Non-perturbative techniques for constraining the cosmological collider


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SG, *Philcox, Hill, Hui,* *(in prep)*

2209.06228
SG, *Esposito, Philcox, Hui, Hill, Scoccimarro, Abitbol*

2310.12959
SG, *Philcox, Hill, Esposito, Hui*

Introduction

- Spec-S5 surveys will ~~improve constraints on~~ help unveil the physics of inflation
  - Constrain **primordial non-Gaussianity (PNG)** beyond $f_{\rm NL}^{\rm loc.}, f_{\rm NL}^{\rm eq.}, f_{\rm NL}^{\rm ort.},$
- **Cosmological collider:** primordial squeezed bispectrum from massive particles during inflation

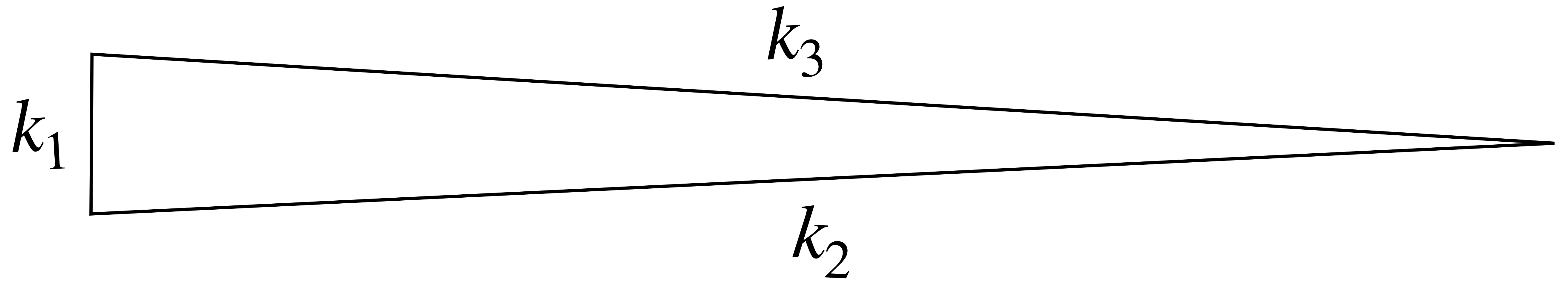
$$\lim_{k_1 \ll k_2 \approx k_3} B_\Phi(k_1, k_2, k_3) = 4 f_{\rm NL}^{\Delta} \left(\frac{k_1}{k_2}\right)^{\Delta} P_\Phi(k_1) P_\Phi(k_2), \ \Delta = 3/2 - \sqrt{9/4 - m^2/H^2}$$

(Arkani-Hamed & Maldacena, 2015)

***Case 1* —** Massless ($m \ll H;\ \ \Delta \approx 0$): local non-Gaussianity

***Case 2* —** Intermediate mass ($0 < m/H \leq 3/2;\ \ 0 < \Delta \leq 3/2$): between $f_{\rm NL}^{\rm loc.}$ and $f_{\rm NL}^{\rm eq.}$ (quasi-single field)

***Case 3* —** Massive ($m/H \geq 3/2$): oscillatory bispectrum



"Squeezed bispectrum"

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How can we constrain this scenario using large-scale structure data?

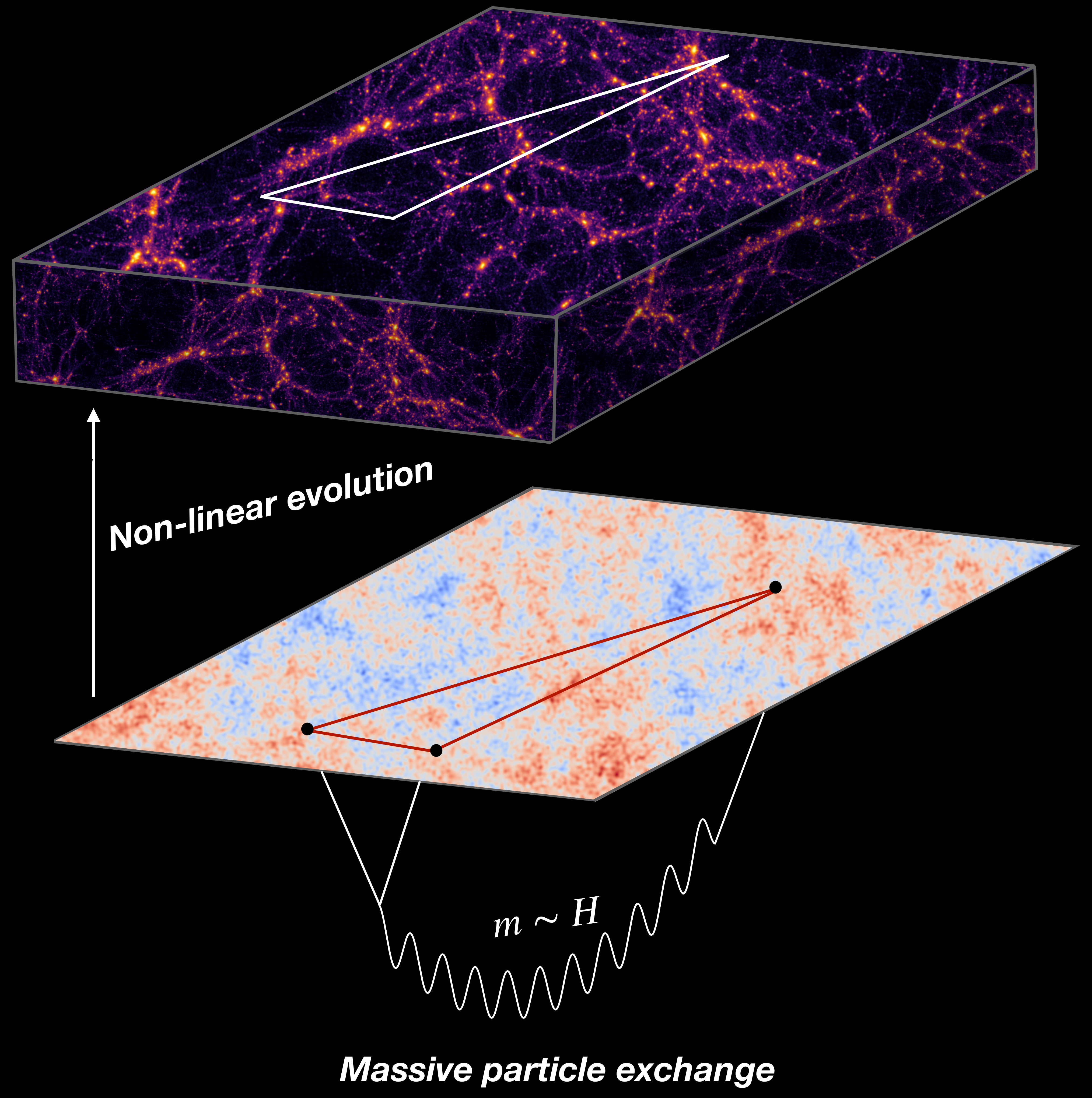
Soft limits of LSS correlation functions

- Late-time LSS correlators contain primordial information
  - **Challenge**: non-linear structure formation
- Exploit two key properties of **soft limits** in LSS
  1. Protected by symmetries

$$\lim_{q \to 0} \left[\frac{B_m(q,k,k')}{P_m(q)} \right] \text{ has no } q^{-\alpha} \text{ poles } (\alpha > 0)$$

Newtonian LSS consistency relation — *Kehagias & Riotto 2012; Peloso & Pietroni, 2013*

  2. Tractable non-perturbative calculations using **separate universe**
- (**1**)+(**2**) used to derive non-perturbative estimators for $f_{NL}^{loc.}$ (**SG**+22/Giri+23)
  - How do we generalize these to cosmo. collider?



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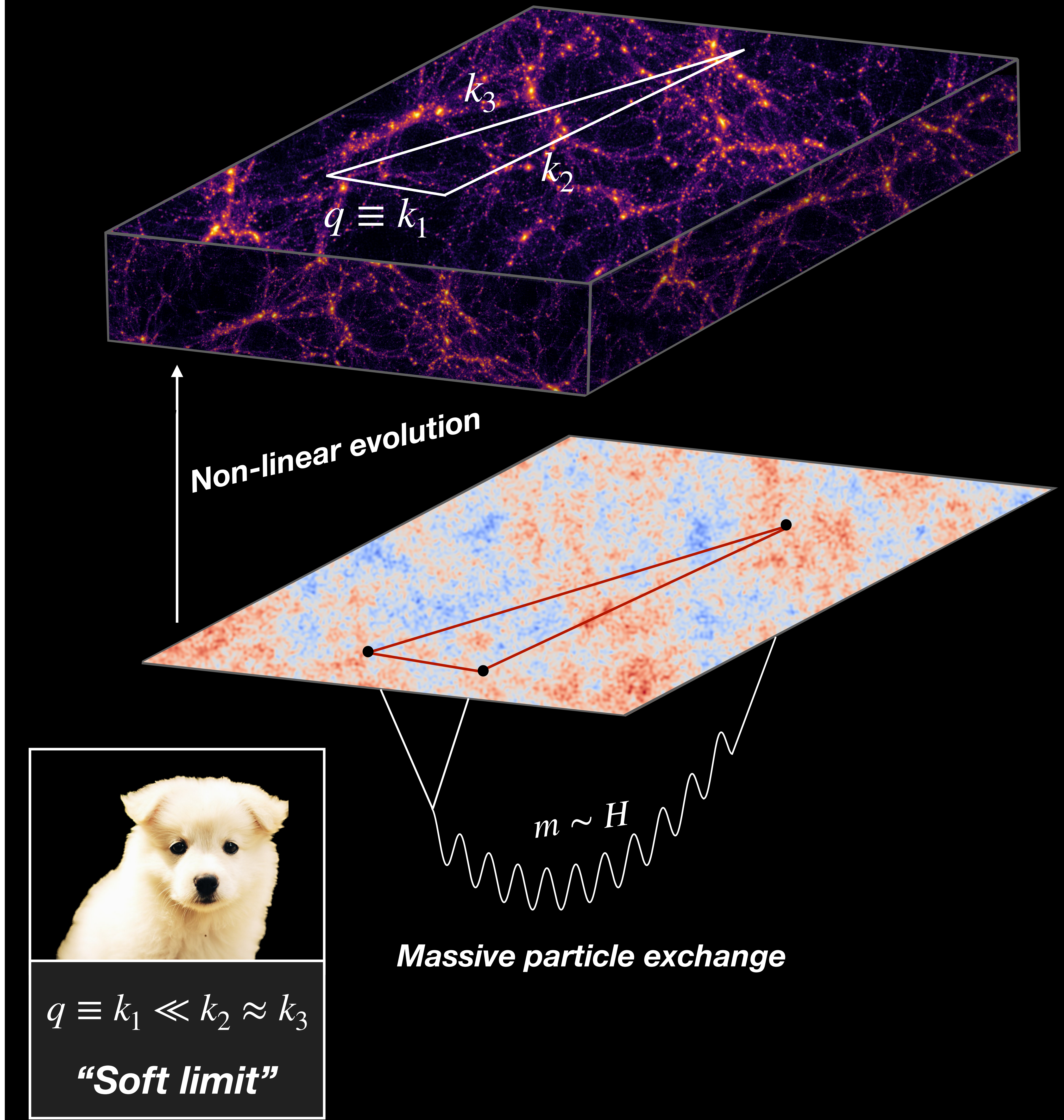

