Non-Gaussian Statistics *for* Non-Gaussian Physics

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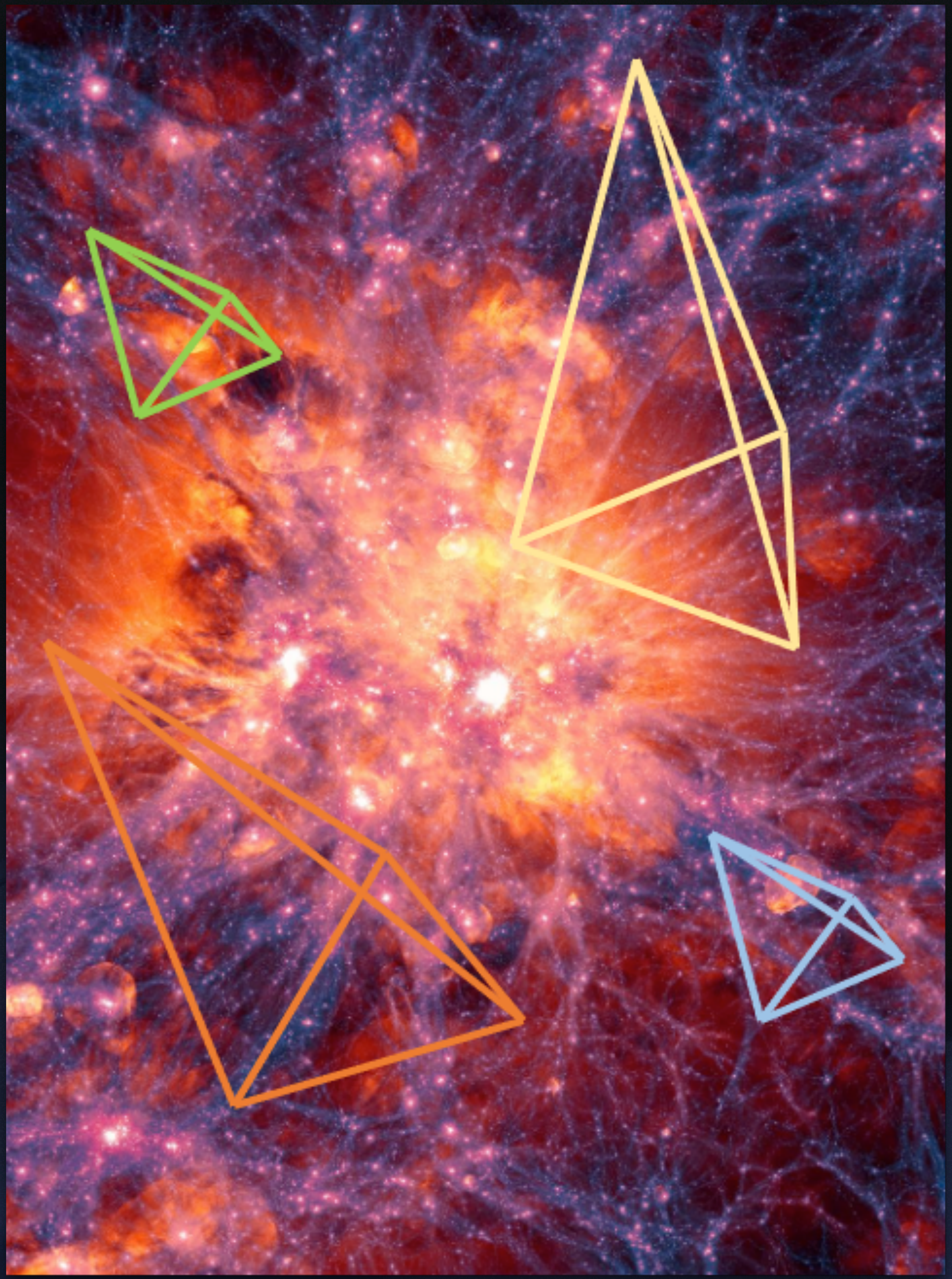


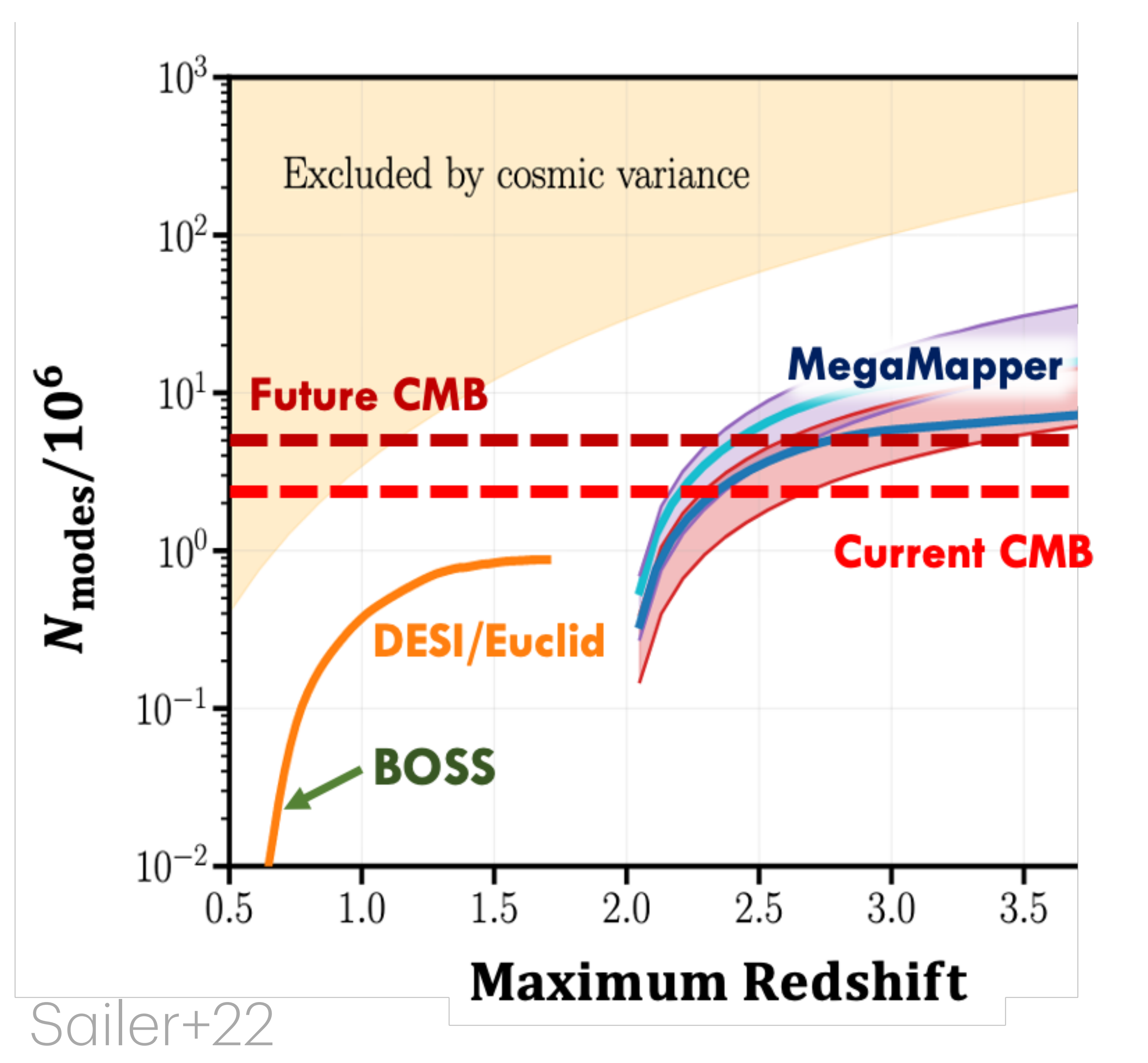
Berkeley Lab, 2024

The Plan

- Future spectroscopic surveys will be a great tool for studying **primordial physics**!

But

- **What** types of physics should we look for?
- **Which** statistics can we find it in?
- **How** can we analyze it robustly?



Let's take a (biased & incomplete) tour of primordial physics!

Sailer+22, Schlegel, White, Chen, Ferraro, etc.

1. Power Spectrum Science: $P_{\zeta\zeta}(k)$

We'll measure **two-point** statistics exceptionally well

This tells us about the **curvature** power spectrum:

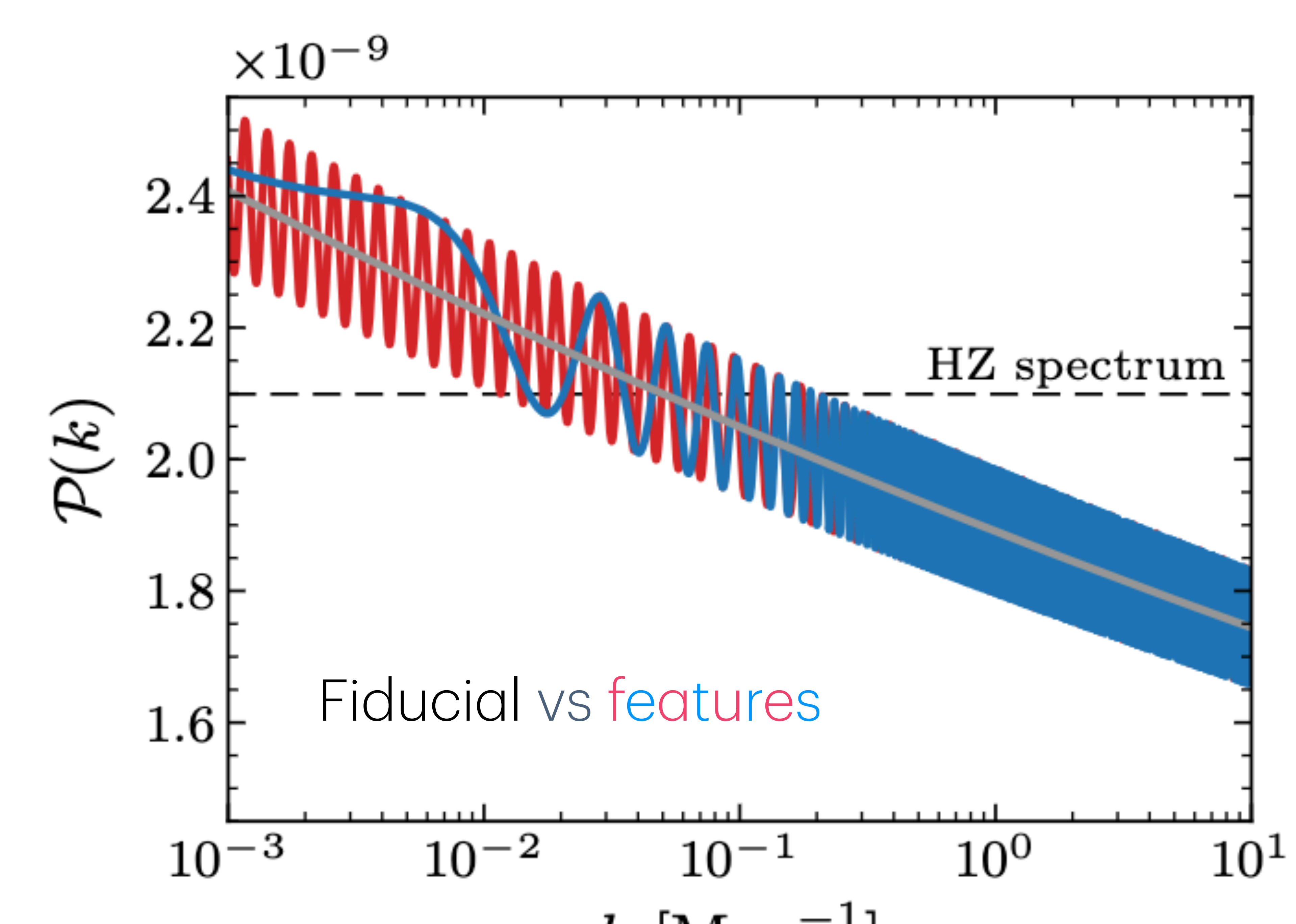
- $A_s, n_s \Rightarrow$ boring inflation paramaters
- $\alpha_s = dn_s/d\log k \Rightarrow$ spectral **running**

$$\alpha_s \sim (n_s - 1)^2 \sim 10^{-3}\,\text{baseline -- hard!!}$$

- Primordial **features**

[Axions! String theory! Quantum Gravity!]

