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Resummation for Photon Isolation

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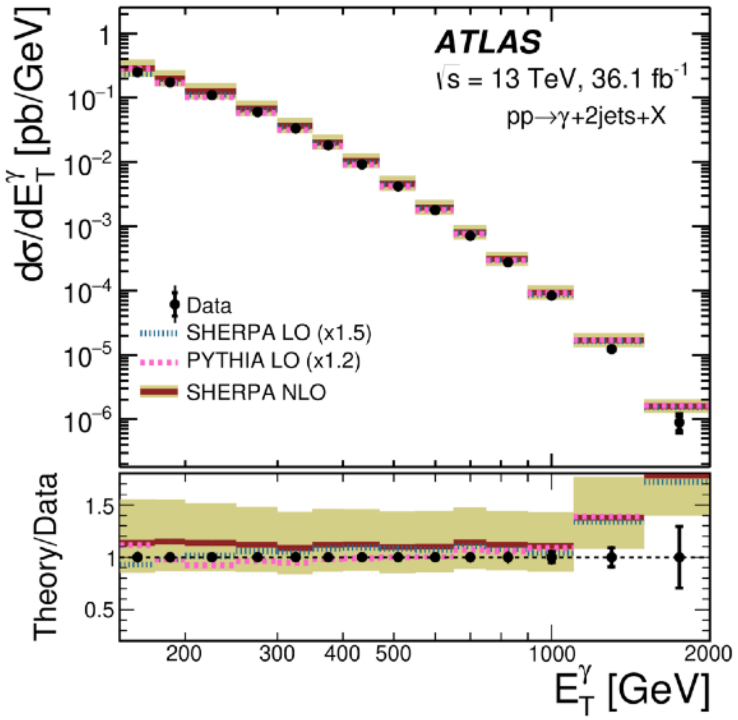
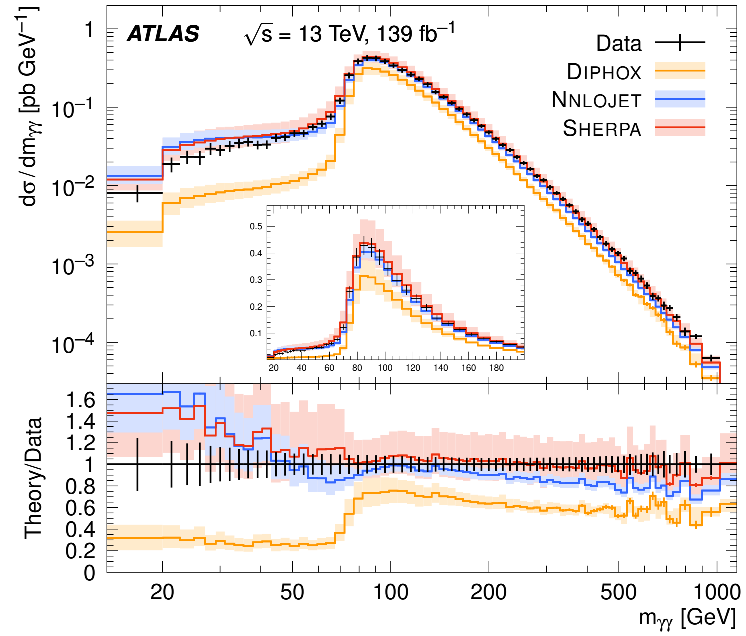
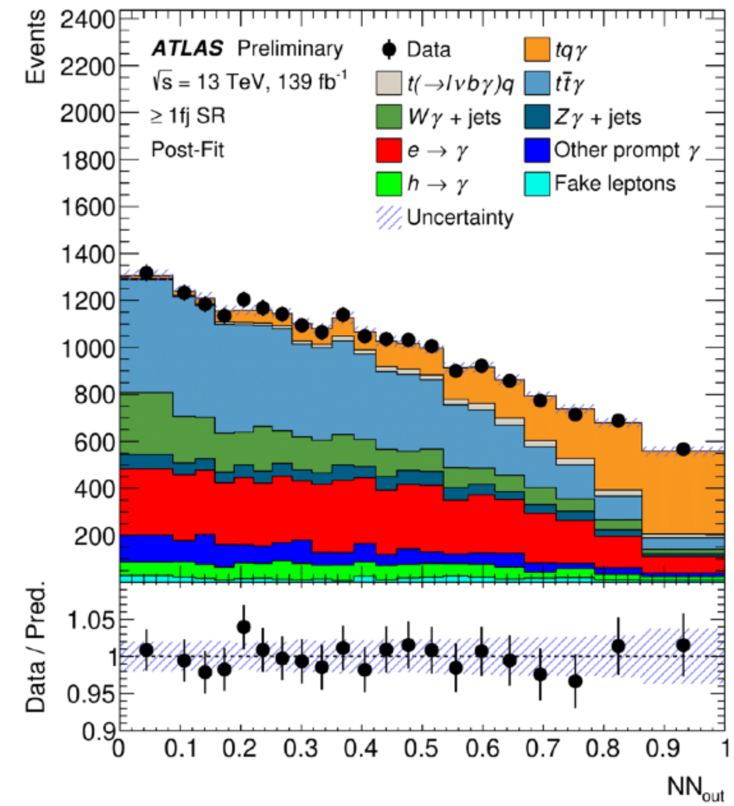
March 30, 2023



photon + 2 jets



diphotons



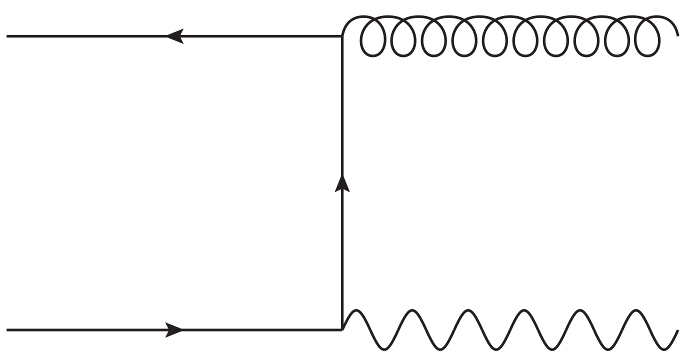
photon + top

Hard photons are a fundamental probe of short-distance physics, of interest both in the context of SM and BSM physics …

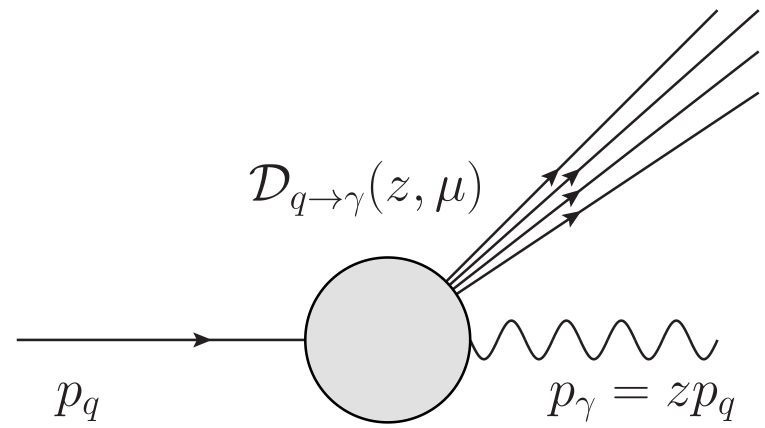
… but also pose experimental and theoretical challenges.

Challenges

- Fragmentation



direct



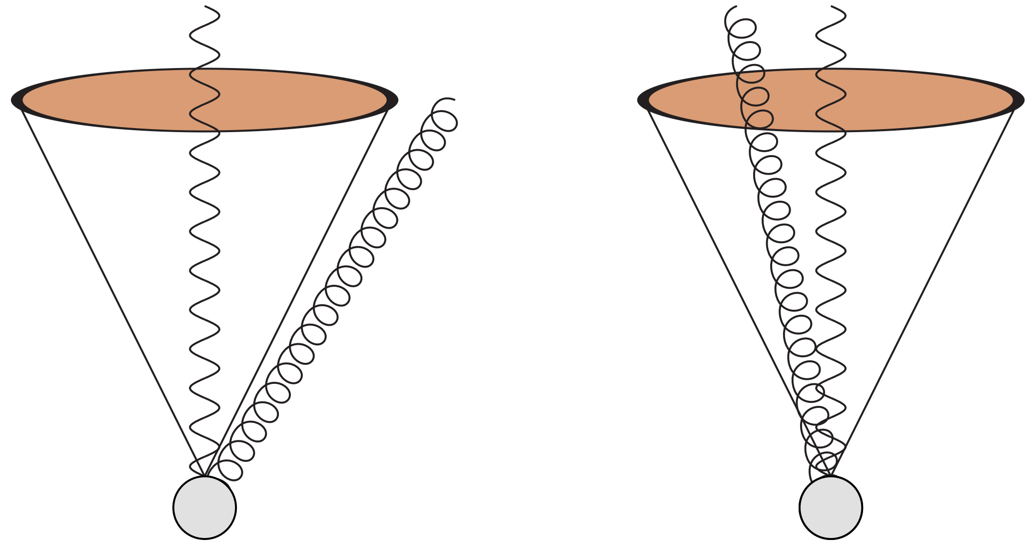
fragmentation

non-perturbative & poorly known

- Background from decays of energetic π^0 and η, producing collimated photon pairs
  - Isolate photon from hadronic radiation to suppress this background → large logs

Fixed-energy cone isolation

Isolation-cone of radius R



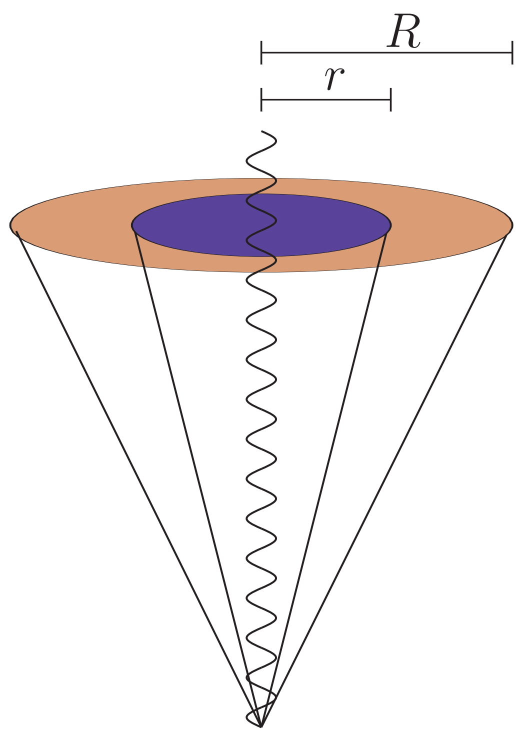
- Traditionally $E^T_{\mathrm{cone}}(R) < E_0 = \epsilon_\gamma E^T_\gamma$
- ATLAS sets $E_0 = \epsilon\, E^T_\gamma + E^T_{\mathrm{th}}$ with

$$\epsilon = 0.0042 \quad \text{with} \quad E^T_{\mathrm{th}} = 4.8\,\mathrm{GeV}$$

- Fragmentation contributes to cross section
- All LHC measurements use fixed cone.