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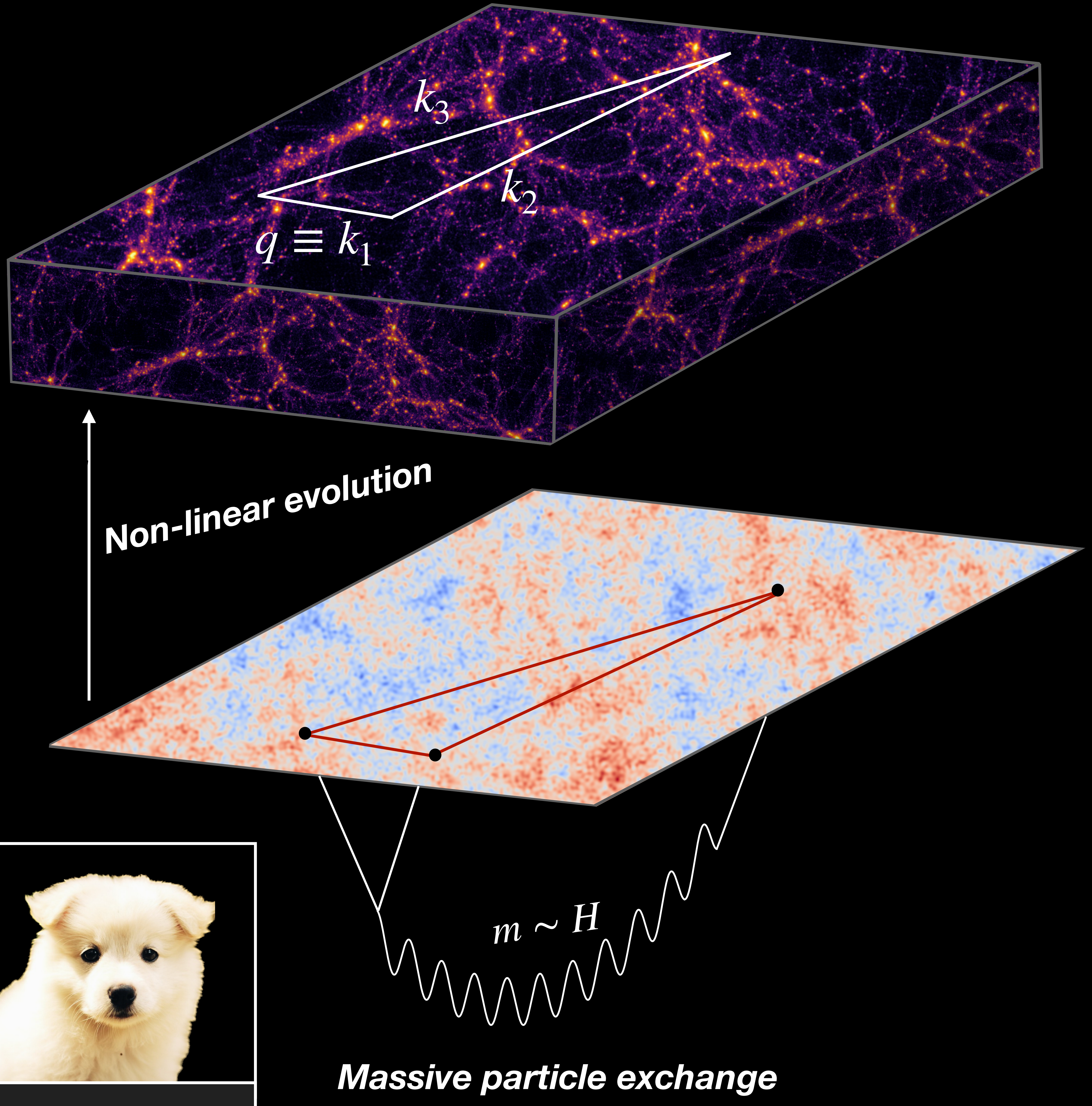
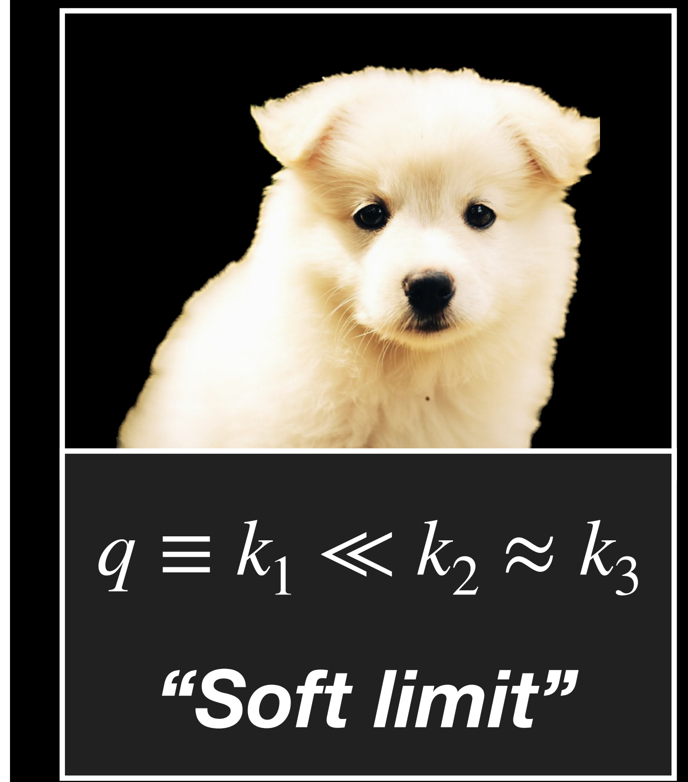
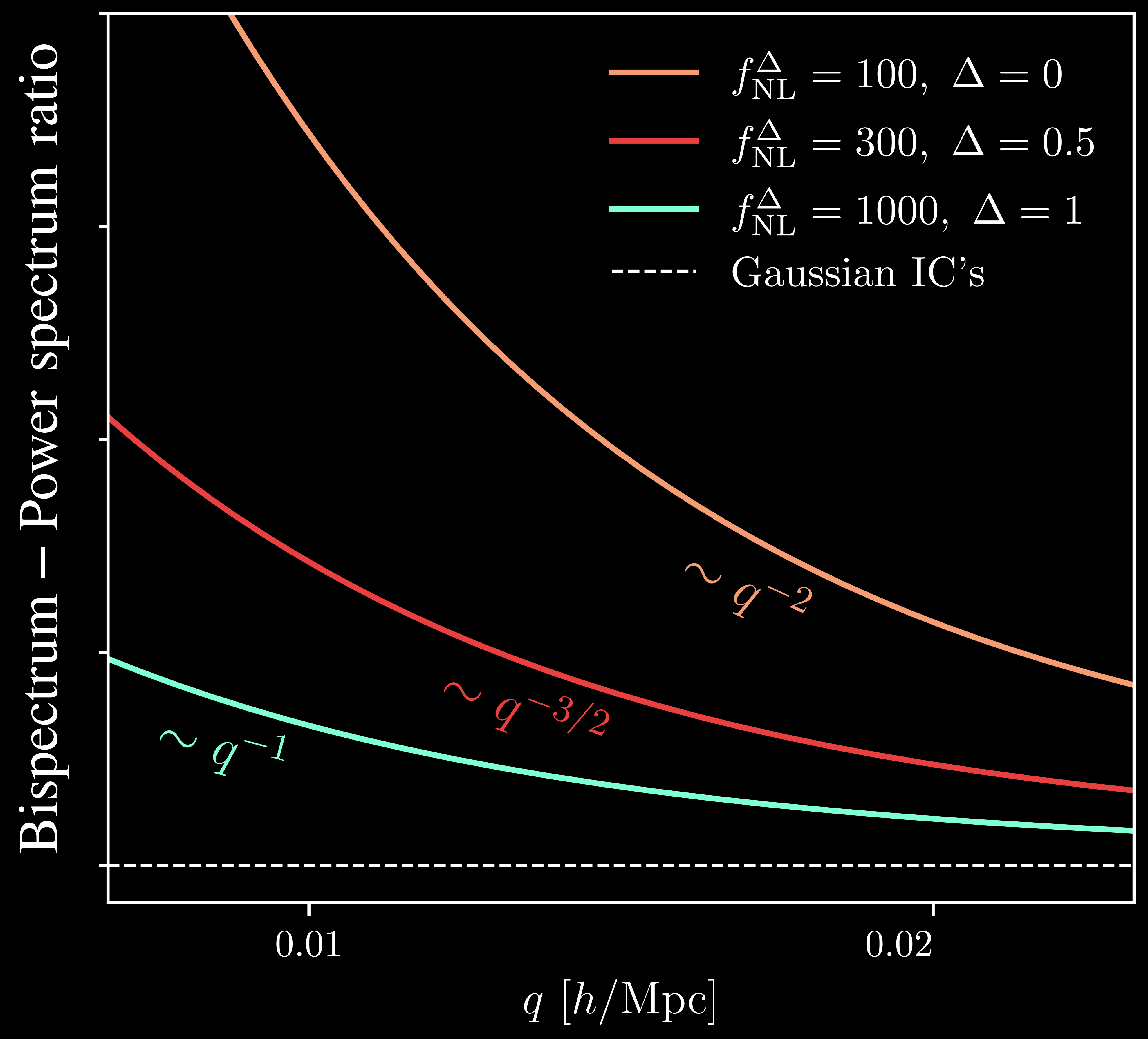




N-body simulations with collider bispectrum



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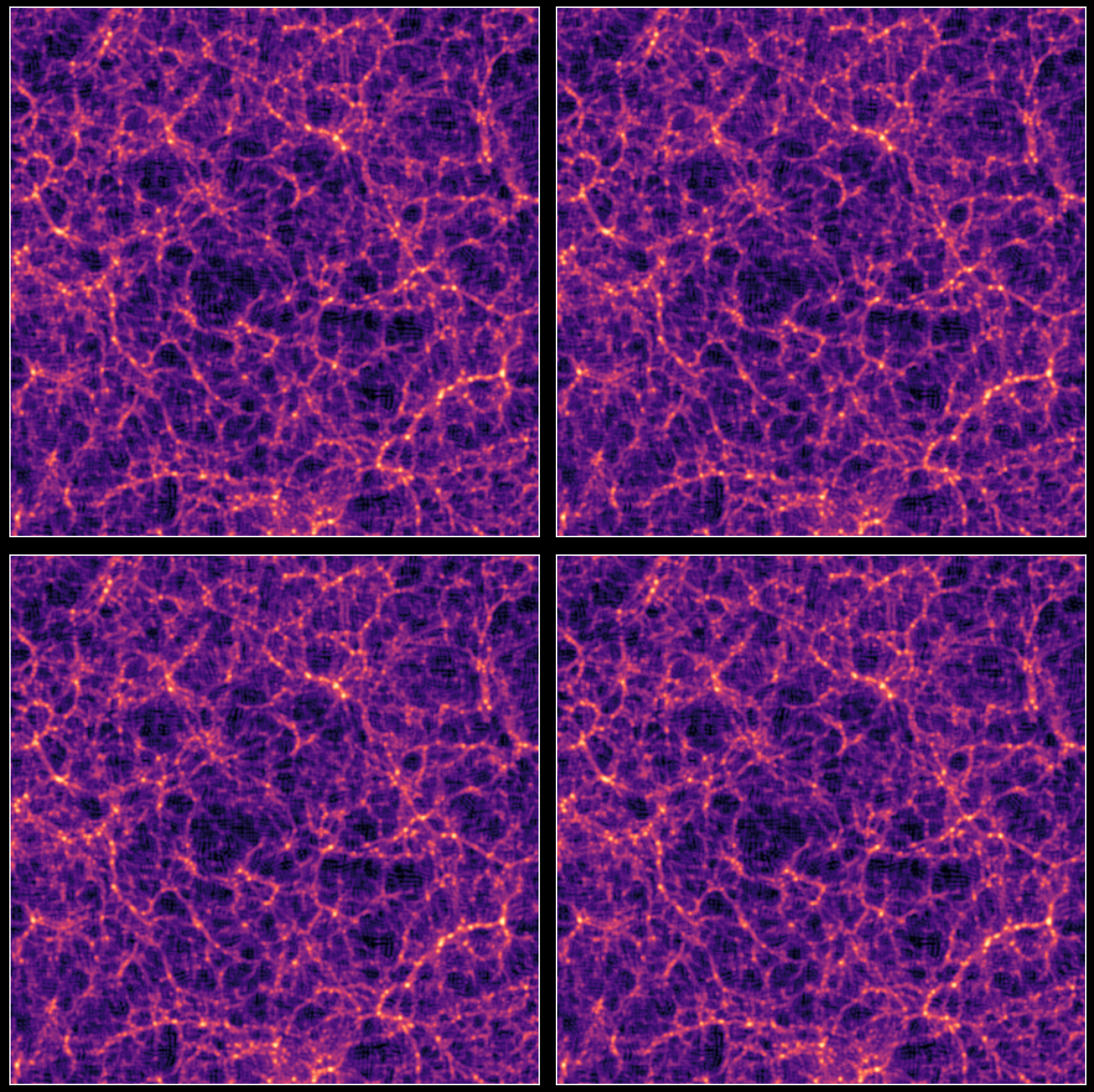
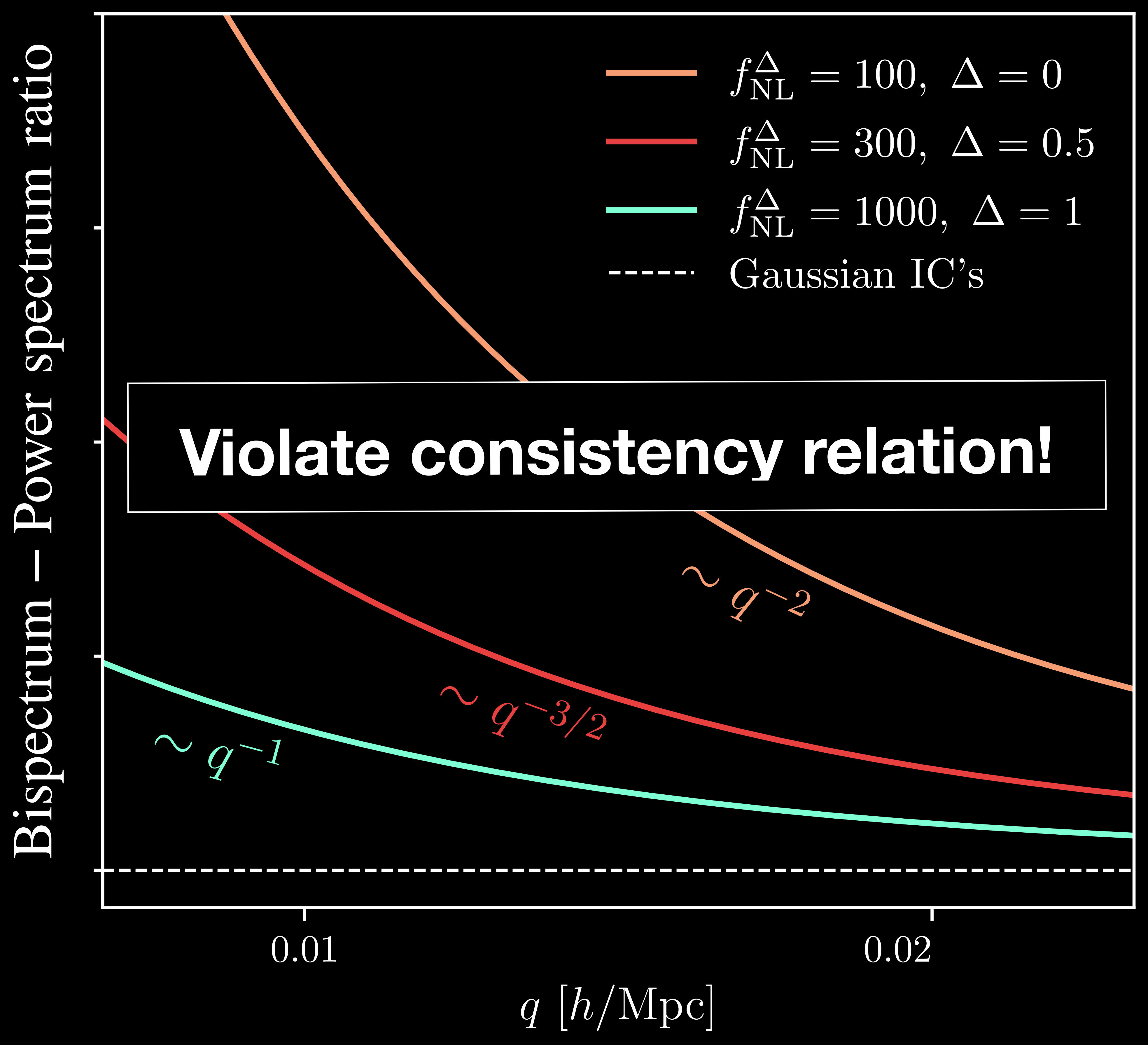


Matter field at $z = 0$

- Added squeezed collider templates to **2LPTPNG**
- Ran suite of simulations with same settings as **QuijotePNG**, but collider primordial bispectrum

N-body simulations with collider bispectrum



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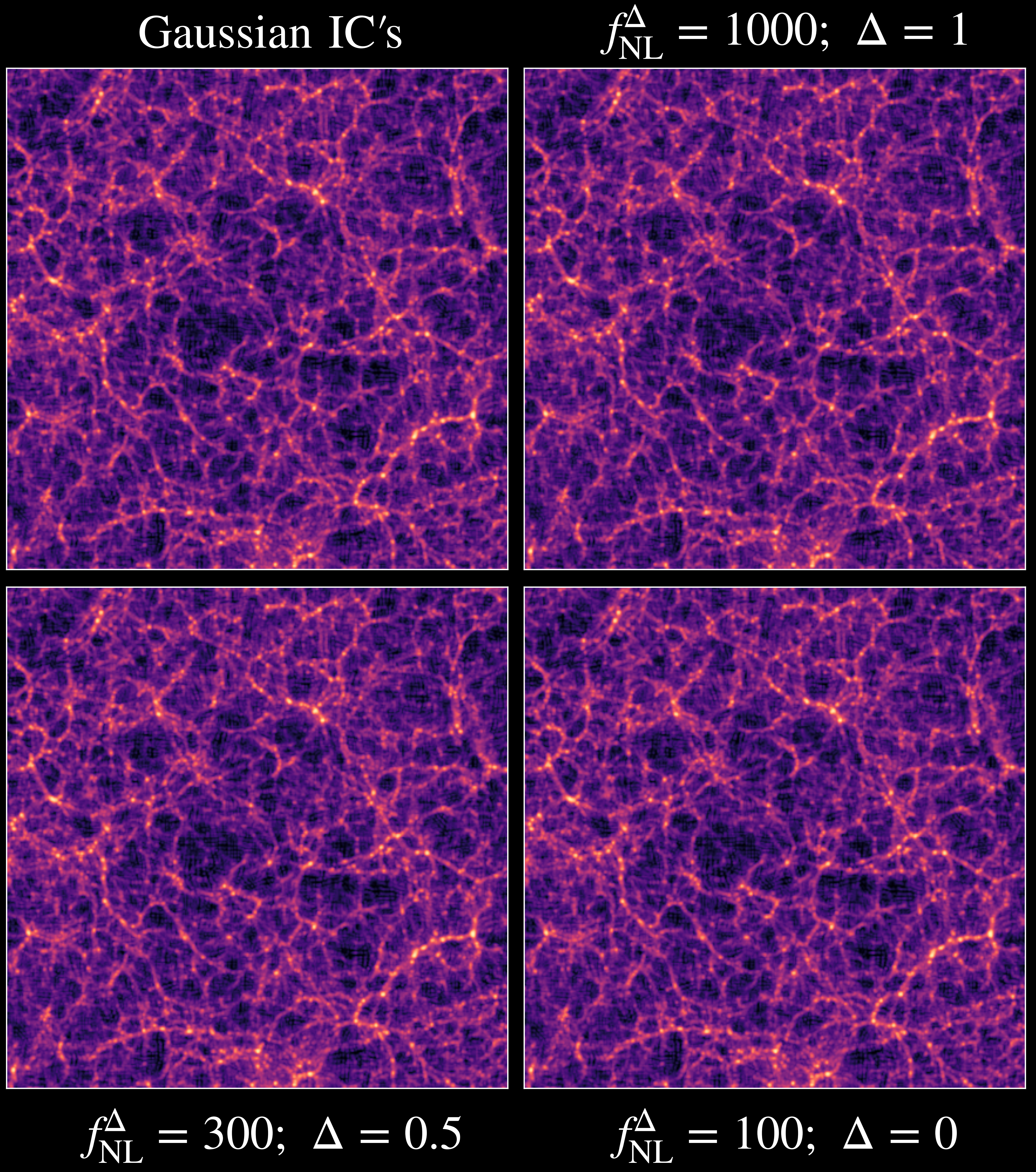

