SCET and Fermi Liquids

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EFT in non-trivial Vacua

	Super-Selection sector (label)	Residual Momenta	Label Changing interactions
HQET:	$P^\mu = m_Q v^\mu$	$k^\mu \sim \Lambda$	$L = J^\mu \bar{h}_v \Gamma_\mu h_{v'}$
NRQCD:	$P^\mu = (m, mv)$	$k^\mu \sim mv^2$	$L = \psi^{\dagger v}_{\vec{p}-\vec{q}/2} \psi^v_{\vec{p}} [\frac{1}{\vec{q}^2}] \chi^{\dagger v}_{-\vec{p}+\vec{q}/2} \chi^v_{-\vec{p}}$
SCET:	$P^\mu = (n \cdot p, p_\perp)$	$k^\mu \sim \lambda \sim \Lambda^2/Q$	$L = \bar{\xi}_{p_+} \xi_{p'_+} A^-_{p_+ - p'_+}$

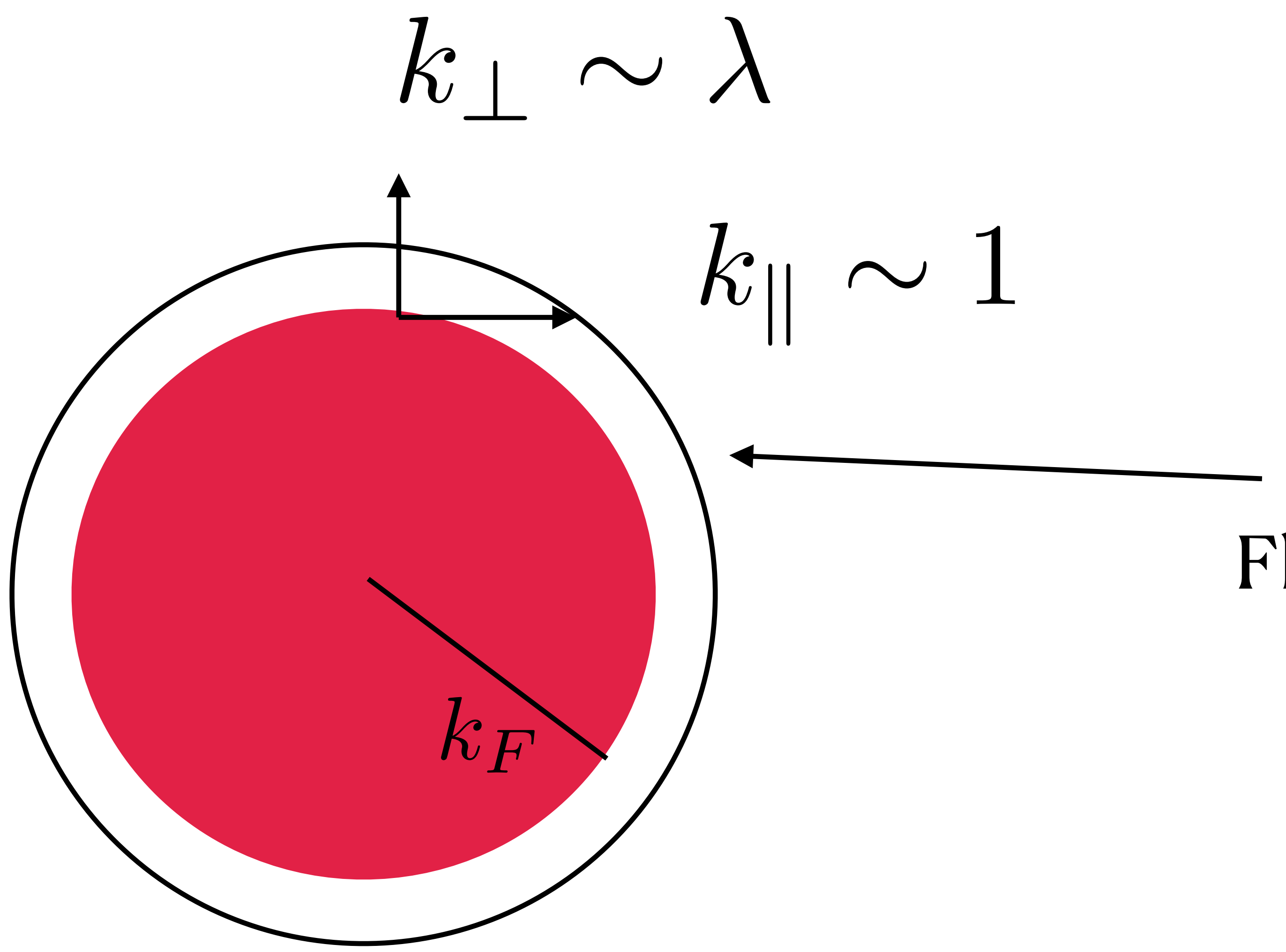
What makes these EFT's unusual is that we are expanding our delocalized stress energy

Can we leverage our expertise on these subjects to study more diverse systems?

