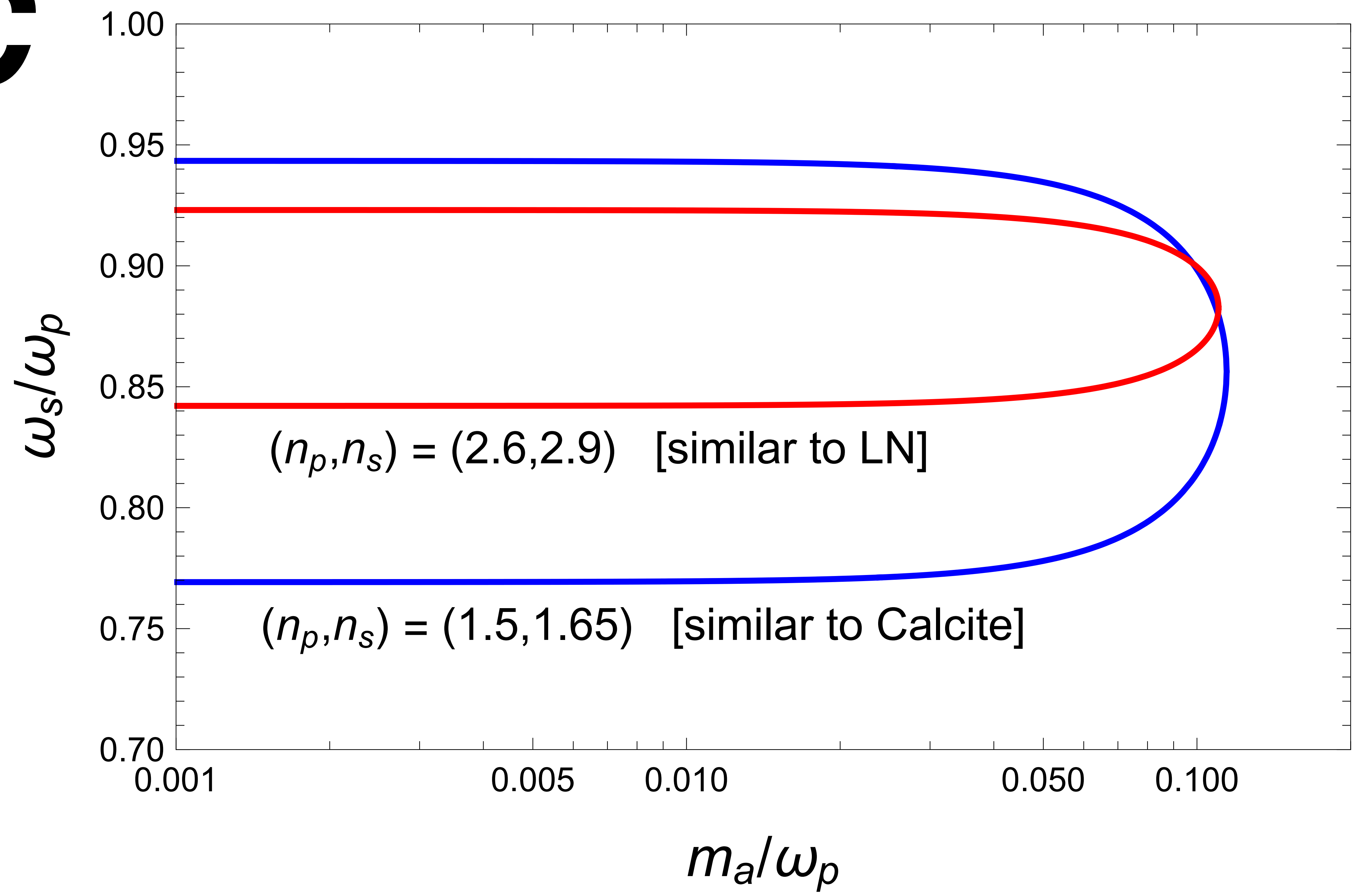


Kinematically, SPDC and dSPDC are O(1) different!

Phase Matching in dSPDC

- In the forward direction, $\theta_s=0$





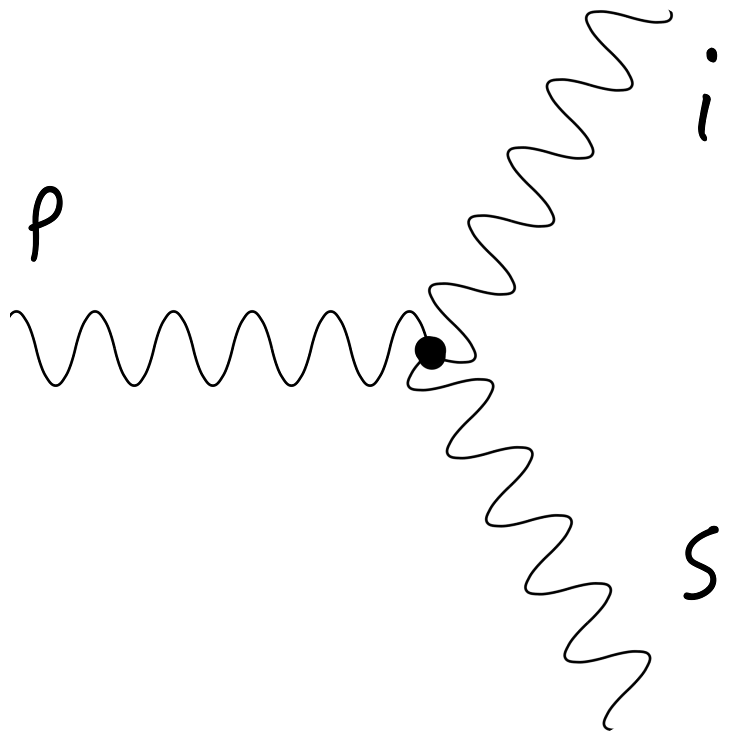
$$\omega_s = \omega_p - \frac{(n_s - n_p)\, n_s \omega_p \pm \sqrt{(n_s-n_p)^2 \omega_p^2 - (n_s^2-1)\, m_\varphi^2}}{n_s^2 - 1} \xrightarrow{m_\varphi \ll \omega_p} \frac{\omega_s}{\omega_p} = \frac{n_p \mp 1}{n_s \mp 1}$$

Can phase match for any $n_s > n_p$.

