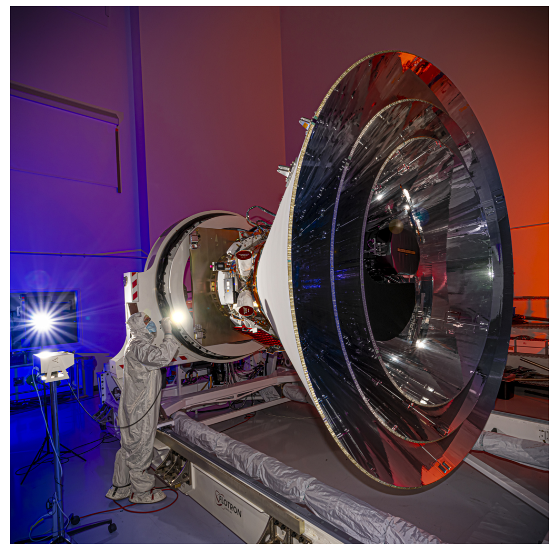
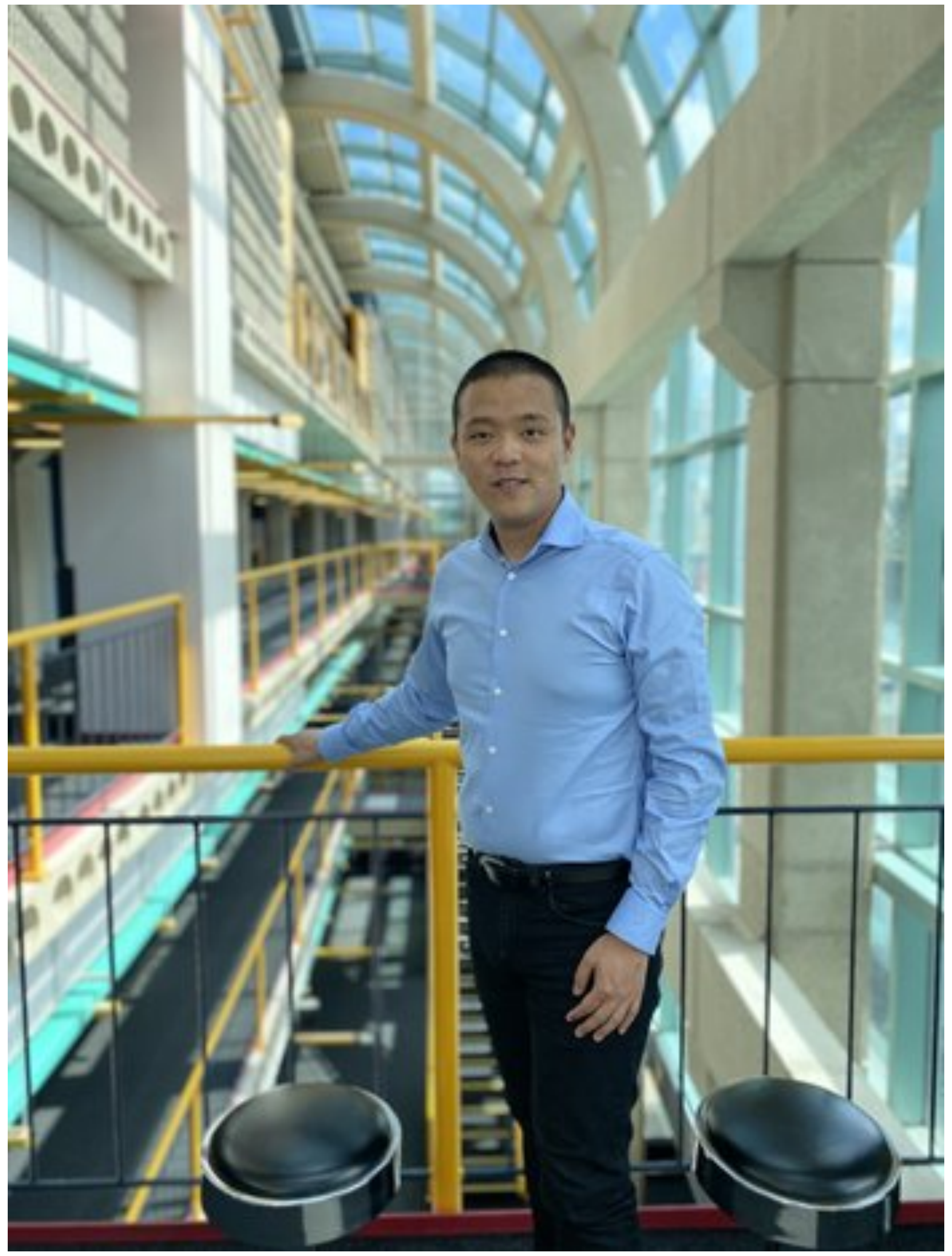