- **Acoustic oscillation** signatures, e.g. N_{eff}



Beutler+19

Sailer+22, Munoz+17, Bahr-Kalus+23, Beutler+19, Chen, Biagetti, Wallisch, Green, etc.

1. Power Spectrum Science: $P_{\zeta\zeta}(k)$

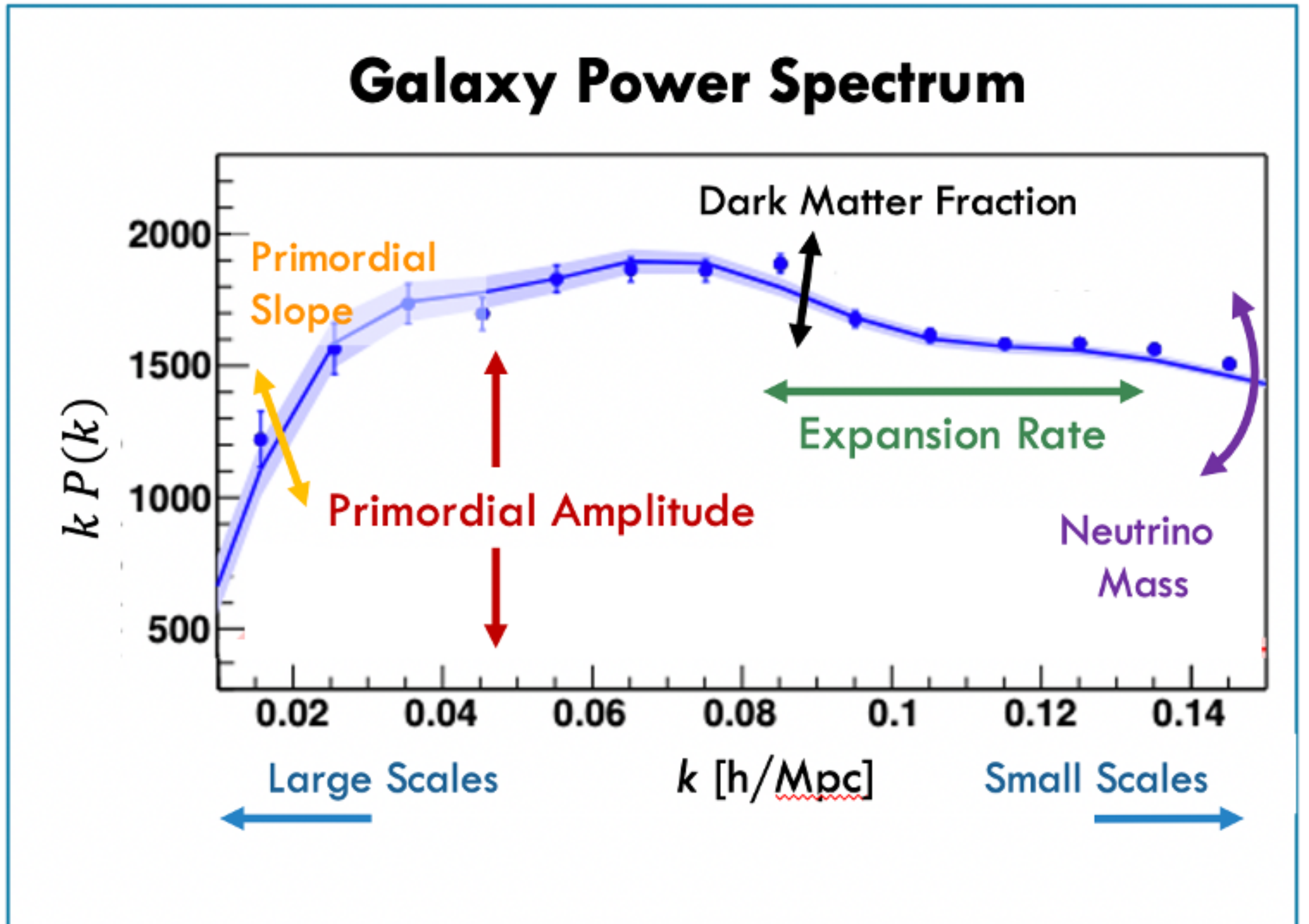
How do we extract this information?

Observables:

- Galaxy power spectrum / two-point correlator
- *Cross-correlations* with lensing & line-intensity mapping
- Some information leaks into **higher-order** statistics!

Methods:

- Perturbation theory (EFTofLSS)
- Simulations



Beutler+14 (adapted)

Many of these methods are ready for Spec-S5 already!!

Philcox, Ivanov, Cabass, Zaldarriaga, Chen, White, Ferraro, Vlah, McDonald, Senatore, Castorina, Zhang, d'Amico, etc.

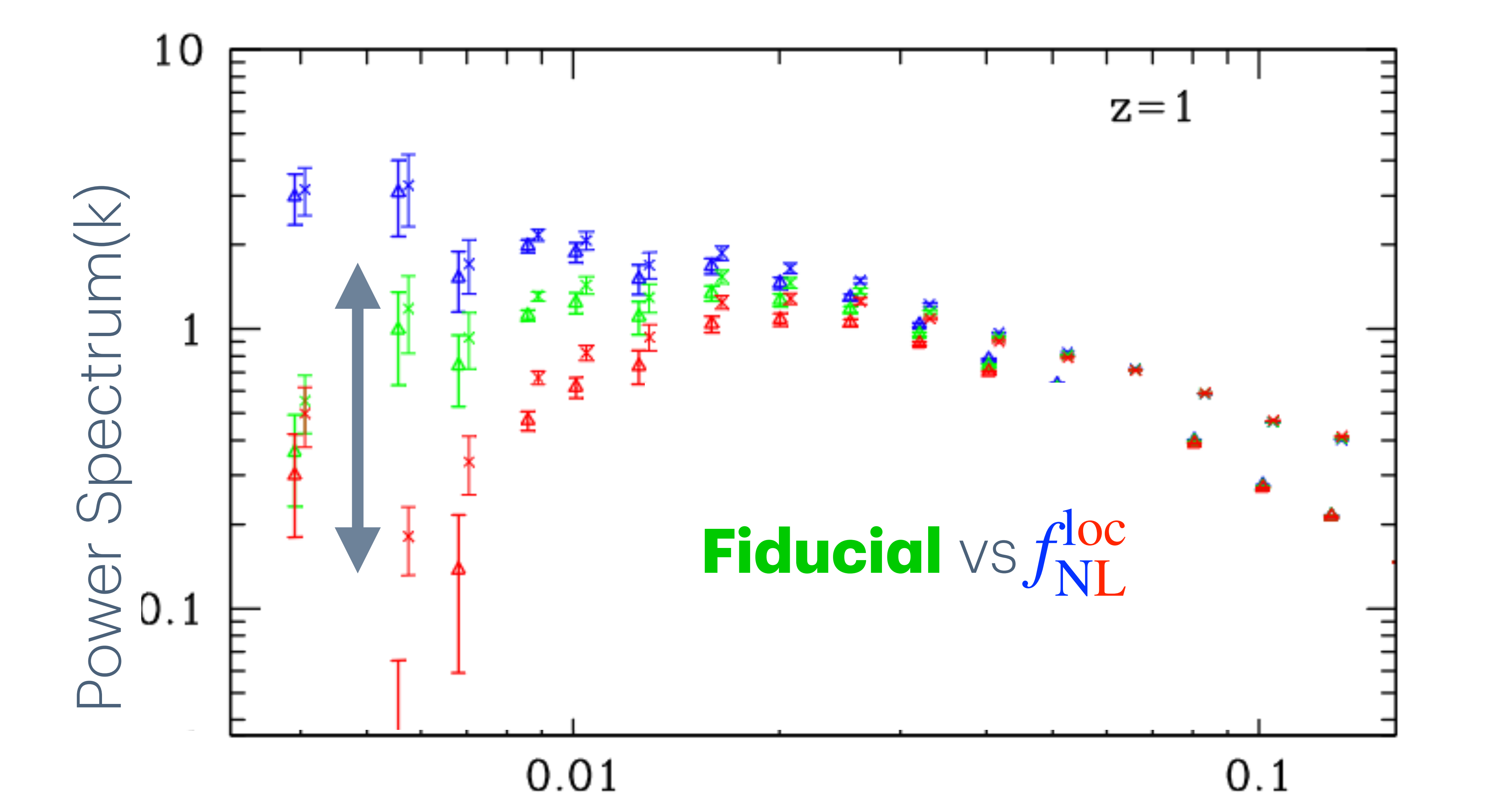
1. Power Spectrum Science: Scale-Dependent Bias

- The galaxy density can couple to the primordial potential

$$\delta_g \supset b_1\delta + b_\phi f_{\rm NL}^{\rm loc}\phi + \cdots$$

- This adds $f_{\rm NL}^{\rm loc}$ information in the **power spectrum** via an *ultra-squeezed* ***bispectrum***

$$P_{gg}(k) \supset 2b_1 b_\phi f_{\rm NL}^{\rm loc}\frac{P_L(k)}{k^2}$$



Desjacques+08

- We can probe **light particles** $(m \ll H)$ in inflation!

Dalal+07, Desjacques+08, Seljak, Barreira, Jeong, etc.

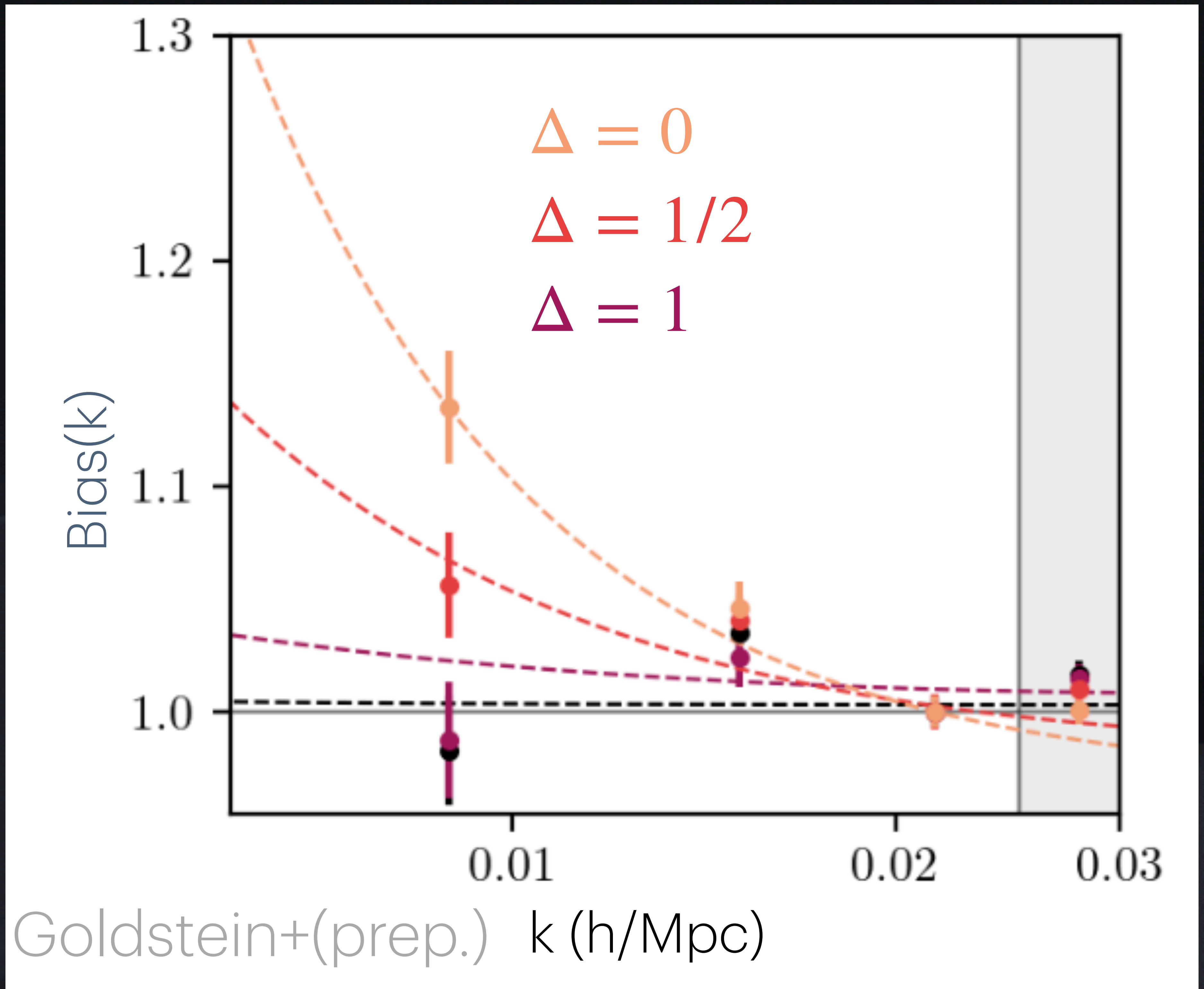
1. Power Spectrum Science: Scale-Dependent Bias

This is much more **generic!**

- Scale-dependent bias also probes **massive-ish** particles in inflation

$$P_{gg}(k) \supset 2b_1 b_\phi^\Delta f_{\rm NL}^\Delta \frac{P_L(k)}{k^{2-\Delta}}$$

for mass parameter $\Delta = 3/2 - \sqrt{9/4 - m^2/H^2}$



- Scale-dependent bias is a **squeezed bispectrum** detector!