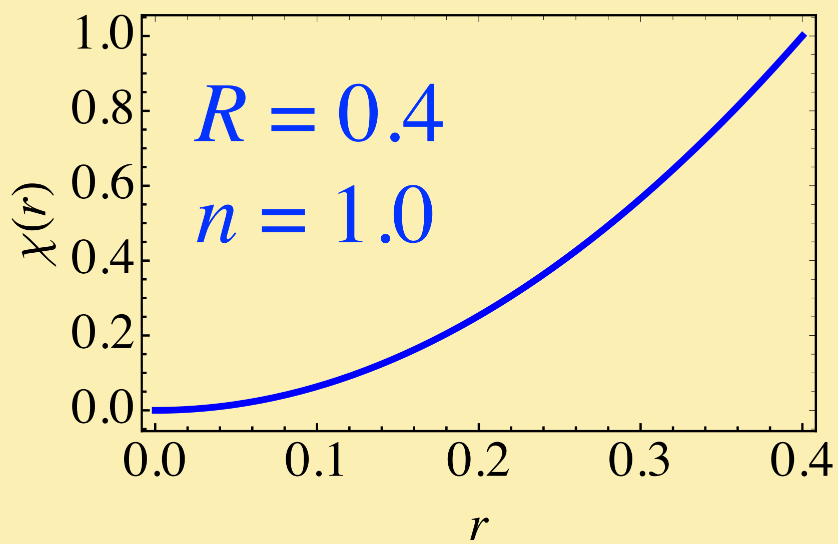
Smooth-cone isolation

Frixione ‘98



$$E^T_{\text{cone}}(r) < E_0(r) = \epsilon_\gamma E^T_\gamma\,\chi(r) \text{ for all } r < R$$

$$\chi(r) = \left(\frac{1-\cos r}{1-\cos R}\right)^n$$



- No energetic collinear radiation → no fragmentation
  - big technical simplification for NNLO computations
- Experimentally not directly realizable. For a study of discretized version, see hep-ph/1003.1241

NLO predictions

Publicly available fixed-cone NLO only from

- Jetphox (Catani et al. ’99), Diphox (Binoth et al. ’99)
  - no longer actively maintained
- MCFM since 2011

Fragmentation functions (and related code) are 25 years old, based on simple models.

Other NLO codes such as MG5_aMC@NLO restricted to smooth-cone isolation.

Have verified (thanks to Alex Huss!) that different codes produce compatible reference cross sections.

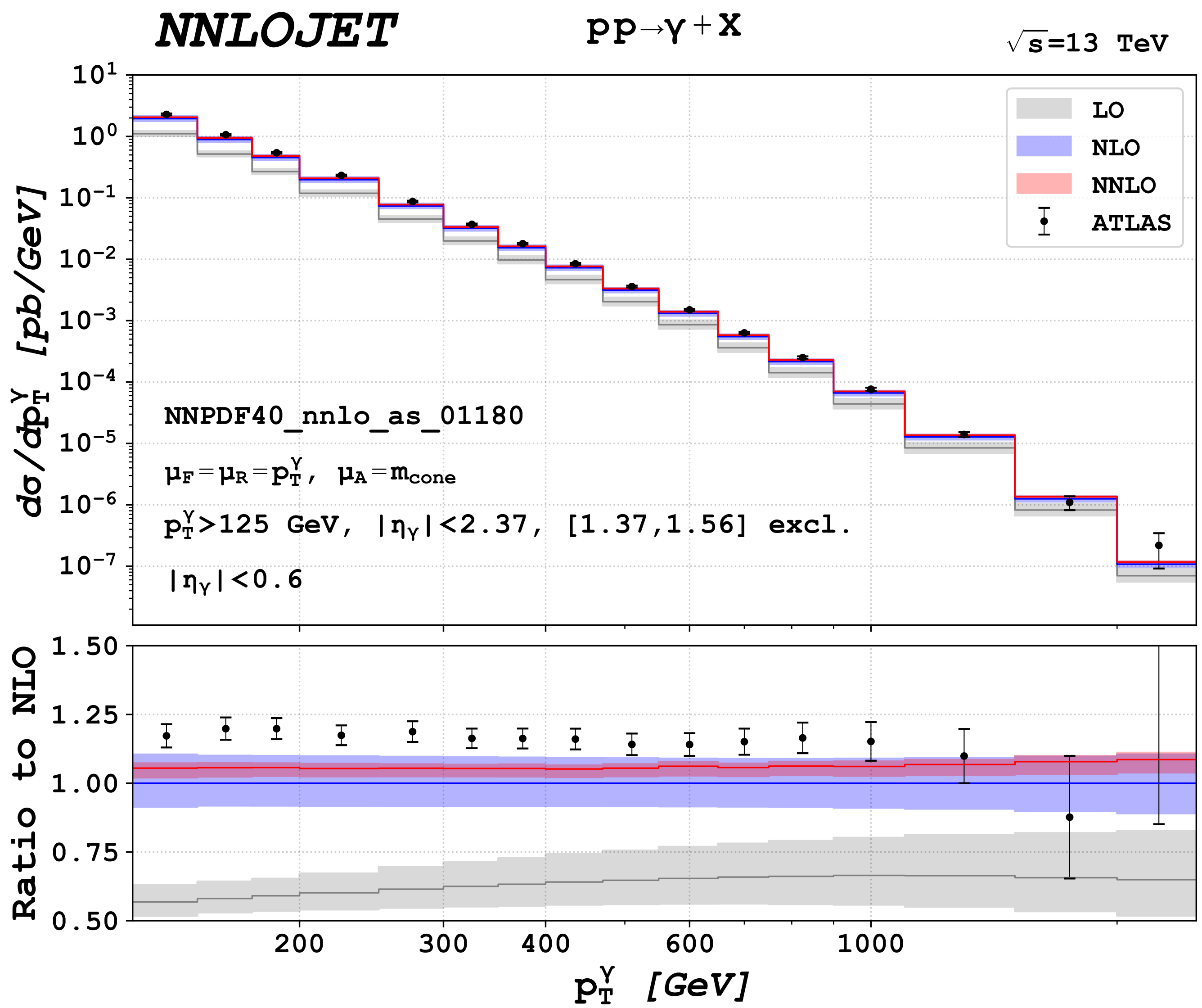
NNLO predictions

- prompt photon Campbell et al. '17, Chen et al. 19
- diphoton Catani et al. 11, Campbell et al. 16, Gehrmann et al. '20
- tri-photon Chawdhry et al. '20, Kallweit et al. '20

but before last year only with smooth-cone isolation

- papers choose values of n and ε_γ that give similar NLO values as fixed-cone isolation
- unknown systematic uncertainty, unsatisfactory in view of the few % accuracy of measurements

Proposal to use fixed cone with smooth cone in the center ``hybrid cone" but not completely satisfactory. Gehrmann et. al. '21



New: First fixed-cone NNLO results Chen, Gehrmann, Glover, Höfer, Huss, Schürmann ’22 using antenna subtraction; extension to fragmentation: Gehrmann, Schürmann ’22.

Outline

Isolation requirement induces small parameters into cross section

- higher-order corrections enhanced by powers of $\ln(\varepsilon_\gamma)$ and $\ln(R)$
- will illustrate at NLO that this can lead to a breakdown of the fixed-order expansion

Factorization of isolation effects for small R using SCET yields

- simple analytic understanding of isolation effects
- resummation of $\ln(\varepsilon_\gamma)$ and $\ln(R)$ using RG evolution
- relation between smooth- and fixed-cone in the limit of small ε_γ

Motivation: pathologies of NLO perturbation theory

For all cross section computations we will use

$E_T^\gamma > E_T^{\min} = 125\,\mathrm{GeV}$	$\lvert\eta_\gamma\rvert < 2.37$
$\alpha_s(M_Z) = 0.119$	$\alpha_{\mathrm{EM}} = 1/132.507$
$\sqrt{s} = 13\,\mathrm{TeV}$	NNPDF23_nlo_as_0119_qed_mc

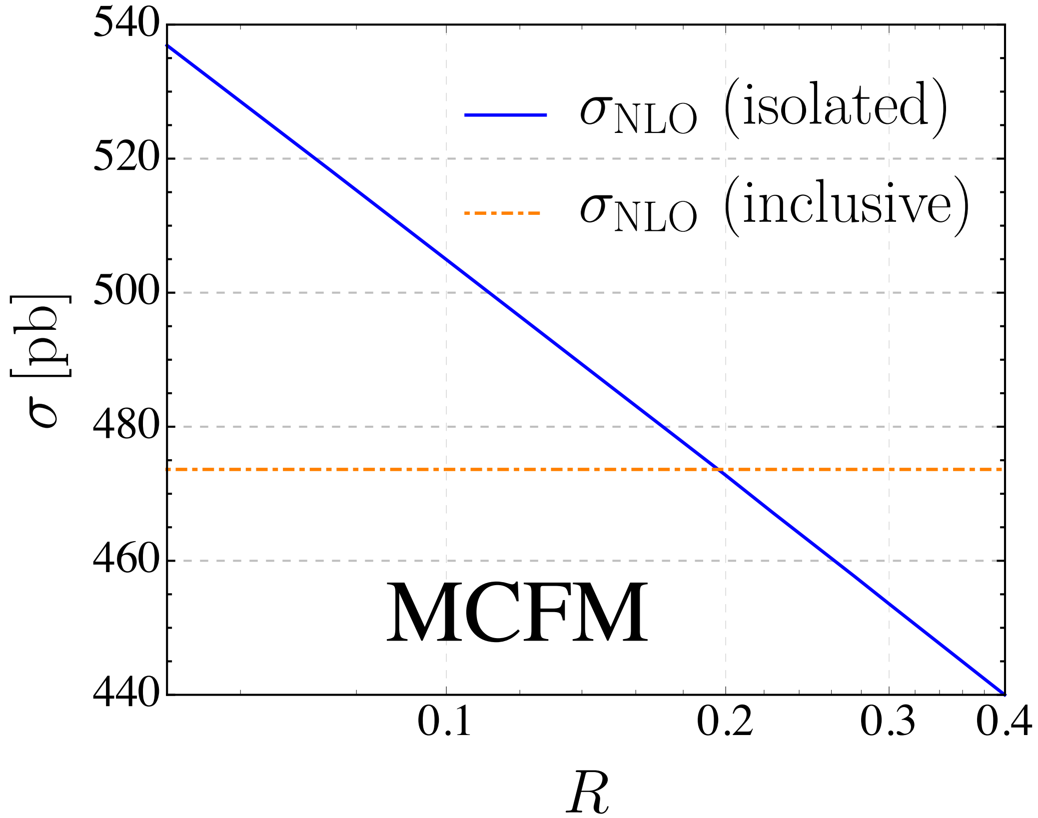
and for fixed-order results we set

$$\mu_f = \mu_r = 125\,\mathrm{GeV}$$

Fixed-cone results involve fragmentation functions and associated scale. For fixed-order, we set

$$\mu_a = 125\,\mathrm{GeV}$$

Fixed-Order Pathologies (I)