Non-perturbative method for wide-angle modeling

Chen Heinrich, with Robin Wen, Henry Gebhardt and Olivier Doré
Caltech Institute of Technology

Fundamental Physics From Future Spectroscopic Surveys, Berkeley, May 2024

Key people



Robin Wen

Henry Gebhardt

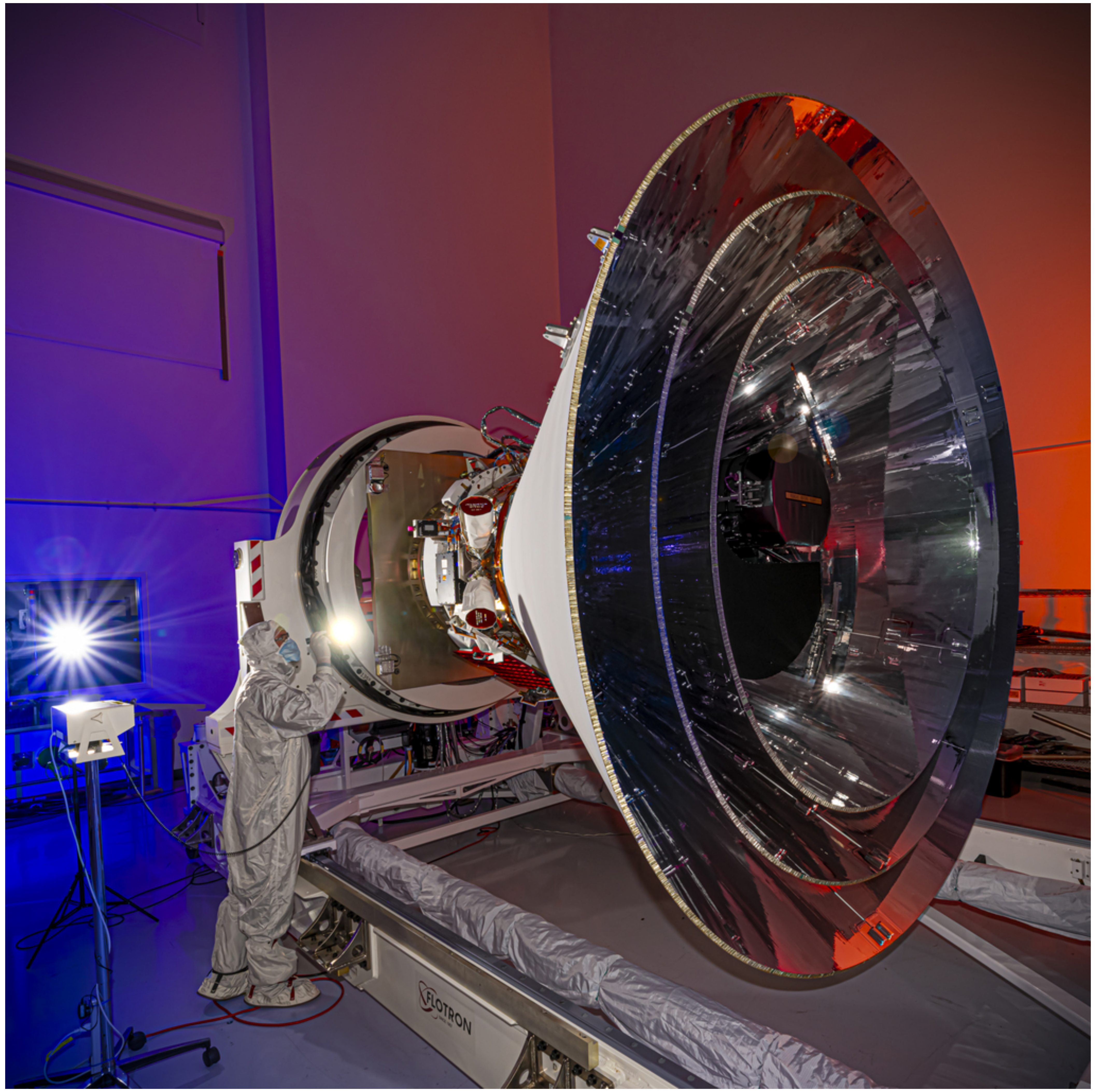
Olivier Doré

Exact Modeling of Power Spectrum Multipole through Spherical Fourier-Bessel Basis

https://arxiv.org/abs/2404.04812

SPHEREx

- Launching Feb 2025
- A key challenge is large-scale modeling.
    - Wide-angle (WA) modeling
    - GR modeling
    - Window function
- Need all three in the signal and covariance, for PS and Bis.



Status in the literature

- **PS Signal: Wide-angle (many methods exist for WA+Window; public code including WA+Window+GR: GAPSE** by Foglieni et al. 2023)
- PS Covariance: (1) Window-convolved covariance often makes plane-parallel approximations — not exact in the WA regime (e.g. Wadekar et al. 2019). (2) No GR.
- Bispectrum Signal:
  - Perturbative method exists for WA and GR (calculation partially demonstrated);
  - Window function on large scales (for squeezed triangles) is an unsolved problem.
- Bispectrum Covariance:
  - Large-volume simulations too expensive for mock-based covariance.
