QCD

- We know it's right
- Issue is how reliable are calculations:
  - do you believe central value
  - Do you believe error estimate

Read these lectures by Giulia Zanderighi

https://www2.physics.ox.ac.uk/sites/default/files/QCDLectures.pdf

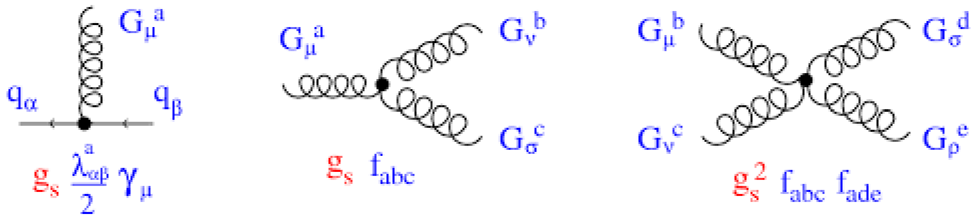
Calculations and approximations in QCD

- perturbation theory
  - How many orders are used?
  - What is the scale?
  - How to estimate next (calculated orders)
- Resummed perturbation theory
  - can yield better estimates for multiple scale problems
- Reliable non-perturbative methods
  - External data inputs needed
  - Effective field theory
  - Lattice
- Unreliable non perturbative models (quoted errors worthless?)
  - Hadronization models
  - Old fashioned models of structure (quark models..)

Perturbation theory

- QCD has one coupling parameter
  - Quark masses come from electro weak theory and are additional inputs
  - Ignore masses for the moment

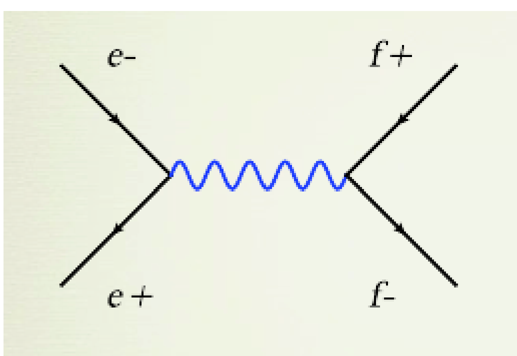
$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$
$$= \bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} ,$$



Using perturbation theory

- Can calculate inclusive quantities in PT
  - Example: total cross section e+e- to hadrons
    - Cannot ask what hadrons are
    - Estimated by e+e1 to quarks and gluons
      - Lowest order qq
      - NLO qqg etc
      - This materializes as hadrons with probability 1

Dimensionless

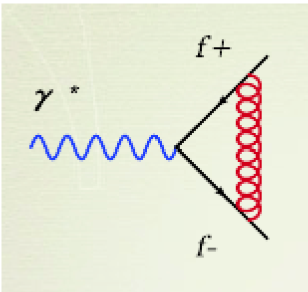


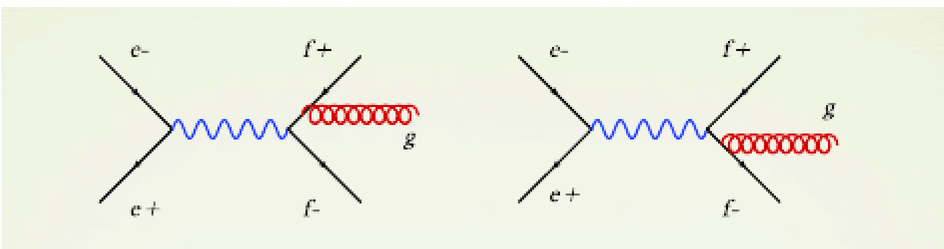
$$R = \frac{\sigma\left(e^+e^- \to \mathrm{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

$$R \to 3 \sum_q Q_q^2$$

Higher orders in PT

- **Now allow for gluons to appear**





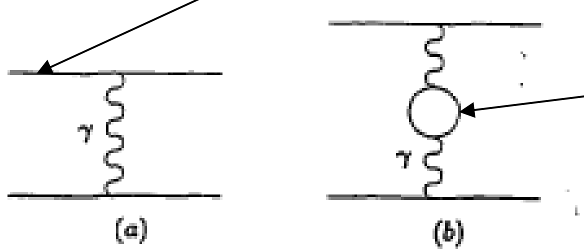
$$R = R_0 + R_1 + \mathcal{O}(\alpha_s^2) = R_0 \times \left\{ 1 + \frac{\alpha_s}{\pi} \right\}$$