Consider a system of free Fermions

$$L = \int d^d x dt (\psi^\dagger \partial_t \psi - \psi^\dagger \frac{\partial^2}{2m} \psi)$$

$$E_F = k_F^2/(2m)$$



$$\psi = \sum_\theta e^{-iE_F t} e^{i\vec{k}_\theta \cdot \vec{x}} \xi_\theta(x) \qquad |\vec{k}_\theta| = k_F$$

Power Counting Parameter: $\lambda = \partial \xi_\theta / E_F$

$$L_{EFT} = \int d^d x dt \sum_\theta \xi_\theta^\dagger (i\partial_t - v_F \partial_\perp) \xi_\theta$$

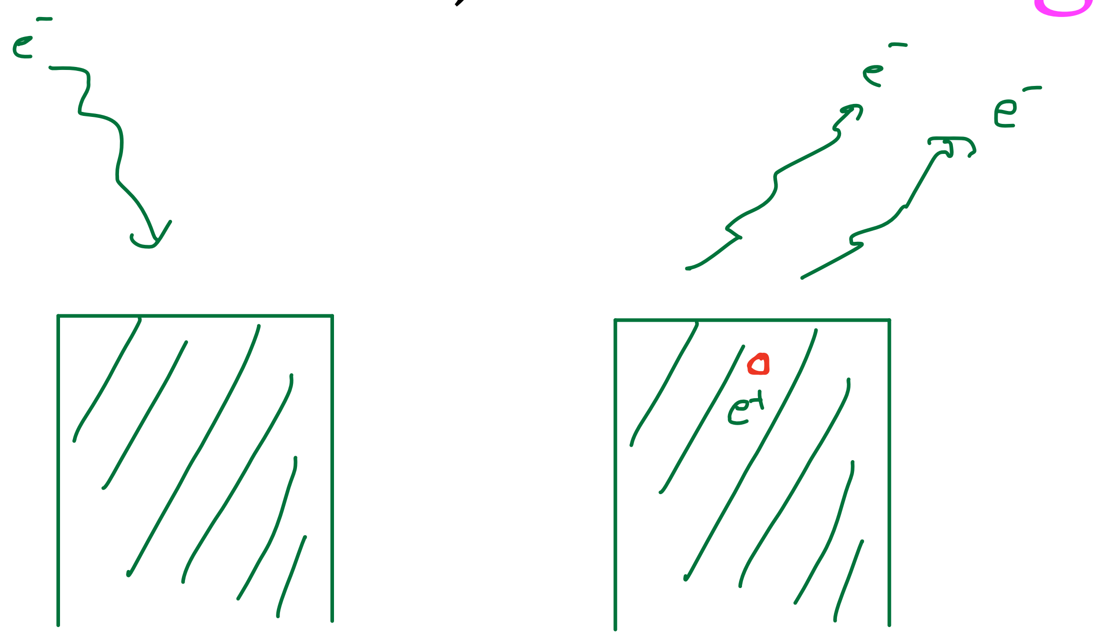
Turn on interactions

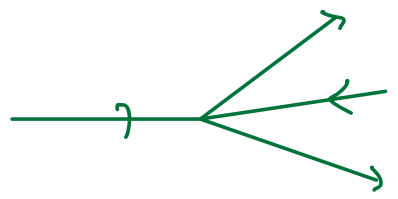
Lack of asymptotic states: relevant degrees of freedom not derivable from first principles

Assume that the relevant DOF have the same quantum numbers as electrons (no phase transitions)

Landau (adiabatic approximation)

These states (``quasi-particles'') are not eigenstate of the Hamiltonian





Criteria for quasi-particle

$$\lim_{E\to 0} \Gamma(E) \leq E^2$$

Check for self-consistency

Let us assume that all interactions between quasi-particles are local: HE3 and to some extent metals

Will see this is not-consistent with the power counting

$$S = \int d^d x dt \prod_i \sum_{\theta_i} g \psi^\dagger_{\theta_1} \psi_{\theta_2} \psi^\dagger_{\theta_3} \psi_{\theta_4}$$

However, MOST kinematic configurations are power suppressed!