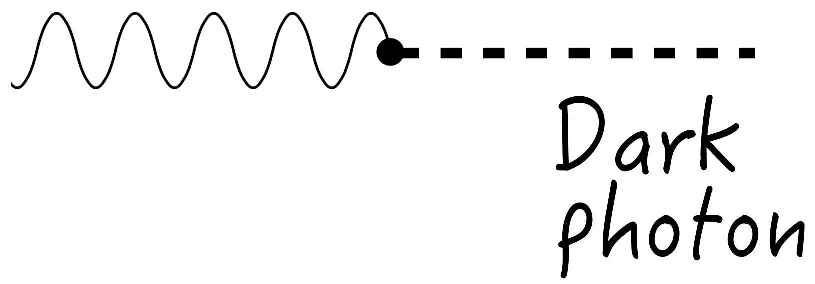
(d)SPDC Interaction and Rates

(d)SPSC Interactions:

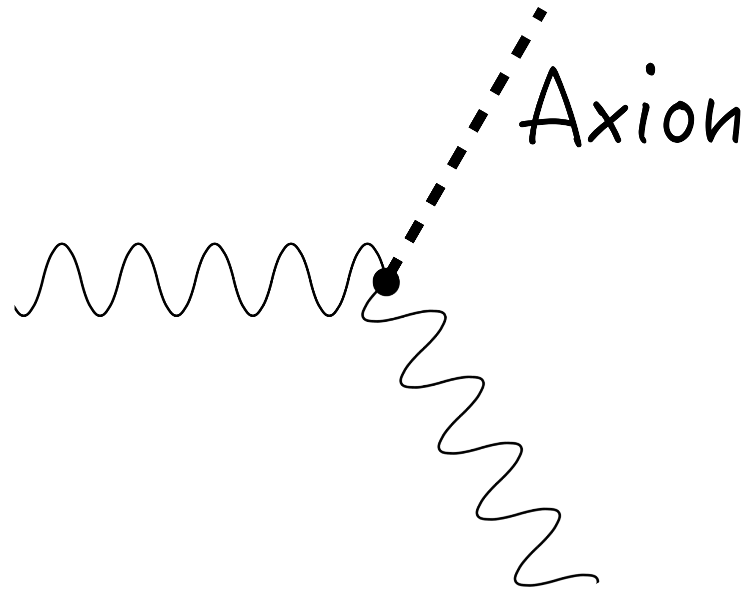
- SPDC and dSPDC can be described using an effective interaction:



  - SPDC (& nonlinear optics): $\mathcal{H}_{SPDC} \supset \chi\, E_p\, E_s\, E_i \; + \cdots$



  - Dark photons (linear extension) $\mathcal{H} \supset \mathcal{H}_{optics} + \varepsilon\, \vec{E}\cdot\vec{E}' + \vec{B}\cdot\vec{B}'$



  - Axion-like particles (nonlinear) $\mathcal{H} \supset \frac{a}{f}\, \vec{E}\cdot\vec{B}$

Note: axions and dark photons have index or refraction 1! (Rather $k_\phi^2 = \omega_\phi^2 + m_\phi^2$)

[in preparation (RH)]

(d)SPSC Interactions:

- Expanding the fields as in quantum optics, SPDC & dSPDC can be treated similarly:

Define φ as an idler photon, axion, or dark photon -

$$H_{(\mathrm{d})\mathrm{SPDC}} = \sum_{k_s} \sum_{k_{\varphi i},\, j} \chi_{\mathrm{eff}} \sqrt{\frac{\omega_p \omega_s \omega_{\varphi i}}{8 n_p^2 n_s^2 n_{\varphi i}^2}} \mathcal{I}_j e^{i \Delta \omega t} a_{k_p} a_{k_s}^{\dagger} b_{k_{\varphi i}}^{\dagger} + \mathrm{h.c.}$$

with $$\mathcal{I}_j \equiv \int dz\, d^2 r\, U^{(p)} U^{(s)\,*} U_j^{(\varphi i)\,*}(z)\, e^{i \Delta k z}$$

and

φi	χ_{eff}	$n_{\varphi i}$
SM idler photon i	$\chi^{(2)}_{jkl} \varepsilon^p_j \varepsilon^s_k \varepsilon^i_l$	n_i
axion a	$\frac{g_{a\gamma}}{\omega_a}(n_s - n_p) \sin \phi_{\mathrm{pol}}$	1
longitudinal dark photon A'_L	$\epsilon \frac{m_{A'_L}}{\omega_{A'_L}} \chi^{(2)}_{jkz} \varepsilon^p_j \varepsilon^s_k$	1

[in preparation (RH)]

(d)SPSC Interactions:

- Expanding the fields as in quantum optics, SPDC & dSPDC can be treated similarly:

Define φ as an idler photon, axion, or dark photon -

$$H_{\rm (d)SPDC} = \sum_{k_s} \sum_{k_{\varphi i},\, j} \chi_{\rm eff} \sqrt{\frac{\omega_p \omega_s \omega_{\varphi i}}{8 n_p^2 n_s^2 n_{\varphi i}^2}} \mathcal{I}_j e^{i\Delta\omega t} a_{k_p} a^\dagger_{k_s} b^\dagger_{k_{\varphi i}} + \text{h.c.}$$

with $$\mathcal{I}_j \equiv \int dz\, d^2r\, U^{(p)} U^{(s)*} U_j^{(\varphi i)*}(z)\, e^{i\Delta k z}$$

Phase matching!