See Sam Goldstein's talk!

Goldstein, Philcox+ (in prep.), Green+23, Creminelli, Zaldarriaga, Schmidt, Maldacena, etc.

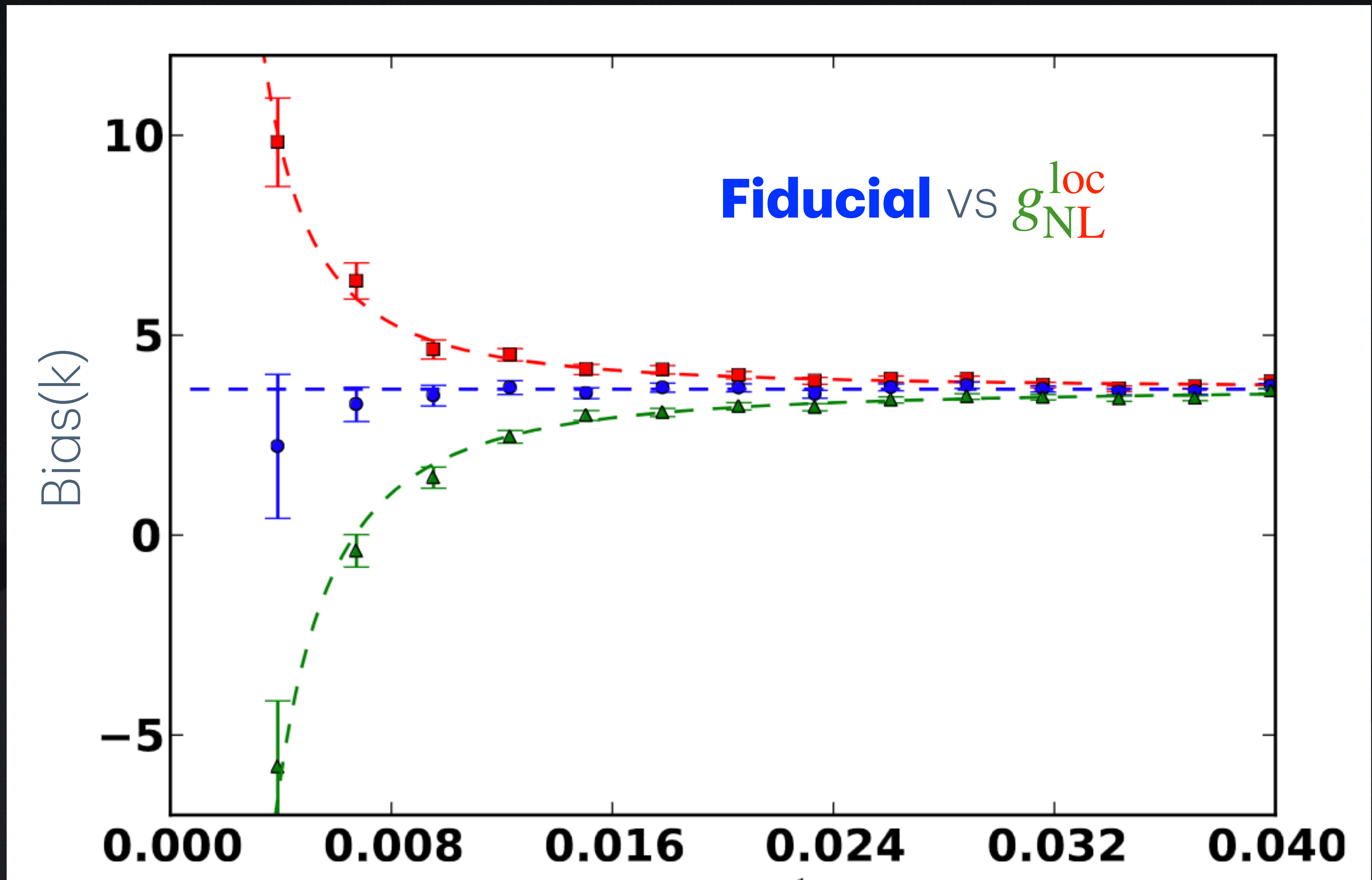
1. Power Spectrum Science: Scale-Dependent Bias

This is much more **generic!**

- Scale-dependent bias also probes **higher-order squeezed** non-Gaussianity

$$P_{gg}(k) \sim g_{\rm NL}^{\rm loc}\frac{P_L(k)}{k^2}, h_{\rm NL}^{\rm loc}\frac{P_L(k)}{k^2}, \ldots$$

Including **four-** and **five**-point functions



Ferraro+12

- Scale-dependent bias is a **local transformation** detector!

$$\zeta \to \zeta + f_{\rm NL}^{\rm loc}\zeta^2 + g_{\rm NL}^{\rm loc}\zeta^3 + h_{\rm NL}^{\rm loc}\zeta^4 + \cdots$$

Jeong+09, Smith+11, Ferraro+12, Coulton, Philcox+ (in prep.)

1. Power Spectrum Science: Scale-Dependent Bias

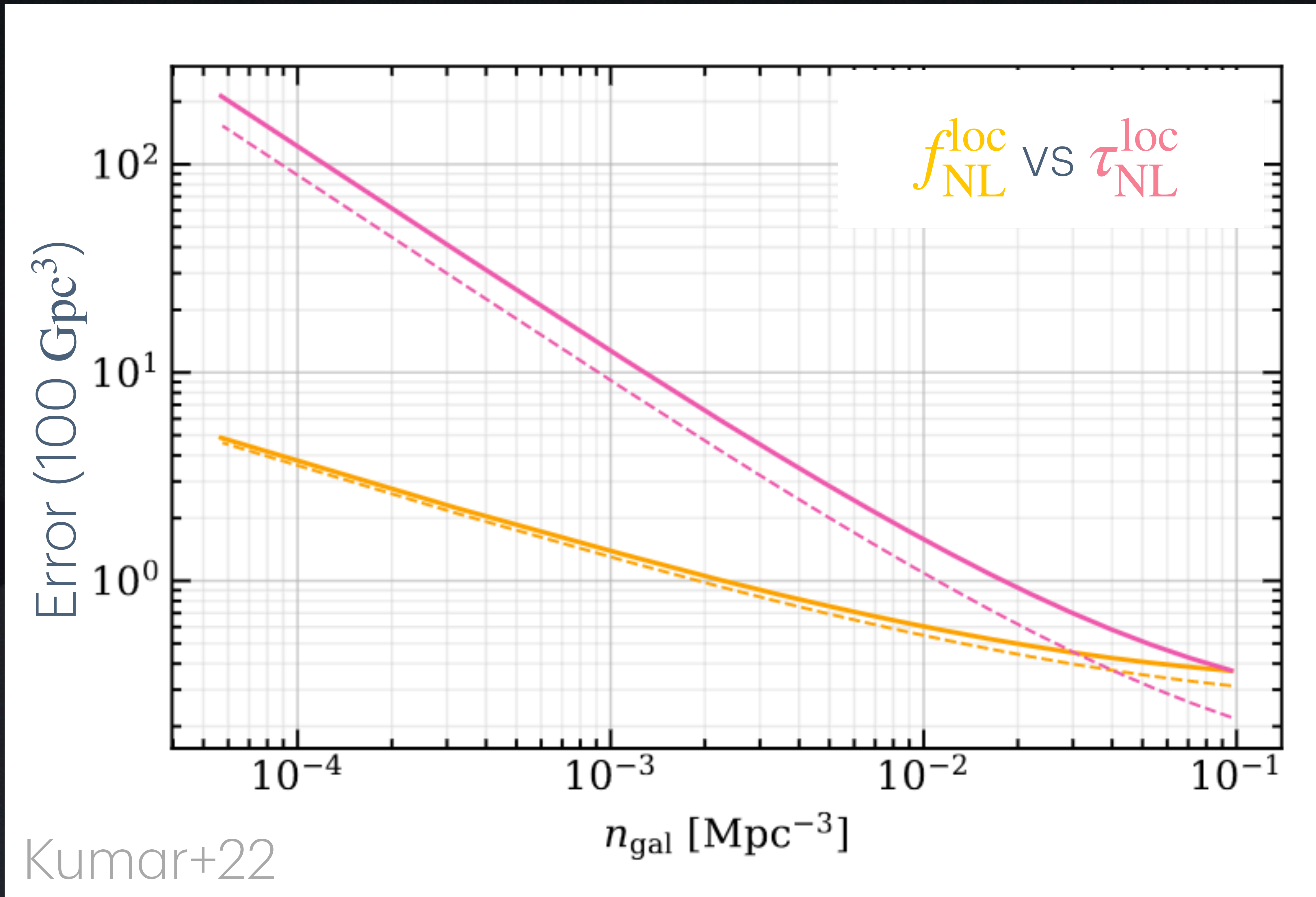
This is much more **generic!**

- We can also probe **collapsed** non-Gaussianity and **isocurvature** modes

$$P_{gg}(k) \sim b_\phi^2 \tau_{\rm NL}^{\rm loc} \frac{P_L(k)}{k^4},\, b_{\rm CIP}^2 P_{\rm CIP}(k), \ldots$$

- Galaxies probe **primordial fields** uncorrelated to ζ

- Scale-dependent bias is a **collapsed trispectrum** detector!