  - Analytic + simulations needed for deriving the covariance.

Wide-angle in the PS

- Current methods:
  - Correlation function
  - Tomographic spherical harmonics (TSH)
  - Power spectrum multipoles (PSM)
    - Perturbatively (see Joshua’s talk previously)
    - Non-perturbatively, through the correlation function (GAPSE code)
- **New method:**
  - **PSM: non-perturbatively through the spherical Fourier-Bessel (SFB) basis.**

SFB-PSM mapping

- SFB:
  - Great for exact modeling of WA on large scales.
  - Larger data vector, may not be feasible on smaller scales.
  - Nonlinear modeling not yet mature.
- PSM:
  - Nonlinear modeling is more mature.
  - Well-tested Yamamoto estimator code.
  - Efficient compression.
- It would be great if we can keep the best of both worlds.

SFB-PSM mapping

$$P_L(k) = \frac{(4\pi)^2(2L+1)}{I_{22}} \sum_{a,b} i^{-a+b}(2a+1)(2b+1) \begin{pmatrix} a & L & b \\ 0 & 0 & 0 \end{pmatrix}^2 C_b^{ab,\mathrm{W}}(k,k)$$

Castorina and White 2017

Generalized SFB

$$\delta^L_{\ell m}(k) = \int_{\mathbf{x}} j_L(kx) Y^*_{\ell m}(\hat{\mathbf{x}}) \delta(\mathbf{x})$$

$$C_\ell^{ab,\mathrm{W}}(k_1,k_2) \equiv \frac{1}{2\ell+1} \sum_m \langle \delta^{a,\mathrm{W}}_{\ell m}(k_1) \delta^{b*,\mathrm{W}}_{\ell m}(k_2) \rangle$$