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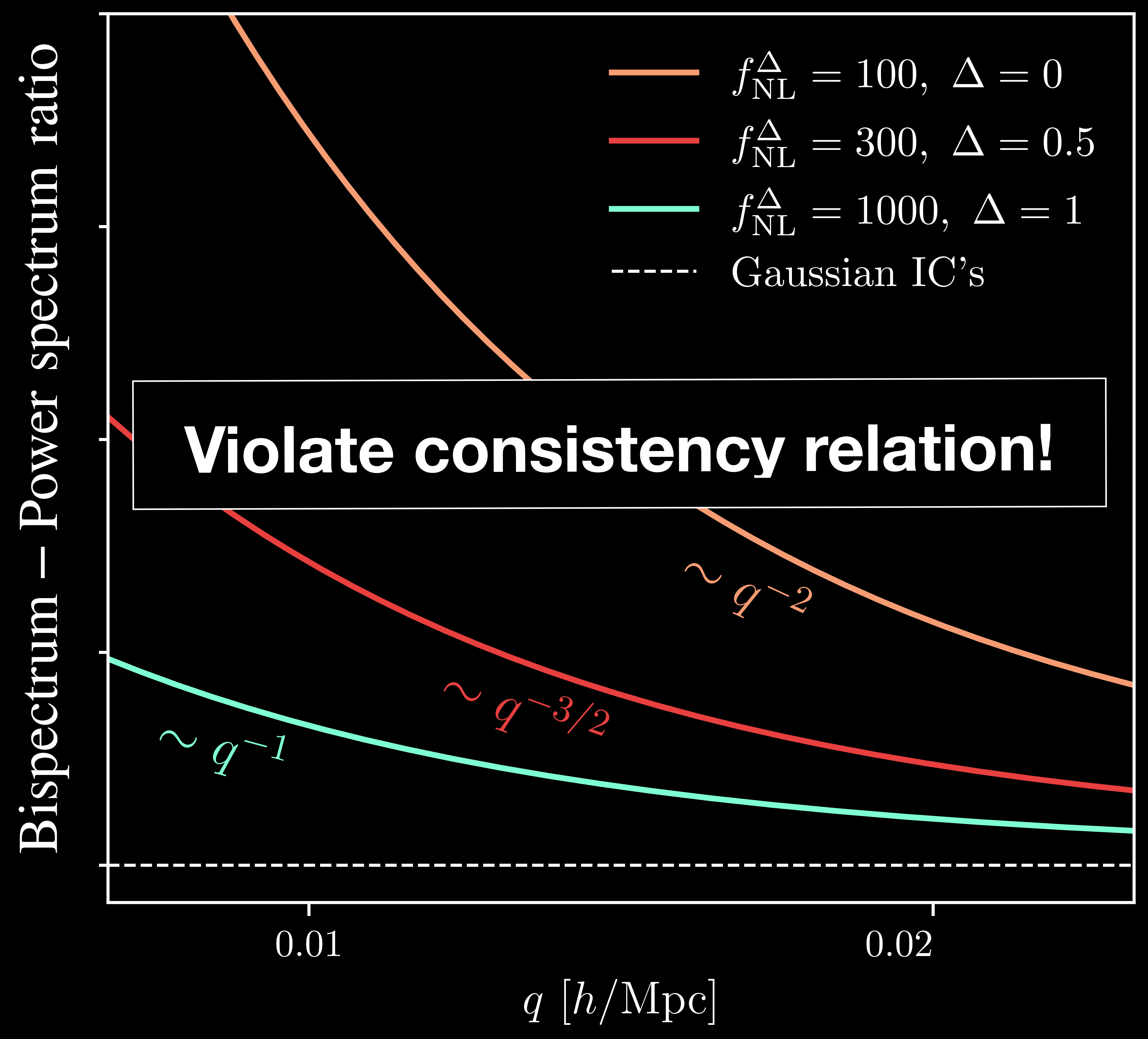

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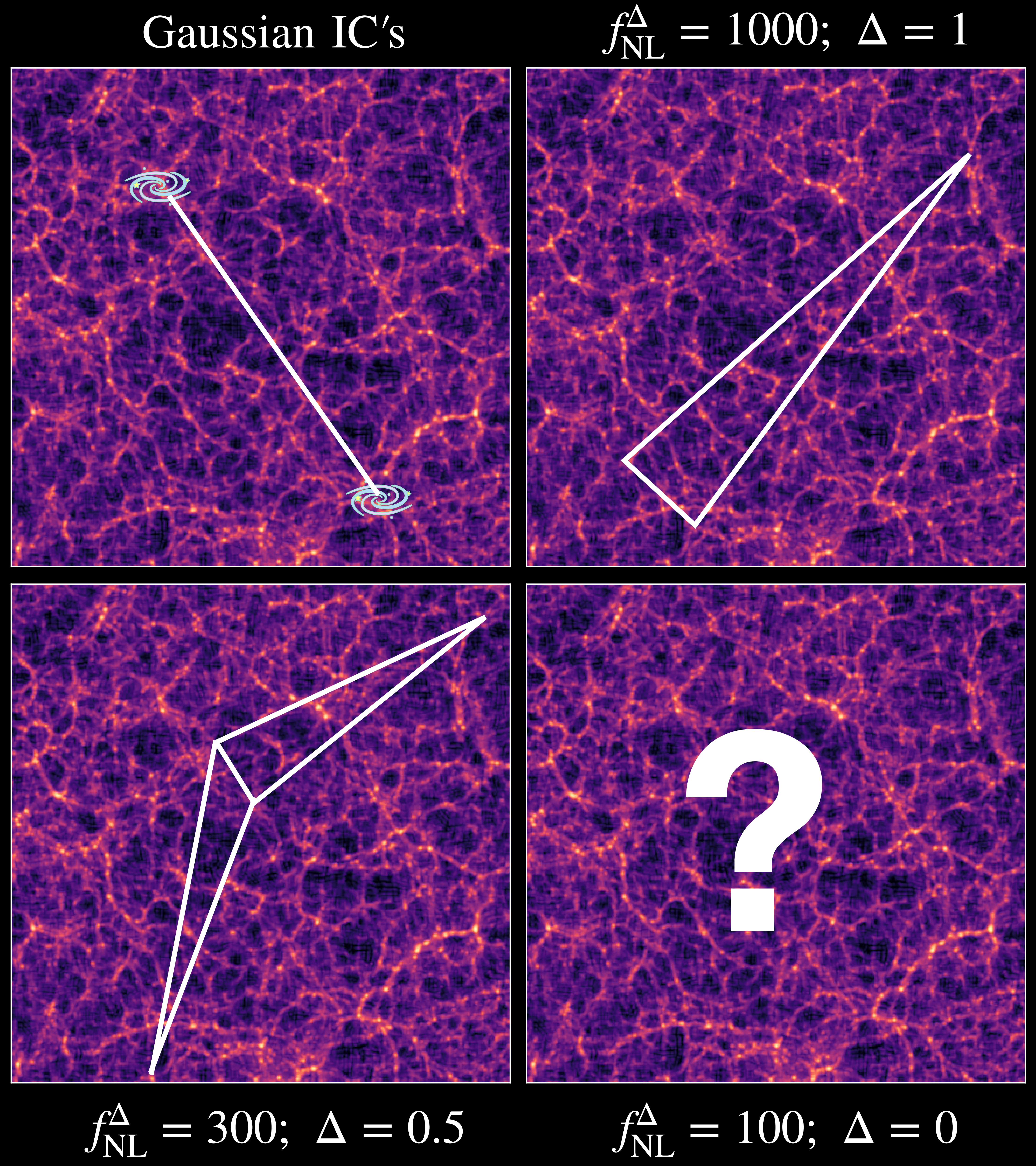

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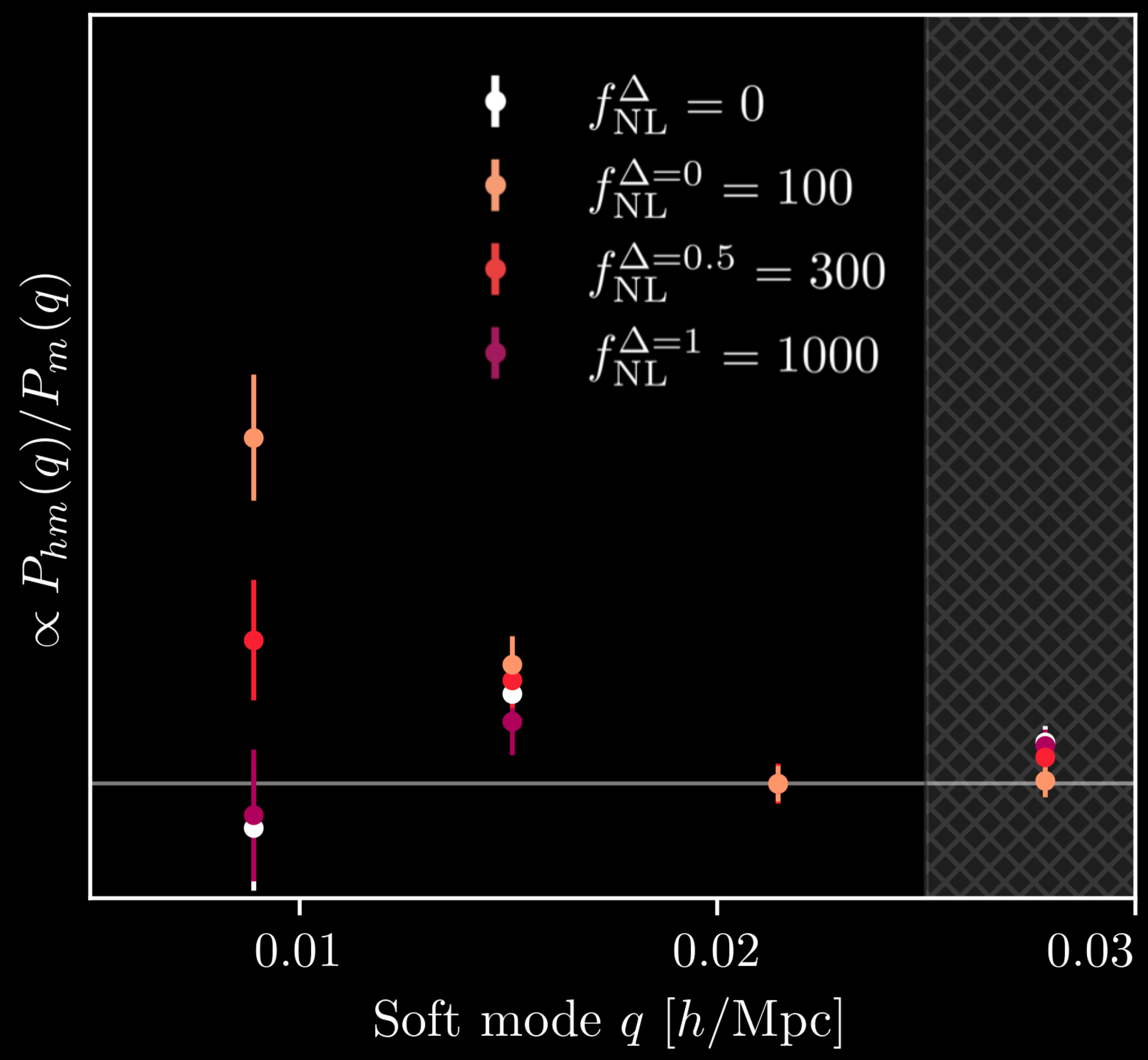
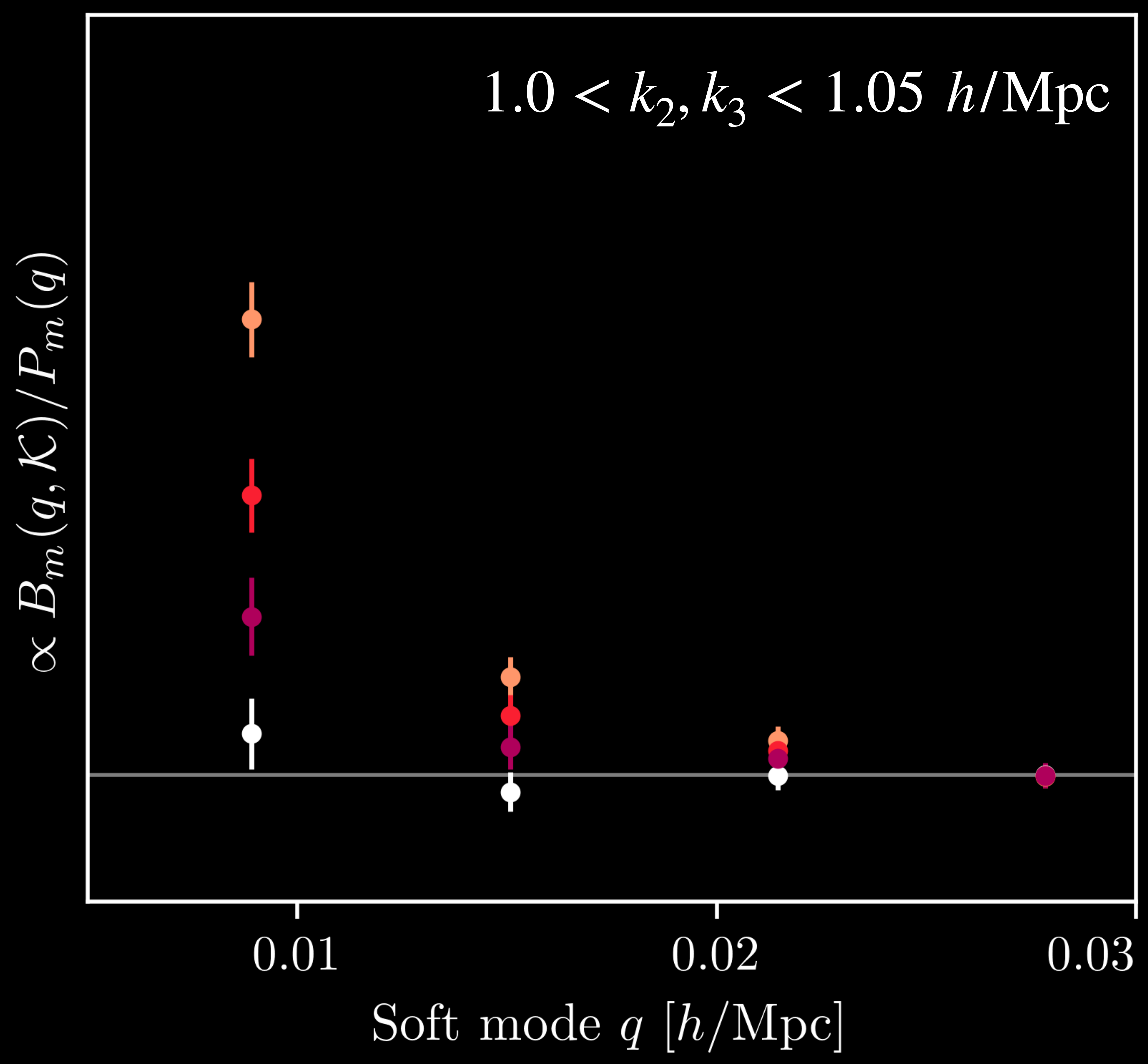
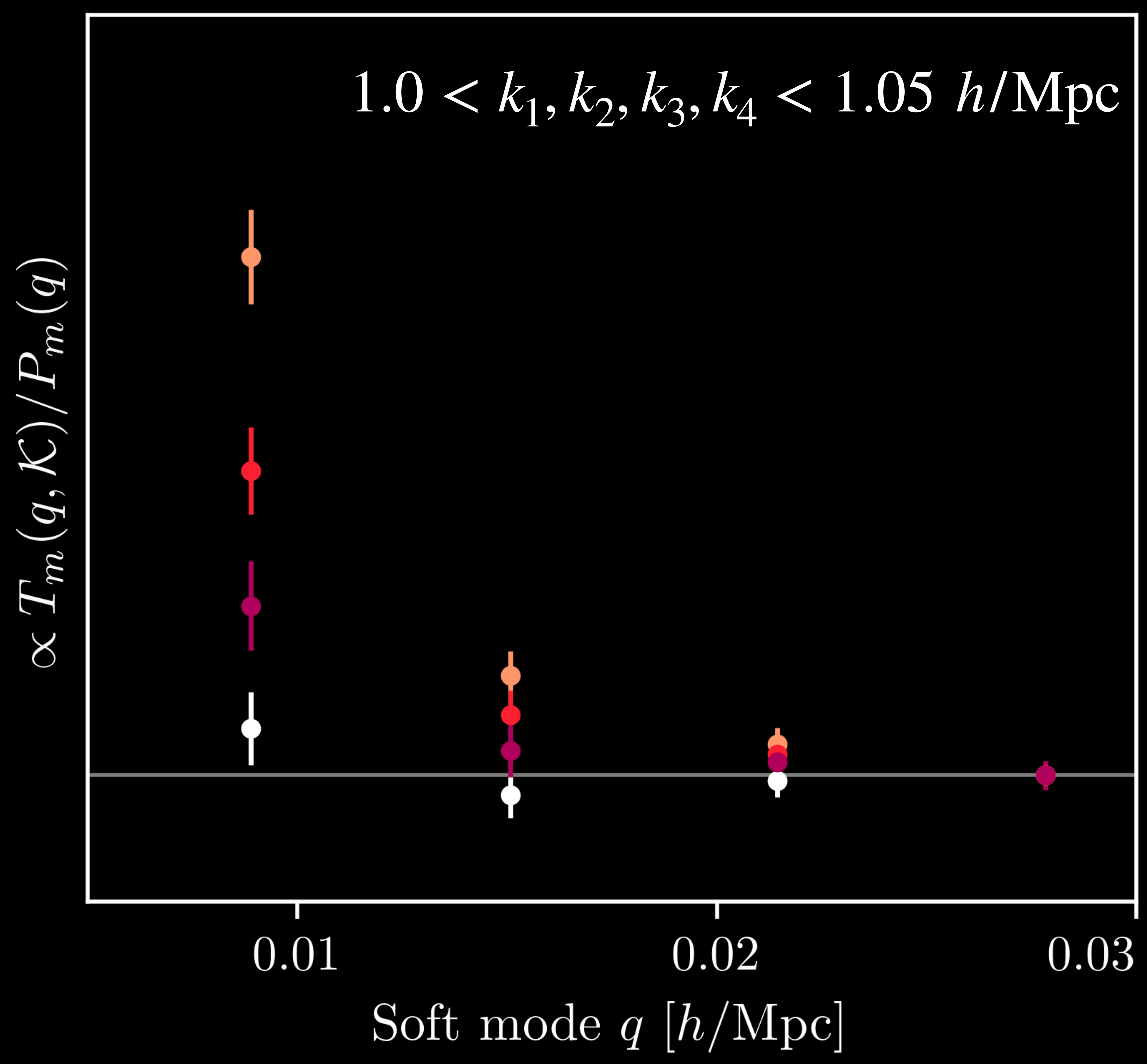
Imprints of the cosmological collider on LSS correlators