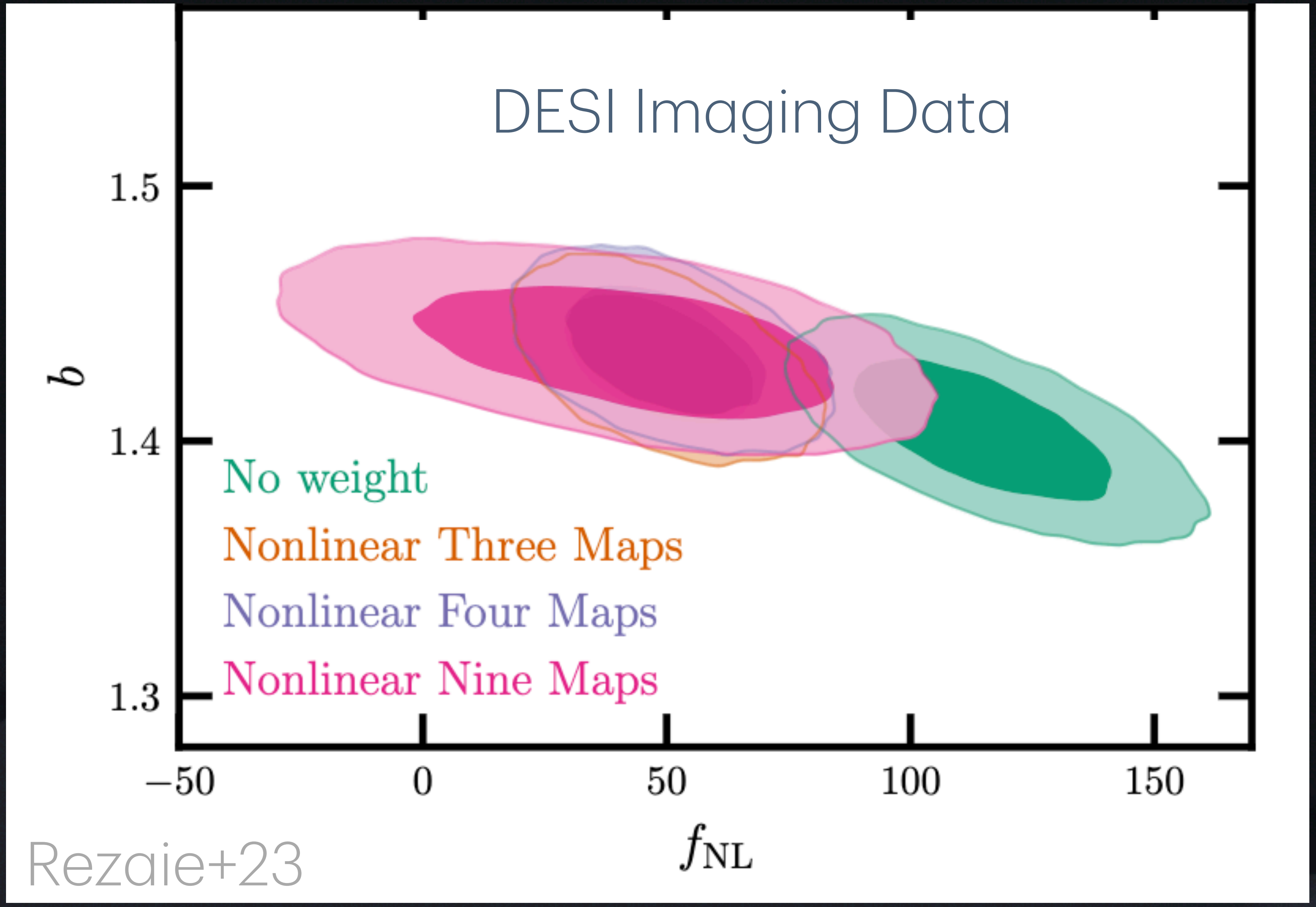


Kumar+22ab, Barriera+22, Vanzan+23, Schmidt, Jeong, Seljak, Desjacques, Ferraro etc.

1. Power Spectrum Science: Scale-Dependent Bias

Observables:

- Galaxy samples with **sample-variance** cancellation
- On *large* scales: error is sensitive to k_{min}
- Cross-correlations can probe **squeezed shapes** e.g.
  - Weak lensing & kSZ velocity fields!



However,

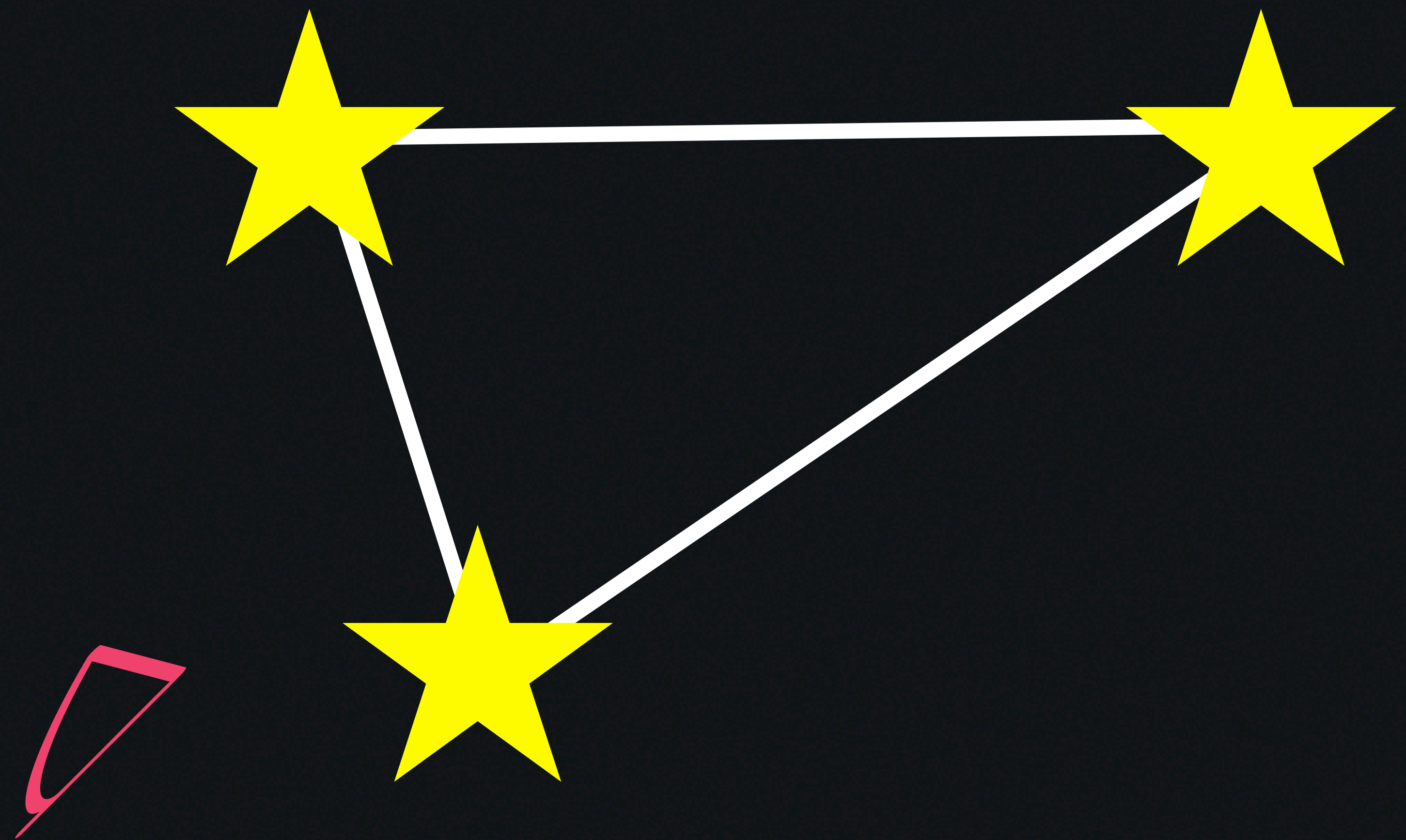
- GR *also* gives large-scale power excess like $f_{\mathrm{NL}}^{\mathrm{loc}} \sim 1$...
- We must carefully model foregrounds!

Dalal+07, Barriera+22, Cabass, Philcox+22, Rezaie+23, Hotinli, Kumar, Kamionkowski, Foglieni, Castorina, Di Dio, etc.

2. Bispectrum Science: Self-Interactions

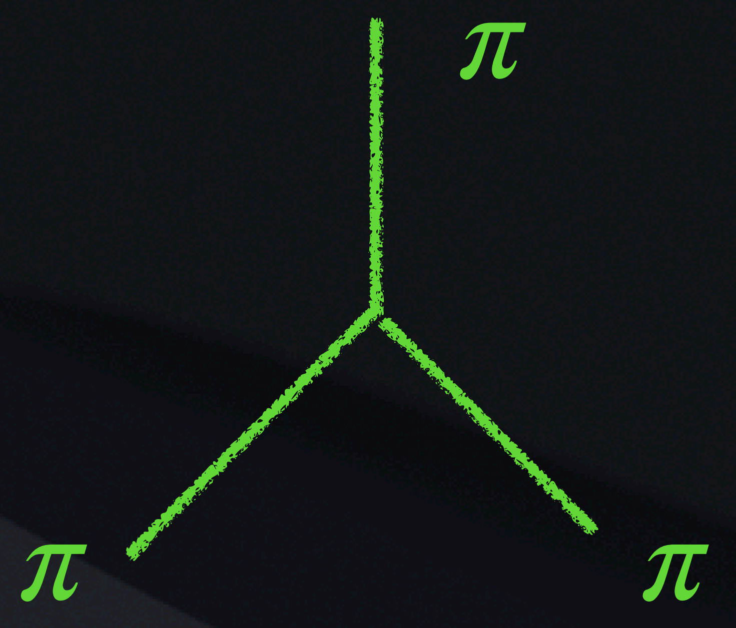
The galaxy bispectrum directly traces the **primordial** bispectrum

$$B_{ggg}(k_1, k_2, k_3) \sim B_{\zeta\zeta\zeta}(k_1, k_2, k_3)$$



- This is a great probe of **self-interactions** in the single-field inflationary model (EFTI)

$$\mathscr{L} \supset \dot{\pi}^3, \dot{\pi}(\nabla \pi)^2$$



Where $\pi \sim \zeta$

- Simple parametrization (assuming shift-symmetries):

$\Rightarrow$ $f_{\rm NL}^{\rm equil}, f_{\rm NL}^{\rm orth}$ probing the two couplings

Planck, d'Amico+22, Cabass, Philcox+22a, Chen+24, Senatore, Smith, Zaldarriaga, etc.

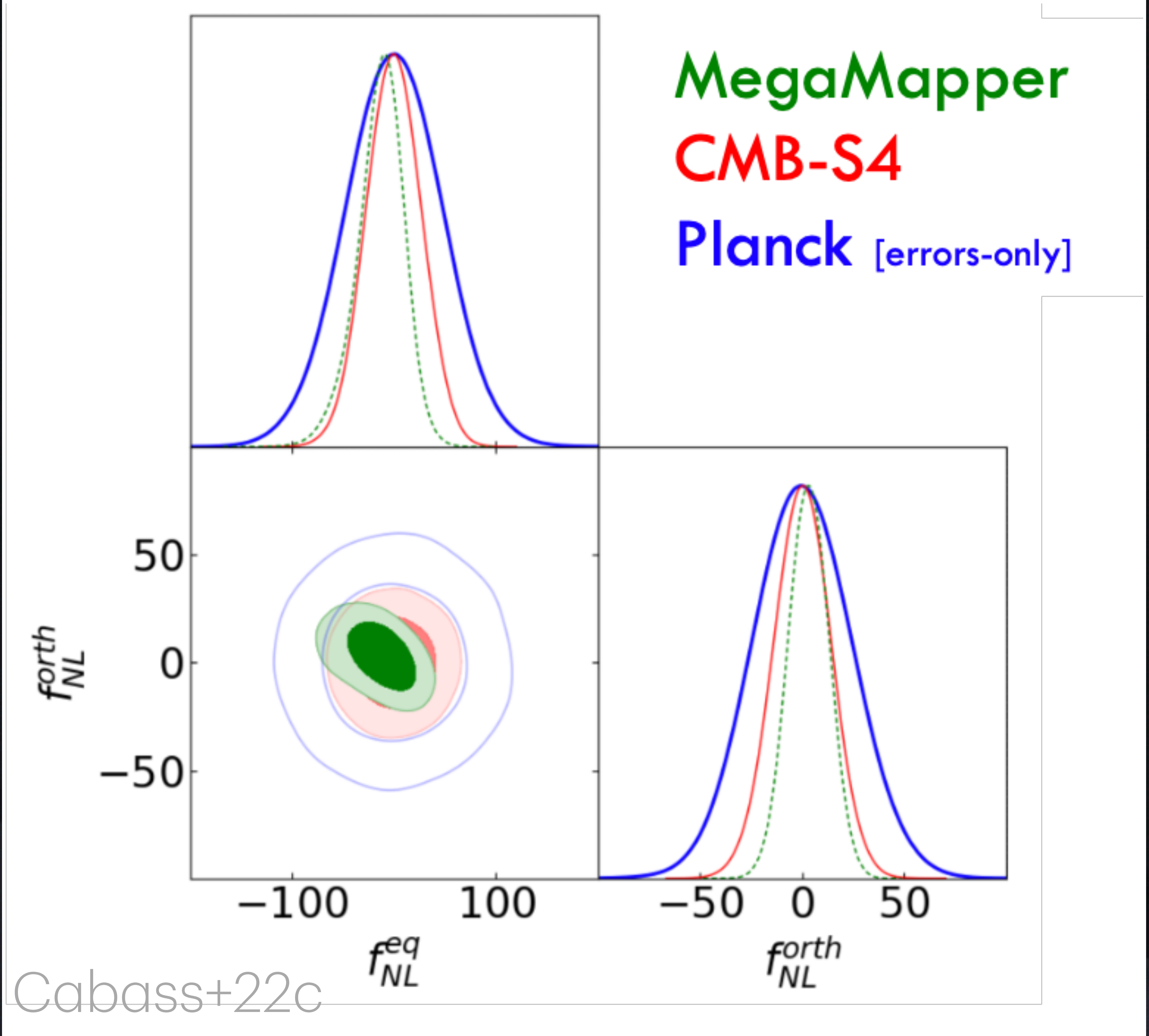
2. Bispectrum Science: Self-Interactions

Analysis is quite **hard**: we need to disentangle galaxy formation and inflationary physics

$$B_{ggg}(k_1, k_2, k_3) \sim B_{\zeta\zeta\zeta}(k_1, k_2, k_3) + B_{\text{quadratic}} + B_{\text{tidal}}$$

EFT(ofLSS) to the rescue!