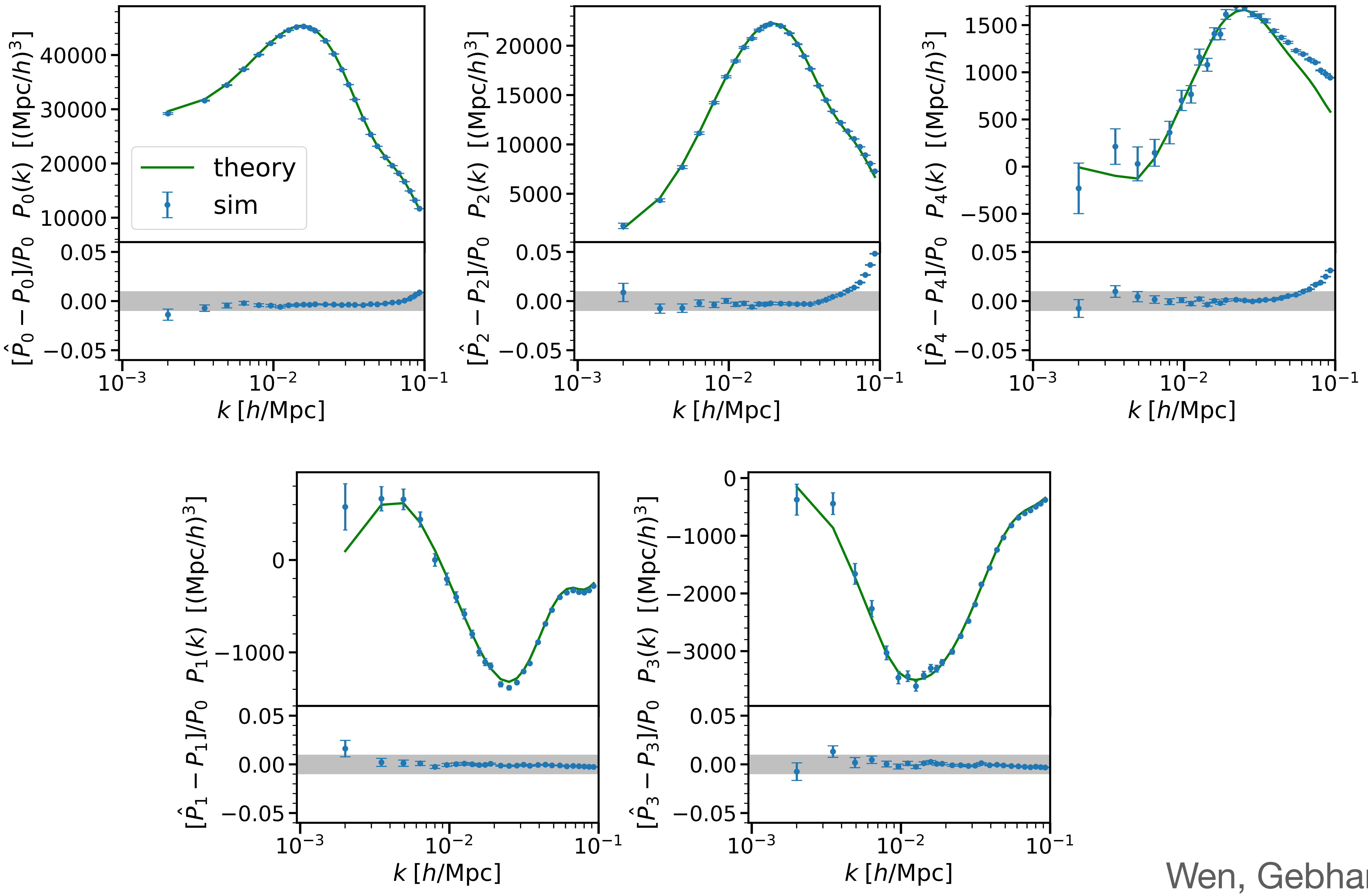
Reduces to the canonical SFB for a = b = ell