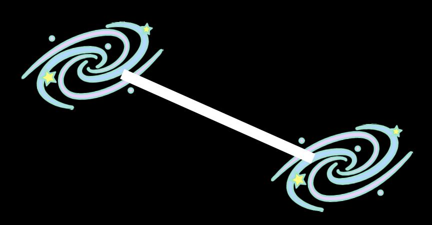


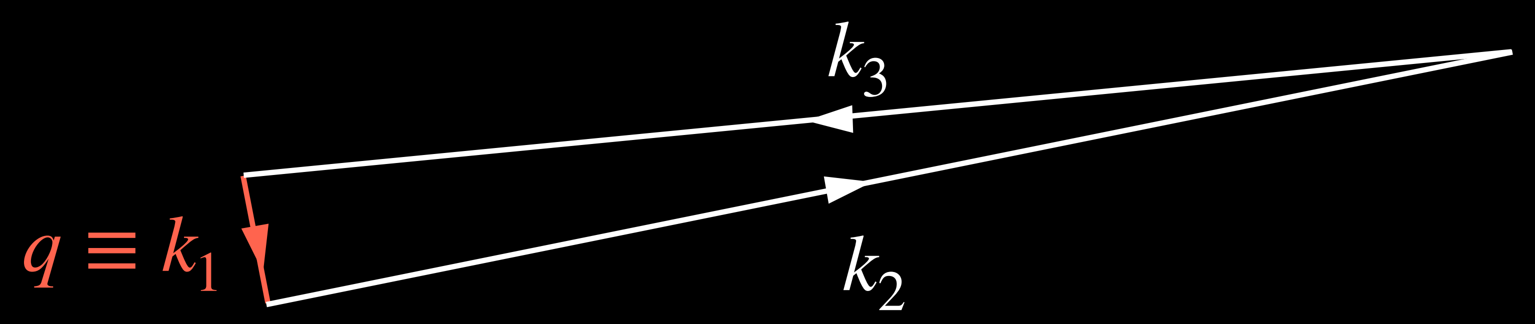


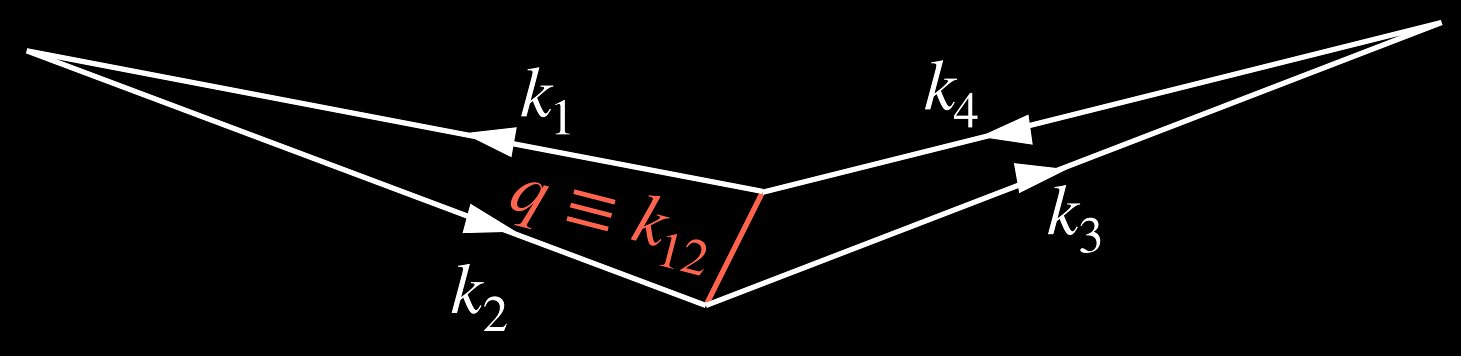
Scale-dependent halo bias

Squeezed matter bispectrum

Collapsed matter trispectrum







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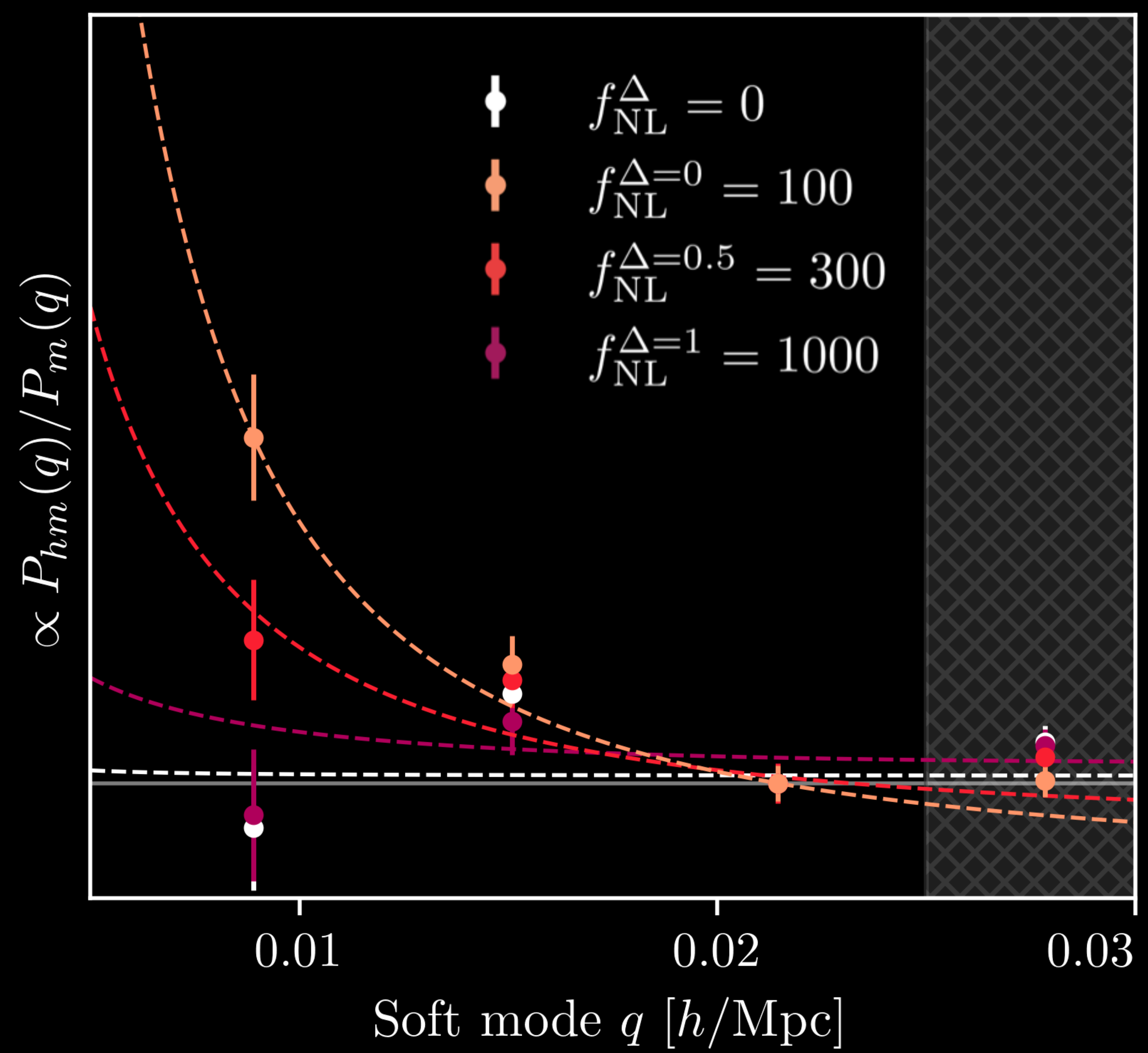