Cannot predict anything until you know what this is

Higher orders in PT

- Could use this process to define α
- But R depends on energy (if you measure it accurately enough)
  - The formula cannot be correct
  - You might expect α to depend on energy $\alpha(\mu)$
    - How does that arise ?
- Alternative to define coupling is in terms of static force between 2 heavy quarks (like QED: 1/127)

Heavy quarks



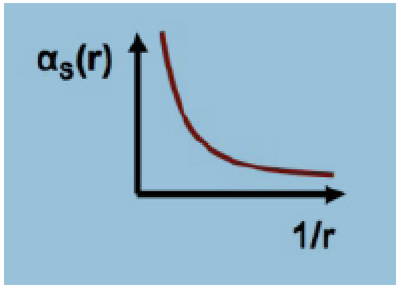
Light quarks and gluons

$F=\alpha/r$

Higher orders in PT

- The loop is divergent (infinite)
- An unphysical scale is introduced (μ) to regulate the divergence: use this coupling
- Now R >>> R(Energy/μ)
  - Now depends on energy
  - R(E2)=R(E1)+Aα(E2)ln(E1/E2)
    - Still have only one constant (now called α(E2))
- Can parametrize energy dependence of α

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + b_0\alpha_s(Q_0^2)\ln\frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln\frac{Q^2}{\Lambda^2}}$$



This is approximate: see PDG for more detail

Higher orders in PT

- When calculating (dimensionless) process with only one energy scale,
- $\sigma(E)=(1/E^2)f(\mu/E,\alpha(\mu))$
  - But μ is unphysical so can use any value?!?
  - If calculation is reliable result will not depend on μ
  - Calculations are more pricise if one works harder!

Need NLO at least to make any prediction

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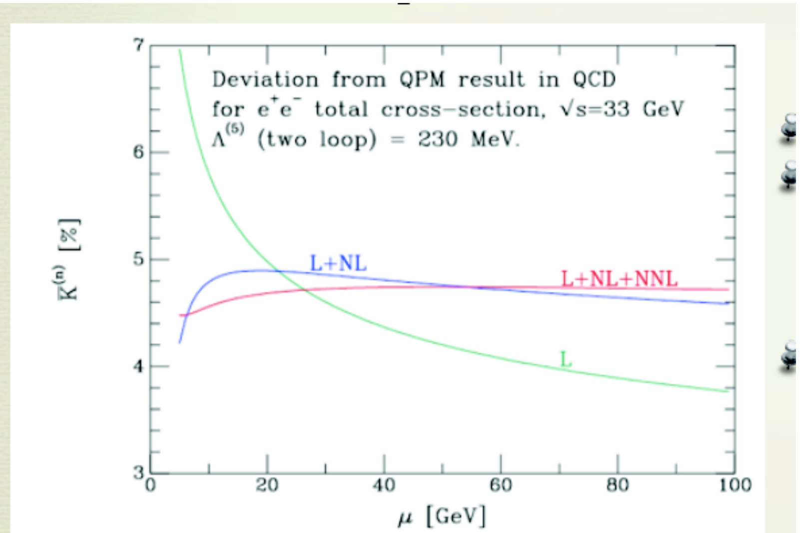
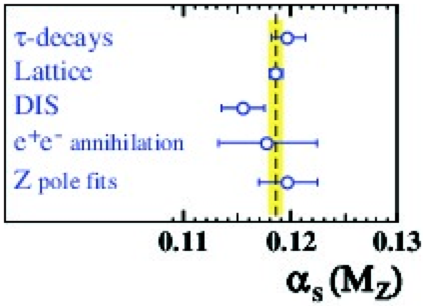