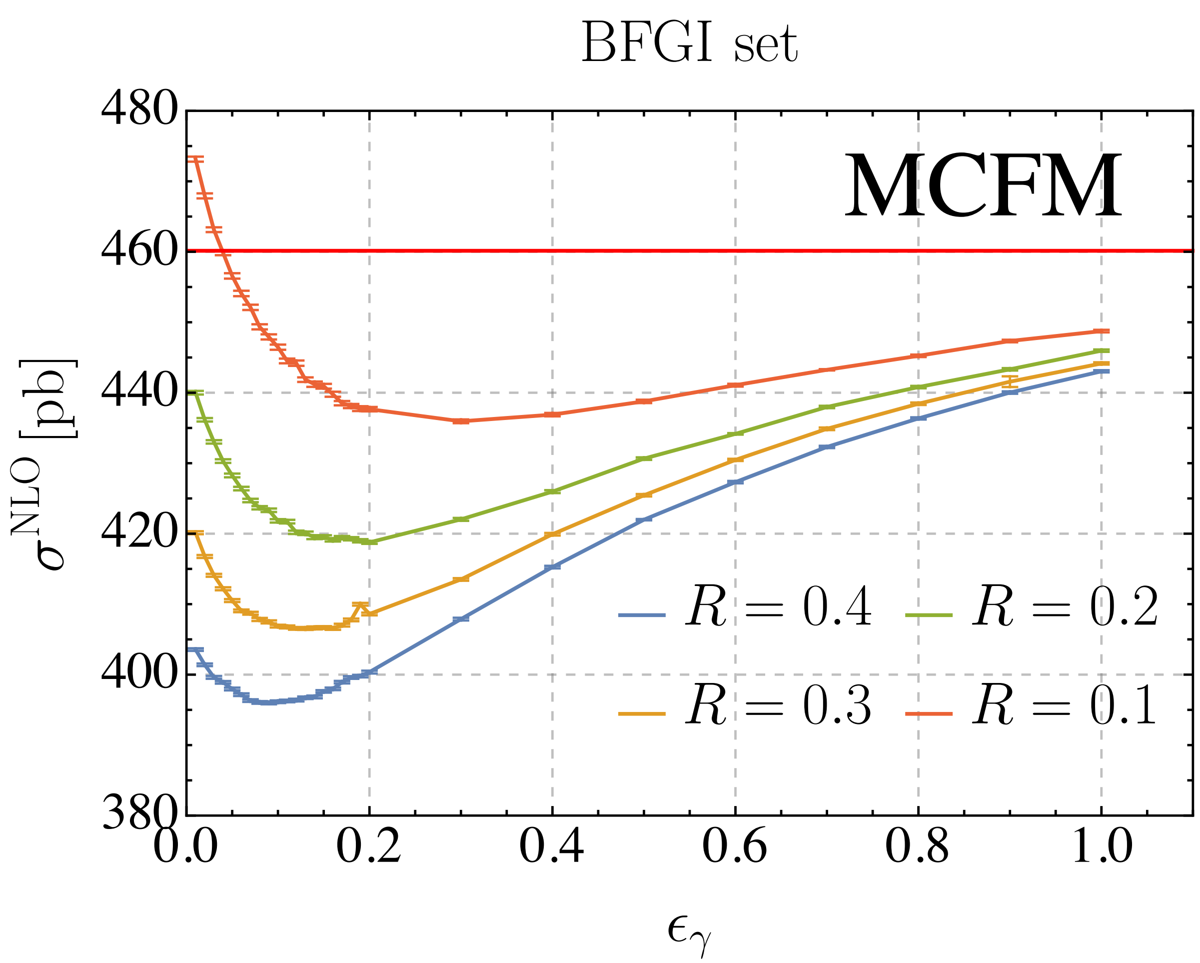


σ(isolated) with smooth-cone, $n = 1$, $\varepsilon_\gamma = 1$

σ(inclusive) with Gehrmann de Ridder, Glover '98 fragmentation functions

- *Should* have: σ(isolated) < σ(inclusive) but at NLO, the isolation dependent part of cross section is proportional to ln(R)
  - Breakdown of FOPT for $R \lesssim 0.2$! $R = 0.2$ is the default value for ATLAS diphoton analyses
  - Same breakdown arises for fixed-cone isolation Catani, Fontannaz, Guillet and Pilon in JHEP 05, 028 (2002)

Fixed-Order Pathologies (II)

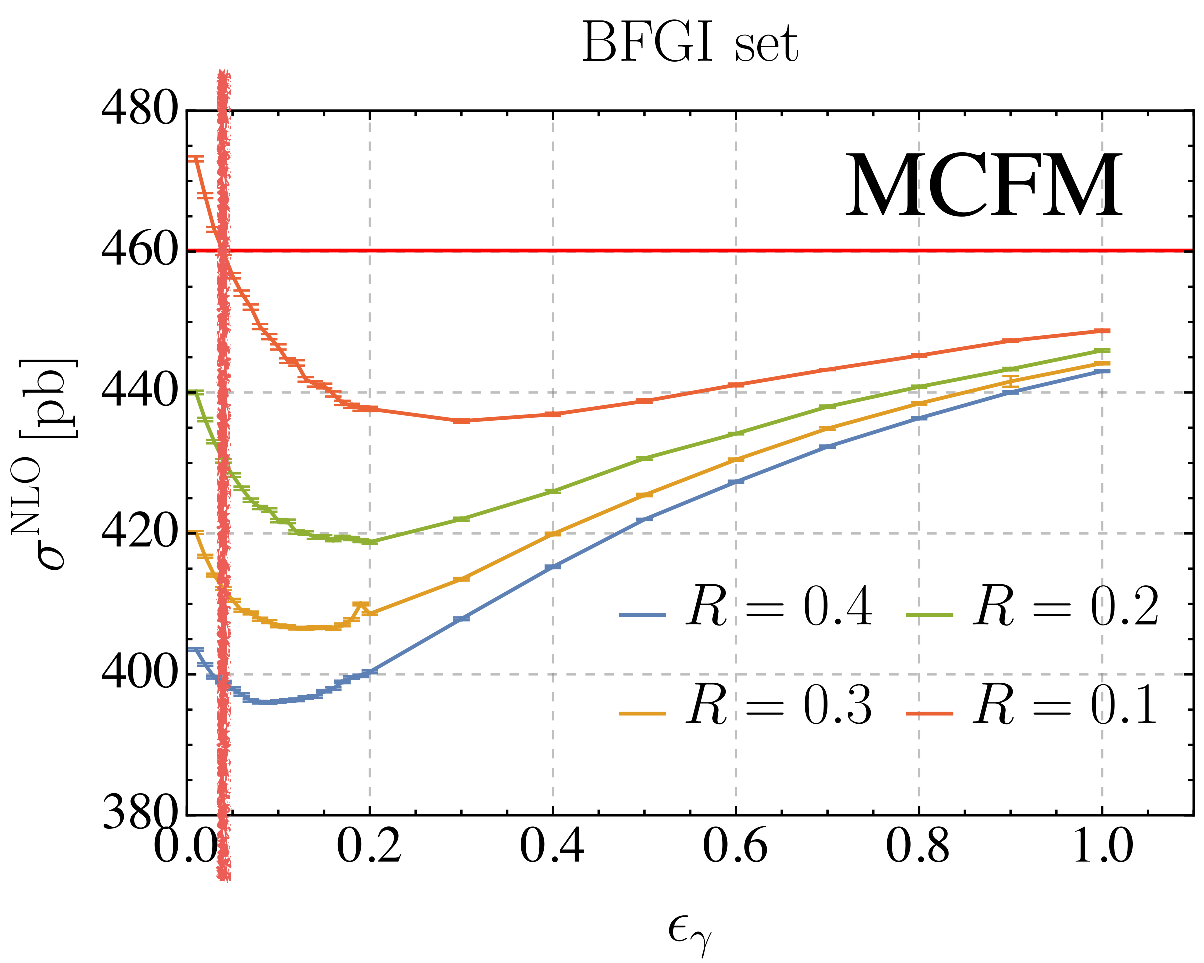


σ(isolated) with fixed-cone isolation.

BFG (Bourhis, Fontannaz and Guillet, '98) fragmentation functions

- σ(isolated) *should* monotonically decrease as ε_γ is lowered
- NLO isolation effects are linear in ε_γ for small ε_γ (soft quark…)
  - coefficient enhanced by ln(*R*), unphysical for small *R*
- ATLAS isolation corresponds to $\varepsilon_\gamma = 0.04$ for $E_T^\gamma = 125\,\mathrm{GeV}$

Fixed-Order Pathologies (II)



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Factorization and resummation for small cone radius R

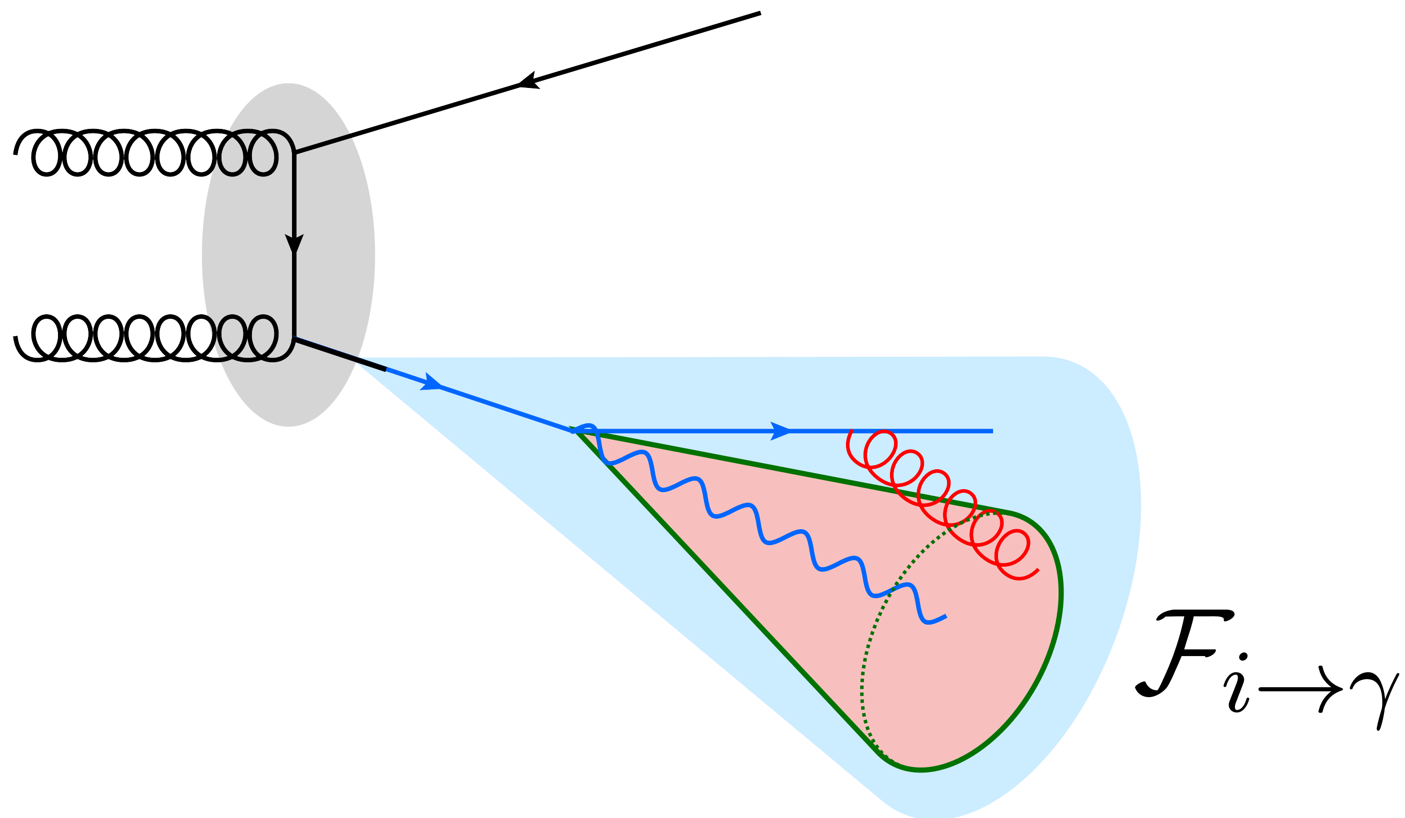
Factorization

Becher, Favrod, Xu, 2208.01554

For small R all isolation effects can be factorized into a cone fragmentation function $\mathcal{F}_{i\to\gamma}$

$$\frac{\mathrm{d}\sigma(E_0,R)}{\mathrm{d}E_\gamma} = \frac{\mathrm{d}\sigma^{\mathrm{dir}}_{\gamma+X}}{\mathrm{d}E_\gamma} + \sum_{i=q,\bar{q},g} \int dz \frac{\mathrm{d}\sigma_{i+X}}{\mathrm{d}E_i} \mathcal{F}_{i\to\gamma}(z,E_\gamma,E_0,R) + \mathcal{O}(R)$$

Analogous to factorization of non-perturbative effects, but $\mathcal{F}_{i\to\gamma}$ includes perturbative part associated with isolation.