Physical intuition

$$P_L(k) = \frac{(4\pi)^2(2L+1)}{I_{22}} \sum_{a,b} i^{-a+b}(2a+1)(2b+1) \begin{pmatrix} a & L & b \\ 0 & 0 & 0 \end{pmatrix}^2 C_b^{ab,\mathrm{W}}(k,k)$$

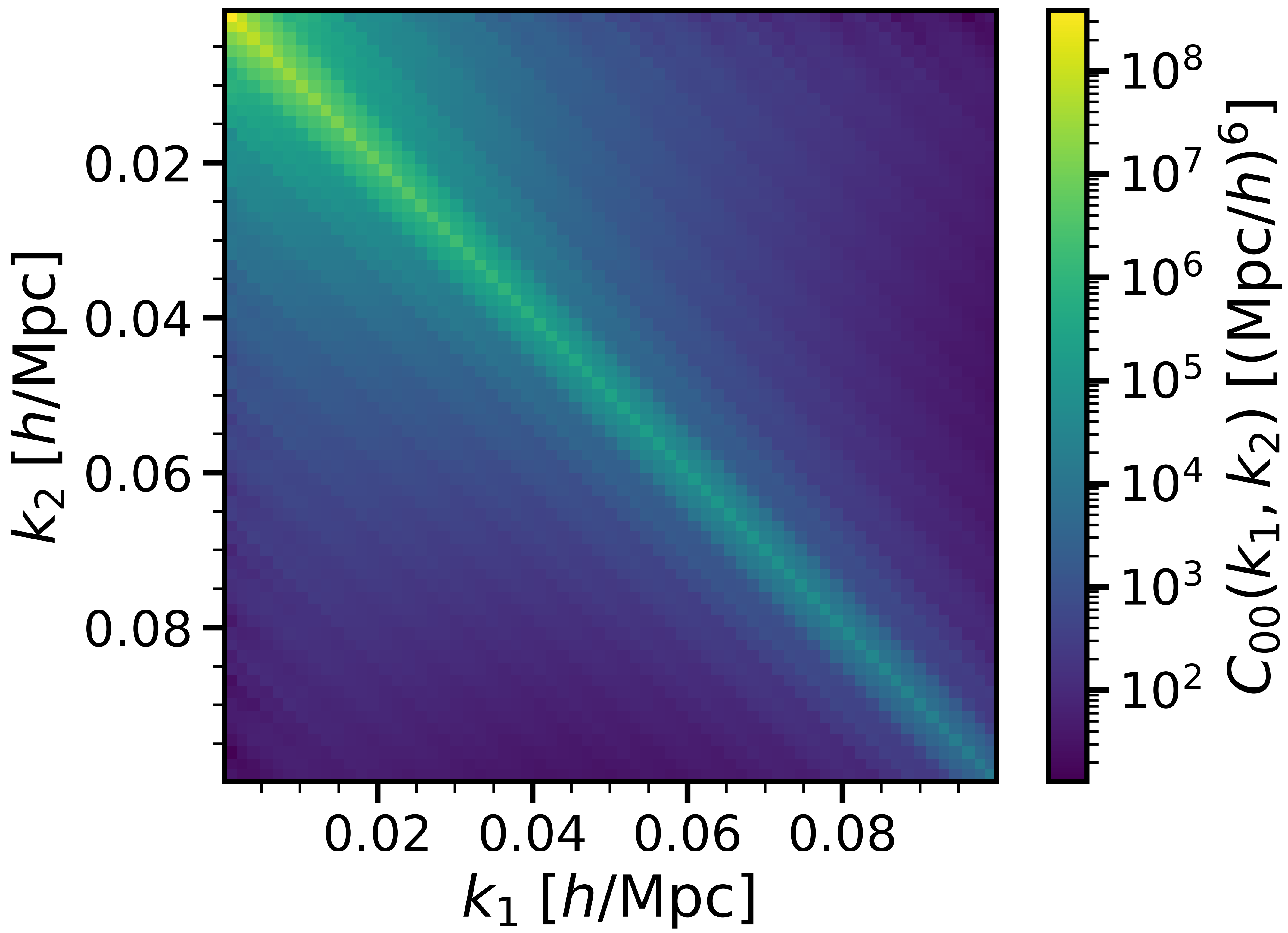
- How is information lost?
- Monopole:
  - Only canonical SFB with the same k: $$P_0(k) = \frac{(4\pi)^2}{I_{22}} \sum_b (2b+1) C_b^{\mathrm{W}}(k,k)$$
  - These are the only modes in an homogenous and isotropic Universe.
  - Monopole: Average over Fourier wavevector orientation; averaging over z-bin.
- Higher multipoles:
  - Off-diagonal components in the canonical SFB are induced by redshift evolution or RSD.
  - Expect those components to be (partially) brought back in higher multipoles.
  - They are folded in through the upper indices of generalized SFB (a, b).