- We can self-consistently model **both** effects to ***marginalize*** over galaxy formation up to $\mathcal{O}(4)$
- Better knowledge of galaxy formation will **considerably** aid this!



Cabass+22c

MegaMapper > CMB-S4, Simons Observatory!

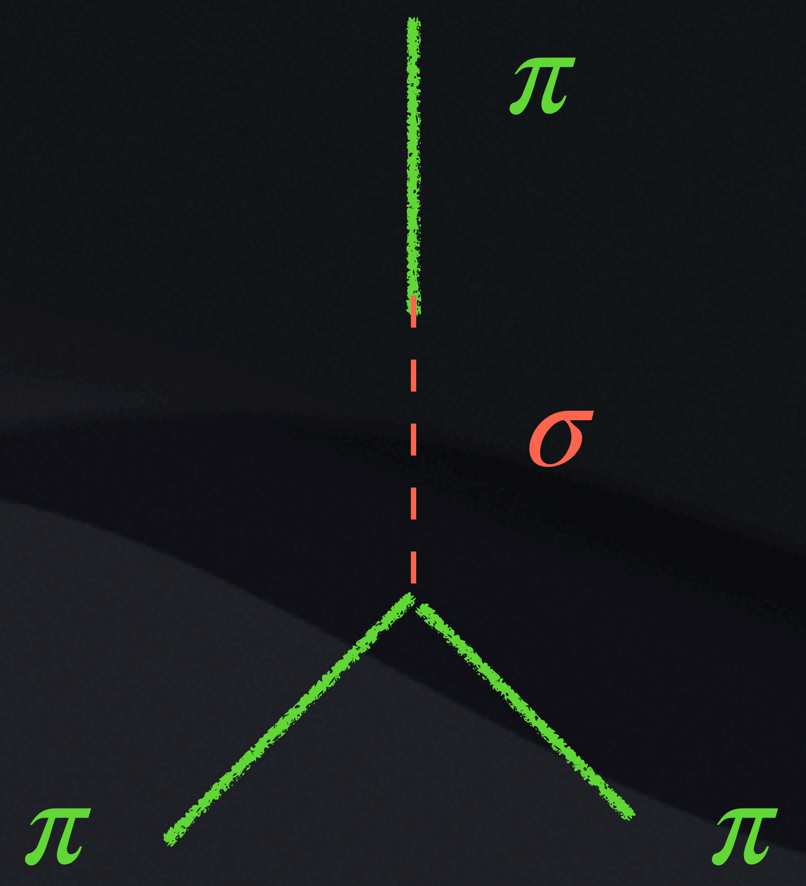
Cabass, Philcox+22ac, d'Amico+22, Assassi, Baumann, Zaldarriaga, Senatore, etc.

2. Bispectrum Science: New Particles

We can also probe **new particles** beyond the squeezed limit *e.g.*

- Massless scalars ($f_{\rm NL}^{\rm loc}$)
- Massive-ish & massive scalars ($m < \frac{3}{2}H,\ m > \frac{3}{2}H$)
- Partially massless states
- Higher-spin physics

$$\mathscr{L} \supset \dot{\pi}\sigma, \dot{\pi}^2\sigma, (\nabla\pi)^2\sigma$$



These have **complex** phenomenology e.g.
$\delta_g \supset b_\phi k^{-1/2} \cos\mu \log k/k_\star$ including oscillations!

Important restriction: new particles must couple to scalars!

Chen+09, Arkani-Hamed+15, Green, Baumann, Dvorkin, Moradinezhad-Dizgah, Lee, etc.

2. Bispectrum Science: New Particles

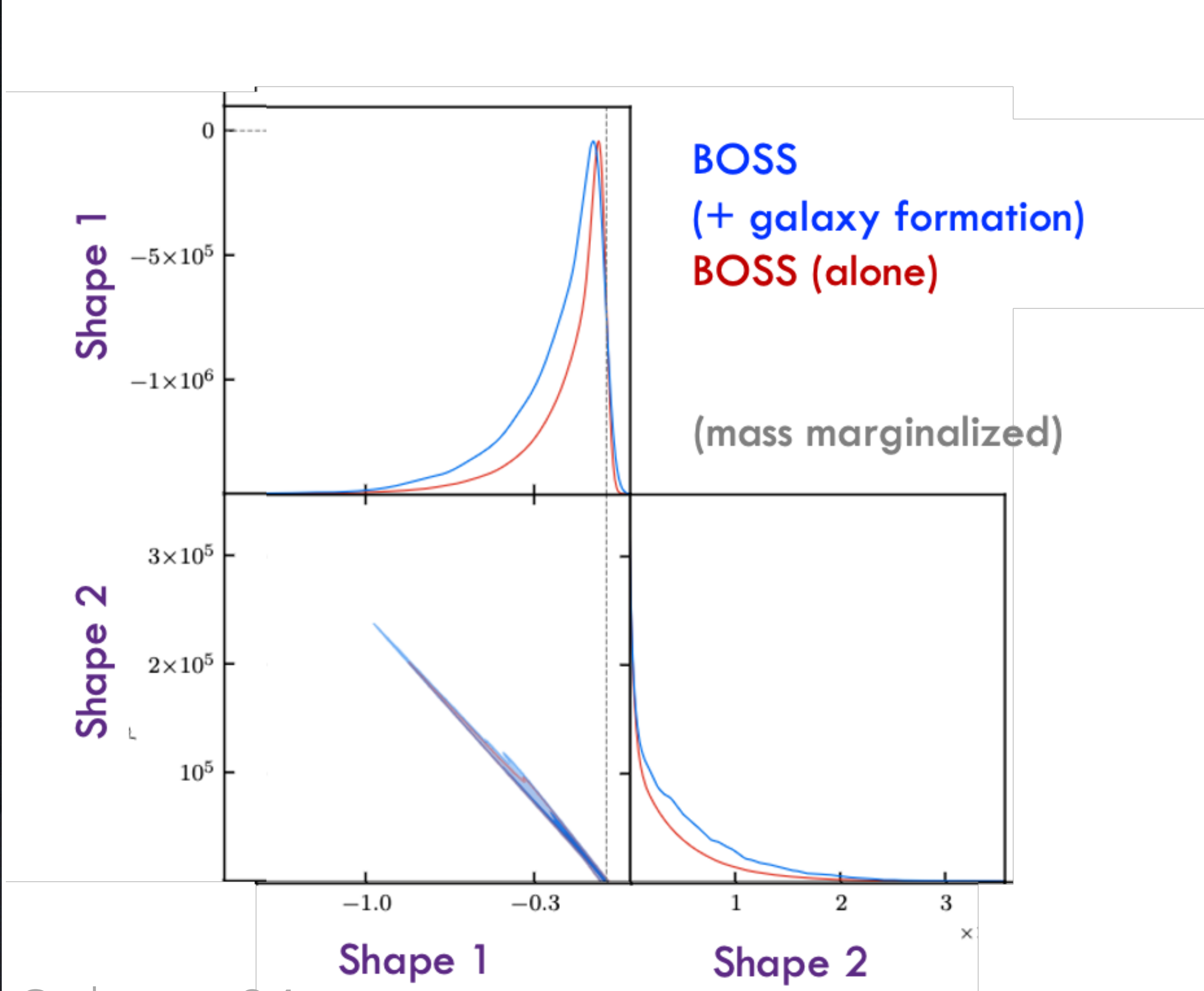
We are **just** beginning to explore these regimes!

- This has required better **EFTofLSS** *and* **inflation** modeling!
- Could do better still with ***non-perturbative*** modeling?

First **massive particle** constraints last month (from CMB *or* LSS)!

There's many other things to probe e.g.

- **Thermal** initial states ($f_{\rm NL}^{\rm folded}$)
- **Dissipative** systems
- **Oscillatory** bispectra (e.g., axions)



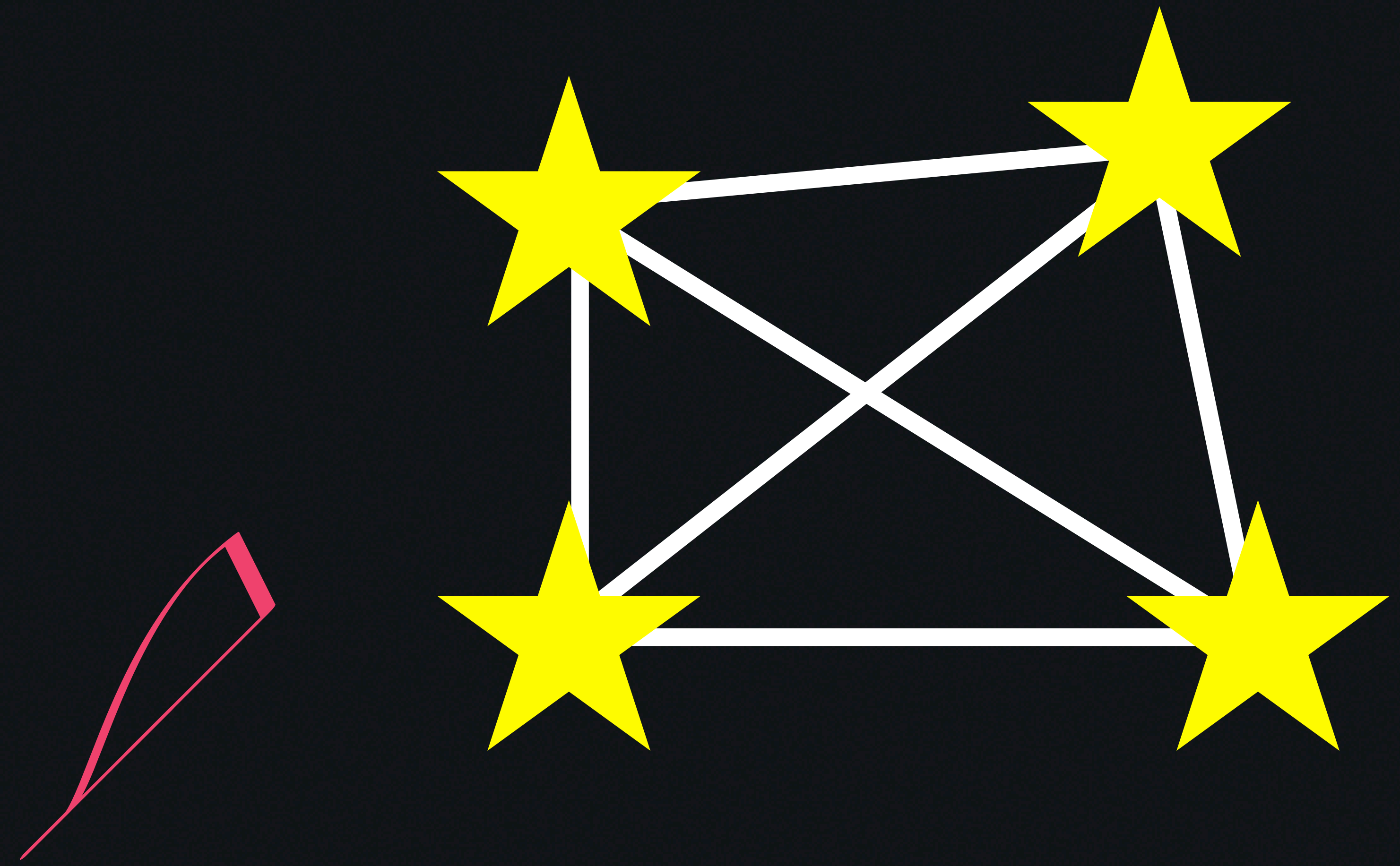
Cabass+24

Constraints on $m > (3/2)H$ particles!