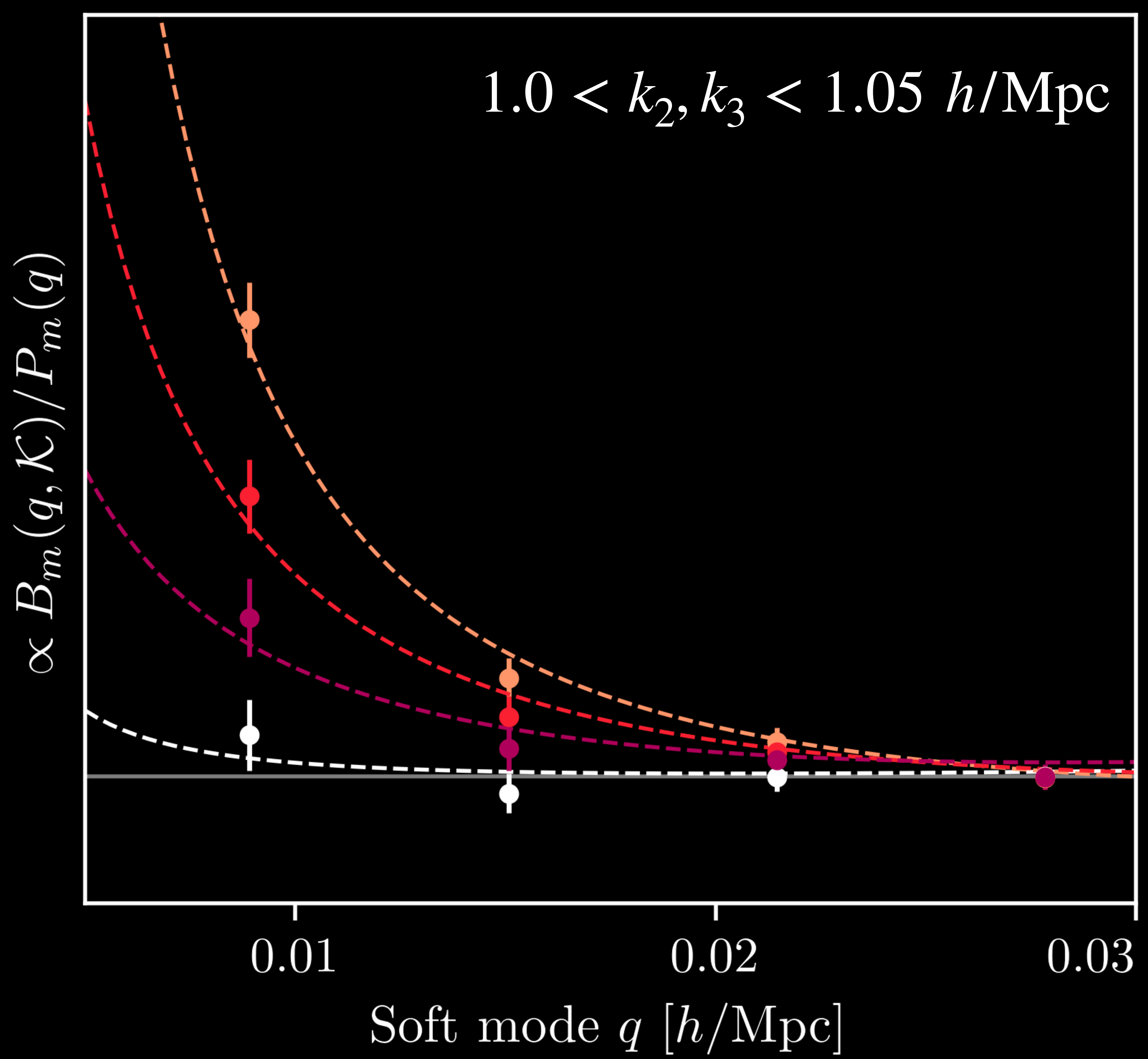

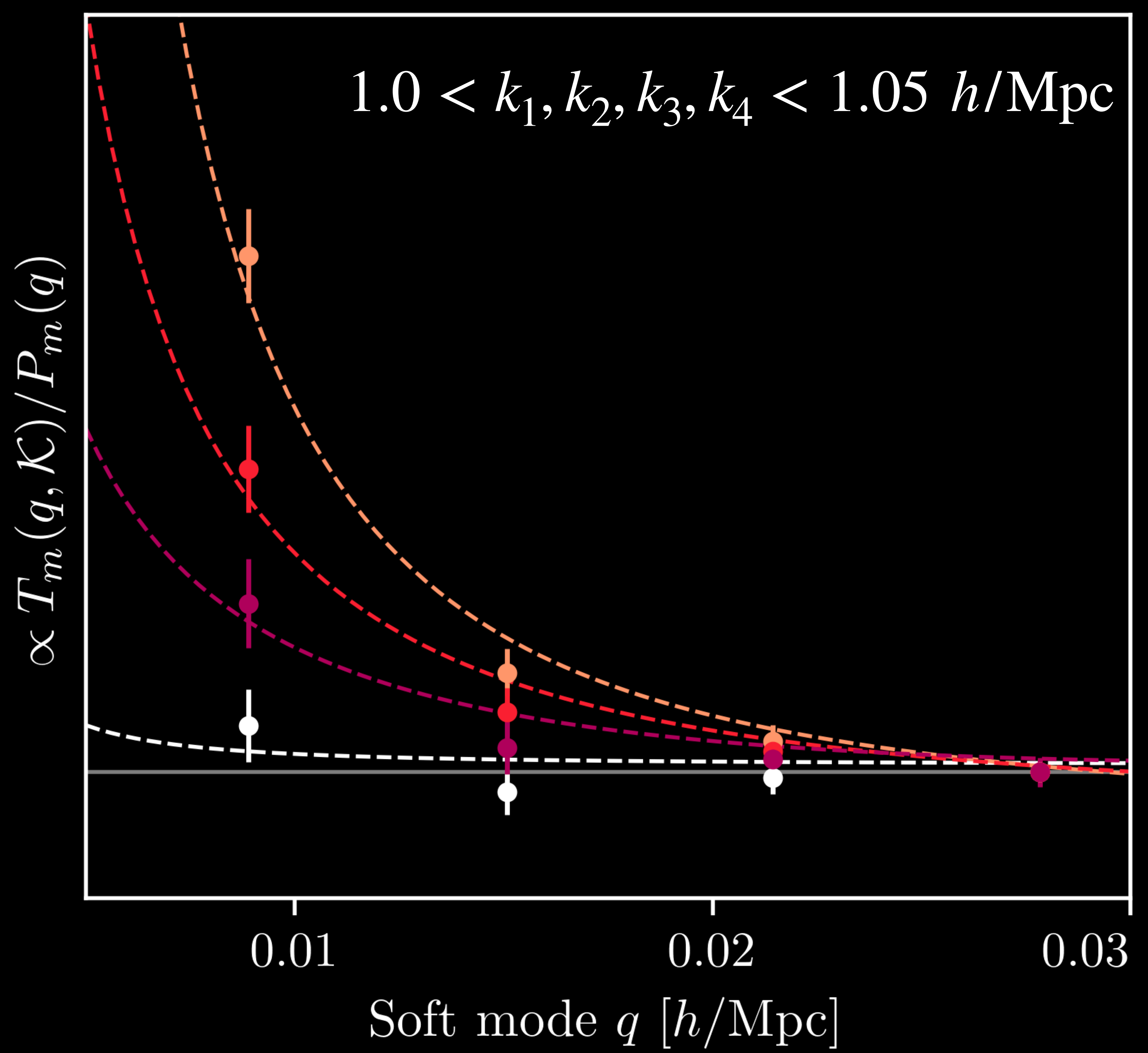

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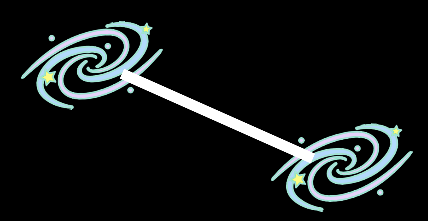

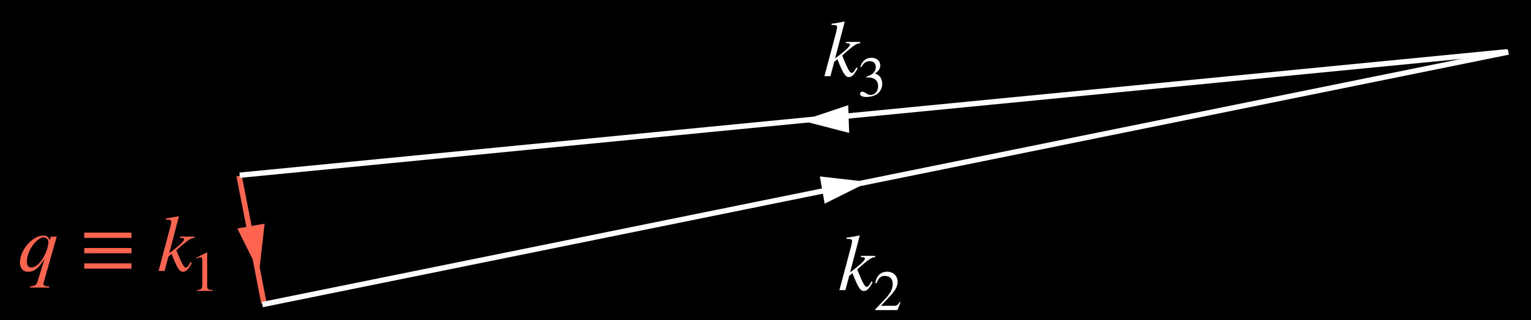

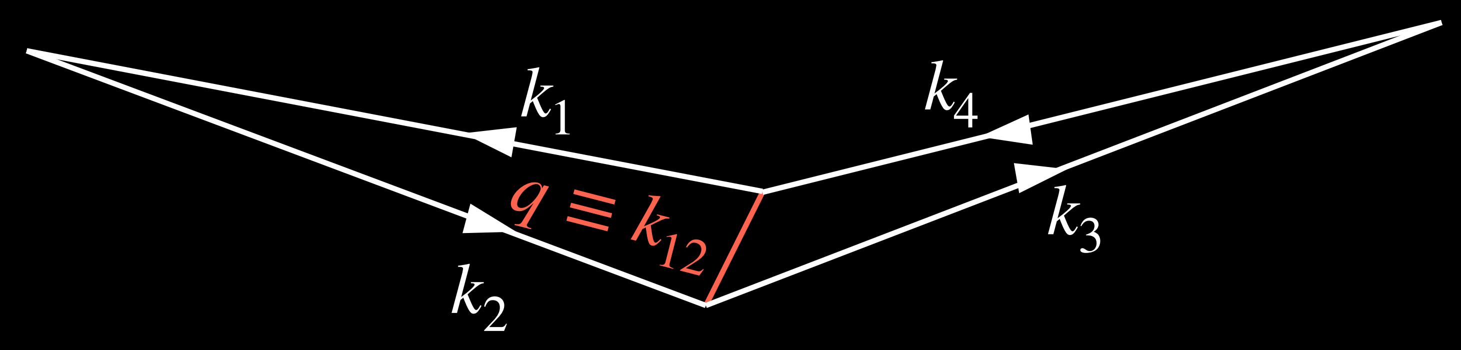

Imprints of the cosmological collider on LSS correlators

Non-perturbative bispectrum model

- Squeezed bispectrum ($q \ll k$) is **modulation** of small scale power spectrum by a long-wavelength gravitational potential $\Phi_L(\mathbf{x})$
  - Contributions from PNG are associated with $\partial P_m(k)/\partial \Phi_L(q)$
  - Contributions from gravitational non-Gaussianity modeled using response approach

$$\lim_{q \ll k_{\rm NL}, k} B_m(q, k, k') = \frac{3\Omega_{m0} H_0^2}{2q^2 T(q) D_{\rm md}(z_q)} P_m(q) \left(\frac{\partial P_m(k)}{\partial \Phi_L(q)} \right) + \bar{a}_0 P_m(q) P_m(k) + \bar{a}_2 \frac{q^2}{k^2} P_m(q) P_m(k)$$

(e.g., Valageas 13; Chiang+17; Esposito+19; Biagetti+22)

- How to estimate this *non-linear* potential derivative?

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Run separate universe simulations

Separate universe and the cosmological collider

- Consider small-scale modes $\delta_m(\boldsymbol{k}_1)$ and $\delta_m(\boldsymbol{k}_2)$ with **fixed amplitude,** but $\boldsymbol{k}_1 < \boldsymbol{k}_2$
- Add in **background** (~const.) potential fluctuation $\Phi_L(\boldsymbol{q})$
- PNG induces mode coupling

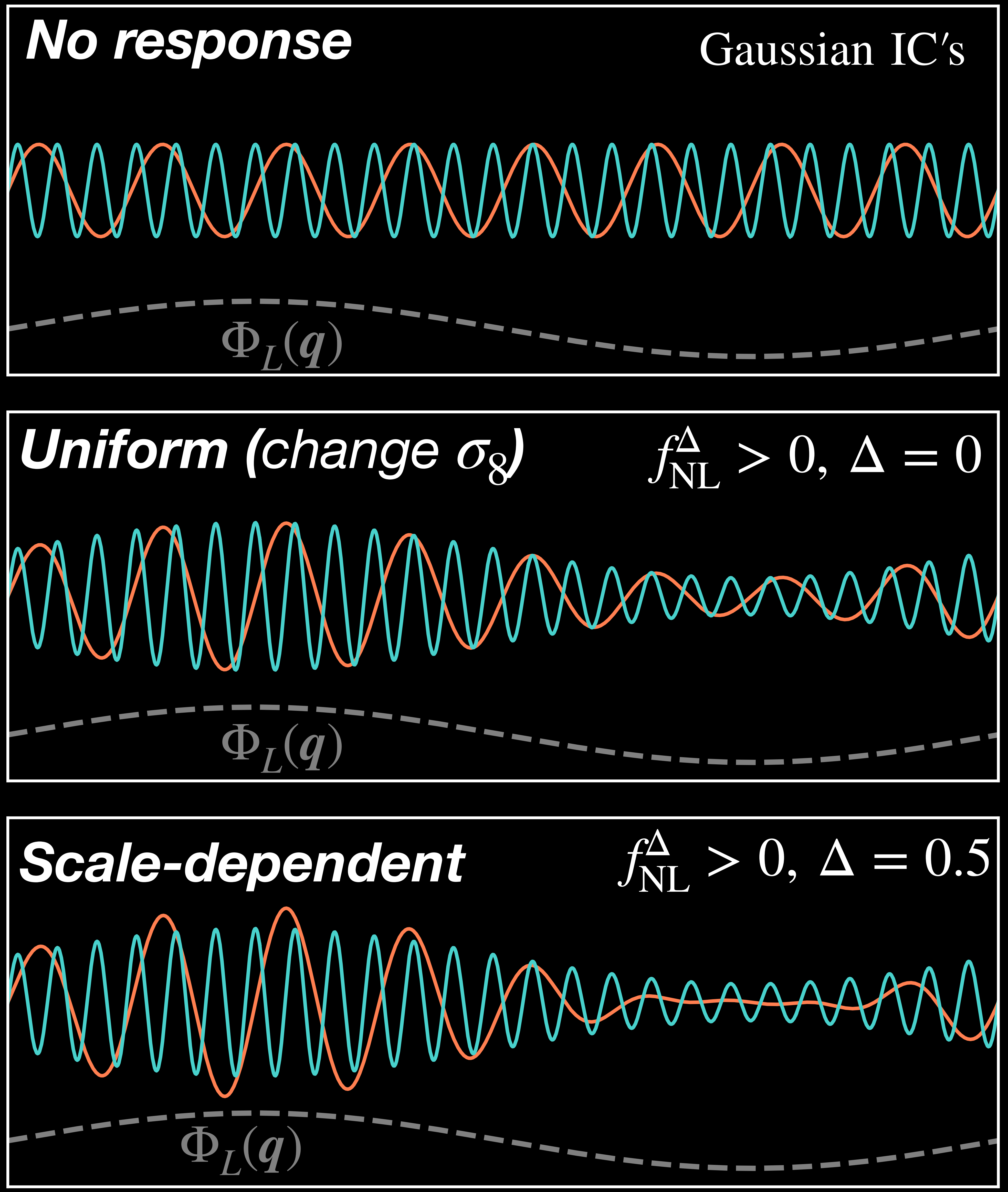
$$\delta_m(\boldsymbol{k}\,|\,\Phi_L(\boldsymbol{q})) = \left(1 + 2f_{\rm NL}^{\Delta}\left(\frac{q}{k}\right)^{\Delta}\Phi_L(\boldsymbol{q})\right)\delta_m(\boldsymbol{k})$$

Schmidt, Jeong, Desjacques, 2012

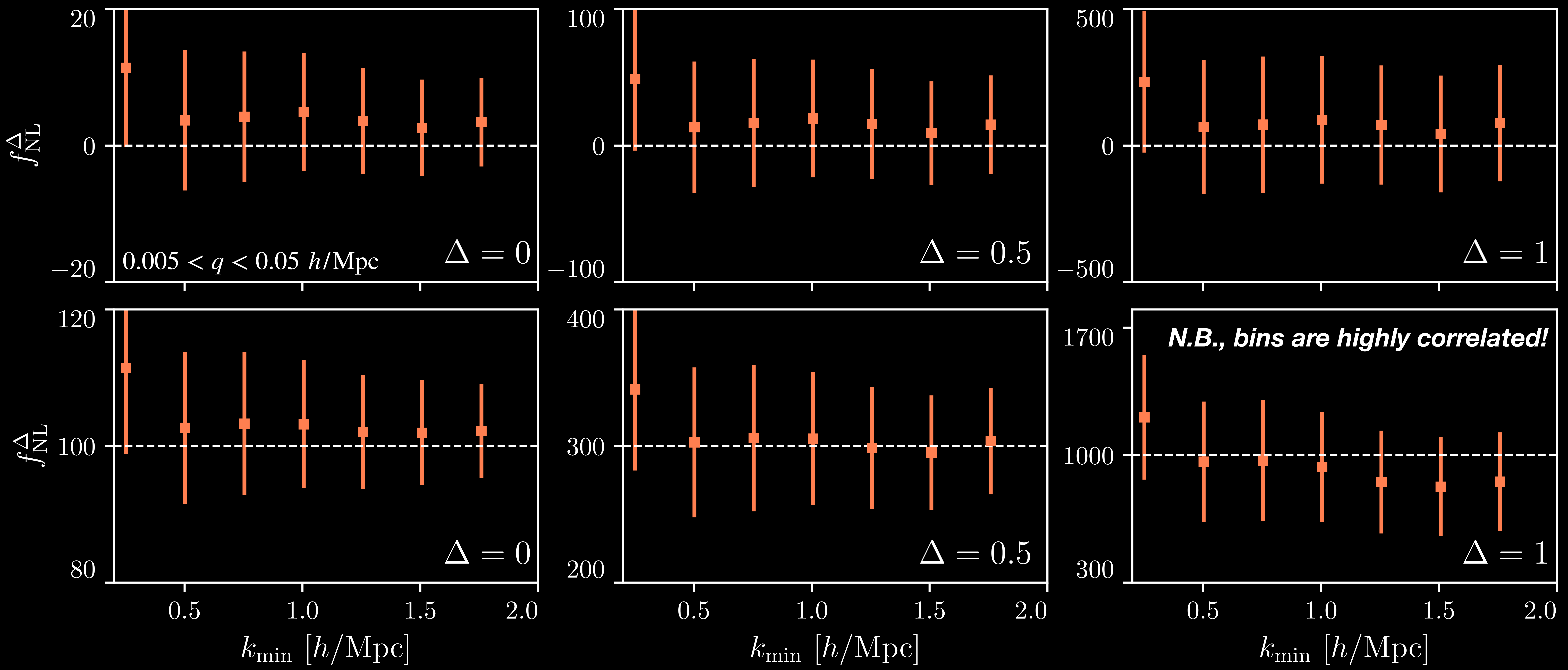
- Can compute $\partial P(k)/\partial\Phi_L(q)$ from sims with modified $P_m^{\rm lin.}$

$$P_m^{\rm lin.}(k\,|\,\epsilon,\Delta) \equiv \left(1 + 2\epsilon k^{-\Delta}\right)P_m^{\rm lin.}(k)\,.$$

$$\frac{\partial P_m(k)}{\partial\,\Phi_L(\mathbf{q})} = 2f_{\rm NL}^{\Delta}q^{\Delta}\frac{\partial P_m(k\,|\,\epsilon,\Delta)}{\partial\epsilon}\bigg|_{\epsilon=0}\,.$$



Constraints from squeezed bispectrum at $z = 0$



Unbiased constraints on f_{NL}^{Δ} using ***non-linear*** squeezed matter bispectrum!