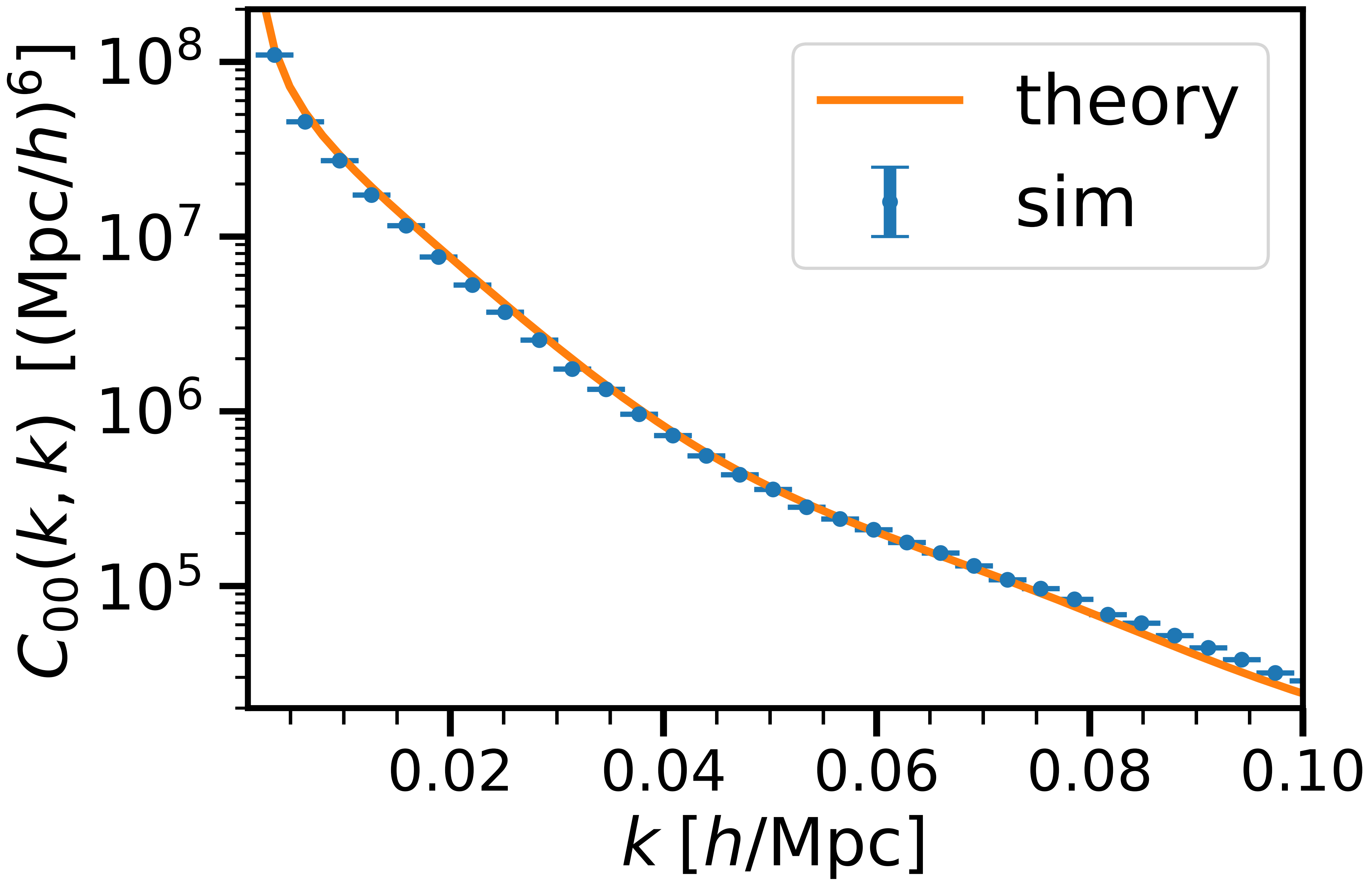
Validation on lognormal simulations



Wen, Gebhardt, CH, Dore 2024

A Nice Consequence: Exact Gaussian covariance





Full sky (radial window only, z = 0.2 - 0.5).

Wen, Gebhardt, CH, Dore 2024

Summary

- We can model the WA effects with windows EXACTLY for the PSM through the SFB-PSM relation.
- We can use the same estimator for large and small scales.
- **Bonus**: we can model EXACTLY the PSM covariance with WA + window.
- Bispectrum:
  - SFB-BSM mapping exists.
  - More hope for computing the bispectrum WA+GR+Window signal, as well as the covariance matrix which depends on this signal.

Backup slides
Covariance expressions