Naively: $d^d x dt \sim 1/\lambda$ $\xi \sim \lambda^{1/2}$

However, we know from the case of Glauber we can have kinematic enhancements

Generic hard scattering

$$\int (\bar{\psi}_{n_1}\psi_{n_2}\bar{\psi}_{n_3}\psi_{n_4})[dx_+ dx_- d^2x_\perp \sim O(1)]$$

But for Glauber Kinematics $\int (\bar{\psi}_{n_1}\psi_{n_1}\bar{\psi}_{n_2}\psi_{n_2})[dx_+ dx_- d^2x_\perp \sim O(1/\lambda^2)]$

Why? Because momenta align: Perp momenta become linearly dependent

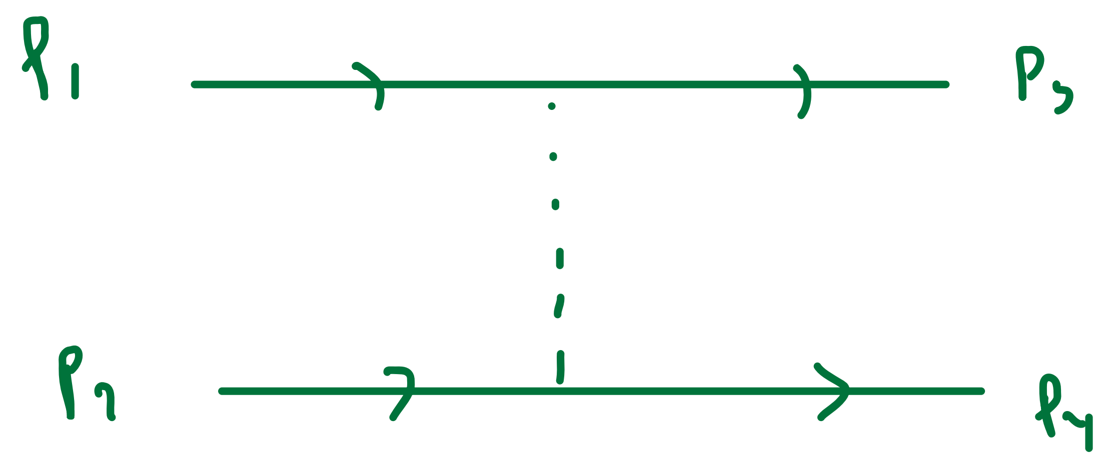
Consider the Fermi Liquid case:

a) Choose p1 and p3 to be equal with vanishing residual momentum, then writing

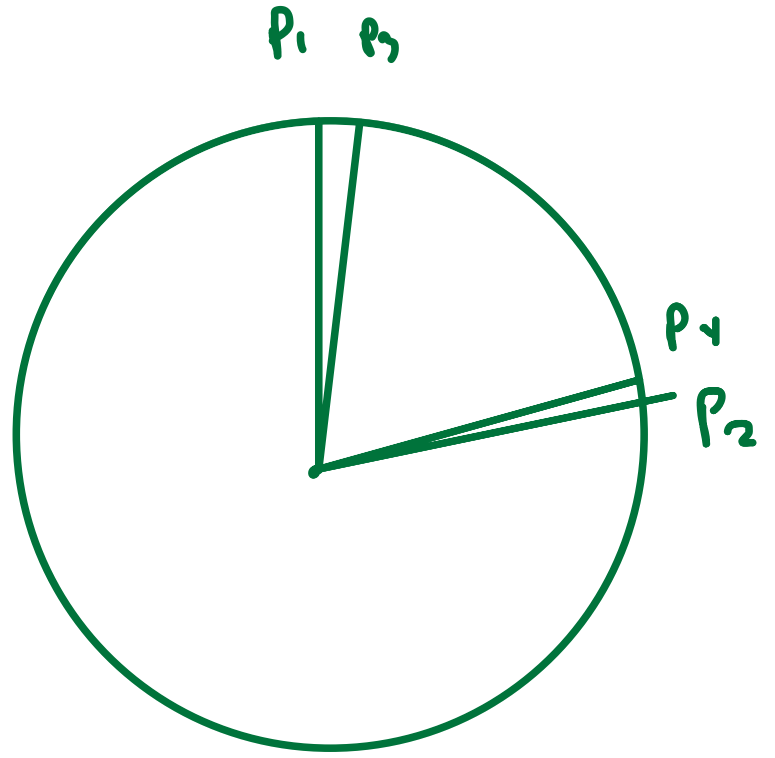
$$p_2 = P_2 + p_2^\perp + p_2^\| \quad p_4 = P_4 + p_4^\perp + p_4^\| \quad (P_2 = P_4)$$