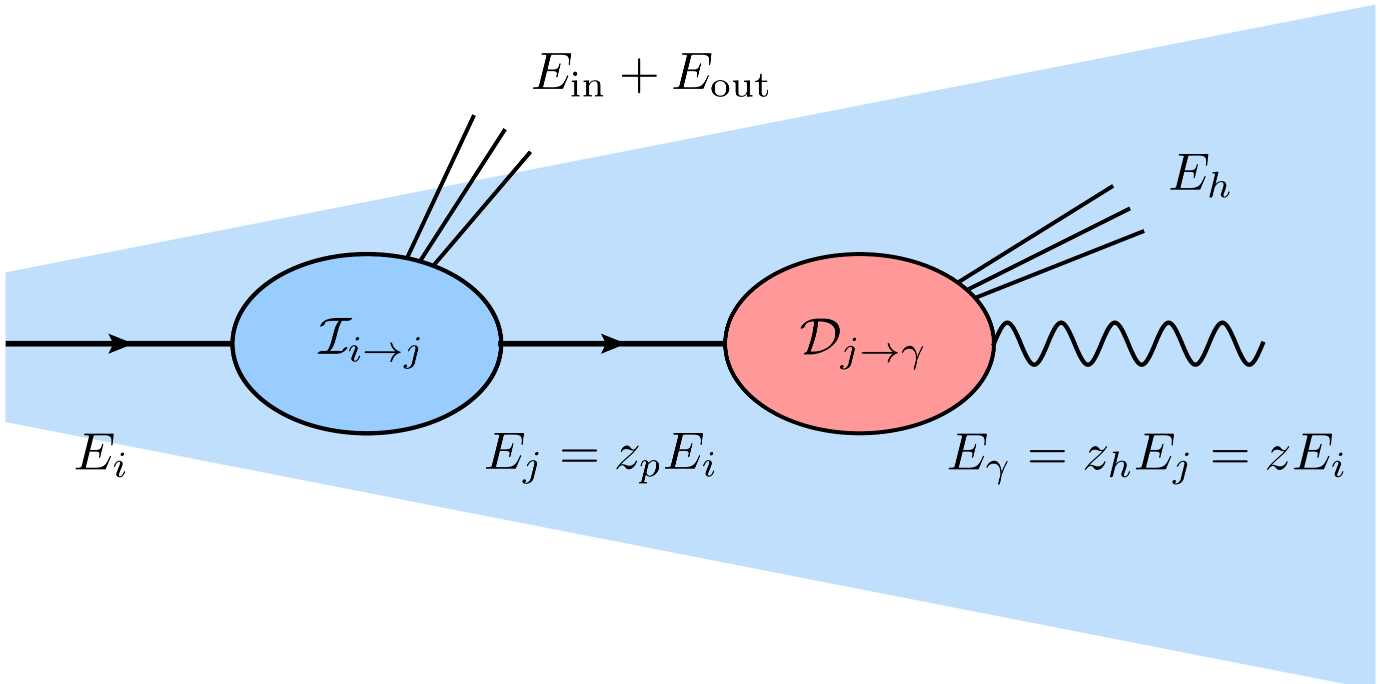
Cone fragmentation function $\mathcal{F}_{i\to\gamma}$ contains all particles collinear to photon

$$\frac{\mathrm{d}\sigma(E_0,R)}{\mathrm{d}E_\gamma} = \frac{\mathrm{d}\sigma^{\mathrm{dir}}_{\gamma+X}}{\mathrm{d}E_\gamma} + \sum_{i=q,\bar{q},g} \int dz \frac{\mathrm{d}\sigma_{i+X}}{\mathrm{d}E_i} \mathcal{F}_{i\to\gamma}(z,E_\gamma,E_0,R) + \mathcal{O}(R)$$

inclusive, direct

all isolation effects + NP frag.

Cone fragmentation function $\mathcal{F}_{i\to\gamma}$



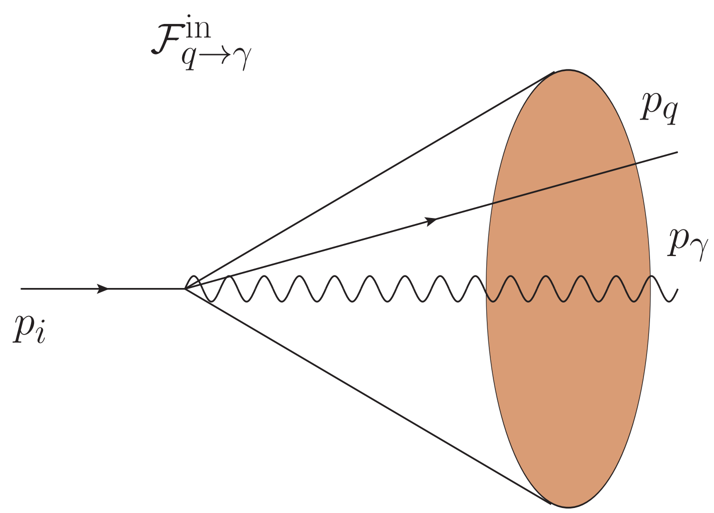
$$\mathcal{F}_{i\to\gamma}(z,\mu) = \sum_{j=\gamma,q,\bar{q},g} \mathcal{I}_{i\to j}(E_\gamma R, E_0 R, \mu) \otimes \mathcal{D}_{j\to\gamma}(\mu)$$

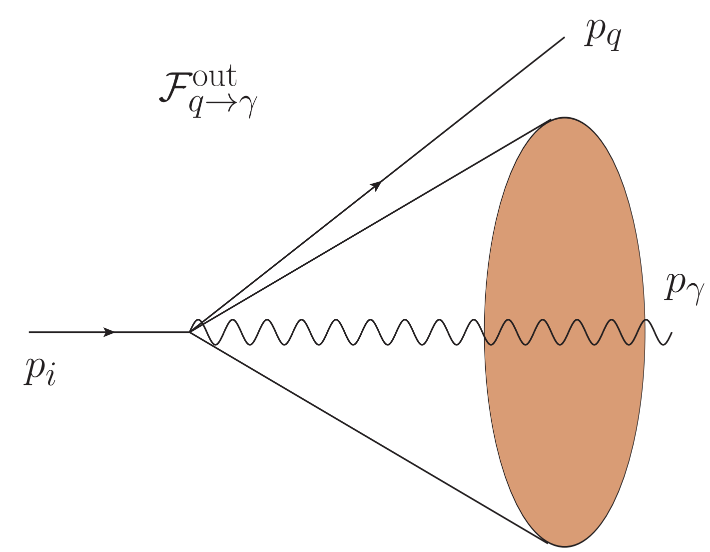
non-pert. fragmentation

perturbative, scales $E_\gamma\, R$ and $E_0\, R$

NLO cone fragmentation functions

$$\mathcal{F}_{q\to\gamma}(z,E_\gamma,E_0,R,\mu)=\mathcal{F}^{\text{in}}_{q\to\gamma}(z,E_\gamma,E_0,R,\mu)+\mathcal{F}^{\text{out}}_{q\to\gamma}(z,R\,E_\gamma,\mu)$$





outside part is independent of isolation

$$\mathcal{F}^{\text{out}}_{q\to\gamma}(z,R\,E_\gamma)=\frac{\alpha_{\text{EM}}\,Q_q^2}{2\pi}\left\{-P(z)\ln\left(\frac{R^2(2E_T^\gamma)^2}{\mu^2}(1-z)^2\right)-z\right\}$$

quark to photon splitting function $P(z)=\dfrac{1+(1-z)^2}{z}$

Inside part

Smooth-cone isolation

$$\mathcal{F}^{\text{in}}_{q\to\gamma}(z, E_\gamma, E_0, R, \mu) = \frac{\alpha_{\text{EM}} Q_q^2}{2\pi} P(z) \frac{1}{n} \ln\left(\frac{z\,\epsilon_\gamma}{1-z}\right) \theta\left(z - \frac{1}{1+\epsilon_\gamma}\right)$$

Note *R* independence!

Fixed-cone isolation

$$\mathcal{F}^{\text{in}}_{i\to\gamma}(z, R, E_\gamma,\, E_0, \mu) = \left[\mathcal{D}_{i\to\gamma}(z,\mu) + \sum_{k=q,\bar{q}} \delta_{ik}\, \mathcal{I}^{\text{in}}_{k\to\gamma}(z, R, E_\gamma, \mu)\right] \theta\left(z - \frac{1}{1+\epsilon_\gamma}\right)$$

$$\mathcal{I}^{\text{in}}_{q\to\gamma}(z, R, E_\gamma, \mu) = \frac{\alpha_{\text{EM}}\, Q_q^2}{2\pi} \left\{P(z) \ln\left(\frac{R^2 (2E_T^\gamma)^2}{\mu^2}(1-z)^2\right) + z\right\}$$

Isolation parameter dependence

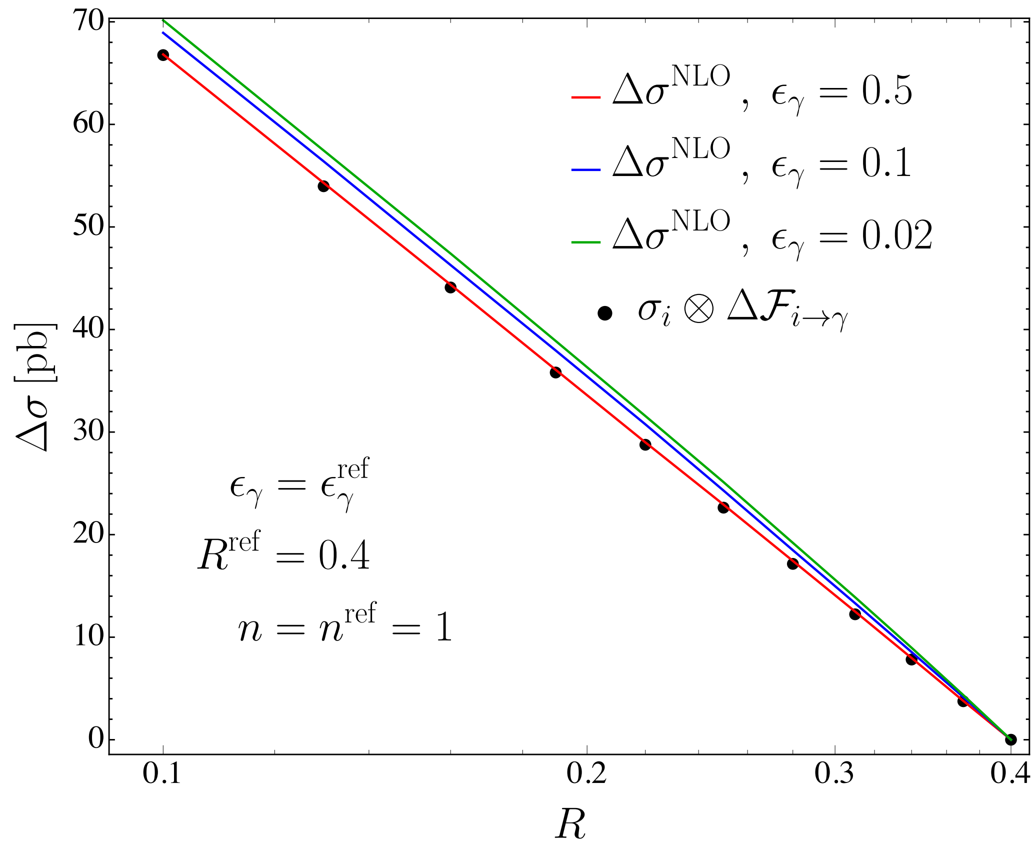
Interesting to look at difference to reference cross section

