Physics 290e: Introduction to QCD

Jan 27, 2016

Outline

- The QCD LaGrangian
- The Running of α_s
- Confinement and Asymptotic Freedom
- A Case Study: $e^+e^- \rightarrow$Hadrons
- Hadron Structure: PDFs
- What We Will Discuss the Semester

The QCD Lagrangian: The matter fields

- Theory of Strong Interactions QCD developed in analogy with QED:
  - Assume color is a continuous rather than a discrete symmetry
  - Postulate local gauge invariance
  - Describe fundamental fermion fields as a 3-vector in color space

$$\psi = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}$$

  - SU(3) is the rotation group for this 3-space

$$\psi'(x) = e^{i\lambda^i \alpha_i/2}$$

where the λ^i are the 8 SU(3) matrices (play the same role as the Pauli matrices do in SU(2))

The QCD Lagrangian: The Gauge Field

- Impose local Gauge Invariance by introducing terms in A_μ and the quark kinetic energy term ∂_μ:

$$
\begin{aligned}
A\mu &\rightarrow A\mu + \partial_\mu \alpha \\
\mathcal{D}_\mu &\equiv \partial_\mu - i\frac{g}{2}\lambda_a A^a_\mu
\end{aligned}
$$

where A_μ is a 3×3 matrix in color space formed from the 8 color fields.

$$A_\mu \equiv \frac{1}{2}\lambda^i b^i_\mu$$

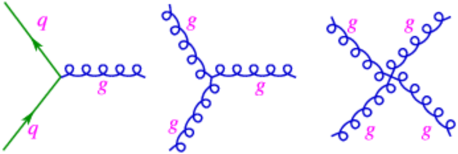
where i goes from 1 to 8 and b_μ plays the same role as the photon field in QED

- The tensor field $G^i_{\mu\nu} = G_{\mu\nu}\lambda^i$ is the QCD equivalent of $F_{\mu\nu}$:

$$G_{\mu\nu} = \frac{1}{ig}[\mathcal{D}_\nu, \mathcal{D}_\mu] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\nu, A_\mu]$$

- Note: unlike QED, the A fields don't commute!
  - Gluons have color charge and interact with each other

The QCD Feynman Diagrams



- qqg vertex looks just like $qq\gamma$ with $e \to g$
- Three and four gluon vertices
  - Three gluon coupling strength gf^{abc}
  - Four gluon coupling strength $g^2 f^{xac} f^{xbd}$
- Here g plays the same role as e in QED

The Running of α_s (I)

- Major success of QCD is ability to explain why strong interactions are strongly coupled at low q^2 (momentum transfer) but quarks act like free particles at high q^2
- Coupling constant α_s *runs*; It is a function of q^2

Low q^2	α_s large	"confinement"
High q^2	α_s small	"asympotic freedom"

- This running is not unique to QCD; Same phenomenon in QED
  - But α runs more slowly and in opposite direction
  - Eg at $q^2 = M_z^2$, $\alpha(M_Z^2) \sim 1/129$
- Running of the coupling constant is a consequence of *renormalization*
- Incorporation of infinities of the theory into the definitions of physical observables such as charge, mass
- Sign of the running in QCD due to gluon self-interactions

The Running of α_s (II)

- QED and QCD relate the value of the coupling constant at one q^2 to that at another through renormalization procedure

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}$$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log\left(\frac{Q^2}{\mu^2}\right)}$$

- In the case of QED, the natural place to measure α is clear: $Q^2 \to 0$
- Since α_s is large at low Q^2, no obvious μ^2 to choose
- It is customary (although a bit bizarre) to define things in terms of the point where α_s becomes large