(Joe Polchinski)

$$\delta^3(p_2^\perp + p_2^\| + p_4^\perp + p_4^\|) = \delta(p_2^\perp + p_4^\perp)\delta^2(p_2^\| + p_4^\|) \qquad \sim O(1/\lambda)$$

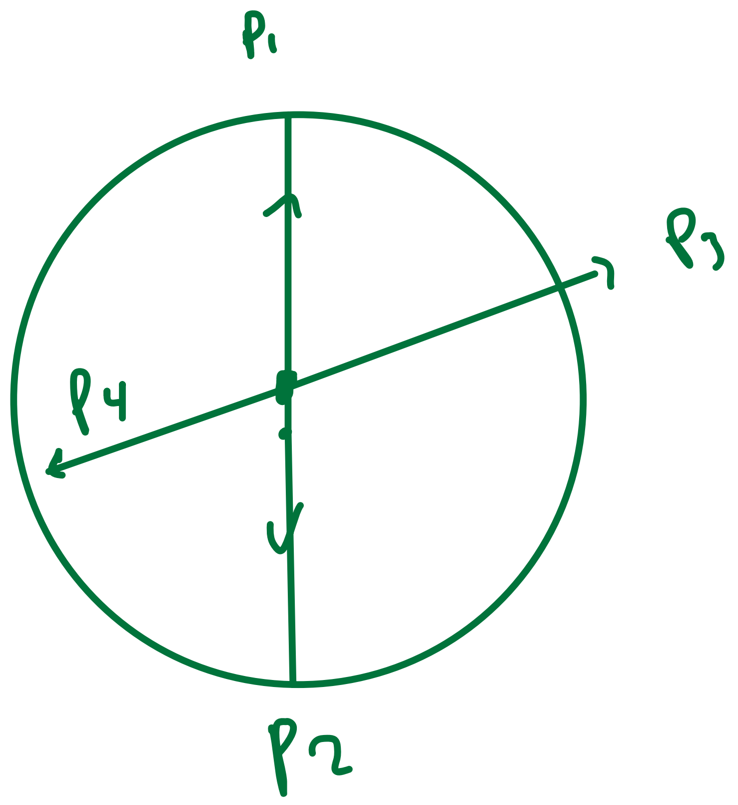


Froward scattering kinematics is leading order due to delta function enhancement



b) Choose p1 and p2 to be equal and OPPOSITE with vanishing residual momentum,

Back to Back BCS “interaction”



Another important distinction between SCET and FLT

$$S_{FS} = \int d^d x dt g(\theta_1 - \theta_2)\psi^\dagger_{\theta_1}\psi_{\theta_1}\psi^\dagger_{\theta_2}\psi_{\theta_2}$$

$$g(k_1^{\parallel}, k_2^{\parallel}) \to g(\theta_1 - \theta_2)$$

Decompose into partial waves

$$S_{BCS} = \int d^d x dt g_{BCS}(\theta_1 - \theta_2)\psi^\dagger_{\theta_1}\psi_{\theta_2}\psi^\dagger_{-\theta_1}\psi_{-\theta_2}$$

A crucial diference with Glauber SCET is that In FLT the box diagram is suppressed (no eikonalization)

In FLT θ (An order one label) is fixed in addition to the magnitude of the momentum.

This would be like fixing $k_\perp \sim O(\lambda^2)$ in the box diagram

Formally we would normally combine label sum with residual momentum integration

$$\sum_\theta \int dk_{||} \to \int dk_\theta$$

But kinematics restrict the label sum leaving to a power suppression. So no corrections to forward scattering at leading order (no running).