and

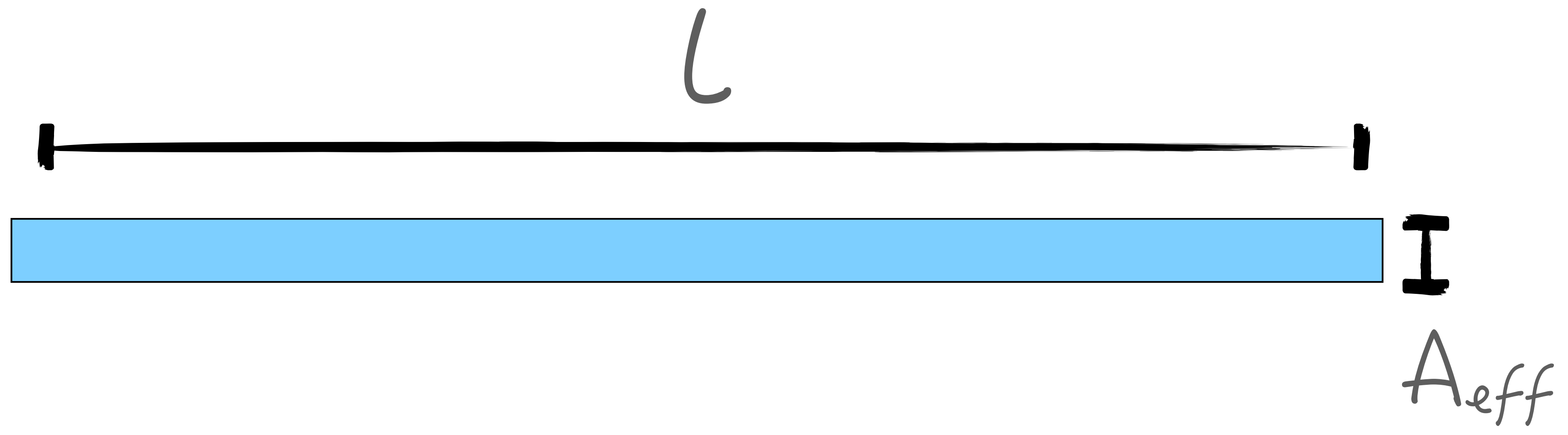
φi	$\chi_{\rm eff}$	$n_{\varphi i}$
SM idler photon i	$\chi^{(2)}_{jkl}\varepsilon^p_j\varepsilon^s_k\varepsilon^i_l$	n_i
axion a	$\frac{g_{a\gamma}}{\omega_a}(n_s - n_p)\sin\phi_{\rm pol}$	1
longitudinal dark photon A'_L	$\epsilon \frac{m_{A'_L}}{\omega_{A'_L}} \chi^{(2)}_{jkz}\varepsilon^p_j\varepsilon^s_k$	1

PRX Quantum 2 (2021) 3, 030340
+ RH (in prep)

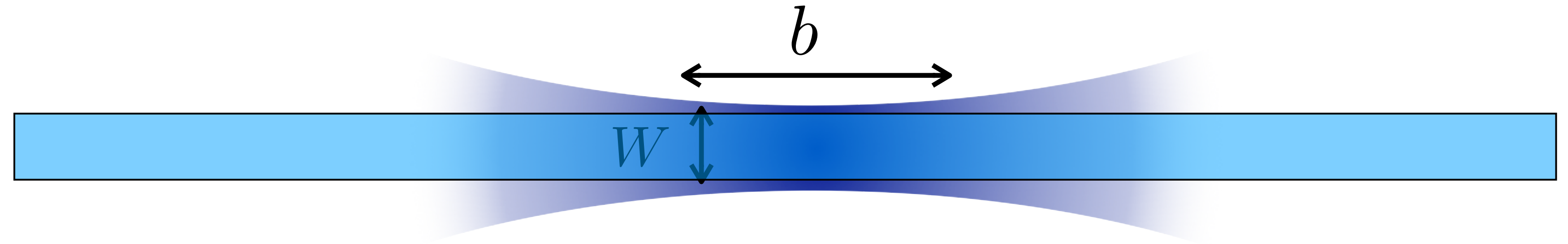
Rates:

$$\Gamma_{\text{SPDC}} \sim \frac{P_p {\chi_{\text{eff}}^{(2)}}^2 \omega_s \omega_i L}{\pi n_p n_s n_i A_{\text{eff}}}$$

For the SM process the mode overlap scales as L/A_{eff} due to interference along full length of the system.



But axions and dark photons are not confined!
Does rate grow with L?



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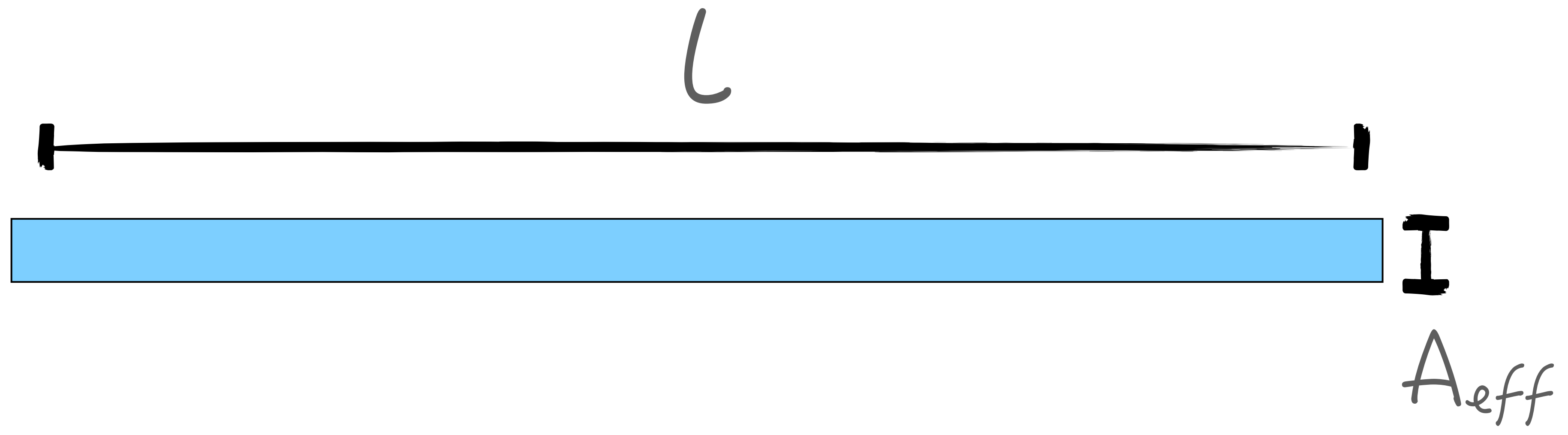

