Atomic Effects Relevant in the Scattering of High Energy Leptons (e.g. neutrinos or muons) off atomic electrons

Pleasure to be here in Berkeley to help celebrate the achievements of Hitoshi and Lawrence.

Discuss how the cross section for scattering of high energy leptons off atomic electrons differs from scattering off free electrons. These differences are quite small but may be important for certain experiments (MUonE). Work described here done in collaboration with Ryan Plestid.

Mark Wise September 2024

Atomic Effects in neutrino (or muon) scattering off atomic electrons

For high energy lepton beams scattering off electrons in a target atom at rest it is an excellent approximation to treat the struck electron as free and at rest.

Corrections from atomic binding arising both from ``energy-momentum conservation, and the intial state many body wave function of the atom [Ryan Plestid and MBW, Phys. Rev. D 110 (2024) 5, 056032 e-Print:2403.12184 (2024)]. For simplicity discuss neutrino scattering

$$\nu(\mathbf{k}) + \mathbf{A}(\mathbf{0}) \to \nu(\mathbf{k}') + \mathbf{e}^-(\mathbf{p}') + \mathbf{B}^+(\mathbf{h}') \ ,$$

Momentum Conservation $\mathbf{k}' + \mathbf{p}' = \mathbf{k} - \mathbf{h}'$

Same as muon scattering off free electron, not at rest but with momentum $\mathbf{p} = -\mathbf{h}'$

$$0 = E_\nu(\mathbf{k}') + E_e(\mathbf{p}') + E_B(\mathbf{h}') - E_\nu(\mathbf{k}) - m_A \qquad E_B(\mathbf{h}') - m_A = -m_e + \epsilon \qquad \epsilon = \epsilon_A - \epsilon_B$$

Energy conservation becomes $0 = E_\nu(\mathbf{k}') + E_e(\mathbf{p}') - E_\nu(\mathbf{k}) - m_e + \epsilon = E_\nu(\mathbf{k}') + E_e(\mathbf{p}') - E_\nu(\mathbf{k}) - \dfrac{\mathbf{h}'^2}{2m_e} - m_e + \epsilon + \dfrac{\mathbf{h}'^2}{2m_e}$

Include effects of order $\mathbf{p}^2/m_e^2$ but neglect terms of order $\mathbf{p}^2/E_{\nu e}^2$

$$\sum_{\text{spins}} |M|_\nu^2 = 128 G_F^2 \times [c_L^2 (p \cdot k)(p' \cdot k) + c_R^2 (p \cdot k')(k \cdot p') - 4 c_L c_R m_e^2 (k \cdot k')]$$

$$\sum_{\text{spins}} |\mathsf{M}|_\mu^2 \simeq 32 e^4 \frac{(k \cdot p')(k' \cdot p) + (k \cdot p)(k' \cdot p') - m_\mu^2 (p \cdot p')}{[(k - k')^2]^2} \ .$$

Use shifted energy conservation and momentum conservation with initial electron three momentum. Expand matrix element in initial electron momentum and energy difference ϵ

Get all corrections of order $\mathbf{p}^2/m_e^2$ and ϵ/m_e

Unpolarized target so nothing linear in the initial atomic electron three momentum

Easy to identify these corrections if you use energy and momentum conservation so numerator and denominators of above manifestly are order the mass of the electron squared.

For averaging over directions of initial electron three momentum $cos(\theta_{ee'}) \simeq cos(\theta_{e\nu'}) \simeq cos(\theta_{e\nu})$

Get corrections of order $\mathbf{p}^2/m_e^2$ and $\epsilon/m_e \sim Z^{4/3}\alpha^2 \sim 6\times 10^{-4}$ for $Z=10$

$$\langle e^- B^+ | \bar{\psi}_e \Gamma_\alpha \psi_e | A\rangle = \int \frac{d^3p}{2E_e(\mathbf{p})} \bar{u}(\mathbf{p}')\Gamma_\alpha u(\mathbf{p})\langle B^+ | \hat{a}_{\mathbf{p}} | A\rangle$$

$$\langle B^+ | \mathbf{a}_{\mathbf{p}} | A\rangle \epsilon = \langle B^+ | [H, \mathbf{a}_{\mathbf{p}}] | A\rangle$$