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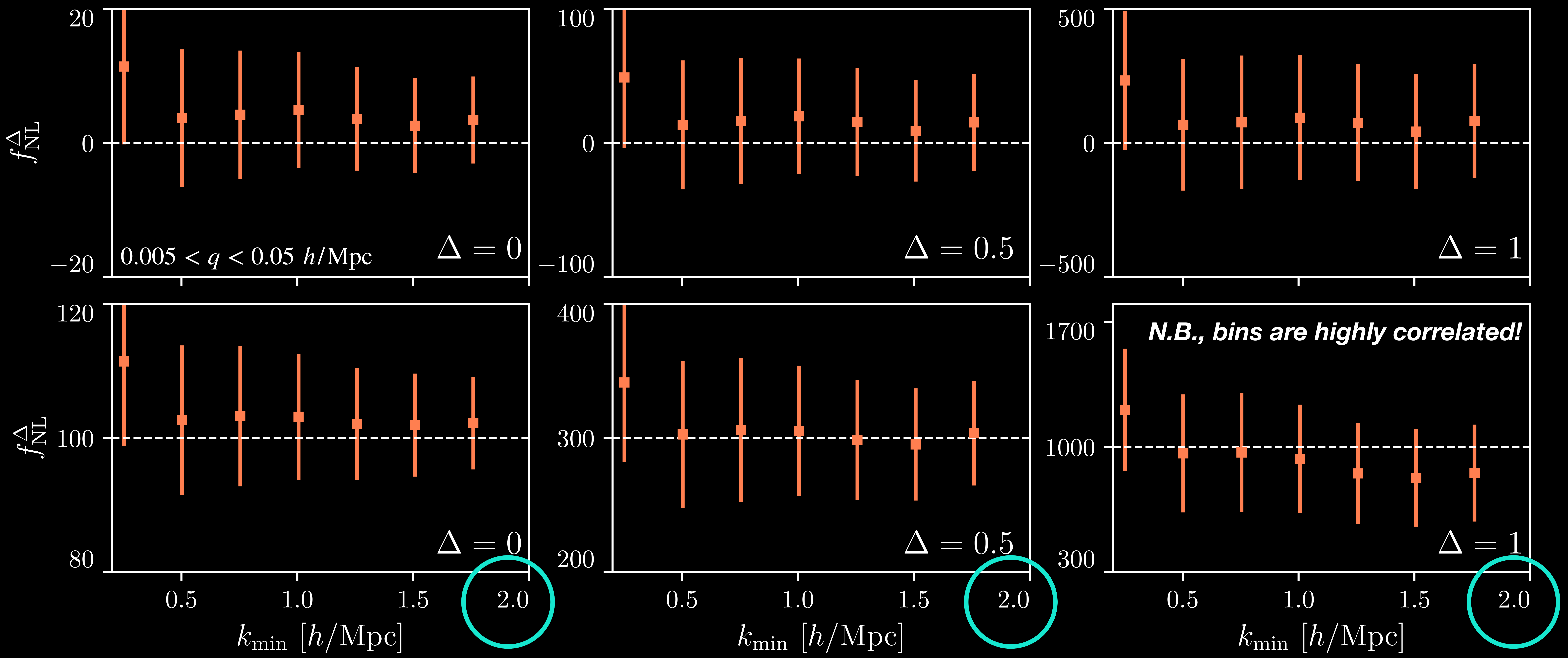


Unbiased constraints on $f_{\rm NL}^{\Delta}$ using ***non-linear*** squeezed matter bispectrum!

Scale-dependent halo bias

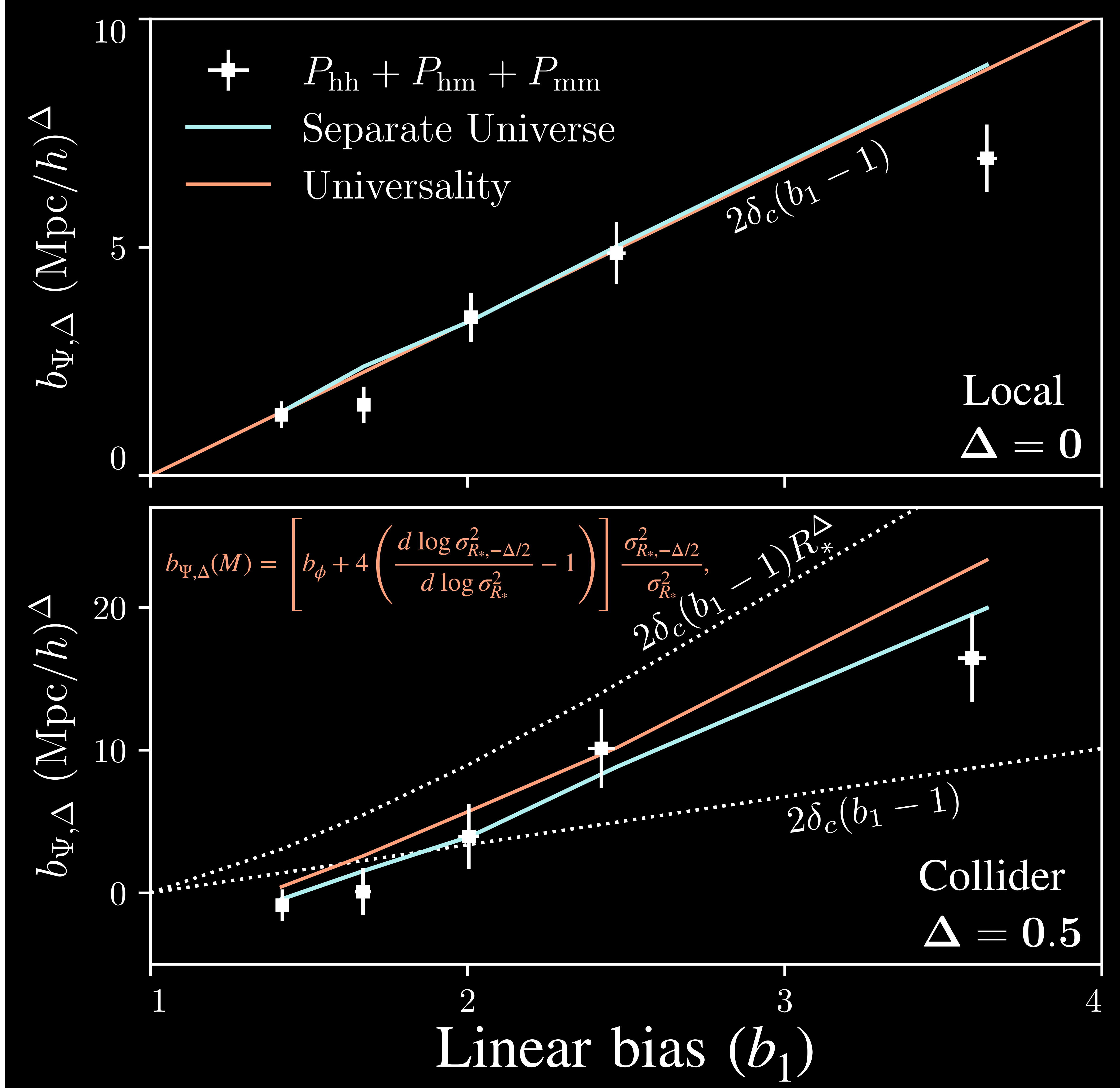
- Squeezed bispectrum leads to **scale-dependent bias** *(Dalal+07, Slosar+08, Desjacques+08)*

f_{NL}^{Δ} degenerate with non-Gaussian bias

$$P_{hm}(q) = \left[b_1 + \frac{3\Omega_{m0}H_0^2}{2D_{\rm md}(z)} \frac{b_{\Psi,\Delta} f_{\rm NL}^{\Delta}}{q^{2-\Delta}} \right] P_m(q)$$

Power depends on Δ

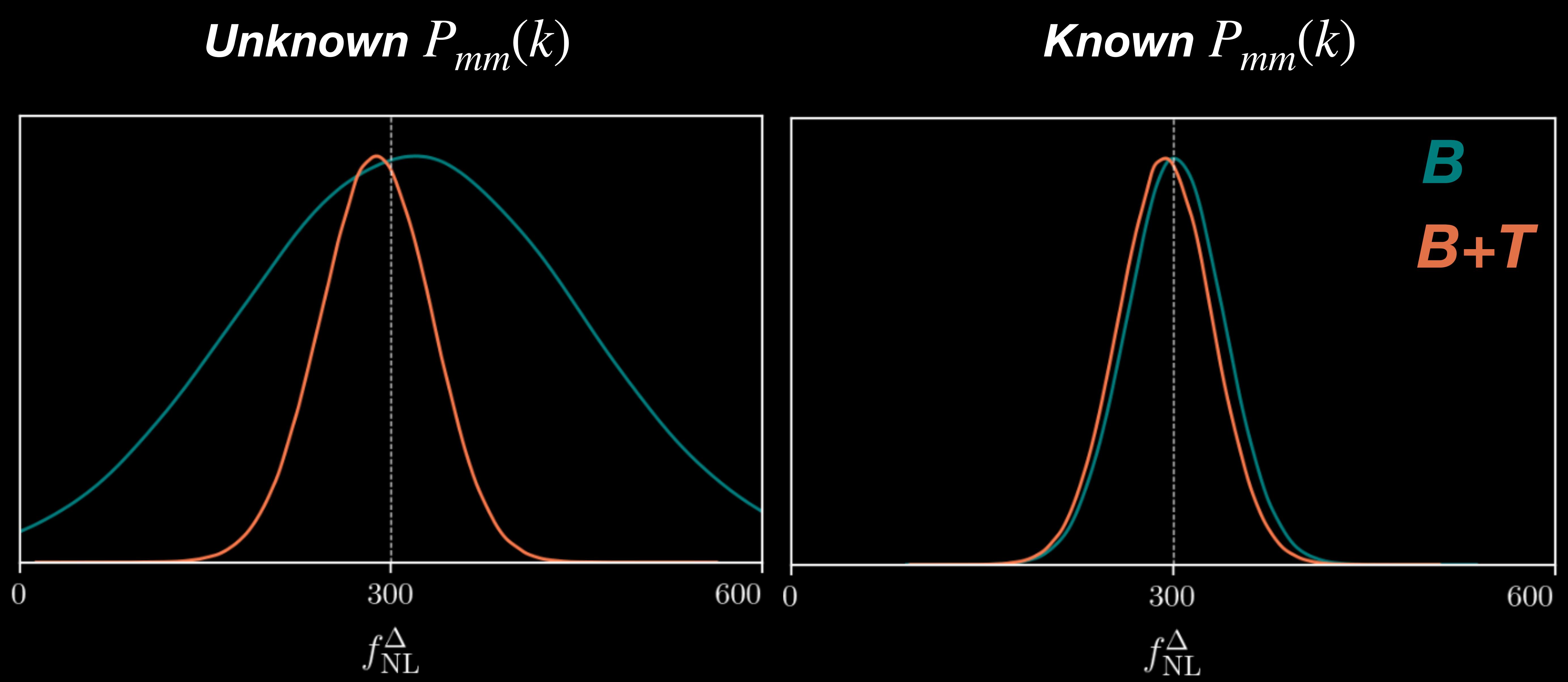
- Fit for $b_{\Psi,\Delta}$ at fixed $f_{\rm NL}^{\Delta}$ and Δ
  - Test predictions for galaxy biasing in non-local PNG *e.g., Shandera+2010, Schmidt & Kamionkowski, 2010 Schmidt+2012*
  - **Galaxies will be more challenging…**

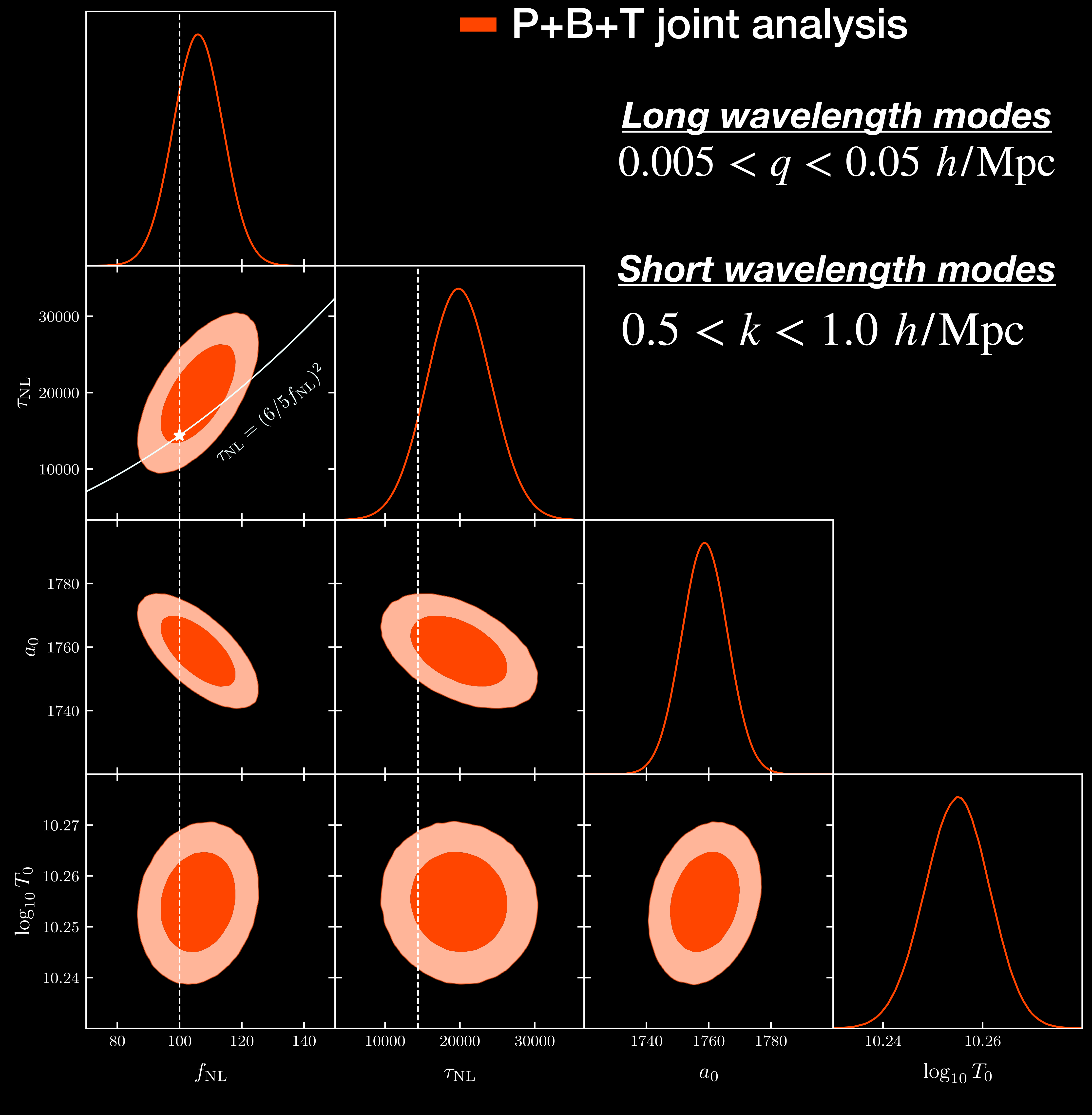


Fits to **halo** catalogues from sims agree with separate universe and universality.