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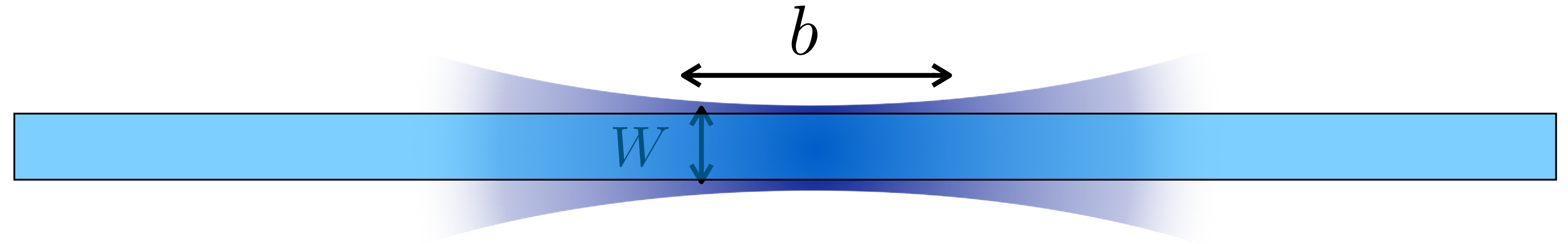
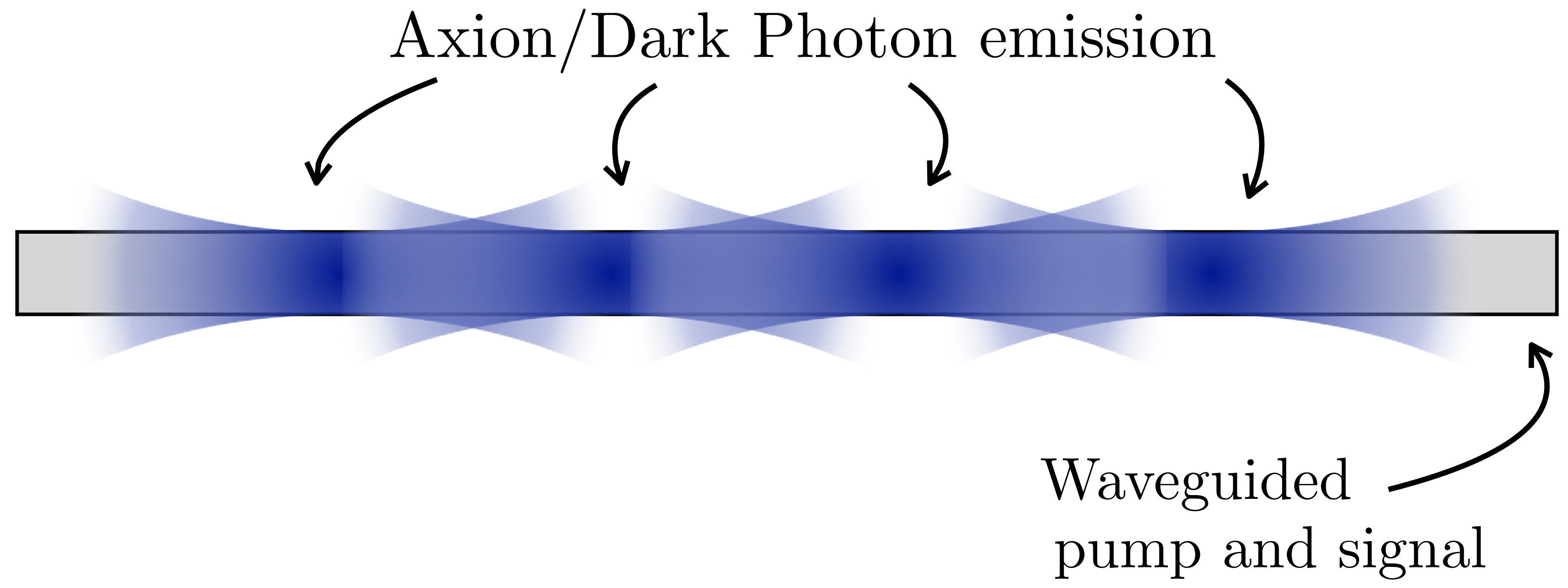


Yes. Due to incoherent sum.



PRX Quantum 2 (2021) 3, 030340

Rates:

$$\Gamma_{\text{SPDC}} \sim \frac{P_p {\chi^{(2)}_{\text{eff}}}^2 \omega_s \omega_i L}{\pi n_p n_s n_i A_{\text{eff}}}$$

Motivates long crystals.

$$\Gamma^{(A'_L)}_{\text{dSPDC}} \sim \epsilon^2 \frac{m^2_{A'}}{\omega^2_{A'}} \frac{P_p {\chi^{(2)}_{A'_L}}^2 \omega_s \omega_{A'} L}{n_p n_s A_{\text{eff}}}$$

$$N^{(A'_L)}_{\text{events}} \sim 10^{21} \left(\epsilon^2 \frac{m^2_{A'}}{\omega^2_{A'}}\right) \left(\frac{P_p}{\text{Watt}}\right) \left(\frac{L}{\text{m}}\right) \left(\frac{t_{\text{int}}}{\text{year}}\right)$$

$$\Gamma^{(\text{axion})}_{\text{dSPDC}} \sim \frac{P_p g^2_{a\gamma\gamma} \omega_s L}{\omega_{\text{axion}} n_p n_s A_{\text{eff}}}$$

$$N^{(\text{axion})}_{\text{events}} \sim 40 \left(\frac{g_{a\gamma}}{10^{-6}\ \text{GeV}^{-1}}\right)^2 \left(\frac{P_p}{\text{Watt}}\right) \left(\frac{L}{\text{m}}\right) \left(\frac{t_{\text{int}}}{\text{year}}\right)$$

	Dark Photon ($m_{A'} = 0.1$ eV)*	Axion-like particle ($m_a = 0.1$ eV)
Current lab limit	$\epsilon < 3 \times 10^{-7}$	$g_{a\gamma} < 10^{-6}$ GeV^{-1}
Example dSPDC setup	$P_p = 1$ W	$P_p = 1$ kW
	$L = 1$ cm	$L = 10$ m
	$\Gamma = 10$/day	$\Gamma = 10$/day
Current Solar limit	$\epsilon < 10^{-10}$	$g_{a\gamma} < 10^{-10}$ GeV^{-1}
Example dSPDC setup	$P_p = 1$ W	$P_p = 100$ kW
	$L = 10$ m	$L = 100$ m
	$\Gamma = 10$/year	$\Gamma = 10$/year