Square sum over final states $|B^+\rangle$ Should use correct spinor $u(\mathbf{p})$

$$\hat{H} = \sum_{\mathbf{p}} \frac{\mathbf{p}^2}{2m_e} a^\dagger_{\mathbf{p}'} a_{\mathbf{p}} + \sum_{\mathbf{p},\mathbf{p}'} V_1(\mathbf{p},\mathbf{p}') a^\dagger_{\mathbf{p}'} a_{\mathbf{p}} + \sum_{\mathbf{p},\mathbf{p}'\mathbf{q},\mathbf{q}'} V_2(\mathbf{p},\mathbf{p}',\mathbf{q},\mathbf{q}') a^\dagger_{\mathbf{p}'} a^\dagger_{\mathbf{q}'} a_{\mathbf{p}'} a_{\mathbf{q}'}$$

Evaluate commutator and get matrix elements of $\langle T_1 \rangle_A$, $\langle V_1 \rangle_A$, $\langle V_2 \rangle_A$,

Use definition of binding energy and viral theorem to get: $\langle \hat{T} \rangle_A = \epsilon_A$, and $\langle V_2 \rangle_A = -2\epsilon_A - \langle V_1 \rangle_A$

For neutrino scattering case $$d\sigma = d\sigma_0 \left[1 + \frac{1}{Z_A m_e} \left(-\frac{7}{3}\epsilon_A - \langle V_1 \rangle_A \right) \right]$$

For Argon correction $\sim d\sigma_0 \times \left(-6 \times 10^{-5}\right)$ much smaller than naive expectation

J. B. Mann Tech. Rep. LA-3691 (Los Almos National Lab (1968)

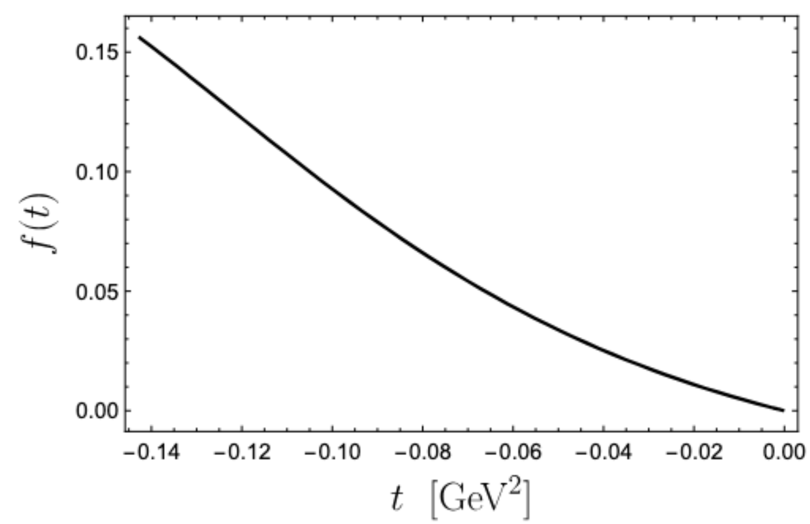
For muon scattering

$$f(t) = \frac{-2m_\mu t}{(s+t-m_\mu^2)^2 + (s-m_\mu^2)^2 + 2m_\mu^2 t} \qquad t \simeq -2p\cdot p' = -2m_e E'$$

MUonE experiment $\sqrt{s} = 405\text{MeV}$

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{1}{\sigma_0}\frac{d\sigma_0}{dt}\left(1 - f(t)c\right)$$

$$c = \frac{1}{Z_A m_e}\left[\frac{11}{3}\epsilon_A + \langle \hat{V}_1 \rangle_A\right] \sim 5\times 10^{-4}$$