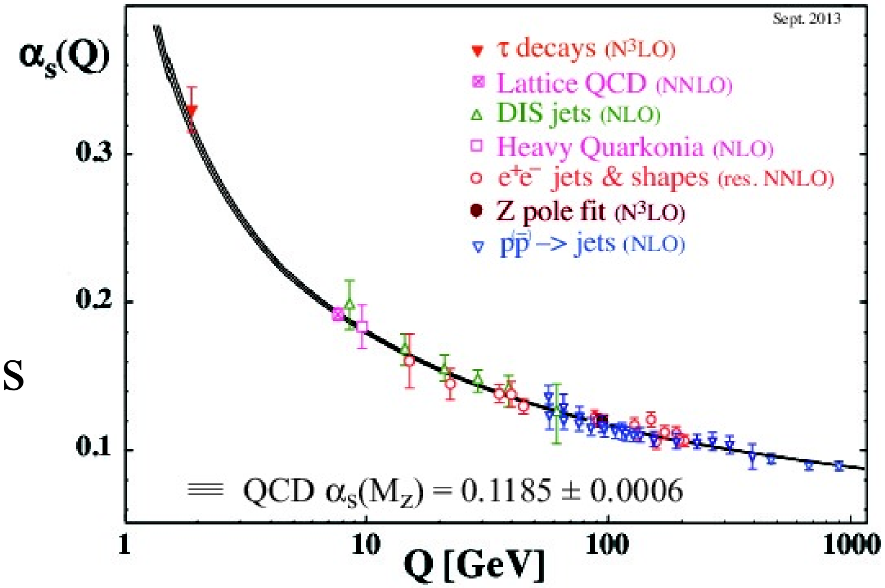
from Keith Ellis

Measured values of strong coupling



Error estimates
From scale and value of α

See PDG review for details

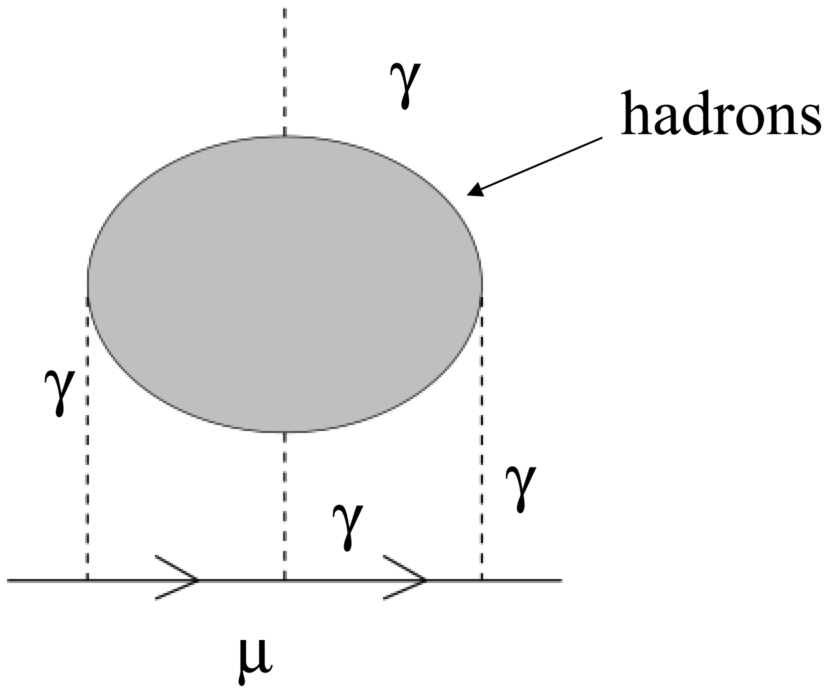


Calculations and approximations in QCD

- Leading order (LO) perturbation theory
  - Higher order corrections
- Resummed perturbation theory
- Non perturbative methods
  - External data inputs
  - Effective field theory
  - Lattice
- Non perturbative models
  - Hadronization models
  - Old fashioned models of structure (quark models..)
- Estimates of the uncertainties