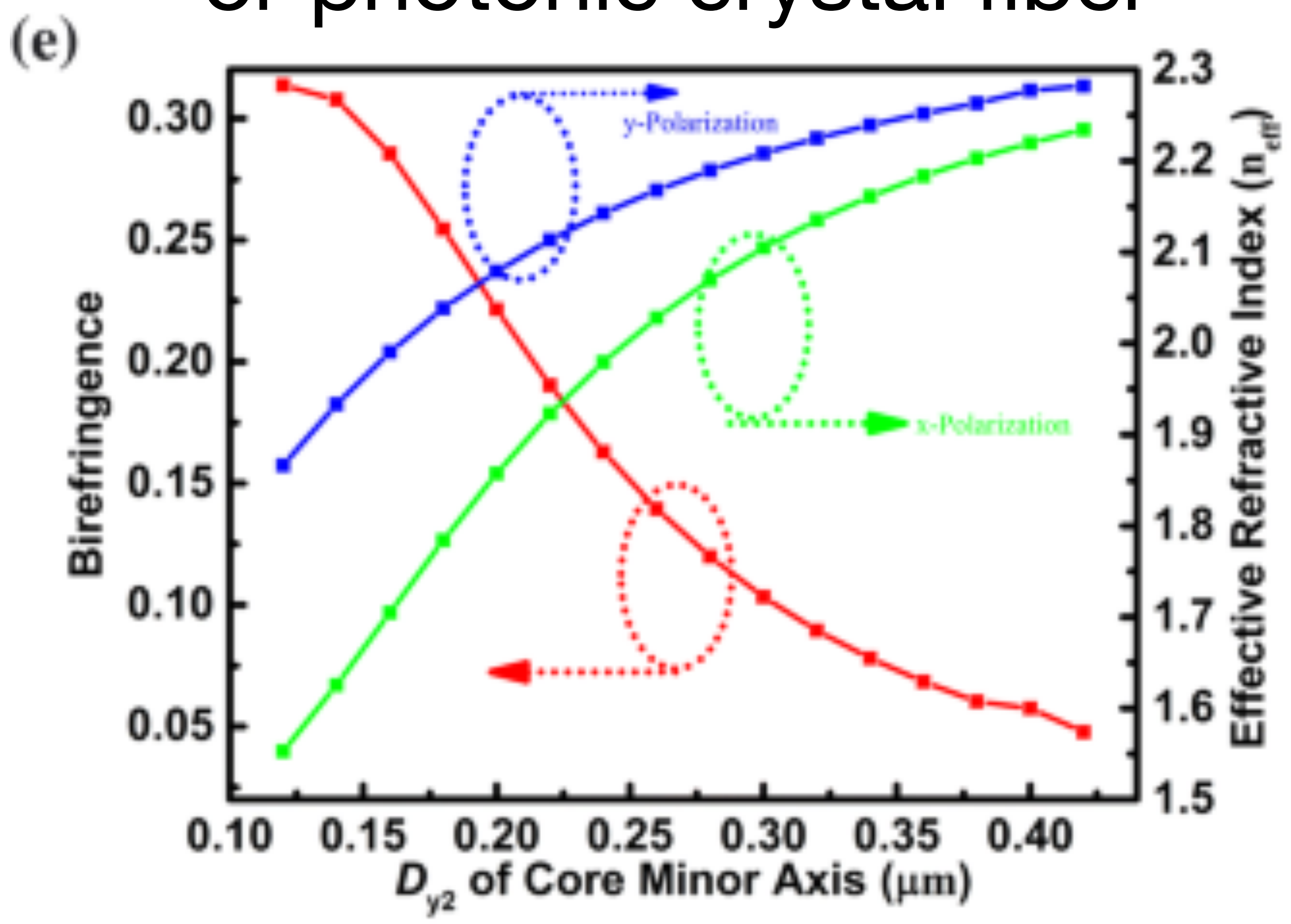
As always probing axion is a challenge.
(Similar in scale to ALPS2)

Dark photon is easier.

*Assumes $\chi^{(2)}$ ~ KTP

Unusual requirements

or photonic crystal fiber

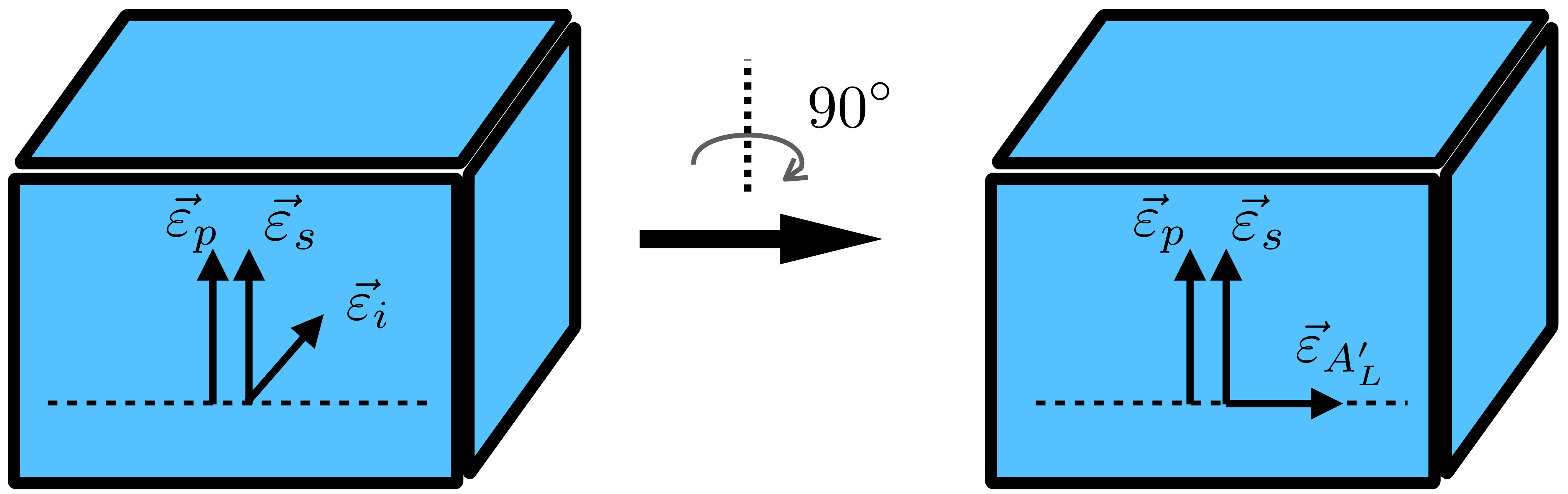


e.g: https://doi.org/10.1007/s00340-019-7273-1

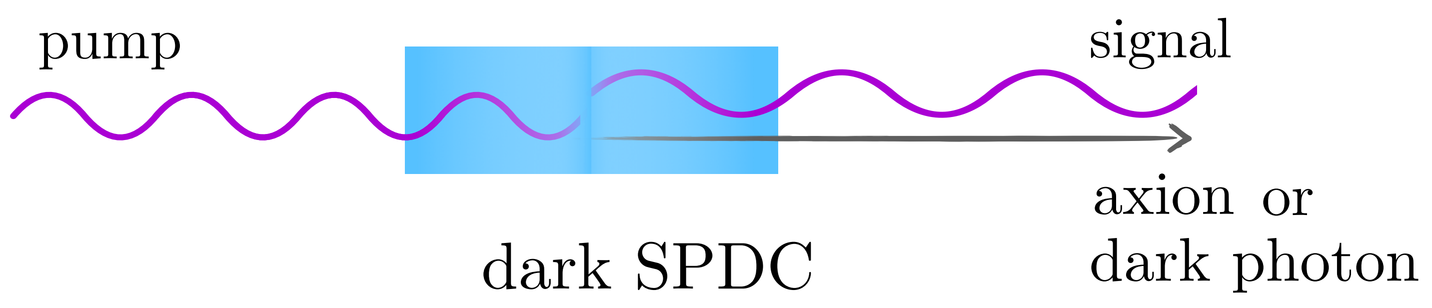
Thanks to Niel Sinclair for discussions.