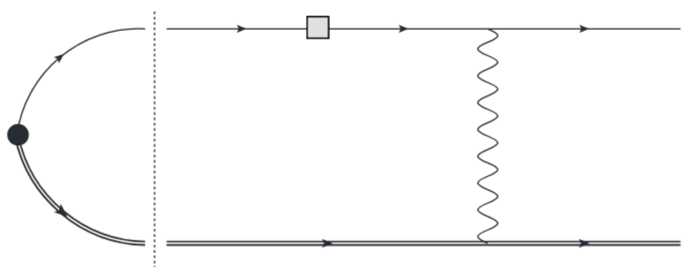


Corrections discussed so far are of order $\dfrac{\mathbf{p}^2}{m_e^2} \sim Z_A^{4/3}\alpha^2$

Further corrections arise from perturbative photon exchange between ``hard'' leptons, and ``soft'' (i.e., non-relativistic) electrons and nuclei. These effects are also inherently absent from calculations for a free electron at rest. Want to work to same order.
For simplicity start with neutrino case.

Discussed in a further paper Ryan Plestid and MBW e-Print: 2405.08110 [hep-ph] those were studied

For example below leaving out the neutrino lines attaching to the square



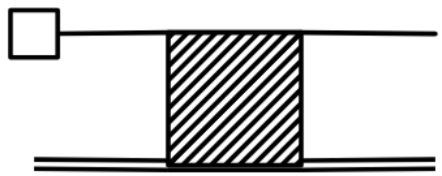
Find dominated by Coulomb exchange with momentum transfer of order the typical atomic scale αm_e. So lets focus on Coulomb exchanges and of course work in Coulomb gauge.

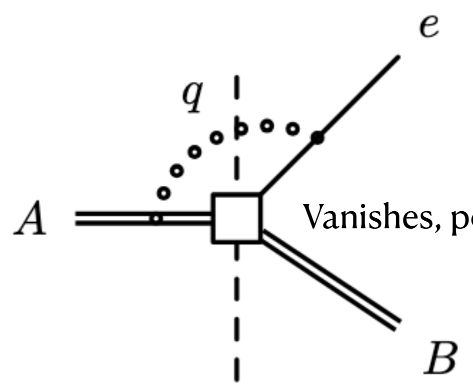
$$\langle \nu' e' B' | H_W | A \nu \rangle = \bar{u}_{k'} \gamma_\mu P_L u_k \langle e' B' | \bar{\psi} \Gamma^\mu \psi | A \rangle$$

Compared with earlier plus superscript on B has changed to prime

$$\langle e' B' | \bar{\psi} \Gamma^\mu \psi | A \rangle = \sum_B \langle e' B' | \bar{\psi} | B \rangle \Gamma^\mu \langle B | \psi | A \rangle$$

$$\langle e' B' | \bar{\psi} | B \rangle \quad =$$





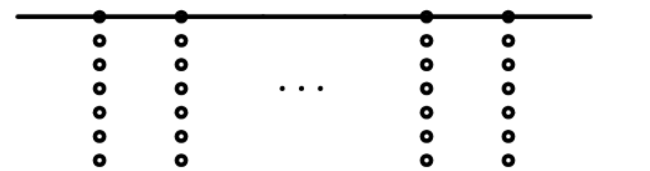
Vanishes, poles on same side of axis

White square is the insertion of $\bar{\psi}$ and the shaded box includes all possible coulomb exchanges. Dominated by virtual momentum of order αm_e.