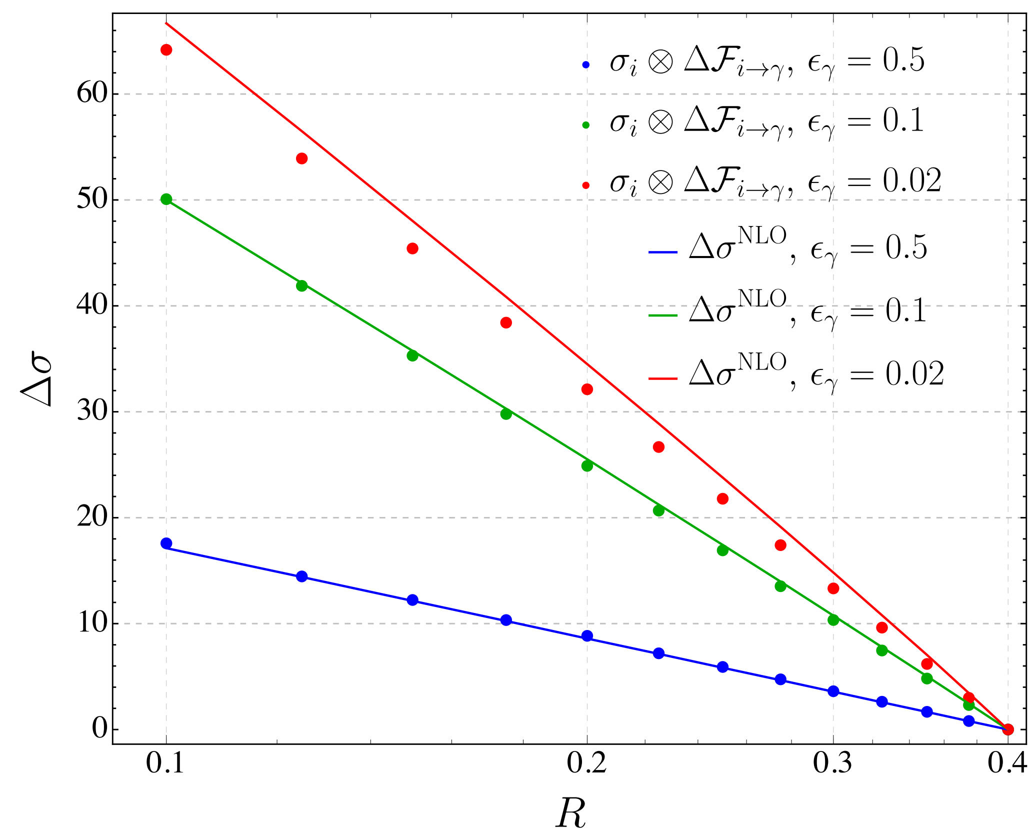
$$\Delta\sigma = \sigma\left(\epsilon_\gamma, n, R\right) - \sigma(\epsilon_\gamma^{\mathrm{ref}}, n^{\mathrm{ref}}, R^{\mathrm{ref}})$$

since direct part drops out:

$$\Delta\sigma = \sum_{i=q,\bar{q}} \int_{E_T^{\mathrm{min}}}^{\infty} dE_i \int_{z_{\mathrm{min}}}^{1} dz \frac{d\sigma_{i+X}}{dE_i} \Delta\mathcal{F}_{i\to\gamma}$$

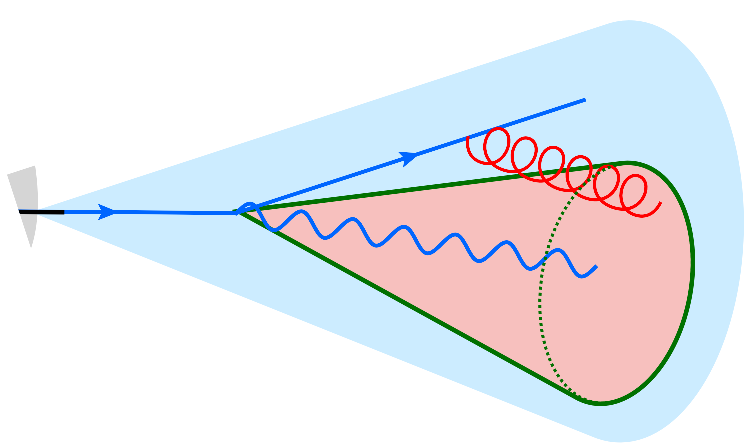
Smooth- vs fixed-cone isolation





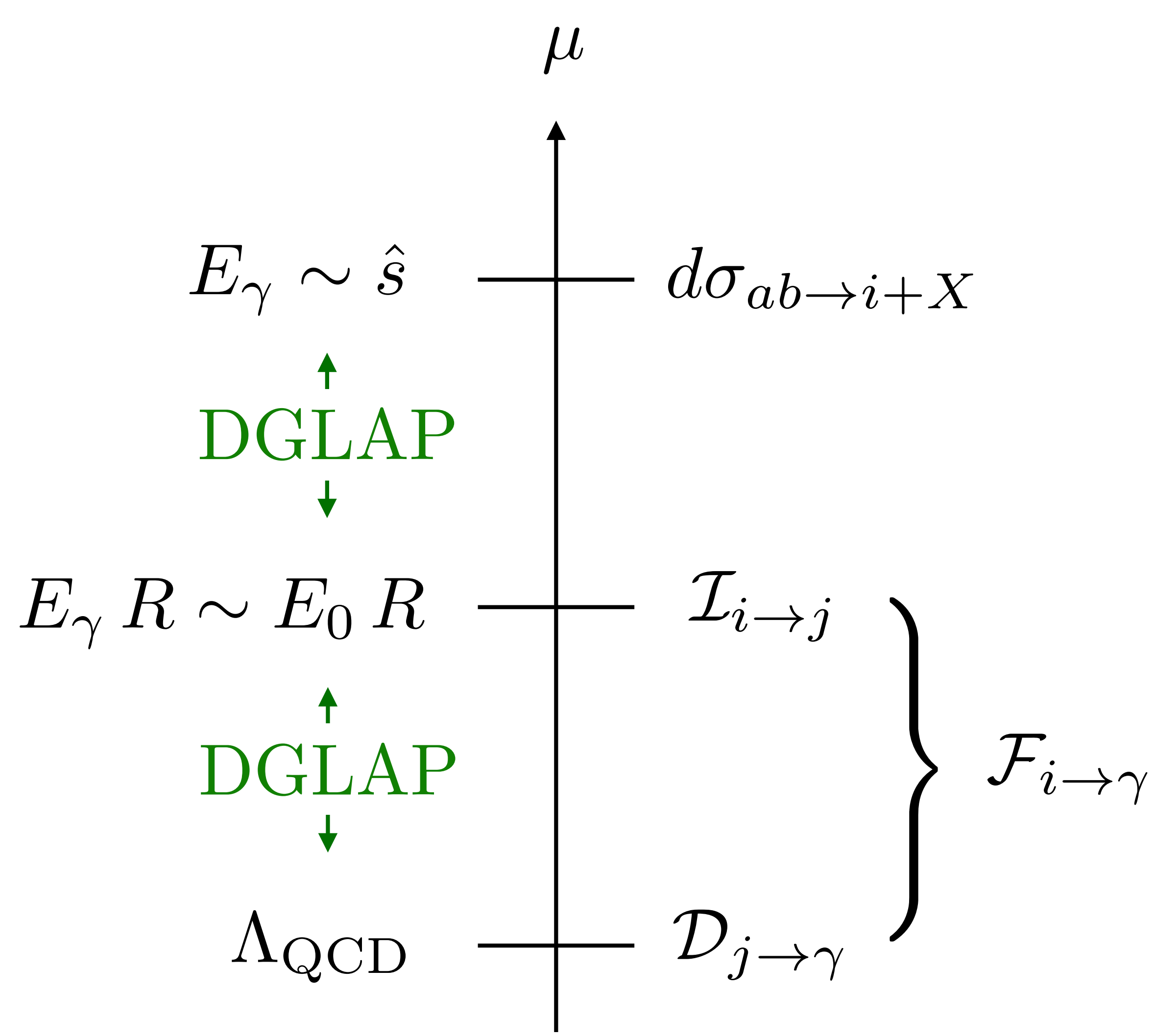
- For fixed cone also inside part of $\mathcal{F}_{i\to\gamma}$ has ln(*R*) contribution, which is ε_γ dependent.
    - For $\varepsilon_\gamma \to 0$ inside part vanishes and one recovers smooth-cone *R*-dep!

- More generally: for small ε_γ the inside part at NLO becomes small
  - Non-perturbative fragmentation suppressed by ε_γ
- and at NLO the following properties hold
  - ln(R) dependence only from outside part
  - All isolation prescriptions become identical!
- but at NNLO differences from out-in terms!