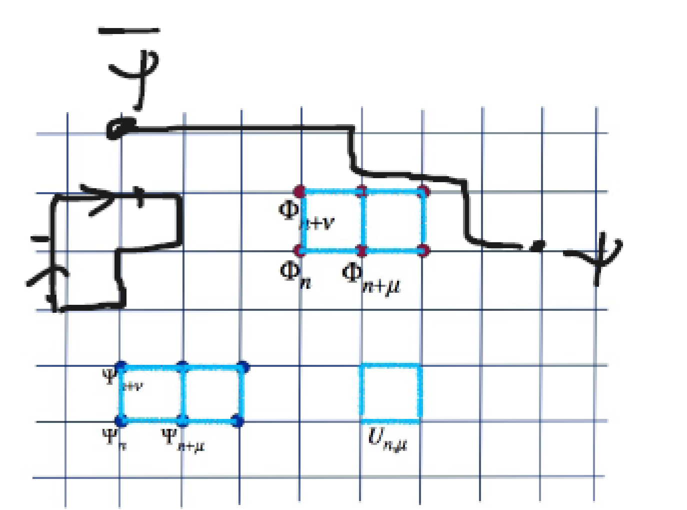
QCD effects in muon (g-2)

- (g-2) is reaching precision that this is important
- Cannot calculate in perturbation theory as α is too big
- Need to know this in order to use (g-2) as test of Standard Model



Lattice

- Calculation technique not dependent on small coupling
- Works by evaluating the path integrals exactly
  - Integrate over gluons at everywhere in space
  - Not possible:
    - Make space discrete: integral-⟶ sum
    - Finite number of space points (quarks sit here)
    - Finite number of links (gluons sit here)



$$\psi(x) \longrightarrow \psi_n, \qquad x = na$$

$$\int dx_i \longrightarrow a\sum_{n_i}$$

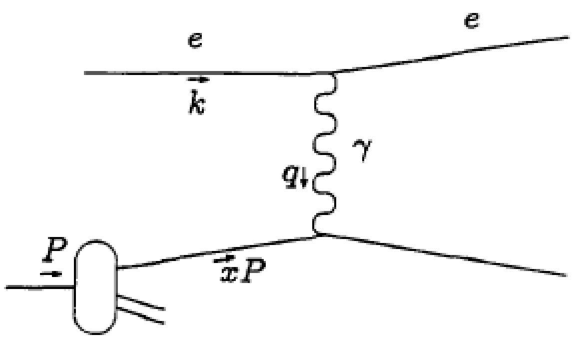
$$\int \mathcal{D}\psi\mathcal{D}\bar{\psi} \longrightarrow \prod_n d\psi_n d\bar{\psi}_n$$

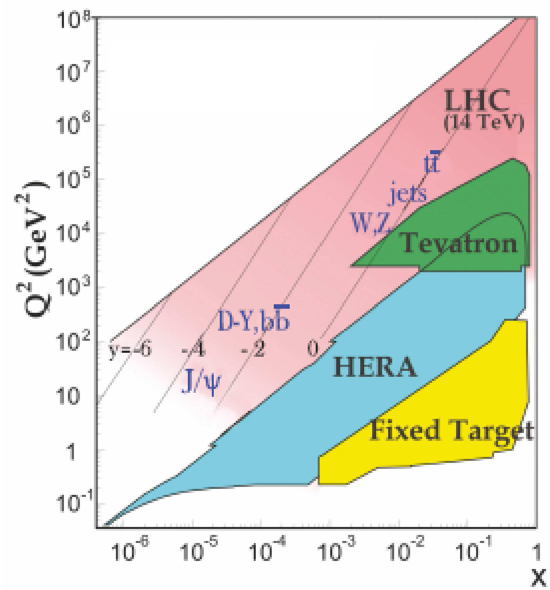
Lattice

- Can calculate everything in principle
- Limited by CPU power
  - Finite number of lattice sites (long distance cut off)
  - Scale is introduced (lattice spacing)
  - Use some data (eg proton mass) to fix coupling at lattice size
  - Map lattice coupling to α in PT
    - Best determination of α
- Limitations
  - Can only compute static quantities
  - Errors due to finite lattice
    - Can try to estimate these