Collapsed trispectrum

- What can we learn from the trispectrum?
  - Higher-order PNG
  - Sample variance cancellation

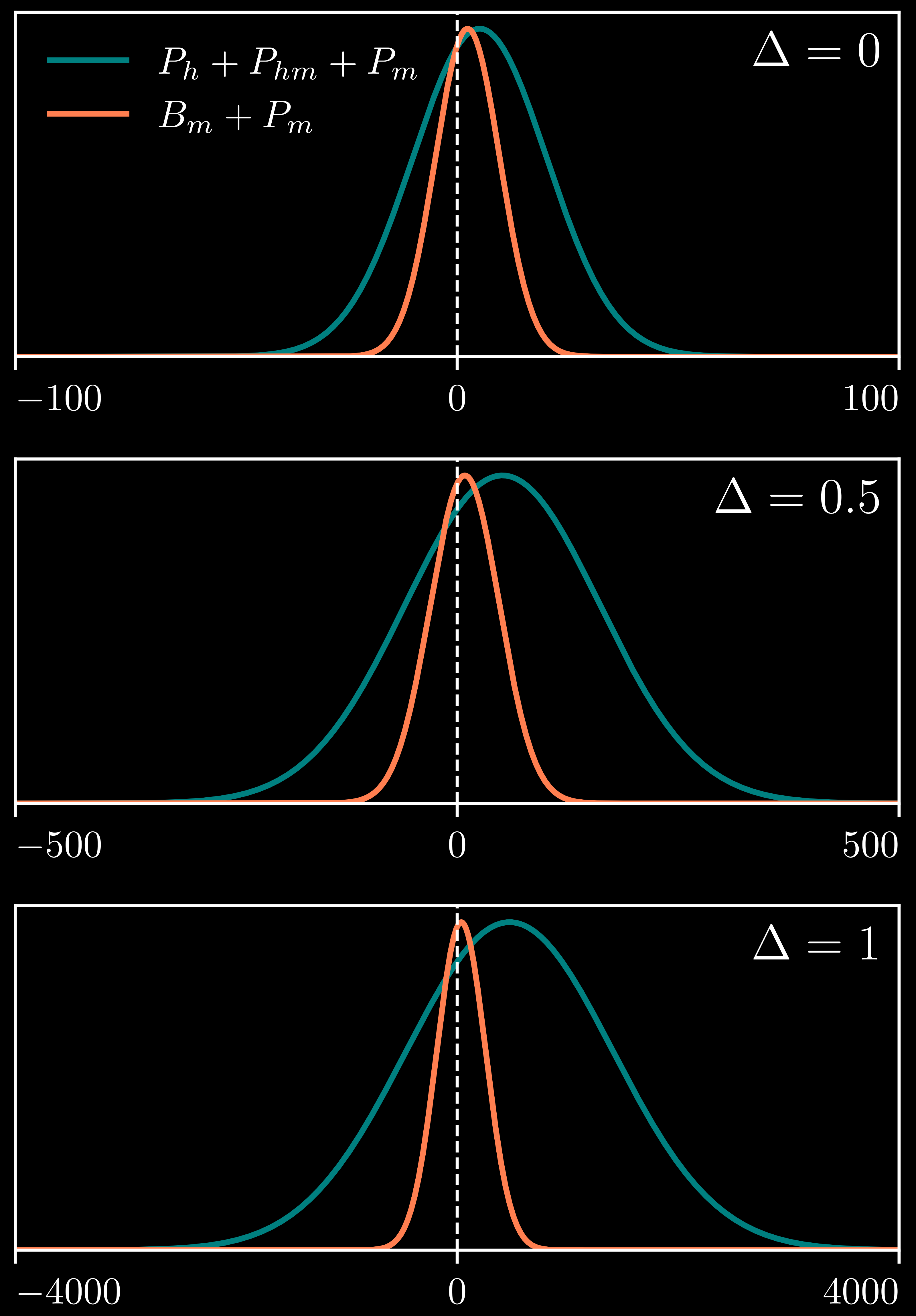




Constraints on $(f_{\rm NL}, \tau_{\rm NL})$ from squeezed bispectrum and collapsed trispectrum using Quijote.

Conclusions

- Particles with $m \sim H$ during inflation leave distinct imprint on LSS correlators ("cosmo. collider")
  - Methods to run **N-body simulations with cosmo. collider** squeezed bispectrum (for $0 < m/H \leq 3/2$)
  - **Non-perturbative** models for squeezed B_m and collapsed T_m
    - Validated up to $k_{\max} = 2\ h/\mathrm{Mpc}$ (useful for deep surveys?)
  - Simulations can be used to test analysis methods based on **scale-dependent bias** *(e.g., Green, Guo, Han, Wallisch, 2023)*
    - Test separate universe halo bias for non-local PNG
- More to do
    - E.g., **oscillatory bispectra**, B_g and T_g, **multi-tracer/$\mathbf{b_{\Psi,\Delta}}$**, CMBxLSS, optimal estimators/field-level inference/ alternative summary statistics, **can we constrain Δ?**



*An unfair, unfinished, but not uninteresting comparison

Backup

Initial conditions for N-body simulations

- How to generate IC's with following **squeezed** bispectrum (🌲-level)

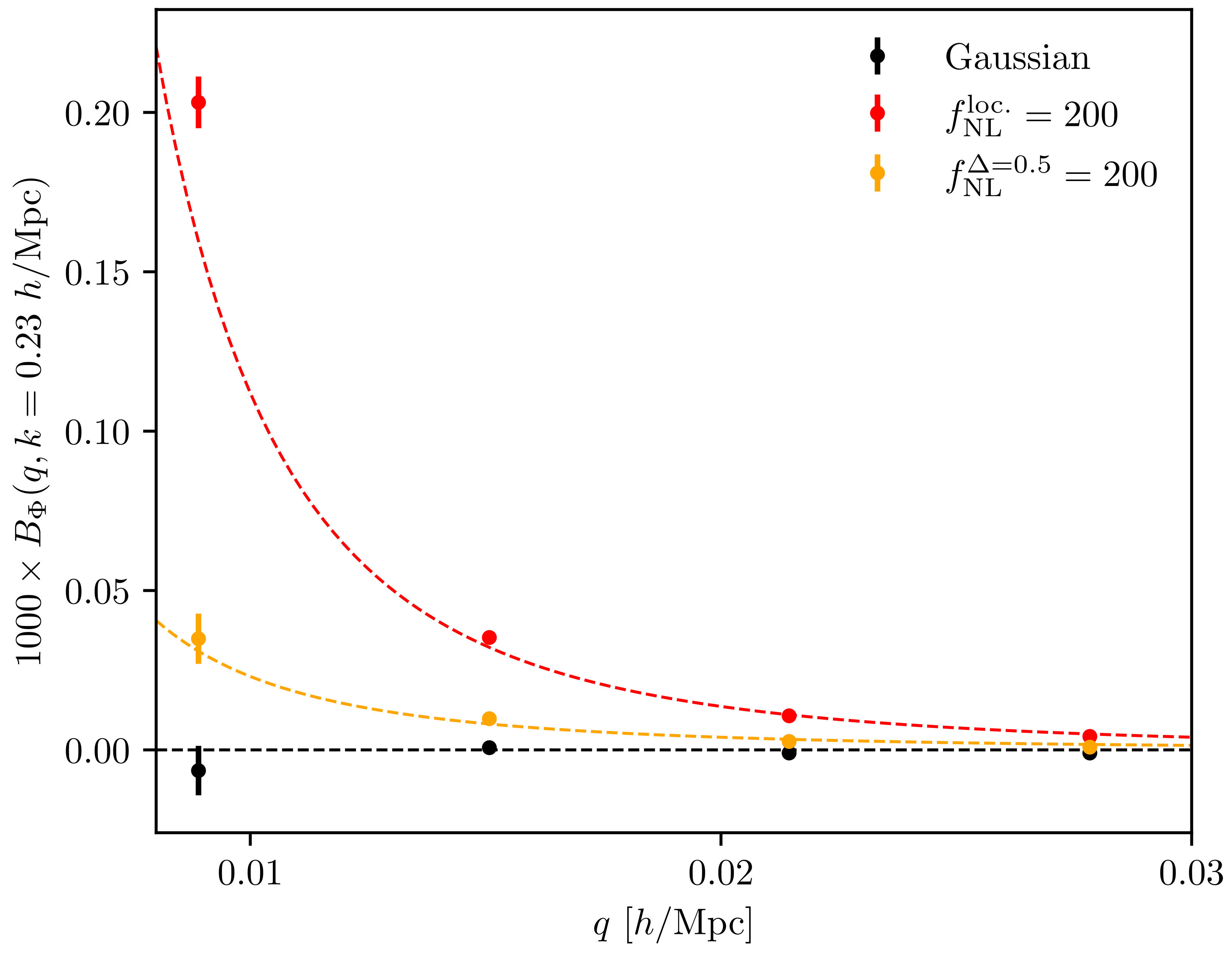
$$\lim_{k_1 \ll k_2 \approx k_3} B_\Phi(k_1, k_2, k_3) = 4 f_{\rm NL}^{\Delta} \left(\frac{k_1}{k_2}\right)^{\Delta} P_\Phi(k_1) P_\Phi(k_2),$$

- Construct Φ quadratically from ϕ_G (e.g., Scoccimarro, Hui, Manera, 2012)

$$\Phi(\mathbf{k}) = \phi_G(\mathbf{k}) + f_{\rm NL}^{\Delta} \left[\Psi_\Delta(\mathbf{k}) - \langle \Psi_\Delta(\mathbf{k}) \rangle\right],$$

$$\Psi_\Delta(\mathbf{k}) = \int_{\mathbf{k}_1, \mathbf{k}_2} (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}_{12})\, K_\Delta(\mathbf{k}_1, \mathbf{k}_2)\, \phi_G(\mathbf{k}_1)\, \phi_G(\mathbf{k}_2),$$

- Choose $K_\Delta(\mathbf{k}_1, \mathbf{k}_2)$ that gives target bispectrum and doesn't "break" P_Φ with loop corrections
  - No fun if I write it out for you!
  - Convolution can be computed with FFTs!