Cabass, Philcox+24, Worth+23ab, Salcedo+24, Green, Pinol, Jazayeri, Pajer, etc.

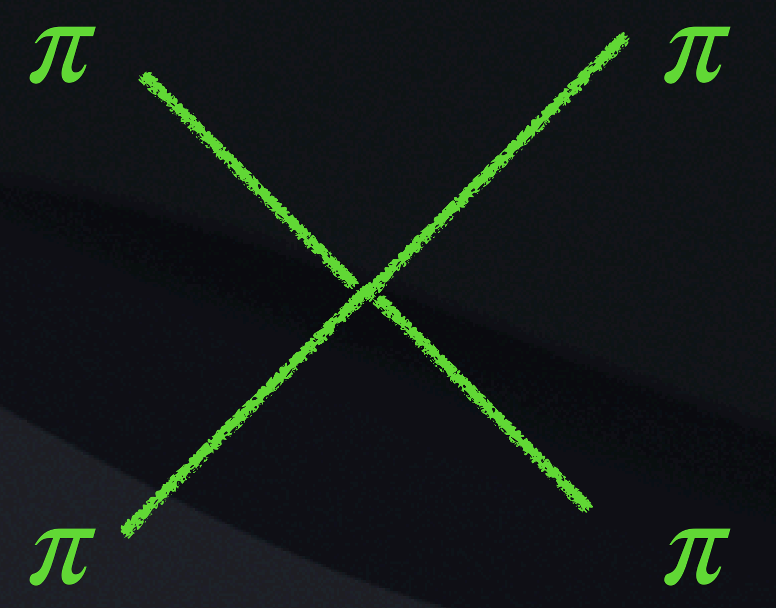
3. Trispectrum Science: Self-Interactions

The galaxy trispectrum directly traces the **primordial** trispectrum



$$T_{gggg}(k_1, k_2, k_3, k_4, k_{12}, k_{34}) \sim T_{\zeta\zeta\zeta\zeta}(k_1, k_2, k_3, k_4, k_{12}, k_{34})$$

Why would we care about this?



- It's quite easy to make a model **without** cubic non-Gaussianity (e.g., $\mathbb{Z}_2$ symmetry!)
- We can probe single-field **EFT** of **Inflation** shapes: e.g. $g_{\rm NL} (\times 3)$

$$\mathscr{L} \supset \dot{\pi}^4, (\nabla\pi)^4, \dot{\pi}^2(\nabla\pi)^2$$

Smith+15, Senatore, *Planck*, etc.

3. Trispectrum Science: New Particles

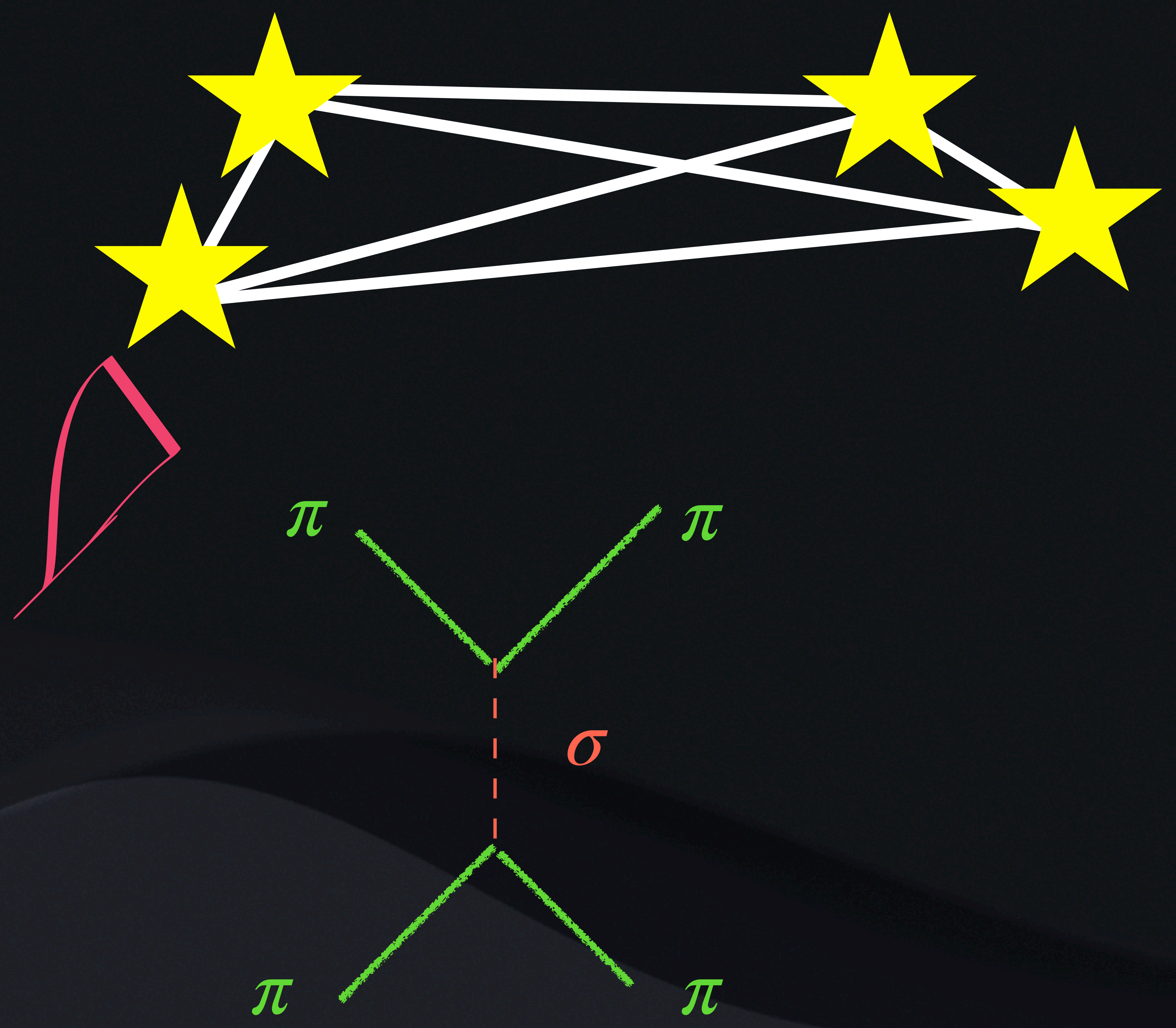
We can **directly** probe particle scattering

- This is much more general: we *don't* need direct $\sigma\phi$ couplings!

  (We can probe all helicity states of σ)

- We retain **kinematic** information which tells us about **mass** and **spin**

$$\mathscr{L} \supset \dot{\pi}^2\sigma, (\nabla\pi)^2\sigma$$



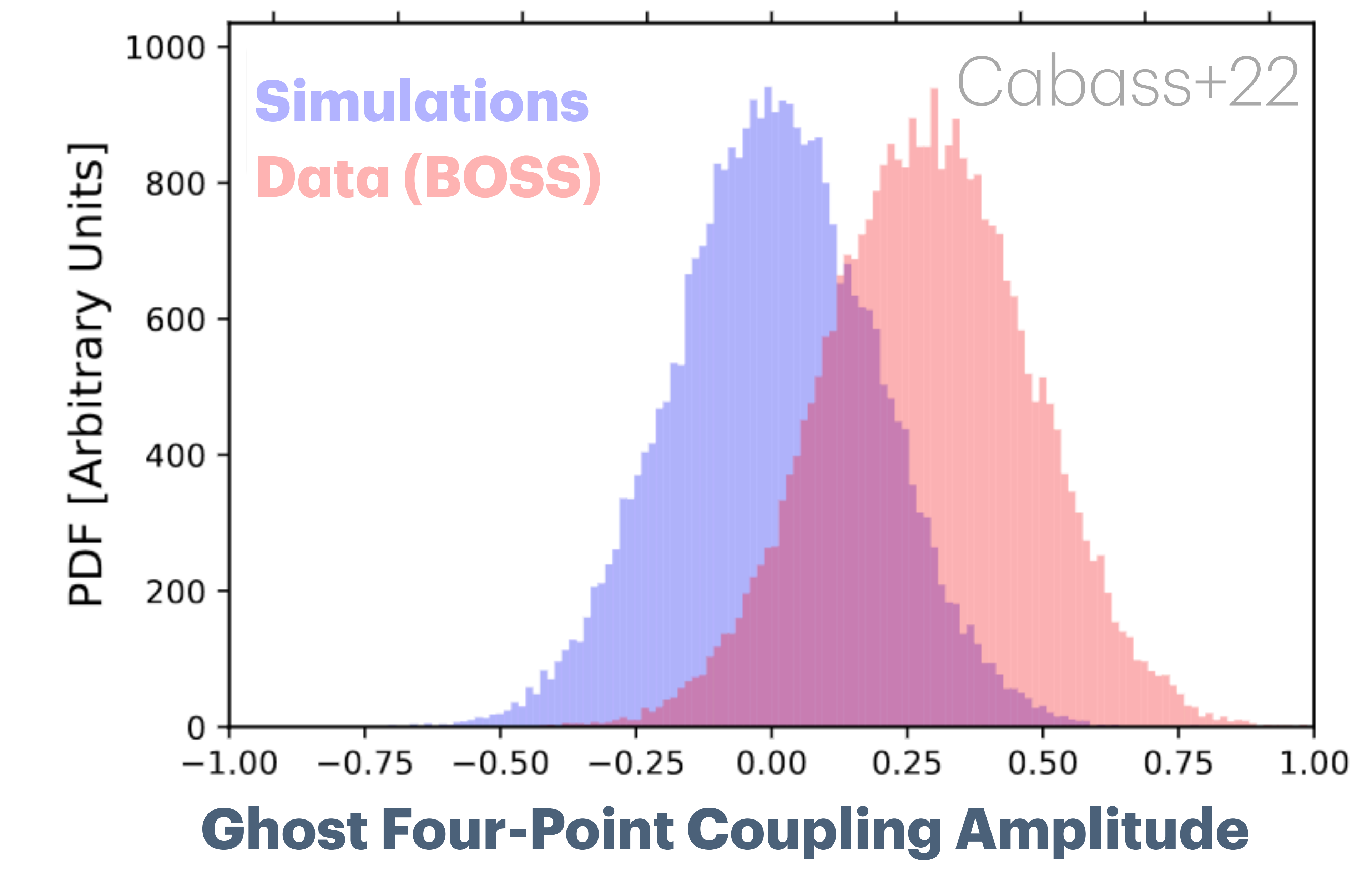
Also a direct probe of **equivalence** and **isocurvature** modes!

Chen+09, Arkani-Hamed+15, Kamionkowski, Hotinli, Kumar, etc.