The same can not be said for the BCS interactions where the angular label sum in unrestricted in the loop

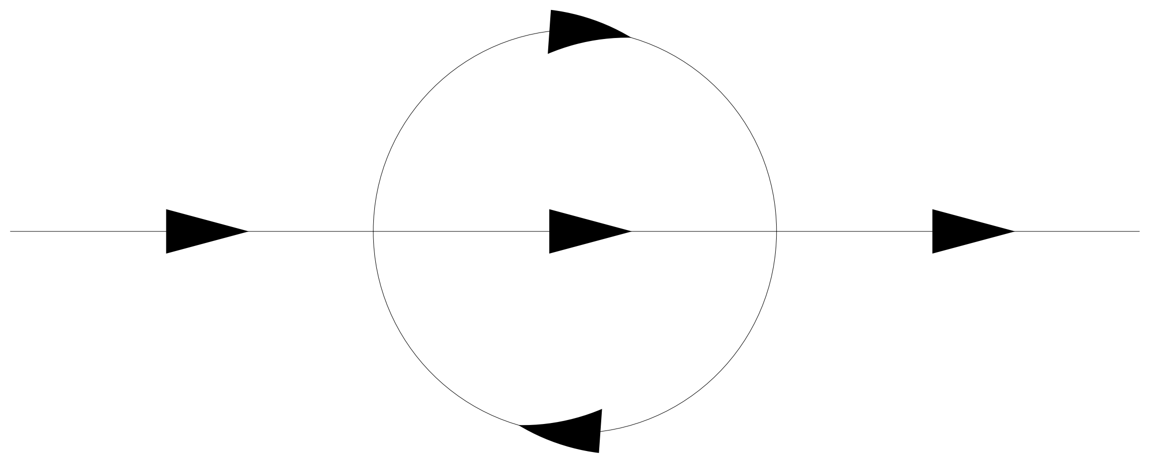
Generates one loop beta function (saturated at one loop)

$$E\frac{dg}{dE} = g^2 N \qquad g(E) = \frac{g(E_0)}{1 + Ng(E_0)\log(E_0/E)}$$

For an attractive interaction coupling grows in IR leading to condensation in particle-particle channel

$U(1) \to \emptyset$ Super-conductivity

Check consistency of the quasi-particle picture

Im  $\Gamma(E) \sim E^2/E_F$

Power suppression of FS loops validates
Landau Theory, limited phase space due to
Pauli Blocking leads to log lived ``particles”

Non-Fermi Liquids

High Tc superconductors in their normal state (strange metal) are at best ``marginal Fermi liquids"

$$\Gamma[E] \sim E$$

No notion of a quasi-particle yet its not a CFT

Finding the proper EFT for such systems is perhaps the most important open problem in theoretical CMT

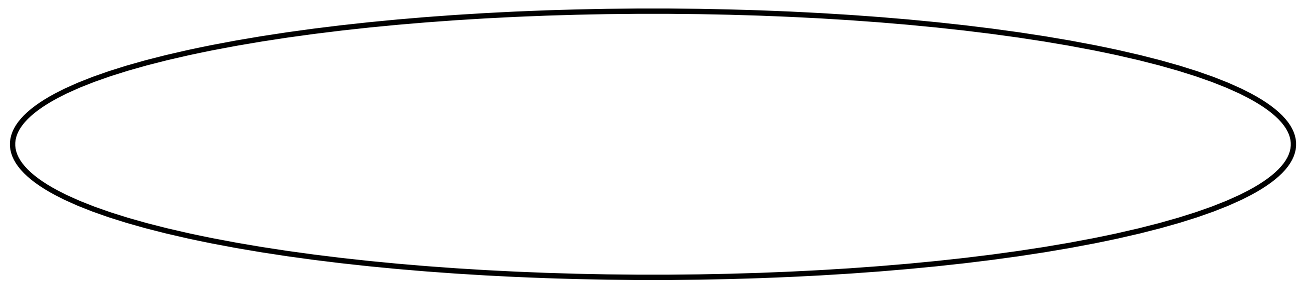
Metals Differ from HE3 in various important ways, but we will focus on the fact that Fermi surface has a much richer structure

There are cases where structure of of Fermi surface leads to further analogies with SCET

- Double Log Running (Sudakov)
- Rapidity Renormalization Group

Shape of the Fermi surface is characterized by :