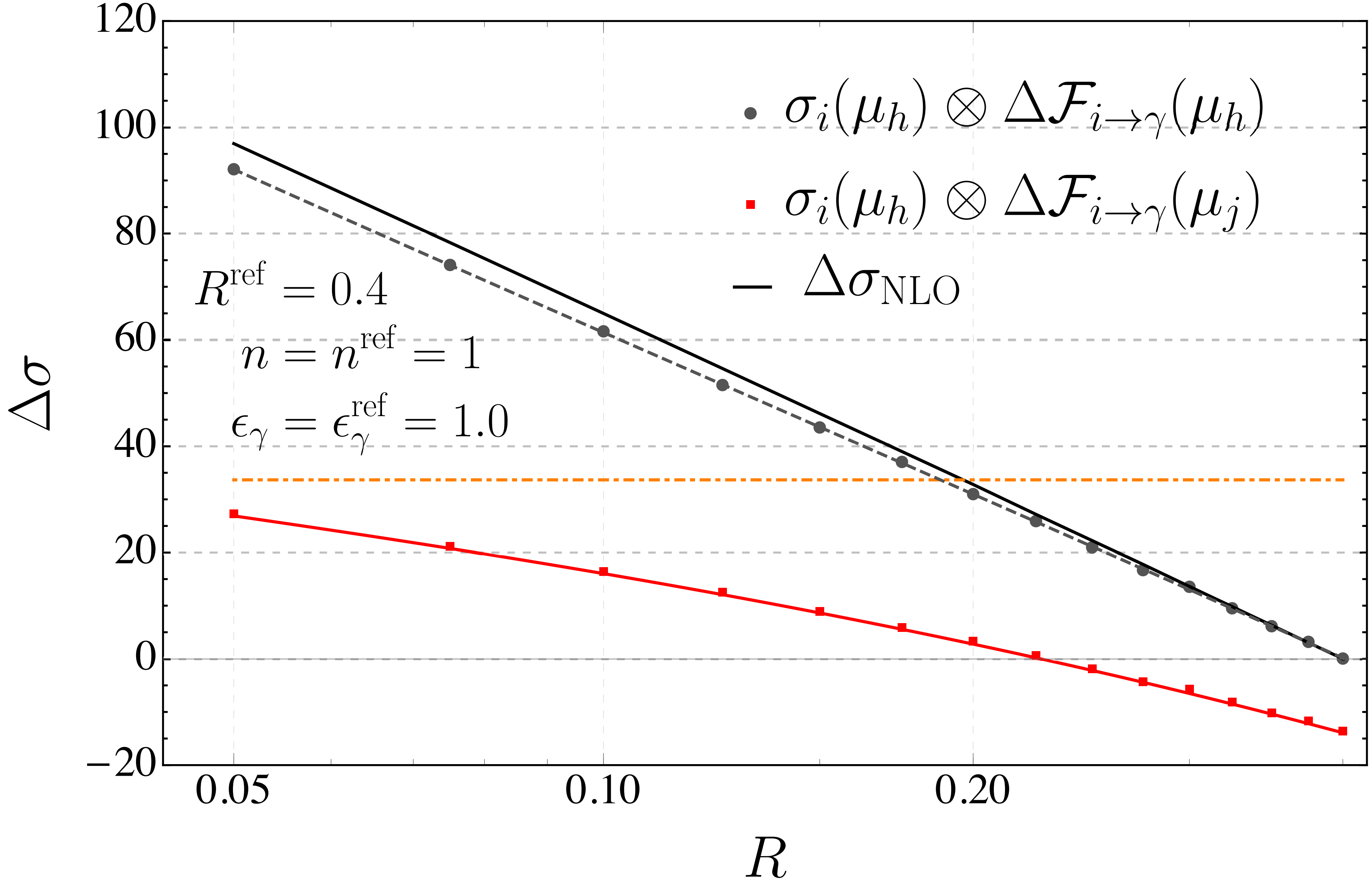


Resummation of ln(R) terms

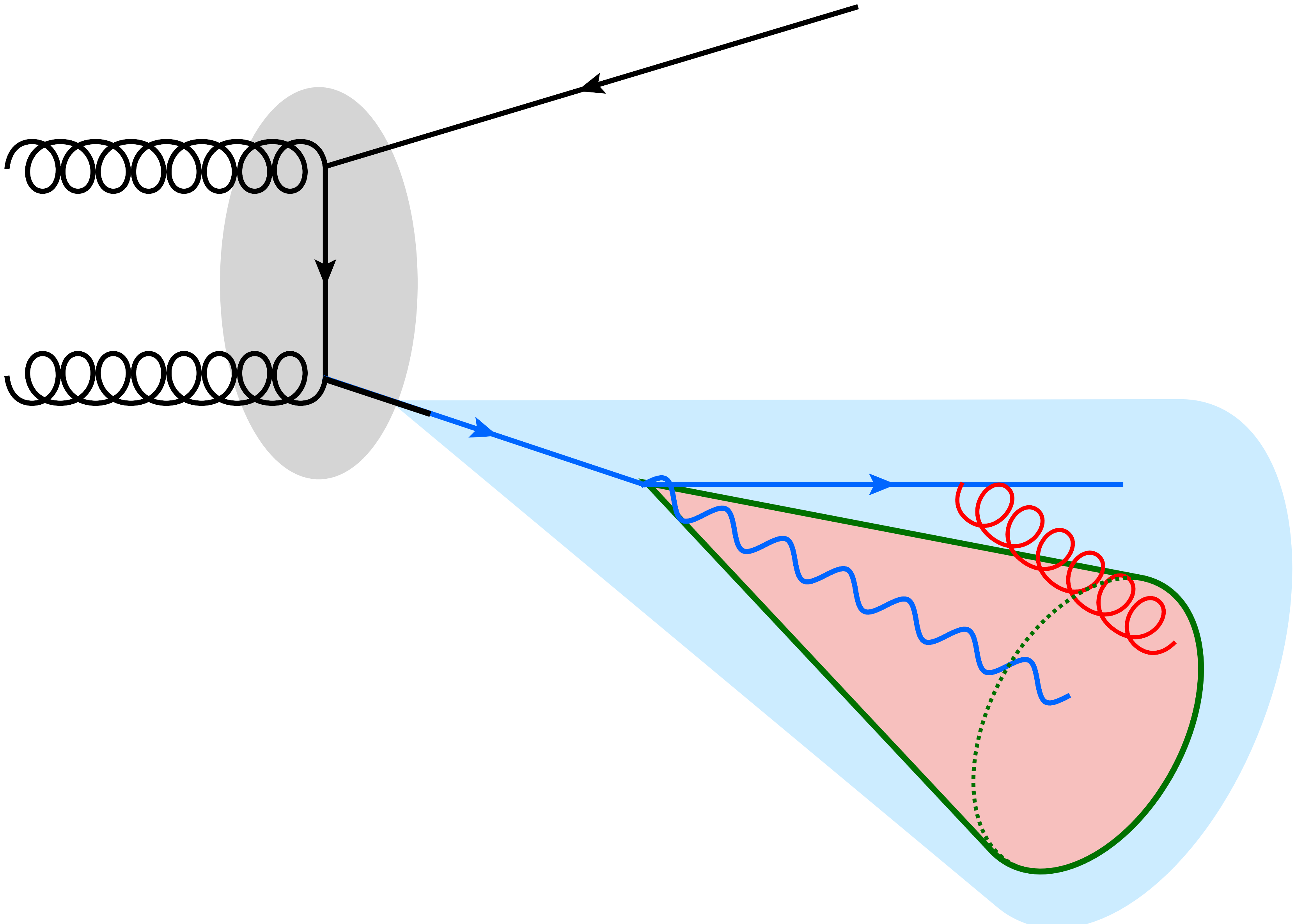
- $\mathcal{F}_{i\to\gamma}$ fullfills same DGLAP evolution equation as standard fragmentation function
- Solve DGLAP equation numerically to resum ln(R) enhanced higher-order contributions
  - Implemented evolution in moment space, interface to tree-level generator



ln(R) resummation



- Plot shows difference to reference NLO cross section
- Resummation cures pathological fixed-order behavior!



Factorization and resummation

for small isolation energy E_0

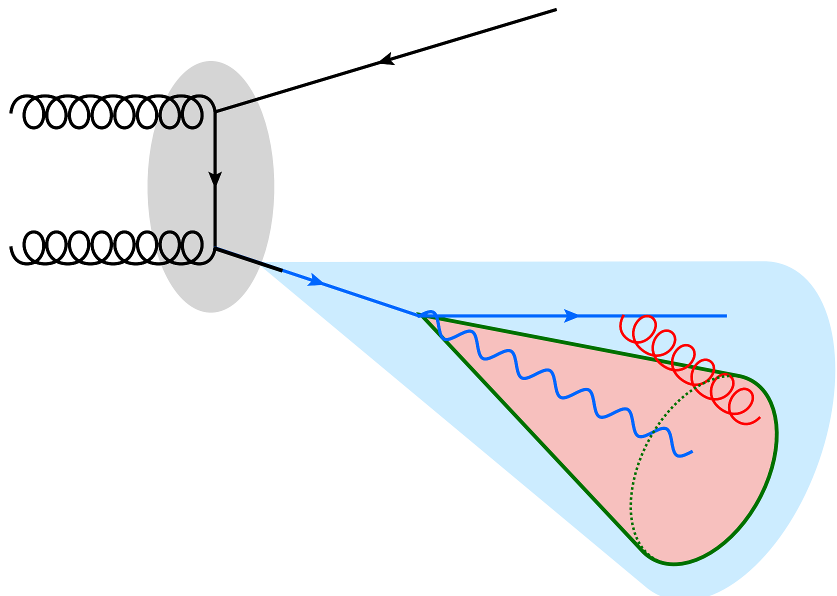
For fixed-cone isolation, the energy inside the cone is always much smaller than the photon energy, e.g.

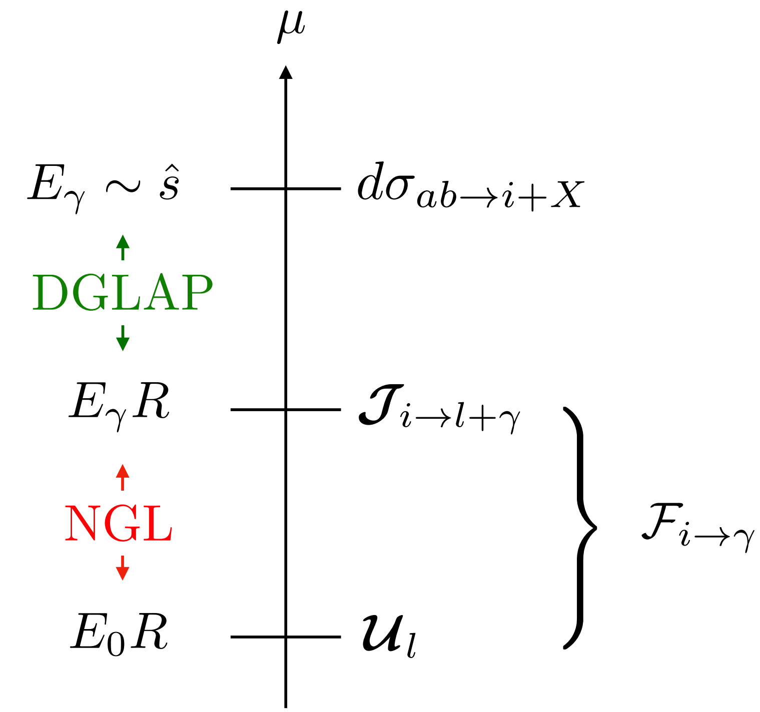
$$E^T_{\rm cone}(R) < E_0 = \epsilon_\gamma E^T_\gamma$$

For ATLAS $E_0 \gtrsim 5$ GeV

- Only soft radiation inside cone
- large non-global logarithms, associated with the energy ratio ε_γ
- perturbation theory at low scale RE_0
- fragmentation is suppressed

Factorization of $\mathcal{F}_{i\to\gamma}$





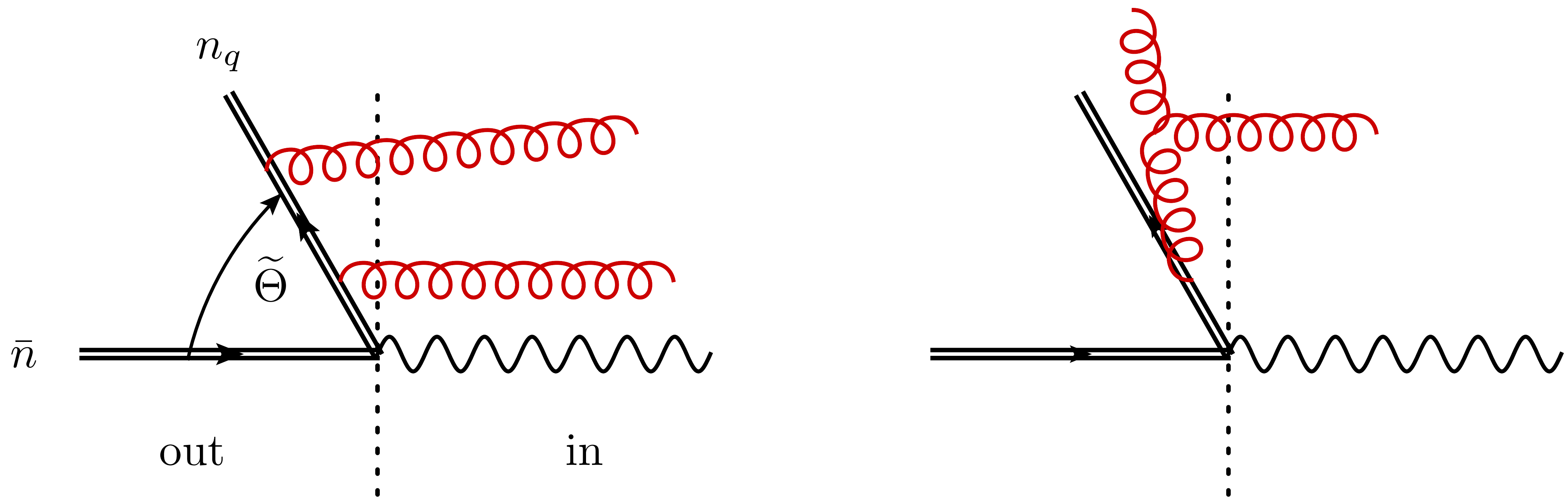
$$\mathcal{F}_{i\to\gamma}(z, R\,E_\gamma, R\,E_0, \mu) = \sum_{l=1}^{\infty} \langle \mathcal{J}_{i\to\gamma+l}(\{\underline{n}\}, R\,E_\gamma, z, \mu) \otimes \mathcal{U}_l\,(\{\underline{n}\},\, R\,E_0, \mu)\rangle$$