$$\mathbf{C}^{\mathrm{G}}_{L_1L_2}(k_1,k_2)=\frac{(2L_1+1)(2L_2+1)}{I_{22}^2}\left[\int_{\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2}\langle F_{L_1}(\mathbf{k}_1)F_0(-\mathbf{k}_1)F_{L_2}(\mathbf{k}_2)F_0(-\mathbf{k}_2)\rangle\right]-\langle\widehat{P}_{L_1}(k_1)\rangle\ \langle\widehat{P}_{L_2}(k_2)\rangle$$

$$\mathbf{C}^{\mathrm{G}}_{L_1L_2}(k_1,k_2)=(4\pi)^4\frac{(2L_1+1)(2L_2+1)}{I_{22}^2}\sum_{a,b,c,d,\ell_1,\ell_2}i^{-a-c+b+d}(2a+1)\begin{pmatrix}a & L_1 & b\\ 0 & 0 & 0\end{pmatrix}^2\begin{pmatrix}c & L_2 & d\\ 0 & 0 & 0\end{pmatrix}^2$$
$$\left[(2c+1)S_{b\ell_1d\ell_2}+(-1)^{L_2}(2d+1)S_{b\ell_1c\ell_2}\right]C^{ad,\mathrm{R}}_{\ell_1}(k_1,k_2)C^{bc,\mathrm{R}}_{\ell_2}(k_1,k_2)\,,$$

Radial window only (full sky):

$$\mathbf{C}^{\mathrm{G}}_{00}(k_1,k_2)=\frac{(4\pi)^4}{I_{22}^2}\sum_b 2(2b+1)\left[C^{\mathrm{R}}_b(k_1,k_2)\right]^2$$

Wen, Gebhardt, CH, Dore 2024