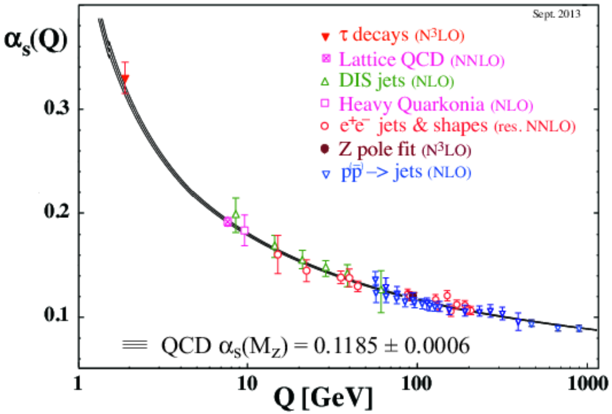
$$\Lambda^2 \equiv \mu^2 \exp\left[\frac{-12\pi}{(33 - 2n_f)\,\alpha_s(\mu^2)}\right]$$

- With this definition

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2)}$$

  - For $Q^2 \gg \Lambda^2$, coupling is small and perturbation theory works
  - For $Q^2 \sim \Lambda^2$, physics is non-perturbative
- Experimentally, $\Lambda \sim$ few hundred MeV

Measurements of α_s

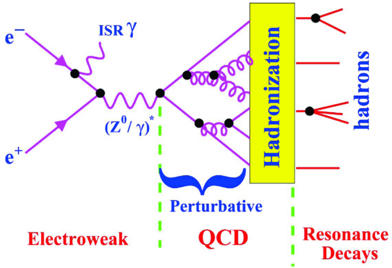


Each one of these measurements, together with discussion of the theoretical and experimental uncertainties, is a good topic for a talk this semester

Implications of the Running of α_s

- α_s small at high q^2: High q^2 processes can be described perturbatively
  - For Deep Inelatic Scattering and $e^+e^- \to hadrons$, the lowest order process is electroweak
  - Higher order perturbative QCD corrections can be added to the basic process
  - For processes such as pp or heavy ion collisions, the lowest order process will be QCD
  - Again, can include QCD perturbative corrections
- α_s large at low q^2: Quarks dress themselves as hadrons with probability=1 and on a time scale long compared to the hard scattering
  - Describe dressing of final quark and antiquark (and gluons if we consider higher order corrections) into a "Fragmentation Function"
  - Process of quarks and gluons turning into hadrons is called *hadronization*
  - If initial state contains hadrons, represent distribution of quarks and gluons within the hadrons with "parton disribution function"

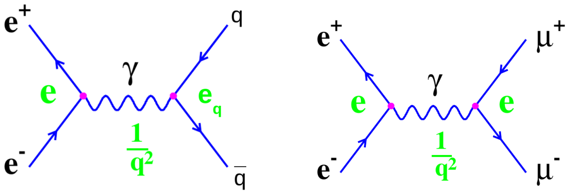
QCD at many scales



- Impulse approximation
  - Short time scale hard scattering
  - Perturbative QCD corrections
  - Long time scale hadronization process
- Approach to the hadronization:
  - Describe distributions individual hadrons statistically
  - Collect hadrons together to approximate the properties of the quarks and gluons they came from

Describe non-perturbative effects using a phenomonological model

A Case Study: $e^+e^- \to hadrons$

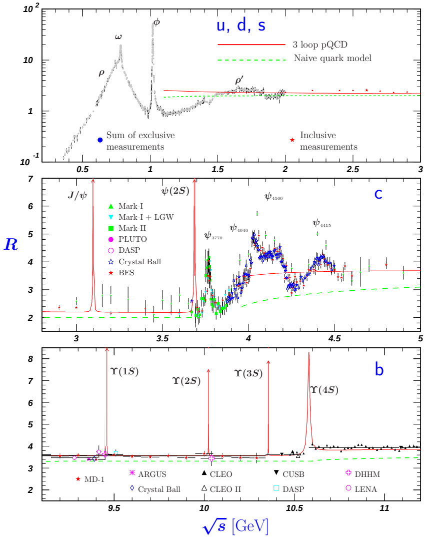


- Describe as $e^+e^- \to q\overline{q}$ where q and $\overline{q}$ turn into hadrons with probability=1
- Same Feynman diagram as $e^+e^- \to \mu^+\mu^-$ except for charge:

$$R = \frac{\sigma(e^+e^- \to hadrons)}{e^+e^- \to \mu^+\mu^-} = N_C \sum_q e_q^2$$