Treat outgoing struck election as ballistic using leading order propagator, neglecting energy transfers.
Similar to HQET it just has one pole

$$\langle e' B' | \bar{\psi} | B \rangle = \bar{u} \langle B' | \hat{W}_{\mathbf{v}'} | B \rangle$$

$$\hat{W}_{\mathbf{v}'} = \exp\left[e^2 \int \frac{d^3 q}{(2\pi)^3} \frac{1}{\mathbf{v}' \cdot \mathbf{q} - i\epsilon} \frac{\hat{\rho}(\mathbf{q})}{\mathbf{q}^2 + \mu_\gamma^2} \right]$$



$$\left\langle \hat{\rho}(\mathbf{q}_1) \hat{\rho}(\mathbf{q}_2) \ \ldots \ \hat{\rho}(\mathbf{q}_{n-1}) \hat{\rho}(\mathbf{q}_n) \right\rangle$$

Unitary operator since $\hat{\rho}(\mathbf{q})^\dagger = \hat{\rho}(-\mathbf{q})$

Using “notation/formulation” discussed in first part of talk

$$\langle B' | \hat{a}_{\mathbf{p}} | A \rangle \to \langle B' | \hat{W}_{\mathbf{v}'} \hat{a}_{\mathbf{p}} | A \rangle$$

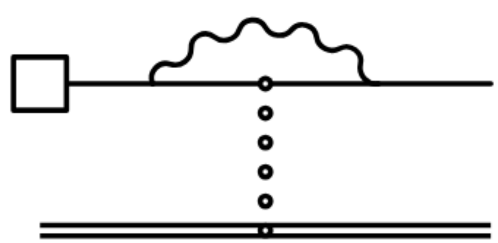
Squaring summing over final atomic debris states the (leading effects of) Coulomb exchanges don’t contribute since $\hat{W}_{\mathbf{v}'}^\dagger \hat{W}_{\mathbf{v}'} = 1.$ Include transverse photons, ...

No contributions at order α^2 of effects that are “fully” enhanced by number of electrons in atom A. Fully means a factor of $\sim Z_A^{2/3}$ for each α.

Just discussed neutrino scattering radiative corrections but also done for muon scattering

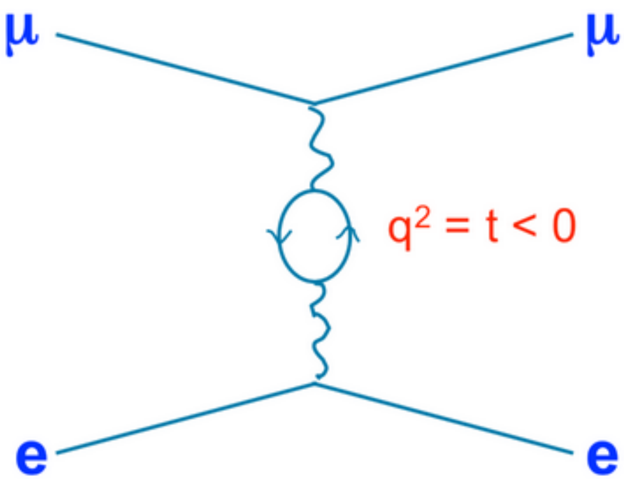
Conclusions

- We have discussed corrections to high energy lepton scattering off atomic electrons that depend on the atomic environment. Small effects of order α^2 but
- for MUonE experiment theory needed at order α^2. This is under control for muon free electron scattering (For a review with references: A. Gurgone [on behalf of MuonE collaboration] e-Print:2401.06491 [hep-ph] 2024.)
- Mostly under control for the order α^2 corrections that care about the fact that the struck electron is in an atom and that final state contains low momentum atomic debris. More explicitly what is included are those order α^2 effects ``fully" enhanced by number of electrons in atom. Below is an example of an effect not yet included



.

MUonE Experiment



CERN: 150 GeV muons incident on electrons in carbon

Order 1% correction to muon electron scattering, and want hadronic vacuum polarization contribution to 1% accuracy