Check IC's with 🌲-level bispectrum

Bispectrum Estimator Validation

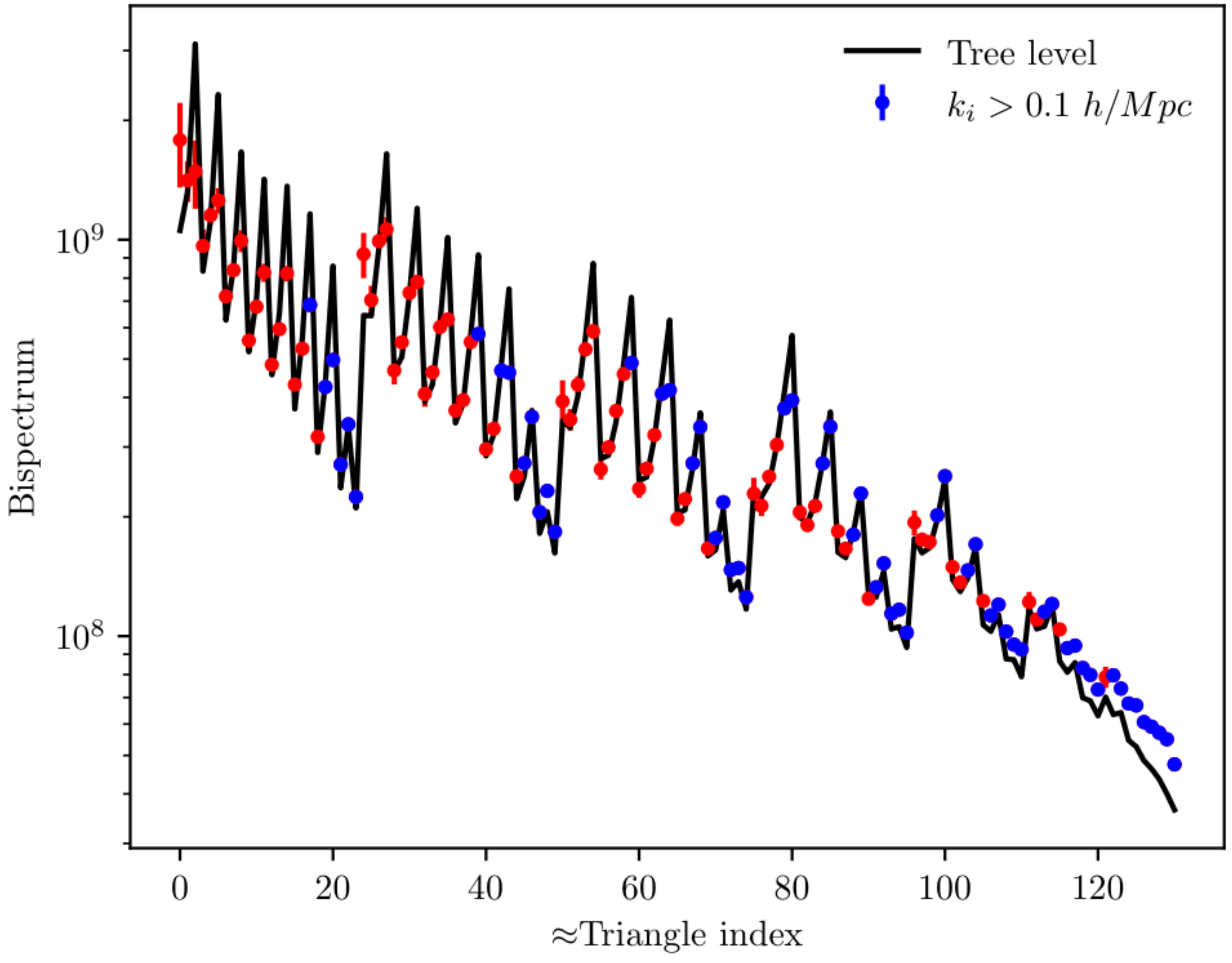


Figure 1: Validation on measurements from Quijote

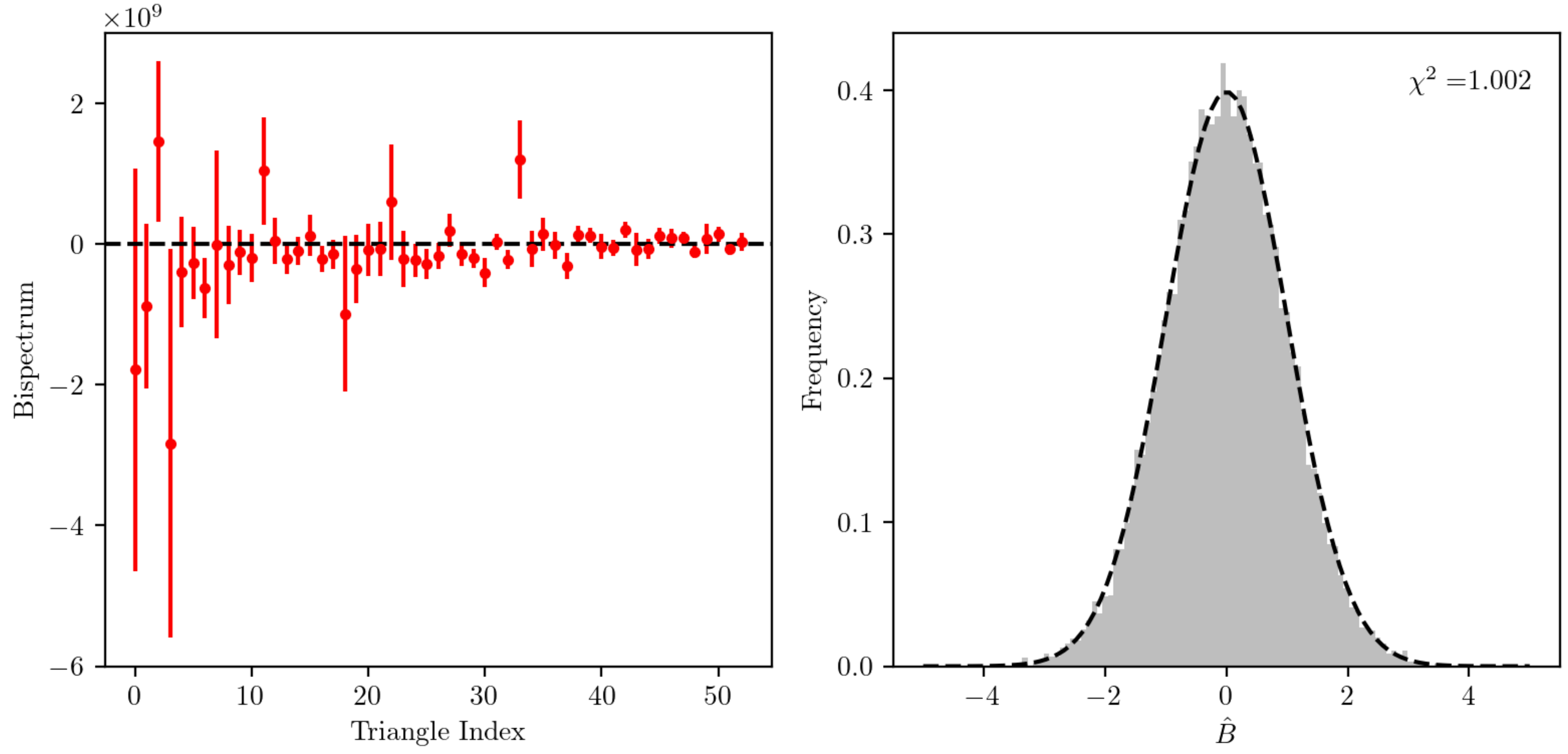


Figure 2: Validation on Gaussian mocks

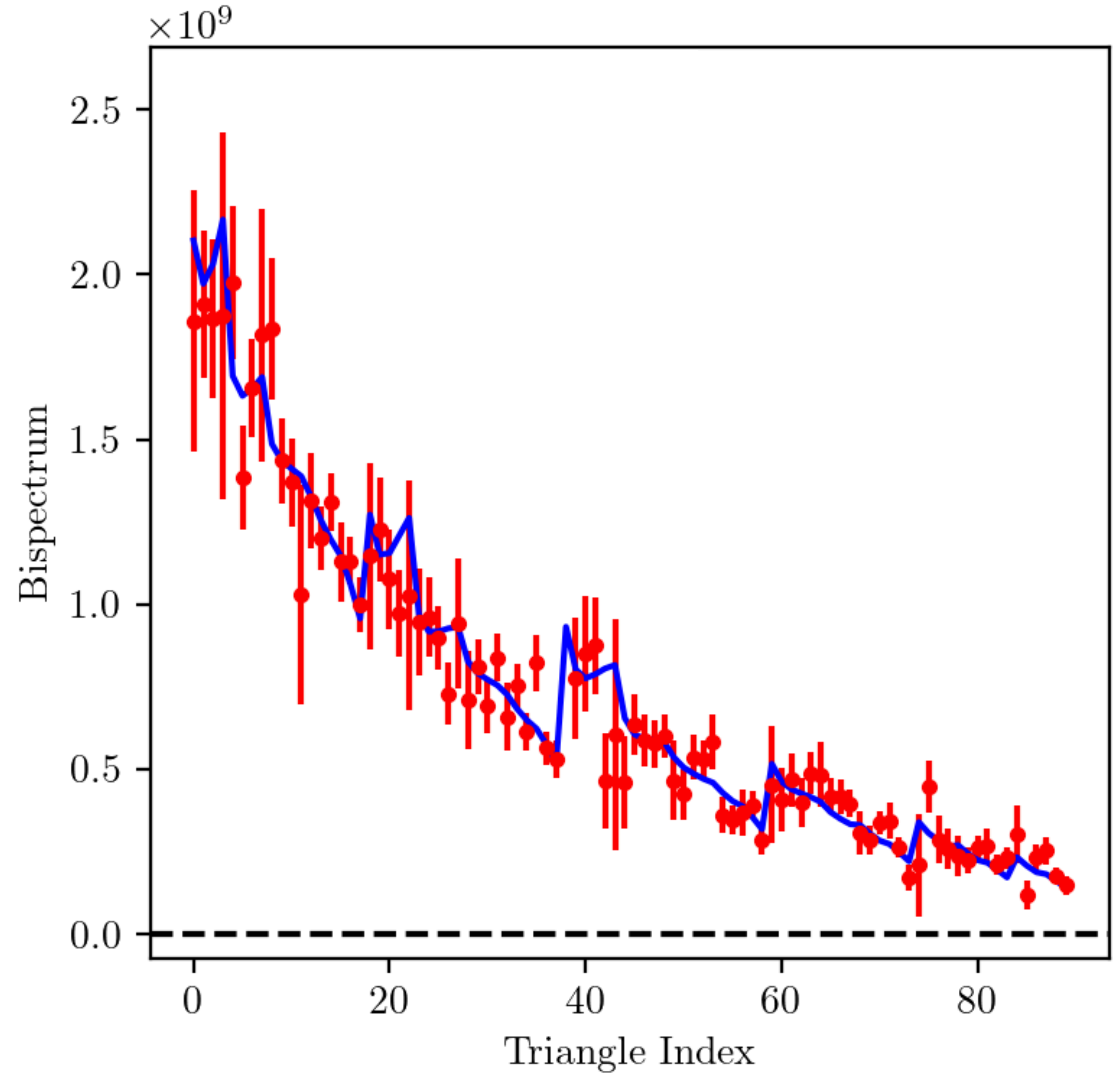


Figure 3: Validation on mocks with $f_{\rm NL}^{\rm loc.}$

Trispectrum Estimator

- Trispectrum estimator in terms of external legs $k_a, \ldots, k_d$ and diagonals k_{ab}, k_{bc} is **not separable**
  - Use estimator integrated over q_{23} (see Appendix A of 2306.11782)

$$\left\langle \hat{T}_{\rm tot}(k_a, k_b, k_c, k_d, k_E) \right\rangle = \frac{1}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) \int_{\boldsymbol{k}_1,\ldots,\boldsymbol{k}_4} \Big\{ W_a(k_1) W_b(k_2) W_c(k_3) W_d(k_4) \left\langle \delta_{\boldsymbol{k}_1} \delta_{\boldsymbol{k}_2} \delta_{\boldsymbol{k}_3} \delta_{\boldsymbol{k}_4} \right\rangle (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}_{12} - \boldsymbol{Q}) (2\pi)^3 \delta_D^{(3)}(\boldsymbol{k}_{34} + \boldsymbol{Q}) \Big\},$$

- Can be efficiently evaluated by computing FFT's of product fields

$$D_{ij}(\boldsymbol{Q}) \equiv \int d^3x \, e^{-i\boldsymbol{Q}\cdot\boldsymbol{x}} \delta_{W_i}(\boldsymbol{x}) \delta_{W_j}(\boldsymbol{x}).$$

$$\boxed{\hat{T}_{\rm tot}(k_a, k_b, k_c, k_d, k_E) = \frac{1}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) D_{ab}(\boldsymbol{Q}) D^*_{cd}(\boldsymbol{Q}),}$$

Biased estimator because it includes disconnected terms!!!

Trispectrum Estimator (continued)

- Estimate and subtract disconnected terms
  - Simple estimator: $\delta^4 - 3\langle\delta^2\rangle^2$
  - More optimal estimator: $\delta^4 - 6\,\delta^2\langle\delta^2\rangle^2 + 3\langle\delta^2\rangle^2$ (See Appendix F. of **Shen, Schaan, Ferraro, 24**)
- Can be efficiently implemented using FFT's

Depends on fiducial P(k)

$$F^P_{ij}(\boldsymbol{x}) \equiv \int_{\boldsymbol{k}} W_i(k)W_j(k)P(k)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

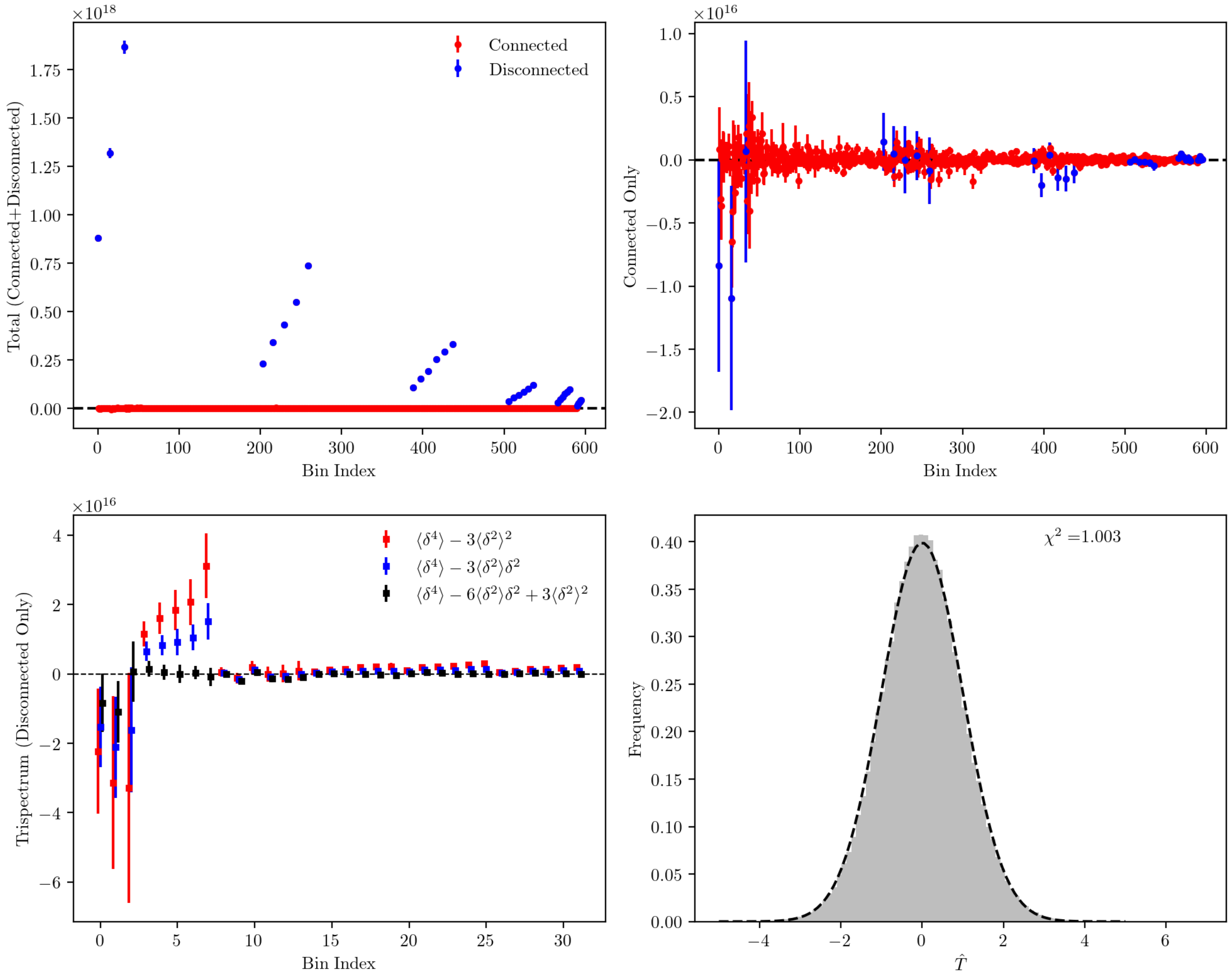
Depends on realization of δ

$$F^\delta_{ij}(\boldsymbol{x}) \equiv \int_{\boldsymbol{k}} W_i(k)W_j(k)|\delta_{\boldsymbol{k}}|^2 e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\hat{T}^{\langle\delta^2\rangle^2}_{\rm disc} = \frac{V}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) \int d^3x\, e^{-i\boldsymbol{Q}\cdot\boldsymbol{x}} \left[F^P_{ac}(\boldsymbol{x})F^P_{bd}(\boldsymbol{x}) + F^P_{ad}(\boldsymbol{x})F^P_{bc}(\boldsymbol{x})\right]$$

$$\hat{T}^{\delta^2\langle\delta^2\rangle}_{\rm disc} = \frac{V}{N_{a,b,c,d,E}} \int_{\boldsymbol{Q}} W_E(Q) \int d^3x\, e^{-i\boldsymbol{Q}\cdot\boldsymbol{x}} [F^P_{ac}(\boldsymbol{x})F^\delta_{bd}(\boldsymbol{x}) + F^\delta_{ac}(\boldsymbol{x})F^P_{bd}(\boldsymbol{x}) + F^P_{ad}(\boldsymbol{x})F^\delta_{bc}(\boldsymbol{x}) + F^\delta_{ad}(\boldsymbol{x})F^P_{bc}(\boldsymbol{x})].$$

Trispectrum Estimator Validation



Potential derivatives

- Compute logarithmic derivative from separate universe sims

$$P_m^{\mathrm{lin.}}(k\,|\,\epsilon,\Delta) \equiv \left(1+2\epsilon k^{-\Delta}\right) P_m^{\mathrm{lin.}}(k)\,.$$

$$\frac{\partial P_m(k)}{\partial\,\Phi_L(\mathbf{q})} = 2f_{\mathrm{NL}}^{\Delta} q^{\Delta} \frac{\partial P_m(k\,|\,\epsilon,\Delta)}{\partial\epsilon}\bigg|_{\epsilon=0}\,.$$

Get from finite differencing SU sims with $\pm\epsilon$

- Strong dependence on Δ

- Same sims used to estimate non-Gaussian bias

$$b_{\Psi,\Delta}(M,z) = \frac{\partial \log \bar{n}_h(M,z)}{\partial\epsilon}\bigg|_{\epsilon=0}$$