where N_C counts number of color degrees of freedom

$e^+e^- \to hadrons$



$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{e^+e^- \to \mu^+\mu^-}$$

$$= N_C \sum_q e_q^2$$

where N_C is number of colors

- Below ~ 3.1 GeV, only u, d, s quarks produced

$$\sum_q e_q^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2$$

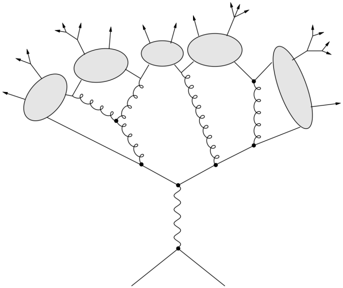
$$= \frac{6}{9} = \frac{2}{3} \Rightarrow N_c = 3$$

- Above 3.1 GeV, charm pairs produced; R increases by $3(\frac{2}{3})^2 = \frac{4}{3}$
- Above 9.4 GeV, bottom pairs produced, R increases by $3(\frac{1}{3})^2 = \frac{1}{3}$

Hadronization and Fragmentation Functions

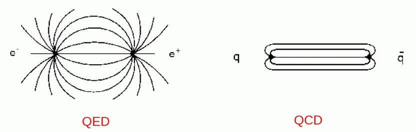
- Define distribution of hadrons using a "fragmentation function":
  - Suppose we want to describe $e^+e^- \to h\,X$ where h is a specific particle (eg π^-)
  - Need probability that a q or $\overline{q}$ will fragment into h
  - Define $D_q^h(z)$ as probability that a quark q will fragment to form a hadron that carries fraction $z = E_h/E_q$ of the initial quark energy
  - We cannot predict $D_q^h(z)$
    - Measure them in one process and then ask are they universal
- These $D_q^h(z)$ are essential for Monte Carlo programs used to predict the hadron level output of a given experiment ("engineering numbers")
- But in the end, what we really care about is how to combine the hadrons to learn about the quarks and gluons they came from

Hadronization as a Showering Process



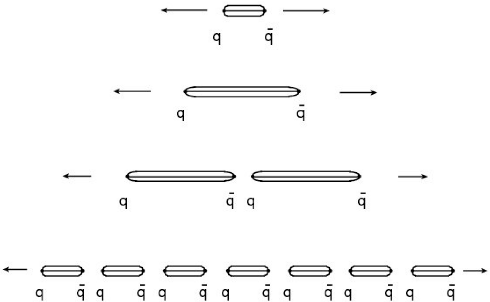
- Similar description to the EM shower
  - Quarks radiate gluons
  - Gluons make $q\overline{q}$ pairs, and can also radiate gluons
- Must in the end produce color singlets
  - Nearby q and $\overline{q}$ combine to form clusters or hadrons
  - Clusters or hadrons then can decay
- Warning: Picture does not make topology of the production clear
  - Gluon radiation peaked in direction of initial partons
  - Expect collimated "jets" of particles following initial partons

Another Way of Thinking About Hadronization



- q and $\overline{q}$ move in opposite directions, creating a color dipole field
- Color Dipole looks different from familiar electric dipole:
  - Confinement: At low energy quarks become confined to hadrons
  - Scale for this confinement, hadronic mass scale: $\Lambda =$ few 100 MeV
  - Coherent effects from multiple gluon emission shield color field far from the colored q and $\overline{q}$
  - Instead of extending through all space, color dipole field is flux tube with limited transverse extent
- Gauss's law in one dimensional field: E independent of x and thus $V(x_1 - x_2) = k(x_1 - x_2)$ where k is a property of the QCD field (often called the "string tension")
  - Experimentally, $k = 1$ GeV/fm $=$ 0.2 GeV^{-2}
  - As the q and $\overline{q}$ separate, the energy in the color field becomes large enough that $q\overline{q}$ pair production can occur
  - This process continues multiple times
  - Neighboring $q\overline{q}$ pairs combine to form hadrons