Processes needing external input

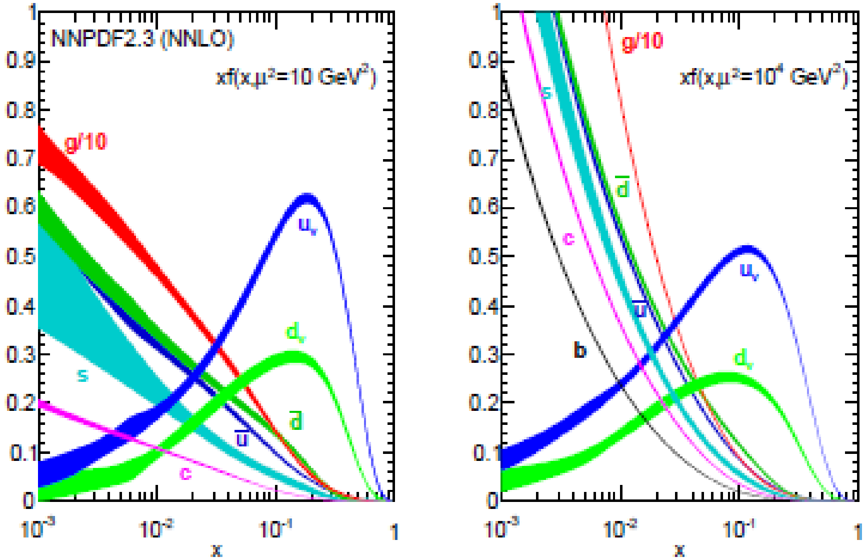
- ep> eX
- Initial state needs to be described





- Process not calculable in PT
  - But Q^2 dependence is calculable
  - Need data at one value, predict rest

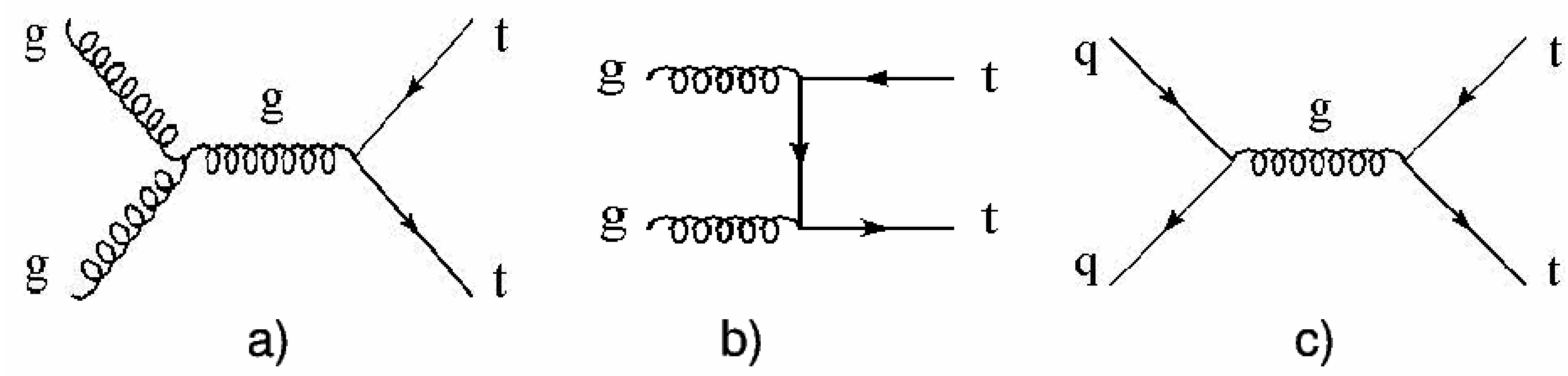
Evolution of PDF's



These PDF's are universal
This is a non trivial result

Production of top quarks at LHC

- Calculate fundamental process
- Put together with PDF
- What is scale?
  - "obviously top mass"



Not obvious

- **Suppose interested in high transverse momenta top**
  - pt>>M
  - Use pt or M? Or average….
- **. $\sigma \sim \alpha^2 + \alpha^3 \log(pt/M) + \alpha^4 \log^2(pt/M)$…..**
  - Problem if $\alpha\log(pt/M) \sim 1$
  - This is a generic issue when 2 (or more scales are present)
  - Terms must be resummed
- **Bottom production**
- **Jets where mass(jet, jet) big but <<< pt**
- **Many others**

Exclusive quantities jets

- Have not yet asked about details of what is seen
  - Avoided issue of hadrons
- Reconstruction of decay products (top, new particles) needs exclusive quantities
- Again factorization helps
  - Use PT to calculate final state of quarks/gluons
  - Then used model to make hadrons
    - Measure quantities not sensitive to this part
- Jet calculations are usually done by showering MC
  - But analytic calculations of "event shapes" are possible