3. Trispectrum Science: In Practice

This is **harder** to analyze! We need

- We need a **full** theory model for the trispectrum including all **third-order biases**
- **Robust** trispectrum estimator accounting for **geometry** effects



Some regimes are **simpler**:

- Parity [Cahn+21; theory has no **additive** biases]
- Collapsed estimators [model with **symmetries**]

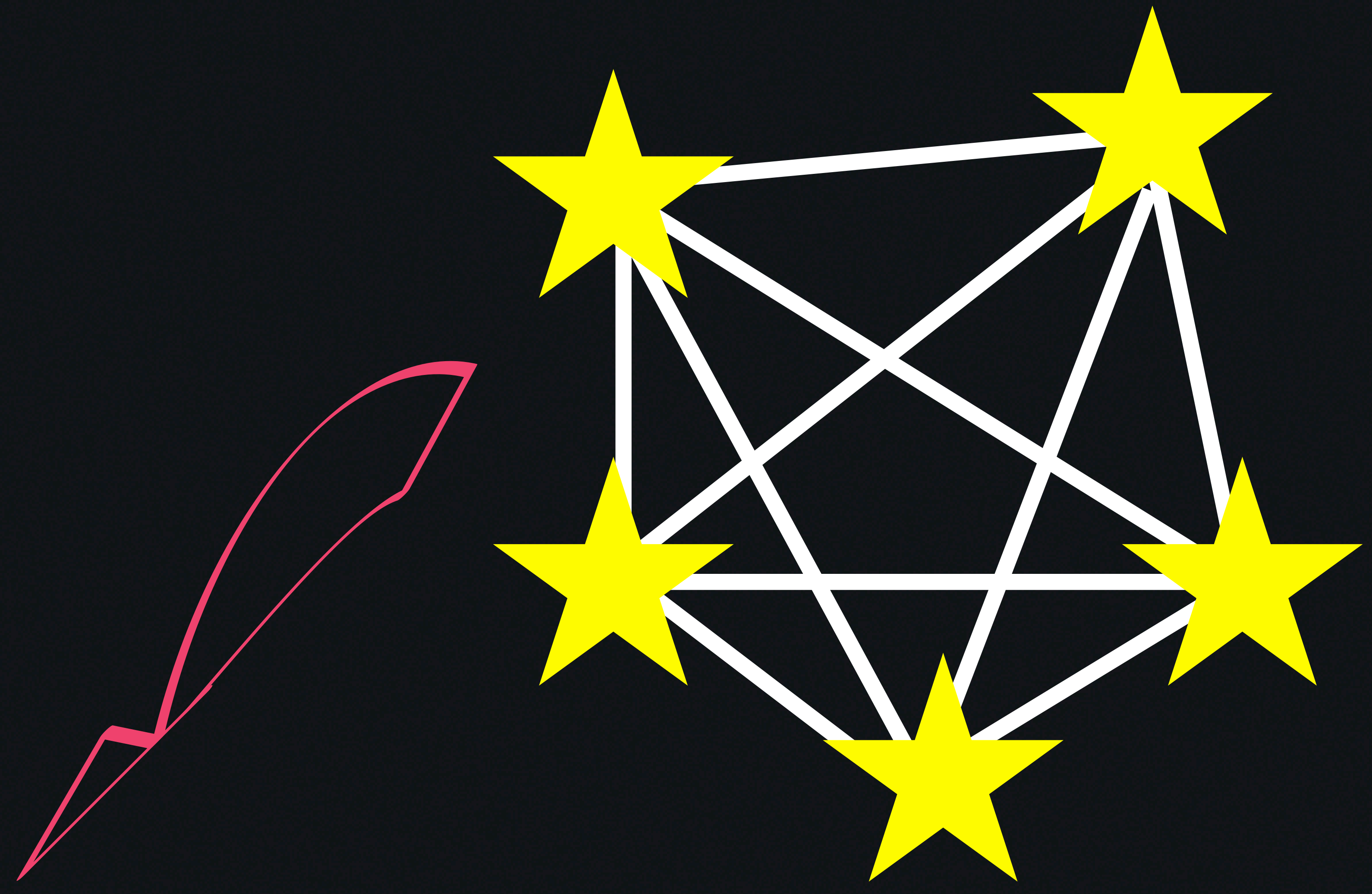
Still lots more work to do!!

Cahn+21, Hou+22, Philcox+22-24, Creque-Sarbinowski+23, Cabass+22, Kamionkowski, Goldstein, etc.

4. Quadspectrum Science

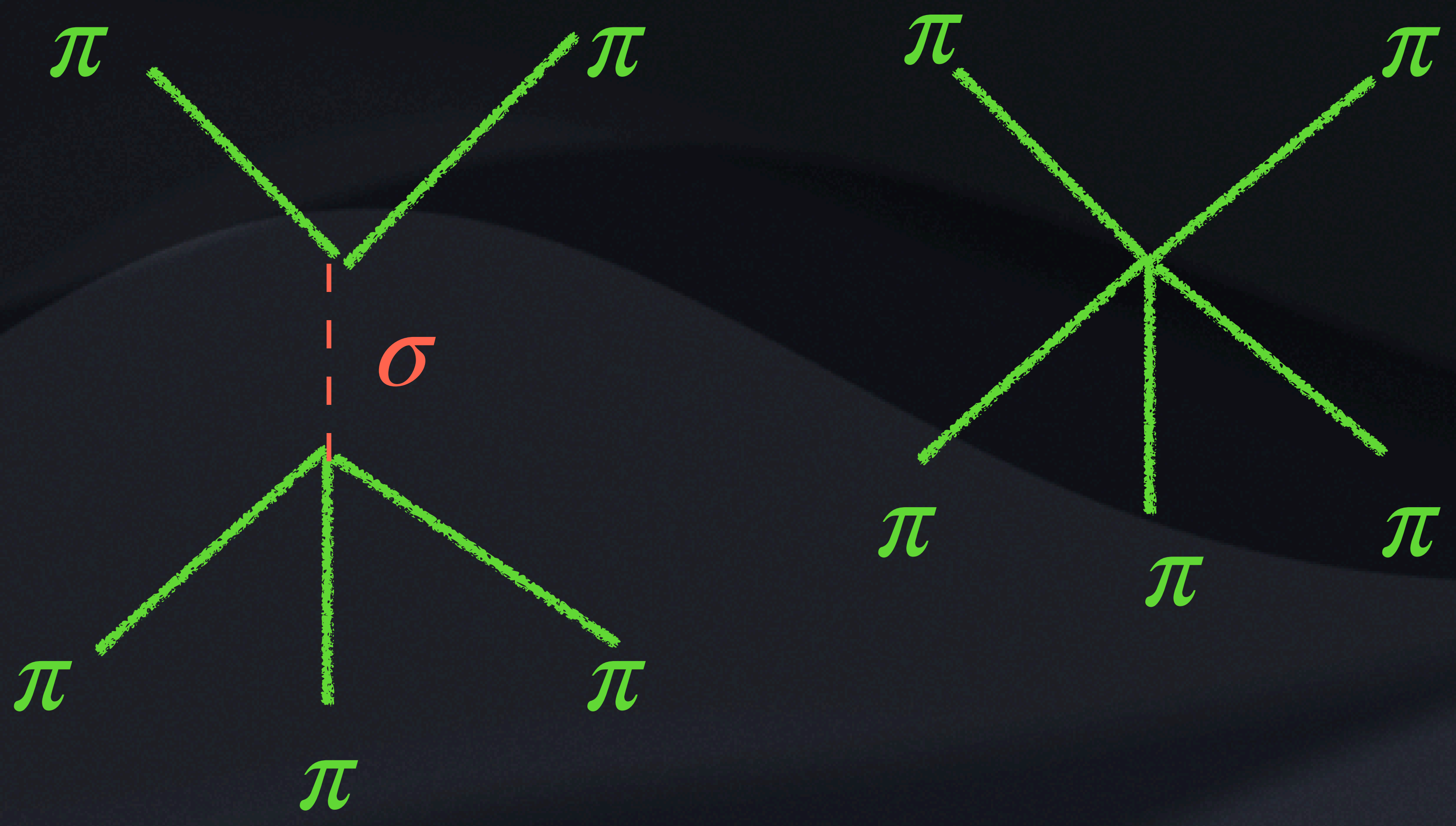
The galaxy quadspectrum directly traces the **primordial** quadspectrum

$$Q_{ggggg}(k_1, k_2, k_3, k_4, k_5, \cdots) \sim Q_{\zeta\zeta\zeta\zeta\zeta}(k_1, k_2, k_3, k_4, k_5, \cdots)$$



Why would we care about this?

- *I don't think we do…*



[references ????]

5. Non-Perturbative Science

If the **galaxy clustering** is non-linear there are many statistics to constrain primordial physics e.g.

- Wavelet statistics?
- CNNs?
- Reconstruction?
- Marked statistics?

These are particularly useful for **squeezed limits** (e.g., $f_{\rm NL}^{\rm loc}$), which impact **many** correlators!

What about if the **primordial physics** is non-linear?

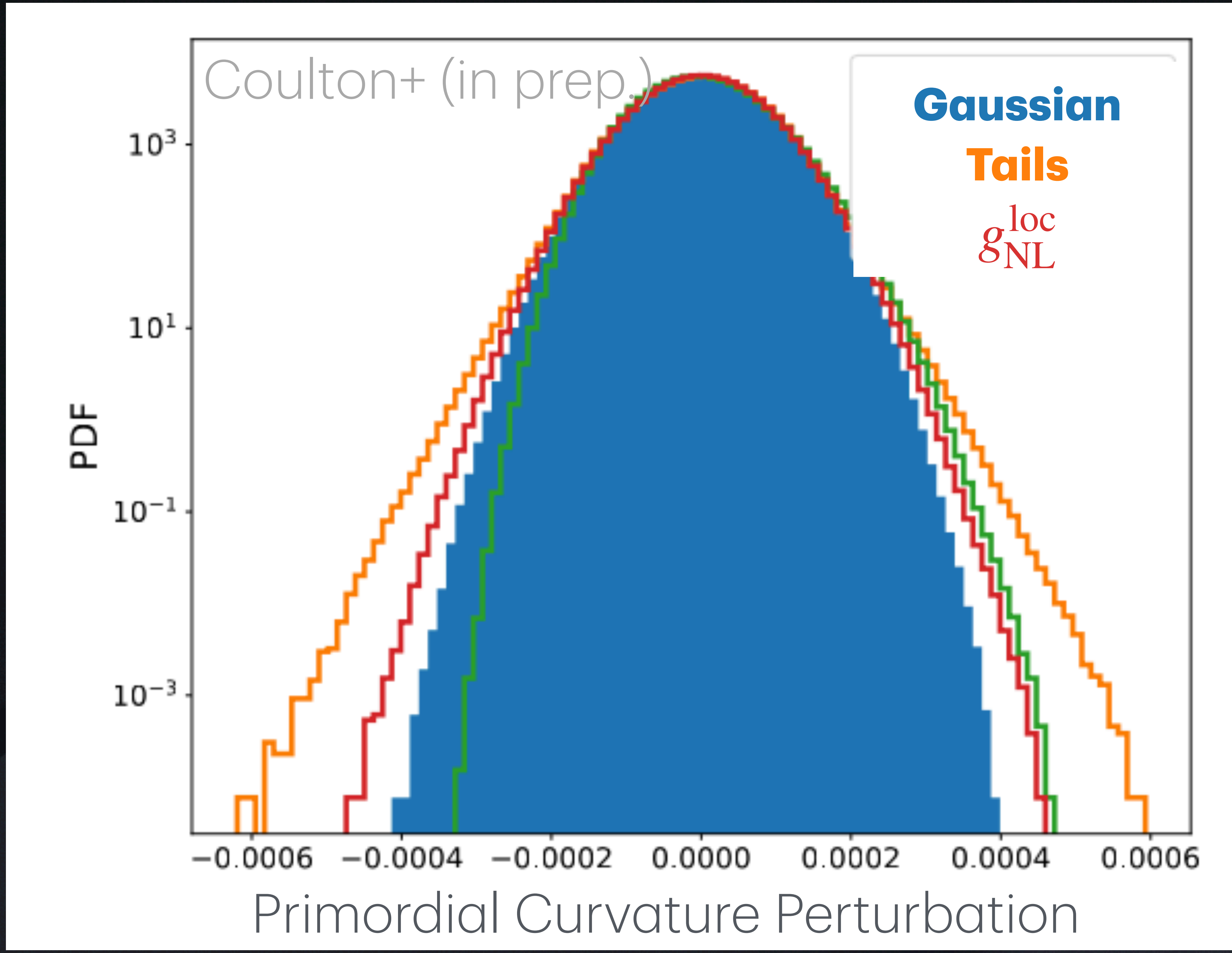
[Many 100s of references]

5. Non-Perturbative Science: Tails

Imagine the primordial PDF has **tails**

$$P[\zeta] \neq e^{-\zeta^2/2\sigma_\zeta^2}$$

- This could be sourced by quantum-**diffusion** processes or **reheating** effects
- This can create N-point functions at **very large N**



Good observables:

- Halo mass function
- Galaxy power spectra (scale-dependent **bias**)

Coulton, Philcox+ (in prep.), Biagetti, Vennin, etc.