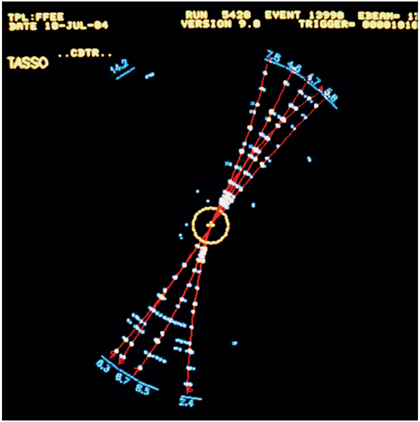
Color Flux Tubes



- Particle production is a stocastic process: the pair production can occur anywhere along the color field
- Quantum numbers are conserved locally in the pair production
- Appearence of the q and $\overline{q}$ is a quantum tunneling phenomenon: $q\overline{q}$ separate eating the color field and appear as physical particles

Here QCD treated as coherent multigluon field
Necessary not only for low q^2 phenomena
but also at high energy or parton density

Jet Production



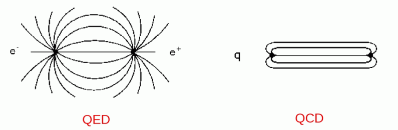
- Probability for producing pair depends quark masses

$$\text{Prob} \propto e^{-m^2 k}$$

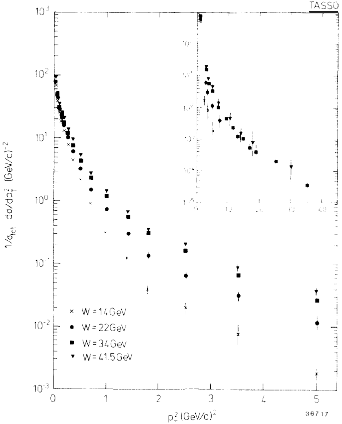
  relative rates of popping different flavors from the field are
  $u : d : s : c = 1 : 1 : 0.37 : 10^{-10}$

- Limited momentum tranverve to $q\overline{q}$ axis
  - If q and $\overline{q}$ each have tranverse momentum $\sim \Lambda$ (think of this as the sigma) the mesons will have $\sim \sqrt{2}\Lambda$
  - Meson transverse momentum (at lowest order) independent of qq center of mass energy
  - As E_{cm} increases, the hadrons collimate: "jets"

Characterizing hadronization using e^+e^- data: Limited Transverse Momentum



- q and $\overline{q}$ move in opposite directions, creating a color dipole field
- Limited p_T wrt jet axis
  - $\sqrt{< p_T^2 >} \sim 350$ MeV
  - Well described by Gaussian distribution
- Range of longitudinal momenta (see next page)



SO [4.1] normalized differential cross section for the square of the momentum component transverse to the jet axis (= sphericity / = 14, 22, 34 and 41.5 GeV.

Characterizing hadronization using e^+e^- data: Rapidity and Longitudinal Momentum

- Define new variable: rapidity

$$y = \frac{1}{2}\ln\frac{E+p_{||}}{E-p_{||}}$$

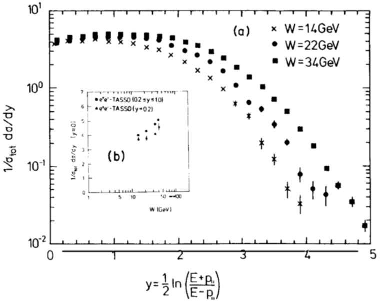
- Phase space with limited transverse momentum:

$$\frac{d^3p}{E} \rightarrow e^{-p_T^2/s\sigma^2}\,dp_T\,\frac{dp_{||}}{E}$$

- But

$$dy = \frac{dp_{||}}{E}$$

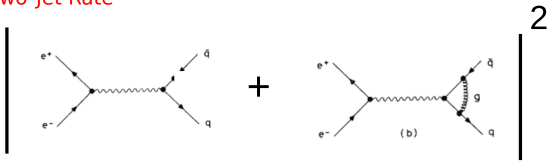
- Rapidity is a longitudinal phase space variable