Jet clustering

Gaol is to define quantities sensitive to energy flow

- Reconstruction of decay products (top, new particles) needs exclusive quantities
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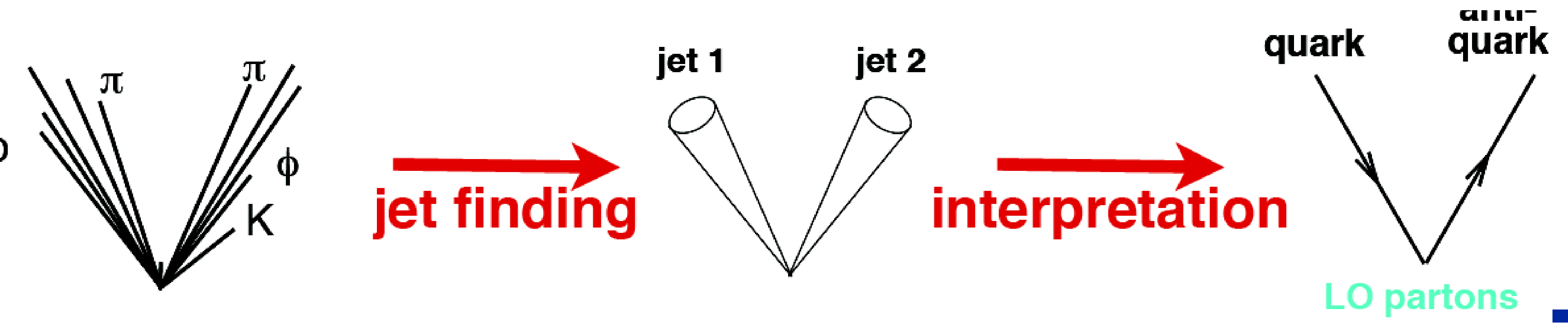
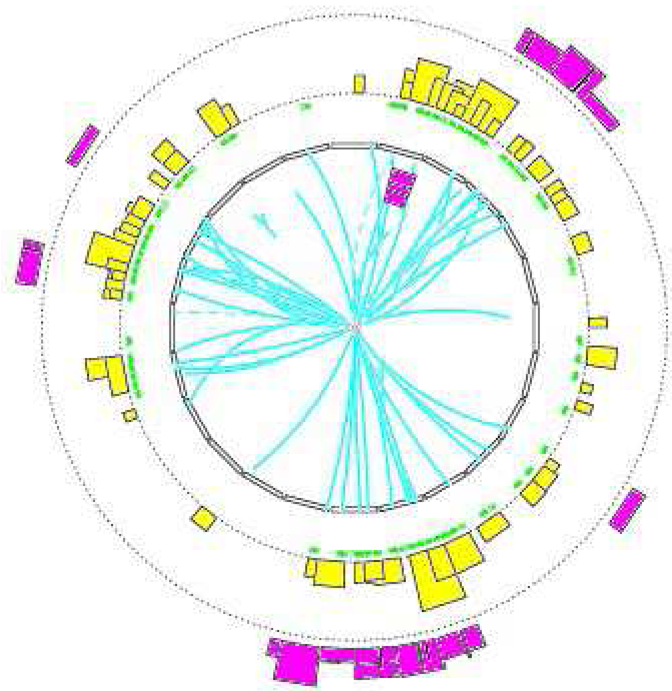


Jet clustering



How many jets are here?

Need a clustering algorithm
Must be applicable to data and to quarks/gluons
Then theory can be compared to data.