energetic collinear
outside cone

soft radiation
inside cone

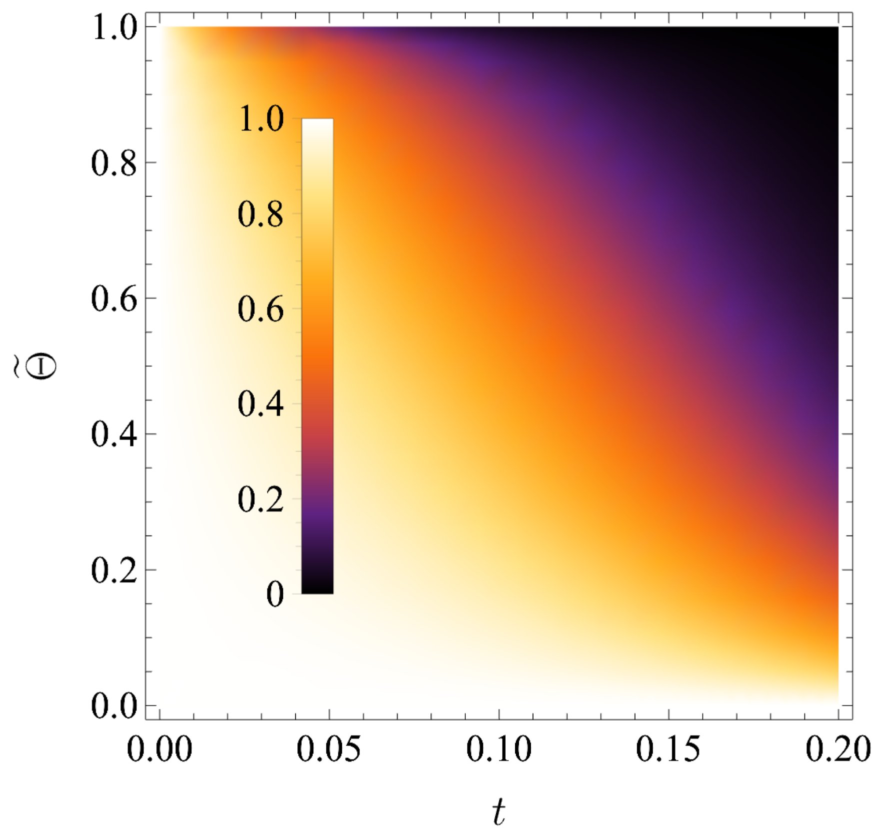
- Resum both ln(ε$_\gamma$) and ln(*R*).
- Lowest scale is *R E*$_0$ ≳ 1 GeV for ATLAS !

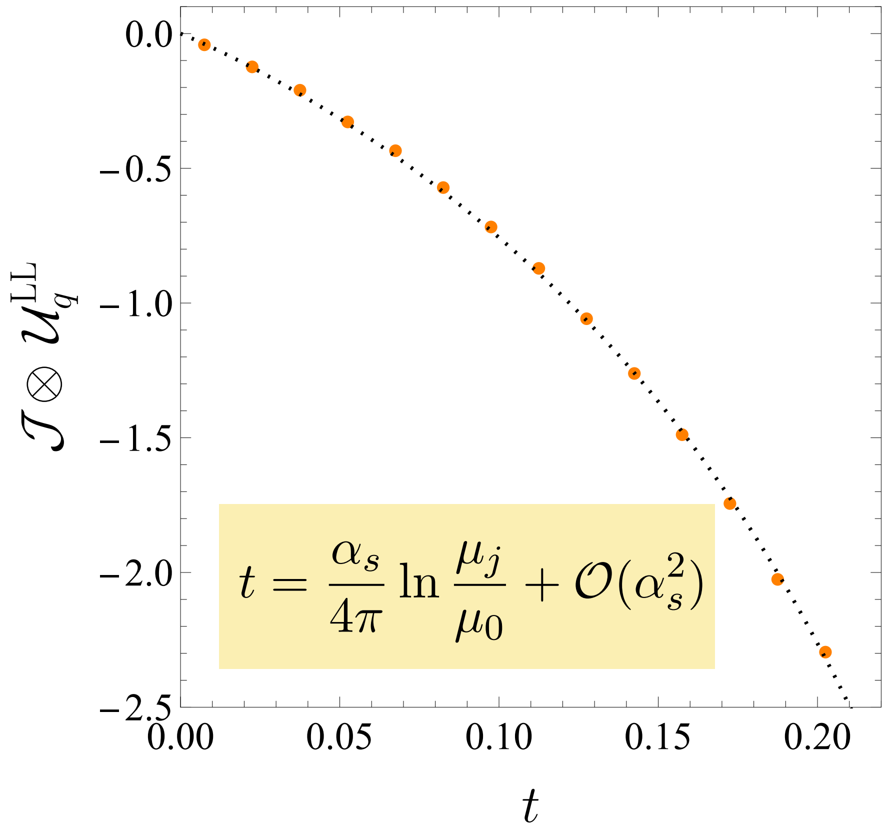
Non-global logarithms (NGLs)



- Isolation cone is prime example of non-global observable
- Complicated pattern of higher-order terms, not captured by standard resummation methods. Even leading NGLs $(\alpha_s L)^n$ do not simply exponentiate! Dasgupta, Salam '02
  - Use `ngl-resum` to resum leading NGLs Balsiger, Becher, Ferroglia '20 after boosting to frame where cone is hemisphere
  - see Nicolas Schalch's talk for progress on resummation of subleading NGLs.

quark-angle



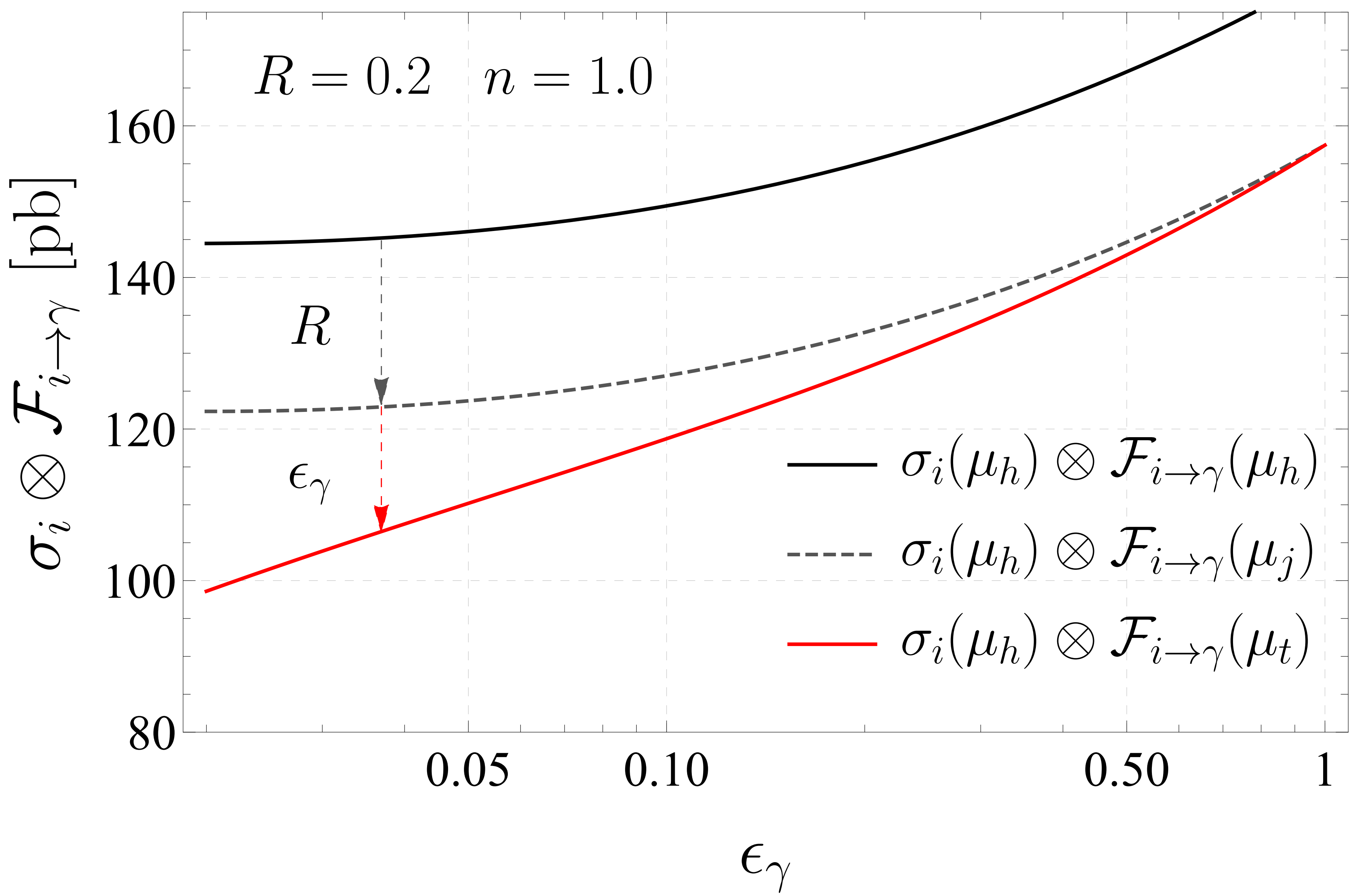


$$\int_0^1 d\widetilde{\Theta}\,\left\langle \boldsymbol{\mathcal{J}}_{q\to\gamma+q}\left(\widetilde{\Theta}, R\,E_\gamma, z, \mu_j\right)\,\mathcal{U}_q^{\mathrm{LL}}(\widetilde{\Theta},t)\right\rangle$$
$$= \frac{Q_i^2\alpha_{\mathrm{EM}}}{2\pi}\Big[-P(z)\ln\left(\frac{\delta^2Q^2}{\mu_j^2}(z-1)^2z^2\right) - z + 2P(z)\int_0^1 d\widetilde{\Theta}\left[\frac{1}{\widetilde{\Theta}}\right]_+\mathcal{U}_q^{\mathrm{LL}}(\widetilde{\Theta},t)\Big]$$

- NGLs much larger than global logs. Two-loop coefficient

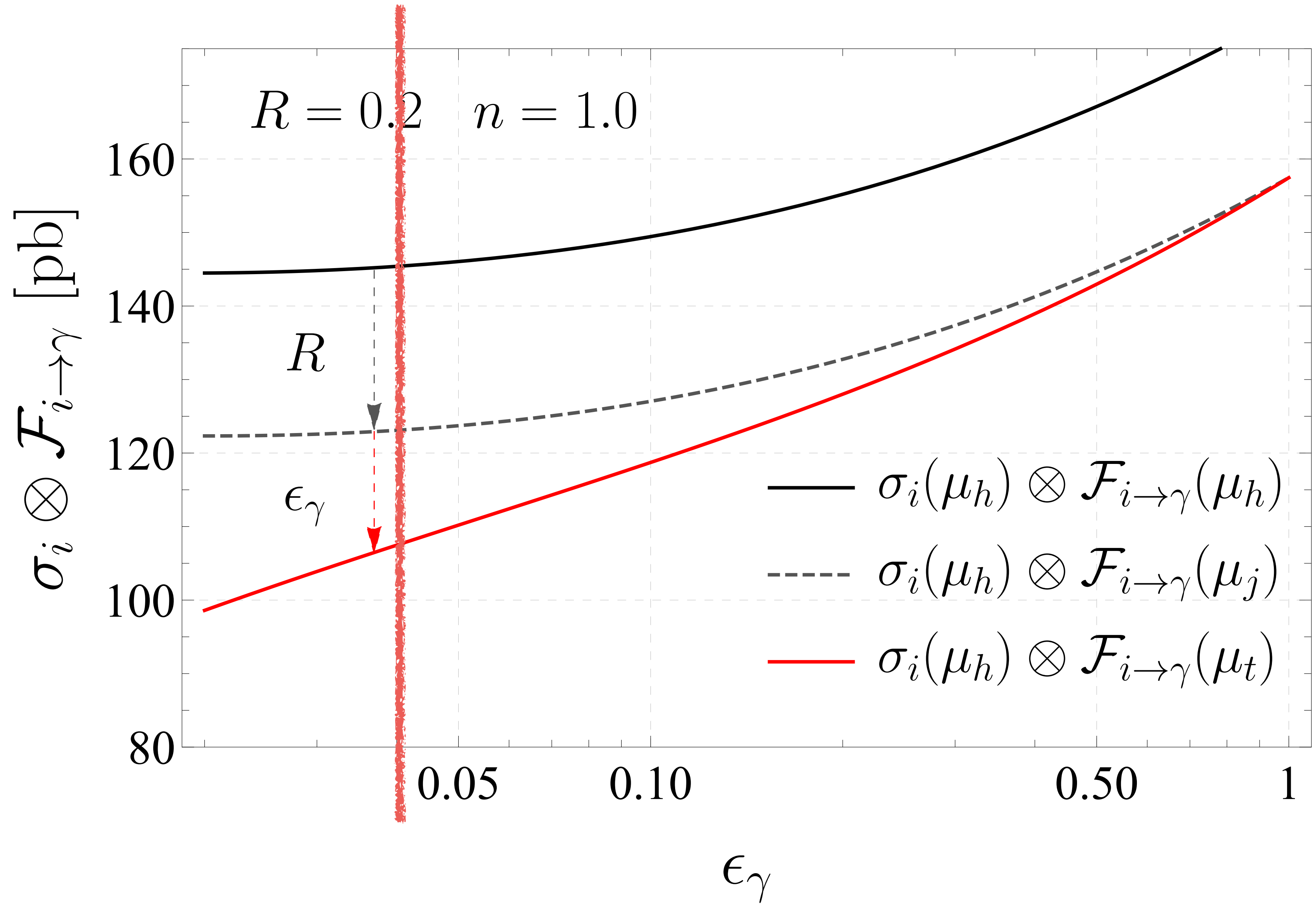
$$-31.5 = -43.7\,(\text{“non-global”}) + 12.2\,(\text{“global”})$$

Resummation of ln(R) and ln(ε_γ)



- For the full cross section, add direct part $\sigma^{\mathrm{dir}} \approx 290\,\mathrm{pb}$
- Note: both resummations lower the cross section!