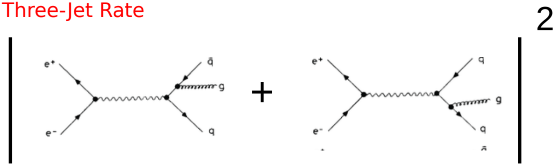
- Particle production flat in rapidity
- y_{max} set by kinematic limit $(E-p_{||}) \geq m_h$
- Height of plateau independent of $\sqrt{s}$
  - Multiplicity increase due to change in y_{max}
  - $< N_h > \sim \ln(\frac{E_{cm}}{m_h})$

QCD corrections to $e^+e^- \rightarrow hadrons$

Two-Jet Rate



Three-Jet Rate



- Two- and Three-jet rates separately diverge
- Sum of the two converge (see next page)
- Can only define sensible three-jet rate with a cutoff in 3^{rd} jet energy and angle

First Order QCD: Jet rates

- Using gluon mass to regularize (very old fashioned approach!):

$$\begin{array}{ll} \text{2 jet :} & \sigma_0(1 + \frac{\alpha_s}{2\pi}\frac{4}{3}\left\{-\ln^2\left(\frac{m_g}{Q}\right) - 3\ln\left(\frac{m_g}{Q}\right) + \frac{\pi^2}{3} - \frac{7}{2}\right\} \\ \text{3 jet :} & \sigma_0\frac{\alpha_s}{2\pi}\frac{4}{3}\left\{\ln^2\left(\frac{m_g}{Q}\right) + 3\ln\left(\frac{m_g}{Q}\right) - \frac{\pi^2}{3} + 5\right\} \\ \text{Sum :} & \sigma_0\left(1 + \frac{\alpha_s}{\pi}\right) \end{array}$$

(see Halzen & Martin pg 244)

- Cancellation of divergences not an accident
- Occurs hroughout gauge theories (QED as well as QCD)
- Cancellation of infrared divergences described using general theorm by Kinoshita, Lee and Nauenberg
- In practice, divergences in 2 and 3 jet rates NOT a problem
  - Can only distinguish two jets if they are separated in angle and both jets have measurable energy.

Calculating the 3-jet rate in region away from singularity

- Define the energy fractions of the 3 jets

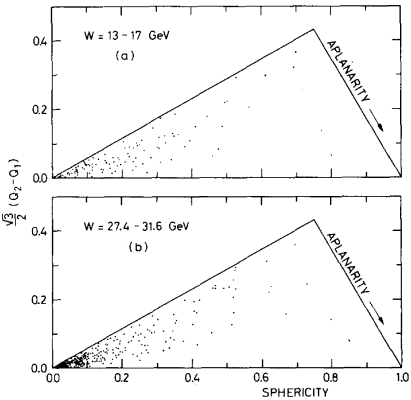
$$x_q = \frac{2E_q}{\sqrt{s}}; \quad x_{\overline{q}} = \frac{2E_{\overline{q}}}{\sqrt{s}}; \quad x_g = \frac{2E_g}{\sqrt{s}};$$

- Conservation of energy: $x_q + x_{\overline{q}} + x_g = 2$
- In practice, don't know which is the q, $\overline{q}$, g
- Order them in momentum

$$\frac{d\sigma_{3\ jet}}{dx_1 dx_2} = \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- Note: σ diverges if $x_1 \to 1$ or $x_2 \to 1$

Searching for 3 jet events using the Sphericity Tensor



$Q_1 + Q_2 + Q_3 = 1$

Sphericity $S = \frac{3}{2}(Q_1 + Q_2)$

Aplanarity $A = \frac{3}{2}Q_1$

- As the energy increases, the narrowing of the jets allows us to look for cases of wide angle gluon emission (3-jet events)
- QCD brem cross section diverges for colinear gluons or when the gluon momentum goes to zero
  - But that is the case where we can't distinguish 2 and 3 jet events anyway
  - Total cross section is finite (QCD corrections to R)
- Can use the sphericity tensor to search for 3-jet events
- Similar searches using a thrust-like variable possible: see next page