- Axion:
  - A long medium. Enhance effective (length)^x(power) with resonators?
  - Highly bi-refringent.
  - As linear as possible.
  - Low dark count (e.g Skipper few pixels of Skipper CCD).

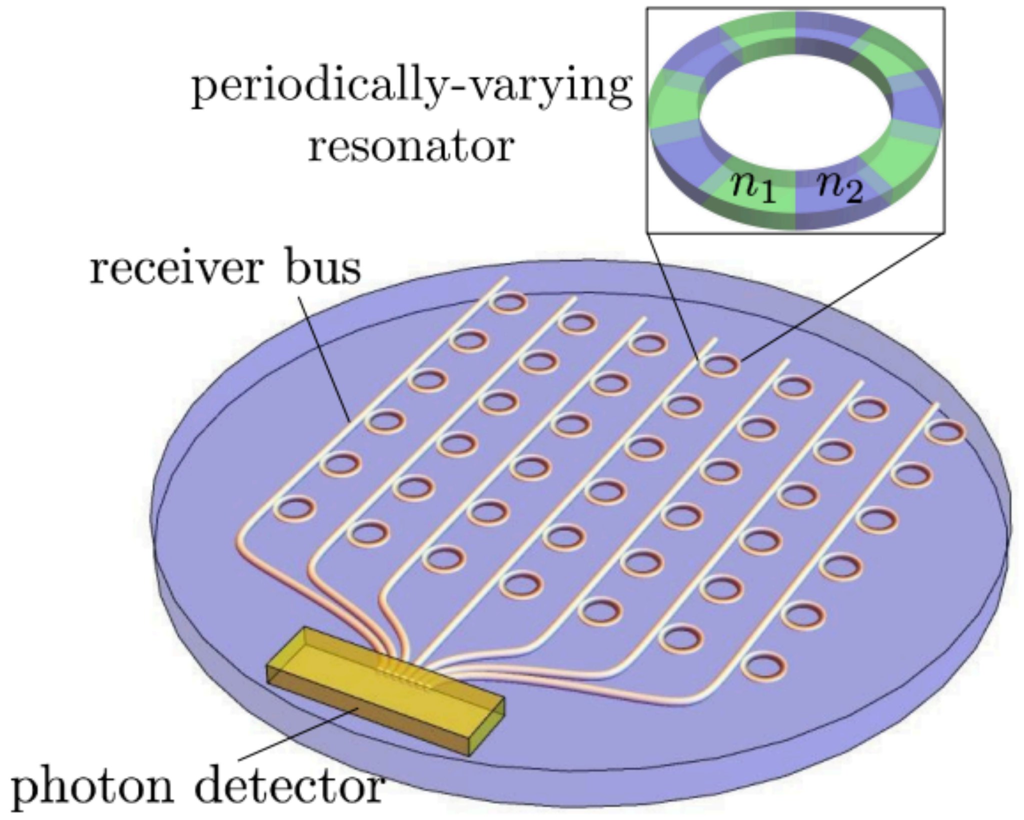
- Dark Photon:
  - A long medium. Ditto.
  - Nonlinear coupling to longitudinal mode! →
  - Bi-refriengence helps to phase match.



To Summarize



- Photonic systems, and other quantum devices are a new tool to search for new physics.
  - Dark SPDC searches for missing energy/momentum
  - Also Dark matter searches
- Also give us an opportunity to model build! (and build).
- Thank you Lawrence and Hitoshi for the impact!



DM searches on a photonics chip
Blinov, Gao, RH, Janish, Sinclair (2024)

Deleted Scenes

Optics

The hierarchy of scales, $\delta x_{atoms} \ll \lambda_{light}$, has several implications:

- Collective (coherent) back reaction:
  - Amplitude for forward scattering off an atom may be small, $O(\alpha)$.
  - Amplitude for forward scattering off of the medium can be $O(1)$.
- The effect of the medium can be described as mean field(s) in a derivative expansion:

  Polarization and magnetization densities $\vec{P}$ and $\vec{M}$.

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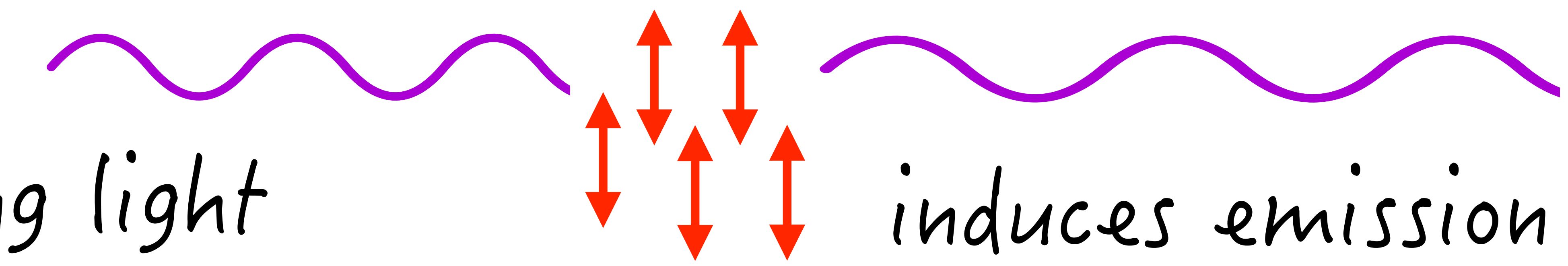
Polarization and magnetization densities $\vec{P}$ and $\vec{M}$.

dipoles in medium react



incoming light

induces emission

Interference in the forward direction
→ **index of refraction!**

$$\vec{P} = \chi \vec{E}$$

Mode Expansions:

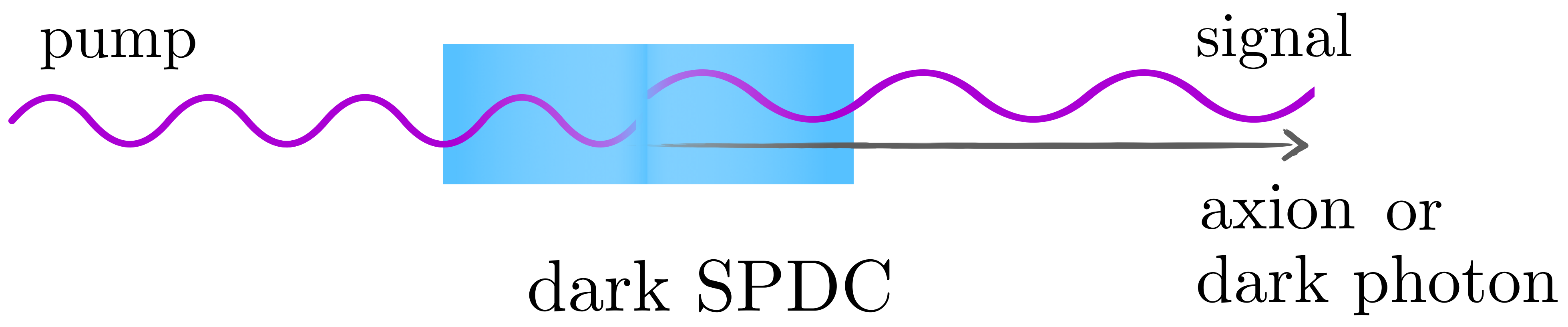
$$\phi_a(x_\mu) = \sum_j \sum_{\vec{k}_a} \frac{1}{\sqrt{2\omega_a}} \left(b_{j,\vec{k}} u^a_{j,\vec{k}}(\vec{x}) e^{i\omega_a t} + b^\dagger_{j,\vec{k}} {u^a_{j,\vec{k}}}^*(\vec{x})(e^{-i\omega_a t} \right)$$

$$\vec{E}(\vec{x},t) = \sum_{\vec{k},\sigma} \sqrt{\frac{\omega_k}{2n^2}} \vec{\varepsilon}_{\vec{k},\sigma} \left[a_{\vec{k},\sigma} u_{\vec{k}}(\vec{x}) e^{-i\omega t} - a^\dagger_{\vec{k},\sigma} u^*_{\vec{k}}(\vec{x}) e^{+i\omega t} \right]$$

$$\vec{B}(\vec{x},t) = \sum_{\vec{k},\sigma} \sqrt{\frac{1}{2n^2\omega_k}} \vec{k} \times \vec{\varepsilon}_{\vec{k},\sigma} \left[a_{\vec{k},\sigma} u_{\vec{k}}(\vec{x}) e^{-i\omega t} - a^\dagger_{\vec{k},\sigma} u^*_{\vec{k}}(\vec{x}) e^{+i\omega t} \right]$$

- In particle physics we use plane waves, $u_k(x) = e^{i(kx-\omega t)}$. Not in optics.

Colinear case:

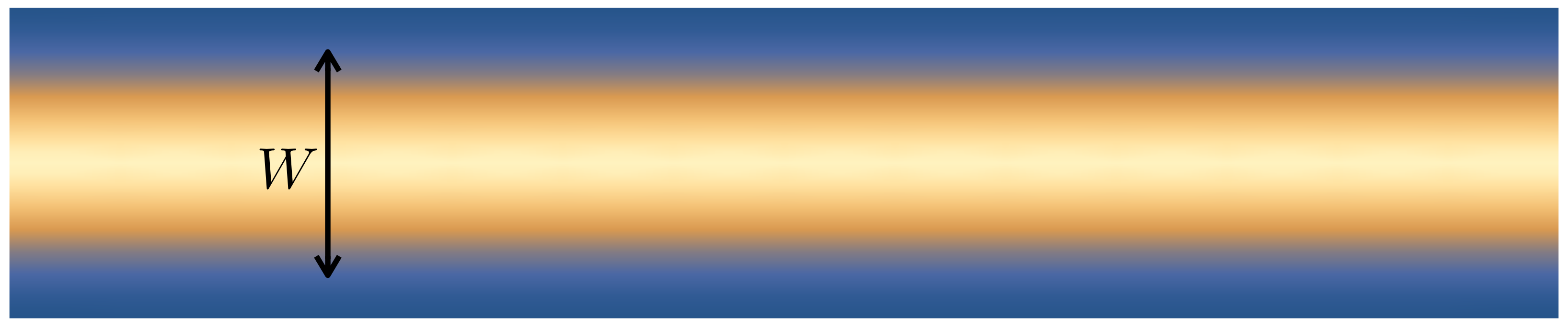


Useful to define: $$u_{j,k}(\vec{x}) = U_{j,k}(\vec{r}, z) e^{ikz}$$

Modes: Bulk vs Waveguide

- Two options in general (with gaussian examples):