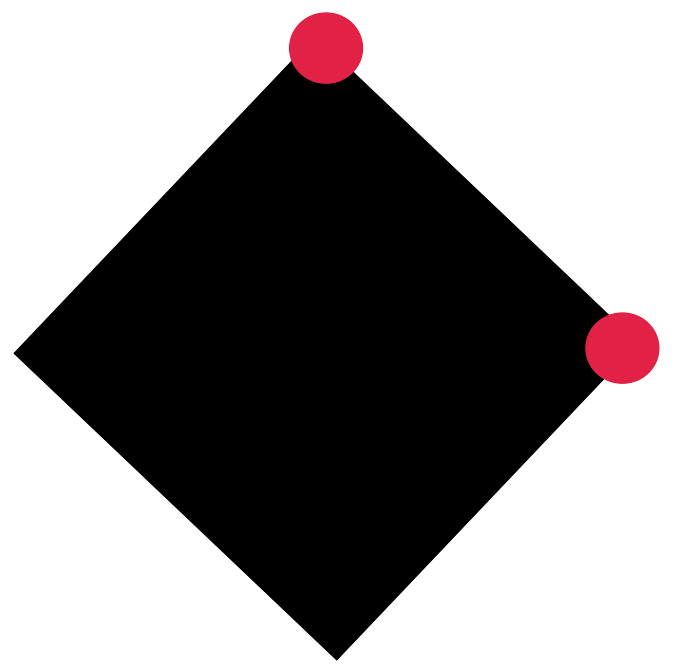
$E(p_F)$



Power Counting can change if there exists a hierarchy in first derivatives (Fermi velocities)

Van Hove Singularity and SCET

Fermi surface of 2-D half-filled Hubbard Model



$$E(k) = \cos(ak_x) + \sin(ak_y)$$

$v_F = \partial E/\partial p = 0$ $\quad \frac{dN}{dE} \sim 1/v_F$ $\quad$ ``Hot-Spot"

Expanding around the hot spot: $\quad E(k) = k_x^2 - k_y^2 \quad = k_+ k_-$

Non-compact Fermi surface leads to new unregulated divergences.

Power Counting: $\quad E \sim \lambda^2$

Three relevant modes

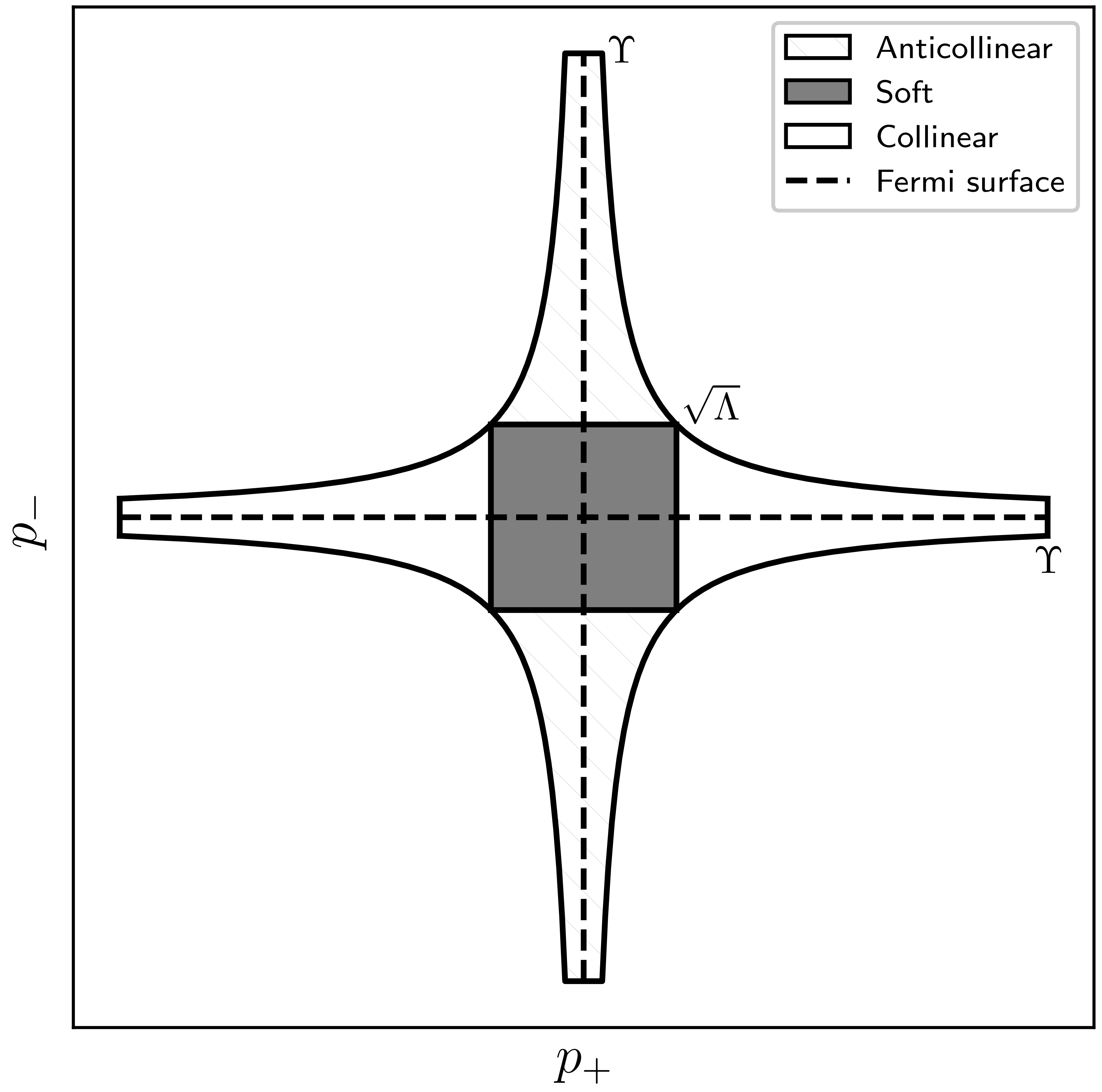
Soft: (λ, λ)