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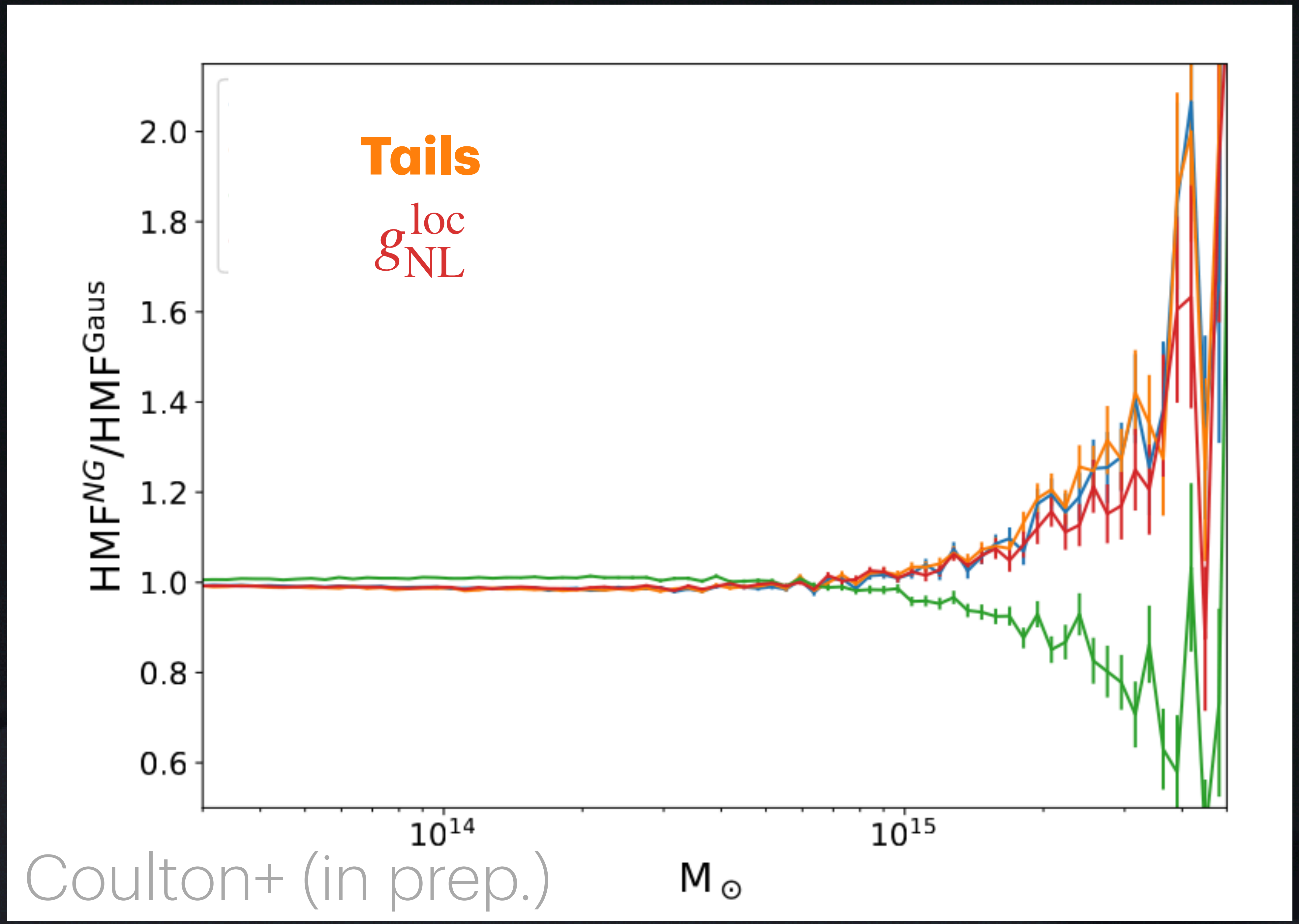


Halo Mass Function Ratio

Coulton, Philcox+ (in prep.), Biagetti, Vennin, etc.

5. Non-Perturbative Science: Massive Particles

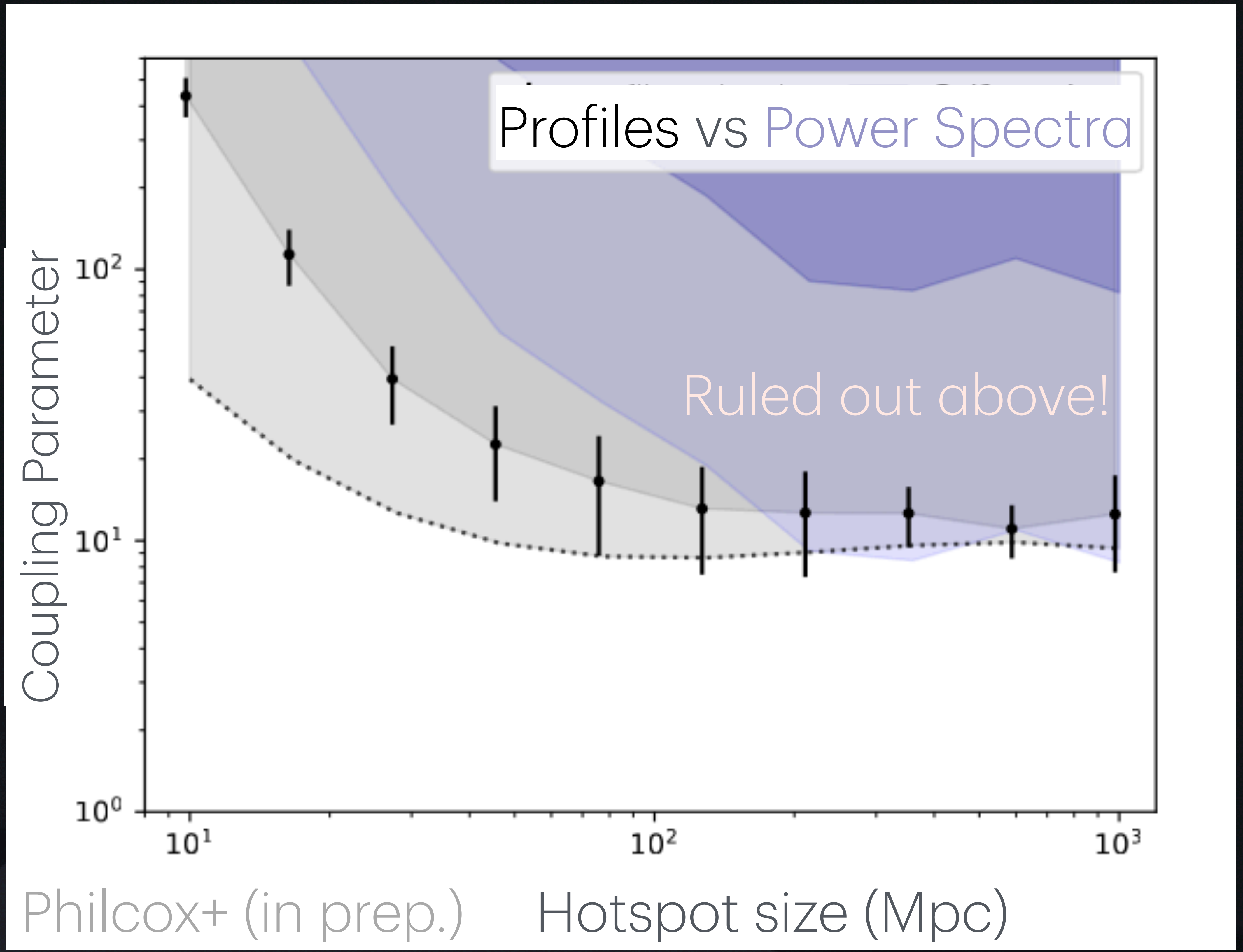
Production of **extremely massive particles** during in inflation is a **rare** event

$$P_{\text{production}} \sim e^{-\pi M^2/\dot{\phi}}$$

but it can be possible with **periodic** particle production or **time-varying masses**

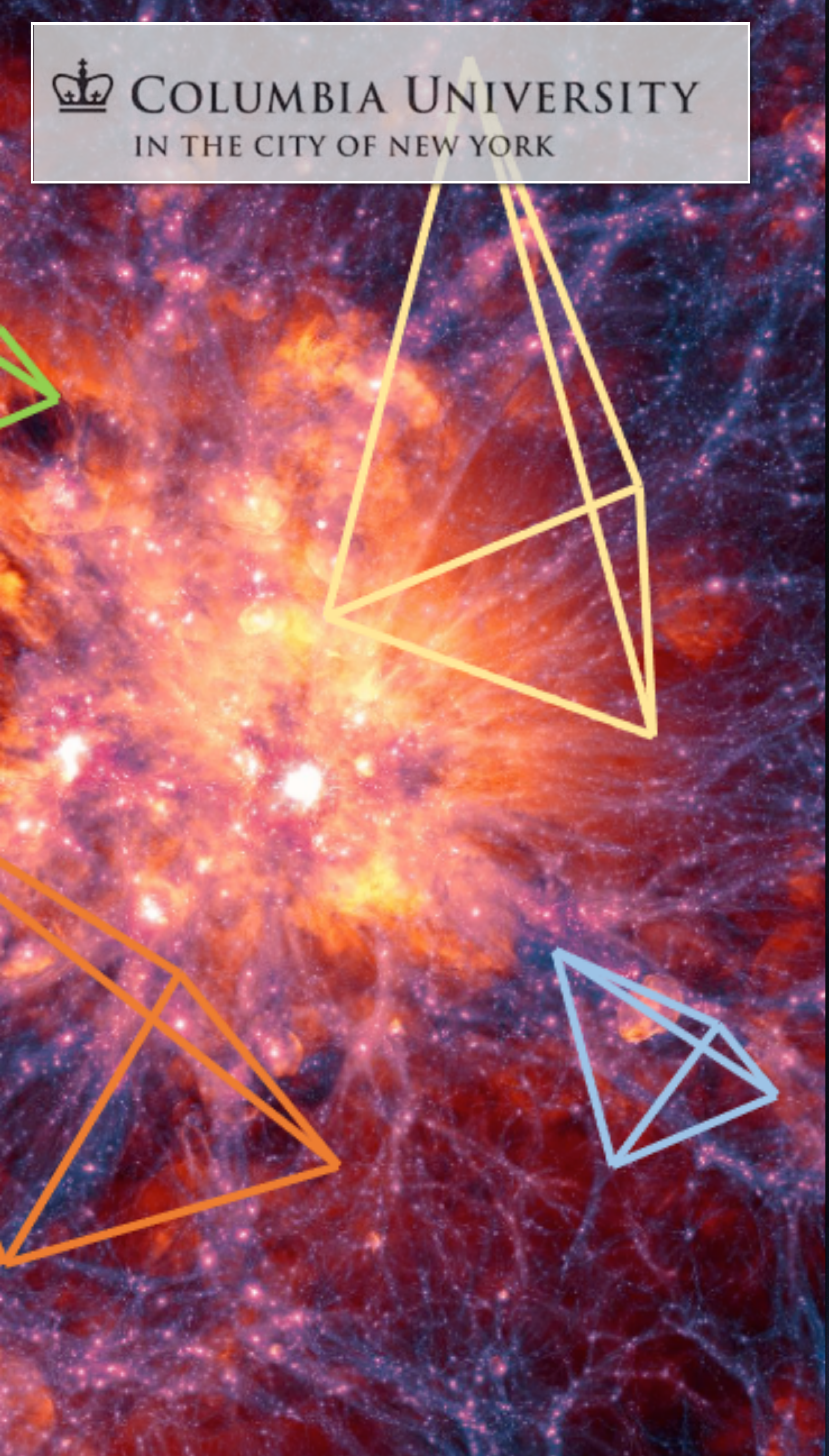
- Rare events produced **localized** signatures in the **potential**
- These are **hotspots** in the CMB ⇒ find with **profile-finding algorithms**
- How can we do this in **galaxy surveys**?

(Rare extreme-mass galaxies? Highly enhanced clustering?)



Time-varying mass constraints (*Planck*)

Philcox+ (this week), Kim+21, 23, Silverstein, Smith, Munchmeyer, Flauger, etc.



Conclusions

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- Future surveys have a lot of primordial physics to discover, and can beat the CMB on almost all fronts!
- For perturbative treatments, measure as many modes as possible!
- For non-perturbative treatments like squeezed limits, small-scales are useful!
- The tools to do this are either available or *actively being developed*!