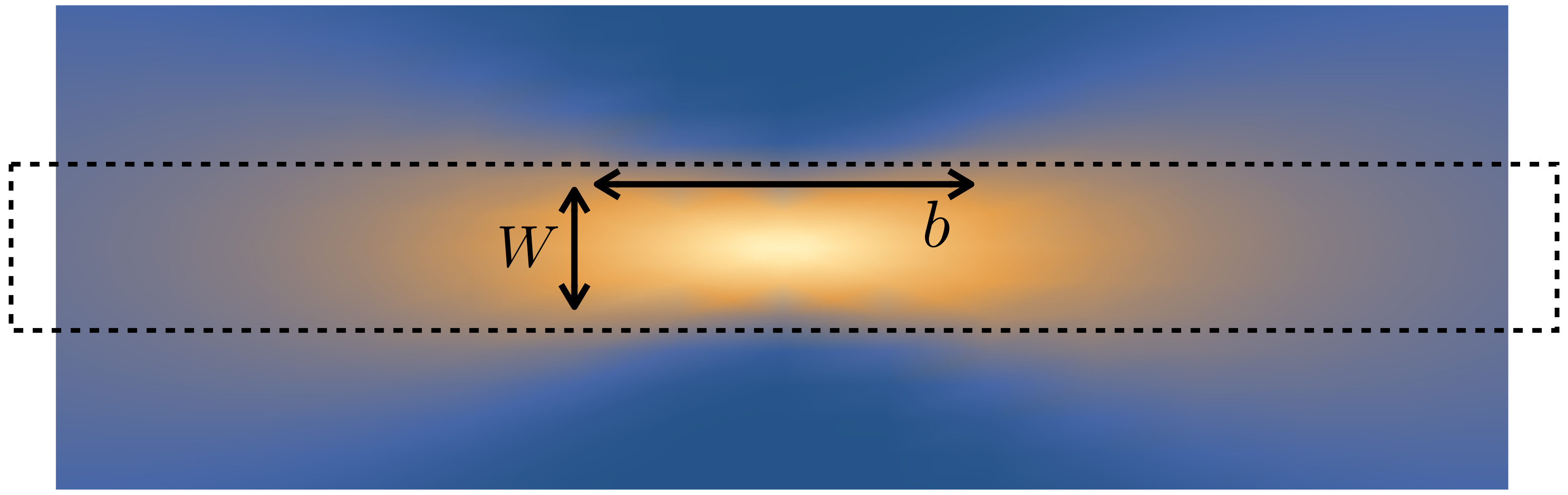
Waveguide mode



$$u_{\vec{k}}^{(\text{waveguide})} = U(\vec{r})e^{ikz}$$

$$U(\vec{r}) = \sqrt{\frac{2}{\pi W^2}}e^{-\frac{r^2}{W^2}}$$

Bulk mode (disperses)



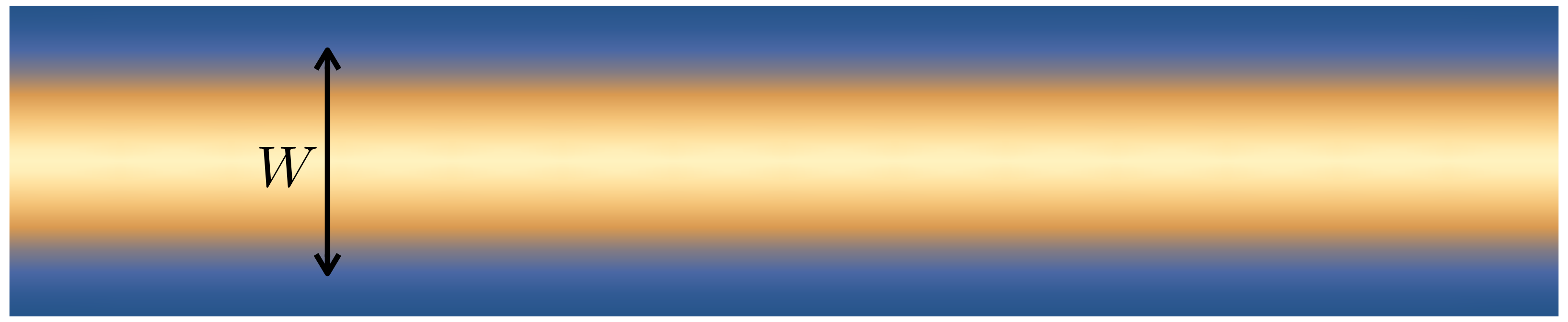
$$u_{\vec{k}}^{(\text{bulk})} = \sqrt{\frac{2}{\pi}}\frac{W}{q(z)}e^{-\frac{r^2}{q(z)}+ikz}$$

where $q(z) = W^2 + 2iz/k$

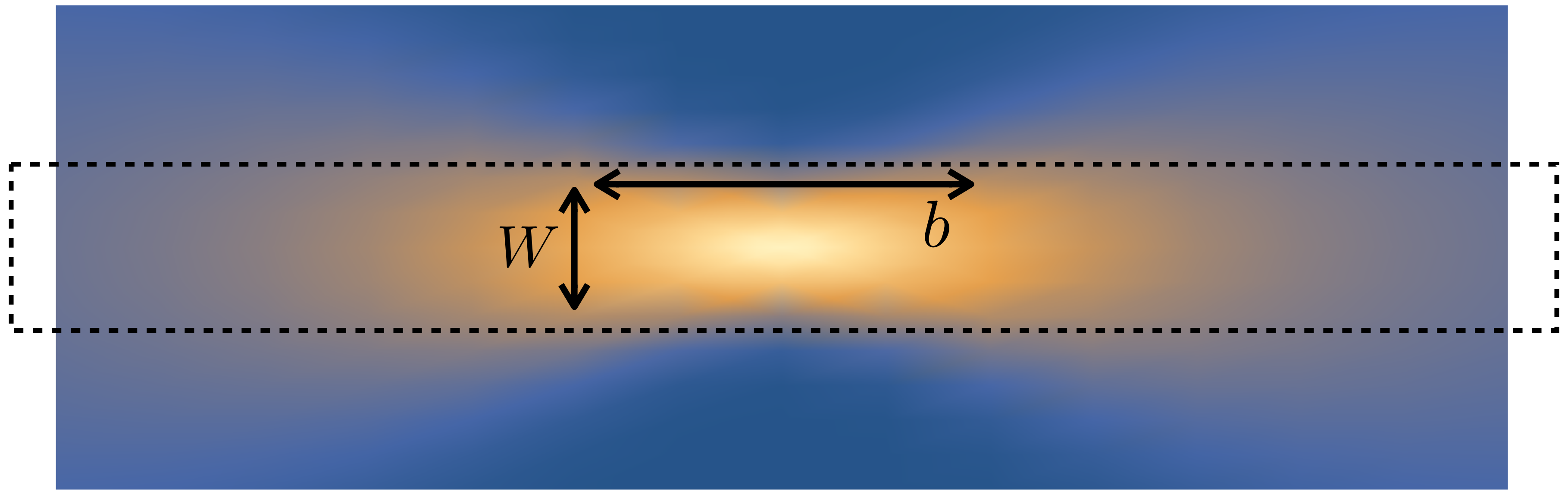
(confocal length $b = W^2k$)

Rate: Bulk vs Waveguide

- In SPDC there is a big difference in mode overlap (and hence rate) b/w waveguide and bulk:



Waveguide mode



bulk mode disperses

Mode overlap grows with L

(diff. rate with L^2)

Rate grows with L

Mode overlap and rate do not grow with L

- Though dSPDC can happen in a waveguide, the dark particle is not confined.

Does dSPDC behave like waveguide or bulk?

[in preparation (RH)]

Rate: Bulk vs Waveguide

- dSPDC can still occur along the whole crystal: **dSPDC Rate does grow with L!**
- Proof in two ways:
  - An "almost orthonormal" anzatz. Good for intuition.
  - An exact calculation w/ full Laguerre-Gauss basis confirms growth of the rate with L. [Following Bennink 2010]