Hard experiment aims for accuracy of 10ppm on shape of cross section

Eur. Phys. J. C (2017) 77:139
DOI 10.1140/epjc/s10052-017-4633-z

THE EUROPEAN PHYSICAL JOURNAL C

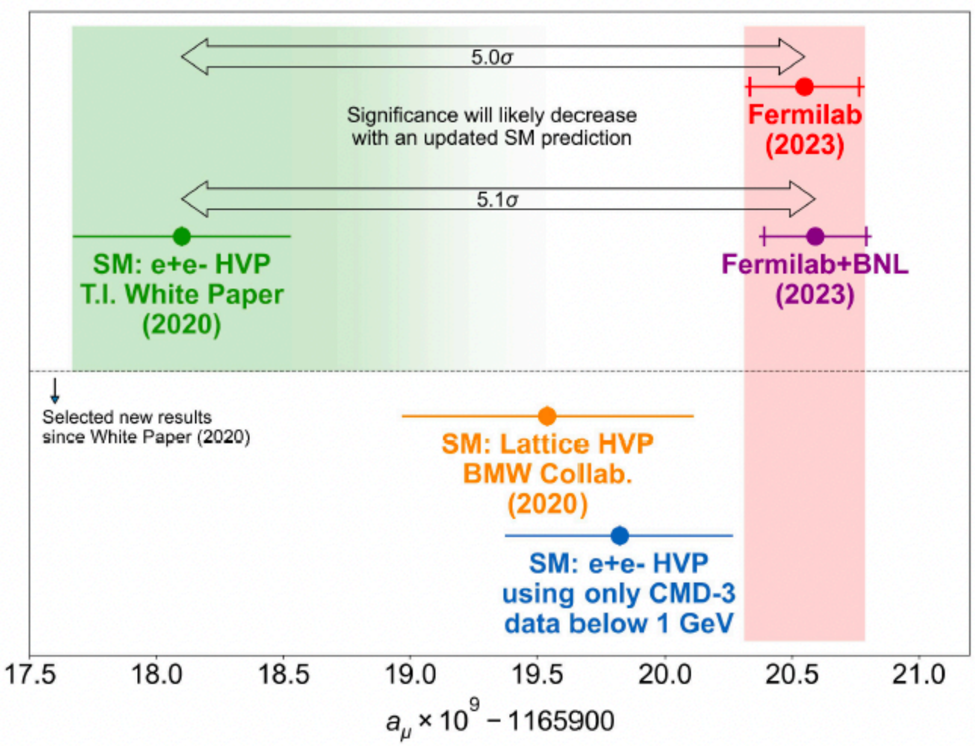
CrossMark

Regular Article - Experimental Physics

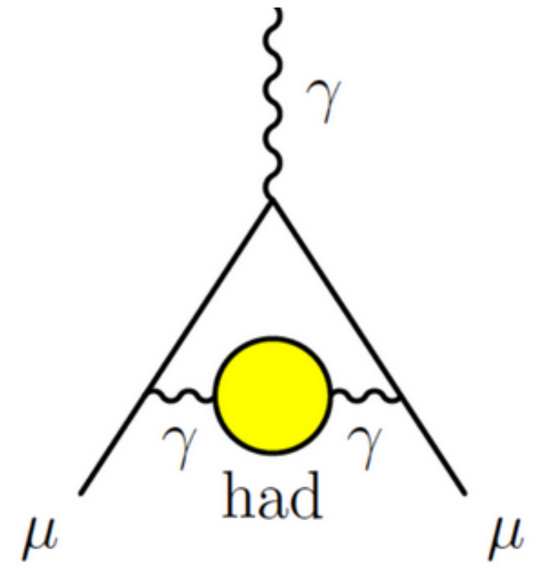
Measuring the leading hadronic contribution to the muon g-2 via μe scattering

G. Abbiendi[1,a], **C. M. Carloni Calame**[2,b], **U. Marconi**[3,c], **C. Matteuzzi**[4,d], **G. Montagna**[2,5,e], **O. Nicrosini**[2,f], **M. Passera**[6,g], **F. Piccinini**[2,h], **R. Tenchini**[7,i], **L. Trentadue**[8,4,j], **G. Venanzoni**[9,k]

Anomalous Magnetic Moment of Muon and Hadronic Physics



X post from Fermilab



$$a_{\mu-HVP} \sim \int dx f(x) \int \frac{ds}{s} Im\Pi(s) \frac{m_\mu^2}{\frac{x^2}{1-x} m_\mu^2 + s}$$