Collinear: $(1, \lambda^2)$

Anti-Collinear: $(\lambda^2, 1)$

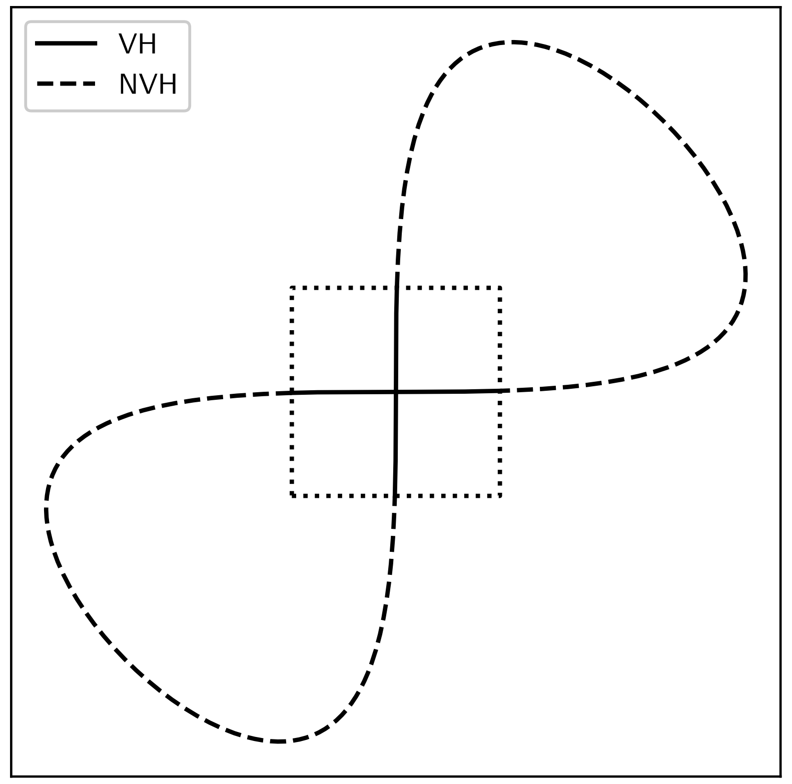
Von Hove Region:

Dim. Reg. does not seem too be an option, but we dont have to concern ourselves with gauge invariance, so cut-offs are sufficient



Υ

Rapidity cut-off compactifies the VH region. Also define a factorization scales between soft and collinear modes (RRG)



NOTE: We must coplemement this region with a Non-Von-Hove region which will also have lead to Υ dependence such that any physical result will be independent of Υ

The largest possible value of Υ is the size of the Brouilln zone which is the large scale in the problem

Power counting parameter $\lambda \sim E/\Upsilon^2$

$$p_c \sim (\Upsilon, E/\Upsilon)$$
$$p_{\bar{c}} \sim (E/\Upsilon, \Upsilon)$$
$$p_s \sim (\sqrt{E}, \sqrt{E})$$