Thrust-like Energy Flow Method

- For each particle define an "energy flow vector"

$$\vec{E}_i = (E_i/|\vec{p}_i|)\, p_i$$

- Unit vector $\hat{e}_1$ analogout to Thrust T is:

$$F_{thrust} = \max \frac{\sum_i |\vec{E}_i \cdot \hat{e}_1}{\sum_i E_i}$$

- Orthogonal axes defined as

$$F_{major} = \max \frac{\sum_i |\vec{E}_i \cdot \hat{e}_2}{\sum_i E_i} \quad \hat{e}_2 \perp \hat{e}_1$$

and

$$\hat{e}_3 = \hat{e}_1 \times \hat{e}_2$$

Global variables such as energy-flow and sphericity
are called "shape-variables"

Jet Finding Algorithms

- Shape variables like Thrust have advantage that they allow tests with minimal sensitivity to hadronization
- But don't allow us to study multijets well
- Need an algorithm to decide how many jets we have and associate particles with the jets
  - Algorithm will have some parameter to handle the infrared divergence (eg a cut-off)

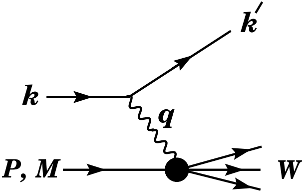
What is important in a jet-finding algorithm?

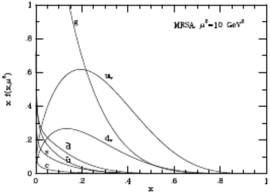
- Should combine particles (or energy clusters) into jets in a way that agrees with what we see "by eye" in straightforward cases
  - Avoid pathologies (turns out this isn't easy)
- Should be insenstive to details of the hadronization
  - If a particle decays, calculation using parent and daughters should give nearly the same answer
- Should be possible to apply same algorithm to the quarks and gluons that are the outgoing "particles" in a QCD calculation (before hadronization)
  - Should not have divergences for colinear or soft emission: "Colinear and Infra-red safe"

Jet finding algorithms a good topic for
a student talk

Parton Distribution Functions

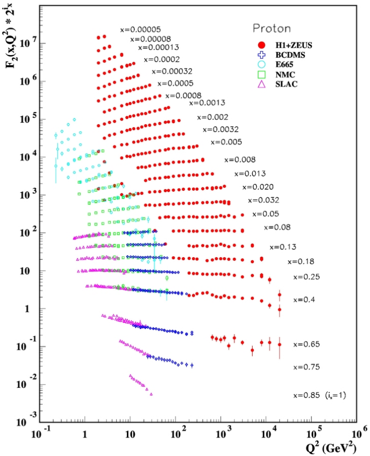
- Have already seen that fragmentation can be described using a phenomenological function
  - Assumed to be process independent (and experimental tests confirm this)
  - Changes logrithmically with momentum transfer (gluon brem)
- Similarly, initial "wave function" of hadrons cannot be calculated but are measured in one process and used in other calculations

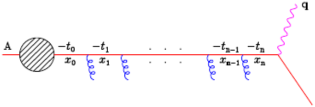




  - Parton distributions functions largely measured in DIS
  - Used as input to calculate cross sections in pp collisions
  - Dependence on momentum transfer can be calculated perturbatively (Altarelli-Parisi). See next page

Modern $F_2(x, Q^2)$ Measurements (Scaling Violations)





- Dependence on Q^2 calculated using coupled differential-integral equations describing gluon brem from quarks and gluon splitting into quarks

Another example where hadronic wave function matters

Measuring CKM matrix elements using B decays

- Heavy b-quark bound inside a hadron
- Effects of binding characterized using "Form Factors"
- Heavy quark effective theory method used to characterize these effects

Lot's of good topics for talks

The Next Few Weeks

- Ian Hinchliffe: QCD Status and Issues
- Feng Yuan: Deep Inelastic Scattering and the Parton Model
- Barbara Jacak Hot QCD
- TBC: Lattice QCD

After that, it's your turn