Resummation of ln(R) and ln(ε_γ)



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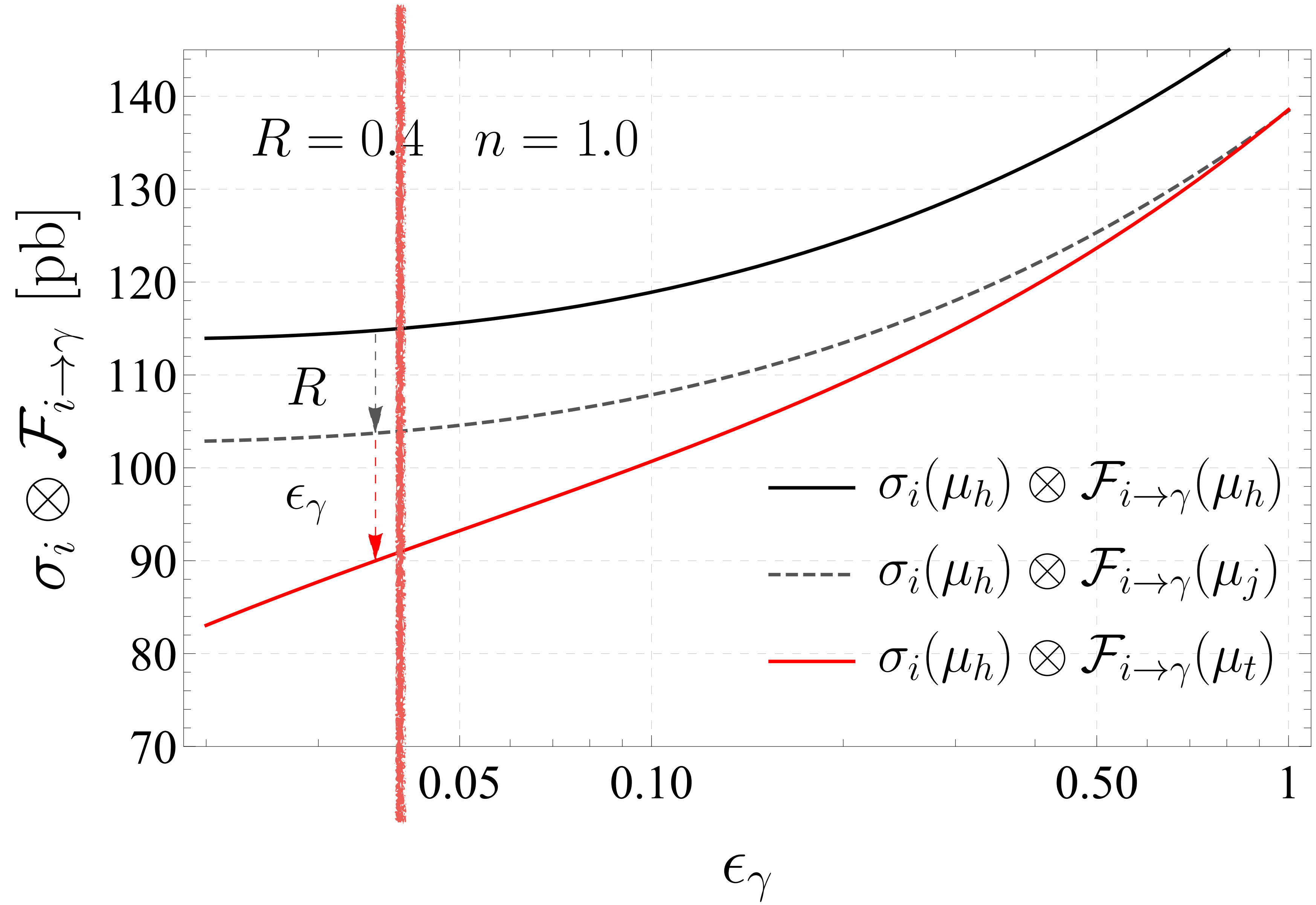

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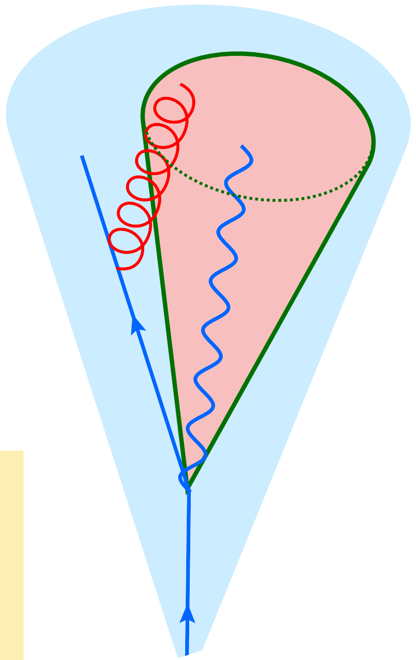
A simple relation

From factorization theorem, we can derive a relation between smooth- and fixed-cone isolation



$$\Delta\sigma = \sigma_{\text{fixedcone}}(R, \epsilon_\gamma) - \sigma_{\text{smoothcone}}(R, \epsilon_\gamma^{\text{ref}}, n)$$

in the limit $R \to 0$ and $\varepsilon_\gamma \to 0$:

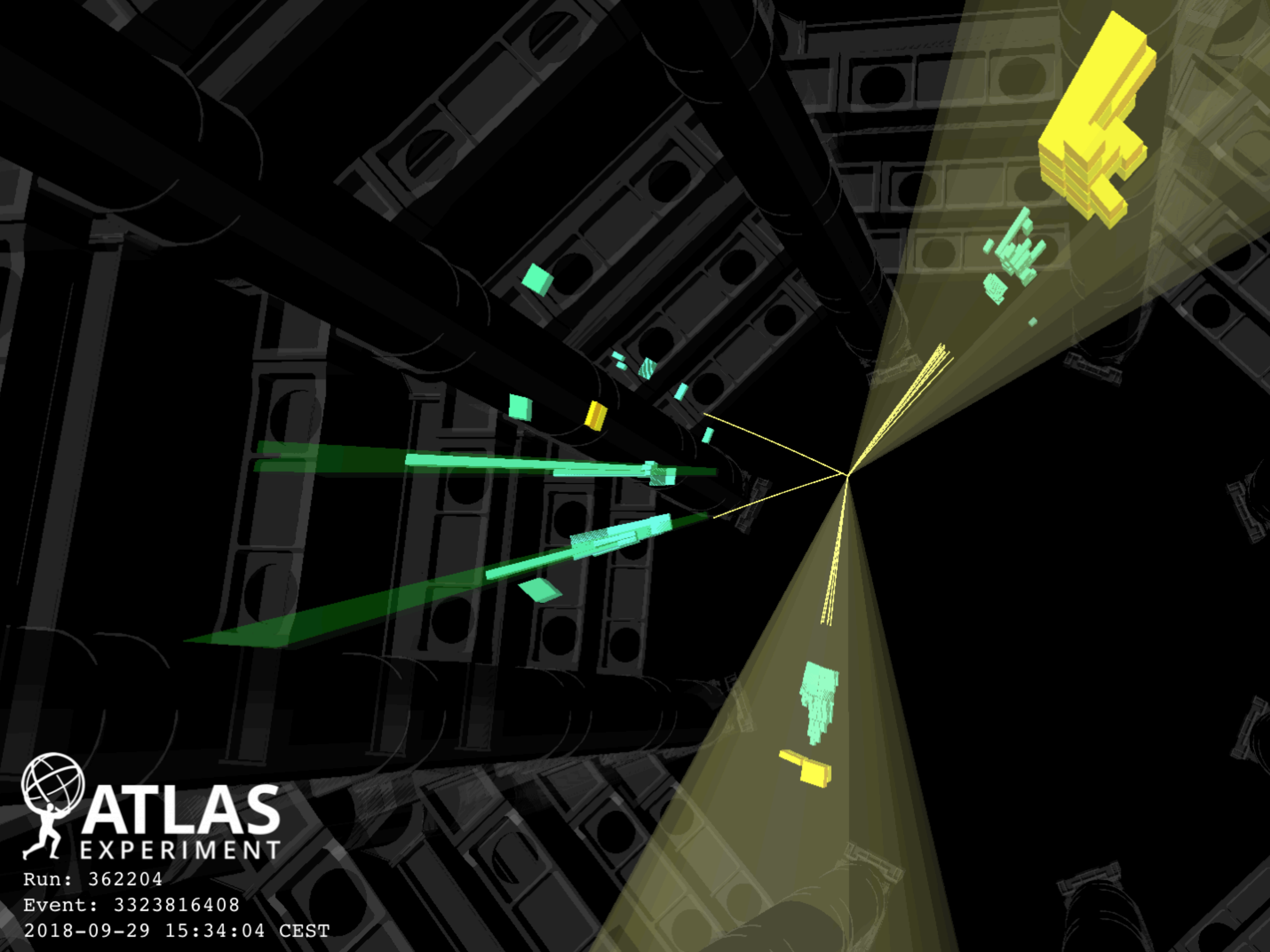
$$\Delta\sigma = \sum_{i=q,\bar{q}} \int_{E_T^{\min}}^{\infty} dE_i \int_{z_{\min}}^{1} dz \frac{d\sigma_{i+X}}{dE_i} \frac{Q_q^2 \alpha_{\text{EM}}}{\pi} \frac{C_F \alpha_s}{4\pi} P(z) \left[\frac{\pi^2}{3} \ln \frac{\epsilon_\gamma}{\epsilon_\gamma^{\text{ref}}} + 2n\,\zeta_3 \right]$$

Can be used to convert NNLO smooth-cone into fixed-cone results. For standard setup and $\epsilon_\gamma = \epsilon_\gamma^{\text{ref}}$

$$\Delta\sigma = -1.3\,\text{pb}$$

Conclusions & Outlook

- Have performed a detailed analysis of QCD effects associated with photon isolation
  - Factorization of isolation effects for small *R*
    - Lowest scale is $R\,E_0 \gtrsim 1$ GeV
  - First resummation of both $\ln(\varepsilon_\gamma)$ and $\ln(R)$ effects
    - numerically relevant for ATLAS isolation, crucial for $R = 0.2$
    - With some effort, could extend accuracy of resummations by one order and match to NNLO
- Formalism provides analytical understanding of isolation
  - Study parameter dependence, convert between isolation schemes, …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