VH modes

NVH modes $p \sim (p_{||}, p_\perp) \sim (\Upsilon, \sqrt{E})$

What type of interactions are allowed at leading power?

$$S_{int} = \int dt \prod_{i=1}^{4} d^2p_i \sum_{\alpha\beta\gamma\delta} g_{\alpha\beta\gamma\delta} \psi^\dagger_\alpha(p_1)\psi^\dagger_\beta(p_2)\psi_\gamma(p_3)\psi_\delta(p_4)$$

$$\psi_\alpha = (\psi_N, \psi_{VH})$$

As before, the NVH kinematics are restricted to be either FS or back-to back

Forward Scattering $\psi_p^{\dagger,NVH}\psi_p^{NVH}\psi_S^{\dagger,VH}\psi_S^{VH}$

BCS $\psi_{-p}^{\dagger NVH}\psi_p^{NVH}\psi_S^{\dagger VH}\psi_S^{VH}$

Pure VH $\psi_{C/S}^{\dagger VH}\psi_{C/S}^{VH}\psi_{C/S}^{\dagger VH}\psi_{C/S}^{VH}$

In general the coupling functions can depend upon the label momenta of the collinear (PDF's) and the NVH transverse momenta

To simplify matters let us focus two possible novel phenomena

- Enhanced Running (increase T_c) Double Log Running (Sudakov)
- Enhanced Running (increase T_c) Marginal Fermi Liquid Behavior

BCS interactions with soft and collinear NVH intermediate states

$$\mathcal{A}_{\rm BCS}(E) = \frac{g_{\rm B}^2}{8\pi^2}\left(\log^2\frac{V_F^2}{\Lambda} - \log^2\frac{V_F^2}{E} - i\pi\log\frac{V_F^2}{E}\right)$$

Not captured by RG (RRG?)

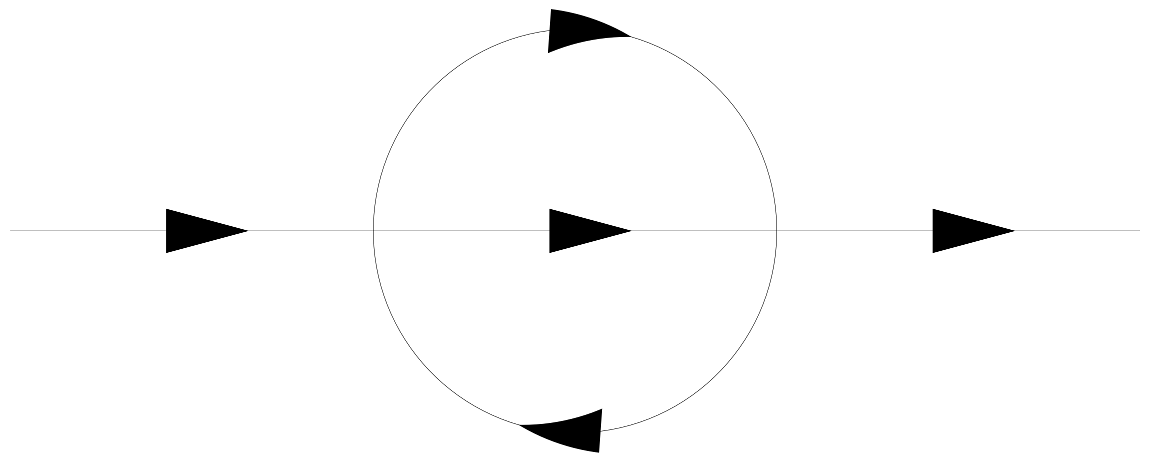
Analogy between SCET and VH-FLT

$$\log(\Lambda/\Upsilon^2) \leftrightarrow \log(\mu/\nu)$$

$$\log(V_F^2/\Upsilon) \leftrightarrow \log(p_+/\nu)$$

Consider Collinear loops

No power suppression as compared to NVH (FS,BCS) regions

Im  $\sim E$

Marginal Fermi Liquid Behavior

While VH singularities reproduce qualitative features of high Tc superconductors, thesis not expected to the the solution to the problem as high Tc superconductors are quite ubiquitous and VH singularities are finely tuned systems, so not generic

Presently it seems that the CMT community has been investigating including ``critical bosons" into the theory to explain NFL behavior, (first principles?).