Jet clustering

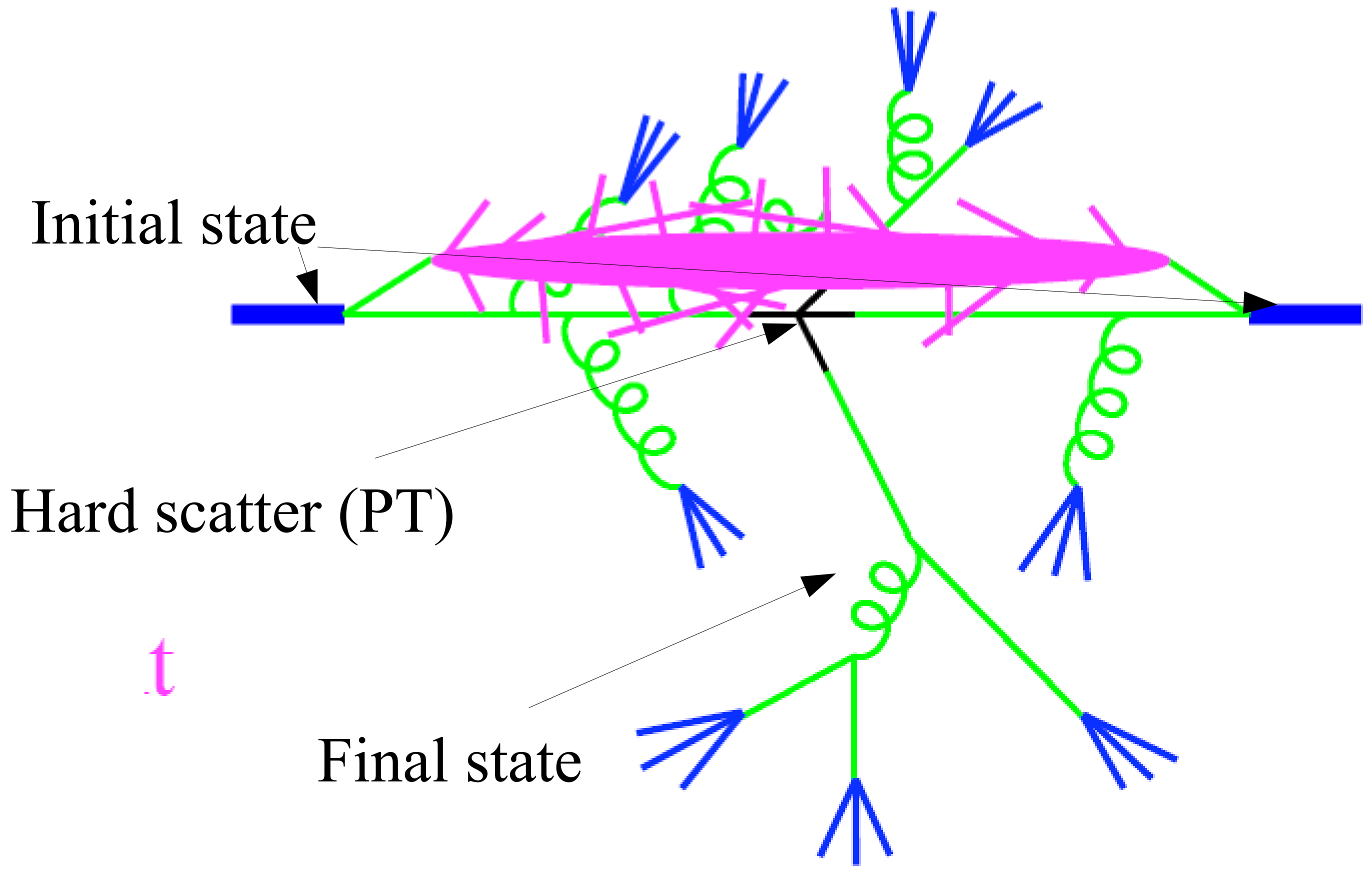
Now we have a multi-scale problem

- Pt1, pt2, pt3, mass of a pair…..
- Which scale to use?
- What about αlog(pt1/pt2)~1
- Showering Monte Carlos attempt to get everything at one
  - PT for fundamental process
  - Initial state showering for energy dependence of PDF
  - Resum large logs from big ratios (m/pt) (pt1/ps2) etc
  - Models of hadronisation
- But all the choices are mixed up
  - Reliable errors are impossible?
  - Easy to do something stupid
  -

Picture of showering MC

Note this a classical picture of a quantum process

- Cannot k



Summary of uncertainties

Strong coupling: If process is α^n, there is n% uncertainty in rate

Choice of μ: Usual to vary factor of 2 from "standard value"

May not know "standard value", this prescription is arbitrary

Multi scale processes: very ambigous

Open issues

- We know that QCD is correct
- We cannot always calculate precisely
  - Structure functions
  - Fragmentation parameters
  - Hadron masses
  - Dense environments (stars and plasma)
  - Decay properties
- There are some oddities
  - Why is nucleon spin not carried by quarks?